1. The amplitude of the induced emf in the loop is

$$\varepsilon_m = A\mu_0 n i_0 \omega = (6.8\times10^{-6}\,\text{m}^2)(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(85400/\text{m})(1.28\text{ A})(212\text{ rad/s})$$
$$=1.98\times10^{-4}\,\text{V}.$$

2. (a) $|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \frac{d}{dt}\left(6.0t^2 + 7.0t\right) = 12t + 7.0 = 12\left(2.0\right) + 7.0 = 31\,\text{mV}.$

(b) Appealing to Lenz's law (especially Fig. 30-5(a)) we see that the current flow in the loop is clockwise. Thus, the current is to left through *R*.

3. (a) We use $\varepsilon = -d\Phi_B/dt = -\pi r^2 dB/dt$. For $0 < t < 2.0$ s:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi(0.12\text{m})^2\left(\frac{0.5\,\text{T}}{2.0\,\text{s}}\right) = -1.1\times10^{-2}\text{ V.}$$

(b) $2.0\text{ s} < t < 4.0\text{ s}$: $\varepsilon \propto dB/dt = 0$.

(c) $4.0\text{ s} < t < 6.0\text{ s}$:

$$\varepsilon = -\pi r^2 \frac{dB}{dt} = -\pi(0.12\text{m})^2\left(\frac{-0.5\text{T}}{6.0\text{s}-4.0\text{s}}\right) = 1.1\times10^{-2}\text{ V.}$$

4. The resistance of the loop is

$$R = \rho \frac{L}{A} = \left(1.69\times10^{-8}\,\Omega\cdot\text{m}\right)\left[\frac{\pi\left(0.10\,\text{m}\right)}{\pi\left(2.5\times10^{-3}\right)^2/4}\right] = 1.1\times10^{-3}\,\Omega.$$

We use $i = |\varepsilon|/R = |d\Phi_B/dt|/R = (\pi r^2/R)|dB/dt|$. Thus

$$\left|\frac{dB}{dt}\right| = \frac{iR}{\pi r^2} = \frac{\left(10\,\text{A}\right)\left(1.1\times10^{-3}\,\Omega\right)}{\pi\left(0.05\,\text{m}\right)^2} = 1.4\ \text{T/s}.$$

5. The total induced emf is given by

$$\begin{aligned}\varepsilon &= -N\frac{d\Phi_B}{dt} = -NA\left(\frac{dB}{dt}\right) = -NA\frac{d}{dt}(\mu_0 ni) = -N\mu_0 nA\frac{di}{dt} = -N\mu_0 n(\pi r^2)\frac{di}{dt} \\ &= -(120)(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})(22000/\mathrm{m})\,\pi\left(0.016\mathrm{m}\right)^2\left(\frac{1.5\ \mathrm{A}}{0.025\ \mathrm{s}}\right) \\ &= 0.16\,\mathrm{V}.\end{aligned}$$

Ohm's law then yields $i = |\varepsilon|/R = 0.016\ \mathrm{V}/5.3\Omega = 0.030\ \mathrm{A}$.

6. Using Faraday's law, the induced emf is

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\frac{d(BA)}{dt} = -B\frac{dA}{dt} = -B\frac{d(\pi r^2)}{dt} = -2\pi r B\frac{dr}{dt}$$

$$= -2\pi(0.12\,\text{m})(0.800\,\text{T})(-0.750\,\text{m/s})$$

$$= 0.452\,\text{V}.$$

7. The flux $\Phi_B = BA\cos\theta$ does not change as the loop is rotated. Faraday's law only leads to a nonzero induced emf when the flux is changing, so the result in this instance is 0.

8. The field (due to the current in the straight wire) is out-of-the-page in the upper half of the circle and is into the page in the lower half of the circle, producing zero net flux, at any time. There is no induced current in the circle.

9. (a) Let L be the length of a side of the square circuit. Then the magnetic flux through the circuit is $\Phi_B = L^2 B/2$, and the induced emf is

$$\varepsilon_i = -\frac{d\Phi_B}{dt} = -\frac{L^2}{2}\frac{dB}{dt}.$$

Now $B = 0.042 - 0.870t$ and $dB/dt = -0.870$ T/s. Thus,

$$\varepsilon_i = \frac{(2.00\ \text{m})^2}{2}(0.870\ \text{T/s}) = 1.74\ \text{V}.$$

The magnetic field is out of the page and decreasing so the induced emf is counterclockwise around the circuit, in the same direction as the emf of the battery. The total emf is

$$\varepsilon + \varepsilon_i = 20.0\ \text{V} + 1.74\ \text{V} = 21.7\ \text{V}.$$

(b) The current is in the sense of the total emf (counterclockwise).

10. Fig. 30-41(b) demonstrates that $\frac{dB}{dt}$ (the slope of that line) is 0.003 T/s. Thus, in absolute value, Faraday's law becomes

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt}$$

where $A = 8 \times 10^{-4}$ m^2. We related the induced emf to resistance and current using Ohm's law. The current is estimated from Fig. 30-41(c) to be $i = \frac{dq}{dt}$ = 0.002 A (the slope of that line). Therefore, the resistance of the loop is

$$R = |\varepsilon| / i = \frac{(8 \text{ x } 10^{-4})(0.003)}{0.002} = 0.0012\ \Omega\ .$$

11. (a) It should be emphasized that the result, given in terms of sin(2π ft), could as easily be given in terms of cos(2π ft) or even cos(2π ft + ϕ) where ϕ is a phase constant as discussed in Chapter 15. The angular position θ of the rotating coil is measured from some reference line (or plane), and which line one chooses will affect whether the magnetic flux should be written as $BA\cos\theta$, $BA\sin\theta$ or $BA\cos(\theta + \phi)$. Here our choice is such that $\Phi_B = BA\cos\theta$. Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ (equivalent to $\theta = 2\pi ft$) if θ is understood to be in radians (and ω would be the angular velocity). Since the area of the rectangular coil is $A=ab$, Faraday's law leads to

$$\varepsilon = -N\frac{d\left(BA\cos\theta\right)}{dt} = -NBA\frac{d\cos\left(2\pi ft\right)}{dt} = N\,Bab2\pi f\sin\left(2\pi ft\right)$$

which is the desired result, shown in the problem statement. The second way this is written ($\varepsilon_0 \sin(2\pi ft)$) is meant to emphasize that the voltage output is sinusoidal (in its time dependence) and has an amplitude of $\varepsilon_0 = 2\pi f N abB$.

(b) We solve ε_0 = 150 V = $2\pi f N abB$ when f = 60.0 rev/s and B = 0.500 T. The three unknowns are N, a, and b which occur in a product; thus, we obtain $N ab$ = 0.796 m^2.

12. (a) Since the flux arises from a dot product of vectors, the result of one sign for B_1 and B_2 and of the opposite sign for B_3 (we choose the minus sign for the flux from B_1 and B_2, and therefore a plus sign for the flux from B_3). The induced emf is

$$\varepsilon = -\Sigma \frac{d\Phi_B}{dt} = A\left(\frac{dB_1}{dt} + \frac{dB_2}{dt} - \frac{dB_3}{dt}\right)$$
$$=(0.10\text{ m})(0.20\text{ m})\left(2.0\times10^{-6}\text{ T/s} + 1.0\times10^{-6}\text{ T/s} - 5.0\times10^{-6}\text{ T/s}\right)$$
$$= -4.0\times10^{-8}\text{ V}.$$

The minus sign meaning that the effect is dominated by the changes in B_3. Its magnitude (using Ohm's law) is $|\varepsilon|/R = 8.0\ \mu\text{A}$.

(b) Consideration of Lenz's law leads to the conclusion that the induced current is therefore counterclockwise.

13. The amount of charge is

$$q(t)=\frac{1}{R}[\Phi_B(0)-\Phi_B(t)]=\frac{A}{R}[B(0)-B(t)]=\frac{1.20\times10^{-3}\,\mathrm{m}^2}{13.0\,\Omega}[1.60\,\mathrm{T}-(-1.60\,\mathrm{T})]$$

$$=2.95\times10^{-2}\,\mathrm{C}\,.$$

14. We note that 1 gauss = 10^{-4} T. The amount of charge is

$$q(t)=\frac{N}{R}[BA\cos 20°-(-BA\cos 20°)]=\frac{2NBA\cos 20°}{R}$$
$$=\frac{2(1000)(0.590\times10^{-4}\text{ T})\pi(0.100\text{ m})^2(\cos 20°)}{85.0\,\Omega+140\,\Omega}=1.55\times10^{-5}\text{ C}.$$

Note that the axis of the coil is at 20°, not 70°, from the magnetic field of the Earth.

15. (a) The frequency is

$$f=\frac{\omega}{2\pi}=\frac{(40 \text{ rev/s})(2\pi \text{ rad/rev})}{2\pi}=40 \text{ Hz}.$$

(b) First, we define angle relative to the plane of Fig. 30-44, such that the semicircular wire is in the $\theta = 0$ position and a quarter of a period (of revolution) later it will be in the $\theta = \pi/2$ position (where its midpoint will reach a distance of a above the plane of the figure). At the moment it is in the $\theta = \pi/2$ position, the area enclosed by the "circuit" will appear to us (as we look down at the figure) to that of a simple rectangle (call this area A_0 which is the area it will again appear to enclose when the wire is in the $\theta = 3\pi/2$ position). Since the area of the semicircle is $\pi a^2/2$ then the area (as it appears to us) enclosed by the circuit, as a function of our angle θ, is

$$A = A_0 + \frac{\pi a^2}{2}\cos\theta$$

where (since θ is increasing at a steady rate) the angle depends linearly on time, which we can write either as $\theta = \omega t$ or $\theta = 2\pi f t$ if we take $t = 0$ to be a moment when the arc is in the $\theta = 0$ position. Since $\vec{B}$ is uniform (in space) and constant (in time), Faraday's law leads to

$$\varepsilon = -\frac{d\Phi_B}{dt} = -B\frac{dA}{dt} = -B\frac{d\left(A_0 + \frac{\pi a^2}{2}\cos\theta\right)}{dt} = -B\frac{\pi a^2}{2}\frac{d\cos(2\pi f t)}{dt}$$

which yields $\varepsilon = B\pi^2 a^2 f \sin(2\pi f t)$. This (due to the sinusoidal dependence) reinforces the conclusion in part (a) and also (due to the factors in front of the sine) provides the voltage amplitude:

$$\varepsilon_m = B\pi^2 a^2 f = (0.020 \text{ T})\pi^2 (0.020 \text{ m})^2 (40/\text{s}) = 3.2\times10^{-3}\text{ V}.$$

16. To have an induced emf, the magnetic field must be perpendicular (or have a nonzero component perpendicular) to the coil, and must be changing with time.

(a) For $\vec{B} = (4.00\times10^{-2}\,\text{T/m})y\hat{\text{k}}$, $dB/dt = 0$ and hence $\varepsilon = 0$.

(b) None.

(c) For $\vec{B} = (6.00\times10^{-2}\,\text{T/s})t\hat{\text{k}}$,

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -(0.400\text{ m} \times 0.250\text{ m})(0.0600\text{ T/s}) = -6.00\text{ mV},$$

or $|\varepsilon| = 6.00$ mV.

(d) Clockwise;

(e) For $\vec{B} = (8.00\times10^{-2}\,\text{T/m}\cdot\text{s})yt\hat{\text{k}}$,

$$\Phi_B = (0.400)(0.0800t)\int y\,dy = 1.00\times10^{-3}\,t,$$

in SI units. The induced emf is $\varepsilon = -d\Phi B/dt = -1.00$ mV, or $|\varepsilon| = 1.00$ mV.

(f) Clockwise.

(g) $\Phi_B = 0 \;\Rightarrow\; \varepsilon = 0$.

(h) None.

(i) $\Phi_B = 0 \;\Rightarrow\; \varepsilon = 0$

(j) None.

17. First we write $\Phi_B = BA \cos \theta$. We note that the angular position θ of the rotating coil is measured from some reference line or plane, and we are implicitly making such a choice by writing the magnetic flux as $BA \cos \theta$ (as opposed to, say, $BA \sin \theta$). Since the coil is rotating steadily, θ increases linearly with time. Thus, $\theta = \omega t$ if θ is understood to be in radians (here, $\omega = 2\pi f$ is the angular velocity of the coil in radians per second, and f = 1000 rev/min ≈ 16.7 rev/s is the frequency). Since the area of the rectangular coil is A = 0.500 × 0.300 = 0.150 m^2, Faraday's law leads to

$$\varepsilon = -N\frac{d(BA\cos\theta)}{dt} = -NBA\frac{d\cos(2\pi ft)}{dt} = NBA2\pi f\,\sin(2\pi ft)$$

which means it has a voltage amplitude of

$$\varepsilon_{\max} = 2\pi fNAB = 2\pi(16.7\,\text{rev/s})(100\,\text{turns})(0.15\,\text{m}^2)(3.5\,\text{T}) = 5.50\times10^3\ \text{V}\ .$$

18. (a) Since $\vec{B} = B\hat{\text{i}}$ uniformly, then only the area "projected" onto the *yz* plane will contribute to the flux (due to the scalar [dot] product). This "projected" area corresponds to one-fourth of a circle. Thus, the magnetic flux Φ_B through the loop is

$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \frac{1}{4}\pi r^2 B \ .$$

Thus,

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = \left|\frac{d}{dt}\left(\frac{1}{4}\pi r^2 B\right)\right| = \frac{\pi r^2}{4}\left|\frac{dB}{dt}\right|$$
$$= \frac{1}{4}\pi(0.10\,\text{m})^2(3.0\times 10^{-3}\,\text{T/s}) = 2.4\times 10^{-5}\,\text{V} \ .$$

(b) We have a situation analogous to that shown in Fig. 30-5(a). Thus, the current in segment *bc* flows from *c* to *b* (following Lenz's law).

19. (a) In the region of the smaller loop the magnetic field produced by the larger loop may be taken to be uniform and equal to its value at the center of the smaller loop, on the axis. Eq. 29-27, with $z = x$ (taken to be much greater than R), gives

$$\vec{B} = \frac{\mu_0 i R^2}{2x^3}\,\hat{\mathrm{i}}$$

where the $+x$ direction is upward in Fig. 30-47. The magnetic flux through the smaller loop is, to a good approximation, the product of this field and the area (πr^2) of the smaller loop:

$$\Phi_B = \frac{\pi \mu_0 i r^2 R^2}{2x^3}.$$

(b) The emf is given by Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\frac{d}{dt}\left(\frac{1}{x^3}\right) = -\left(\frac{\pi\mu_0 i r^2 R^2}{2}\right)\left(-\frac{3}{x^4}\frac{dx}{dt}\right) = \frac{3\pi\mu_0 i r^2 R^2 v}{2x^4}.$$

(c) As the smaller loop moves upward, the flux through it decreases, and we have a situation like that shown in Fig. 30-5(b). The induced current will be directed so as to produce a magnetic field that is upward through the smaller loop, in the same direction as the field of the larger loop. It will be counterclockwise as viewed from above, in the same direction as the current in the larger loop.

20. Since $\frac{d\cos\phi}{dt} = -\frac{d\phi}{dt}\sin\phi$, Faraday's law (with $N = 1$) becomes (in absolute value)

$$\varepsilon = -\frac{d\Phi_B}{dt} = -BA\,\frac{d\phi}{dt}\sin\phi$$

which yields $|\varepsilon| = 0.018$ V.

21. (a) Eq. 29-10 gives the field at the center of the large loop with R = 1.00 m and current $i(t)$. This is approximately the field throughout the area (A = 2.00 × 10^{-4} m^2) enclosed by the small loop. Thus, with $B = \mu_0 i/2R$ and $i(t) = i_0 + kt$, where i_0 = 200 A and

$$k = (-200\text{ A} - 200\text{ A})/1.00\text{ s} = -400\text{ A/s},$$

we find

(a) $B(t=0) = \dfrac{\mu_0 i_0}{2R} = \dfrac{(4\pi\times10^{-7}\text{ H/m})(200\text{A})}{2(1.00\text{m})} = 1.26\times10^{-4}\text{ T},$

(b) $B(t=0.500\text{s}) = \dfrac{(4\pi\times10^{-7}\text{ H/m})[200\text{A}-(400\text{A/s})(0.500\text{s})]}{2(1.00\text{m})} = 0.$

(c) $B(t=1.00\text{s}) = \dfrac{(4\pi\times10^{-7}\text{ H/m})[200\text{A}-(400\text{A/s})(1.00\text{s})]}{2(1.00\text{m})} = -1.26\times10^{-4}\text{ T},$

or $|B(t=1.00\text{s})| = 1.26\times10^{-4}\text{ T}.$

(d) yes, as indicated by the flip of sign of B(t) in (c).

(e) Let the area of the small loop be a. Then $\Phi_B = Ba$, and Faraday's law yields

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d(Ba)}{dt} = -a\frac{dB}{dt} = -a\left(\frac{\Delta B}{\Delta t}\right)\\ &= -(2.00\times10^{-4}\text{m}^2)\left(\frac{-1.26\times10^{-4}\text{ T}-1.26\times10^{-4}\text{ T}}{1.00\text{ s}}\right)\\ &= 5.04\times10^{-8}\text{ V}.\end{aligned}$$

22. (a) First, we observe that a large portion of the figure contributes flux which "cancels out." The field (due to the current in the long straight wire) through the part of the rectangle above the wire is out of the page (by the right-hand rule) and below the wire it is into the page. Thus, since the height of the part above the wire is $b - a$, then a strip below the wire (where the strip borders the long wire, and extends a distance $b - a$ away from it) has exactly the equal-but-opposite flux which cancels the contribution from the part above the wire. Thus, we obtain the non-zero contributions to the flux:

$$\Phi_B = \int B\,dA = \int_{b-a}^{a}\left(\frac{\mu_0 i}{2\pi r}\right)(b\,dr) = \frac{\mu_0 i b}{2\pi}\ln\left(\frac{a}{b-a}\right).$$

Faraday's law, then, (with SI units and 3 significant figures understood) leads to

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left[\frac{\mu_0 i b}{2\pi}\ln\left(\frac{a}{b-a}\right)\right] = -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{di}{dt}\\ &= -\frac{\mu_0 b}{2\pi}\ln\left(\frac{a}{b-a}\right)\frac{d}{dt}\left(\frac{9}{2}t^2 - 10t\right)\\ &= \frac{-\mu_0 b(9t-10)}{2\pi}\ln\left(\frac{a}{b-a}\right).\end{aligned}$$

With $a = 0.120$ m and $b = 0.160$ m, then, at $t = 3.00$ s, the magnitude of the emf induced in the rectangular loop is

$$|\varepsilon| = \frac{(4\pi\times10^{-7})(0.16)(9(3)-10)}{2\pi}\ln\left(\frac{0.12}{0.16-0.12}\right) = 5.98\times10^{-7}\text{ V}.$$

(b) We note that $di/dt > 0$ at $t = 3$ s. The situation is roughly analogous to that shown in Fig. 30-5(c). From Lenz's law, then, the induced emf (hence, the induced current) in the loop is counterclockwise.

23. (a) Consider a (thin) strip of area of height dy and width $\ell = 0.020\,\text{m}$. The strip is located at some $0 < y < \ell$. The element of flux through the strip is

$$d\Phi_B = B\,dA = \left(4t^2 y\right)\left(\ell\,dy\right)$$

where SI units (and 2 significant figures) are understood. To find the total flux through the square loop, we integrate:

$$\Phi_B = \int d\Phi_B = \int_0^\ell \left(4t^2 y\ell\right)dy = 2t^2\ell^3 \ .$$

Thus, Faraday's law yields

$$|\varepsilon| = \left|\frac{d\Phi_B}{dt}\right| = 4t\ell^3 \ .$$

At $t = 2.5$ s, we find the magnitude of the induced emf is 8.0×10^{-5} V.

(b) Its "direction" (or "sense'') is clockwise, by Lenz's law.

24. (a) We assume the flux is entirely due to the field generated by the long straight wire (which is given by Eq. 29-17). We integrate according to Eq. 30-1, not worrying about the possibility of an overall minus sign since we are asked to find the absolute value of the flux.

$$|\Phi_B| = \int_{r-b/2}^{r+b/2} \left(\frac{\mu_0 i}{2\pi r}\right)(a\,dr) = \frac{\mu_0 i a}{2\pi} \ln\left(\frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right).$$

When $r = 1.5b$, we have

$$|\Phi_B| = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.7\text{A})(0.022\text{m})}{2\pi}\ln(2.0) = 1.4\times10^{-8}\,\text{Wb}.$$

(b) Implementing Faraday's law involves taking a derivative of the flux in part (a), and recognizing that $\frac{dr}{dt} = v$. The magnitude of the induced emf divided by the loop resistance then gives the induced current:

$$\begin{aligned} i_{\text{loop}} &= \left|\frac{\varepsilon}{R}\right| = -\frac{\mu_0 i a}{2\pi R}\left|\frac{d}{dt}\ln\left(\frac{r+\frac{b}{2}}{r-\frac{b}{2}}\right)\right| = \frac{\mu_0 i a b v}{2\pi R[r^2-(b/2)^2]} \\ &= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(4.7\text{A})(0.022\text{m})(0.0080\text{m})(3.2\times10^{-3}\,\text{m/s})}{2\pi(4.0\times10^{-4}\,\Omega)[2(0.0080\text{m})^2]} \\ &= 1.0\times10^{-5}\,\text{A}. \end{aligned}$$

25. (a) We refer to the (very large) wire length as L and seek to compute the flux per meter: Φ_B/L. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of anti-parallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at what we will call $x = \ell/2$, where $\ell = 20\,\text{mm} = 0.020\,\text{m}$); the net field at any point $0 < x < \ell/2$ is the same at its "mirror image" point $\ell - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = \ell$. We make use of the symmetry by integrating over $0 < x < \ell/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{\ell/2} B\,dA = 2\int_0^{d/2} B\left(L\,dx\right) + 2\int_{d/2}^{\ell/2} B\left(L\,dx\right)$$

where $d = 0.0025$ m is the diameter of each wire. We will use $R = d/2$, and r instead of x in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{L} &= 2\int_0^R \left(\frac{\mu_0 i}{2\pi R^2} r + \frac{\mu_0 i}{2\pi(\ell - r)}\right) dr + 2\int_R^{\ell/2} \left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(\ell - r)}\right) dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{\ell - R}{\ell}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{\ell - R}{R}\right) \\ &= 0.23\times10^{-5}\,\text{T}\cdot\text{m} + 1.08\times10^{-5}\,\text{T}\cdot\text{m}\end{aligned}$$

which yields $\Phi_B/L = 1.3 \times 10^{-5}$ T·m or 1.3×10^{-5} Wb/m.

(b) The flux (per meter) existing within the regions of space occupied by one or the other wires was computed above to be 0.23×10^{-5} T·m. Thus,

$$\frac{0.23\times10^{-5}\,\text{T}\cdot\text{m}}{1.3\times10^{-5}\,\text{T}\cdot\text{m}} = 0.17 = 17\%\,.$$

(c) What was described in part (a) as a symmetry plane at $x = \ell/2$ is now (in the case of parallel currents) a plane of vanishing field (the fields subtract from each other in the region between them, as the right-hand rule shows). The flux in the $0 < x < \ell/2$ region is now of opposite sign of the flux in the $\ell/2 < x < \ell$ region which causes the total flux (or, in this case, flux per meter) to be zero.

26. Noting that $|\Delta B| = B$, we find the thermal energy is

$$P_{\text{thermal}}\Delta t = \frac{\varepsilon^2 \Delta t}{R} = \frac{1}{R}\left(-\frac{d\Phi_B}{dt}\right)^2 \Delta t = \frac{1}{R}\left(-A\frac{\Delta B}{\Delta t}\right)^2 \Delta t = \frac{A^2 B^2}{R\Delta t}$$
$$= \frac{(2.00\times10^{-4}\,\text{m}^2)^2(17.0\times10^{-6}\,\text{T})^2}{(5.21\times10^{-6}\,\Omega)(2.96\times10^{-3}\,\text{s})}$$
$$= 7.50\times10^{-10}\,\text{J}.$$

27. Thermal energy is generated at the rate $P = \varepsilon^2/R$ (see Eq. 27-23). Using Eq. 27-16, the resistance is given by $R = \rho L/A$, where the resistivity is 1.69×10^{-8} Ω·m (by Table 27-1) and $A = \pi d^2/4$ is the cross-sectional area of the wire (d = 0.00100 m is the wire thickness). The area *enclosed* by the loop is

$$A_{\text{loop}} = \pi r_{\text{loop}}^2 = \pi\left(\frac{L}{2\pi}\right)^2$$

since the length of the wire (L = 0.500 m) is the circumference of the loop. This enclosed area is used in Faraday's law (where we ignore minus signs in the interest of finding the magnitudes of the quantities):

$$\varepsilon = \frac{d\Phi_B}{dt} = A_{\text{loop}}\frac{dB}{dt} = \frac{L^2}{4\pi}\frac{dB}{dt}$$

where the rate of change of the field is dB/dt = 0.0100 T/s. Consequently, we obtain

$$P = \frac{\left(\frac{L^2}{4\pi}\frac{dB}{dt}^2\right)}{4\rho L/\pi d^2} = \frac{d^2 L^3}{64\pi\rho}\left(\frac{dB}{dt}\right)^2 = 3.68\times10^{-6}\,\text{W} .$$

28. Eq. 27-23 gives ε^2/R as the rate of energy transfer into thermal forms (dE_{th}/dt, which, from Fig. 30-51(c), is roughly 40 nJ/s). Interpreting ε as the induced emf (in absolute value) in the single-turn loop ($N = 1$) from Faraday's law, we have

$$\varepsilon = \frac{d\Phi_B}{dt} = A\frac{dB}{dt} .$$

Eq. 29-23 gives $B = \mu_o ni$ for the solenoid (and note that the field is zero outside of the solenoid – which implies that $A = A_{coil}$), so our expression for the magnitude of the induced emf becomes

$$\varepsilon = \mu_o\, n\, A_{coil}\frac{di_{coil}}{dt}$$

where Fig. 30-51(b) suggests that $di_{coil}/dt = 0.5$ A/s. With $n = 8000$ (in SI units) and $A_{coil} = \pi(0.02)^2$ (note that the loop radius does not come into the computations of this problem, just the coil's), we find V = 6.3 microvolts. Returning to our earlier observations, we can now solve for the resistance: $R = \varepsilon^2/(dE_{th}/dt) = 1.0$ mΩ.

29. (a) Eq. 30-8 leads to

$$\varepsilon = BLv = (0.350 \text{ T})(0.250 \text{ m})(0.55 \text{ m/s}) = 0.0481 \text{ V} .$$

(b) By Ohm's law, the induced current is i = 0.0481 V/18.0 Ω = 0.00267 A. By Lenz's law, the current is clockwise in Fig. 30-52.

(c) Eq. 26-22 leads to $P = i^2R$ = 0.000129 W.

30. Noting that $F_{\text{net}} = BiL - mg = 0$, we solve for the current:

$$i = \frac{mg}{BL} = \frac{|\varepsilon|}{R} = \frac{1}{R}\left|\frac{d\Phi_B}{dt}\right| = \frac{B}{R}\left|\frac{dA}{dt}\right| = \frac{Bv_t L}{R} ,$$

which yields $v_t = mgR/B^2L^2$.

31. (a) Eq. 30-8 leads to

$$\varepsilon = BLv = (1.2\,\text{T})(0.10\ \text{m})(5.0\ \text{m/s}) = 0.60\ \text{V}\ .$$

(b) By Lenz's law, the induced emf is clockwise. In the rod itself, we would say the emf is directed up the page.

(c) By Ohm's law, the induced current is i = 0.60 V/0.40 Ω = 1.5 A.

(d) The direction is clockwise.

(e) Eq. 27-22 leads to $P = i^2R$ = 0.90 W.

(f) From Eq. 29-2, we find that the force on the rod associated with the uniform magnetic field is directed rightward and has magnitude

$$F = iLB = (1.5\ \text{A})(0.10\ \text{m})(1.2\ \text{T}) = 0.18\ \text{N}\ .$$

To keep the rod moving at constant velocity, therefore, a leftward force (due to some external agent) having that same magnitude must be continuously supplied to the rod.

(g) Using Eq. 7-48, we find the power associated with the force being exerted by the external agent: $P = Fv$ = (0.18 N)(5.0 m/s) = 0.90 W, which is the same as our result from part (e).

32. (a) The "height" of the triangular area enclosed by the rails and bar is the same as the distance traveled in time v: $d = vt$, where $v = 5.20$ m/s. We also note that the "base" of that triangle (the distance between the intersection points of the bar with the rails) is $2d$. Thus, the area of the triangle is

$$A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(2vt)(vt) = v^2t^2 \ .$$

Since the field is a uniform $B = 0.350$ T, then the magnitude of the flux (in SI units) is

$$\Phi_B = BA = (0.350)(5.20)^2t^2 = 9.46t^2.$$

At $t = 3.00$ s, we obtain $\Phi_B = 85.2$ Wb.

(b) The magnitude of the emf is the (absolute value of) Faraday's law:

$$\varepsilon = \frac{d\Phi_B}{dt} = 9.46\frac{dt^2}{dt} = 18.9t$$

in SI units. At $t = 3.00$ s, this yields $\varepsilon = 56.8$ V.

(c) Our calculation in part (b) shows that $n = 1$.

33. (a) Letting x be the distance from the right end of the rails to the rod, we find an expression for the magnetic flux through the area enclosed by the rod and rails. By Eq. 29-17, the field is $B = \mu_0 i/2\pi r$, where r is the distance from the long straight wire. We consider an infinitesimal horizontal strip of length x and width dr, parallel to the wire and a distance r from it; it has area $A = x\,dr$ and the flux $d\Phi_B = (\mu_0 ix/2\pi r)dr$. By Eq. 30-1, the total flux through the area enclosed by the rod and rails is

$$\Phi_B = \frac{\mu_0 ix}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 ix}{2\pi}\ln\left(\frac{a+L}{a}\right).$$

According to Faraday's law the emf induced in the loop is

$$\begin{aligned}\varepsilon &= \frac{d\Phi_B}{dt} = \frac{\mu_0 i}{2\pi}\frac{dx}{dt}\ln\left(\frac{a+L}{a}\right) = \frac{\mu_0 iv}{2\pi}\ln\left(\frac{a+L}{a}\right)\\ &= \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(100\,\text{A})(5.00\,\text{m/s})}{2\pi}\ln\left(\frac{1.00\,\text{cm}+10.0\,\text{cm}}{1.00\,\text{cm}}\right) = 2.40\times10^{-4}\,\text{V}.\end{aligned}$$

(b) By Ohm's law, the induced current is

$$i_\ell = \varepsilon/R = (2.40\times10^{-4}\,\text{V})/(0.400\,\Omega) = 6.00\times10^{-4}\,\text{A}.$$

Since the flux is increasing the magnetic field produced by the induced current must be into the page in the region enclosed by the rod and rails. This means the current is clockwise.

(c) Thermal energy is being generated at the rate

$$P = i_\ell^2 R = (6.00\times10^{-4}\,\text{A})^2(0.400\,\Omega) = 1.44\times10^{-7}\,\text{W}.$$

(d) Since the rod moves with constant velocity, the net force on it is zero. The force of the external agent must have the same magnitude as the magnetic force and must be in the opposite direction. The magnitude of the magnetic force on an infinitesimal segment of the rod, with length dr at a distance r from the long straight wire, is

$$dF_B = i_\ell B\,dr = (\mu_0 i_\ell i/2\pi r)\,dr.$$

We integrate to find the magnitude of the total magnetic force on the rod:

$$
\begin{aligned}
F_B &= \frac{\mu_0 i_\ell i}{2\pi}\int_a^{a+L}\frac{dr}{r} = \frac{\mu_0 i_\ell i}{2\pi}\ln\left(\frac{a+L}{a}\right) \\
&= \frac{\left(4\pi\times10^{-7}\,\mathrm{T\cdot m/A}\right)\left(6.00\times10^{-4}\,\mathrm{A}\right)\left(100\,\mathrm{A}\right)}{2\pi}\ln\left(\frac{1.00\,\mathrm{cm}+10.0\,\mathrm{cm}}{1.00\,\mathrm{cm}}\right) \\
&= 2.87\times10^{-8}\ \mathrm{N}.
\end{aligned}
$$

Since the field is out of the page and the current in the rod is upward in the diagram, the force associated with the magnetic field is toward the right. The external agent must therefore apply a force of 2.87×10^{-8} N, to the left.

(e) By Eq. 7-48, the external agent does work at the rate

$$P = Fv = (2.87 \times 10^{-8}\ \mathrm{N})(5.00\ \mathrm{m/s}) = 1.44 \times 10^{-7}\ \mathrm{W}.$$

This is the same as the rate at which thermal energy is generated in the rod. All the energy supplied by the agent is converted to thermal energy.

34. (a) For path 1, we have

$$\oint_1 \vec{E}\cdot d\vec{s} = -\frac{d\vec{\Phi}_{B1}}{dt} = \frac{d}{dt}\left(B_1 A_1\right) = A_1 \frac{dB_1}{dt} = \pi r_1^2 \frac{dB_1}{dt} = \pi\left(0.200\,\text{m}\right)^2\left(-8.50\times10^{-3}\ \text{T/s}\right)$$
$$= -1.07\times10^{-3}\ \text{V}$$

(b) For path 2, the result is

$$\oint_2 \vec{E}\cdot d\vec{s} = -\frac{d\vec{\Phi}_{B2}}{dt} = \pi r_2^2 \frac{dB_2}{dt} = \pi\left(0.300\,\text{m}\right)^2\left(-8.50\times10^{-3}\ \text{T/s}\right) = -2.40\times10^{-3}\ \text{V}$$

(c) For path 3, we have

$$\oint_3 \vec{E}\cdot d\vec{s} = \oint_1 \vec{E}\cdot d\vec{s} - \oint_2 \vec{E}\cdot d\vec{s} = -1.07\times10^{-3}\ \text{V} - \left(-2.4\times10^{-3}\ \text{V}\right) = 1.33\times10^{-3}\ \text{V}$$

35. (a) The point at which we are evaluating the field is inside the solenoid, so Eq. 30-25 applies. The magnitude of the induced electric field is

$$E = \frac{1}{2}\frac{dB}{dt}r = \frac{1}{2}\left(6.5\times10^{-3}\ \text{T/s}\right)\left(0.0220\ \text{m}\right) = 7.15\times10^{-5}\ \text{V/m}.$$

(b) Now the point at which we are evaluating the field is outside the solenoid and Eq. 30-27 applies. The magnitude of the induced field is

$$E = \frac{1}{2}\frac{dB}{dt}\frac{R^2}{r} = \frac{1}{2}\left(6.5\times10^{-3}\ \text{T/s}\right)\frac{\left(0.0600\ \text{m}\right)^2}{\left(0.0820\ \text{m}\right)} = 1.43\times10^{-4}\ \text{V/m}.$$

36. From the "kink" in the graph of Fig. 30-57, we conclude that the radius of the circular region is 2.0 cm. For values of r less than that, we have (from the absolute value of Eq. 30-20)

$$E(2\pi\, r\,) = \frac{d\Phi_B}{dt} = A\,\frac{dB}{dt} = \pi r^2\,\frac{dB}{dt} = \pi r^2 a\, t$$

which means that $E/r = a/2$. This corresponds to the slope of that graph (the linear portion for small values of r) which we estimate to be 0.015 (in SI units). Thus, $a = 0.030$ T/s.

37. The magnetic field B can be expressed as

$$B(t) = B_0 + B_1 \sin(\omega t + \phi_0),$$

where B_0 = (30.0 T + 29.6 T)/2 = 29.8 T and B_1 = (30.0 T – 29.6 T)/2 = 0.200 T. Then from Eq. 30-25

$$E = \frac{1}{2}\left(\frac{dB}{dt}\right)r = \frac{r}{2}\frac{d}{dt}\left[B_0 + B_1 \sin(\omega t + \phi_0)\right] = \frac{1}{2}B_1\omega r \cos(\omega t + \phi_0).$$

We note that $\omega = 2\pi f$ and that the factor in front of the cosine is the maximum value of the field. Consequently,

$$E_{\max} = \frac{1}{2}B_1(2\pi f)r = \frac{1}{2}(0.200\,\mathrm{T})(2\pi)(15\,\mathrm{Hz})(1.6\times10^{-2}\ \mathrm{m}) = 0.15\ \mathrm{V/m}.$$

38. (a) We interpret the question as asking for N multiplied by the flux through one turn:

$$\Phi_{\text{turns}} = N\Phi_B = NBA = NB\left(\pi r^2\right) = (30.0)\left(2.60\times10^{-3}\ \text{T}\right)(\pi)(0.100\,\text{m})^2 = 2.45\times10^{-3}\ \text{Wb}.$$

(b) Eq. 30-33 leads to

$$L = \frac{N\Phi_B}{i} = \frac{2.45\times10^{-3}\ \text{Wb}}{3.80\,\text{A}} = 6.45\times10^{-4}\ \text{H}.$$

39. Since $N\Phi_B = Li$, we obtain

$$\Phi_B = \frac{Li}{N} = \frac{\left(8.0\times10^{-3}\ \text{H}\right)\left(5.0\times10^{-3}\ \text{A}\right)}{400} = 1.0\times10^{-7}\,\text{Wb}.$$

40. (a) We imagine dividing the one-turn solenoid into N small circular loops placed along the width W of the copper strip. Each loop carries a current $\Delta i = i/N$. Then the magnetic field inside the solenoid is

$$B = \mu_0 n\Delta i = \mu_0(N/W)(i/N) = \mu_0 i/W = \frac{(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.035\text{A})}{0.16\text{m}} = 2.7\times10^{-7}\,\text{T}.$$

(b) Eq. 30-33 leads to

$$L = \frac{\Phi_B}{i} = \frac{\pi R^2 B}{i} = \frac{\pi R^2\left(\mu_0 i/W\right)}{i} = \frac{\pi\mu_0 R^2}{W} = \frac{\pi(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(0.018\text{m})^2}{0.16\text{m}} = 8.0\times10^{-9}\,\text{H}.$$

41. We refer to the (very large) wire length as ℓ and seek to compute the flux per meter: Φ_B / ℓ. Using the right-hand rule discussed in Chapter 29, we see that the net field in the region between the axes of antiparallel currents is the addition of the magnitudes of their individual fields, as given by Eq. 29-17 and Eq. 29-20. There is an evident reflection symmetry in the problem, where the plane of symmetry is midway between the two wires (at $x = d/2$); the net field at any point $0 < x < d/2$ is the same at its "mirror image" point $d - x$. The central axis of one of the wires passes through the origin, and that of the other passes through $x = d$. We make use of the symmetry by integrating over $0 < x < d/2$ and then multiplying by 2:

$$\Phi_B = 2\int_0^{d/2} B\,dA = 2\int_0^a B(\ell\,dx) + 2\int_a^{d/2} B(\ell\,dx)$$

where $d = 0.0025$ m is the diameter of each wire. We will use r instead of x in the following steps. Thus, using the equations from Ch. 29 referred to above, we find

$$\begin{aligned}\frac{\Phi_B}{\ell} &= 2\int_0^a \left(\frac{\mu_0 i}{2\,\pi a^2} r + \frac{\mu_0 i}{2\pi(d-r)}\right) dr + 2\int_a^{d/2} \left(\frac{\mu_0 i}{2\pi r} + \frac{\mu_0 i}{2\pi(d-r)}\right) dr \\ &= \frac{\mu_0 i}{2\pi}\left(1 - 2\ln\left(\frac{d-a}{d}\right)\right) + \frac{\mu_0 i}{\pi}\ln\left(\frac{d-a}{a}\right)\end{aligned}$$

where the first term is the flux within the wires and will be neglected (as the problem suggests). Thus, the flux is approximately $\Phi_B \approx \mu_0 i\ell/\pi \ln((d-a)/a)$. Now, we use Eq. 30-33 (with $N = 1$) to obtain the inductance per unit length:

$$\frac{L}{\ell} = \frac{\Phi_B}{\ell i} = \frac{\mu_0}{\pi}\ln\left(\frac{d-a}{a}\right) = \frac{(4\pi\times10^{-7}\,\mathrm{T\cdot m/A})}{\pi}\ln\left(\frac{142-1.53}{1.53}\right) = 1.81\times10^{-6}\,\mathrm{H/m}.$$

42. (a) Speaking anthropomorphically, the coil wants to fight the changes—so if it wants to push current rightward (when the current is already going rightward) then i must be in the process of decreasing.

(b) From Eq. 30-35 (in absolute value) we get

$$L=\left|\frac{\varepsilon}{di/dt}\right|=\frac{17\,\text{V}}{2.5\,\text{kA/s}}=6.8\times10^{-4}\ \text{H}.$$

43. Since $\varepsilon = -L(di/dt)$, we may obtain the desired induced emf by setting

$$\frac{di}{dt} = -\frac{\varepsilon}{L} = -\frac{60\,\text{V}}{12\,\text{H}} = -5.0\,\text{A/s},$$

or $|di/dt| = 5.0\,\text{A/s}$. We might, for example, uniformly reduce the current from 2.0 A to zero in 40 ms.

44. During periods of time when the current is varying linearly with time, Eq. 30-35 (in absolute values) becomes $|\varepsilon| = L|\Delta i/\Delta t|$. For simplicity, we omit the absolute value signs in the following.

(a) For $0 < t < 2$ ms

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(7.0\,\text{A}-0)}{2.0\times10^{-3}\,\text{s}} = 1.6\times10^{4}\,\text{V}.$$

(b) For 2 ms $< t <$ 5 ms

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(5.0\,\text{A}-7.0\,\text{A})}{(5.0-2.0)10^{-3}\,\text{s}} = 3.1\times10^{3}\,\text{V}.$$

(c) For 5 ms $< t <$ 6 ms

$$\varepsilon = L\frac{\Delta i}{\Delta t} = \frac{(4.6\,\text{H})(0-5.0\,\text{A})}{(6.0-5.0)10^{-3}\,\text{s}} = 2.3\times10^{4}\,\text{V}.$$

45. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Since the (independent) voltages for series elements add ($V_1 + V_2$), then inductances in series must *add*, $L_{eq} = L_1 + L_2$, just as was the case for resistances. Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §30-12). The requirement is that magnetic field lines from one inductor should not have significant presence in any other.

(b) Just as with resistors, $L_{eq} = \sum_{n=1}^{N} L_n$.

46. (a) Voltage is proportional to inductance (by Eq. 30-35) just as, for resistors, it is proportional to resistance. Now, the (independent) voltages for parallel elements are equal ($V_1 = V_2$), and the currents (which are generally functions of time) add ($i_1(t) + i_2(t) = i(t)$). This leads to the Eq. 27-21 for resistors. We note that this condition on the currents implies

$$\frac{di_1(t)}{dt} + \frac{di_2(t)}{dt} = \frac{di(t)}{dt}.$$

Thus, although the inductance equation Eq. 30-35 involves the rate of change of current, as opposed to current itself, the conditions that led to the parallel resistor formula also applies to inductors. Therefore,

$$\frac{1}{L_{\text{eq}}} = \frac{1}{L_1} + \frac{1}{L_2}.$$

Note that to ensure the independence of the voltage values, it is important that the inductors not be too close together (the related topic of mutual inductance is treated in §30-12). The requirement is that the field of one inductor not have significant influence (or "coupling'') in the next.

(b) Just as with resistors, $\frac{1}{L_{\text{eq}}} = \sum_{n=1}^{N} \frac{1}{L_n}$.

47. Using the results from Problems 45 and 46, the equivalent resistance is

$$\begin{aligned} L_{\mathrm{eq}} &= L_1 + L_4 + L_{23} = L_1 + L_4 + \frac{L_2 L_3}{L_2 + L_3} \\ &= 30.0\,\mathrm{mH} + 15.0\,\mathrm{mH} + \frac{(50.0\,\mathrm{mH})(20.0\,\mathrm{mH})}{50.0\,\mathrm{mH} + 20.0\,\mathrm{mH}} \\ &= 59.3\ \mathrm{mH}. \end{aligned}$$

48. The steady state value of the current is also its maximum value, $\mathcal{E}/R$, which we denote as i_m. We are told that $i = i_m/3$ at t_0 = 5.00 s. Eq. 30-41 becomes $i = i_m\left(1 - e^{-t_0/\tau_L}\right)$ which leads to

$$\tau_L = -\frac{t_0}{\ln(1 - i/i_m)} = -\frac{5.00\,\text{s}}{\ln(1 - 1/3)} = 12.3\ \text{s}.$$

49. Starting with zero current at $t = 0$ (the moment the switch is closed) the current in the circuit increases according to

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$ is the inductive time constant and ε is the battery emf. To calculate the time at which $i = 0.9990\varepsilon/R$, we solve for t:

$$0.990\frac{\varepsilon}{R} = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) \Rightarrow \ln(0.0010) = -(t/\tau) \quad \Rightarrow \quad t/\tau_L = 6.91.$$

50. (a) Immediately after the switch is closed $\varepsilon - \varepsilon_L = iR$. But $i = 0$ at this instant, so $\varepsilon_L = \varepsilon$, or $\varepsilon_L/\varepsilon = 1.00$

(b) $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L} = \varepsilon e^{-2.0\tau_L/\tau_L} = \varepsilon e^{-2.0} = 0.135\varepsilon,$ or $\varepsilon_L/\varepsilon = 0.135$.

(c) From $\varepsilon_L(t) = \varepsilon e^{-t/\tau_L}$ we obtain

$$\frac{t}{\tau_L} = \ln\left(\frac{\varepsilon}{\varepsilon_L}\right) = \ln 2 \Rightarrow t = \tau_L \ln 2 = 0.693\tau_L \quad \Rightarrow \quad t/\tau_L = 0.693.$$

51. The current in the circuit is given by $i = i_0 e^{-t/\tau_L}$, where i_0 is the current at time $t = 0$ and τ_L is the inductive time constant (L/R). We solve for τ_L. Dividing by i_0 and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{i}{i_0}\right) = -\frac{t}{\tau_L}.$$

This yields

$$\tau_L = -\frac{t}{\ln(i/i_0)} = -\frac{1.0\,\text{s}}{\ln\left((10\times10^{-3}\,\text{A})/(1.0\,\text{A})\right)} = 0.217\,\text{s}.$$

Therefore, $R = L/\tau_L = 10\ \text{H}/0.217\ \text{s} = 46\ \Omega$.

52. (a) The inductor prevents a fast build-up of the current through it, so immediately after the switch is closed, the current in the inductor is zero. It follows that

$$i_1 = \frac{\varepsilon}{R_1 + R_2} = \frac{100\,\text{V}}{10.0\,\Omega + 20.0\,\Omega} = 3.33\,\text{A}.$$

(b) $i_2 = i_1 = 3.33\,\text{A}.$

(c) After a suitably long time, the current reaches steady state. Then, the emf across the inductor is zero, and we may imagine it replaced by a wire. The current in R_3 is $i_1 - i_2$. Kirchhoff's loop rule gives

$$\varepsilon - i_1 R_1 - i_2 R_2 = 0 \qquad \text{and} \qquad \varepsilon - i_1 R_1 - (i_1 - i_2) R_3 = 0.$$

We solve these simultaneously for i_1 and i_2, and find

$$i_1 = \frac{\varepsilon(R_2 + R_3)}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{(100\,\text{V})(20.0\,\Omega + 30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 4.55\,\text{A},$$

(d) and

$$i_2 = \frac{\varepsilon R_3}{R_1R_2 + R_1R_3 + R_2R_3} = \frac{(100\,\text{V})(30.0\,\Omega)}{(10.0\,\Omega)(20.0\,\Omega) + (10.0\,\Omega)(30.0\,\Omega) + (20.0\,\Omega)(30.0\,\Omega)}$$
$$= 2.73\,\text{A}.$$

(e) The left-hand branch is now broken. We take the current (immediately) as zero in that branch when the switch is opened (that is, $i_1 = 0$).

(f) The current in R_3 changes less rapidly because there is an inductor in its branch. In fact, immediately after the switch is opened it has the same value that it had before the switch was opened. That value is 4.55 A – 2.73 A = 1.82 A. The current in R_2 is the same but in the opposite direction as that in R_3, i.e., $i_2 = -1.82$ A.

A long time later after the switch is reopened, there are no longer any sources of emf in the circuit, so all currents eventually drop to zero. Thus,

(g) $i_1 = 0$, and

(h) $i_2 = 0$.

53. (a) If the battery is switched into the circuit at $t = 0$, then the current at a later time t is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where $\tau_L = L/R$. Our goal is to find the time at which $i = 0.800\varepsilon/R$. This means

$$0.800 = 1 - e^{-t/\tau_L} \Rightarrow e^{-t/\tau_L} = 0.200 .$$

Taking the natural logarithm of both sides, we obtain $-(t/\tau_L) = \ln(0.200) = -1.609$. Thus

$$t = 1.609\tau_L = \frac{1.609L}{R} = \frac{1.609(6.30\times10^{-6}\,\mathrm{H})}{1.20\times10^{3}\,\Omega} = 8.45\times10^{-9}\,\mathrm{s} .$$

(b) At $t = 1.0\tau_L$ the current in the circuit is

$$i = \frac{\varepsilon}{R}\left(1 - e^{-1.0}\right) = \left(\frac{14.0\,\mathrm{V}}{1.20\times10^{3}\Omega}\right)\left(1 - e^{-1.0}\right) = 7.37\times10^{-3}\ \mathrm{A} .$$

54. From the graph we get $\Phi/i = 2\times10^{-4}$ in SI units. Therefore, with $N = 25$, we find the self-inductance is $L = N\Phi/i = 5\times10^{-3}$ H. From the derivative of Eq. 30-41 (or a combination of that equation and Eq. 30-39) we find (using the symbol V to stand for the battery emf)

$$\frac{di}{dt} = \frac{\mathrm{V}}{R}\frac{R}{L}\,e^{-t/\tau_L} = \frac{\mathrm{V}}{L}\,e^{-t/\tau_L} = 7.1\times10^{2}\ \mathrm{A/s}\ .$$

55. Applying the loop theorem

$$\varepsilon - L\left(\frac{di}{dt}\right) = iR \ ,$$

we solve for the (time-dependent) emf, with SI units understood:

$$\varepsilon = L\frac{di}{dt} + iR = L\frac{d}{dt}(3.0+5.0t) + (3.0+5.0t)R = (6.0)(5.0) + (3.0+5.0t)(4.0)$$
$$= (42+20t).$$

56. (a) Our notation is as follows: h is the height of the toroid, a its inner radius, and b its outer radius. Since it has a square cross section, $h = b - a = 0.12$ m – 0.10 m = 0.02 m. We derive the flux using Eq. 29-24 and the self-inductance using Eq. 30-33:

$$\Phi_B = \int_a^b B\,dA = \int_a^b \left(\frac{\mu_0 Ni}{2\pi r}\right) h\,dr = \frac{\mu_0 Nih}{2\pi}\ln\left(\frac{b}{a}\right)$$

and

$$L = N\Phi_B/i = (\mu_0 N^2 h/2\pi)\ln(b/a).$$

Now, since the inner circumference of the toroid is $l = 2\pi a = 2\pi(10 \text{ cm}) \approx 62.8$ cm, the number of turns of the toroid is roughly $N \approx 62.8$ cm/1.0 mm = 628. Thus

$$L = \frac{\mu_0 N^2 h}{2\pi}\ln\left(\frac{b}{a}\right) \approx \frac{\left(4\pi\times 10^{-7}\text{ H/m}\right)(628)^2(0.02\text{m})}{2\pi}\ln\left(\frac{12}{10}\right) = 2.9\times 10^{-4}\text{H}.$$

(b) Noting that the perimeter of a square is four times its sides, the total length ℓ of the wire is $\ell = (628)4(2.0\,\text{cm}) = 50\,\text{m}$, the resistance of the wire is

$$R = (50 \text{ m})(0.02\ \Omega/\text{m}) = 1.0\ \Omega.$$

Thus

$$\tau_L = \frac{L}{R} = \frac{2.9\times 10^{-4}\text{H}}{1.0\Omega} = 2.9\times 10^{-4}\text{s}.$$

57. (a) We assume i is from left to right through the closed switch. We let i_1 be the current in the resistor and take it to be downward. Let i_2 be the current in the inductor, also assumed downward. The junction rule gives $i = i_1 + i_2$ and the loop rule gives $i_1R - L(di_2/dt) = 0$. According to the junction rule, $(di_1/dt) = -(di_2/dt)$. We substitute into the loop equation to obtain

$$L\frac{di_1}{dt} + i_1R = 0.$$

This equation is similar to Eq. 30-46, and its solution is the function given as Eq. 30-47:

$$i_1 = i_0 e^{-Rt/L},$$

where i_0 is the current through the resistor at $t = 0$, just after the switch is closed. Now just after the switch is closed, the inductor prevents the rapid build-up of current in its branch, so at that moment $i_2 = 0$ and $i_1 = i$. Thus $i_0 = i$, so

$$i_1 = ie^{-Rt/L} \qquad \text{and} \qquad i_2 = i - i_1 = i\left(1 - e^{-Rt/L}\right).$$

(b) When $i_2 = i_1$,

$$e^{-Rt/L} = 1 - e^{-Rt/L} \Rightarrow e^{-Rt/L} = \frac{1}{2}.$$

Taking the natural logarithm of both sides (and using $\ln(1/2) = -\ln 2$) we obtain

$$\left(\frac{Rt}{L}\right) = \ln 2 \Rightarrow t = \frac{L}{R}\ln 2.$$

58. Let $U_B(t) = \frac{1}{2} Li^2(t)$. We require the energy at time t to be half of its final value: $U(t) = \frac{1}{2} U_B(t \to \infty) = \frac{1}{4} Li_f^2$. This gives $i(t) = i_f \big/ \sqrt{2}$. But $i(t) = i_f(1 - e^{-t/\tau_L})$, so

$$1 - e^{-t/\tau_L} = \frac{1}{\sqrt{2}} \quad \Rightarrow \quad \frac{t}{\tau_L} = -\ln\left(1 - \frac{1}{\sqrt{2}}\right) = 1.23.$$

59. From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt}=\frac{d\left(\frac{1}{2}Li^2\right)}{dt}=Li\frac{di}{dt}=L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right)=\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}$$

where $\tau_L = L/R$ has been used. From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

We equate this to dU_B/dt, and solve for the time:

$$\frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2 = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L} \Rightarrow t = \tau_L \ln 2 = (37.0\,\text{ms})\ln 2 = 25.6\,\text{ms}.$$

60. (a) From Eq. 30-49 and Eq. 30-41, the rate at which the energy is being stored in the inductor is

$$\frac{dU_B}{dt} = \frac{d\left(\frac{1}{2}Li^2\right)}{dt} = Li\frac{di}{dt} = L\left(\frac{\varepsilon}{R}\left(1-e^{-t/\tau_L}\right)\right)\left(\frac{\varepsilon}{R}\frac{1}{\tau_L}e^{-t/\tau_L}\right) = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)e^{-t/\tau_L}.$$

Now,

$$\tau_L = L/R = 2.0\text{ H}/10\ \Omega = 0.20\text{ s}$$

and $\varepsilon = 100$ V, so the above expression yields $dU_B/dt = 2.4 \times 10^2$ W when $t = 0.10$ s.

(b) From Eq. 26-22 and Eq. 30-41, the rate at which the resistor is generating thermal energy is

$$P_{\text{thermal}} = i^2 R = \frac{\varepsilon^2}{R^2}\left(1-e^{-t/\tau_L}\right)^2 R = \frac{\varepsilon^2}{R}\left(1-e^{-t/\tau_L}\right)^2.$$

At $t = 0.10$ s, this yields $P_{\text{thermal}} = 1.5 \times 10^2$ W.

(c) By energy conservation, the rate of energy being supplied to the circuit by the battery is

$$P_{\text{battery}} = P_{\text{thermal}} + \frac{dU_B}{dt} = 3.9 \times 10^2\text{ W}.$$

We note that this result could alternatively have been found from Eq. 28-14 (with Eq. 30-41).

61. (a) If the battery is applied at time $t = 0$ the current is given by

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right),$$

where ε is the emf of the battery, R is the resistance, and τ_L is the inductive time constant (L/R). This leads to

$$e^{-t/\tau_L} = 1 - \frac{iR}{\varepsilon} \Rightarrow -\frac{t}{\tau_L} = \ln\left(1 - \frac{iR}{\varepsilon}\right).$$

Since

$$\ln\left(1 - \frac{iR}{\varepsilon}\right) = \ln\left[1 - \frac{\left(2.00\times10^{-3}\,\mathrm{A}\right)\left(10.0\times10^{3}\,\Omega\right)}{50.0\,\mathrm{V}}\right] = -0.5108,$$

the inductive time constant is

$$\tau_L = t/0.5108 = (5.00 \times 10^{-3}\ \mathrm{s})/0.5108 = 9.79 \times 10^{-3}\ \mathrm{s}$$

and the inductance is

$$L = \tau_L R = \left(9.79\times10^{-3}\,\mathrm{s}\right)\left(10.0\times10^{3}\,\Omega\right) = 97.9\,\mathrm{H}.$$

(b) The energy stored in the coil is

$$U_B = \frac{1}{2}Li^2 = \frac{1}{2}(97.9\,\mathrm{H})\left(2.00\times10^{-3}\,\mathrm{A}\right)^2 = 1.96\times10^{-4}\,\mathrm{J}.$$

62. (a) The energy delivered by the battery is the integral of Eq. 28-14 (where we use Eq. 30-41 for the current):

$$\int_0^t P_{\text{battery}}\,dt = \int_0^t \frac{\varepsilon^2}{R}\left(1-e^{-Rt/L}\right)dt = \frac{\varepsilon^2}{R}\left[t+\frac{L}{R}\left(e^{-Rt/L}-1\right)\right]$$

$$= \frac{(10.0\,\text{V})^2}{6.70\ \Omega}\left[2.00\ \text{s}+\frac{(5.50\,\text{H})\left(e^{-(6.70\ \Omega)(2.00\ \text{s})/5.50\ \text{H}}-1\right)}{6.70\ \Omega}\right]$$

$$= 18.7\ \text{J}.$$

(b) The energy stored in the magnetic field is given by Eq. 30-49:

$$U_B = \frac{1}{2}Li^2(t) = \frac{1}{2}L\left(\frac{\varepsilon}{R}\right)^2\left(1-e^{-Rt/L}\right)^2$$

$$= \frac{1}{2}(5.50\,\text{H})\left(\frac{10.0\,\text{V}}{6.70\,\Omega}\right)^2\left[1-e^{-(6.70\ \Omega)(2.00\ \text{s})/5.50\ \text{H}}\right]^2$$

$$= 5.10\ \text{J}.$$

(c) The difference of the previous two results gives the amount “lost” in the resistor:

18.7 J – 5.10 J = 13.6 J.

63. (a) At any point the magnetic energy density is given by $u_B = B^2/2\mu_0$, where B is the magnitude of the magnetic field at that point. Inside a solenoid $B = \mu_0 ni$, where n, for the solenoid of this problem, is (950 turns)/(0.850 m) = $1.118 \times 10^3\ \text{m}^{-1}$. The magnetic energy density is

$$u_B = \frac{1}{2}\mu_0 n^2 i^2 = \frac{1}{2}\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.118\times10^3\ \text{m}^{-1}\right)^2\left(6.60\ \text{A}\right)^2 = 34.2\ \text{J/m}^3\ .$$

(b) Since the magnetic field is uniform inside an ideal solenoid, the total energy stored in the field is $U_B = u_B$, where is the volume of the solenoid. is calculated as the product of the cross-sectional area and the length. Thus

$$U_B = \left(34.2\ \text{J/m}^3\right)\left(17.0\times10^{-4}\ \text{m}^2\right)\left(0.850\ \text{m}\right) = 4.94\times10^{-2}\ \text{J}\ .$$

64. The magnetic energy stored in the toroid is given by $U_B = \frac{1}{2}Li^2$, where L is its inductance and i is the current. By Eq. 30-54, the energy is also given by $U_B = u_B$, where u_B is the average energy density and is the volume. Thus

$$i = \sqrt{\frac{2u_B}{L}} = \sqrt{\frac{2\left(70.0\,\mathrm{J/m^3}\right)\left(0.0200\,\mathrm{m^3}\right)}{90.0\times10^{-3}\,\mathrm{H}}} = 5.58\,\mathrm{A}\ .$$

65. We set $u_E = \frac{1}{2}\varepsilon_0 E^2 = u_B = \frac{1}{2} B^2/\mu_0$ and solve for the magnitude of the electric field:

$$E = \frac{B}{\sqrt{\varepsilon_0 \mu_0}} = \frac{0.50\,\text{T}}{\sqrt{(8.85\times10^{-12}\ \text{F/m})(4\pi\times10^{-7}\ \text{H/m})}} = 1.5\times10^{8}\ \text{V/m}\,.$$

66. (a) The magnitude of the magnetic field at the center of the loop, using Eq. 29-9, is

$$B = \frac{\mu_0 i}{2R} = \frac{(4\pi \times 10^{-7}\ \text{H/m})(100\,\text{A})}{2(50 \times 10^{-3}\ \text{m})} = 1.3 \times 10^{-3}\ \text{T}\ .$$

(b) The energy per unit volume in the immediate vicinity of the center of the loop is

$$u_B = \frac{B^2}{2\mu_0} = \frac{(1.3 \times 10^{-3}\ \text{T})^2}{2(4\pi \times 10^{-7}\ \text{H/m})} = 0.63\ \text{J/m}^3\ .$$

67. (a) The energy per unit volume associated with the magnetic field is

$$u_B = \frac{B^2}{2\mu_0} = \frac{1}{2\mu_0}\left(\frac{\mu_0 i}{2R}\right)^2 = \frac{\mu_0 i^2}{8R^2} = \frac{(4\pi\times10^{-7}\text{ H/m})(10\text{ A})^2}{8(2.5\times10^{-3}\text{ m}/2)^2} = 1.0\text{ J/m}^3 .$$

(b) The electric energy density is

$$u_E = \frac{1}{2}\varepsilon_0 E^2 = \frac{\varepsilon_0}{2}(\rho J)^2 = \frac{\varepsilon_0}{2}\left(\frac{iR}{\ell}\right)^2 = \frac{1}{2}(8.85\times10^{-12}\text{ F/m})\left[(10\text{A})(3.3\Omega/10^3\text{ m})\right]^2$$
$$= 4.8\times10^{-15}\text{ J/m}^3 .$$

Here we used $J = i/A$ and $R = \rho\ell/A$ to obtain $\rho J = iR/\ell$.

68. We use $\varepsilon_2 = -M\, di_1/dt \approx M|\Delta i/\Delta t|$ to find M:

$$M = \left|\frac{\varepsilon}{\Delta i_1/\Delta t}\right| = \frac{30\times10^3\ \text{V}}{6.0\,\text{A}/\left(2.5\times10^{-3}\ \text{s}\right)} = 13\,\text{H}\ .$$

69. (a) Eq. 30-65 yields

$$M = \frac{\varepsilon_1}{|di_2/dt|} = \frac{25.0\,\text{mV}}{15.0\,\text{A/s}} = 1.67\,\text{mH} \ .$$

(b) Eq. 30-60 leads to

$$N_2\Phi_{21} = Mi_1 = (1.67\,\text{mH})(3.60\,\text{A}) = 6.00\,\text{mWb} \ .$$

70. (a) The flux in coil 1 is

$$\frac{L_1 i_1}{N_1} = \frac{(25\,\text{mH})(6.0\,\text{mA})}{100} = 1.5\,\mu\text{Wb}.$$

(b) The magnitude of the self-induced emf is

$$L_1 \frac{di_1}{dt} = (25\,\text{mH})(4.0\ \text{A/s}) = 1.0\times 10^2\,\text{mV}.$$

(c) In coil 2, we find

$$\Phi_{21} = \frac{M i_1}{N_2} = \frac{(3.0\,\text{mH})(6.0\,\text{mA})}{200} = 90\,\text{nWb}\ .$$

(d) The mutually induced emf is

$$\varepsilon_{21} = M\frac{di_1}{dt} = (3.0\,\text{mH})(4.0\ \text{A/s}) = 12\,\text{mV}.$$

71. (a) We assume the current is changing at (nonzero) rate di/dt and calculate the total emf across both coils. First consider the coil 1. The magnetic field due to the current in that coil points to the right. The magnetic field due to the current in coil 2 also points to the right. When the current increases, both fields increase and both changes in flux contribute emf's in the same direction. Thus, the induced emf's are

$$\varepsilon_1 = -(L_1 + M)\frac{di}{dt} \text{ and } \varepsilon_2 = -(L_2 + M)\frac{di}{dt}.$$

Therefore, the total emf across both coils is

$$\varepsilon = \varepsilon_1 + \varepsilon_2 = -(L_1 + L_2 + 2M)\frac{di}{dt}$$

which is exactly the emf that would be produced if the coils were replaced by a single coil with inductance $L_{eq} = L_1 + L_2 + 2M$.

(b) We imagine reversing the leads of coil 2 so the current enters at the back of coil rather than the front (as pictured in the diagram). Then the field produced by coil 2 at the site of coil 1 is opposite to the field produced by coil 1 itself. The fluxes have opposite signs. An increasing current in coil 1 tends to increase the flux in that coil, but an increasing current in coil 2 tends to decrease it. The emf across coil 1 is

$$\varepsilon_1 = -(L_1 - M)\frac{di}{dt}.$$

Similarly, the emf across coil 2 is

$$\varepsilon_2 = -(L_2 - M)\frac{di}{dt}.$$

The total emf across both coils is

$$\varepsilon = -(L_1 + L_2 - 2M)\frac{di}{dt}.$$

This the same as the emf that would be produced by a single coil with inductance

$$L_{eq} = L_1 + L_2 - 2M.$$

72. The coil-solenoid mutual inductance is

$$M = M_{cs} = \frac{N\Phi_{cs}}{i_s} = \frac{N\left(\mu_0 i_s n \pi R^2\right)}{i_s} = \mu_0 \pi R^2 nN \ .$$

As long as the magnetic field of the solenoid is entirely contained within the cross-section of the coil we have $\Phi_{sc} = B_s A_s = B_s \pi R^2$, regardless of the shape, size, or possible lack of close-packing of the coil.

73. The flux over the loop cross section due to the current i in the wire is given by

$$\Phi = \int_a^{a+b} B_{\text{wire}} l dr = \int_a^{a+b} \frac{\mu_0 i l}{2\pi r} dr = \frac{\mu_0 i l}{2\pi} \ln\left(1+\frac{b}{a}\right).$$

Thus,

$$M = \frac{N\Phi}{i} = \frac{N\mu_0 l}{2\pi} \ln\left(1+\frac{b}{a}\right).$$

From the formula for M obtained above, we have

$$M = \frac{(100)(4\pi\times10^{-7}\text{ H/m})(0.30\text{ m})}{2\pi} \ln\left(1+\frac{8.0}{1.0}\right) = 1.3\times10^{-5}\text{ H}.$$

74. (a) The current is given by Eq. 30-41

$$i = (\varepsilon/R)(1 - e^{-t/\tau_L}) = 2.00 \text{ A},$$

where $L = 0.018$ H and $\varepsilon = 12$ V. If $R = 1.00\ \Omega$ (so $\tau_L = L/R = 0.018$ s), we obtain $t = 0.00328$ s when we solve this equation.

(b) For $R = 5.00\ \Omega$ we find $t = 0.00645$ s.

(c) If we set $R = 6.00\ \Omega$ then $\varepsilon/R = 2.00$ A so $e^{-t/\tau_L} = 0$, which means $t = \infty$.

(d) The trend in our answers to parts (a), (b) and (c) lead us to expect the smaller the resistance then the smaller to value of t. If we consider what happens to Eq. 30-39 in the extreme case where $R \to 0$, we find that the time-derivative of the current becomes equal to the emf divided by the self-inductance, which leads to a linear dependence of current on time: $i = (\varepsilon/L)t$. In fact, this is what one have obtained starting from Eq. 30-41 and considering its $R \to 0$ limit. Thus, this case seems self-consistent, so we conclude that it is meaningful and that $R = 0$ is actually a valid answer here.

(e) Thus $t = Li/\varepsilon = 0.00300$ s in this "least-time" scenario.

75. Faraday's law (for a single turn, with B changing in time) gives

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -\pi r^2 \frac{dB}{dt}.$$

In this problem, we find $\frac{dB}{dt} = -\frac{B_o}{\tau} e^{-t/\tau}$. Thus, $\varepsilon = \pi r^2 \frac{B_o}{\tau} e^{-t/\tau}$.

76. From the datum at $t = 0$ in Fig. 30-69(b) we see 0.0015 A = $V_{battery}/R$, which implies that the resistance is R = (6 μV)/(0.0015 A) = 0.004 Ω. Now, the value of the current during 10 s < t < 20 s leads us to equate $(V_{battery} + \varepsilon_{nduced})/R$ = 0.0005 A. This shows that the induced emf is $\varepsilon_{induced}$ = –4 μV. Now we use Faraday's law:

$$\varepsilon = -\frac{d\Phi_B}{dt} = -A\frac{dB}{dt} = -A\,a\,.$$

Plugging in $\varepsilon = -4\times10^{-6}$ V and $A = 5\times10^{-4}\ \text{m}^2$, we obtain a = 0.0080 T/s.

77. Using Ohm's law, we relate the induced current to the emf and (the absolute value of) Faraday's law:

$$i = \varepsilon / R = \frac{1}{R}\frac{d\Phi_B}{dt} \ .$$

As the loop is crossing the boundary between regions 1 and 2 (so that "x" amount of its length is in region 2 while "$D - x$" amount of its length remains in region 1) the flux is

$$\Phi_B = xHB_2 + (D - x)HB_1 = DHB_1 + xH(B_2 - B_1)$$

which means

$$\frac{d\Phi_B}{dt} = \frac{dx}{dt}H(B_2 - B_1) = vH(B_2 - B_1) \ \Rightarrow \quad i = vH(B_2 - B_1)/R.$$

Similar considerations hold (replacing "B_1" with 0 and "B_2" with B_1) for the loop crossing initially from the zero-field region (to the left of Fig. 30-70(a)) into region 1.

(a) In this latter case, appeal to Fig. 30-70(b) leads to

$$3.0 \times 10^{-6}\ \text{A} = (0.40\ \text{m/s})(0.015\ \text{m})\ B_1/(0.020\ \Omega)$$

which yields $B_1 = 10\ \mu\text{T}$.

(b) Lenz's law considerations lead us to conclude that the direction of the region 1 field is *out of the page*.

(c) Similarly, $i = vH(B_2 - B_1)/R$ leads to $\vec{B}_2 = 3.3\ \mu\text{T}$,

(d) The direction of $\vec{B}_2$ is out of the page.

78. The energy stored when the current is i is

$$U_B = \tfrac{1}{2} L i^2$$

where L is the self-inductance. The rate at which this is developed is

$$\frac{d U_B}{d t} = L i \frac{d i}{d t}$$

where i is given by Eq. 30-41 and di/dt is gotten by taking the derivative of that equation (or by using Eq. 30-37). Thus, using the symbol V to stand for the battery voltage (12.0 volts) and R for the resistance (20.0 Ω), we have

$$\frac{d U_B}{d t} = \frac{\mathrm{V}^2}{R} (1 - e^{-t/\tau_L}) e^{-t/\tau_L} = 1.15 \text{ W} .$$

79. (a) Before the fuse blows, the current through the resistor remains zero. We apply the loop theorem to the battery-fuse-inductor loop: $\varepsilon - L\,di/dt = 0$. So $i = \varepsilon t/L$. As the fuse blows at $t = t_0$, $i = i_0 = 3.0$ A. Thus,

$$t_0 = \frac{i_0 L}{\varepsilon} = \frac{(3.0\,\text{A})(5.0\,\text{H})}{10\,\text{V}} = 1.5\ \text{s}.$$

(b) We do not show the graph here; qualitatively, it would be similar to Fig. 30-15.

80. Since $A = \ell^2$, we have $dA/dt = 2\ell\, d\ell/dt$. Thus, Faraday's law, with $N = 1$, becomes (in absolute value)

$$\varepsilon = \frac{d\Phi_B}{dt} = B\frac{dA}{dt} = 2\ell\, B\, \frac{d\ell}{dt}$$

which yields $\varepsilon = 0.0029$ V.

81. We write $i = i_0 e^{-t/\tau_L}$ and note that i = 10% i_0. We solve for t:

$$t = \tau_L \ln\left(\frac{i_0}{i}\right) = \frac{L}{R}\ln\left(\frac{i_0}{i}\right) = \frac{2.00\,\text{H}}{3.00\,\Omega}\ln\left(\frac{i_0}{0.100\,i_0}\right) = 1.54\,\text{s}\,.$$

82. It is important to note that the x that is used in the graph of Fig. 30-72(b) is not the x at which the energy density is being evaluated. The x in Fig. 30-72(b) is the location of wire 2. The energy density (Eq. 30-54) is being evaluated at the coordinate origin throughout this problem. We note the curve in Fig. 30-72(b) has a zero; this implies that the magnetic fields (caused by the individual currents) are in opposite directions (at the origin), which further implies that the currents have the same direction. Since the magnitudes of the fields must be equal (for them to cancel) when the x of Fig. 30-72(b) is equal to 0.20 m, then we have (using Eq. 29-4) $B_1 = B_2$, or

$$\frac{\mu_o i_1}{2\pi d} = \frac{\mu_o i_2}{2\pi(0.20 \text{ m})}$$

which leads to $d = \frac{1}{3}(0.20 \text{ m})$ once we substitute $i_1 = \frac{1}{3} i_2$ and simplify. We can also use the given fact that when the energy density is completely caused by B_1 (this occurs when x becomes infinitely large because then $B_2 = 0$) its value is $u_B = 1.96 \times 10^{-9}$ (in SI units) in order to solve for B_1:

$$B_1 = \sqrt{2\mu_o u_B} \; .$$

(a) This combined with $B_1 = \dfrac{\mu_o i_1}{2\pi d}$ allows us to find wire 1's current: $i_1 \approx 23$ mA.

(b) Since $i_2 = 3i_1$ then $i_2 = 70$ mA (approximately).

83. (a) As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit. Thus, at $t = 0$ any current through the battery is also that through the 20 Ω and 10 Ω resistors. Hence,

$$i = \frac{\varepsilon}{30\Omega} = 0.40\,\text{A}$$

which results in a voltage drop across the 10 Ω resistor equal to (0.40)(10) = 4.0 V. The inductor must have this same voltage across it $|\varepsilon_L|$, and we use (the absolute value of) Eq. 30-35:

$$\frac{di}{dt} = \frac{|\varepsilon_L|}{L} = \frac{4.0}{0.010} = 400\,\text{A/s}.$$

(b) Applying the loop rule to the outer loop, we have

$$\varepsilon - (0.50\,\text{A})(20\,\Omega) - |\varepsilon_L| = 0.$$

Therefore, $|\varepsilon_L| = 2.0$ V, and Eq. 30-35 leads to

$$\frac{di}{dt} = \frac{|\varepsilon_L|}{L} = \frac{2.0}{0.010} = 200\,\text{A/s}.$$

(c) As $t \to \infty$, the inductor has $\varepsilon_L = 0$ (since the current is no longer changing). Thus, the loop rule (for the outer loop) leads to

$$\varepsilon - i(20\,\Omega) - |\varepsilon_L| = 0 \Rightarrow i = 0.60\,\text{A}.$$

84. (a) From Eq. 30-35, we find $L = (3.00\text{ mV})/(5.00\text{ A/s}) = 0.600\text{ mH}$.

(b) Since $N\Phi = iL$ (where $\Phi = 40.0\ \mu\text{Wb}$ and $i = 8.00$ A), we obtain $N = 120$.

85. (a) The magnitude of the average induced emf is

$$\varepsilon_{\text{avg}} = \left|\frac{-d\Phi_B}{dt}\right| = \left|\frac{\Delta\Phi_B}{\Delta t}\right| = \frac{BA_i}{t} = \frac{(2.0\,\text{T})(0.20\,\text{m})^2}{0.20\,\text{s}} = 0.40\,\text{V}.$$

(b) The average induced current is

$$i_{\text{avg}} = \frac{\varepsilon_{\text{avg}}}{R} = \frac{0.40\,\text{V}}{20\times10^{-3}\,\Omega} = 20\,\text{A}.$$

86. In absolute value, Faraday's law (for a single turn, with B changing in time) gives

$$\frac{d\Phi_B}{dt} = A\frac{dB}{dt} = \pi R^2 \frac{dB}{dt}$$

for the magnitude of the induced emf. Dividing it by R^2 then allows us to relate this to the slope of the graph in Fig. 30-74(b) [particularly the first part of the graph], which we estimate to be 80 μV/m^2.

(a) Thus, $\frac{dB_1}{dt} = (80\ \mu\text{V/m}^2)/\pi \approx 25\ \mu\text{T/s}$.

(b) Similar reasoning for region 2 (corresponding to the slope of the second part of the graph in Fig. 30-74(b)) leads to an emf equal to

$$\pi r_1^2\left(\frac{dB_1}{dt} - \frac{dB_2}{dt}\right) + \pi R^2 \frac{dB_2}{dt} .$$

which means the second slope (which we estimate to be 40 μV/m^2) is equal to $\pi\frac{dB_2}{dt}$.

Therefore, $\frac{dB_2}{dt} = (40\ \mu\text{V/m}^2)/\pi \approx 13\ \mu\text{T/s}$.

(c) Considerations of Lenz's law leads to the conclusion that B_2 is increasing.

87. The induced electric field E as a function of r is given by $E(r) = (r/2)(dB/dt)$.

(a) The acceleration of the electron released at point a is

$$\vec{a}_a = \frac{eE}{m}\hat{\mathrm{i}} = \frac{er}{2m}\left(\frac{dB}{dt}\right)\hat{\mathrm{i}} = \frac{\left(1.60\times10^{-19}\,\mathrm{C}\right)\left(5.0\times10^{-2}\,\mathrm{m}\right)\left(10\times10^{-3}\,\mathrm{T/s}\right)}{2\left(9.11\times10^{-27}\,\mathrm{kg}\right)}\hat{\mathrm{i}} = (4.4\times10^{7}\,\mathrm{m/s^2})\hat{\mathrm{i}}.$$

(b) At point b we have $a_b \propto r_b = 0$.

(c) The acceleration of the electron released at point c is

$$\vec{a}_c = -\vec{a}_a = -(4.4\times10^{7}\,\mathrm{m/s^2})\hat{\mathrm{i}}.$$

88. Because of the decay of current (Eq. 30-45) that occurs after the switches are closed on B, the flux will decay according to

$$\Phi_1 = \Phi_{1o} e^{-t/\tau_{L1}}, \ \Phi_2 = \Phi_{2o} e^{-t/\tau_{L2}} \quad .$$

where each time-constant is given by Eq. 30-42. Setting the fluxes equal to each other and solving for time leads to

$$t = \frac{\ln\left(\dfrac{\Phi_{2o}}{\Phi_{1o}}\right)}{\dfrac{R_2}{L_2} - \dfrac{R_1}{L_1}} = \frac{\ln(1.5)}{\dfrac{30}{0.003} - \dfrac{25}{0.005}} \quad .$$

Thus, $t = 81.1$ μs.

89. (a) When switch S is just closed, $V_1 = \varepsilon$ and $i_1 = \varepsilon/R_1 = 10\ \text{V}/5.0\ \Omega = 2.0\ \text{A}$.

(b) Since now $\varepsilon_L = \varepsilon$, we have $i_2 = 0$.

(c) $i_s = i_1 + i_2 = 2.0\ \text{A} + 0 = 2.0\ \text{A}$.

(d) Since $V_L = \varepsilon$, $V_2 = \varepsilon - \varepsilon_L = 0$.

(e) $V_L = \varepsilon = 10\ \text{V}$.

(f) $di_2/dt = V_L/L = \varepsilon/L = 10\ \text{V} / 5.0\ \text{H} = 2.0\ \text{A/s}$.

(g) After a long time, we still have $V_1 = \varepsilon$, so $i_1 = 2.0\ \text{A}$.

(h) Since now $V_L = 0$, $i_2 = \varepsilon/R_2 = 10\ \text{V}/10\ \Omega = 1.0\ \text{A}$.

(i) $i_s = i_1 + i_2 = 2.0\ \text{A} + 1.0\ \text{A} = 3.0\ \text{A}$.

(j) Since $V_L = 0$, $V_2 = \varepsilon - V_L = \varepsilon = 10\ \text{V}$.

(k) $V_L = 0$.

(l) $di_2/dt = V_L/L = 0$.

90. Eq. 30-41 applies, and the problem requires

$$iR = L\frac{di}{dt} = \varepsilon - iR$$

at some time t (where Eq. 30-39 has been used in that last step). Thus, we have $2iR = \varepsilon$, or

$$2[(\varepsilon / R)(1 - e^{-t/\tau_L})]R = \varepsilon$$

where Eq. 30-42 gives the inductive time constant as $\tau_L = L/R$. We note that the emf ε cancels out of that final equation, and we are able to rearrange (and take natural log) and solve. We obtain $t = 0.520$ ms.

91. Taking the derivative of Eq. 30-41, we have

$$\frac{di}{dt} = (\varepsilon / R\tau_L)\, e^{-t/\tau_L} = (\varepsilon / L)\, e^{-t/\tau_L} \quad .$$

With $\tau_L = L/R$ (Eq. 30-42), L = 0.023 H and ε = 12 V, t = 0.00015 s, and di/dt = 280 A/s, we obtain e^{-t/τ_L} = 0.537. Taking the natural log and rearranging leads to R = 95.4 Ω.

92. We use the expression for the flux Φ_B over the toroid cross-section derived in our solution to problem 52 to obtain the coil-toroid mutual inductance:

$$M_{ct} = \frac{N_c \Phi_{ct}}{i_t} = \frac{N_c}{i_t} \frac{\mu_0 i_t N_t h}{2\pi} \ln\left(\frac{b}{a}\right) = \frac{\mu_0 N_1 N_2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

where $N_t = N_1$ and $N_c = N_2$.

93. From the given information, we find

$$\frac{dB}{dt} = \frac{0.030\,\text{T}}{0.015\,\text{s}} = 2.0\,\text{T/s}.$$

Thus, with $N = 1$ and $\cos 30° = \sqrt{3}/2$, and using Faraday's law with Ohm's law, we have

$$i = \frac{|\varepsilon|}{R} = \frac{N\pi r^2}{R}\frac{\sqrt{3}}{2}\frac{dB}{dt} = 0.021\,\text{A}.$$

94. The self-inductance and resistance of the coil may be treated as a "pure" inductor in series with a "pure" resistor, in which case the situation described in the problem may be addressed by using Eq. 30-41. The derivative of that solution is

$$\frac{di}{dt} = (\varepsilon / R\tau_L)\, e^{-t/\tau_L} = (\varepsilon / L)\, e^{-t/\tau_L} \ .$$

With τ_L = 0.28 ms (by Eq. 30-42), L = 0.050 H and ε = 45 V, we obtain di/dt = 12 A/s when t = 1.2 ms.

95. (a) The energy density is

$$u_B = \frac{B_e^2}{2\mu_0} = \frac{\left(50\times 10^{-6}\ \mathrm{T}\right)^2}{2\left(4\pi\times 10^{-7}\ \mathrm{H/m}\right)} = 1.0\times 10^{-3}\ \mathrm{J/m^3}.$$

(b) The volume of the shell of thickness h is $\mathcal{V} \approx 4\pi R_e^2 h$, where R_e is the radius of the Earth. So

$$U_B \approx \mathcal{V} u_B \approx 4\pi\left(6.4\times 10^6\,\mathrm{m}\right)^2\left(16\times 10^3\,\mathrm{m}\right)\left(1.0\times 10^{-3}\ \mathrm{J/m^3}\right) = 8.4\times 10^{15}\ \mathrm{J}.$$

96. (a) From Eq. 30-28, we have

$$L = N\Phi/i = (150)(50 \times 10^{-9})/(0.002) = 3.75 \text{ mH}.$$

(b) The answer for L (which should be considered the *constant* of proportionality in Eq. 30-35) does not change; it is still 3.75 mH.

(c) The equations of Chapter 28 display a simple proportionality between magnetic field and the current that creates it. Thus, if the current has doubled, so has the field (and consequently the flux). The answer is 2(50) = 100 nWb.

(d) The magnitude of the induced emf is (from Eq. 30-35)

$$L\frac{di}{dt}\Big|_{\max} = (0.00375 \text{ H})(0.003 \text{ A})(377 \text{ rad/s}) = 0.00424 \text{ V}.$$

97. (a) At $t = 0.50$, the magnetic field is decreasing at a rate of 3/2 mT/s, leading to

$$i = \frac{|\varepsilon|}{R} = \frac{A|dB/dt|}{R} = \frac{(3.0)(3/2)}{9.0} = 0.50\,\text{mA}\ .$$

(b) By Lenz's law, the current is counterclockwise.

(c) At $t = 1.5$ s, the magnetic field is decreasing at a rate of 3/2 mT/s, same as that in (a). Thus, $i = 0.50\,\text{mA}$.

(d) By Lenz's law, the current is counterclockwise.

(e) For $t = 3.0$ s, there is no change in flux and therefore no induced current.

(f) None.

98. For $t < 0$, no current goes through L_2, so $i_2 = 0$ and $i_1 = \varepsilon/R$. As the switch is opened there will be a very brief sparking across the gap. i_1 drops while i_2 increases, both very quickly. The loop rule can be written as

$$\varepsilon - i_1 R - L_1 \frac{di_1}{dt} - i_2 R - L_2 \frac{di_2}{dt} = 0 ,$$

where the initial value of i_1 at $t = 0$ is given by ε/R and that of i_2 at $t = 0$ is 0. We consider the situation shortly after $t = 0$. Since the sparking is very brief, we can reasonably assume that both i_1 and i_2 get equalized quickly, before they can change appreciably from their respective initial values. Here, the loop rule requires that $L_1(di_1/dt)$, which is large and negative, must roughly cancel $L_2(di_2/dt)$, which is large and positive:

$$L_1 \frac{di_1}{dt} \approx -L_2 \frac{di_2}{dt} .$$

Let the common value reached by i_1 and i_2 be i, then

$$\frac{di_1}{dt} \approx \frac{\Delta i_1}{\Delta t} = \frac{i - \varepsilon/R}{\Delta t}$$

and

$$\frac{di_2}{dt} \approx \frac{\Delta i_2}{\Delta t} = \frac{i - 0}{\Delta t} .$$

The equations above yield

$$L_1\left(i - \frac{\varepsilon}{R}\right) = -L_2\left(i - 0\right) \Rightarrow i = \frac{\varepsilon L_1}{L_2 R_1 + L_1 R_2} = \frac{L_1}{L_1 + L_2}\frac{\varepsilon}{R} .$$

99. (a) As the switch closes at $t = 0$, the current being zero in the inductor serves as an initial condition for the building-up of current in the circuit. Thus, at $t = 0$ the current through the battery is also zero.

(b) With no current anywhere in the circuit at $t = 0$, the loop rule requires the emf of the inductor ε_L to cancel that of the battery ($\varepsilon = 40$ V). Thus, the absolute value of Eq. 30-35 yields

$$\frac{di_{\text{bat}}}{dt} = \frac{|\varepsilon_L|}{L} = \frac{40}{0.050} = 8.0\times10^2 \text{ A/s} .$$

(c) This circuit becomes equivalent to that analyzed in §30-9 when we replace the parallel set of 20000 Ω resistors with $R = 10000$ Ω. Now, with $\tau_L = L/R = 5 \times 10^{-6}$ s, we have $t/\tau_L = 3/5$, and we apply Eq. 30-41:

$$i_{\text{bat}} = \frac{\varepsilon}{R}\left(1-e^{-3/5}\right) \approx 1.8\times10^{-3}\text{ A}.$$

(d) The rate of change of the current is figured from the loop rule (and Eq. 30-35):

$$\varepsilon - i_{\text{bat}}R - |\varepsilon_L| = 0 .$$

Using the values from part (c), we obtain $|\varepsilon_L| \approx 22$ V. Then,

$$\frac{di_{\text{bat}}}{dt} = \frac{|\varepsilon_L|}{L} = \frac{22}{0.050} \approx 4.4\times10^2 \text{ A/s} .$$

(e) As $t \to \infty$, the circuit reaches a steady state condition, so that $di_{\text{bat}}/dt = 0$ and $\varepsilon_L = 0$. The loop rule then leads to

$$\varepsilon - i_{\text{bat}}R - |\varepsilon_L| = 0 \;\Rightarrow\; i_{\text{bat}} = \frac{40}{10000} = 4.0\times10^{-3}\text{ A}.$$

(f) As $t \to \infty$, the circuit reaches a steady state condition, $di_{\text{bat}}/dt = 0$.

100. (a) $i_0 = \varepsilon / R$ = 100 V/10 Ω = 10 A.

(b) $U_B = \frac{1}{2} L i_0^2 = \frac{1}{2}(2.0\,\text{H})(10\,\text{A})^2 = 1.0 \times 10^2\,\text{J}$.

101. (a) The magnetic flux Φ_B through the loop is given by

$$\Phi_B = 2B\left(\pi r^2/2\right)\left(\cos 45°\right) = \pi r^2 B\big/\sqrt{2}\ .$$

Thus

$$\begin{aligned}\varepsilon &= -\frac{d\Phi_B}{dt} = -\frac{d}{dt}\left(\frac{\pi r^2 B}{\sqrt{2}}\right) = -\frac{\pi r^2}{\sqrt{2}}\left(\frac{\Delta B}{\Delta t}\right) = -\frac{\pi\left(3.7\times10^{-2}\,\text{m}\right)^2}{\sqrt{2}}\left(\frac{0-76\times10^{-3}\,\text{T}}{4.5\times10^{-3}\,\text{s}}\right)\\ &= 5.1\times10^{-2}\,\text{V}.\end{aligned}$$

(a) The direction of the induced current is clockwise when viewed along the direction of $\vec{B}$.

102. Using Eq. 30-41

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right)$$

where τ_L = 2.0 ns, we find

$$t = \tau_L \ln\left(\frac{1}{1 - iR/\varepsilon}\right) \approx 1.0\,\text{ns}.$$

103. The area enclosed by any turn of the coil is πr^2 where r = 0.15 m, and the coil has N = 50 turns. Thus, the magnitude of the induced emf, using Eq. 30-5, is

$$|\varepsilon| = N\pi r^2 \left|\frac{dB}{dt}\right| = \left(3.53\,\text{m}^2\right)\left|\frac{dB}{dt}\right|$$

where $\left|\frac{dB}{dt}\right| = (0.0126\,\text{T/s})|\cos\omega t|$. Thus, using Ohm's law, we have

$$i = \frac{|\varepsilon|}{R} = \frac{(3.53)(0.0126)}{4.0}|\cos\omega t| \ .$$

When t = 0.020 s, this yields i = 0.011 A.

104. (a) $L = \Phi/i = 26 \times 10^{-3}$ Wb/5.5 A = 4.7×10^{-3} H.

(b) We use Eq. 30-41 to solve for t:

$$t = -\tau_L \ln\left(1 - \frac{iR}{\varepsilon}\right) = -\frac{L}{R}\ln\left(1 - \frac{iR}{\varepsilon}\right) = -\frac{4.7\times10^{-3}\,\text{H}}{0.75\,\Omega}\ln\left[1 - \frac{(2.5\,\text{A})(0.75\,\Omega)}{6.0\,\text{V}}\right]$$

$$= 2.4\times10^{-3}\,\text{s}.$$

105. (a) We use $U_B = \frac{1}{2}Li^2$ to solve for the self-inductance:

$$L = \frac{2U_B}{i^2} = \frac{2\left(25.0\times10^{-3}\ \text{J}\right)}{\left(60.0\times10^{-3}\ \text{A}\right)^2} = 13.9\ \text{H}.$$

(b) Since $U_B \propto i^2$, for U_B to increase by a factor of 4, i must increase by a factor of 2. Therefore, i should be increased to 2(60.0 mA) = 120 mA.

106. (a) The self-inductance per meter is

$$\frac{L}{\ell} = \mu_0 n^2 A = \left(4\pi\times10^{-7}\text{ H/m}\right)\left(100\text{ turns/cm}\right)^2\left(\pi\right)\left(1.6\text{ cm}\right)^2 = 0.10\text{ H/m}.$$

(b) The induced emf per meter is

$$\frac{\varepsilon}{\ell} = \frac{L}{\ell}\frac{di}{dt} = \left(0.10\text{ H/m}\right)\left(13\text{ A/s}\right) = 1.3\text{ V/m}.$$

107. Using Eq. 30-41, we find

$$i = \frac{\varepsilon}{R}\left(1 - e^{-t/\tau_L}\right) \Rightarrow \tau_L = \frac{t}{\ln\left(\frac{1}{1 - iR/\varepsilon}\right)} = 22.4\,\mathrm{s}.$$

Thus, from Eq. 30-42 (the definition of the time constant), we obtain

$$L = (22.4\ \mathrm{s})(2.0\ \Omega) = 45\ \mathrm{H}.$$

108. (a) As the switch closes at $t = 0$, the current being zero in the inductors serves as an initial condition for the building-up of current in the circuit. Thus, the current through any element of this circuit is also zero at that instant. Consequently, the loop rule requires the emf (ε_{L1}) of the $L_1 = 0.30$ H inductor to cancel that of the battery. We now apply (the absolute value of) Eq. 30-35

$$\frac{di}{dt} = \frac{|\varepsilon_{L1}|}{L_1} = \frac{6.0}{0.30} = 20\,\text{A/s}.$$

(b) What is being asked for is essentially the current in the battery when the emf's of the inductors vanish (as $t \to \infty$). Applying the loop rule to the outer loop, with $R_1 = 8.0\ \Omega$, we have

$$\varepsilon - i R_1 - |\varepsilon_{L1}| - |\varepsilon_{L2}| = 0 \Rightarrow i = \frac{6.0\,\text{V}}{R_1} = 0.75\,\text{A}.$$