1. (a) All the energy in the circuit resides in the capacitor when it has its maximum charge. The current is then zero. If Q is the maximum charge on the capacitor, then the total energy is

$$U = \frac{Q^2}{2C} = \frac{\left(2.90\times10^{-6}\,\mathrm{C}\right)^2}{2\left(3.60\times10^{-6}\,\mathrm{F}\right)} = 1.17\times10^{-6}\,\mathrm{J}.$$

(b) When the capacitor is fully discharged, the current is a maximum and all the energy resides in the inductor. If I is the maximum current, then $U = LI^2/2$ leads to

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.168\times10^{-6}\,\mathrm{J}\right)}{75\times10^{-3}\,\mathrm{H}}} = 5.58\times10^{-3}\,\mathrm{A}.$$

2. According to $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$, the current amplitude is

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.00\times10^{-6}\,\text{C}}{\sqrt{(1.10\times10^{-3}\,\text{H})(4.00\times10^{-6}\,\text{F})}} = 4.52\times10^{-2}\,\text{A}.$$

3. We find the capacitance from $U = \frac{1}{2}Q^2/C$:

$$C = \frac{Q^2}{2U} = \frac{\left(1.60\times 10^{-6}\,\text{C}\right)^2}{2\left(140\times 10^{-6}\,\text{J}\right)} = 9.14\times 10^{-9}\ \text{F}.$$

4. (a) The period is $T = 4(1.50\ \mu\text{s}) = 6.00\ \mu\text{s}$.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{6.00\mu\text{s}} = 1.67 \times 10^5\ \text{Hz}.$$

(c) The magnetic energy does not depend on the direction of the current (since $U_B \propto i^2$), so this will occur after one-half of a period, or $3.00\ \mu\text{s}$.

5. (a) We recall the fact that the period is the reciprocal of the frequency. It is helpful to refer also to Fig. 31-1. The values of t when plate A will again have maximum positive charge are multiples of the period:

$$t_A = nT = \frac{n}{f} = \frac{n}{2.00\times 10^3\ \text{Hz}} = n(5.00\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n=1) $t_A = 5.00\,\mu$s.

(b) We note that it takes $t = \frac{1}{2}T$ for the charge on the other plate to reach its maximum positive value for the first time (compare steps a and e in Fig. 31-1). This is when plate A acquires its most negative charge. From that time onward, this situation will repeat once every period. Consequently,

$$t = \frac{1}{2}T + (n-1)T = \frac{1}{2}(2n-1)T = \frac{(2n-1)}{2f} = \frac{(2n-1)}{2\left(2\times 10^3\,\text{Hz}\right)} = (2n-1)(2.50\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n=1) $t = 2.50\,\mu$s.

(c) At $t = \frac{1}{4}T$, the current and the magnetic field in the inductor reach maximum values for the first time (compare steps a and c in Fig. 31-1). Later this will repeat every half-period (compare steps c and g in Fig. 31-1). Therefore,

$$t_L = \frac{T}{4} + \frac{(n-1)T}{2} = (2n-1)\frac{T}{4} = (2n-1)(1.25\,\mu\text{s}),$$

where n = 1, 2, 3, 4, …. The earliest time is (n=1) $t = 1.25\,\mu$s.

6. (a) The angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{F/x}{m}} = \sqrt{\frac{8.0\,\text{N}}{\left(2.0\times 10^{-13}\,\text{m}\right)\left(0.50\,\text{kg}\right)}} = 89\,\text{rad/s}.$$

(b) The period is $1/f$ and $f = \omega/2\pi$. Therefore,

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{89\,\text{rad/s}} = 7.0\times 10^{-2}\,\text{s}.$$

(c) From $\omega = (LC)^{-1/2}$, we obtain

$$C = \frac{1}{\omega^2 L} = \frac{1}{\left(89\,\text{rad/s}\right)^2\left(5.0\,\text{H}\right)} = 2.5\times 10^{-5}\,\text{F}.$$

7. (a) The mass m corresponds to the inductance, so m = 1.25 kg.

(b) The spring constant k corresponds to the reciprocal of the capacitance. Since the total energy is given by $U = Q^2/2C$, where Q is the maximum charge on the capacitor and C is the capacitance,

$$C = \frac{Q^2}{2U} = \frac{\left(175\times10^{-6}\text{ C}\right)^2}{2\left(5.70\times10^{-6}\text{ J}\right)} = 2.69\times10^{-3}\text{ F}$$

and

$$k = \frac{1}{2.69\times10^{-3}\text{ m/N}} = 372\text{ N/m.}$$

(c) The maximum displacement corresponds to the maximum charge, so $x_{\text{max}} = 1.75\times10^{-4}$ m.

(d) The maximum speed v_{max} corresponds to the maximum current. The maximum current is

$$I = Q\omega = \frac{Q}{\sqrt{LC}} = \frac{175\times10^{-6}\text{ C}}{\sqrt{(1.25\text{ H})\left(2.69\times10^{-3}\text{ F}\right)}} = 3.02\times10^{-3}\text{ A.}$$

Consequently, $v_{\text{max}} = 3.02\times10^{-3}$ m/s.

8. We apply the loop rule to the entire circuit:

$$\varepsilon_{\text{total}} = \varepsilon_{L_1} + \varepsilon_{C_1} + \varepsilon_{R_1} + \cdots = \sum_j \left(\varepsilon_{L_j} + \varepsilon_{C_j} + \varepsilon_{R_j} \right) = \sum_j \left(L_j \frac{di}{dt} + \frac{q}{C_j} + iR_j \right)$$

$$= L\frac{di}{dt} + \frac{q}{C} + iR \qquad \text{with} \quad L = \sum_j L_j, \ \frac{1}{C} = \sum_j \frac{1}{C_j}, \ R = \sum_j R_j$$

where we require $\varepsilon_{\text{total}} = 0$. This is equivalent to the simple *LRC* circuit shown in Fig. 31-24(b).

9. The time required is t = $T/4$, where the period is given by $T = 2\pi/\omega = 2\pi\sqrt{LC}$. Consequently,

$$t = \frac{T}{4} = \frac{2\pi\sqrt{LC}}{4} = \frac{2\pi\sqrt{(0.050\,\text{H})(4.0\times10^{-6}\,\text{F})}}{4} = 7.0\times10^{-4}\,\text{s}.$$

10. We find the inductance from $f = \omega / 2\pi = \left(2\pi\sqrt{LC}\right)^{-1}$.

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \left(10\times 10^3 \text{ Hz}\right)^2 \left(6.7\times 10^{-6} \text{ F}\right)} = 3.8\times 10^{-5} \text{ H}.$$

11. (a) $Q = CV_{max} = (1.0 \times 10^{-9}\ \text{F})(3.0\ \text{V}) = 3.0 \times 10^{-9}$ C.

(b) From $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ we get

$$I = \frac{Q}{\sqrt{LC}} = \frac{3.0\times10^{-9}\ \text{C}}{\sqrt{(3.0\times10^{-3}\ \text{H})(1.0\times10^{-9}\ \text{F})}} = 1.7\times10^{-3}\ \text{A}.$$

(c) When the current is at a maximum, the magnetic field is at maximum:

$$U_{B,\max} = \frac{1}{2}LI^2 = \frac{1}{2}(3.0\times10^{-3}\ \text{H})(1.7\times10^{-3}\ \text{A})^2 = 4.5\times10^{-9}\ \text{J}.$$

12. The capacitors C_1 and C_2 can be used in four different ways: (1) C_1 only; (2) C_2 only; (3) C_1 and C_2 in parallel; and (4) C_1 and C_2 in series.

(a) The smallest oscillation frequency is

$$f_3 = \frac{1}{2\pi\sqrt{L(C_1+C_2)}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\text{ H})(2.0\times10^{-6}\text{ F}+5.0\times10^{-6}\text{ F})}} = 6.0\times10^{2}\text{ Hz}$$

(b) The second smallest oscillation frequency is

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\text{ H})(5.0\times10^{-6}\text{ F})}} = 7.1\times10^{2}\text{ Hz}$$

(c) The second largest oscillation frequency is

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}} = \frac{1}{2\pi\sqrt{(1.0\times10^{-2}\text{ H})(2.0\times10^{-6}\text{ F})}} = 1.1\times10^{3}\text{ Hz}$$

(d) The largest oscillation frequency is

$$f_4 = \frac{1}{2\pi\sqrt{LC_1C_2/(C_1+C_2)}} = \frac{1}{2\pi}\sqrt{\frac{2.0\times10^{-6}\text{ F}+5.0\times10^{-6}\text{ F}}{(1.0\times10^{-2}\text{ H})(2.0\times10^{-6}\text{ F})(5.0\times10^{-6}\text{ F})}} = 1.3\times10^{3}\text{ Hz}$$

13. (a) After the switch is thrown to position *b* the circuit is an *LC* circuit. The angular frequency of oscillation is $\omega = 1/\sqrt{LC}$. Consequently,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(54.0\times10^{-3}\text{ H})(6.20\times10^{-6}\text{ F})}} = 275\text{ Hz}.$$

(b) When the switch is thrown, the capacitor is charged to $V = 34.0$ V and the current is zero. Thus, the maximum charge on the capacitor is $Q = VC = (34.0\text{ V})(6.20\times10^{-6}\text{ F}) = 2.11\times10^{-4}$ C. The current amplitude is

$$I = \omega Q = 2\pi f Q = 2\pi(275\text{ Hz})(2.11\times10^{-4}\text{ C}) = 0.365\text{A}.$$

14. For the first circuit $\omega = (L_1C_1)^{-1/2}$, and for the second one $\omega = (L_2C_2)^{-1/2}$. When the two circuits are connected in series, the new frequency is

$$\omega' = \frac{1}{\sqrt{L_{\text{eq}}C_{\text{eq}}}} = \frac{1}{\sqrt{(L_1+L_2)C_1C_2/(C_1+C_2)}} = \frac{1}{\sqrt{(L_1C_1C_2+L_2C_2C_1)/(C_1+C_2)}}$$
$$= \frac{1}{\sqrt{L_1C_1}}\frac{1}{\sqrt{(C_1+C_2)/(C_1+C_2)}} = \omega,$$

where we use $\omega^{-1} = \sqrt{L_1C_1} = \sqrt{L_2C_2}$.

15. (a) Since the frequency of oscillation f is related to the inductance L and capacitance C by $f = 1/2\pi\sqrt{LC}$, the smaller value of C gives the larger value of f. Consequently, $f_{\max} = 1/2\pi\sqrt{LC_{\min}}$, $f_{\min} = 1/2\pi\sqrt{LC_{\max}}$, and

$$\frac{f_{\max}}{f_{\min}} = \frac{\sqrt{C_{\max}}}{\sqrt{C_{\min}}} = \frac{\sqrt{365\,\text{pF}}}{\sqrt{10\,\text{pF}}} = 6.0.$$

(b) An additional capacitance C is chosen so the ratio of the frequencies is

$$r = \frac{1.60\,\text{MHz}}{0.54\,\text{MHz}} = 2.96.$$

Since the additional capacitor is in parallel with the tuning capacitor, its capacitance adds to that of the tuning capacitor. If C is in picofarads, then

$$\frac{\sqrt{C + 365\,\text{pF}}}{\sqrt{C + 10\,\text{pF}}} = 2.96.$$

The solution for C is

$$C = \frac{(365\,\text{pF}) - (2.96)^2(10\,\text{pF})}{(2.96)^2 - 1} = 36\,\text{pF}.$$

(c) We solve $f = 1/2\pi\sqrt{LC}$ for L. For the minimum frequency C = 365 pF + 36 pF = 401 pF and f = 0.54 MHz. Thus

$$L = \frac{1}{(2\pi)^2 C f^2} = \frac{1}{(2\pi)^2 (401 \times 10^{-12}\,\text{F})(0.54 \times 10^6\,\text{Hz})^2} = 2.2 \times 10^{-4}\,\text{H}.$$

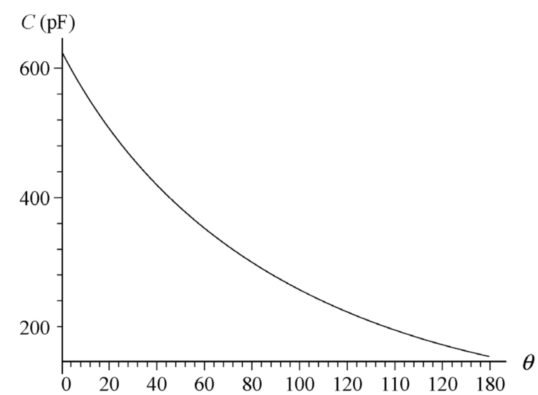
16. The linear relationship between θ (the knob angle in degrees) and frequency f is

$$f = f_0\left(1+\frac{\theta}{180°}\right) \Rightarrow \theta = 180°\left(\frac{f}{f_0}-1\right)$$

where $f_0 = 2 \times 10^5$ Hz. Since $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, we are able to solve for C in terms of θ:

$$C = \frac{1}{4\pi^2 L f_0^2\left(1+\frac{\theta}{180°}\right)^2} = \frac{81}{400000\pi^2\left(180°+\theta\right)^2}$$

with SI units understood. After multiplying by 10^{12} (to convert to picofarads), this is plotted, below.



17. (a) The total energy U is the sum of the energies in the inductor and capacitor:

$$U = U_E + U_B = \frac{q^2}{2C} + \frac{i^2 L}{2} = \frac{\left(3.80\times10^{-6}\,\text{C}\right)^2}{2\left(7.80\times10^{-6}\,\text{F}\right)} + \frac{\left(9.20\times10^{-3}\,\text{A}\right)^2\left(25.0\times10^{-3}\,\text{H}\right)}{2} = 1.98\times10^{-6}\,\text{J}.$$

(b) We solve $U = Q^2/2C$ for the maximum charge:

$$Q = \sqrt{2CU} = \sqrt{2\left(7.80\times10^{-6}\,\text{F}\right)\left(1.98\times10^{-6}\,\text{J}\right)} = 5.56\times10^{-6}\,\text{C}.$$

(c) From $U = I^2L/2$, we find the maximum current:

$$I = \sqrt{\frac{2U}{L}} = \sqrt{\frac{2\left(1.98\times10^{-6}\,\text{J}\right)}{25.0\times10^{-3}\,\text{H}}} = 1.26\times10^{-2}\,\text{A}.$$

(d) If q_0 is the charge on the capacitor at time $t = 0$, then $q_0 = Q\cos\phi$ and

$$\phi = \cos^{-1}\left(\frac{q}{Q}\right) = \cos^{-1}\left(\frac{3.80\times10^{-6}\,\text{C}}{5.56\times10^{-6}\,\text{C}}\right) = \pm 46.9°.$$

For $\phi = +46.9°$ the charge on the capacitor is decreasing, for $\phi = -46.9°$ it is increasing. To check this, we calculate the derivative of q with respect to time, evaluated for $t = 0$. We obtain $-\omega Q\sin\phi$, which we wish to be positive. Since sin(+46.9°) is positive and sin(–46.9°) is negative, the correct value for increasing charge is $\phi = -46.9°$.

(e) Now we want the derivative to be negative and $\sin\phi$ to be positive. Thus, we take $\phi = +46.9°$.

18. (a) Since the percentage of energy stored in the electric field of the capacitor is $(1-75.0\%) = 25.0\%$, then

$$\frac{U_E}{U} = \frac{q^2/2C}{Q^2/2C} = 25.0\%$$

which leads to $q/Q = \sqrt{0.250} = 0.500$.

(b) From

$$\frac{U_B}{U} = \frac{Li^2/2}{LI^2/2} = 75.0\%,$$

we find $i/I = \sqrt{0.750} = 0.866$.

19. (a) The charge (as a function of time) is given by $q = Q \sin \omega t$, where Q is the maximum charge on the capacitor and ω is the angular frequency of oscillation. A sine function was chosen so that $q = 0$ at time $t = 0$. The current (as a function of time) is

$$i = \frac{dq}{dt} = \omega Q \cos \omega t,$$

and at $t = 0$, it is $I = \omega Q$. Since $\omega = 1/\sqrt{LC}$,

$$Q = I\sqrt{LC} = (2.00\,\text{A})\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 1.80\times10^{-4}\,\text{C}.$$

(b) The energy stored in the capacitor is given by

$$U_E = \frac{q^2}{2C} = \frac{Q^2 \sin^2 \omega t}{2C}$$

and its rate of change is

$$\frac{dU_E}{dt} = \frac{Q^2 \omega \sin \omega t \cos \omega t}{C}$$

We use the trigonometric identity $\cos \omega t \sin \omega t = \frac{1}{2}\sin(2\omega t)$ to write this as

$$\frac{dU_E}{dt} = \frac{\omega Q^2}{2C}\sin(2\omega t).$$

The greatest rate of change occurs when $\sin(2\omega t) = 1$ or $2\omega t = \pi/2$ rad. This means

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4}\sqrt{LC} = \frac{\pi}{4}\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 7.07\times10^{-5}\,\text{s}.$$

(c) Substituting $\omega = 2\pi/T$ and $\sin(2\omega t) = 1$ into $dU_E/dt = (\omega Q^2/2C)\sin(2\omega t)$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{2\pi Q^2}{2TC} = \frac{\pi Q^2}{TC}.$$

Now $T = 2\pi\sqrt{LC} = 2\pi\sqrt{(3.00\times10^{-3}\,\text{H})(2.70\times10^{-6}\,\text{F})} = 5.655\times10^{-4}\,\text{s}$, so

$$\left(\frac{dU_E}{dt}\right)_{max} = \frac{\pi\left(1.80\times10^{-4}\text{ C}\right)^2}{\left(5.655\times10^{-4}\text{ s}\right)\left(2.70\times10^{-6}\text{ F}\right)} = 66.7\text{ W}.$$

We note that this is a positive result, indicating that the energy in the capacitor is indeed increasing at $t = T/8$.

20. (a) We use $U = \frac{1}{2}LI^2 = \frac{1}{2}Q^2/C$ to solve for L:

$$L = \frac{1}{C}\left(\frac{Q}{I}\right)^2 = \frac{1}{C}\left(\frac{CV_{\max}}{I}\right)^2 = C\left(\frac{V_{\max}}{I}\right)^2 = \left(4.00\times10^{-6}\,\text{F}\right)\left(\frac{1.50\,\text{V}}{50.0\times10^{-3}\,\text{A}}\right)^2 = 3.60\times10^{-3}\,\text{H}.$$

(b) Since $f = \omega/2\pi$, the frequency is

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\left(3.60\times10^{-3}\,\text{H}\right)\left(4.00\times10^{-6}\,\text{F}\right)}} = 1.33\times10^{3}\,\text{Hz}.$$

(c) Referring to Fig. 31-1, we see that the required time is one-fourth of a period (where the period is the reciprocal of the frequency). Consequently,

$$t = \frac{1}{4}T = \frac{1}{4f} = \frac{1}{4\left(1.33\times10^{3}\ \text{Hz}\right)} = 1.88\times10^{-4}\ \text{s}.$$

21. (a) We compare this expression for the current with $i = I\sin(\omega t+\phi_0)$. Setting $(\omega t+\phi) = 2500t + 0.680 = \pi/2$, we obtain $t = 3.56 \times 10^{-4}$ s.

(b) Since $\omega = 2500$ rad/s $= (LC)^{-1/2}$,

$$L = \frac{1}{\omega^2 C} = \frac{1}{(2500\,\text{rad/s})^2(64.0\times10^{-6}\,\text{F})} = 2.50\times10^{-3}\ \text{H}.$$

(c) The energy is

$$U = \frac{1}{2}LI^2 = \frac{1}{2}(2.50\times10^{-3}\,\text{H})(1.60\,\text{A})^2 = 3.20\times10^{-3}\ \text{J}.$$

22. (a) From $V = IX_C$ we find $\omega = I/CV$. The period is then $T = 2\pi/\omega = 2\pi CV/I = 46.1$ μs.

(b) $\frac{1}{2}CV^2 = 6.88$ nJ.

(c) The answer is again 6.88 nJ (see Fig. 31-4).

(d) We apply Eq. 30-35 as $V = L(di/dt)_{max}$. We can substitute $L = CV^2/I^2$ (combining what we found in part (a) with Eq. 31-4) into Eq. 30-35 (as written above) and solve for $(di/dt)_{max}$. Our result is 1.02×10^3 A/s.

(e) The derivative of $U = \frac{1}{2}Li^2$ leads to $dU/dt = LI^2\omega \sin(\omega t)\cos(\omega t) = \frac{1}{2}LI^2\omega \sin(2\omega t)$. Therefore, $(dU/dt)_{max} = \frac{1}{2}LI^2\omega = \frac{1}{2}IV = 0.938$ mW.

23. The loop rule, for just two devices in the loop, reduces to the statement that the magnitude of the voltage across one of them must equal the magnitude of the voltage across the other. Consider that the capacitor has charge q and a voltage (which we'll consider positive in this discussion) $V = q/C$. Consider at this moment that the current in the inductor at this moment is directed in such a way that the capacitor charge is increasing (so $i = +dq/dt$). Eq. 30-35 then produces a positive result equal to the V across the capacitor: $V = -L(di/dt)$, and we interpret the fact that $-di/dt > 0$ in this discussion to mean that $d(dq/dt)/dt = d^2q/dt^2 < 0$ represents a "deceleration" of the charge-buildup process on the capacitor (since it is approaching its maximum value of charge). In this way we can "check" the signs in Eq. 31-11 (which states $q/C = -L\,d^2q/dt^2$) to make sure we have implemented the loop rule correctly.

24. The charge q after N cycles is obtained by substituting $t = NT = 2\pi N/\omega'$ into Eq. 31-25:

$$
\begin{aligned}
q &= Qe^{-Rt/2L}\cos\left(\omega' t+\phi\right) = Qe^{-RNT/2L}\cos\left[\omega'\left(2\pi N/\omega'\right)+\phi\right] \\
&= Qe^{-RN\left(2\pi\sqrt{L/C}\right)/2L}\cos\left(2\pi N+\phi\right) \\
&= Qe^{-N\pi R\sqrt{C/L}}\cos\phi .
\end{aligned}
$$

We note that the initial charge (setting $N = 0$ in the above expression) is $q_0 = Q\cos\phi$, where $q_0 = 6.2\ \mu\text{C}$ is given (with 3 significant figures understood). Consequently, we write the above result as $q_N = q_0 e^{-N\pi R\sqrt{C/L}}$.

(a) For $N = 5$,

$$q_5 = \left(6.2\,\mu\text{C}\right)e^{-5\pi(7.2\Omega)\sqrt{0.0000032\text{ F}/12\text{ H}}} = 5.85\,\mu\text{C}.$$

(b) For $N = 10$,

$$q_{10} = \left(6.2\,\mu\text{C}\right)e^{-10\pi(7.2\Omega)\sqrt{0.0000032\text{ F}/12\text{ H}}} = 5.52\,\mu\text{C}.$$

(c) For $N = 100$,

$$q_{100} = \left(6.2\,\mu\text{C}\right)e^{-100\pi(7.2\Omega)\sqrt{0.0000032\text{ F}/12\text{ H}}} = 1.93\,\mu\text{C}.$$

25. Since $\omega \approx \omega'$, we may write $T = 2\pi/\omega$ as the period and $\omega = 1/\sqrt{LC}$ as the angular frequency. The time required for 50 cycles (with 3 significant figures understood) is

$$t = 50T = 50\left(\frac{2\pi}{\omega}\right) = 50\left(2\pi\sqrt{LC}\right) = 50\left(2\pi\sqrt{\left(220\times10^{-3}\text{ H}\right)\left(12.0\times10^{-6}\text{ F}\right)}\right)$$
$$= 0.5104\text{ s}.$$

The maximum charge on the capacitor decays according to $q_{\max} = Qe^{-Rt/2L}$ (this is called the *exponentially decaying amplitude* in §31-5), where Q is the charge at time $t = 0$ (if we take $\phi = 0$ in Eq. 31-25). Dividing by Q and taking the natural logarithm of both sides, we obtain

$$\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{Rt}{2L}$$

which leads to

$$R = -\frac{2L}{t}\ln\left(\frac{q_{\max}}{Q}\right) = -\frac{2\left(220\times10^{-3}\text{ H}\right)}{0.5104\text{ s}}\ln\left(0.99\right) = 8.66\times10^{-3}\ \Omega.$$

26. The assumption stated at the end of the problem is equivalent to setting $\phi = 0$ in Eq. 31-25. Since the maximum energy in the capacitor (each cycle) is given by $q_{max}^2/2C$, where q_{max} is the maximum charge (during a given cycle), then we seek the time for which

$$\frac{q_{max}^2}{2C} = \frac{1}{2}\frac{Q^2}{2C} \Rightarrow q_{max} = \frac{Q}{\sqrt{2}}.$$

Now q_{max} (referred to as the *exponentially decaying amplitude* in §31-5) is related to Q (and the other parameters of the circuit) by

$$q_{max} = Qe^{-Rt/2L} \Rightarrow \ln\left(\frac{q_{max}}{Q}\right) = -\frac{Rt}{2L}.$$

Setting $q_{max} = Q/\sqrt{2}$, we solve for t:

$$t = -\frac{2L}{R}\ln\left(\frac{q_{max}}{Q}\right) = -\frac{2L}{R}\ln\left(\frac{1}{\sqrt{2}}\right) = \frac{L}{R}\ln 2 .$$

The identities $\ln(1/\sqrt{2}) = -\ln\sqrt{2} = -\frac{1}{2}\ln 2$ were used to obtain the final form of the result.

27. Let t be a time at which the capacitor is fully charged in some cycle and let $q_{\max 1}$ be the charge on the capacitor then. The energy in the capacitor at that time is

$$U(t) = \frac{q_{\max 1}^2}{2C} = \frac{Q^2}{2C} e^{-Rt/L}$$

where

$$q_{\max 1} = Q e^{-Rt/2L}$$

(see the discussion of the *exponentially decaying amplitude* in §31-5). One period later the charge on the fully charged capacitor is

$$q_{\max 2} = Q e^{-R(t+T)2/L} \qquad \text{where } T = \frac{2\pi}{\omega'} ,$$

and the energy is

$$U(t+T) = \frac{q_{\max 2}^2}{2C} = \frac{Q^2}{2C} e^{-R(t+T)/L} .$$

The fractional loss in energy is

$$\frac{|\Delta U|}{U} = \frac{U(t) - U(t+T)}{U(t)} = \frac{e^{-Rt/L} - e^{-R(t+T)/L}}{e^{-Rt/L}} = 1 - e^{-RT/L} .$$

Assuming that RT/L is very small compared to 1 (which would be the case if the resistance is small), we expand the exponential (see Appendix E). The first few terms are:

$$e^{-RT/L} \approx 1 - \frac{RT}{L} + \frac{R^2T^2}{2L^2} + \cdots .$$

If we approximate $\omega \approx \omega'$, then we can write T as $2\pi/\omega$. As a result, we obtain

$$\frac{|\Delta U|}{U} \approx 1 - \left(1 - \frac{RT}{L} + \cdots\right) \approx \frac{RT}{L} = \frac{2\pi R}{\omega L}.$$

28. (a) We use $I = \varepsilon/X_c = \omega_d C\varepsilon$:

$$I = \omega_d C\varepsilon_m = 2\pi f_d C\varepsilon_m = 2\pi(1.00\times10^3\,\text{Hz})(1.50\times10^{-6}\,\text{F})(30.0\ \text{V}) = 0.283\ \text{A}\ .$$

(b) $I = 2\pi(8.00 \times 10^3\ \text{Hz})(1.50 \times 10^{-6}\ \text{F})(30.0\ \text{V}) = 2.26\ \text{A}$.

29. (a) The current amplitude I is given by $I = V_L/X_L$, where $X_L = \omega_d L = 2\pi f_d L$. Since the circuit contains only the inductor and a sinusoidal generator, $V_L = \varepsilon_m$. Therefore,

$$I = \frac{V_L}{X_L} = \frac{\varepsilon_m}{2\pi f_d L} = \frac{30.0\,\text{V}}{2\pi(1.00\times10^3\,\text{Hz})(50.0\times10^{-3}\,\text{H})} = 0.0955\,\text{A} = 95.5\text{ mA}.$$

(b) The frequency is now eight times larger than in part (a), so the inductive reactance X_L is eight times larger and the current is one-eighth as much. The current is now

$$I = (0.0955\text{ A})/8 = 0.0119\text{ A} = 11.9\text{ mA}.$$

30. (a) The current through the resistor is

$$I = \frac{\mathcal{E}_m}{R} = \frac{30.0\,\text{V}}{50.0\,\Omega} = 0.600\ \text{A}\,.$$

(b) Regardless of the frequency of the generator, the current is the same, $I = 0.600\ \text{A}$.

31. (a) The inductive reactance for angular frequency ω_d is given by $X_L = \omega_d L$, and the capacitive reactance is given by $X_C = 1/\omega_d C$. The two reactances are equal if $\omega_d L = 1/\omega_d C$, or $\omega_d = 1/\sqrt{LC}$. The frequency is

$$f_d = \frac{\omega_d}{2\pi} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(6.0\times10^{-3}\,\text{H})(10\times10^{-6}\,\text{F})}} = 6.5\times10^{2}\ \text{Hz}.$$

(b) The inductive reactance is

$$X_L = \omega_d L = 2\pi f_d L = 2\pi(650\ \text{Hz})(6.0\times10^{-3}\ \text{H}) = 24\ \Omega.$$

The capacitive reactance has the same value at this frequency.

(c) The natural frequency for free LC oscillations is $f = \omega/2\pi = 1/2\pi\sqrt{LC}$, the same as we found in part (a).

32. (a) The circuit consists of one generator across one inductor; therefore, $\mathcal{E}_m = V_L$. The current amplitude is

$$I = \frac{\mathcal{E}_m}{X_L} = \frac{\mathcal{E}_m}{\omega_d L} = \frac{25.0 \text{ V}}{(377 \text{ rad/s})(12.7 \text{ H})} = 5.22\times10^{-3} \text{ A} .$$

(b) When the current is at a maximum, its derivative is zero. Thus, Eq. 30-35 gives $\mathcal{E}_L = 0$ at that instant. Stated another way, since $\mathcal{E}(t)$ and $i(t)$ have a 90° phase difference, then $\mathcal{E}(t)$ must be zero when $i(t) = I$. The fact that $\phi = 90° = \pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\mathcal{E} = -\frac{1}{2}\mathcal{E}_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that $\mathcal{E}$ is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I \sin\left(2n\pi - \frac{5\pi}{6} - \frac{\pi}{2}\right) = (5.22\times10^{-3}\text{ A})\left(\frac{\sqrt{3}}{2}\right) = 4.51\times10^{-3} \text{ A} .$$

33. (a) The generator emf is a maximum when $\sin(\omega_d t - \pi/4) = 1$ or $\omega_d t - \pi/4 = (\pi/2) \pm 2n\pi$ [n = integer]. The first time this occurs after $t = 0$ is when $\omega_d t - \pi/4 = \pi/2$ (that is, $n = 0$). Therefore,

$$t = \frac{3\pi}{4\omega_d} = \frac{3\pi}{4(350 \text{ rad/s})} = 6.73 \times 10^{-3} \text{ s}.$$

(b) The current is a maximum when $\sin(\omega_d t - 3\pi/4) = 1$, or $\omega_d t - 3\pi/4 = (\pi/2) \pm 2n\pi$ [n = integer]. The first time this occurs after $t = 0$ is when $\omega_d t - 3\pi/4 = \pi/2$ (as in part (a), $n = 0$). Therefore,

$$t = \frac{5\pi}{4\omega_d} = \frac{5\pi}{4(350 \text{ rad/s})} = 1.12 \times 10^{-2} \text{ s}.$$

(c) The current lags the emf by $+\pi/2$ rad, so the circuit element must be an inductor.

(d) The current amplitude I is related to the voltage amplitude V_L by $V_L = IX_L$, where X_L is the inductive reactance, given by $X_L = \omega_d L$. Furthermore, since there is only one element in the circuit, the amplitude of the potential difference across the element must be the same as the amplitude of the generator emf: $V_L = \varepsilon_m$. Thus, $\varepsilon_m = I\omega_d L$ and

$$L = \frac{\varepsilon_m}{I\omega_d} = \frac{30.0\text{V}}{(620 \times 10^{-3}\text{A})(350 \text{ rad/s})} = 0.138 \text{ H}.$$

34. (a) The circuit consists of one generator across one capacitor; therefore, $\varepsilon_m = V_C$. Consequently, the current amplitude is

$$I = \frac{\varepsilon_m}{X_C} = \omega C \varepsilon_m = (377 \text{ rad/s})(4.15\times10^{-6}\text{F})(25.0 \text{ V}) = 3.91\times10^{-2}\text{A} .$$

(b) When the current is at a maximum, the charge on the capacitor is changing at its largest rate. This happens not when it is fully charged ($\pm q_{\max}$), but rather as it passes through the (momentary) states of being uncharged (q = 0). Since $q = CV$, then the voltage across the capacitor (and at the generator, by the loop rule) is zero when the current is at a maximum. Stated more precisely, the time-dependent emf $\varepsilon(t)$ and current $i(t)$ have a $\phi = -90°$ phase relation, implying $\varepsilon(t) = 0$ when $i(t) = I$. The fact that $\phi = -90°$ $= -\pi/2$ rad is used in part (c).

(c) Consider Eq. 32-28 with $\varepsilon = -\frac{1}{2}\varepsilon_m$. In order to satisfy this equation, we require $\sin(\omega_d t) = -1/2$. Now we note that the problem states that ε is increasing *in magnitude*, which (since it is already negative) means that it is becoming more negative. Thus, differentiating Eq. 32-28 with respect to time (and demanding the result be negative) we must also require $\cos(\omega_d t) < 0$. These conditions imply that ωt must equal $(2n\pi - 5\pi/6)$ [n = integer]. Consequently, Eq. 31-29 yields (for all values of n)

$$i = I \sin\left(2n\pi - \frac{5\pi}{6} + \frac{\pi}{2}\right) = (3.91\times10^{-3}\text{ A})\left(-\frac{\sqrt{3}}{2}\right) = -3.38\times10^{-2}\text{ A},$$

or $|i| = 3.38\times10^{-2}$ A.

35. (a) Now $X_C = 0$, while $R = 200\ \Omega$ and $X_L = \omega L = 2\pi f_d L = 86.7\ \Omega$ remain unchanged. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{(200\ \Omega)^2 + (86.7\ \Omega)^2} = 218\ \Omega\ .$$

(b) The phase angle is, from Eq. 31-65,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 0}{200\ \Omega}\right) = 23.4°\ .$$

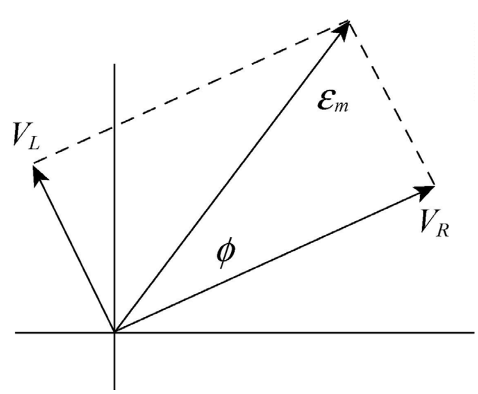
(c) The current amplitude is now found to be

$$I = \frac{\mathcal{E}_m}{Z} = \frac{36.0\ \text{V}}{218\ \Omega} = 0.165\ \text{A}\ .$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.165\ \text{A})(200\Omega) \approx 33\,\text{V}$$
$$V_L = IX_L = (0.165\,\text{A})(86.7\Omega) \approx 14.3\,\text{V}$$

This is an inductive circuit, so $\mathcal{E}_m$ leads I. The phasor diagram is drawn to scale below.



36. (a) The graph shows that the resonance angular frequency is 25000 rad/s, which means (using Eq. 31-4)

$$C = (\omega^2 L)^{-1} = [(25000)^2 \times 200 \times 10^{-6}]^{-1} = 8.0\ \mu\text{F}.$$

(b) The graph also shows that the current amplitude at resonance is 4.0 A, but at resonance the impedance Z becomes purely resistive ($Z = R$) so that we can divide the emf amplitude by the current amplitude at resonance to find R: 8.0/4.0 = 2.0 Ω.

37. (a) Now $X_L = 0$, while $R = 200\ \Omega$ and $X_C = 1/2\pi f_d C = 177\ \Omega$. Therefore, the impedance is

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{(200\,\Omega)^2 + (177\,\Omega)^2} = 267\,\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 177\,\Omega}{200\,\Omega}\right) = -41.5°$$

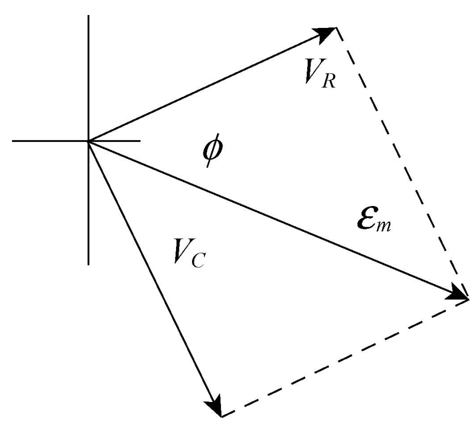
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\ \text{V}}{267\,\Omega} = 0.135\ \text{A}\ .$$

(d) We first find the voltage amplitudes across the circuit elements:

$$V_R = IR = (0.135\,\text{A})(200\Omega) \approx 27.0\,\text{V}$$
$$V_C = IX_C = (0.135\,\text{A})(177\Omega) \approx 23.9\,\text{V}$$

The circuit is capacitive, so I leads ε_m. The phasor diagram is drawn to scale next.



38. (a) The circuit has a resistor and a capacitor (but no inductor). Since the capacitive reactance decreases with frequency, then the asymptotic value of Z must be the resistance: $R = 500\ \Omega$.

(b) We describe three methods here (each using information from different points on the graph):

<u>method 1</u>: At $\omega_d = 50$ rad/s, we have $Z \approx 700\ \Omega$ which gives $C = (\omega_d\sqrt{Z^2 - R^2}\,)^{-1} = 41\ \mu\text{F}$.

<u>method 2</u>: At $\omega_d = 50$ rad/s, we have $X_C \approx 500\ \Omega$ which gives $C = (\omega_d X_C)^{-1} = 40\ \mu\text{F}$.

<u>method 3</u>: At $\omega_d = 250$ rad/s, we have $X_C \approx 100\ \Omega$ which gives $C = (\omega_d X_C)^{-1} = 40\ \mu\text{F}$.

39. (a) The capacitive reactance is

$$X_C = \frac{1}{\omega_d C} = \frac{1}{2\pi f_d C} = \frac{1}{2\pi(60.0\text{ Hz})(70.0\times10^{-6}\text{F})} = 37.9\,\Omega\,.$$

The inductive reactance 86.7 Ω is unchanged. The new impedance is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(200\,\Omega)^2 + (37.9\,\Omega - 86.7\,\Omega)^2} = 206\,\Omega.$$

(b) The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{86.7\,\Omega - 37.9\,\Omega}{200\,\Omega}\right) = 13.7^\circ.$$

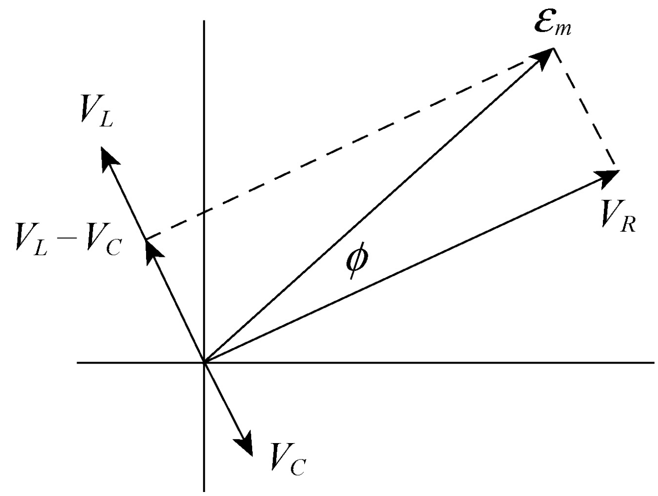
(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{36.0\text{ V}}{206\,\Omega} = 0.175\,\text{A}.$$

(d) We first find the voltage amplitudes across the circuit elements:

$$\begin{aligned} V_R &= IR = (0.175\text{ A})(200\,\Omega) = 35.0\text{ V} \\ V_L &= IX_L = (0.175\text{ A})(86.7\,\Omega) = 15.2\text{ V} \\ V_C &= IX_C = (0.175\text{ A})(37.9\,\Omega) = 6.62\,\text{V} \end{aligned}$$

Note that $X_L > X_C$, so that ε_m leads *I*. The phasor diagram is drawn to scale below.


ε_m
V_L
$V_L - V_C$
ϕ
V_R
V_C

40. (a) Since $Z=\sqrt{R^2+X_L^2}$ and $X_L = \omega_d L$, then as $\omega_d \to 0$ we find $Z \to R = 40\ \Omega$.

(b) $L = X_L/\omega_d = slope = 60$ mH.

41. The resistance of the coil is related to the reactances and the phase constant by Eq. 31-65. Thus,

$$\frac{X_L - X_C}{R} = \frac{\omega_d L - 1/\omega_d C}{R} = \tan\phi ,$$

which we solve for *R*:

$$R = \frac{1}{\tan\phi}\left(\omega_d L - \frac{1}{\omega_d C}\right) = \frac{1}{\tan 75°}\left[(2\pi)(930\,\text{Hz}(8.8\times10^{-2}\,\text{H}) - \frac{1}{(2\pi)(930\,\text{Hz})(0.94\times10^{-6}\text{F}}\right]$$

$$= 89\,\Omega.$$

42. A phasor diagram very much like Fig. 31-11(c) leads to the condition:

$$V_L - V_C = (6.00 \text{ V})\sin(30^\circ) = 3.00 \text{ V}.$$

With the magnitude of the capacitor voltage at 5.00 V, this gives a inductor voltage magnitude equal to 8.00 V.

43. (a) Yes, the voltage amplitude across the inductor can be much larger than the amplitude of the generator emf.

(b) The amplitude of the voltage across the inductor in an *RLC* series circuit is given by $V_L = IX_L = I\omega_d L$. At resonance, the driving angular frequency equals the natural angular frequency: $\omega_d = \omega = 1/\sqrt{LC}$. For the given circuit

$$X_L = \frac{L}{\sqrt{LC}} = \frac{1.0\text{ H}}{\sqrt{(1.0\text{ H})(1.0\times10^{-6}\text{F})}} = 1000\ \Omega .$$

At resonance the capacitive reactance has this same value, and the impedance reduces simply: $Z = R$. Consequently,

$$I = \left.\frac{\varepsilon_m}{Z}\right|_{\text{resonance}} = \frac{\varepsilon_m}{R} = \frac{10\text{ V}}{10\ \Omega} = 1.0\text{ A} .$$

The voltage amplitude across the inductor is therefore

$$V_L = IX_L = (1.0\text{ A})(1000\ \Omega) = 1.0\times10^{3}\text{ V}$$

which is much larger than the amplitude of the generator emf.

44. (a) With both switches closed (which effectively removes the resistor from the circuit), the impedance is just equal to the (net) reactance and is equal to (12 V)/(0.447 A) = 26.85 Ω. With switch 1 closed but switch 2 open, we have the same (net) reactance as just discussed, but now the resistor is part of the circuit; using Eq. 31-65 we find

$$R = X_{\text{net}} / \tan\phi = 26.85/\tan(15^\circ) = 100\ \Omega.$$

(b) For the first situation described in the problem (both switches open) we can reverse our reasoning of part (a) and find $X_{\text{net first}} = R\tan(-30.9^\circ) = -59.96\ \Omega$. We observe that the effect of switch 1 implies

$$X_C = X_{\text{net}} - X_{\text{net firsrt}} = 26.85 - (-59.96) = 86.81\ \Omega.$$

Then Eq. 31-39 leads to $C = 1/\omega X_C = 30.6\ \mu\text{F}$.

(c) Since $X_{\text{net}} = X_L - X_C$, then we find $L = X_L/\omega = 301$ mH

45. (a) For a given amplitude ε_m of the generator emf, the current amplitude is given by

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}.$$

We find the maximum by setting the derivative with respect to ω_d equal to zero:

$$\frac{dI}{d\omega_d} = -(E)_m [R^2 + (\omega_d L - 1/\omega_d C)^2]^{-3/2} \left[\omega_d L - \frac{1}{\omega_d C}\right]\left[L + \frac{1}{\omega_d^2 C}\right].$$

The only factor that can equal zero is $\omega_d L - (1/\omega_d C)$; it does so for $\omega_d = 1/\sqrt{LC} = \omega$. For this

$$\omega_d = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(1.00 \text{ H})(20.0\times 10^{-6}\text{ F})}} = 224 \text{ rad/s}.$$

(b) When $\omega_d = \omega$, the impedance is $Z = R$, and the current amplitude is

$$I = \frac{\varepsilon_m}{R} = \frac{30.0 \text{ V}}{5.00\ \Omega} = 6.00 \text{ A}.$$

(c) We want to find the (positive) values of ω_d for which $I = \varepsilon_m / 2R$:

$$\frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}} = \frac{\varepsilon_m}{2R}.$$

This may be rearranged to yield

$$\left(\omega_d L - \frac{1}{\omega_d C}\right)^2 = 3R^2.$$

Taking the square root of both sides (acknowledging the two ± roots) and multiplying by $\omega_d C$, we obtain

$$\omega_d^2 (LC) \pm \omega_d \left(\sqrt{3}CR\right) - 1 = 0.$$

Using the quadratic formula, we find the smallest positive solution

$$\omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{-\sqrt{3}(20.0\times10^{-6}\ \text{F})(5.00\ \Omega)}{2(1.00\ \text{H})(20.0\times10^{-6}\text{F})}$$

$$+\frac{\sqrt{3(20.0\times10^{-6}\ \text{F})^2(5.00\ \Omega)^2 + 4(1.00\ \text{H})(20.0\times10^{-6}\text{F})}}{2(1.00\ \text{H})(20.0\times10^{-6}\text{F})}$$

$$= 219\ \text{rad/s}\ ,$$

(d) and the largest positive solution

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC} = \frac{+\sqrt{3}(20.0\times10^{-6}\ \text{F})(5.00\,\Omega)}{2(1.00\ \text{H})(20.0\times10^{-6}\text{F})}$$

$$+\frac{\sqrt{3(20.0\times10^{-6}\ \text{F})^2(5.00\ \Omega)^2 + 4(1.00\ \text{H})(20.0\times10^{-6}\text{F})}}{2(1.00\ \text{H})(20.0\times10^{-6}\text{F})}$$

$$= 228\ \text{rad/s}\ .$$

(e) The fractional width is

$$\frac{\omega_1 - \omega_2}{\omega_0} = \frac{228\,\text{rad/s} - 219\,\text{rad/s}}{224\ \text{rad/s}} = 0.040.$$

46. (a) The capacitive reactance is

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(400\ \text{Hz})(24.0\times 10^{-6}\text{F})} = 16.6\ \Omega\ .$$

(b) The impedance is

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi fL - X_C)^2} \\ &= \sqrt{(220\,\Omega)^2 + [2\pi(400\ \text{Hz})(150\times 10^{-3}\ \text{H}) - 16.6\ \Omega]^2} = 422\ \Omega\ . \end{aligned}$$

(c) The current amplitude is

$$I = \frac{\varepsilon_m}{Z} = \frac{220\,\text{V}}{422\,\Omega} = 0.521\ \text{A}\ .$$

(d) Now $X_C \propto C_{\text{eq}}^{-1}$. Thus, X_C increases as C_{eq} decreases.

(e) Now $C_{\text{eq}} = C/2$, and the new impedance is

$$Z = \sqrt{(220\ \Omega)^2 + [2\pi(400\ \text{Hz})(150\times 10^{-3}\ \text{H}) - 2(16.6\ \Omega)]^2} = 408\ \Omega < 422\ \Omega\ .$$

Therefore, the impedance decreases.

(f) Since $I \propto Z^{-1}$, it increases.

47. We use the expressions found in Problem 45:

$$\omega_1 = \frac{+\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}$$

$$\omega_2 = \frac{-\sqrt{3}CR + \sqrt{3C^2R^2 + 4LC}}{2LC}$$

We also use Eq. 31-4. Thus,

$$\frac{\Delta\omega_d}{\omega} = \frac{\omega_1 - \omega_2}{\omega} = \frac{2\sqrt{3}CR\sqrt{LC}}{2LC} = R\sqrt{\frac{3C}{L}}.$$

For the data of Problem 45,

$$\frac{\Delta\omega_d}{\omega} = (5.00\,\Omega)\sqrt{\frac{3(20.0\times10^{-6}\text{ F})}{1.00\text{ H}}} = 3.87\times10^{-2}.$$

This is in agreement with the result of Problem 45. The method of Problem 45, however, gives only one significant figure since two numbers close in value are subtracted ($\omega_1 - \omega_2$). Here the subtraction is done algebraically, and three significant figures are obtained.

48. (a) A sketch of the phasors would be very much like Fig. 31-10(c) but with the label "I_L" on the green arrow replaced with "V_R."

(b) We have $V_R = V_L$, which implies

$$I\,R = I\,X_L \;\rightarrow\; R = \omega_d\,L$$

which yields $f = \omega_d/2\pi = R/2\pi L = 318$ Hz.

(c) $\phi = \tan^{-1}(V_L/V_R) = +45°$.

(d) $\omega_d = R/L = 2.00\times10^3$ rad/s.

(e) $I = (6\text{ V})/\sqrt{R^2 + X_L^2} = 3/(40\sqrt{2}) \approx 53.0$ mA.

49. (a) Since $L_{eq} = L_1 + L_2$ and $C_{eq} = C_1 + C_2 + C_3$ for the circuit, the resonant frequency is

$$\omega = \frac{1}{2\pi\sqrt{L_{eq}C_{eq}}} = \frac{1}{2\pi\sqrt{(L_1 + L_2)(C_1 + C_2 + C_3)}}$$

$$= \frac{1}{2\pi\sqrt{(1.70\times10^{-3}\text{ H} + 2.30\times10^{-3}\text{ H})(4.00\times10^{-6}\text{ F} + 2.50\times10^{-6}\text{ F} + 3.50\times10^{-6}\text{ F})}}$$

$$= 796\,\text{Hz}.$$

(b) The resonant frequency does not depend on R so it will not change as R increases.

(c) Since $\omega \propto (L_1 + L_2)^{-1/2}$, it will decrease as L_1 increases.

(d) Since $\omega \propto C_{eq}^{-1/2}$ and C_{eq} decreases as C_3 is removed, ω will increase.

50. Since the impedance of the voltmeter is large, it will not affect the impedance of the circuit when connected in parallel with the circuit. So the reading will be 100 V in all three cases.

51. The average power dissipated in resistance R when the current is alternating is given by $P_{\text{avg}} = I_{\text{rms}}^2 R,$ where I_{rms} is the root-mean-square current. Since $I_{\text{rms}} = I/\sqrt{2}$, where I is the current amplitude, this can be written $P_{\text{avg}} = I^2R/2$. The power dissipated in the same resistor when the current i_d is direct is given by $P = i_d^2 R.$ Setting the two powers equal to each other and solving, we obtain

$$i_d = \frac{I}{\sqrt{2}} = \frac{2.60\,\text{A}}{\sqrt{2}} = 1.84\,\text{A}.$$

52. The amplitude (peak) value is

$$V_{\max} = \sqrt{2}V_{\text{rms}} = \sqrt{2}(100\,\text{V}) = 141\,\text{V}.$$

53. (a) Using Eq. 31-61, the impedance is

$$Z = \sqrt{(12.0\,\Omega)^2 + (1.30\,\Omega - 0)^2} = 12.1\,\Omega.$$

(b) We use the result of problem 54:

$$P_{\text{avg}} = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2} = \frac{(120\,\text{V})^2 (12.0\,\Omega)}{(12.07\,\Omega)^2} = 1.186\times10^3\ \text{W} \approx 1.19\times10^3\ \text{W}.$$

54. This circuit contains no reactances, so $\varepsilon_{\text{rms}} = I_{\text{rms}}R_{\text{total}}$. Using Eq. 31-71, we find the average dissipated power in resistor R is

$$P_R = I_{\text{rms}}^2 R = \left(\frac{\varepsilon_m}{r+R}\right)^2 R.$$

In order to maximize P_R we set the derivative equal to zero:

$$\frac{dP_R}{dR} = \frac{\varepsilon_m^2\left[(r+R)^2 - 2(r+R)R\right]}{(r+R)^4} = \frac{\varepsilon_{\text{m}}^2(r-R)}{(r+R)^3} = 0 \Rightarrow R = r$$

55. (a) The power factor is cos ϕ, where ϕ is the phase constant defined by the expression $i = I \sin(\omega t - \phi)$. Thus, $\phi = -42.0°$ and $\cos \phi = \cos(-42.0°) = 0.743$.

(b) Since $\phi < 0$, $\omega t - \phi > \omega t$. The current leads the emf.

(c) The phase constant is related to the reactance difference by $\tan \phi = (X_L - X_C)/R$. We have $\tan \phi = \tan(-42.0°) = -0.900$, a negative number. Therefore, $X_L - X_C$ is negative, which leads to $X_C > X_L$. The circuit in the box is predominantly capacitive.

(d) If the circuit were in resonance X_L would be the same as X_C, tan ϕ would be zero, and ϕ would be zero. Since ϕ is not zero, we conclude the circuit is not in resonance.

(e) Since tan ϕ is negative and finite, neither the capacitive reactance nor the resistance are zero. This means the box must contain a capacitor and a resistor.

(f) The inductive reactance may be zero, so there need not be an inductor.

(g) Yes, there is a resistor.

(h) The average power is

$$P_{\text{avg}} = \frac{1}{2}\varepsilon_m I \cos\phi = \frac{1}{2}(75.0\,\text{V})(1.20\,\text{A})(0.743) = 33.4\,\text{W}.$$

(i) The answers above depend on the frequency only through the phase constant ϕ, which is given. If values were given for R, L and C then the value of the frequency would also be needed to compute the power factor.

56. (a) The power consumed by the light bulb is $P = I^2R/2$. So we must let $P_{max}/P_{min} = (I/I_{min})^2 = 5$, or

$$\left(\frac{I}{I_{min}}\right)^2 = \left(\frac{\varepsilon_m / Z_{min}}{\varepsilon_m / Z_{max}}\right)^2 = \left(\frac{Z_{max}}{Z_{min}}\right)^2 = \left(\frac{\sqrt{R^2 + (\omega L_{max})^2}}{R}\right)^2 = 5.$$

We solve for L_{max}:

$$L_{max} = \frac{2R}{\omega} = \frac{2(120\,\text{V})^2 / 1000\,\text{W}}{2\pi(60.0\,\text{Hz})} = 7.64 \times 10^{-2}\,\text{H}.$$

(b) Yes, one could use a variable resistor.

(c) Now we must let

$$\left(\frac{R_{max} + R_{bulb}}{R_{bulb}}\right)^2 = 5,$$

or

$$R_{max} = \left(\sqrt{5} - 1\right) R_{bulb} = \left(\sqrt{5} - 1\right) \frac{(120\,\text{V})^2}{1000\,\text{W}} = 17.8\ \Omega.$$

(d) This is not done because the resistors would consume, rather than temporarily store, electromagnetic energy.

57. We shall use

$$P_{\text{avg}} = \frac{\varepsilon_m^2 R}{2Z^2} = \frac{\varepsilon_m^2 R}{2\left[R^2 + (\omega_d L - 1/\omega_d C)^2\right]}.$$

where $Z = \sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}$ is the impedance.

(a) Considered as a function of *C,* P_{avg} has its largest value when the factor $R^2 + (\omega_d L - 1/\omega_d C)^2$ has the smallest possible value. This occurs for $\omega_d L = 1/\omega_d C$, or

$$C = \frac{1}{\omega_d^2 L} = \frac{1}{(2\pi)^2 (60.0\,\text{Hz})^2 (60.0\times 10^{-3}\,\text{H})} = 1.17\times 10^{-4}\ \text{F}.$$

The circuit is then at resonance.

(b) In this case, we want Z^2 to be as large as possible. The impedance becomes large without bound as *C* becomes very small. Thus, the smallest average power occurs for $C = 0$ (which is not very different from a simple open switch).

(c) When $\omega_d L = 1/\omega_d C$, the expression for the average power becomes

$$P_{\text{avg}} = \frac{\varepsilon_m^2}{2R},$$

so the maximum average power is in the resonant case and is equal to

$$P_{\text{avg}} = \frac{(30.0\,\text{V})^2}{2(5.00\,\Omega)} = 90.0\,\text{W}.$$

(d) At maximum power, the reactances are equal: $X_L = X_C$. The phase angle ϕ in this case may be found from

$$\tan\phi = \frac{X_L - X_C}{R} = 0,$$

which implies $\phi = 0°$.

(e) At maximum power, the power factor is $\cos\phi = \cos 0° = 1$,

(f) The minimum average power is $P_{\text{avg}} = 0$ (as it would be for an open switch).

(g) On the other hand, at minimum power $X_C \propto 1/C$ is infinite, which leads us to set $\tan\phi = -\infty$. In this case, we conclude that $\phi = -90°$.

(h) At minimum power, the power factor is $\cos\phi = \cos(-90°) = 0$.

58. The current in the circuit satisfies $i(t) = I\sin(\omega_d t - \phi)$, where

$$I = \frac{\varepsilon_m}{Z} = \frac{\varepsilon_m}{\sqrt{R^2 + (\omega_d L - 1/\omega_d C)^2}}$$
$$= \frac{45.0\text{ V}}{\sqrt{(16.0\,\Omega)^2 + \{(3000\text{ rad/s})(9.20\text{ mH}) - 1/[(3000\text{ rad/s})(31.2\,\mu\text{F})]\}^2}}$$
$$= 1.93\text{ A}$$

and

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{\omega_d L - 1/\omega_d C}{R}\right)$$
$$= \tan^{-1}\left[\frac{(3000\text{ rad/s})(9.20\text{ mH})}{16.0\,\Omega} - \frac{1}{(3000\text{ rad/s})(16.0\,\Omega)(31.2\,\mu\text{F})}\right]$$
$$= 46.5°.$$

(a) The power supplied by the generator is

$$P_g = i(t)\varepsilon(t) = I\sin(\omega_d t - \phi)\varepsilon_m \sin\omega_d t$$
$$= (1.93\text{ A})(45.0\text{ V})\sin[(3000\text{ rad/s})(0.442\text{ ms})]\sin[(3000\text{ rad/s})(0.442\text{ ms}) - 46.5°]$$
$$= 41.4\text{ W}.$$

(b) The rate at which the energy in the capacitor changes is

$$P_c = -\frac{d}{dt}\left(\frac{q^2}{2C}\right) = -i\frac{q}{C} = -iV_c$$
$$= -I\sin(\omega_d t - \phi)\left(\frac{I}{\omega_d C}\right)\cos(\omega_d t - \phi) = -\frac{I^2}{2\omega_d C}\sin[2(\omega_d t - \phi)]$$
$$= -\frac{(1.93\text{ A})^2}{2(3000\text{ rad/s})(31.2\times10^{-6}\text{ F})}\sin[2(3000\text{ rad/s})(0.442\text{ ms}) - 2(46.5°)]$$
$$= -17.0\text{ W}.$$

(c) The rate at which the energy in the inductor changes is

$$P_L = \frac{d}{dt}\left(\frac{1}{2}Li^2\right) = Li\frac{di}{dt} = LI\sin(\omega_d t - \phi)\frac{d}{dt}\left[I\sin(\omega_d t - \phi)\right] = \frac{1}{2}\omega_d LI^2 \sin\left[2(\omega_d t - \phi)\right]$$
$$= \frac{1}{2}(3000\,\text{rad/s})(1.93\,\text{A})^2(9.20\,\text{mH})\sin\left[2(3000\,\text{rad/s})(0.442\,\text{ms}) - 2(46.5°)\right]$$
$$= 44.1\ \text{W}.$$

(d) The rate at which energy is being dissipated by the resistor is

$$P_R = i^2R = I^2R\sin^2(\omega_d t - \phi) = (1.93\,\text{A})^2(16.0\,\Omega)\sin^2\left[(3000\,\text{rad/s})(0.442\,\text{ms}) - 46.5°\right]$$
$$= 14.4\ \text{W}.$$

(e) Equal. $P_L + P_R + P_c = 44.1\,\text{W} - 17.0\ \text{W} + 14.4\,\text{W} = 41.5\ \text{W} = P_g$.

59. (a) The rms current is

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2}}$$

$$= \frac{75.0\,\text{V}}{\sqrt{(15.0\,\Omega)^2 + \{2\pi(550\,\text{Hz})(25.0\,\text{mH}) - 1/[2\pi(550\,\text{Hz})(4.70\,\mu\text{F})]\}^2}}$$

$$= 2.59\,\text{A}.$$

(b) The rms voltage across R is

$$V_{ab} = I_{\text{rms}} R = (2.59\,\text{A})(15.0\,\Omega) = 38.8\,\text{V}.$$

(c) The rms voltage across C is

$$V_{bc} = I_{\text{rms}} X_C = \frac{I_{\text{rms}}}{2\pi fC} = \frac{2.59\,\text{A}}{2\pi(550\,\text{Hz})(4.70\,\mu\text{F})} = 159\,\text{V}.$$

(d) The rms voltage across L is

$$V_{cd} = I_{\text{rms}} X_L = 2\pi I_{\text{rms}} fL = 2\pi(2.59\,\text{A})(550\,\text{Hz})(25.0\,\text{mH}) = 224\,\text{V}.$$

(e) The rms voltage across C and L together is

$$V_{bd} = |V_{bc} - V_{cd}| = |159.5\,\text{V} - 223.7\,\text{V}| = 64.2\,\text{V}$$

(f) The rms voltage across R, C and L together is

$$V_{ad} = \sqrt{V_{ab}^2 + V_{bd}^2} = \sqrt{(38.8\,\text{V})^2 + (64.2\,\text{V})^2} = 75.0\,\text{V}$$

(g) For R,

$$P_R = \frac{V_{ab}^2}{R} = \frac{(38.8\,\text{V})^2}{15.0\,\Omega} = 100\,\text{W}.$$

(h) No energy dissipation in C.

(i) No energy dissipation in L.

60. We use Eq. 31-79 to find

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (100\,\text{V})\left(\frac{500}{50}\right) = 1.00\times 10^3\ \text{V}.$$

61. (a) The stepped-down voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (120\,\text{V})\left(\frac{10}{500}\right) = 2.4\,\text{V}.$$

(b) By Ohm's law, the current in the secondary is

$$I_s = \frac{V_s}{R_s} = \frac{2.4\,\text{V}}{15\,\Omega} = 0.16\,\text{A}.$$

We find the primary current from Eq. 31-80:

$$I_p = I_s\left(\frac{N_s}{N_p}\right) = (0.16\,\text{A})\left(\frac{10}{500}\right) = 3.2\times10^{-3}\,\text{A}.$$

(c) As shown above, the current in the secondary is $I_s = 0.16\,\text{A}$.

62. For step-up trasnformer:

(a) The smallest value of the ratio V_s / V_p is achieved by using T_2T_3 as primary and T_1T_3 as secondary coil: V_{13}/V_{23} = (800 + 200)/800 = 1.25.

(b) The second smallest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_2T_3 as secondary coil: V_{23}/V_{13} = 800/200 = 4.00.

(c) The largest value of the ratio V_s / V_p is achieved by using T_1T_2 as primary and T_1T_3 as secondary coil: V_{13}/V_{12} = (800 + 200)/200 = 5.00.

For the step-down transformer, we simply exchange the primary and secondary coils in each of the three cases above.

(d) The smallest value of the ratio V_s / V_p is 1/5.00 = 0.200.

(e) The second smallest value of the ratio V_s / V_p is 1/4.00 = 0.250.

(f) The largest value of the ratio V_s / V_p is 1/1.25 = 0.800.

63. (a) The rms current in the cable is $I_{\text{rms}} = P/V_t = 250\times10^3\,\text{W}/\left(80\times10^3\ \text{V}\right) = 3.125\,\text{A}$. The rms voltage drop is then $\Delta V = I_{\text{rms}}R = (3.125\,\text{A})(2)(0.30\,\Omega) = 1.9\,\text{V}$.

(b) The rate of energy dissipation is $P_d = I_{\text{rms}}^2R = (3.125\,\text{A})(2)(0.60\,\Omega) = 5.9\,\text{W}$.

(c) Now $I_{\text{rms}} = 250\times10^3\ \text{W}/\left(8.0\times10^3\ \text{V}\right) = 31.25\,\text{A}$, so $\Delta V = (31.25\,\text{A})(0.60\,\Omega) = 19\,\text{V}$.

(d) $P_d = (3.125\,\text{A})^2(0.60\,\Omega) = 5.9\times10^2\ \text{W}$.

(e) $I_{\text{rms}} = 250\times10^3\ \text{W}/\left(0.80\times10^3\ \text{V}\right) = 312.5\ \text{A}$, so $\Delta V = (312.5\,\text{A})(0.60\,\Omega) = 1.9\times10^2\ \text{V}$.

(f) $P_d = (312.5\,\text{A})^2\,(0.60\ \Omega) = 5.9\times10^4\ \text{W}$.

64. (a) The amplifier is connected across the primary windings of a transformer and the resistor R is connected across the secondary windings.

(b) If I_s is the rms current in the secondary coil then the average power delivered to R is $P_{\text{avg}} = I_s^2 R$. Using $I_s = \left(N_p / N_s\right) I_p$, we obtain

$$P_{\text{avg}} = \left(\frac{I_p N_p}{N_s}\right)^2 R.$$

Next, we find the current in the primary circuit. This is effectively a circuit consisting of a generator and two resistors in series. One resistance is that of the amplifier (r), and the other is the equivalent resistance R_{eq} of the secondary circuit. Therefore,

$$I_p = \frac{\varepsilon_{\text{rms}}}{r + R_{\text{eq}}} = \frac{\varepsilon_{\text{rms}}}{r + \left(N_p / N_s\right)^2 R}$$

where Eq. 31-82 is used for R_{eq}. Consequently,

$$P_{\text{avg}} = \frac{\varepsilon^2 \left(N_p / N_s\right)^2 R}{\left[r + \left(N_p / N_s\right)^2 R\right]^2}.$$

Now, we wish to find the value of N_p/N_s such that P_{avg} is a maximum. For brevity, let $x = (N_p/N_s)^2$. Then

$$P_{\text{avg}} = \frac{\varepsilon^2 R x}{(r + xR)^2},$$

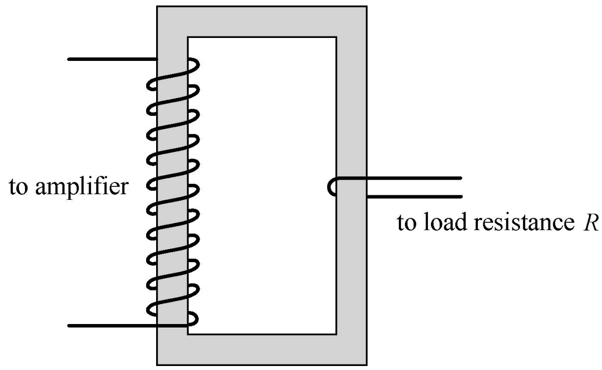
and the derivative with respect to x is

$$\frac{dP_{\text{avg}}}{dx} = \frac{\varepsilon^2 R(r - xR)}{(r + xR)^3}.$$

This is zero for $x = r / R = (1000\,\Omega)/(10\,\Omega) = 100$. We note that for small x, P_{avg} increases linearly with x, and for large x it decreases in proportion to $1/x$. Thus $x = r/R$ is indeed a maximum, not a minimum. Recalling $x = (N_p/N_s)^2$, we conclude that the maximum power is achieved for

$$N_p / N_s = \sqrt{x} = 10.$$

The diagram that follows is a schematic of a transformer with a ten to one turns ratio. An actual transformer would have many more turns in both the primary and secondary coils.



65. (a) We consider the following combinations: $\Delta V_{12} = V_1 - V_2$, $\Delta V_{13} = V_1 - V_3$, and $\Delta V_{23} = V_2 - V_3$. For ΔV_{12},

$$\Delta V_{12} = A\sin(\omega_d t) - A\sin(\omega_d t - 120°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 120°}{2}\right) = \sqrt{3}A\cos(\omega_d t - 60°)$$

where we use $\sin\alpha - \sin\beta = 2\sin[(\alpha-\beta)/2]\cos[(\alpha+\beta)/2]$ and $\sin 60° = \sqrt{3}/2$. Similarly,

$$\begin{aligned}\Delta V_{13} &= A\sin(\omega_d t) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{240°}{2}\right)\cos\left(\frac{2\omega_d t - 240°}{2}\right)\\ &= \sqrt{3}A\cos(\omega_d t - 120°)\end{aligned}$$

and

$$\begin{aligned}\Delta V_{23} &= A\sin(\omega_d t - 120°) - A\sin(\omega_d t - 240°) = 2A\sin\left(\frac{120°}{2}\right)\cos\left(\frac{2\omega_d t - 360°}{2}\right)\\ &= \sqrt{3}A\cos(\omega_d t - 180°)\end{aligned}$$

All three expressions are sinusoidal functions of t with angular frequency ω_d.

(b) We note that each of the above expressions has an amplitude of $\sqrt{3}A$.

66. We start with Eq. 31-76:

$$P_{\text{avg}} = \varepsilon_{\text{rms}} I_{\text{rms}} \cos\phi = \varepsilon_{\text{rms}} \left(\frac{\varepsilon_{\text{rms}}}{Z}\right)\left(\frac{R}{Z}\right) = \frac{\varepsilon_{\text{rms}}^2 R}{Z^2}.$$

For a purely resistive circuit, $Z = R$, and this result reduces to Eq. 27-23 (with V replaced with ε_{rms}). This is also the case for a series *RLC* circuit at resonance. The average rate for dissipating energy is, of course, zero if $R = 0$, as would be the case for a purely inductive circuit.

67. (a) The effective resistance R_{eff} satisfies $I_{\text{rms}}^2 R_{\text{eff}} = P_{\text{mechanical}}$, or

$$R_{\text{eff}} = \frac{P_{\text{mechanical}}}{I_{\text{rms}}^2} = \frac{(0.100\,\text{hp})(746\,\text{W/hp})}{(0.650\,\text{A})^2} = 177\,\Omega.$$

(b) This is not the same as the resistance R of its coils, but just the effective resistance for power transfer from electrical to mechanical form. In fact $I_{\text{rms}}^2 R$ would not give $P_{\text{mechanical}}$ but rather the rate of energy loss due to thermal dissipation.

69. The rms current in the motor is

$$I_{\text{rms}} = \frac{\varepsilon_{\text{rms}}}{Z} = \frac{\varepsilon_{\text{rms}}}{\sqrt{R^2 + X_L^2}} = \frac{420\,\text{V}}{\sqrt{(45.0\,\Omega)^2 + (32.0\,\Omega)^2}} = 7.61\,\text{A}.$$

70. (a) A sketch of the phasors would be very much like Fig. 31-9(c) but with the label "I_C" on the green arrow replaced with "V_R."

(b) We have $I R = I X_C$, or

$$I R = I X_C \rightarrow R = \frac{1}{\omega_d C}$$

which yields $f = \omega_d/2\pi = 1/2\pi RC = 159$ Hz.

(c) $\phi = \tan^{-1}(-V_C/V_R) = -45°$.

(d) $\omega_d = 1/RC = 1.00 \times 10^3$ rad/s.

(e) $I = (12 \text{ V})/\sqrt{R^2 + X_C^2} = 6/(25\sqrt{2}) \approx 170$ mA.

71. (a) The energy stored in the capacitor is given by $U_E = q^2 / 2C$. Since q is a periodic function of t with period T, so must be U_E. Consequently, U_E will not be changed over one complete cycle. Actually, U_E has period $T/2$, which does not alter our conclusion.

(b) Similarly, the energy stored in the inductor is $U_B = \frac{1}{2} i^2 L$. Since i is a periodic function of t with period T, so must be U_B.

(c) The energy supplied by the generator is

$$P_{\text{avg}} T = \left(I_{\text{rms}} \varepsilon_{\text{rms}} \cos\phi\right) T = \left(\frac{1}{2} T\right) \varepsilon_m I \cos\phi$$

where we substitute $I_{\text{rms}} = I / \sqrt{2}$ and $\varepsilon_{\text{rms}} = \varepsilon_m / \sqrt{2}$.

(d) The energy dissipated by the resistor is

$$P_{\text{avg,resistor}}\, T = \left(I_{\text{rms}} V_R\right) T = I_{\text{rms}} \left(I_{\text{rms}} R\right) T = \left(\frac{1}{2} T\right) I^2 R.$$

(e) Since $\varepsilon_m I \cos\phi = \varepsilon_m I \left(V_R / \varepsilon_m\right) = \varepsilon_m I \left(IR / \varepsilon_m\right) = I^2 R$, the two quantities are indeed the same.

72. (a) Eq. 31-39 gives $f = \omega/2\pi = (2\pi C X_C)^{-1} = 8.84$ kHz.

(b) Because of its inverse relationship with frequency, then the reactance will go down by a factor of 2 when f increases by a factor of 2. The answer is $X_C = 6.00\ \Omega$.

73. (a) The impedance is $Z = \dfrac{\varepsilon_m}{I} = \dfrac{125\,\text{V}}{3.20\,\text{A}} = 39.1\ \Omega.$

(b) From $V_R = IR = \varepsilon_m \cos\phi,$ we get

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(125\,\text{V})\cos(0.982\,\text{rad})}{3.20\,\text{A}} = 21.7\,\Omega.$$

(c) Since $X_L - X_C \propto \sin\phi = \sin(-0.982\,\text{rad}),$ we conclude that $X_L < X_C$. The circuit is predominantly capacitive.

74. (a) Eq. 31-4 directly gives $1/\sqrt{LC} \approx 5.77\times10^3$ rad/s.

(b) Eq. 16-5 then yields $T = 2\pi/\omega = 1.09$ ms.

(c) Although we do not show the graph here, we describe it: it is a cosine curve with amplitude 200 μC and period given in part (b).

75. (a) The phase constant is given by

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{R}\right) = \tan^{-1}\left(\frac{V_L - V_L/2.00}{V_L/2.00}\right) = \tan^{-1}(1.00) = 45.0^\circ.$$

(b) We solve R from $\varepsilon_m \cos\phi = IR$:

$$R = \frac{\varepsilon_m \cos\phi}{I} = \frac{(30.0\text{ V})(\cos 45^\circ)}{300\times10^{-3}\text{ A}} = 70.7\ \Omega.$$

76. From Eq. 31-4, we have $C = (\omega^2 L)^{-1} = ((2\pi f)^2 L)^{-1} = 1.59\ \mu\text{F}$.

77. (a) We solve L from Eq. 31-4, using the fact that $\omega = 2\pi f$:

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 \left(10.4\times10^3 \text{ Hz}\right)^2 \left(340\times10^{-6} \text{ F}\right)} = 6.89\times10^{-7} \text{ H}.$$

(b) The total energy may be figured from the inductor (when the current is at maximum):

$$U = \frac{1}{2}LI^2 = \frac{1}{2}\left(6.89\times10^{-7} \text{ H}\right)\left(7.20\times10^{-3} \text{ A}\right)^2 = 1.79\times10^{-11} \text{ J}.$$

(c) We solve for Q from $U = \frac{1}{2}Q^2 / C$:

$$Q = \sqrt{2CU} = \sqrt{2\left(340\times10^{-6} \text{ F}\right)\left(1.79\times10^{-11} \text{ J}\right)} = 1.10\times10^{-7} \text{ C}.$$

78. (a) With a phase constant of 45º the (net) reactance must equal the resistance in the circuit, which means the circuit impedance becomes $Z = R\sqrt{2} \Rightarrow R = Z/\sqrt{2} = 707\ \Omega$.

(b) Since $f = 8000$ Hz then $\omega_d = 2\pi(8000)$ rad/s. The net reactance (which, as observed, must equal the resistance) is therefore $X_L - X_C = \omega_d L - (\omega_d C)^{-1} = 707\ \Omega$. We are also told that the resonance frequency is 6000 Hz, which (by Eq. 31-4) means $C = (\omega^2 L)^{-1} = ((2\pi(6000))^2 L)^{-1}$. Substituting this in for C in our previous expression (for the net reactance) we obtain an equation that can be solved for the self-inductance. Our result is $L = 32.2$ mH.

(c) $C = ((2\pi(6000))^2 L)^{-1} = 21.9$ nF.

79. (a) Let $\omega t - \pi/4 = \pi/2$ to obtain $t = 3\pi/4\omega = 3\pi/\left[4(350\,\text{rad/s})\right] = 6.73\times10^{-3}$ s.

(b) Let $\omega t + \pi/4 = \pi/2$ to obtain $t = \pi/4\omega = \pi/\left[4(350\,\text{rad/s})\right] = 2.24\times10^{-3}$ s.

(c) Since i leads ε in phase by $\pi/2$, the element must be a capacitor.

(d) We solve C from $X_C = (\omega C)^{-1} = \varepsilon_m / I$:

$$C = \frac{I}{\varepsilon_m \omega} = \frac{6.20\times10^{-3}\text{ A}}{(30.0\text{ V})(350\,\text{rad/s})} = 5.90\times10^{-5}\text{ F}.$$

80. Resonance occurs when the inductive reactance equals the capacitive reactance. Reactances of a certain type add (in series) just like resistances did in Chapter 28. Thus, since the resonance ω values are the same for both circuits, we have for each circuit:

$$\omega L_1 = \frac{1}{\omega C_1}, \quad \omega L_2 = \frac{1}{\omega C_2}$$

and adding these equations we find

$$\omega(L_1 + L_2) = \frac{1}{\omega}\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$

$$\omega L_{eq} = \frac{1}{\omega C_{eq}} \Rightarrow \text{resonance in the combined circuit.}$$

81. (a) From Eq. 31-4, we have $L = (\omega^2 C)^{-1} = ((2\pi f)^2 C)^{-1} = 2.41\ \mu\text{H}$.

(b) The total energy is the maximum energy on either device (see Fig. 31-4). Thus, we have $U_{\max} = \frac{1}{2}LI^2 = 21.4$ pJ.

(c) Of several methods available to do this part, probably the one most "in the spirit" of this problem (considering the energy that was calculated in part (b)) is to appeal to $U_{\max} = \frac{1}{2}Q^2/C$ (from Chapter 26) to find the maximum charge: $Q = \sqrt{2CU_{\max}} = 82.2$ nC.

82. (a) From Eq. 31-65, we have

$$\phi = \tan^{-1}\left(\frac{V_L - V_C}{V_R}\right) = \tan^{-1}\left(\frac{V_L - (V_L / 1.50)}{(V_L / 2.00)}\right)$$

which becomes $\tan^{-1} 2/3 = 33.7°$ or 0.588 rad.

(b) Since $\phi > 0$, it is inductive ($X_L > X_C$).

(c) We have $V_R = IR = 9.98$ V, so that $V_L = 2.00V_R = 20.0$ V and $V_C = V_L/1.50 = 13.3$ V. Therefore, from Eq. 31-60.

$$\varepsilon_m = \sqrt{V_R^2 + (V_L - V_C)^2}$$

we find $\varepsilon_m = 12.0\ \text{V}$.

83. When switch S_1 is closed and the others are open, the inductor is essentially out of the circuit and what remains is an RC circuit. The time constant is $\tau_C = RC$. When switch S_2 is closed and the others are open, the capacitor is essentially out of the circuit. In this case, what we have is an LR circuit with time constant $\tau_L = L/R$. Finally, when switch S_3 is closed and the others are open, the resistor is essentially out of the circuit and what remains is an LC circuit that oscillates with period $T = 2\pi\sqrt{LC}$. Substituting $L = R\tau_L$ and $C = \tau_C/R$, we obtain $T = 2\pi\sqrt{\tau_C \tau_L}$.

84. (a) The impedance is $Z = (80.0 \text{ V})/(1.25 \text{ A}) = 64.0\ \Omega$.

(b) We can write $\cos\phi = R/Z \Rightarrow R = (64.0\ \Omega)\cos(0.650 \text{ rad}) = 50.9\ \Omega$.

(c) Since the "current leads the emf" the circuit is capacitive.

85. (a) We find L from $X_L = \omega L = 2\pi f L$:

$$f = \frac{X_L}{2\pi L} = \frac{1.30\times10^{3}\,\Omega}{2\pi\left(45.0\times10^{-3}\,\text{H}\right)} = 4.60\times10^{3}\,\text{Hz}.$$

(b) The capacitance is found from $X_C = (\omega C)^{-1} = (2\pi f C)^{-1}$:

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi\left(4.60\times10^{3}\,\text{Hz}\right)\left(1.30\times10^{3}\,\Omega\right)} = 2.66\times10^{-8}\,\text{F}.$$

(c) Noting that $X_L \propto f$ and $X_C \propto f^{-1}$, we conclude that when f is doubled, X_L doubles and X_C reduces by half. Thus, $X_L = 2(1.30 \times 10^3\ \Omega) = 2.60 \times 10^3\ \Omega$.

(d) $X_C = 1.30 \times 10^3\ \Omega/2 = 6.50 \times 10^2\ \Omega$.

86. (a) Using $\omega = 2\pi f$, $X_L = \omega L$, $X_C = 1/\omega C$ and $\tan(\phi) = (X_L - X_C)/R$, we find

$$\phi = \tan^{-1}[(16.022 - 33.157)/40.0] = -0.40473 \approx -0.405 \text{ rad.}$$

(b) Eq. 31-63 gives $I = 120/\sqrt{40^2 + (16\text{-}33)^2} = 2.7576 \approx 2.76$ A.

(c) $X_C > X_L \Rightarrow$ capacitive.

87. When the switch is open, we have a series *LRC* circuit involving just the one capacitor near the upper right corner. Eq. 31-65 leads to

$$\frac{\omega_d L - \frac{1}{\omega_d C}}{R} = \tan\phi_o = \tan(-20°) = -\tan 20°.$$

Now, when the switch is in position 1, the equivalent capacitance in the circuit is 2*C*. In this case, we have

$$\frac{\omega_d L - \frac{1}{2\omega_d C}}{R} = \tan\phi_1 = \tan 10.0°.$$

Finally, with the switch in position 2, the circuit is simply an *LC* circuit with current amplitude

$$I_2 = \frac{\varepsilon_m}{Z_{LC}} = \frac{\varepsilon_m}{\sqrt{\left(\omega_d L - \frac{1}{\omega_d C}\right)^2}} = \frac{\varepsilon_m}{\frac{1}{\omega_d C} - \omega_d L}$$

where we use the fact that $(\omega_d C)^{-1} > \omega_d L$ in simplifying the square root (this fact is evident from the description of the first situation, when the switch was open). We solve for *L, R* and *C* from the three equations above, and the results are

(a) $R = \dfrac{-\varepsilon_m}{I_2 \tan\phi_o} = \dfrac{120\text{V}}{(2.00\text{ A})\tan 20.0°} = 165\,\Omega.$

(b) $L = \dfrac{\varepsilon_m}{\omega_d I_2}\left(1 - 2\dfrac{\tan\phi_1}{\tan\phi_o}\right) = \dfrac{120\text{ V}}{2\pi(60.0\text{ Hz})(2.00\text{ A})}\left(1 + 2\dfrac{\tan 10.0°}{\tan 20.0°}\right) = 0.313\text{ H}.$

(c) $C = \dfrac{I_2}{2\omega_d \varepsilon_m\left(1 - \frac{\tan\phi_1}{\tan\phi_o}\right)} = \dfrac{2.00\text{ A}}{2(2\pi)(60.0\text{ Hz})(120\text{ V})\left(1 + \frac{\tan 10.0°}{\tan 20.0°}\right)} = 1.49\times10^{-5}\text{ F}$

88. From $U_{max} = \frac{1}{2}LI^2$ we get $I = 0.115$ A.

89. (a) At any time, the total energy U in the circuit is the sum of the energy U_E in the capacitor and the energy U_B in the inductor. When $U_E = 0.500U_B$ (at time t), then $U_B = 2.00U_E$ and $U = U_E + U_B = 3.00U_E$. Now, U_E is given by $q^2/2C$, where q is the charge on the capacitor at time t. The total energy U is given by $Q^2/2C$, where Q is the maximum charge on the capacitor. Thus, $Q^2/2C = 3.00q^2/2C$ or $q = Q/\sqrt{3.00}$ = $0.577Q$.

(b) If the capacitor is fully charged at time $t = 0$, then the time-dependent charge on the capacitor is given by $q = Q\cos\omega t$. This implies that the condition $q = 0.577Q$ is satisfied when $\cos\omega t$ =0.557, or ωt = 0.955 rad. Since $\omega = 2\pi/T$ (where T is the period of oscillation), $t = 0.955T/2\pi = 0.152T$, or $t/T = 0.152$.

90. (a) The computations are as follows:

$$X_L = 2\pi f_d L = 60.82\ \Omega$$

$$X_C = (2\pi f_d C)^{-1} = 32.88\ \Omega$$

$$Z = \sqrt{20^2 + (61\text{-}33)^2} = 34.36\ \Omega$$

$$I = \varepsilon / Z = 2.62\ \text{A} \quad \Rightarrow \quad I_{\text{rms}} = I/\sqrt{2} = 1.85\ \text{A}\ .$$

Therefore, $V_{R\,\text{rms}} = I_{\text{rms}} R = 37.0$ V.

(b) $V_{C\,\text{rms}} = I_{\text{rms}} X_C = 60.9$ V.

(c) $V_{L\,\text{rms}} = I_{\text{rms}} X_L = 113$ V.

(d) $P_{\text{avg}} = (I_{\text{rms}})^2 R = 68.6$ W.

91. (a) Eqs. 31-4 and 31-14 lead to

$$Q = \frac{1}{\omega} = I\sqrt{LC} = 1.27\times10^{-6}\,\text{C} .$$

(b) We choose the phase constant in Eq. 31-12 to be $\phi = -\pi/2$, so that $i_0 = I$ in Eq. 31-15). Thus, the energy in the capacitor is

$$U_E = \frac{q^2}{2C} = \frac{Q^2}{2C}(\sin\omega t)^2 .$$

Differentiating and using the fact that 2 sin θ cos θ = sin 2θ, we obtain

$$\frac{dU_E}{dt} = \frac{Q^2}{2C}\omega\sin 2\omega t .$$

We find the maximum value occurs whenever sin $2\omega t = 1$, which leads (with n = *odd* integer) to

$$t = \frac{1}{2\omega}\frac{n\pi}{2} = \frac{n\pi}{4\omega} = \frac{n\pi}{4}\sqrt{LC} = 8.31\times10^{-5}\,\text{s},\ 2.49\times10^{-4}\,\text{s},\ldots .$$

The earliest time is $t = 8.31\times10^{-5}$ s.

(c) Returning to the above expression for dU_E/dt with the requirement that $\sin 2\omega t = 1$, we obtain

$$\left(\frac{dU_E}{dt}\right)_{\max} = \frac{Q^2}{2C}\omega = \frac{\left(I\sqrt{LC}\right)^2}{2C}\frac{I}{\sqrt{LC}} = \frac{I^2}{2}\sqrt{\frac{L}{C}} = 5.44\times10^{-3}\,\text{J/s} .$$

92. (a) We observe that ω = 6597 rad/s, and, consequently, X_L = 594 Ω and X_C = 303 Ω. Since $X_L > X_C$, the phase angle is positive: $\phi = +60.0°$.

From Eq. 31-65, we obtain $R = \frac{X_L - X_C}{\tan\phi} = 168\,\Omega$.

(b) Since we are already on the "high side" of resonance, increasing *f* will only decrease the current further, but *decreasing f* brings us closer to resonance and, consequently, large values of *I*.

(c) Increasing *L* increases X_L, but we already have $X_L > X_C$. Thus, if we wish to move closer to resonance (where X_L must equal X_C), we need to *decrease* the value of *L*.

(d) To change the present condition of $X_C < X_L$ to something closer to $X_C = X_L$ (resonance, large current), we can increase X_C. Since X_C depends inversely on *C*, this means *decreasing C*.

93. (a) We observe that ω_d = 12566 rad/s. Consequently, X_L = 754 Ω and X_C = 199 Ω. Hence, Eq. 31-65 gives

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = 1.22 \text{ rad} .$$

(b) We find the current amplitude from Eq. 31-60:

$$I = \frac{\varepsilon_m}{\sqrt{R^2 + (X_L - X_C)^2}} = 0.288 \text{ A} .$$

94. From Eq. 31-60, we have

$$\left(\frac{220\,\text{V}}{3.00\,\text{A}}\right)^2 = R^2 + X_L^2 \Rightarrow X_L = 69.3\,\Omega\ .$$

95. From Eq. 31-4, with $\omega = 2\pi f = 4.49 \times 10^3$ rad / s, we obtain

$$L = \frac{1}{\omega^2 C} = 7.08 \times 10^{-3} \text{ H}.$$

96. (a) From Eq. 31-4, with $\omega = 2\pi f$, C = 2.00 nF and L = 2.00 mH, we have

$$f = \frac{1}{2\pi\sqrt{LC}} = 7.96\times10^4 \text{ Hz.}$$

(b) The maximum current in the oscillator is

$$i_{\max} = I_C = \frac{V_C}{X_C} = \omega C v_{\max} = 4.00\times10^{-3} \text{ A.}$$

(c) Using Eq. 30-49, we find the maximum magnetic energy:

$$U_{B,\max} = \frac{1}{2} L i_{\max}^2 = 1.60\times10^{-8} \text{ J.}$$

(d) Adapting Eq. 30-35 to the notation of this chapter,

$$v_{\max} = L\left|\frac{di}{dt}\right|_{\max}$$

which yields a (maximum) time rate of change (for i) equal to 2.00×10^3 A/s.

97. Reading carefully, we note that the driving frequency of the source is permanently set at the resonance frequency of the *initial* circuit (with switches open); it is set at $\omega_d = 1/\sqrt{LC} = 1.58 \times 10^4$ rad/s and does not correspond to the resonance frequency once the switches are closed. In our table, below, C_{eq} is in μF, f is in kHz, and R_{eq} and Z are in Ω. Steady state conditions are assumed in calculating the current amplitude (which is in amperes); this I is the current through the source (or through the inductor), as opposed to the (generally smaller) current in one of the resistors. Resonant frequencies f are computed with $\omega = 2\pi f$. Reducing capacitor and resistor combinations is explained in chapters 26 and 28, respectively.

switch	(a) C_{eq}(μF)	(b) f(kHz)	(c) R_{eq}(Ω)	(d) Z(Ω)	(e) I (A)
S_1	4.00	1.78	12.0	19.8	0.605
S_2	5.00	1.59	12.0	22.4	0.535
S_3	5.00	1.59	6.0	19.9	0.603
S_4	5.00	1.59	4.0	19.4	0.619

98. (a) We note that we obtain the maximum value in Eq. 31-28 when we set

$$t = \frac{\pi}{2\omega_d} = \frac{1}{4f} = \frac{1}{4(60)} = 0.00417 \text{ s}$$

or 4.17 ms. The result is $\varepsilon_m \sin(\pi/2) = \varepsilon_m \sin(90°) = 36.0 \text{ V}$.

(b) At t = 4.17 ms, the current is

$$i = I\sin(\omega_d t - \phi) = I\sin(90° - (-24.3°)) = (0.164\text{A})\cos(24.3°) = 0.1495\text{A} \approx 0.150 \text{ A}.$$

using Eq. 31-29 and the results of the Sample Problem. Ohm's law directly gives

$$v_R = iR = (0.1495\text{A})(200\Omega) = 29.9\text{V}.$$

(c) The capacitor voltage phasor is 90° less than that of the current. Thus, at t = 4.17 ms, we obtain

$$v_C = I\sin(90° - (-24.3°) - 90°)X_C = IX_C\sin(24.3°) = (0.164\text{A})(177\Omega)\sin(24.3°) = 11.9\text{V}.$$

(d) The inductor voltage phasor is 90° more than that of the current. Therefore, at t = 4.17 ms, we find

$$\begin{aligned} v_L &= I\sin(90° - (-24.3°) + 90°)X_L = -IX_L\sin(24.3°) = -(0.164\text{A})(86.7\Omega)\sin(24.3°) \\ &= -5.85\text{V}. \end{aligned}$$

(e) Our results for parts (b), (c) and (d) add to give 36.0 V, the same as the answer for part (a).

99. (a) Since $T = 2\pi/\omega = 2\pi\sqrt{LC},$ we may rewrite the power on the exponential factor as

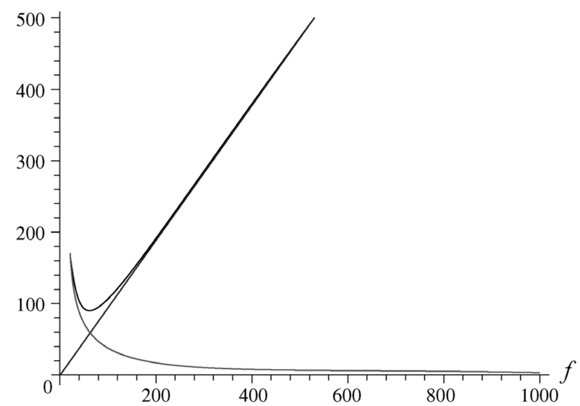
$$-\pi R\sqrt{\frac{C}{L}}\frac{t}{T} = -\pi R\sqrt{\frac{C}{L}}\frac{t}{2\pi\sqrt{LC}} = -\frac{Rt}{2L}.$$

Thus $e^{-Rt/2L} = e^{-\pi R\sqrt{C/L}(t/T)}$.

(b) Since $-\pi R\sqrt{C/L}(t/T)$ must be dimensionless (as is t/T), $R\sqrt{C/L}$ must also be dimensionless. Thus, the SI unit of $\sqrt{C/L}$ must be Ω^{-1}. In other words, the SI unit for $\sqrt{L/C}$ is Ω.

(c) Since the amplitude of oscillation reduces by a factor of $e^{-\pi R\sqrt{C/L}(T/T)} = e^{-\pi R\sqrt{C/L}}$ after each cycle, the condition is equivalent to $\pi R\sqrt{C/L} \ll 1$, or $R \ll \sqrt{L/C}$.

100. (a) The curves are shown in the graph below. We have also included here the impedance curve (which is asked for later in the problem statement). The curve sloping towards zero at high frequencies is X_C, and the linearly rising line is X_L. The vertical axis is in ohms. For simplicity of notation, we have omitted the "*d*" subscript from *f*.



(b) The reactance curves cross each other (to the extent that we can estimate from our graph) at a value near 60 Hz. A more careful calculation (setting the reactances equal to each other) leads to the resonance value: $f = 61.26 \approx 61$ Hz.

(c) Z is at its lowest value at resonance: $Z_{resonance} = R = 90\ \Omega$.

(d) As noted in our solution of part (b), the resonance value is $f = 61.26 \approx 61$ Hz.