1. Charge flows until the potential difference across the capacitor is the same as the potential difference across the battery. The charge on the capacitor is then $q = CV$, and this is the same as the total charge that has passed through the battery. Thus,

$$q = (25 \times 10^{-6}\ \text{F})(120\ \text{V}) = 3.0 \times 10^{-3}\ \text{C}.$$

2. (a) The capacitance of the system is

$$C = \frac{q}{\Delta V} = \frac{70\,\text{pC}}{20\,\text{V}} = 3.5\,\text{pF}.$$

(b) The capacitance is independent of q; it is still 3.5 pF.

(c) The potential difference becomes

$$\Delta V = \frac{q}{C} = \frac{200\,\text{pC}}{3.5\,\text{pF}} = 57\,\text{V}.$$

3. (a) The capacitance of a parallel-plate capacitor is given by $C = \varepsilon_0 A/d$, where A is the area of each plate and d is the plate separation. Since the plates are circular, the plate area is $A = \pi R^2$, where R is the radius of a plate. Thus,

$$C = \frac{\varepsilon_0 \pi R^2}{d} = \frac{\left(8.85\times10^{-12}\ \text{F/m}\right)\pi\left(8.2\times10^{-2}\ \text{m}\right)^2}{1.3\times10^{-3}\ \text{m}} = 1.44\times10^{-10}\ \text{F} = 144\ \text{pF}.$$

(b) The charge on the positive plate is given by $q = CV$, where V is the potential difference across the plates. Thus,

$$q = (1.44 \times 10^{-10}\ \text{F})(120\ \text{V}) = 1.73 \times 10^{-8}\ \text{C} = 17.3\ \text{nC}.$$

4. We use $C = A\varepsilon_0/d$.

(a) Thus,

$$d = \frac{A\varepsilon_0}{C} = \frac{\left(1.00\,\text{m}^2\right)\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)}{1.00\,\text{F}} = 8.85\times10^{-12}\ \text{m}.$$

(b) Since d is much less than the size of an atom (~ 10^{-10} m), this capacitor cannot be constructed.

5. Assuming conservation of volume, we find the radius of the combined spheres, then use $C = 4\pi\varepsilon_0 R$ to find the capacitance. When the drops combine, the volume is doubled. It is then $V = 2(4\pi/3)R^3$. The new radius R' is given by

$$\frac{4\pi}{3}\left(R'\right)^3 = 2\frac{4\pi}{3}R^3 \quad \Rightarrow \quad R' = 2^{1/3}R.$$

The new capacitance is

$$C' = 4\pi\varepsilon_0 R' = 4\pi\varepsilon_0 2^{1/3} R = 5.04\pi\varepsilon_0 R.$$

With R = 2.00 mm, we obtain $C = 5.04\pi\left(8.85\times10^{-12}\ \text{F/m}\right)\left(2.00\times10^{-3}\ \text{m}\right) = 2.80\times10^{-13}\ \text{F}$.

6. (a) We use Eq. 25-17:

$$C = 4\pi\varepsilon_0 \frac{ab}{b-a} = \frac{(40.0\,\text{mm})(38.0\,\text{mm})}{\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)(40.0\,\text{mm}-38.0\,\text{mm})} = 84.5\,\text{pF}.$$

(b) Let the area required be A. Then $C = \varepsilon_0 A/(b - a)$, or

$$A = \frac{C(b-a)}{\varepsilon_0} = \frac{(84.5\,\text{pF})(40.0\,\text{mm}-38.0\,\text{mm})}{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)} = 191\,\text{cm}^2.$$

7. The equivalent capacitance is given by $C_{eq} = q/V$, where q is the total charge on all the capacitors and V is the potential difference across any one of them. For N identical capacitors in parallel, $C_{eq} = NC$, where C is the capacitance of one of them. Thus, $NC = q/V$ and

$$N = \frac{q}{VC} = \frac{1.00\,\text{C}}{(110\,\text{V})\left(1.00\times10^{-6}\,\text{F}\right)} = 9.09\times10^{3}.$$

8. The equivalent capacitance is

$$C_{\text{eq}} = C_3 + \frac{C_1 C_2}{C_1 + C_2} = 4.00\mu\text{F} + \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 7.33\,\mu\text{F}.$$

9. The equivalent capacitance is

$$C_{\text{eq}} = \frac{(C_1 + C_2)C_3}{C_1 + C_2 + C_3} = \frac{(10.0\,\mu\text{F} + 5.00\,\mu\text{F})(4.00\,\mu\text{F})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 3.16\,\mu\text{F}.$$

10. The charge that passes through meter *A* is

$$q = C_{eq}V = 3CV = 3(25.0\,\mu\text{F})(4200\,\text{V}) = 0.315\,\text{C}.$$

11. (a) and (b) The original potential difference V_1 across C_1 is

$$V_1 = \frac{C_{eq}V}{C_1 + C_2} = \frac{(3.16\,\mu\text{F})(100.0\,\text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 21.1\,\text{V}.$$

Thus ΔV_1 = 100.0 V – 21.1 V = 78.9 V and

$$\Delta q_1 = C_1 \Delta V_1 = (10.0\ \mu\text{F})(78.9\ \text{V}) = 7.89 \times 10^{-4}\ \text{C}.$$

12. (a) The potential difference across C_1 is $V_1 = 10.0$ V. Thus,

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 1.00 \times 10^{-4}\ \text{C}.$$

(b) Let $C = 10.0\ \mu$F. We first consider the three-capacitor combination consisting of C_2 and its two closest neighbors, each of capacitance C. The equivalent capacitance of this combination is

$$C_{\text{eq}} = C + \frac{C_2 C}{C + C_2} = 1.50\ C.$$

Also, the voltage drop across this combination is

$$V = \frac{CV_1}{C + C_{eq}} = \frac{CV_1}{C + 1.50\ C} = 0.40V_1.$$

Since this voltage difference is divided equally between C_2 and the one connected in series with it, the voltage difference across C_2 satisfies $V_2 = V/2 = V_1/5$. Thus

$$q_2 = C_2V_2 = (10.0\mu\text{F})\left(\frac{10.0\text{V}}{5}\right) = 2.00\times10^{-5}\ \text{C}.$$

13. The charge initially on the charged capacitor is given by $q = C_1V_0$, where $C_1 = 100$ pF is the capacitance and $V_0 = 50$ V is the initial potential difference. After the battery is disconnected and the second capacitor wired in parallel to the first, the charge on the first capacitor is $q_1 = C_1V$, where $V = 35$ V is the new potential difference. Since charge is conserved in the process, the charge on the second capacitor is $q_2 = q - q_1$, where C_2 is the capacitance of the second capacitor. Substituting C_1V_0 for q and C_1V for q_1, we obtain $q_2 = C_1 (V_0 - V)$. The potential difference across the second capacitor is also V, so the capacitance is

$$C_2 = \frac{q_2}{V} = \frac{V_0 - V}{V} C_1 = \frac{50\,\text{V} - 35\,\text{V}}{35\,\text{V}} (100\,\text{pF}) = 43\,\text{pF}.$$

14. The two 6.0 μF capacitors are in parallel and are consequently equivalent to $C_{\text{eq}} = 12\ \mu\text{F}$. Thus, the total charge stored (before the squeezing) is

$$q_{\text{total}} = C_{\text{eq}} V_{\text{battery}} = 120\ \mu\text{C}\ .$$

(a) and (b) As a result of the squeezing, one of the capacitors is now 12 μF (due to the inverse proportionality between C and d in Eq. 25-9) which represents an increase of $6.0\ \mu\text{F}$ and thus a charge increase of

$$\Delta q_{\text{total}} = \Delta C_{\text{eq}} V_{\text{battery}} = (6.0\ \mu\text{F})(10\ \text{V}) = 60\ \mu\text{C}\ .$$

15. (a) First, the equivalent capacitance of the two 4.00 μF capacitors connected in series is given by 4.00 μF/2 = 2.00 μF. This combination is then connected in parallel with two other 2.00-μF capacitors (one on each side), resulting in an equivalent capacitance C = 3(2.00 μF) = 6.00 μF. This is now seen to be in series with another combination, which consists of the two 3.0-μF capacitors connected in parallel (which are themselves equivalent to C' = 2(3.00 μF) = 6.00 μF). Thus, the equivalent capacitance of the circuit is

$$C_{eq} = \frac{CC'}{C+C'} = \frac{(6.00\mu\text{F})(6.00\mu\text{F})}{6.00\mu\text{F}+6.00\mu\text{F}} = 3.00\mu\text{F}.$$

(b) Let V = 20.0 V be the potential difference supplied by the battery. Then

$$q = C_{eq}V = (3.00\ \mu\text{F})(20.0\ \text{V}) = 6.00 \times 10^{-5}\ \text{C}.$$

(c) The potential difference across C_1 is given by

$$V_1 = \frac{CV}{C+C'} = \frac{(6.00\mu\text{F})(20.0\text{V})}{6.00\mu\text{F}+6.00\mu\text{F}} = 10.0\text{V}.$$

(d) The charge carried by C_1 is $q_1 = C_1V_1$= (3.00 μF)(10.0 V) = 3.00 × 10^{-5} C.

(e) The potential difference across C_2 is given by $V_2 = V - V_1$ = 20.0 V – 10.0 V = 10.0 V.

(f) The charge carried by C_2 is $q_2 = C_2V_2$ = (2.00 μF)(10.0 V) = 2.00 × 10^{-5} C.

(g) Since this voltage difference V_2 is divided equally between C_3 and the other 4.00-μF capacitors connected in series with it, the voltage difference across C_3 is given by $V_3 = V_2/2$ = 10.0 V/2 = 5.00 V.

(h) Thus, $q_3 = C_3V_3$ = (4.00 μF)(5.00 V) = 2.00 × 10^{-5} C.

16. We determine each capacitance from the slope of the appropriate line in the graph. Thus, C_1 = (12 μC)/(2.0 V) = 6.0 μF. Similarly, C_2 = 4.0 μF and C_3 = 2.0 μF. The total equivalent capacitance is

$$C_{123} = ((C_1)^{-1} + (C_3 + C_2)^{-1})^{-1} = 3.0\ \mu\text{F}.$$

This implies that the charge on capacitor 1 is (3.0 μF)(6.0 V) = 18 μC. The voltage across capacitor 1 is therefore (18 μC)/(6.0 μF) = 3.0 V. From the discussion in section 25-4, we conclude that the voltage across capacitor 2 must be 6.0 V – 3.0 V = 3.0 V. Consequently, the charge on capacitor 2 is (4.0 μF)(3.0 V) = 12 μC.

17. (a) After the switches are closed, the potential differences across the capacitors are the same and the two capacitors are in parallel. The potential difference from a to b is given by $V_{ab} = Q/C_{eq}$, where Q is the net charge on the combination and C_{eq} is the equivalent capacitance. The equivalent capacitance is $C_{eq} = C_1 + C_2 = 4.0 \times 10^{-6}$ F. The total charge on the combination is the net charge on either pair of connected plates. The charge on capacitor 1 is

$$q_1 = C_1 V = \left(1.0\times10^{-6}\text{ F}\right)\left(100\text{ V}\right) = 1.0\times10^{-4}\text{ C}$$

and the charge on capacitor 2 is

$$q_2 = C_2 V = \left(3.0\times10^{-6}\text{ F}\right)\left(100\text{ V}\right) = 3.0\times10^{-4}\text{ C},$$

so the net charge on the combination is 3.0×10^{-4} C – 1.0×10^{-4} C = 2.0×10^{-4} C. The potential difference is

$$V_{ab} = \frac{2.0\times10^{-4}\text{ C}}{4.0\times10^{-6}\text{ F}} = 50\text{ V}.$$

(b) The charge on capacitor 1 is now $q_1 = C_1V_{ab} = (1.0 \times 10^{-6}\text{ F})(50\text{ V}) = 5.0 \times 10^{-5}$ C.

(c) The charge on capacitor 2 is now $q_2 = C_2V_{ab} = (3.0 \times 10^{-6}\text{ F})(50\text{ V}) = 1.5 \times 10^{-4}$ C.

18. Eq. 23-14 applies to each of these capacitors. Bearing in mind that $\sigma = q/A$, we find the total charge to be

$$q_{\text{total}} = q_1 + q_2 = \sigma_1 A_1 + \sigma_2 A_2 = \varepsilon_o E_1 A_1 + \varepsilon_o E_2 A_2 = 3.6 \text{ pC}$$

where we have been careful to convert cm^2 to m^2 by dividing by 10^4.

19. (a) and (b) We note that the charge on C_3 is q_3 = 12 μC – 8.0 μC = 4.0 μC. Since the charge on C_4 is q_4 = 8.0 μC, then the voltage across it is q_4/C_4 = 2.0 V. Consequently, the voltage V_3 across C_3 is 2.0 V $\Rightarrow C_3 = q_3/V_3$ = 2.0 μF.

Now C_3 and C_4 are in parallel and are thus equivalent to 6 μF capacitor which would then be in series with C_2; thus, Eq 25-20 leads to an equivalence of 2.0 μF which is to be thought of as being in series with the unknown C_1. We know that the total effective capacitance of the circuit (in the sense of what the battery "sees" when it is hooked up) is (12 μC)/V_{battery} = 4μF/3. Using Eq 25-20 again, we find

$$\frac{1}{2\ \mu\text{F}} + \frac{1}{C_1} = \frac{3}{4\ \mu\text{F}} \Rightarrow C_1 = 4.0\ \mu\text{F}\ .$$

20. We note that the total equivalent capacitance is $C_{123} = [(C_3)^{-1} + (C_1 + C_2)^{-1}]^{-1} = 6\ \mu\text{F}$.

(a) Thus, the charge that passed point *a* is $C_{123}\, V_{\text{batt}} = (6\ \mu\text{F})(12\ \text{V}) = 72\ \mu\text{C}$. Dividing this by the value $e = 1.60 \times 10^{-19}$ C gives the number of electrons: 4.5×10^{14}, which travel to the left – towards the positive terminal of the battery.

(b) The equivalent capacitance of the parallel pair is $C_{12} = C_1 + C_2 = 12\ \mu\text{F}$. Thus, the voltage across the pair (which is the same as the voltage across C_1 and C_2 individually) is

$$\frac{72\ \mu\text{C}}{12\ \mu\text{F}} = 6\ \text{V}\,.$$

Thus, the charge on C_1 is $(4\ \mu\text{F})(6\ \text{V}) = 24\ \mu\text{C}$, and dividing this by *e* gives the number of electrons (1.5×10^{14}) which have passed (upward) though point *b*.

(c) Similarly, the charge on C_2 is $(8\ \mu\text{F})(6\ \text{V}) = 48\ \mu\text{C}$, and dividing this by *e* gives the number of electrons (3.0×10^{14}) which have passed (upward) though point *c*.

(d) Finally, since C_3 is in series with the battery, its charge is the same that passed through the battery (the same as passed through the switch). Thus, 4.5×10^{14} electrons passed rightward though point *d*. By leaving the rightmost plate of C_3, that plate is then the positive plate of the fully charged capacitor – making its leftmost plate (the one closest to the negative terminal of the battery) the negative plate, as it should be.

(e) As stated in (b), the electrons travel up through point *b*.

(f) As stated in (c), the electrons travel up through point *c*.

21. The charges on capacitors 2 and 3 are the same, so these capacitors may be replaced by an equivalent capacitance determined from

$$\frac{1}{C_{eq}} = \frac{1}{C_2} + \frac{1}{C_3} = \frac{C_2 + C_3}{C_2 C_3}.$$

Thus, $C_{eq} = C_2C_3/(C_2 + C_3)$. The charge on the equivalent capacitor is the same as the charge on either of the two capacitors in the combination and the potential difference across the equivalent capacitor is given by q_2/C_{eq}. The potential difference across capacitor 1 is q_1/C_1, where q_1 is the charge on this capacitor. The potential difference across the combination of capacitors 2 and 3 must be the same as the potential difference across capacitor 1, so $q_1/C_1 = q_2/C_{eq}$. Now some of the charge originally on capacitor 1 flows to the combination of 2 and 3. If q_0 is the original charge, conservation of charge yields $q_1 + q_2 = q_0 = C_1 V_0$, where V_0 is the original potential difference across capacitor 1.

(a) Solving the two equations

$$\frac{q_1}{C_1} = \frac{q_2}{C_{eq}} \qquad \text{and} \qquad q_1 + q_2 = C_1 V_0$$

for q_1 and q_2, we obtain

$$q_1 = \frac{C_1^2 V_0}{C_{eq} + C_1} = \frac{C_1^2 V_0}{\dfrac{C_2 C_3}{C_2 + C_3} + C_1} = \frac{C_1^2 (C_2 + C_3) V_0}{C_1 C_2 + C_1 C_3 + C_2 C_3}.$$

With $V_0 = 12.0$ V, $C_1 = 4.00\ \mu$F, $C_2 = 6.00\ \mu$F and $C_3 = 3.00\ \mu$F, we find $C_{eq} = 2.00\ \mu$F and $q_1 = 32.0\ \mu$C.

(b) The charge on capacitors 2 is

$$q_2 = C_1 V_0 - q_1 = (4.00 \mu\text{F})(12.0\text{V}) - 32.0 \mu\text{F} = 16.0 \mu\text{F}$$

(c) The charge on capacitor 3 is the same as that on capacitor 2:

$$q_3 = C_1 V_0 - q_1 = (4.00 \mu\text{F})(12.0\text{V}) - 32.0 \mu\text{F} = 16.0 \mu\text{F}$$

22. Initially the capacitors C_1, C_2, and C_3 form a combination equivalent to a single capacitor which we denote C_{123}. This obeys the equation

$$\frac{1}{C_1} + \frac{1}{C_2 + C_3} = \frac{1}{C_{123}}.$$

Hence, using $q = C_{123}V$ and the fact that $q = q_1 = C_1 V_1$, we arrive at

$$V_1 = \frac{C_2 + C_3}{C_1 + C_2 + C_3} V.$$

(a) As $C_3 \to \infty$ this expression becomes $V_1 = V$. Since the problem states that V_1 approaches 10 volts in this limit, so we conclude $V = 10$ V.

(b) and (c) At $C_3 = 0$, the graph indicates $V_1 = 2.0$ V. The above expression consequently implies $C_1 = 4C_2$. Next we note that the graph shows that, at $C_3 = 6.0$ μF, the voltage across C_1 is exactly half of the battery voltage. Thus,

$$\frac{1}{2} = \frac{C_2 + 6.0\ \mu\text{F}}{C_1 + C_2 + 6.0\ \mu\text{F}} = \frac{C_2 + 6.0\ \mu\text{F}}{4C_2 + C_2 + 6.0\ \mu\text{F}}$$

which leads to $C_2 = 2.0$ μF. We conclude, too, that $C_1 = 8.0$ μF.

23. (a) In this situation, capacitors 1 and 3 are in series, which means their charges are necessarily the same:

$$q_1 = q_3 = \frac{C_1 C_3 V}{C_1 + C_3} = \frac{(1.00\mu\text{F})\,(3.00\mu\text{F})\,(12.0\text{V})}{1.00\mu\text{F}+3.00\mu\text{F}} = 9.00\mu\text{C}.$$

(b) Capacitors 2 and 4 are also in series:

$$q_2 = q_4 = \frac{C_2 C_4 V}{C_2 + C_4} = \frac{(2.00\mu\text{F})\,(4.00\mu\text{F})\,(12.0\text{V})}{2.00\mu\text{F}+4.00\mu\text{F}} = 16.0\mu\text{C}.$$

(c) $q_3 = q_1 = 9.00\mu\text{C}.$

(d) $q_4 = q_2 = 16.0\mu\text{C}.$

(e) With switch 2 also closed, the potential difference V_1 across C_1 must equal the potential difference across C_2 and is

$$V_1 = \frac{C_3 + C_4}{C_1 + C_2 + C_3 + C_4} V = \frac{(3.00\mu\text{F}+4.00\mu\text{F})(12.0\text{V})}{1.00\mu\text{F}+2.00\mu\text{F}+3.00\mu\text{F}+4.00\mu\text{F}} = 8.40\text{V}.$$

Thus, $q_1 = C_1 V_1 = (1.00\ \mu\text{F})(8.40\ \text{V}) = 8.40\ \mu\text{C}$.

(f) Similarly, $q_2 = C_2 V_1 = (2.00\ \mu\text{F})(8.40\ \text{V}) = 16.8\ \mu\text{C}$.

(g) $q_3 = C_3(V - V_1) = (3.00\ \mu\text{F})(12.0\ \text{V} - 8.40\ \text{V}) = 10.8\ \mu\text{C}$.

(h) $q_4 = C_4(V - V_1) = (4.00\ \mu\text{F})(12.0\ \text{V} - 8.40\ \text{V}) = 14.4\ \mu\text{C}$.

24. Let $\mathcal{V}$ = 1.00 m^3. Using Eq. 25-25, the energy stored is

$$U = u\mathcal{V} = \frac{1}{2}\varepsilon_0 E^2 \mathcal{V} = \frac{1}{2}\left(8.85\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N}\cdot\mathrm{m}^2}\right)(150\ \mathrm{V/m})^2\left(1.00\ \mathrm{m}^3\right) = 9.96\times10^{-8}\ \mathrm{J}.$$

25. The energy stored by a capacitor is given by $U = \frac{1}{2}CV^2$, where V is the potential difference across its plates. We convert the given value of the energy to Joules. Since a Joule is a watt·second, we multiply by (10^3 W/kW) (3600 s/h) to obtain $10\ \text{kW}\cdot\text{h} = 3.6\times10^7\ \text{J}$. Thus,

$$C = \frac{2U}{V^2} = \frac{2\left(3.6\times10^7\ \text{J}\right)}{\left(1000\ \text{V}\right)^2} = 72\ \text{F}.$$

26. (a) The capacitance is

$$C=\frac{\varepsilon_0 A}{d}=\frac{\left(8.85\times10^{-12}\,\frac{\mathrm{C}^2}{\mathrm{N\cdot m}^2}\right)\left(40\times10^{-4}\,\mathrm{m}^2\right)}{1.0\times10^{-3}\,\mathrm{m}}=3.5\times10^{-11}\,\mathrm{F}=35\,\mathrm{pF}.$$

(b) $q = CV = (35\text{ pF})(600\text{ V}) = 2.1\times10^{-8}\text{ C} = 21\text{ nC}$.

(c) $U=\frac{1}{2}CV^2=\frac{1}{2}(35\,\mathrm{pF})(21\,\mathrm{nC})^2=6.3\times10^{-6}\,\mathrm{J}=6.3\,\mu\mathrm{J}$.

(d) $E = V/d = 600\text{ V}/1.0\times10^{-3}\text{ m} = 6.0\times10^{5}\text{ V/m}$.

(e) The energy density (energy per unit volume) is

$$u=\frac{U}{Ad}=\frac{6.3\times10^{-6}\,\mathrm{J}}{\left(40\times10^{-4}\mathrm{m}^2\right)\left(1.0\times10^{-3}\mathrm{m}\right)}=1.6\ \mathrm{J/m}^3.$$

27. The total energy is the sum of the energies stored in the individual capacitors. Since they are connected in parallel, the potential difference V across the capacitors is the same and the total energy is

$$U = \frac{1}{2}(C_1 + C_2)V^2 = \frac{1}{2}\left(2.0\times10^{-6}\ \text{F} + 4.0\times10^{-6}\ \text{F}\right)(300\ \text{V})^2 = 0.27\ \text{J}.$$

28. (a) The potential difference across C_1 (the same as across C_2) is given by

$$V_1 = V_2 = \frac{C_3 V}{C_1 + C_2 + C_3} = \frac{(15.0\mu\text{F})(100\text{V})}{10.0\mu\text{F}+5.00\mu\text{F}+15.0\mu\text{F}} = 50.0\text{V}.$$

Also, $V_3 = V - V_1 = V - V_2 = 100\text{ V} - 50.0\text{ V} = 50.0\text{ V}$. Thus,

$$\begin{aligned} q_1 &= C_1 V_1 = (10.0\mu\text{F})(50.0\text{V}) = 5.00\times10^{-4}\,\text{C} \\ q_2 &= C_2 V_2 = (5.00\mu\text{F})(50.0\text{V}) = 2.50\times10^{-4}\,\text{C} \\ q_3 &= q_1 + q_2 = 5.00\times10^{-4}\,\text{C} + 2.50\times10^{-4}\,\text{C} = 7.50\times10^{-4}\,\text{C}. \end{aligned}$$

(b) The potential difference V_3 was found in the course of solving for the charges in part (a). Its value is $V_3 = 50.0$ V.

(c) The energy stored in C_3 is

$$U_3 = \frac{1}{2} C_3 V_3^2 = \frac{1}{2}(15.0\mu\text{F})(50.0\text{V})^2 = 1.88\times10^{-2}\,\text{J}.$$

(d) From part (a), we have $q_1 = 5.00\times10^{-4}\,\text{C}$, and

(e) $V_1 = 50.0$ V.

(f) The energy stored in C_1 is

$$U_1 = \frac{1}{2} C_1 V_1^2 = \frac{1}{2}(10.0\mu\text{F})(50.0\text{V})^2 = 1.25\times10^{-2}\,\text{J}.$$

(g) Again, from part (a), $q_2 = 2.50\times10^{-4}\,\text{C}$, and

(h) $V_2 = 50.0$ V.

(i) The energy stored in C_2 is

$$U_2 = \frac{1}{2} C_2 V_2^2 = \frac{1}{2}(5.00\mu\text{F})(50.0\text{V})^2 = 6.25\times10^{-3}\,\text{J}.$$

29. The energy per unit volume is

$$u=\frac{1}{2}\varepsilon_0E^2=\frac{1}{2}\varepsilon_0\left(\frac{e}{4\pi\varepsilon_0 r^2}\right)^2=\frac{e^2}{32\pi^2\varepsilon_0 r^4}\ .$$

(a) At $r=1.00\times10^{-3}\,\text{m}$, with $e=1.60\times10^{-19}\,\text{C}$ and $\varepsilon_0=8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2$, we have $u=9.16\times10^{-18}\ \text{J/m}^3$.

(b) Similarly, at $r=1.00\times10^{-6}\,\text{m}$, $u=9.16\times10^{-6}\ \text{J/m}^3$,

(c) at $r=1.00\times10^{-9}\,\text{m}$, $u=9.16\times10^{6}\ \text{J/m}^3$, and

(d) at $r=1.00\times10^{-12}\,\text{m}$, $u=9.16\times10^{18}\ \text{J/m}^3$.

(e) From the expression above $u \propto r^{-4}$. Thus, for $r \to 0$, the energy density $u \to \infty$.

30. (a) The charge q_3 in the Figure is $q_3 = C_3V = (4.00\,\mu\text{F})(100\ \text{V}) = 4.00\times10^{-4}\ \text{C}$.

(b) $V_3 = V = 100$ V.

(c) Using $U_i = \frac{1}{2}C_iV_i^2$, we have $U_3 = \frac{1}{2}C_3V_3^2 = 2.00\times10^{-2}\,\text{J}$.

(d) From the Figure,

$$q_1 = q_2 = \frac{C_1C_2V}{C_1 + C_2} = \frac{(10.0\,\mu\text{F})(5.00\,\mu\text{F})(100\ \text{V})}{10.0\,\mu\text{F} + 5.00\,\mu\text{F}} = 3.33\times10^{-4}\text{C}.$$

(e) $V_1 = q_1/C_1 = 3.33 \times 10^{-4}$ C/10.0 μF = 33.3 V.

(f) $U_1 = \frac{1}{2}C_1V_1^2 = 5.55\times10^{-3}\,\text{J}$.

(g) From part (d), we have $q_2 = q_1 = 3.33\times10^{-4}\text{C}$.

(h) $V_2 = V - V_1 = 100$ V – 33.3 V = 66.7 V.

(i) $U_2 = \frac{1}{2}C_2V_2^2 = 1.11\times10^{-2}\,\text{J}$.

31. (a) Let q be the charge on the positive plate. Since the capacitance of a parallel-plate capacitor is given by $\varepsilon_0 A/d_i$, the charge is $q = CV = \varepsilon_0 AV_i/d_i$. After the plates are pulled apart, their separation is d_f and the potential difference is V_f. Then $q = \varepsilon_0 AV_f/2d_f$ and

$$V_f = \frac{d_f}{\varepsilon_0 A} q = \frac{d_f}{\varepsilon_0 A} \frac{\varepsilon_0 A}{d_i} V_i = \frac{d_f}{d_i} V_i.$$

With $d_i = 3.00\times10^{-3}\,\text{m}$, $V_i = 6.00$ V and $d_f = 8.00\times10^{-3}\,\text{m}$, we have $V_f = 16.0\ \text{V}$.

(b) The initial energy stored in the capacitor is (in SI units)

$$U_i = \frac{1}{2}CV_i^2 = \frac{\varepsilon_0 AV_i^2}{2d_i} = \frac{(8.85\times10^{-12})(8.50\times10^{-4})(6.00)^2}{2(3.00\times10^{-3})} = 4.51\times10^{-11}\ \text{J}.$$

(c) The final energy stored is

$$U_f = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}V_f^2 = \frac{1}{2}\frac{\varepsilon_0 A}{d_f}\left(\frac{d_f}{d_i}V_i\right)^2 = \frac{d_f}{d_i}\left(\frac{\varepsilon_0 AV_i^2}{d_i}\right) = \frac{d_f}{d_i}U_i.$$

With $d_f/d_i = 8.00/3.00$, we have $U_f = 1.20\times10^{-10}\ \text{J}$.

(d) The work done to pull the plates apart is the difference in the energy:

$$W = U_f - U_i = 7.52\times10^{-11}\ \text{J}.$$

32. We use $E = q / 4\pi\varepsilon_0 R^2 = V / R$. Thus

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{1}{2}\varepsilon_0 \left(\frac{V}{R}\right)^2 = \frac{1}{2}\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{8000\,\text{V}}{0.050\text{ m}}\right)^2 = 0.11\text{ J/m}^3.$$

33. (a) They each store the same charge, so the maximum voltage is across the smallest capacitor. With 100 V across 10 μF, then the voltage across the 20 μF capacitor is 50 V and the voltage across the 25 μF capacitor is 40 V. Therefore, the voltage across the arrangement is 190 V.

(b) Using Eq. 25-21 or Eq. 25-22, we sum the energies on the capacitors and obtain U_{total} = 0.095 J.

34. If the original capacitance is given by $C = \varepsilon_0 A/d$, then the new capacitance is $C' = \varepsilon_0 \kappa A/2d$. Thus $C'/C = \kappa/2$ or

$$\kappa = 2C'/C = 2(2.6 \text{ pF}/1.3 \text{ pF}) = 4.0.$$

35. The capacitance with the dielectric in place is given by $C = \kappa C_0$, where C_0 is the capacitance before the dielectric is inserted. The energy stored is given by $U = \frac{1}{2}CV^2 = \frac{1}{2}\kappa C_0 V^2$, so

$$\kappa = \frac{2U}{C_0 V^2} = \frac{2(7.4\times10^{-6}\,\text{J})}{(7.4\times10^{-12}\,\text{F})(652\text{V})^2} = 4.7.$$

According to Table 25-1, you should use Pyrex.

36. (a) We use $C = \varepsilon_0 A/d$ to solve for d:

$$d = \frac{\varepsilon_0 A}{C} = \frac{\left(8.85\times10^{-12}\ \frac{\mathrm{C}^2}{\mathrm{N\cdot m^2}}\right)(0.35\ \mathrm{m}^2)}{50\times10^{-12}\,\mathrm{F}} = 6.2\times10^{-2}\ \mathrm{m}.$$

(b) We use $C \propto \kappa$. The new capacitance is $C' = C(\kappa/\kappa_{air}) = (50\ \mathrm{pf})(5.6/1.0) = 2.8\times10^2\ \mathrm{pF}$.

37. The capacitance of a cylindrical capacitor is given by

$$C = \kappa C_0 = \frac{2\pi\kappa\varepsilon_0 L}{\ln(b/a)},$$

where C_0 is the capacitance without the dielectric, κ is the dielectric constant, L is the length, a is the inner radius, and b is the outer radius. The capacitance per unit length of the cable is

$$\frac{C}{L} = \frac{2\pi\kappa\varepsilon_0}{\ln(b/a)} = \frac{2\pi(2.6)(8.85\times10^{-12}\ \text{F/m})}{\ln[(0.60\ \text{mm})/(0.10\ \text{mm})]} = 8.1\times10^{-11}\,\text{F/m} = 81\ \text{pF/m}.$$

38. Each capacitor has 12.0 V across it, so Eq. 25-1 yields the charge values once we know C_1 and C_2. From Eq. 25-9,

$$C_2 = \frac{\varepsilon_0 A}{d} = 2.21 \times 10^{-11} \text{ F} ,$$

and from Eq. 25-27,

$$C_1 = \frac{\kappa\varepsilon_0 A}{d} = 6.64 \times 10^{-11} \text{ F} .$$

This leads to $q_1 = C_1V_1 = 8.00 \times 10^{-10}$ C and $q_2 = C_2V_2 = 2.66 \times 10^{-10}$ C. The addition of these gives the desired result: $q_{tot} = 1.06 \times 10^{-9}$ C. Alternatively, the circuit could be reduced to find the q_{tot}.

39. The capacitance is given by $C = \kappa C_0 = \kappa\varepsilon_0 A/d$, where C_0 is the capacitance without the dielectric, κ is the dielectric constant, A is the plate area, and d is the plate separation. The electric field between the plates is given by $E = V/d$, where V is the potential difference between the plates. Thus, $d = V/E$ and $C = \kappa\varepsilon_0 AE/V$. Thus,

$$A = \frac{CV}{\kappa\varepsilon_0 E}.$$

For the area to be a minimum, the electric field must be the greatest it can be without breakdown occurring. That is,

$$A = \frac{(7.0\times10^{-8}\,\text{F})(4.0\times10^{3}\,\text{V})}{2.8(8.85\times10^{-12}\,\text{F/m})(18\times10^{6}\ \text{V/m})} = 0.63\ \text{m}^2.$$

40. (a) We use Eq. 25-14:

$$C = 2\pi\varepsilon_0\kappa\frac{L}{\ln(b/a)} = \frac{(4.7)(0.15\text{ m})}{2\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\ln(3.8\text{ cm}/3.6\text{ cm})} = 0.73\text{ nF}.$$

(b) The breakdown potential is (14 kV/mm) (3.8 cm – 3.6 cm) = 28 kV.

41. Using Eq. 25-29, with $\sigma = q/A$, we have

$$\left|\vec{E}\right| = \frac{q}{\kappa\varepsilon_0 A} = 200\times10^3 \text{ N/C}$$

which yields $q = 3.3 \times 10^{-7}$ C. Eq. 25-21 and Eq. 25-27 therefore lead to

$$U = \frac{q^2}{2C} = \frac{q^2 d}{2\kappa\varepsilon_0 A} = 6.6\times10^{-5} \text{ J} .$$

42. The capacitor can be viewed as two capacitors C_1 and C_2 in parallel, each with surface area $A/2$ and plate separation d, filled with dielectric materials with dielectric constants κ_1 and κ_2, respectively. Thus, (in SI units),

$$C = C_1 + C_2 = \frac{\varepsilon_0 (A/2)\kappa_1}{d} + \frac{\varepsilon_0 (A/2)\kappa_2}{d} = \frac{\varepsilon_0 A}{d}\left(\frac{\kappa_1 + \kappa_2}{2}\right)$$

$$= \frac{(8.85\times 10^{-12})(5.56\times 10^{-4})}{5.56\times 10^{-3}} \frac{7.00 + 12.00}{2} = 8.41\times 10^{-12} \text{ F.}$$

43. We assume there is charge q on one plate and charge $-q$ on the other. The electric field in the lower half of the region between the plates is

$$E_1 = \frac{q}{\kappa_1 \varepsilon_0 A},$$

where A is the plate area. The electric field in the upper half is

$$E_2 = \frac{q}{\kappa_2 \varepsilon_0 A}.$$

Let $d/2$ be the thickness of each dielectric. Since the field is uniform in each region, the potential difference between the plates is

$$V = \frac{E_1 d}{2} + \frac{E_2 d}{2} = \frac{qd}{2\varepsilon_0 A}\left[\frac{1}{\kappa_1} + \frac{1}{\kappa_2}\right] = \frac{qd}{2\varepsilon_0 A}\frac{\kappa_1 + \kappa_2}{\kappa_1 \kappa_2},$$

so

$$C = \frac{q}{V} = \frac{2\varepsilon_0 A}{d}\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}.$$

This expression is exactly the same as that for C_{eq} of two capacitors in series, one with dielectric constant κ_1 and the other with dielectric constant κ_2. Each has plate area A and plate separation $d/2$. Also we note that if $\kappa_1 = \kappa_2$, the expression reduces to $C = \kappa_1 \varepsilon_0 A/d$, the correct result for a parallel-plate capacitor with plate area A, plate separation d, and dielectric constant κ_1.

With $A = 7.89\times10^{-4}\,\text{m}^2$, $d = 4.62\times10^{-3}\,\text{m}$, $\kappa_1 = 11.0$ and $\kappa_2 = 12.0$, the capacitance is, (in SI units)

$$C = \frac{2(8.85\times10^{-12})(7.89\times10^{-4})}{4.62\times10^{-3}}\frac{(11.0)(12.0)}{11.0+12.0} = 1.73\times10^{-11}\,\text{F}.$$

44. Let $C_1 = \varepsilon_0(A/2)\kappa_1/2d = \varepsilon_0 A\kappa_1/4d$, $C_2 = \varepsilon_0(A/2)\kappa_2/d = \varepsilon_0 A\kappa_2/2d$, and $C_3 = \varepsilon_0 A\kappa_3/2d$. Note that C_2 and C_3 are effectively connected in series, while C_1 is effectively connected in parallel with the C_2-C_3 combination. Thus,

$$C = C_1 + \frac{C_2 C_3}{C_2 + C_3} = \frac{\varepsilon_0 A\kappa_1}{4d} + \frac{(\varepsilon_0 A/d)(\kappa_2/2)(\kappa_3/2)}{\kappa_2/2 + \kappa_3/2} = \frac{\varepsilon_0 A}{4d}\left(\kappa_1 + \frac{2\kappa_2\kappa_3}{\kappa_2 + \kappa_3}\right).$$

With $A = 1.05\times10^{-3}\,\mathrm{m}^2$, $d = 3.56\times10^{-3}\,\mathrm{m}$, $\kappa_1 = 21.0$, $\kappa_2 = 42.0$ and $\kappa_3 = 58.0$, the capacitance is, (in SI units)

$$C = \frac{(8.85\times10^{-12})(1.05\times10^{-3})}{4(3.56)\times10^{-3}}\left(21.0 + \frac{2(42.0)(58.0)}{42.0 + 58.0}\right) = 4.55\times10^{-11}\,\mathrm{F}.$$

45. (a) The electric field in the region between the plates is given by $E = V/d$, where V is the potential difference between the plates and d is the plate separation. The capacitance is given by $C = \kappa\varepsilon_0 A/d$, where A is the plate area and κ is the dielectric constant, so $d = \kappa\varepsilon_0 A / C$ and

$$E = \frac{VC}{\kappa\varepsilon_0 A} = \frac{(50\text{ V})(100\times10^{-12}\text{ F})}{5.4(8.85\times10^{-12}\text{ F/m})(100\times10^{-4}\text{ m}^2)} = 1.0\times10^4\text{ V/m}.$$

(b) The free charge on the plates is $q_f = CV = (100 \times 10^{-12}\text{ F})(50\text{ V}) = 5.0 \times 10^{-9}\text{ C}$.

(c) The electric field is produced by both the free and induced charge. Since the field of a large uniform layer of charge is $q/2\varepsilon_0 A$, the field between the plates is

$$E = \frac{q_f}{2\varepsilon_0 A} + \frac{q_f}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A} - \frac{q_i}{2\varepsilon_0 A},$$

where the first term is due to the positive free charge on one plate, the second is due to the negative free charge on the other plate, the third is due to the positive induced charge on one dielectric surface, and the fourth is due to the negative induced charge on the other dielectric surface. Note that the field due to the induced charge is opposite the field due to the free charge, so they tend to cancel. The induced charge is therefore

$$\begin{aligned} q_i &= q_f - \varepsilon_0 AE = 5.0\times10^{-9}\text{ C} - (8.85\times10^{-12}\text{ F/m})(100\times10^{-4}\text{ m}^2)(1.0\times10^4\text{ V/m}) \\ &= 4.1\times10^{-9}\text{ C} = 4.1\text{ nC}. \end{aligned}$$

46. (a) The electric field E_1 in the free space between the two plates is $E_1 = q/\varepsilon_0 A$ while that inside the slab is $E_2 = E_1/\kappa = q/\kappa\varepsilon_0 A$. Thus,

$$V_0 = E_1(d-b) + E_2 b = \left(\frac{q}{\varepsilon_0 A}\right)\left(d - b + \frac{b}{\kappa}\right),$$

and the capacitance is

$$C = \frac{q}{V_0} = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(115\times10^{-4}\,\text{m}^2\right)(2.61)}{(2.61)(0.0124\,\text{m}-0.00780\,\text{m})+(0.00780\,\text{m})} = 13.4\,\text{pF}.$$

(b) $q = CV = (13.4 \times 10^{-12}\text{ F})(85.5\text{ V}) = 1.15\text{ nC}$.

(c) The magnitude of the electric field in the gap is

$$E_1 = \frac{q}{\varepsilon_0 A} = \frac{1.15\times10^{-9}\ \text{C}}{\left(8.85\times10^{-12}\,\frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(115\times10^{-4}\ \text{m}^2\right)} = 1.13\times10^{4}\ \text{N/C}.$$

(d) Using Eq. 25-34, we obtain

$$E_2 = \frac{E_1}{\kappa} = \frac{1.13\times10^{4}\ \text{N/C}}{2.61} = 4.33\times10^{3}\ \text{N/C}.$$

47. (a) According to Eq. 25-17 the capacitance of an air-filled spherical capacitor is given by

$$C_0 = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right).$$

When the dielectric is inserted between the plates the capacitance is greater by a factor of the dielectric constant κ. Consequently, the new capacitance is

$$C = 4\pi\kappa\varepsilon_0\left(\frac{ab}{b-a}\right) = \frac{23.5}{8.99\times10^9}\frac{(0.0120)(0.0170)}{0.0170-0.0120} = 0.107 \text{ nF}.$$

(b) The charge on the positive plate is

$$q = CV = (0.107 \text{ nF})(73.0 \text{ V}) = 7.79 \text{ nC}.$$

(c) Let the charge on the inner conductor be $-q$. Immediately adjacent to it is the induced charge q'. Since the electric field is less by a factor $1/\kappa$ than the field when no dielectric is present, then $-q + q' = -q/\kappa$. Thus,

$$q' = \frac{\kappa-1}{\kappa}q = 4\pi(\kappa-1)\varepsilon_0\frac{ab}{b-a}V$$
$$= \left(\frac{23.5-1.00}{23.5}\right)(7.79 \text{ nC}) = 7.45 \text{ nC}.$$

48. (a) We apply Gauss's law with dielectric: $q/\varepsilon_0 = \kappa EA$, and solve for κ:

$$\kappa = \frac{q}{\varepsilon_0 EA} = \frac{8.9\times10^{-7}\ \text{C}}{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(1.4\times10^{-6}\ \text{V/m}\right)\left(100\times10^{-4}\,\text{m}^2\right)} = 7.2.$$

(b) The charge induced is

$$q' = q\left(1-\frac{1}{\kappa}\right) = \left(8.9\times10^{-7}\ \text{C}\right)\left(1-\frac{1}{7.2}\right) = 7.7\times10^{-7}\ \text{C}.$$

49. (a) Initially, the capacitance is

$$C_0 = \frac{\varepsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.12\ \text{m}^2)}{1.2\times10^{-2}\ \text{m}} = 89\ \text{pF}.$$

(b) Working through Sample Problem 25-7 algebraically, we find:

$$C = \frac{\varepsilon_0 A\kappa}{\kappa(d-b)+b} = \frac{\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)(0.12\,\text{m}^2)(4.8)}{(4.8)(1.2-0.40)(10^{-2}\ \text{m})+(4.0\times10^{-3}\ \text{m})} = 1.2\times10^2\ \text{pF}.$$

(c) Before the insertion, $q = C_0V$ (89 pF)(120 V) = 11 nC.

(d) Since the battery is disconnected, q will remain the same after the insertion of the slab, with q = 11 nC.

(e) $E = q/\varepsilon_0 A = 11\times10^{-9}\ \text{C}/(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2})(0.12\ \text{m}^2) = 10\ \text{kV/m}.$

(f) $E' = E/\kappa$ = (10 kV/m)/4.8 = 2.1 kV/m.

(g) $V = E(d - b) + E'b$ = (10 kV/m)(0.012 m – 0.0040 m) + (2.1 kV/m)(0.40 × 10^{-3} m) = 88 V.

(h) The work done is

$$W_{\text{ext}} = \Delta U = \frac{q^2}{2}\left(\frac{1}{C}-\frac{1}{C_0}\right) = \frac{(11\times10^{-9}\ \text{C})^2}{2}\left(\frac{1}{89\times10^{-12}\ \text{F}}-\frac{1}{120\times10^{-12}\ \text{F}}\right) = -1.7\times10^{-7}\ \text{J}.$$

50. (a) Eq. 25-22 yields

$$U = \frac{1}{2}CV^2 = \frac{1}{2}\left(200\times10^{-12}\ \text{F}\right)\left(7.0\times10^{3}\ \text{V}\right)^2 = 4.9\times10^{-3}\ \text{J}.$$

(b) Our result from part (a) is much less than the required 150 mJ, so such a spark should not have set off an explosion.

51.One way to approach this is to note that – since they are identical – the voltage is evenly divided between them. That is, the voltage across each capacitor is $V = (10/n)$ volt. With $C = 2.0 \times 10^{-6}$ F, the electric energy stored by each capacitor is $\frac{1}{2}CV^2$. The total energy stored by the capacitors is n times that value, and the problem requires the total be equal to 25×10^{-6} J. Thus,

$$\frac{n}{2}(2.0 \times 10^{-6})\left(\frac{10}{n}\right)^2 = 25 \times 10^{-6}$$

leads to $n = 4$.

52. Initially the capacitors C_1, C_2, and C_3 form a series combination equivalent to a single capacitor which we denote C_{123}. Solving the equation

$$\frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{1}{C_{123}} ,$$

we obtain C_{123} = 2.40 μF. With V = 12.0 V, we then obtain $q = C_{123}V$ = 28.8 μC. In the final situation, C_2 and C_4 are in parallel and are thus effectively equivalent to $C_{24} = 12.0\ \mu\text{F}$. Similar to the previous computation, we use

$$\frac{1}{C_1} + \frac{1}{C_{24}} + \frac{1}{C_3} = \frac{1}{C_{1234}}$$

and find C_{1234} = 3.00 μF. Therefore, the final charge is $q = C_{1234}V$ = 36.0 μC.

(a) This represents a change (relative to the initial charge) of Δq = 7.20 μC.

(b) The capacitor C_{24} which we imagined to replace the parallel pair C_2 and C_4 is in series with C_1 and C_3 and thus also has the final charge q =36.0 μC found above. The voltage across C_{24} would be $V_{24} = q/C_{24}$ = 36.0/12.0 = 3.00 V. This is the same voltage across each of the parallel pair. In particular, V_4 = 3.00 V implies that $q_4 = C_4 V_4$ = 18.0 μC.

(c) The battery supplies charges only to the plates where it is connected. The charges on the rest of the plates are due to electron transfers between them, in accord with the new distribution of voltages across the capacitors. So, the battery does not directly supply the charge on capacitor 4.

53. In series, their equivalent capacitance (and thus their total energy stored) is smaller than either one individually (by Eq. 25-20). In parallel, their equivalent capacitance (and thus their total energy stored) is larger than either one individually (by Eq. 25-19). Thus, the middle two values quoted in the problem must correspond to the individual capacitors. We use Eq. 25-22 and find

(a) $100\ \mu\text{J} = \frac{1}{2} C_1 (10\ \text{V})^2 \quad \Rightarrow \quad C_1 = 2.0\ \mu\text{F}$

(b) $300\ \mu\text{J} = \frac{1}{2} C_2 (10\ \text{V})^2 \quad \Rightarrow \quad C_2 = 6.0\ \mu\text{F}$.

54. We note that the voltage across C_3 is $V_3 = (12\text{ V} - 2\text{ V} - 5\text{ V}) = 5\text{ V}$. Thus, its charge is $q_3 = C_3 V_3 = 4\ \mu\text{C}$.

(a) Therefore, since C_1, C_2 and C_3 are in series (so they have the same charge), then

$$C_1 = \frac{4\ \mu\text{C}}{2\text{ V}} = 2.0\ \mu\text{F} .$$

(b) Similarly, $C_2 = 4/5 = 0.80\ \mu\text{F}$.

55. (a) The number of (conduction) electrons per cubic meter is $n = 8.49 \times 10^{28}\,\text{m}^3$. The volume in question is the face area multiplied by the depth: $A{\cdot}d$. The total number of electrons which have moved to the face is

$$N = \frac{-3.0 \text{ x } 10^{-6}\,\text{C}}{-1.6 \text{ x } 10^{-19}\,\text{C}} \approx 1.9 \times 10^{13} \ .$$

Using the relation $N = nAd$, we obtain $d = 1.1 \times 10^{-12}$ m, a remarkably small distance!

56. Initially, the total equivalent capacitance is $C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 3.0$ μF, and the charge on the positive plate of each one is (3.0 μF)(10 V) = 30 μC. Next, the capacitor (call is C_1) is squeezed as described in the problem, with the effect that the new value of C_1 is 12 μF (see Eq. 25-9). The new total equivalent capacitance then becomes

$$C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 4.0 \ \mu\text{F},$$

and the new charge on the positive plate of each one is (4.0 μF)(10 V) = 40 μC.

(a) Thus we see that the charge transferred from the battery as a result of the squeezing is 40 μC – 30 μC = 10 μC.

(b) The total increase in positive charge (on the respective positive plates) stored on the capacitors is twice the value found in part (a) (since we are dealing with two capacitors in series): 20 μC.

57. (a) Put five such capacitors in series. Then, the equivalent capacitance is 2.0 μF/5 = 0.40 μF. With each capacitor taking a 200-V potential difference, the equivalent capacitor can withstand 1000 V.

(b) As one possibility, you can take three identical arrays of capacitors, each array being a five-capacitor combination described in part (a) above, and hook up the arrays in parallel. The equivalent capacitance is now $C_{eq} = 3(0.40\ \mu\text{F}) = 1.2\ \mu\text{F}$. With each capacitor taking a 200-V potential difference the equivalent capacitor can withstand 1000 V.

58. Equation 25-14 leads to C_1 = 2.53 pF and C_1 = 2.17 pF. Initially, the total equivalent capacitance is

$$C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 1.488 \text{ pF},$$

and the charge on the positive plate of each one is (1.488 pF)(10 V) = 14.88 pC. Next, capacitor 2 is modified as described in the problem, with the effect that the new value of C_2 is 2.17 pF (again using Eq. 25-14). The new total equivalent capacitance is

$$C_{12} = [(C_1)^{-1} + (C_2)^{-1}]^{-1} = 1.170 \text{ pF},$$

and the new charge on the positive plate of each one is (1.170 pF)(10 V) = 11.70 pC. Thus we see that the charge transferred from the battery (considered in absolute value) as a result of the modification is 14.88 pC – 11.70 pC = 3.18 pC.

(a) This charge, divided by *e* gives the number of electrons that pass point *P*. Thus,

$$\frac{3.18 \times 10^{-12}}{1.6 \times 10^{-19}} = 2.0 \times 10^{7} .$$

(b) These electrons move rightwards in the figure (that is, away from the battery) since the positive plates (the ones closest to point *P*) of the capacitors have suffered a *decease* in their positive charges. The usual reason for a metal plate to be positive is that it has more protons than electrons. Thus, in this problem some electrons have “returned” to the positive plates (making them less positive).

59. (a) We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1V_{\text{bat}} = 40\ \mu\text{C}$, and q_1 and q_2 are the charges on C_1 and C_2 after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2.$$

Reducing the right portion of the circuit (the C_3, C_4 parallel pair which are in series with C_2) we have an equivalent capacitance of $C' = 8.0\ \mu\text{F}$ which has charge $q' = q_2$ and potential difference equal to that of C_1. Thus, $V_1 = V'$, or

$$\frac{q_1}{C_1} = \frac{q_2}{C'}$$

which yields $4q_1 = q_2$. Therefore, $Q = q_1 + 4q_1$. This leads to $q_1 = 8.0\ \mu\text{C}$ and consequently to $q_2 = 32\ \mu\text{C}$.

(b) From Eq. 25-1, we have $V_2 = (32\ \mu\text{C})(16\ \mu\text{F}) = 2.0$ V.

60. (a) We calculate the charged surface area of the cylindrical volume as follows:

$$A = 2\pi rh + \pi r^2 = 2\pi(0.20\ \text{m})(0.10\ \text{m}) + \pi(0.20\ \text{m})^2 = 0.25\ \text{m}^2$$

where we note from the figure that although the bottom is charged, the top is not. Therefore, the charge is $q = \sigma A = -0.50\ \mu\text{C}$ on the exterior surface, and consequently (according to the assumptions in the problem) that same charge q is induced in the interior of the fluid.

(b) By Eq. 25-21, the energy stored is

$$U = \frac{q^2}{2C} = \frac{(5.0\times10^{-7}\ \text{C})^2}{2(35\times10^{-12}\ \text{F})} = 3.6\times10^{-3}\ \text{J}.$$

(c) Our result is within a factor of three of that needed to cause a spark. Our conclusion is that it will probably not cause a spark; however, there is not enough of a safety factor to be sure.

61. (a) In the top right portion of the circuit is a pair of 4.00 μF which we reduce to a single 8.00 μF capacitor (which is then in series with the bottom capacitor that the problem is asking about). The further reduction with the bottom 4.00 μF capacitor results in an equivalence of $\frac{8}{3}$ μF, which clearly has the battery voltage across it -- and therefore has charge ($\frac{8}{3}$ μF)(9.00 V) = 24.0 μC. This is seen to be the same as the charge on the bottom capacitor.

(b) The voltage across the bottom capacitor is

$$V = \frac{q}{C} = \frac{24.0\ \mu\text{C}}{4.00\ \mu\text{F}} = 6.00\ \text{V} \quad .$$

62. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = C_1V_{\text{bat}} = 100\ \mu\text{C}$, and q_1, q_2 and q_3 are the charges on C_1, C_2 and C_3 after the switch is thrown to the right and equilibrium is reached, then

$$Q = q_1 + q_2 + q_3 \ .$$

Since the parallel pair C_2 and C_3 are identical, it is clear that $q_2 = q_3$. They are in parallel with C_1 so that $V_1=V_3$, or

$$\frac{q_1}{C_1} = \frac{q_3}{C_3}$$

which leads to $q_1 = q_3/2$. Therefore,

$$Q = \left(\frac{1}{2}q_3\right) + q_3 + q_3$$

which yields $q_3 = 40\ \mu\text{C}$ and consequently $q_1 = 20\ \mu\text{C}$.

63. The pair C_3 and C_4 are in parallel and consequently equivalent to 30 μF. Since this numerical value is identical to that of the others (with which it is in series, with the battery), we observe that each has one-third the battery voltage across it. Hence, 3.0 V is across C_4, producing a charge

$$q_4 = C_4 V_4 = (15\ \mu\text{F})(3.0\ \text{V}) = 45\ \mu\text{C}\ .$$

64. (a) We reduce the parallel group C_2, C_3 and C_4, and the parallel pair C_5 and C_6, obtaining equivalent values $C' = 12\ \mu\text{F}$ and $C'' = 12\ \mu\text{F}$, respectively. We then reduce the series group C_1, C' and C'' to obtain an equivalent capacitance of $C_{eq} = 3\ \mu\text{F}$ hooked to the battery. Thus, the charge stored in the system is

$$q_{sys} = C_{eq} V_{bat} = 36\ \mu\text{C} \ .$$

(b) Since $q_{sys} = q_1$ then the voltage across C_1 is

$$V_1 = \frac{q_1}{C_1} = \frac{36\ \mu\text{C}}{6.0\ \mu\text{F}} = 6.0\ \text{V} \ .$$

The voltage across the series-pair C' and C'' is consequently $V_{bat} - V_1 = 6.0$ V. Since $C' = C''$, we infer $V' = V'' = 6.0/2 = 3.0$ V, which, in turn, is equal to V_4, the potential across C_4. Therefore,

$$q_4 = C_4 V_4 = (4.0\ \mu\text{F})(3.0\ \text{V}) = 12\ \mu\text{C} \ .$$

65. (a) The potential across C_1 is 10 V, so the charge on it is

$$q_1 = C_1V_1 = (10.0\ \mu\text{F})(10.0\ \text{V}) = 100\ \mu\text{C}.$$

(b) Reducing the right portion of the circuit produces an equivalence equal to 6.00 μF, with 10.0 V across it. Thus, a charge of 60.0 μC is on it -- and consequently also on the bottom right capacitor. The bottom right capacitor has, as a result, a potential across it equal to

$$V = \frac{q}{C} = \frac{60\ \mu\text{C}}{10\ \mu\text{F}} = 6.00\ \text{V}$$

which leaves 10.0 V – 6.00 V = 4.00 V across the group of capacitors in the upper right portion of the circuit. Inspection of the arrangement (and capacitance values) of that group reveals that this 4.00 V must be equally divided by C_2 and the capacitor directly below it (in series with it). Therefore, with 2.00 V across C_2 we find

$$q_2 = C_2V_2 = (10.0\ \mu\text{F})(2.00\ \text{V}) = 20.0\ \mu\text{C}\ .$$

66. The pair C_1 and C_2 are in parallel, as are the pair C_3 and C_4; they reduce to equivalent values 6.0 μF and 3.0 μF, respectively. These are now in series and reduce to 2.0 μF, across which we have the battery voltage. Consequently, the charge on the 2.0 μF equivalence is (2.0 μF)(12 V) = 24 μC. This charge on the 3.0 μF equivalence (of C_3 and C_4) has a voltage of

$$V = \frac{q}{C} = \frac{24\ \mu\text{C}}{3\ \mu\text{F}} = 8.0\ \text{V} \quad .$$

Finally, this voltage on capacitor C_4 produces a charge (2.0 μF)(8.0 V) = 16 μC.

67. For maximum capacitance the two groups of plates must face each other with maximum area. In this case the whole capacitor consists of (n – 1) identical single capacitors connected in parallel. Each capacitor has surface area A and plate separation d so its capacitance is given by $C_0 = \varepsilon_0 A/d$. Thus, the total capacitance of the combination is (in SI units)

$$C = (n-1)C_0 = \frac{(n-1)\varepsilon_0 A}{d} = \frac{(8-1)(8.85\times10^{-12})(1.25\times10^{-4})}{3.40\times10^{-3}} = 2.28\times10^{-12}\,\text{F}.$$

68. (a) Here D is not attached to anything, so that the $6C$ and $4C$ capacitors are in series (equivalent to $2.4C$). This is then in parallel with the $2C$ capacitor, which produces an equivalence of $4.4C$. Finally the $4.4C$ is in series with C and we obtain

$$C_{\text{eq}} = \frac{(C)(4.4\,C)}{C+4.4\,C} = 0.82\,C = 41\,\mu\text{F}$$

where we have used the fact that $C = 50\ \mu\text{F}$.

(b) Now, B is the point which is not attached to anything, so that the $6C$ and $2C$ capacitors are now in series (equivalent to $1.5C$), which is then in parallel with the $4C$ capacitor (and thus equivalent to $5.5C$). The $5.5C$ is then in series with the C capacitor; consequently,

$$C_{\text{eq}} = \frac{(C)(5.5\,C)}{C+5.5\,C} = 0.85C = 42\,\mu\text{F} .$$

69. (a) In the first case the two capacitors are effectively connected in series, so the output potential difference is $V_{out} = CV_{in}/2C = V_{in}/2 = 50.0$ V.

(b) In the second case the lower diode acts as a wire so $V_{out} = 0$.

70. The voltage across capacitor 1 is

$$V_1 = \frac{q_1}{C_1} = \frac{30\,\mu\text{C}}{10\,\mu\text{F}} = 3.0 \text{ V} .$$

Since $V_1 = V_2$, the total charge on capacitor 2 is

$$q_2 = C_2V_2 = (20\,\mu\text{F})(2\text{ V}) = 60\,\mu\text{C} ,$$

which means a total of 90 μC of charge is on the pair of capacitors C_1 and C_2. This implies there is a total of 90 μC of charge also on the C_3 and C_4 pair. Since $C_3 = C_4$, the charge divides equally between them, so $q_3 = q_4 = 45$ μC. Thus, the voltage across capacitor 3 is

$$V_3 = \frac{q_3}{C_3} = \frac{45\,\mu\text{C}}{20\,\mu\text{F}} = 2.3\text{ V} .$$

Therefore, $|V_A - V_B| = V_1 + V_3 = 5.3$ V.

71. (a) The equivalent capacitance is

$$C_{\text{eq}} = \frac{C_1 C_2}{C_1 + C_2} = \frac{(6.00\,\mu\text{F})(4.00\,\mu\text{F})}{6.00\,\mu\text{F} + 4.00\,\mu\text{F}} = 2.40\,\mu\text{F} .$$

(b) $q_1 = C_{\text{eq}}V = (2.40\ \mu\text{F})(200\ \text{V}) = 4.80 \times 10^{-4}\ \text{C}$.

(c) $V_1 = q_1/C_1 = 4.80 \times 10^{-4}\ \text{C}/6.00\ \mu\text{F} = 80.0\ \text{V}$.

(d) $q_2 = q_1 = 4.80 \times 10^{-4}\ \text{C}$.

(e) $V_2 = V - V_1 = 200\ \text{V} - 80.0\ \text{V} = 120\ \text{V}$.

72. (a) Now $C_{eq} = C_1 + C_2 = 6.00\ \mu\text{F} + 4.00\ \mu\text{F} = 10.0\ \mu\text{F}$.

(b) $q_1 = C_1 V = (6.00\ \mu\text{F})(200\ \text{V}) = 1.20 \times 10^{-3}$ C.

(c) V_1=200 V.

(d) $q_2 = C_2 V = (4.00\ \mu\text{F})(200\ \text{V}) = 8.00 \times 10^{-4}$ C.

(e) $V_2 = V_1 = 200$ V.

73. We cannot expect simple energy conservation to hold since energy is presumably dissipated either as heat in the hookup wires or as radio waves while the charge oscillates in the course of the system "settling down" to its final state (of having 40 V across the parallel pair of capacitors C and 60 μF). We do expect charge to be conserved. Thus, if Q is the charge originally stored on C and q_1, q_2 are the charges on the parallel pair after "settling down," then

$$\begin{aligned} Q &= q_1 + q_2 \\ C(100\,\text{V}) &= C(40\,\text{V}) + (60\,\mu\text{F})(40\,\text{V}) \end{aligned}$$

which leads to the solution $C = 40$ μF.

74. We first need to find an expression for the energy stored in a cylinder of radius R and length L, whose surface lies between the inner and outer cylinders of the capacitor ($a < R < b$). The energy density at any point is given by $u = \frac{1}{2}\varepsilon_0 E^2$, where E is the magnitude of the electric field at that point. If q is the charge on the surface of the inner cylinder, then the magnitude of the electric field at a point a distance r from the cylinder axis is given by

$$E = \frac{q}{2\pi\varepsilon_0 Lr}$$

(see Eq. 25-12), and the energy density at that point is given by

$$u = \frac{1}{2}\varepsilon_0 E^2 = \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2}.$$

The energy in the cylinder is the volume integral

$$U_R = \int u\,d\mathcal{V}.$$

Now, $d\mathcal{V} = 2\pi rL\,dr$, so

$$U_R = \int_a^R \frac{q^2}{8\pi^2\varepsilon_0 L^2 r^2} 2\pi rL\,dr = \frac{q^2}{4\pi\varepsilon_0 L}\int_a^R \frac{dr}{r} = \frac{q^2}{4\pi\varepsilon_0 L}\ln\frac{R}{a}.$$

To find an expression for the total energy stored in the capacitor, we replace R with b:

$$U_b = \frac{q^2}{4\pi\varepsilon_0 L}\ln\frac{b}{a}.$$

We want the ratio U_R/U_b to be 1/2, so

$$\ln\frac{R}{a} = \frac{1}{2}\ln\frac{b}{a}$$

or, since $\frac{1}{2}\ln(b/a) = \ln\left(\sqrt{b/a}\right)$, $\ln(R/a) = \ln\left(\sqrt{b/a}\right)$. This means $R/a = \sqrt{b/a}$ or $R = \sqrt{ab}$.

75. (a) Since the field is constant and the capacitors are in parallel (each with 600 V across them) with identical distances ($d = 0.00300$ m) between the plates, then the field in A is equal to the field in B:

$$\left|\vec{E}\right| = \frac{V}{d} = 2.00\times10^{5}\ \text{V/m}\ .$$

(b) $|\vec{E}| = 2.00\times10^{5}\ \text{V/m}$. See the note in part (a).

(c) For the air-filled capacitor, Eq. 25-4 leads to

$$\sigma = \frac{q}{A} = \varepsilon_0\left|\vec{E}\right| = 1.77\times10^{-6}\ \text{C/m}^2\ .$$

(d) For the dielectric-filled capacitor, we use Eq. 25-29:

$$\sigma = \kappa\varepsilon_0\left|\vec{E}\right| = 4.60\times10^{-6}\ \text{C/m}^2\ .$$

(e) Although the discussion in the textbook (§25-8) is in terms of the charge being held fixed (while a dielectric is inserted), it is readily adapted to this situation (where comparison is made of two capacitors which have the same *voltage* and are identical except for the fact that one has a dielectric). The fact that capacitor B has a relatively large charge but only produces the field that A produces (with its smaller charge) is in line with the point being made (in the text) with Eq. 25-34 and in the material that follows. Adapting Eq. 25-35 to this problem, we see that the difference in charge densities between parts (c) and (d) is due, in part, to the (negative) layer of charge at the top surface of the dielectric; consequently,

$$\sigma' = \left(1.77\times10^{-6}\right) - \left(4.60\times10^{-6}\right) = -2.83\times10^{-6}\ \text{C/m}^2\ .$$

76. (a) The equivalent capacitance is $C_{eq} = C_1C_2/(C_1 + C_2)$. Thus the charge q on each capacitor is

$$q = q_1 = q_2 = C_{eq}V = \frac{C_1C_2V}{C_1 + C_2} = \frac{(2.00\mu\text{F})(8.00\mu\text{F})(300\text{V})}{2.00\mu\text{F} + 8.00\mu\text{F}} = 4.80\times10^{-4}\,\text{C}.$$

(b) The potential difference is $V_1 = q/C_1 = 4.80 \times 10^{-4}$ C/2.0 μF = 240 V.

(c) As noted in part (a), $q_2 = q_1 = 4.80\times10^{-4}\,\text{C}$.

(d) $V_2 = V - V_1 = 300$ V – 240 V = 60.0 V.

Now we have $q'_1/C_1 = q'_2/C_2 = V'$ (V' being the new potential difference across each capacitor) and $q'_1 + q'_2 = 2q$. We solve for q'_1, q'_2 and V:

(e) $q'_1 = \dfrac{2C_1q}{C_1 + C_2} = \dfrac{2(2.00\mu\text{F})(4.80\times10^{-4}C)}{2.00\mu\,\text{F} + 8.00\mu\,\text{F}} = 1.92\times10^{-4}\,\text{C}.$

(f) $V_1' = \dfrac{q_1'}{C_1} = \dfrac{1.92\times10^{-4}\,C}{2.00\,\mu\text{F}} = 96.0\,\text{V}.$

(g) $q'_2 = 2q - q_1 = 7.68\times10^{-4}\,C.$

(h) $V_2' = V_1' = 96.0\,\text{V}.$

(i) In this circumstance, the capacitors will simply discharge themselves, leaving $q_1 = 0$,

(j) $V_1 = 0$,

(k) $q_2 = 0$,

(l) and $V_2 = V_1 = 0$.

77. We use $U = \frac{1}{2}CV^2$. As V is increased by ΔV, the energy stored in the capacitor increases correspondingly from U to $U + \Delta U$: $U + \Delta U = \frac{1}{2}C(V + \Delta V)^2$. Thus, $(1 + \Delta V/V)^2 = 1 + \Delta U/U$, or

$$\frac{\Delta V}{V} = \sqrt{1 + \frac{\Delta U}{U}} - 1 = \sqrt{1 + 10\%} - 1 = 4.9\% \ .$$

78. (a) The voltage across C_1 is 12 V, so the charge is

$$q_1 = C_1 V_1 = 24 \mu\text{C} .$$

(b) We reduce the circuit, starting with C_4 and C_3 (in parallel) which are equivalent to $4\ \mu\text{F}$. This is then in series with C_2, resulting in an equivalence equal to $\frac{4}{3}\mu\text{F}$ which would have 12 V across it. The charge on this $\frac{4}{3}\mu\text{F}$ capacitor (and therefore on C_2) is $(\frac{4}{3}\mu\text{F})(12\,\text{V}) = 16\,\mu\text{C}$. Consequently, the voltage across C_2 is

$$V_2 = \frac{q_2}{C_2} = \frac{16\,\mu\text{C}}{2\,\mu\text{F}} = 8\ \text{V}.$$

This leaves 12 – 8 = 4 V across C_4 (similarly for C_3).

79. We reduce the circuit, starting with C_1 and C_2 (in series) which are equivalent to 4 μF. This is then parallel to C_3 and results in a total of 8 μF, which is now in series with C_4 and can be further reduced. However, the final step in the reduction is not necessary, as we observe that the 8 μF equivalence from the top 3 capacitors has the same capacitance as C_4 and therefore the same voltage; since they are in series, that voltage is then 12/2 = 6.0 V.

80. We use $C = \varepsilon_0 \kappa A/d \propto \kappa/d$. To maximize C we need to choose the material with the greatest value of κ/d. It follows that the mica sheet should be chosen.

81. We may think of this as two capacitors in series C_1 and C_2, the former with the κ_1 = 3.00 material and the latter with the κ_2 = 4.00 material. Upon using Eq. 25-9, Eq. 25-27 and then reducing C_1 and C_2 to an equivalent capacitance (connected directly to the battery) with Eq. 25-20, we obtain

$$C_{eq} = \left(\frac{\kappa_1 \kappa_2}{\kappa_1 + \kappa_2}\right)\frac{\varepsilon_0 A}{d} = 1.52 \times 10^{-10}\ \text{F} \quad .$$

Therefore, $q = C_{eq}V = 1.06 \times 10^{-9}$ C.

82. (a) The length d is effectively shortened by b so $C' = \varepsilon_0 A/(d - b) = 0.708$ pF.

(b) The energy before, divided by the energy after inserting the slab is

$$\frac{U}{U'} = \frac{q^2/2C}{q^2/2C'} = \frac{C'}{C} = \frac{\varepsilon_0 A/(d-b)}{\varepsilon_0 A/d} = \frac{d}{d-b} = \frac{5.00}{5.00-2.00} = 1.67.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{q^2}{2}\left(\frac{1}{C'} - \frac{1}{C}\right) = \frac{q^2}{2\varepsilon_0 A}(d - b - d) = -\frac{q^2 b}{2\varepsilon_0 A} = -5.44 \text{ J}.$$

(d) Since $W < 0$ the slab is sucked in.

83. (a) $C' = \varepsilon_0 A/(d - b) = 0.708$ pF, the same as part (a) in problem 82.

(b) Now,

$$\frac{U}{U'} = \frac{\frac{1}{2}CV^2}{\frac{1}{2}C'V^2} = \frac{C}{C'} = \frac{\varepsilon_0 A/d}{\varepsilon_0 A/(d-b)} = \frac{d-b}{d} = \frac{5.00-2.00}{5.00} = 0.600.$$

(c) The work done is

$$W = \Delta U = U' - U = \frac{1}{2}(C' - C)V^2 = \frac{\varepsilon_0 A}{2}\left(\frac{1}{d-b} - \frac{1}{d}\right)V^2 = \frac{\varepsilon_0 A b V^2}{2d(d-b)} = 1.02\times 10^{-9}\ \text{J}.$$

(d) In Problem 82 where the capacitor is disconnected from the battery and the slab is sucked in, F is certainly given by $-dU/dx$. However, that relation does not hold when the battery is left attached because the force on the slab is not conservative. The charge distribution in the slab causes the slab to be sucked into the gap by the charge distribution on the plates. This action causes an increase in the potential energy stored by the battery in the capacitor.

84. We do not employ energy conservation since, in reaching equilibrium, some energy is dissipated either as heat or radio waves. Charge is conserved; therefore, if $Q = 48\ \mu$C, and q_1 and q_3 are the charges on C_1 and C_3 after the switch is thrown to the right (and equilibrium is reached), then

$$Q = q_1 + q_3.$$

We note that $V_{1\text{ and }2} = V_3$ because of the parallel arrangement, and $V_1 = \frac{1}{2}V_{1\text{ and }2}$ since they are identical capacitors. This leads to

$$2V_1 = V_3$$
$$2\frac{q_1}{C_1} = \frac{q_3}{C_3}$$
$$2q_1 = q_3$$

where the last step follows from multiplying both sides by 2.00 μF. Therefore,

$$Q = q_1 + (2q_1)$$

which yields $q_1 = 16.0\ \mu$C and $q_3 = 32.0\ \mu$C.