1. In air, light travels at roughly $c = 3.0 \times 10^8$ m/s. Therefore, for $t = 1.0$ ns, we have a distance of

$$d = ct = (3.0\times10^8 \text{ m/s})(1.0\times10^{-9} \text{ s}) = 0.30 \text{ m}.$$

2. (a) From Fig. 33-2 we find the smaller wavelength in question to be about 515 nm,

(b) and the larger wavelength to be approximately 610 nm.

(c) From Fig. 33-2 the wavelength at which the eye is most sensitive is about 555 nm.

(d) Using the result in (c), we have

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{555\ \text{nm}} = 5.41\times10^{14}\ \text{Hz}.$$

(e) The period is $(5.41 \times 10^{14}\ \text{Hz})^{-1} = 1.85 \times 10^{-15}$ s.

3. (a) The frequency of the radiation is

$$f = \frac{c}{\lambda} = \frac{3.0\times10^{8}\ \text{m/s}}{(1.0\times10^{5})(6.4\times10^{6}\ \text{m})} = 4.7\times10^{-3}\ \text{Hz}.$$

(b) The period of the radiation is

$$T = \frac{1}{f} = \frac{1}{4.7\times10^{-3}\ \text{Hz}} = 212\ \text{s} = 3\,\text{min}\,32\ \text{s}.$$

4. Since $\Delta\lambda \ll \lambda$, we find Δf is equal to

$$\left|\Delta\left(\frac{c}{\lambda}\right)\right| \approx \frac{c\Delta\lambda}{\lambda^2} = \frac{(3.0\times 10^8 \text{ m/s})(0.0100\times 10^{-9} \text{ m})}{(632.8\times 10^{-9} \text{ m})^2} = 7.49\times 10^9 \text{ Hz}.$$

5. If f is the frequency and λ is the wavelength of an electromagnetic wave, then $f\lambda = c$. The frequency is the same as the frequency of oscillation of the current in the LC circuit of the generator. That is, $f = 1/2\pi\sqrt{LC}$, where C is the capacitance and L is the inductance. Thus

$$\frac{\lambda}{2\pi\sqrt{LC}} = c.$$

The solution for L is

$$L = \frac{\lambda^2}{4\pi^2 Cc^2} = \frac{\left(550\times10^{-9}\ \text{m}\right)^2}{4\pi^2\left(17\times10^{-12}\ \text{F}\right)\left(2.998\times10^{8}\ \text{m/s}\right)^2} = 5.00\times10^{-21}\ \text{H}.$$

This is exceedingly small.

6. The emitted wavelength is

$$\lambda = \frac{c}{f} = 2\pi c\sqrt{LC} = 2\pi\left(2.998\times10^{8}\ \text{m/s}\right)\sqrt{\left(0.253\times10^{-6}\ \text{H}\right)\left(25.0\times10^{-12}\ \text{F}\right)} = 4.74\,\text{m}.$$

7. The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{3.20\times10^{-4}\ \text{V/m}}{2.998\times10^{8}\ \text{m/s}} = 1.07\times10^{-12}\ \text{T}.$$

8. (a) The amplitude of the magnetic field is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\text{V/m}}{2.998\times10^{8}\,\text{m/s}} = 6.67\times10^{-9}\,\text{T} \approx 6.7\times10^{-9}\,\text{T}..$$

(b) Since the $\vec{E}$-wave oscillates in the z direction and travels in the x direction, we have $B_x = B_z = 0$. So, the oscillation of the magnetic field is parallel to the y axis.

(c) The direction (+x) of the electromagnetic wave propagation is determined by $\vec{E}\times\vec{B}$. If the electric field points in +z, then the magnetic field must point in the –y direction.

With SI units understood, we may write

$$B_y = B_m \cos\left[\pi\times10^{15}\left(t-\frac{x}{c}\right)\right] = \frac{2.0\cos\left[10^{15}\pi\left(t-x/c\right)\right]}{3.0\times10^{8}}$$

$$= \left(6.7\times10^{-9}\right)\cos\left[10^{15}\pi\left(t-\frac{x}{c}\right)\right]$$

9. If P is the power and Δt is the time interval of one pulse, then the energy in a pulse is

$$E = P\Delta t = \left(100\times 10^{12}\text{ W}\right)\left(1.0\times 10^{-9}\text{ s}\right) = 1.0\times 10^{5}\text{ J}.$$

10. The intensity of the signal at Proxima Centauri is

$$I = \frac{P}{4\pi r^2} = \frac{1.0\times 10^6 \text{ W}}{4\pi\left[(4.3\,\text{ly})(9.46\times 10^{15} \text{ m/ly})\right]^2} = 4.8\times 10^{-29} \text{ W/m}^2.$$

11. The intensity is the average of the Poynting vector:

$$I = S_{\text{avg}} = \frac{cB_m^2}{2\mu_0} = \frac{\left(3.0\times10^{8}\ \text{m/s}\right)\left(1.0\times10^{-4}\ \text{T}\right)^2}{2\left(1.26\times10^{-6}\ \text{H/m}\right)^2} = 1.2\times10^{6}\ \text{W/m}^2.$$

12. (a) The amplitude of the magnetic field in the wave is

$$B_m = \frac{E_m}{c} = \frac{5.00\ \text{V/m}}{2.998\times10^{8}\ \text{m/s}} = 1.67\times10^{-8}\ \text{T}.$$

(b) The intensity is the average of the Poynting vector:

$$I = S_{\text{avg}} = \frac{E_m^2}{2\mu_0 c} = \frac{(5.00\ \text{V/m})^2}{2(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(2.998\times10^{8}\ \text{m/s})} = 3.31\times10^{-2}\ \text{W/m}^2.$$

13. (a) We use $I = E_m^2/2\mu_0 c$ to calculate E_m:

$$E_m = \sqrt{2\mu_0 I_c} = \sqrt{2\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.40\times10^{3}\ \text{W/m}^2\right)\left(2.998\times10^{8}\ \text{m/s}\right)}$$
$$= 1.03\times10^{3}\ \text{V/m}.$$

(b) The magnetic field amplitude is therefore

$$B_m = \frac{E_m}{c} = \frac{1.03\times10^{4}\ \text{V/m}}{2.998\times10^{8}\ \text{m/s}} = 3.43\times10^{-6}\ \text{T}.$$

14. (a) The power received is

$$P_r = \left(1.0\times10^{-12}\ \text{W}\right)\frac{\pi(300\ \text{m})^2/4}{4\pi\left(6.37\times10^{6}\ \text{m}\right)^2} = 1.4\times10^{-22}\ \text{W}.$$

(b) The power of the source would be

$$P = 4\pi r^2 I = 4\pi\left[\left(2.2\times10^{4}\ \text{ly}\right)\left(9.46\times10^{15}\ \text{m/ly}\right)\right]^2\left[\frac{1.0\times10^{-12}\ \text{W}}{4\pi\left(6.37\times10^{6}\ \text{m}\right)^2}\right] = 1.1\times10^{15}\ \text{W}.$$

15. (a) The magnetic field amplitude of the wave is

$$B_m = \frac{E_m}{c} = \frac{2.0\,\text{V/m}}{2.998\times10^{8}\,\text{m/s}} = 6.7\times10^{-9}\,\text{T}.$$

(b) The intensity is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(2.0\,\text{V/m})^2}{2(4\pi\times10^{-7}\,\text{T}\cdot\text{m/A})(2.998\times10^{8}\,\text{m/s})} = 5.3\times10^{-3}\,\text{W/m}^2.$$

(c) The power of the source is

$$P = 4\pi r^2 I_{\text{avg}} = 4\pi(10\,\text{m})^2\left(5.3\times10^{-3}\,\text{W/m}^2\right) = 6.7\,\text{W}.$$

16. From the equation immediately preceding Eq. 33-12, we see that the maximum value of $\partial B/\partial t$ is ωB_m. We can relate B_m to the intensity: $B_m = E_m/c = \sqrt{2c\mu_o I}/c$, and relate the intensity to the power P (and distance r) using Eq. 33-27. Finally, we relate ω to wavelength λ using $\omega = kc = 2\pi c/\lambda$. Putting all this together, we obtain

$$\left(\frac{\partial B}{\partial t}\right)_{\max} = \sqrt{\frac{2\mu_o P}{4\pi c}}\,\frac{2\pi c}{\lambda r} = 3.44\times10^{6}\ \text{T/s}.$$

17. (a) The average rate of energy flow per unit area, or intensity, is related to the electric field amplitude E_m by $I = E_m^2 / 2\mu_0 c$, so

$$E_m = \sqrt{2\mu_0 cI} = \sqrt{2(4\pi \times 10^{-7}\ \text{H/m})(2.998 \times 10^8\ \text{m/s})(10 \times 10^{-6}\ \text{W/m}^2)}$$
$$= 8.7 \times 10^{-2}\ \text{V/m}.$$

(b) The amplitude of the magnetic field is given by

$$B_m = \frac{E_m}{c} = \frac{8.7 \times 10^{-2}\ \text{V/m}}{2.998 \times 10^8\ \text{m/s}} = 2.9 \times 10^{-10}\ \text{T}.$$

(c) At a distance r from the transmitter, the intensity is $I = P / 2\pi r^2$, where P is the power of the transmitter over the hemisphere having a surface area $2\pi r^2$. Thus

$$P = 2\pi r^2 I = 2\pi \left(10 \times 10^3\ \text{m}\right)^2 \left(10 \times 10^{-6}\ \text{W/m}^2\right) = 6.3 \times 10^3\ \text{W}.$$

18. (a) The expression $E_y = E_m \sin(kx - \omega t)$ it fits the requirement "at point P … [it] is decreasing with time" if we imagine P is just to the right ($x > 0$) of the coordinate origin (but at a value of x less than $\pi/2k = \lambda/4$ which is where there would be a maximum, at t = 0). It is important to bear in mind, in this description, that the wave is moving to the right. Specifically, $x_P = \frac{1}{k}\sin^{-1}(1/4)$ so that $E_y = (1/4)\,E_m$ at $t = 0$, there. Also, $E_y = 0$ with our choice of expression for E_y. Therefore, part (a) is answered simply by solving for x_P. Since $k = 2\pi f/c$ we find

$$x_P = \frac{c}{2\pi f}\sin^{-1}(1/4) = 30.1 \text{ nm}.$$

(b) If we proceed to the right on the x axis (still studying this "snapshot" of the wave at t = 0) we find another point where $E_y = 0$ at a distance of one-half wavelength from the previous point where $E_y = 0$. Thus (since $\lambda = c/f$) the next point is at $x = \frac{1}{2}\lambda = \frac{1}{2}c/f$ and is consequently a distance $c/2f - x_P$ = 345 nm to the right of P.

19. The plasma completely reflects all the energy incident on it, so the radiation pressure is given by $p_r = 2I/c$, where I is the intensity. The intensity is $I = P/A$, where P is the power and A is the area intercepted by the radiation. Thus

$$p_r = \frac{2P}{Ac} = \frac{2\left(1.5\times10^{9}\,\mathrm{W}\right)}{\left(1.00\times10^{-6}\,\mathrm{m}^2\right)\left(2.998\times10^{8}\,\mathrm{m/s}\right)} = 1.0\times10^{7}\,\mathrm{Pa}.$$

20. The radiation pressure is

$$p_r = \frac{I}{c} = \frac{10\,\text{W/m}^2}{2.998\times10^8\ \text{m/s}} = 3.3\times10^{-8}\ \text{Pa}.$$

21. Since the surface is perfectly absorbing, the radiation pressure is given by $p_r = I/c$, where I is the intensity. Since the bulb radiates uniformly in all directions, the intensity a distance r from it is given by $I = P/4\pi r^2$, where P is the power of the bulb. Thus

$$p_r = \frac{P}{4\pi r^2 c} = \frac{500\,\text{W}}{4\pi(1.5\text{ m})^2(2.998\times10^8\text{ m/s})} = 5.9\times10^{-8}\text{ Pa}.$$

22. (a) The radiation pressure produces a force equal to

$$F_r = p_r\left(\pi R_e^2\right) = \left(\frac{I}{c}\right)\left(\pi R_e^2\right) = \frac{\pi\left(1.4\times10^3\ \mathrm{W/m^2}\right)\left(6.37\times10^6\ \mathrm{m}\right)^2}{2.998\times10^8\ \mathrm{m/s}} = 6.0\times10^8\ \mathrm{N}.$$

(b) The gravitational pull of the Sun on Earth is

$$F_{\mathrm{grav}} = \frac{GM_sM_e}{d_{es}^2} = \frac{\left(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}\right)\left(2.0\times10^{30}\ \mathrm{kg}\right)\left(5.98\times10^{24}\ \mathrm{kg}\right)}{\left(1.5\times10^{11}\ \mathrm{m}\right)^2}$$

$$= 3.6\times10^{22}\ \mathrm{N},$$

which is much greater than F_r.

23. (a) Since $c = \lambda f$, where λ is the wavelength and *f* is the frequency of the wave,

$$f = \frac{c}{\lambda} = \frac{2.998\times10^{8}\text{ m/s}}{3.0\text{ m}} = 1.0\times10^{8}\text{ Hz}.$$

(b) The angular frequency is

$$\omega = 2\pi f = 2\pi(1.0\times10^{8}\text{ Hz}) = 6.3\times10^{8}\text{ rad/s}.$$

(c) The angular wave number is

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{3.0\text{ m}} = 2.1\text{ rad/m}.$$

(d) The magnetic field amplitude is

$$B_m = \frac{E_m}{c} = \frac{300\text{ V/m}}{2.998\times10^{8}\text{ m/s}} = 1.0\times10^{-6}\text{ T}.$$

(e) $\vec{B}$ must be in the positive *z* direction when $\vec{E}$ is in the positive *y* direction in order for $\vec{E}\times\vec{B}$ to be in the positive *x* direction (the direction of propagation).

(f) The intensity of the wave is

$$I = \frac{E_m^2}{2\mu_0 c} = \frac{(300\text{ V/m})^2}{2(4\pi\times10^{-7}\text{ H/m})(2.998\times10^{8}\text{ m/s})} = 119\text{ W/m}^2 \approx 1.2\times10^{2}\text{ W/m}^2.$$

(g) Since the sheet is perfectly absorbing, the rate per unit area with which momentum is delivered to it is *I*/*c*, so

$$\frac{dp}{dt} = \frac{IA}{c} = \frac{(119\text{ W/m}^2)(2.0\text{ m}^2)}{2.998\times10^{8}\text{ m/s}} = 8.0\times10^{-7}\text{ N}.$$

(h) The radiation pressure is

$$p_r = \frac{dp/dt}{A} = \frac{8.0\times10^{-7}\text{ N}}{2.0\text{ m}^2} = 4.0\times10^{-7}\text{ Pa}.$$

24. (a) We note that the cross section area of the beam is $\pi d^2/4$, where d is the diameter of the spot ($d = 2.00\lambda$). The beam intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{5.00\times 10^{-3}\text{ W}}{\pi\left[(2.00)(633\times 10^{-9}\text{ m})\right]^2/4} = 3.97\times 10^{9}\text{ W/m}^2.$$

(b) The radiation pressure is

$$p_r = \frac{I}{c} = \frac{3.97\times 10^{9}\text{ W/m}^2}{2.998\times 10^{8}\text{ m/s}} = 13.2\text{ Pa}.$$

(c) In computing the corresponding force, we can use the power and intensity to eliminate the area (mentioned in part (a)). We obtain

$$F_r = \left(\frac{\pi d^2}{4}\right)p_r = \left(\frac{P}{I}\right)p_r = \frac{(5.00\times 10^{-3}\text{ W})(13.2\text{ Pa})}{3.97\times 10^{9}\text{ W/m}^2} = 1.67\times 10^{-11}\text{ N}.$$

(d) The acceleration of the sphere is

$$a = \frac{F_r}{m} = \frac{F_r}{\rho(\pi d^3/6)} = \frac{6(1.67\times 10^{-11}\text{ N})}{\pi(5.00\times 10^{3}\text{ kg/m}^3)[(2.00)(633\times 10^{-9}\text{ m})]^3}$$
$$= 3.14\times 10^{3}\text{ m/s}^2.$$

25. Let f be the fraction of the incident beam intensity that is reflected. The fraction absorbed is $1 - f$. The reflected portion exerts a radiation pressure of

$$p_r = \frac{2fI_0}{c}$$

and the absorbed portion exerts a radiation pressure of

$$p_a = \frac{(1-f)I_0}{c},$$

where I_0 is the incident intensity. The factor 2 enters the first expression because the momentum of the reflected portion is reversed. The total radiation pressure is the sum of the two contributions:

$$p_{\text{total}} = p_r + p_a = \frac{2fI_0 + (1-f)I_0}{c} = \frac{(1+f)I_0}{c}.$$

To relate the intensity and energy density, we consider a tube with length ℓ and cross-sectional area A, lying with its axis along the propagation direction of an electromagnetic wave. The electromagnetic energy inside is $U = uA\ell$, where u is the energy density. All this energy passes through the end in time $t = \ell / c$, so the intensity is

$$I = \frac{U}{At} = \frac{uA\ell c}{A\ell} = uc.$$

Thus $u = I/c$. The intensity and energy density are positive, regardless of the propagation direction. For the partially reflected and partially absorbed wave, the intensity just outside the surface is $I = I_0 + fI_0 = (1 + f)I_0$, where the first term is associated with the incident beam and the second is associated with the reflected beam. Consequently, the energy density is

$$u = \frac{I}{c} = \frac{(1+f)I_0}{c},$$

the same as radiation pressure.

26. The mass of the cylinder is $m = \rho(\pi D^2/4)H$, where D is the diameter of the cylinder. Since it is in equilibrium

$$F_{\text{net}} = mg - F_r = \frac{\pi H D^2 g\rho}{4} - \left(\frac{\pi D^2}{4}\right)\left(\frac{2I}{c}\right) = 0.$$

We solve for H:

$$H = \frac{2I}{gc\rho} = \left(\frac{2P}{\pi D^2/4}\right)\frac{1}{gc\rho}$$
$$= \frac{2(4.60\,\text{W})}{[\pi(2.60\times10^{-3}\,\text{m})^2/4](9.8\,\text{m/s}^2)(3.0\times10^8\,\text{m/s})(1.20\times10^3\,\text{kg/m}^3)}$$
$$= 4.91\times10^{-7}\,\text{m}.$$

27. If the beam carries energy U away from the spaceship, then it also carries momentum $p = U/c$ away. Since the total momentum of the spaceship and light is conserved, this is the magnitude of the momentum acquired by the spaceship. If P is the power of the laser, then the energy carried away in time t is $U = Pt$. We note that there are 86400 seconds in a day. Thus, $p = Pt/c$ and, if m is mass of the spaceship, its speed is

$$v = \frac{p}{m} = \frac{Pt}{mc} = \frac{(10\times 10^3\ \text{W})(86400\ \text{s})}{(1.5\times 10^3\ \text{kg})(2.998\times 10^8\ \text{m/s})} = 1.9\times 10^{-3}\ \text{m/s}.$$

28. We require $F_{\text{grav}} = F_r$ or

$$G\frac{mM_s}{d_{es}^2} = \frac{2IA}{c},$$

and solve for the area A:

$$A = \frac{cGmM_s}{2Id_{es}^2} = \frac{(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1500\ \text{kg})(1.99\times10^{30}\ \text{kg})(2.998\times10^{8}\ \text{m/s})}{2(1.40\times10^{3}\ \text{W/m}^2)(1.50\times10^{11}\ \text{m})^2}$$
$$= 9.5\times10^{5}\ \text{m}^2 = 0.95\ \text{km}^2.$$

29. Eq. 33-27 suggests that the slope in an intensity versus inverse-square-distance graph (I plotted versus r^{-2}) is $P/4\pi$. We estimate the slope to be about 20 (in SI units) which means the power is $P = 4\pi(30) \approx 2.5 \times 10^2$ W.

30. (a) The upward force supplied by radiation pressure in this case (Eq. 33-32) must be equal to the magnitude of the pull of gravity (mg). For a sphere, the "projected" area (which is a factor in Eq. 33-32) is that of a circle $A = \pi r^2$ (not the entire surface area of the sphere) and the volume (needed because the mass is given by the density multiplied by the volume: $m = \rho V$) is $V = \frac{4}{3}\pi r^3$. Finally, the intensity is related to the power P of the light source and another area factor $4\pi R^2$, given by Eq. 33-27. In this way, with ρ = 19000 in SI units, equating the forces leads to

$$P = 4\pi R^2 c\, \rho \tfrac{4}{3}\pi r^3 g \,/\pi r^2 \;= 4.68 \times 10^{11} \text{ W}.$$

(b) Any chance disturbance could move the sphere from being directly above the source, and then the two force vectors would no longer be along the same axis.

31. The angle between the direction of polarization of the light incident on the first polarizing sheet and the polarizing direction of that sheet is $\theta_1 = 70°$. If I_0 is the intensity of the incident light, then the intensity of the light transmitted through the first sheet is

$$I_1 = I_0 \cos^2 \theta_1 = (43 \text{ W/m}^2) \cos^2 70° = 5.03 \text{ W/m}^2.$$

The direction of polarization of the transmitted light makes an angle of 70° with the vertical and an angle of $\theta_2 = 20°$ with the horizontal. θ_2 is the angle it makes with the polarizing direction of the second polarizing sheet. Consequently, the transmitted intensity is

$$I_2 = I_1 \cos^2 \theta_2 = (5.03 \text{ W/m}^2) \cos^2 20° = 4.4 \text{ W/m}^2.$$

32. In this case, we replace $I_0 \cos^2 70°$ by $\frac{1}{2} I_0$ as the intensity of the light after passing through the first polarizer. Therefore,

$$I_f = \frac{1}{2} I_0 \cos^2 (90° - 70°) = \frac{1}{2} (43 \text{ W/m}^2)(\cos^2 20°) = 19 \text{ W/m}^2 .$$

33. Let I_0 be the intensity of the unpolarized light that is incident on the first polarizing sheet. The transmitted intensity is $I_1 = \frac{1}{2} I_0$, and the direction of polarization of the transmitted light is θ_1 = 40° counterclockwise from the y axis in the diagram. The polarizing direction of the second sheet is θ_2 = 20° clockwise from the y axis, so the angle between the direction of polarization that is incident on that sheet and the polarizing direction of the sheet is 40° + 20° = 60°. The transmitted intensity is

$$I_2 = I_1 \cos^2 60° = \frac{1}{2} I_0 \cos^2 60°,$$

and the direction of polarization of the transmitted light is 20° clockwise from the y axis. The polarizing direction of the third sheet is θ_3 = 40° counterclockwise from the y axis. Consequently, the angle between the direction of polarization of the light incident on that sheet and the polarizing direction of the sheet is 20° + 40° = 60°. The transmitted intensity is

$$I_3 = I_2 \cos^2 60° = \frac{1}{2} I_0 \cos^4 60° = 3.1 \times 10^{-2}.$$

Thus, 3.1% of the light's initial intensity is transmitted.

34. After passing through the first polarizer the initial intensity I_0 reduces by a factor of 1/2. After passing through the second one it is further reduced by a factor of $\cos^2(\pi - \theta_1 - \theta_2) = \cos^2(\theta_1 + \theta_2)$. Finally, after passing through the third one it is again reduced by a factor of $\cos^2(\pi - \theta_2 - \theta_3) = \cos^2(\theta_2 + \theta_3)$. Therefore,

$$\frac{I_f}{I_0} = \frac{1}{2}\cos^2(\theta_1 + \theta_2)\cos^2(\theta_2 + \theta_3) = \frac{1}{2}\cos^2(50° + 50°)\cos^2(50° + 50°)$$
$$= 4.5 \times 10^{-4}.$$

Thus, 0.045% of the light's initial intensity is transmitted.

35. (a) Since the incident light is unpolarized, half the intensity is transmitted and half is absorbed. Thus the transmitted intensity is $I = 5.0$ mW/m^2. The intensity and the electric field amplitude are related by $I = E_m^2 / 2\mu_0 c$, so

$$E_m = \sqrt{2\mu_0 cI} = \sqrt{2(4\pi\times10^{-7}\text{ H/m})(3.00\times10^{8}\text{ m/s})(5.0\times10^{-3}\text{ W/m}^2)}$$
$$= 1.9\text{ V/m}.$$

(b) The radiation pressure is $p_r = I_a/c$, where I_a is the absorbed intensity. Thus

$$p_r = \frac{5.0\times10^{-3}\text{ W/m}^2}{3.00\times10^{8}\text{ m/s}} = 1.7\times10^{-11}\text{ Pa}.$$

36. We examine the point where the graph reaches zero: $\theta_2 = 160°$. Since the polarizers must be "crossed" for the intensity to vanish, then $\theta_1 = 160° - 90° = 70°$. Now we consider the case $\theta_2 = 90°$ (which is hard to judge from the graph). Since θ_1 is still equal to 70º, then the angle between the polarizers is now $\Delta\theta = 20°$. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is $\frac{1}{2}\cos^2(\Delta\theta) = 0.442 \approx 44\%$.

37. As the polarized beam of intensity I_0 passes the first polarizer, its intensity is reduced to $I_0 \cos^2 \theta$. After passing through the second polarizer which makes a 90° angle with the first filter, the intensity is $I = (I_0 \cos^2\theta)\sin^2\theta = I_0/10$ which implies $\sin^2\theta\cos^2\theta = 1/10$, or $\sin\theta\cos\theta = \sin 2\theta/2 = 1/\sqrt{10}$. This leads to $\theta = 70°$ or $20°$.

38. We note the points at which the curve is zero (θ_2 = 0° and 90°) in Fig. 33-44(b). We infer that sheet 2 is perpendicular to one of the other sheets at θ_2 = 0°, and that it is perpendicular to the *other* of the other sheets when θ_2 = 90°. Without loss of generality, we choose θ_1 = 0°, θ_3 = 90°. Now, when θ_2 = 30°, it will be $\Delta\theta$ = 30° relative to sheet 1 and $\Delta\theta'$ = 60° relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}\cos^2(\Delta\theta)\cos^2(\Delta\theta') = 9.4\% .$$

39. Let I_0 be the intensity of the incident beam and f be the fraction that is polarized. Thus, the intensity of the polarized portion is $f I_0$. After transmission, this portion contributes $f I_0 \cos^2 \theta$ to the intensity of the transmitted beam. Here θ is the angle between the direction of polarization of the radiation and the polarizing direction of the filter. The intensity of the unpolarized portion of the incident beam is $(1-f)I_0$ and after transmission, this portion contributes $(1 - f)I_0/2$ to the transmitted intensity. Consequently, the transmitted intensity is

$$I = f I_0 \cos^2 \theta + \frac{1}{2}(1-f)I_0.$$

As the filter is rotated, $\cos^2 \theta$ varies from a minimum of 0 to a maximum of 1, so the transmitted intensity varies from a minimum of

$$I_{\min} = \frac{1}{2}(1-f)I_0$$

to a maximum of

$$I_{\max} = f I_0 + \frac{1}{2}(1-f)I_0 = \frac{1}{2}(1+f)I_0.$$

The ratio of $I_{\max}$ to $I_{\min}$ is

$$\frac{I_{\max}}{I_{\min}} = \frac{1+f}{1-f}.$$

Setting the ratio equal to 5.0 and solving for f, we get $f = 0.67$.

40. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2 \theta_2 \cos^2 (90° - \theta_2) \ .$$

Using trig identities, we rewrite this as

$$\frac{I}{I_0} = \frac{1}{8} \sin^2 (2\theta_2) \ .$$

(a) Therefore we find $\theta_2 = \frac{1}{2} \sin^{-1} \sqrt{0.40} = 19.6°$.

(b) Since the first expression we wrote is symmetric under the exchange: $\theta_2 \leftrightarrow 90° - \theta_2$, then we see that the angle's complement, 70.4°, is also a solution.

41. (a) The fraction of light which is transmitted by the glasses is

$$\frac{I_f}{I_0} = \frac{E_f^2}{E_0^2} = \frac{E_v^2}{E_v^2 + E_h^2} = \frac{E_v^2}{E_v^2 + (2.3E_v)^2} = 0.16.$$

(b) Since now the horizontal component of $\vec{E}$ will pass through the glasses,

$$\frac{I_f}{I_0} = \frac{E_h^2}{E_v^2 + E_h^2} = \frac{(2.3E_v)^2}{E_v^2 + (2.3E_v)^2} = 0.84.$$

42. We note the points at which the curve is zero ($\theta_2 = 60°$ and 140°) in Fig. 33-44(b). We infer that sheet 2 is perpendicular to one of the other sheets at $\theta_2 = 60°$, and that it is perpendicular to the *other* of the other sheets when $\theta_2 = 140°$. Without loss of generality, we choose $\theta_1 = 150°$, $\theta_3 = 50°$. Now, when $\theta_2 = 90°$, it will be $|\Delta\theta| = 60°$ relative to sheet 1 and $|\Delta\theta'| = 40°$ relative to sheet 3. Therefore,

$$\frac{I_f}{I_i} = \frac{1}{2}(\cos(\Delta\theta))^2\,(\cos(\Delta\theta'))^2 = 7.3\% \,.$$

43. (a) The rotation cannot be done with a single sheet. If a sheet is placed with its polarizing direction at an angle of 90° to the direction of polarization of the incident radiation, no radiation is transmitted. It can be done with two sheets. We place the first sheet with its polarizing direction at some angle θ, between 0 and 90°, to the direction of polarization of the incident radiation. Place the second sheet with its polarizing direction at 90° to the polarization direction of the incident radiation. The transmitted radiation is then polarized at 90° to the incident polarization direction. The intensity is $I_0 \cos^2\theta \cos^2(90° - \theta) = I_0 \cos^2\theta \sin^2\theta$, where I_0 is the incident radiation. If θ is not 0 or 90°, the transmitted intensity is not zero.

(b) Consider n sheets, with the polarizing direction of the first sheet making an angle of $\theta = 90°/n$ relative to the direction of polarization of the incident radiation. The polarizing direction of each successive sheet is rotated $90°/n$ in the same sense from the polarizing direction of the previous sheet. The transmitted radiation is polarized, with its direction of polarization making an angle of 90° with the direction of polarization of the incident radiation. The intensity is

$$I = I_0 \cos^{2n}(90°/n).$$

We want the smallest integer value of n for which this is greater than $0.60I_0$. We start with $n = 2$ and calculate $\cos^{2n}(90°/n)$. If the result is greater than 0.60, we have obtained the solution. If it is less, increase n by 1 and try again. We repeat this process, increasing n by 1 each time, until we have a value for which $\cos^{2n}(90°/n)$ is greater than 0.60. The first one will be $n = 5$.

44. The angle of incidence for the light ray on mirror *B* is $90° - \theta$. So the outgoing ray r' makes an angle $90° - (90° - \theta) = \theta$ with the vertical direction, and is antiparallel to the incoming one. The angle between i and r' is therefore 180°.

45. The law of refraction states

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 .$$

We take medium 1 to be the vacuum, with $n_1 = 1$ and $\theta_1 = 32.0°$. Medium 2 is the glass, with $\theta_2 = 21.0°$. We solve for n_2:

$$n_2 = n_1 \frac{\sin\theta_1}{\sin\theta_2} = (1.00)\left(\frac{\sin 32.0°}{\sin 21.0°}\right) = 1.48.$$

46. (a) For the angles of incidence and refraction to be equal, the graph in Fig. 33-48(b) would consist of a "$y = x$" line at 45º in the plot. Instead, the curve for material 1 falls under such a "$y = x$" line, which tells us that all refraction angles are less than incident ones. With $\theta_2 < \theta_1$ Snell's law implies $n_2 > n_1$.

(b) Using the same argument as in (a), the value of n_2 for material 2 is also greater than that of water (n_1).

(c) It's easiest to examine the right end-point of each curve. With $\theta_1 = 90°$ and $\theta_2 = ¾(90°)$, and with $n_1 = 1.33$ (Table 33-1) we find, from Snell's law, $n_2 = 1.4$ for material 1.

(d) Similarly, with $\theta_1 = 90°$ and $\theta_2 = ½(90°)$, we obtain $n_2 = 1.9$.

47. Note that the normal to the refracting surface is vertical in the diagram. The angle of refraction is $\theta_2 = 90°$ and the angle of incidence is given by tan $\theta_1 = L/D$, where D is the height of the tank and L is its width. Thus

$$\theta_1 = \tan^{-1}\left(\frac{L}{D}\right) = \tan^{-1}\left(\frac{1.10 \text{ m}}{0.850 \text{ m}}\right) = 52.31°.$$

The law of refraction yields

$$n_1 = n_2 \frac{\sin\theta_2}{\sin\theta_1} = (1.00)\left(\frac{\sin 90°}{\sin 52.31°}\right) = 1.26,$$

where the index of refraction of air was taken to be unity.

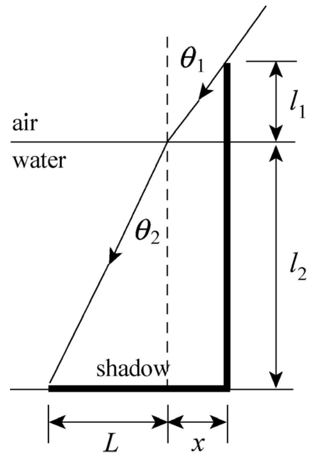
48. (a) For the angles of incidence and refraction to be equal, the graph in Fig. 33-48(b) would consist of a "$y = x$" line at 45º in the plot. Instead, the curve for material 1 falls under such a "$y = x$" line, which tells us that all refraction angles are less than incident ones. With $\theta_2 < \theta_1$ Snell's law implies $n_2 > n_1$.

(b) Using the same argument as in (a), the value of n_2 for material 2 is also greater than that of water (n_1).

(c) It's easiest to examine the topmost point of each curve. With $\theta_2 = 90º$ and $\theta_1 = ½(90º)$, and with $n_2 = 1.33$ (Table 33-1) we find $n_1 = 1.9$ from Snell's law.

(d) Similarly, with $\theta_2 = 90º$ and $\theta_1 = ¾(90º)$, we obtain $n_1 = 1.4$.

49. Consider a ray that grazes the top of the pole, as shown in the diagram that follows.



Here $\theta_1 = 90° - \theta = 35°$, $\ell_1 = 0.50$ m, and $\ell_2 = 1.50$ m. The length of the shadow is $x + L$. x is given by

$$x = \ell_1 \tan\theta_1 = (0.50 \text{ m})\tan 35° = 0.35 \text{ m}.$$

According to the law of refraction, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. We take $n_1 = 1$ and $n_2 = 1.33$ (from Table 33-1). Then,

$$\theta_2 = \sin^{-1}\left(\frac{\sin\theta_1}{n_2}\right) = \sin^{-1}\left(\frac{\sin 35.0°}{1.33}\right) = 25.55°.$$

L is given by

$$L = \ell_2 \tan\theta_2 = (1.50 \text{ m})\tan 25.55° = 0.72 \text{ m}.$$

The length of the shadow is 0.35 m + 0.72 m = 1.07 m.

50. (a) A simple implication of Snell's law is that $\theta_2 = \theta_1$ when $n_1 = n_2$. Since the angle of incidence is shown in Fig. 33-52(a) to be 30º, then we look for a point in Fig. 33-52(b) where $\theta_2 = 30º$. This seems to occur when $n_2 = 1.7$. By inference, then, $n_1 = 1.7$.

(b) From $1.7\sin(60º) = 2.4\sin(\theta_2)$ we get $\theta_2 = 38°$.

51. (a) Approximating $n = 1$ for air, we have

$$n_1 \sin\theta_1 = (1)\sin\theta_5 \Rightarrow 56.9° = \theta_5$$

and with the more accurate value for n_{air} in Table 33-1, we obtain 56.8°.

(b) Eq. 33-44 leads to

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_4 \sin\theta_4$$

so that

$$\theta_4 = \sin^{-1}\left(\frac{n_1}{n_4}\sin\theta_1\right) = 35.3°.$$

52. (a) We use subscripts *b* and *r* for the blue and red light rays. Snell's law gives

$$\theta_{2b} = \sin^{-1}\left(\frac{1}{1.343}\sin(70°)\right) = 44.403°$$
$$\theta_{2r} = \sin^{-1}\left(\frac{1}{1.331}\sin(70°)\right) = 44.911°$$

for the refraction angles at the first surface (where the normal axis is vertical). These rays strike the second surface (where *A* is) at complementary angles to those just calculated (since the normal axis is horizontal for the second surface). Taking this into consideration, we again use Snell's law to calculate the second refractions (with which the light re-enters the air):

$$\theta_{3b} = \sin^{-1}[1.343\sin(90° - \theta_{2b})] = 73.636°$$
$$\theta_{3r} = \sin^{-1}[1.331\sin(90° - \theta_{2r})] = 70.497°$$

which differ by 3.1° (thus giving a rainbow of angular width 3.1°).

(b) Both of the refracted rays emerges from the bottom side with the same angle (70°) with which they were incident on the topside (the occurrence of an intermediate reflection [from side 2] does not alter this overall fact: light comes into the block at the same angle that it emerges with from the opposite parallel side). There is thus no difference (the difference is 0°) and thus there is no rainbow in this case.

53. We label the light ray's point of entry *A*, the vertex of the prism *B*, and the light ray's exit point *C*. Also, the point in Fig. 33-55 where ψ is defined (at the point of intersection of the extrapolations of the incident and emergent rays) is denoted *D*. The angle indicated by *ADC* is the supplement of ψ, so we denote it $\psi_s = 180° - \psi$. The angle of refraction in the glass is $\theta_2 = \frac{1}{n}\sin\theta$. The angles between the interior ray and the nearby surfaces is the complement of θ_2, so we denote it $\theta_{2c} = 90° - \theta_2$. Now, the angles in the triangle *ABC* must add to 180°:

$$180° = 2\theta_{2c} + \phi \Rightarrow \theta_2 = \frac{\phi}{2}.$$

Also, the angles in the triangle *ADC* must add to 180°:

$$180° = 2(\theta - \theta_2) + \psi_s \Rightarrow \theta = 90° + \theta_2 - \frac{1}{2}\psi_s$$

which simplifies to $\theta = \theta_2 + \frac{1}{2}\psi$. Combining this with our previous result, we find $\theta = \frac{1}{2}(\phi + \psi)$. Thus, the law of refraction yields

$$n = \frac{\sin(\theta)}{\sin(\theta_2)} = \frac{\sin\left(\frac{1}{2}(\phi + \psi)\right)}{\sin\left(\frac{1}{2}\phi\right)}.$$

54. The critical angle is $\theta_c = \sin^{-1}\left(\frac{1}{n}\right) = \sin^{-1}\left(\frac{1}{1.8}\right) = 34°$.

55. Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. Thus, the diameter *D* of the circle in question is

$$D = 2h\tan\theta_c = 2h\tan\left[\sin^{-1}\left(\frac{1}{n_w}\right)\right] = 2(80.0\,\text{cm})\tan\left[\sin^{-1}\left(\frac{1}{1.33}\right)\right] = 182\text{ cm.}$$

56. (a) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1 - \left(\frac{n_3}{n_2}\right)^2} = \sqrt{(n_2)^2 - (n_3)^2}$$

leads to $\theta = 26.8°$.

(b) Increasing θ leads to a decrease of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle; therefore, there will be some transmission of light into material 3.

57. (a) In the notation of this problem, Eq. 33-47 becomes

$$\theta_c = \sin^{-1}\frac{n_3}{n_2}$$

which yields $n_3 = 1.39$ for $\theta_c = \phi = 60°$.

(b) Applying Eq. 33-44 law to the interface between material 1 and material 2, we have

$$n_2 \sin 30° = n_1 \sin\theta$$

which yields $\theta = 28.1°$.

(c) Decreasing θ will increase ϕ and thus cause the ray to strike the interface (between materials 2 and 3) at an angle larger than θ_c. Therefore, no transmission of light into material 3 can occur.

58. (a) The angle of incidence $\theta_{B,1}$ at B is the complement of the critical angle at A; its sine is

$$\sin\theta_{B,1} = \cos\theta_c = \sqrt{1 - \left(\frac{n_3}{n_2}\right)^2}$$

so that the angle of refraction $\theta_{B,2}$ at B becomes

$$\theta_{B,2} = \sin^{-1}\left(\frac{n_2}{n_3}\sqrt{1 - \left(\frac{n_3}{n_2}\right)^2}\right) = \sin^{-1}\sqrt{\left(\frac{n_2}{n_3}\right)^2 - 1} = 35.1° .$$

(b) From $n_1 \sin\theta = n_2 \sin\theta_c = n_2(n_3/n_2)$, we find

$$\theta = \sin^{-1}\frac{n_3}{n_1} = 49.9° .$$

(c) The angle of incidence $\theta_{A,1}$ at A is the complement of the critical angle at B; its sine is

$$\sin\theta_{A,1} = \cos\theta_c = \sqrt{1 - \left(\frac{n_3}{n_2}\right)^2}$$

so that the angle of refraction $\theta_{A,2}$ at A becomes

$$\theta_{A,2} = \sin^{-1}\left(\frac{n_2}{n_3}\sqrt{1 - \left(\frac{n_3}{n_2}\right)^2}\right) = \sin^{-1}\sqrt{\left(\frac{n_2}{n_3}\right)^2 - 1} = 35.1° .$$

(d) From

$$n_1 \sin\theta = n_2 \sin\theta_{A,1} = n_2\sqrt{1 - \left(\frac{n_3}{n_2}\right)^2} = \sqrt{(n_2)^2 - (n_3)^2}$$

we find

$$\theta = \sin^{-1}\frac{\sqrt{(n_2)^2 - (n_3)^2}}{n_1} = 26.1° .$$

(e) The angle of incidence $\theta_{B,1}$ at B is the complement of the Brewster angle at A; its sine is

$$\sin\theta_{B,1} = \frac{n_2}{\sqrt{(n_2)^2 + (n_3)^2}}$$

so that the angle of refraction $\theta_{B,2}$ at B becomes

$$\theta_{B,2} = \sin^{-1}\left(\frac{(n_2)^2}{n_3\sqrt{(n_2)^2+(n_3)^2}}\right) = 60.7^\circ \ .$$

(f) From

$$n_1 \sin\theta = n_2 \sin\theta_{\text{Brewster}} = n_2 \frac{n_3}{\sqrt{(n_2)^2+(n_3)^2}}$$

we find

$$\theta = \sin^{-1}\frac{n_2 n_3}{n_1\sqrt{(n_2)^2+(n_3)^2}} = 35.3^\circ \ .$$

59. When examining Fig. 33-59, it is important to note that the angle (measured from the central axis) for the light ray in air, θ, is not the angle for the ray in the glass core, which we denote θ'. The law of refraction leads to

$$\sin\theta' = \frac{1}{n_1}\sin\theta$$

assuming $n_{\text{air}} = 1$. The angle of incidence for the light ray striking the coating is the complement of θ', which we denote as θ'_{comp} and recall that

$$\sin\theta'_{\text{comp}} = \cos\theta' = \sqrt{1-\sin^2\theta'}.$$

In the critical case, θ'_{comp} must equal θ_c specified by Eq. 33-47. Therefore,

$$\frac{n_2}{n_1} = \sin\theta'_{\text{comp}} = \sqrt{1-\sin^2\theta'} = \sqrt{1-\left(\frac{1}{n_1}\sin\theta\right)^2}$$

which leads to the result: $\sin\theta = \sqrt{n_1^2 - n_2^2}$. With $n_1 = 1.58$ and $n_2 = 1.53$, we obtain

$$\theta = \sin^{-1}\left(1.58^2 - 1.53^2\right) = 23.2°.$$

60. (a) We note that the upper-right corner is at an angle (measured from the point where the light enters, and measured relative to a normal axis established at that point [the normal at that point would be horizontal in Fig. 33-60) is at $\tan^{-1}(2/3) = 33.7°$. The angle of refraction is given by

$$n_{\text{air}} \sin 40° = 1.56 \sin \theta_2$$

which yields $\theta_2 = 24.33°$ if we use the common approximation $n_{\text{air}} = 1.0$, and yields $\theta_2 = 24.34°$ if we use the more accurate value for n_{air} found in Table 33-1. The value is less than 33.7° which means that the light goes to side 3.

(b) The ray strikes a point on side 3 which is 0.643 cm below that upper-right corner, and then (using the fact that the angle is symmetrical upon reflection) strikes the top surface (side 2) at a point 1.42 cm to the left of that corner. Since 1.42 cm is certainly less than 3 cm we have a self-consistency check to the effect that the ray does indeed strike side 2 as its second reflection (if we had gotten 3.42 cm instead of 1.42 cm, then the situation would be quite different).

(c) The normal axes for sides 1 and 3 are both horizontal, so the angle of incidence (in the plastic) at side 3 is the same as the angle of refraction was at side 1. Thus,

$$1.56 \sin 24.3^\circ = n_{air} \sin \theta_{air} \quad \Rightarrow \quad \theta_{air} = 40^\circ .$$

(d) It strikes the top surface (side 2) at an angle (measured from the normal axis there, which in this case would be a vertical axis) of $90^\circ - \theta_2 = 66^\circ$ which is much greater than the critical angle for total internal reflection ($\sin^{-1}(n_{air}/1.56) = 39.9^\circ$). Therefore, no refraction occurs when the light strikes side 2.

(e) In this case, we have $n_{air} \sin 70^\circ = 1.56 \sin \theta_2$ which yields $\theta_2 = 37.04^\circ$ if we use the common approximation $n_{air} = 1.0$, and yields $\theta_2 = 37.05^\circ$ if we use the more accurate value for n_{air} found in Table 33-1. This is greater than the 33.7º mentioned above (regarding the upper-right corner), so the ray strikes side 2 instead of side 3.

(f) After bouncing from side 2 (at a point fairly close to that corner) to goes to side 3.

(g) When it bounced from side 2, its angle of incidence (because the normal axis for side 2 is orthogonal to that for side 1) is $90^\circ - \theta_2 = 53^\circ$ which is much greater than the critical angle for total internal reflection (which, again, is $\sin^{-1}(n_{air}/1.56) = 39.9^\circ$). Therefore, no refraction occurs when the light strikes side 2.

(h) For the same reasons implicit in the calculation of part (c), the refracted ray emerges from side 3 with the same angle (70°) that it entered side 1 at (we see that the occurrence of an intermediate reflection [from side 2] does not alter this overall fact: light comes into the block at the same angle that it emerges with from the opposite parallel side.

61. (a) No refraction occurs at the surface *ab*, so the angle of incidence at surface *ac* is $90° - \phi$. For total internal reflection at the second surface, $n_g \sin(90° - \phi)$ must be greater than n_a. Here n_g is the index of refraction for the glass and n_a is the index of refraction for air. Since $\sin(90° - \phi) = \cos\phi$, we want the largest value of ϕ for which $n_g \cos\phi \geq n_a$. Recall that $\cos\phi$ decreases as ϕ increases from zero. When ϕ has the largest value for which total internal reflection occurs, then $n_g \cos\phi = n_a$, or

$$\phi = \cos^{-1}\left(\frac{n_a}{n_g}\right) = \cos^{-1}\left(\frac{1}{1.52}\right) = 48.9°.$$

The index of refraction for air is taken to be unity.

(b) We now replace the air with water. If $n_w = 1.33$ is the index of refraction for water, then the largest value of ϕ for which total internal reflection occurs is

$$\phi = \cos^{-1}\left(\frac{n_w}{n_g}\right) = \cos^{-1}\left(\frac{1.33}{1.52}\right) = 29.0°.$$

62. (a) We refer to the entry point for the original incident ray as point *A* (which we take to be on the left side of the prism, as in Fig. 33-55), the prism vertex as point *B*, and the point where the interior ray strikes the right surface of the prism as point *C*. The angle between line *AB* and the interior ray is β (the complement of the angle of refraction at the first surface), and the angle between the line *BC* and the interior ray is α (the complement of its angle of incidence when it strikes the second surface). When the incident ray is at the minimum angle for which light is able to exit the prism, the light exits along the second face. That is, the angle of refraction at the second face is 90°, and the angle of incidence there for the interior ray is the critical angle for total internal reflection. Let θ_1 be the angle of incidence for the original incident ray and θ_2 be the angle of refraction at the first face, and let θ_3 be the angle of incidence at the second face. The law of refraction, applied to point *C*, yields $n \sin \theta_3 = 1$, so

$$\sin \theta_3 = 1/n = 1/1.60 = 0.625 \Rightarrow \theta_3 = 38.68°.$$

The interior angles of the triangle *ABC* must sum to 180°, so $\alpha + \beta = 120°$. Now, $\alpha = 90° - \theta_3 = 51.32°$, so $\beta = 120° - 51.32° = 69.68°$. Thus, $\theta_2 = 90° - \beta = 21.32°$. The law of refraction, applied to point *A*, yields

$$\sin \theta_1 = n \sin \theta_2 = 1.60 \sin 21.32° = 0.5817.$$

Thus $\theta_1 = 35.6°$.

(b) We apply the law of refraction to point *C*. Since the angle of refraction there is the same as the angle of incidence at *A*, $n \sin \theta_3 = \sin \theta_1$. Now, $\alpha + \beta = 120°$, $\alpha = 90° - \theta_3$, and $\beta = 90° - \theta_2$, as before. This means $\theta_2 + \theta_3 = 60°$. Thus, the law of refraction leads to

$$\sin\theta_1 = n\ \sin(60° - \theta_2) \Rightarrow \sin\theta_1 = n\sin 60°\cos\theta_2 - n\cos 60°\sin\theta_2$$

where the trigonometric identity $\sin(A - B) = \sin A \cos B - \cos A \sin B$ is used. Next, we apply the law of refraction to point *A*:

$$\sin\theta_1 = n\sin\theta_2 \Rightarrow \sin\theta_2 = (1/n)\sin\theta_1$$

which yields $\cos\theta_2 = \sqrt{1-\sin^2\theta_2} = \sqrt{1-(1/n^2)\sin^2\theta_1}$. Thus,

$$\sin\theta_1 = n\sin 60°\sqrt{1-(1/n)^2\sin^2\theta_1} - \cos 60°\sin\theta_1$$

or

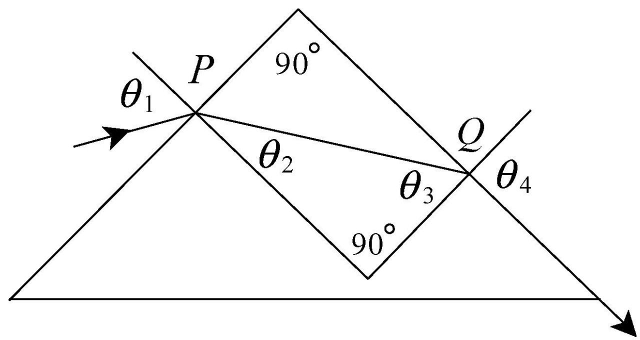
$$(1+\cos 60°)\sin\theta_1 = \sin 60°\sqrt{n^2-\sin^2\theta_1}.$$

Squaring both sides and solving for sin θ_1, we obtain

$$\sin\theta_1 = \frac{n\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}} = \frac{1.60\sin 60°}{\sqrt{(1+\cos 60°)^2 + \sin^2 60°}} = 0.80$$

and θ_1 = 53.1°.

63. (a) A ray diagram is shown below.



Let θ_1 be the angle of incidence and θ_2 be the angle of refraction at the first surface. Let θ_3 be the angle of incidence at the second surface. The angle of refraction there is θ_4 = 90°. The law of refraction, applied to the second surface, yields $n \sin \theta_3 = \sin \theta_4 = 1$. As shown in the diagram, the normals to the surfaces at P and Q are perpendicular to each other. The interior angles of the triangle formed by the ray and the two normals must sum to 180°, so $\theta_3 = 90° - \theta_2$ and

$$\sin\theta_3 = \sin(90°-\theta_2) = \cos\theta_2 = \sqrt{1-\sin^2\theta_2}.$$

According to the law of refraction, applied at Q, $n\sqrt{1-\sin^2\theta_2} = 1.$ The law of refraction, applied to point P, yields $\sin \theta_1 = n \sin \theta_2$, so $\sin \theta_2 = (\sin \theta_1)/n$ and

$$n\sqrt{1-\frac{\sin^2\theta_1}{n^2}} = 1.$$

Squaring both sides and solving for n, we get

$$n = \sqrt{1+\sin^2\theta_1}.$$

(b) The greatest possible value of $\sin^2 \theta_1$ is 1, so the greatest possible value of n is $n_{max} = \sqrt{2} = 1.41.$

(c) For a given value of n, if the angle of incidence at the first surface is greater than θ_1, the angle of refraction there is greater than θ_2 and the angle of incidence at the second face is less than θ_3 (= 90° – θ_2). That is, it is less than the critical angle for total internal reflection, so light leaves the second surface and emerges into the air.

(d) If the angle of incidence at the first surface is less than θ_1, the angle of refraction there is less than θ_2 and the angle of incidence at the second surface is greater than θ_3. This is greater than the critical angle for total internal reflection, so all the light is reflected at Q.

64. (a) We use Eq. 33-49: $\theta_B = \tan^{-1} n_w = \tan^{-1}(1.33) = 53.1°$.

(b) Yes, since n_w depends on the wavelength of the light.

65. The angle of incidence θ_B for which reflected light is fully polarized is given by Eq. 33-48 of the text. If n_1 is the index of refraction for the medium of incidence and n_2 is the index of refraction for the second medium, then

$$\theta_B = \tan^{-1}(n_2 / n_1) = \tan^{-1}(1.53/1.33) = 49.0°.$$

66. Since the layers are parallel, the angle of refraction regarding the first surface is the same as the angle of incidence regarding the second surface (as is suggested by the notation in Fig. 33-63). We recall that as part of the derivation of Eq. 33-49 (Brewster's angle), the refracted angle is the complement of the incident angle:

$$\theta_2 = (\theta_1)_c = 90° - \theta_1 .$$

We apply Eq. 33-49 to both refractions, setting up a product:

$$\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right) = (\tan\theta_{B\,1\to 2})(\tan\,\theta_{B\,2\to 3}) \quad\Rightarrow\quad \frac{n_3}{n_1} = (\tan\,\theta_1)(\tan\,\theta_2).$$

Now, since θ_2 is the complement of θ_1 we have

$$\tan\theta_2 = \tan(\theta_1)_c = \frac{1}{\tan\theta_1}.$$

Therefore, the product of tangents cancel and we obtain $n_3/n_1 = 1$. Consequently, the third medium is air: $n_3 = 1.0$.

67. Since some of the angles in Fig. 33-64 are measured from vertical axes and some are measured from horizontal axes, we must be very careful in taking differences. For instance, the angle difference between the first polarizer struck by the light and the second is 110º (or 70º depending on how we measure it; it does not matter in the final result whether we put $\Delta\theta_1 = 70º$ or put $\Delta\theta_1 = 110º$). Similarly, the angle difference between the second and the third is $\Delta\theta_2 = 40º$, and between the third and the fourth is $\Delta\theta_3 = 40º$, also. Accounting for the "automatic" reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is the incident intensity multiplied by

$$\frac{1}{2}\cos^2(\Delta\theta_1)\,\cos^2(\Delta\theta_2)\,\cos^2(\Delta\theta_3).$$

Thus, the light that emerges from the system has intensity equal to 0.50 W/m^2.

68. (a) Suppose there are a total of N transparent layers (N = 5 in our case). We label these layers from left to right with indices 1, 2, …, N. Let the index of refraction of the air be n_0. We denote the initial angle of incidence of the light ray upon the air-layer boundary as θ_i and the angle of the emerging light ray as θ_f. We note that, since all the boundaries are parallel to each other, the angle of incidence θ_j at the boundary between the j-th and the (j + 1)-th layers is the same as the angle between the transmitted light ray and the normal in the j-th layer. Thus, for the first boundary (the one between the air and the first layer)

$$\frac{n_1}{n_0} = \frac{\sin\theta_i}{\sin\theta_1},$$

for the second boundary

$$\frac{n_2}{n_1} = \frac{\sin\theta_1}{\sin\theta_2},$$

and so on. Finally, for the last boundary

$$\frac{n_0}{n_N} = \frac{\sin\theta_N}{\sin\theta_f},$$

Multiplying these equations, we obtain

$$\left(\frac{n_1}{n_0}\right)\left(\frac{n_2}{n_1}\right)\left(\frac{n_3}{n_2}\right)\cdots\left(\frac{n_0}{n_N}\right) = \left(\frac{\sin\theta_i}{\sin\theta_1}\right)\left(\frac{\sin\theta_1}{\sin\theta_2}\right)\left(\frac{\sin\theta_2}{\sin\theta_3}\right)\cdots\left(\frac{\sin\theta_N}{\sin\theta_f}\right).$$

We see that the L.H.S. of the equation above can be reduced to n_0/n_0 while the R.H.S. is equal to $\sin\theta_i/\sin\theta_f$. Equating these two expressions, we find

$$\sin\theta_f = \left(\frac{n_0}{n_0}\right)\sin\theta_i = \sin\theta_i,$$

which gives $\theta_i = \theta_f$. So for the two light rays in the problem statement, the angle of the emerging light rays are both the same as their respective incident angles. Thus, $\theta_f = 0$ for ray a,

(b) and $\theta_f = 20°$ for ray b.

(c) In this case, all we need to do is to change the value of n_0 from 1.0 (for air) to 1.5 (for glass). This does not change the result above. That is, we still have $\theta_f = 0$ for ray a,

(d) and $\theta_f = 20°$ for ray *b*.

Note that the result of this problem is fairly general. It is independent of the number of layers and the thickness and index of refraction of each layer.

69. (a) The Sun is far enough away that we approximate its rays as "parallel" in this Figure. That is, if the sunray makes angle θ from horizontal when the bird is in one position, then it makes the same angle θ when the bird is any other position. Therefore, its shadow on the ground moves as the bird moves: at 15 m/s.

(b) If the bird is in a position, a distance $x > 0$ from the wall, such that its shadow is on the wall at a distance $0 \geq y \geq h$ from the top of the wall, then it is clear from the Figure that $\tan\theta = y/x$. Thus,

$$\frac{dy}{dt} = \frac{dx}{dt}\tan\theta = (-15 \text{ m/s})\tan 30° = -8.7 \text{ m/s},$$

which means that the distance y (which was measured as a positive number downward from the top of the wall) is shrinking at the rate of 8.7 m/s.

(c) Since $\tan\theta$ grows as $0 \leq \theta < 90°$ increases, then a larger value of $|dy/dt|$ implies a larger value of θ. The Sun is higher in the sky when the hawk glides by.

(d) With $|dy/dt| = 45$ m/s, we find

$$v_{\text{hawk}} = \left|\frac{dx}{dt}\right| = \frac{\left|\frac{dy}{dt}\right|}{\tan\theta}$$

so that we obtain $\theta = 72°$ if we assume $v_{\text{hawk}} = 15$ m/s.

70. (a) From $n_1\sin\theta_1 = n_2\sin\theta_2$ and $n_2\sin\theta_2 = n_3\sin\theta_3$, we find $n_1\sin\theta_1 = n_3\sin\theta_3$. This has a simple implication: that $\theta_1 = \theta_3$ when $n_1 = n_3$. Since we are given $\theta_1 = 40°$ in Fig. 33-67(a) then we look for a point in Fig. 33-67(b) where $\theta_3 = 40°$. This seems to occur at $n_3 = 1.6$, so we infer that $n_1 = 1.6$.

(b) Our first step in our solution to part (a) shows that information concerning n_2 disappears (cancels) in the manipulation. Thus, we cannot tell; we need more information.

(c) From $1.6\sin 70° = 2.4\sin\theta_3$ we obtain $\theta_3 = 39°$.

71. (a) Reference to Fig. 33-24 may help in the visualization of why there appears to be a "circle of light" (consider revolving that picture about a vertical axis). The depth and the radius of that circle (which is from point *a* to point *f* in that figure) is related to the tangent of the angle of incidence. The diameter of the circle in question is given by $d = 2h \tan \theta_c$. For water $n = 1.33$, so Eq. 33-47 gives $\sin \theta_c = 1/1.33$, or $\theta_c = 48.75°$. Thus,

$$d = 2h \tan \theta_c = 2(2.00 \text{ m})(\tan 48.75°) = 4.56 \text{ m}.$$

(b) The diameter *d* of the circle will increase if the fish descends (increasing *h*).

72. (a) Snell's law gives $n_{air} \sin(50°) = n_{2b} \sin \theta_{2b}$ and $n_{air} \sin(50°) = n_{2r} \sin \theta_{2r}$ where we use subscripts b and r for the blue and red light rays. Using the common approximation for air's index ($n_{air} = 1.0$) we find the two angles of refraction to be 30.176° and 30.507°. Therefore, $\Delta\theta = 0.33°$.

(b) Both of the refracted rays emerges from the other side with the same angle (50°) with which they were incident on the first side (generally speaking, light comes into a block at the same angle that it emerges with from the opposite parallel side). There is thus no difference (the difference is 0°) and thus there is no dispersion in this case.

73. (a) The wave is traveling in the $-y$ direction (see §16-5 for the significance of the relative sign between the spatial and temporal arguments of the wave function).

(b) Figure 33-5 may help in visualizing this. The direction of propagation (along the y axis) is perpendicular to $\vec{B}$ (presumably along the x axis, since the problem gives B_x and no other component) and both are perpendicular to $\vec{E}$ (which determines the axis of polarization). Thus, the wave is z-polarized.

(c) Since the magnetic field amplitude is B_m = 4.00 μT, then (by Eq. 33-5) E_m = 1199 V/m $\approx 1.20\times10^3$ V/m. Dividing by $\sqrt{2}$ yields E_{rms} = 848 V/m. Then, Eq. 33-26 gives

$$I = \frac{I}{c\mu_0} E_{\text{rms}}^2 = 1.91\times10^3 \text{ W / m}^2.$$

(d) Since $kc = \omega$ (equivalent to $c = f\lambda$), we have

$$k = \frac{2.00\times10^{15}}{c} = 6.67\times10^6 \text{ m}^{-1}.$$

Summarizing the information gathered so far, we have (with SI units understood)

$$E_z = (1.2\times10^3)\sin\left(\left(6.67\times10^6\right)y + \left(2.00\times10^{15}\right)t\right).$$

(e) $\lambda = 2\pi/k$ = 942 nm.

(f) This is an infrared light.

74. (a) The condition (in Eq. 33-44) required in the critical angle calculation is $\theta_3 = 90°$. Thus (with $\theta_2 = \theta_c$, which we don't compute here),

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3$$

leads to $\theta_1 = \theta = \sin^{-1} n_3/n_1 = 54.3°$.

(b) Yes. Reducing θ leads to a reduction of θ_2 so that it becomes less than the critical angle; therefore, there will be some transmission of light into material 3.

(c) We note that the complement of the angle of refraction (in material 2) is the critical angle. Thus,

$$n_1 \sin\theta = n_2 \cos\theta_c = n_2\sqrt{1-\left(\frac{n_3}{n_2}\right)^2} = \sqrt{n_2^2 - n_3^2}$$

leads to $\theta = 51.1°$.

(d) No. Reducing θ leads to an increase of the angle with which the light strikes the interface between materials 2 and 3, so it becomes greater than the critical angle. Therefore, there will be no transmission of light into material 3.

75. Let $\theta_1 = 45°$ be the angle of incidence at the first surface and θ_2 be the angle of refraction there. Let θ_3 be the angle of incidence at the second surface. The condition for total internal reflection at the second surface is $n \sin \theta_3 \geq 1$. We want to find the smallest value of the index of refraction n for which this inequality holds. The law of refraction, applied to the first surface, yields $n \sin \theta_2 = \sin \theta_1$. Consideration of the triangle formed by the surface of the slab and the ray in the slab tells us that $\theta_3 = 90° - \theta_2$. Thus, the condition for total internal reflection becomes $1 \leq n \sin(90° - \theta_2) = n \cos \theta_2$. Squaring this equation and using $\sin^2 \theta_2 + \cos^2 \theta_2 = 1$, we obtain $1 \leq n^2 (1 - \sin^2 \theta_2)$. Substituting $\sin \theta_2 = (1/n) \sin \theta_1$ now leads to

$$1 \leq n^2 \left(1 - \frac{\sin^2 \theta_1}{n^2}\right) = n^2 - \sin^2 \theta_1.$$

The largest value of n for which this equation is true is given by $1 = n^2 - \sin^2 \theta_1$. We solve for n:

$$n = \sqrt{1 + \sin^2 \theta_1} = \sqrt{1 + \sin^2 45°} = 1.22.$$

76. We write $m = \rho\varsigma$ where $\varsigma = 4\pi R^3/3$ is the volume. Plugging this into $F = ma$ and then into Eq. 33-32 (with $A = \pi R^2$, assuming the light is in the form of plane waves), we find

$$\rho \frac{4\pi R^3}{3} a = \frac{I\pi R^2}{c}.$$

This simplifies to

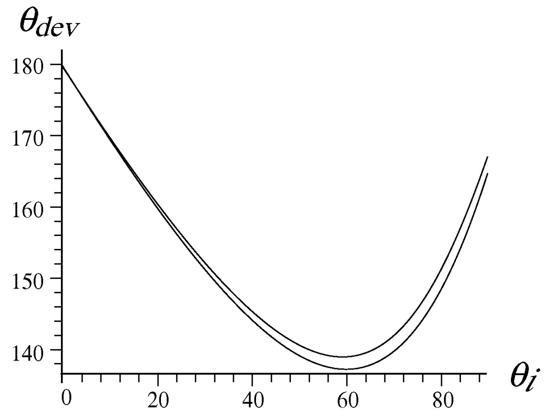
$$a = \frac{3I}{4\rho c R}$$

which yields $a = 1.5 \times 10^{-9}$ m/s^2.

77. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution to the overall deviation is the reflection. Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after the reflection) is $\delta\theta_2 = 180° - 2\theta_r$. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 180° + 2\theta_i - 4\theta_r.$$

(b) We substitute $\theta_r = \sin^{-1}(\frac{1}{n}\sin\theta_i)$ into the expression derived in part (a), using the two given values for n. The higher curve is for the blue light.



(c) We can expand the graph and try to estimate the minimum, or search for it with a more sophisticated numerical procedure. We find that the θ_{dev} minimum for red light is $137.63° \approx 137.6°$, and this occurs at $\theta_i = 59.52°$.

(d) For blue light, we find that the θ_{dev} minimum is $139.35° \approx 139.4°$, and this occurs at $\theta_i = 59.52°$.

(e) The difference in θ_{dev} in the previous two parts is 1.72°.

78. (a) The first contribution to the overall deviation is at the first refraction: $\delta\theta_1 = \theta_i - \theta_r$. The next contribution(s) to the overall deviation is (are) the reflection(s). Noting that the angle between the ray right before reflection and the axis normal to the back surface of the sphere is equal to θ_r, and recalling the law of reflection, we conclude that the angle by which the ray turns (comparing the direction of propagation before and after [each] reflection) is $\delta\theta_r = 180° - 2\theta_r$. Thus, for k reflections, we have $\delta\theta_2 = k\theta_r$ to account for these contributions. The final contribution is the refraction suffered by the ray upon leaving the sphere: $\delta\theta_3 = \theta_i - \theta_r$ again. Therefore,

$$\theta_{\text{dev}} = \delta\theta_1 + \delta\theta_2 + \delta\theta_3 = 2(\theta_i - \theta_r) + k(180° - 2\theta_r) = k(180°) + 2\theta_i - 2(k+1)\theta_r.$$

(b) For k = 2 and n = 1.331 (given in problem 67), we search for the second-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 230.37° $\approx 230.4°$, and this occurs at θ_i = 71.90°.

(c) Similarly, we find that the second-order θ_{dev} minimum for blue light (for which n = 1.343) is 233.48° $\approx 233.5°$, and this occurs at θ_i = 71.52°.

(d) The difference in θ_{dev} in the previous two parts is approximately 3.1°.

(e) Setting k = 3, we search for the third-order rainbow angle numerically. We find that the θ_{dev} minimum for red light is 317.5°, and this occurs at θ_i = 76.88°.

(f) Similarly, we find that the third-order θ_{dev} minimum for blue light is 321.9°, and this occurs at θ_i = 76.62°.

(g) The difference in θ_{dev} in the previous two parts is 4.4°.

79. (a) and (b) At the Brewster angle, $\theta_{incident} + \theta_{refracted} = \theta_B + 32.0° = 90.0°$, so $\theta_B = 58.0°$ and $n_{glass} = \tan\theta_B = \tan 58.0° = 1.60$.

80. We take the derivative with respect to x of both sides of Eq. 33-11:

$$\frac{\partial}{\partial x}\left(\frac{\partial E}{\partial x}\right)=\frac{\partial^2 E}{\partial x^2}=\frac{\partial}{\partial x}\left(-\frac{\partial B}{\partial t}\right)=-\frac{\partial^2 B}{\partial x\partial t}.$$

Now we differentiate both sides of Eq. 33-18 with respect to t:

$$\frac{\partial}{\partial t}\left(-\frac{\partial B}{\partial x}\right)=-\frac{\partial^2 B}{\partial x\partial t}=\frac{\partial}{\partial t}\left(\varepsilon_0\mu_0\frac{\partial E}{\partial t}\right)=\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}.$$

Substituting $\partial^2 E/\partial x^2=-\partial^2 B/\partial x\partial t$ from the first equation above into the second one, we get

$$\varepsilon_0\mu_0\frac{\partial^2 E}{\partial t^2}=\frac{\partial^2 E}{\partial x^2}\qquad\Rightarrow\qquad\frac{\partial^2 E}{\partial t^2}=\frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 E}{\partial x^2}=c^2\frac{\partial^2 E}{\partial x^2}.$$

Similarly, we differentiate both sides of Eq. 33-11 with respect to t

$$\frac{\partial^2 E}{\partial x\partial t}=-\frac{\partial^2 B}{\partial t^2},$$

and differentiate both sides of Eq. 33-18 with respect to x

$$-\frac{\partial^2 B}{\partial x^2}=\varepsilon_0\mu_0-\frac{\partial^2 E}{\partial x\partial t}.$$

Combining these two equations, we get

$$\frac{\partial^2 B}{\partial t^2}=\frac{1}{\varepsilon_0\mu_0}\frac{\partial^2 B}{\partial x^2}=c^2\frac{\partial^2 B}{\partial x^2}.$$

81. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2 \theta_1' \cos^2 \theta_2'$$

where $\theta_1' = 90° - \theta_1 = 60°$ and $\theta_2' = 90° - \theta_2 = 60°$. This yields $I/I_0 = 0.031$.

82. (a) An incident ray which is normal to the water surface is not refracted, so the angle at which it strikes the first mirror is $\theta_1 = 45°$. According to the law of reflection, the angle of reflection is also 45°. This means the ray is horizontal as it leaves the first mirror, and the angle of incidence at the second mirror is $\theta_2 = 45°$. Since the angle of reflection at the second mirror is also 45° the ray leaves that mirror normal again to the water surface. There is no refraction at the water surface, and the emerging ray is parallel to the incident ray.

(b) We imagine that the incident ray makes an angle θ_1 with the normal to the water surface. The angle of refraction θ_2 is found from $\sin\theta_1 = n\sin\theta_2$, where n is the index of refraction of the water. The normal to the water surface and the normal to the first mirror make an angle of 45°. If the normal to the water surface is continued downward until it meets the normal to the first mirror, the triangle formed has an interior angle of 180° – 45° = 135° at the vertex formed by the normal. Since the interior angles of a triangle must sum to 180°, the angle of incidence at the first mirror satisfies $\theta_3 + \theta_2 + 135° = 180°$, so $\theta_3 = 45° - \theta_2$. Using the law of reflection, the angle of reflection at the first mirror is also $45° - \theta_2$. We note that the triangle formed by the ray and the normals to the two mirrors is a right triangle. Consequently,

$$\theta_3 + \theta_4 + 90° = 180° \;\Rightarrow\; \theta_4 = 90° - \theta_3 = 90° - 45° + \theta_2 = 45° + \theta_2.$$

The angle of reflection at the second mirror is also $45° + \theta_2$. Now, we continue the normal to the water surface downward from the exit point of the ray to the second mirror. It makes an angle of 45° with the mirror. Consider the triangle formed by the second mirror, the ray, and the normal to the water surface. The angle at the intersection of the normal and the mirror is 180° – 45° = 135°. The angle at the intersection of the ray and the mirror is

$$90° - \theta_4 = 90° - (45° + \theta_2) = 45° - \theta_2.$$

The angle at the intersection of the ray and the water surface is θ_5. These three angles must sum to 180°, so $135° + 45° - \theta_2 + \theta_5 = 180°$. This means $\theta_5 = \theta_2$. Finally, we use the law of refraction to find θ_6:

$$\sin\theta_6 = n\sin\theta_5 \Rightarrow \sin\theta_6 = n\,\sin\theta_2,$$

since $\theta_5 = \theta_2$. Finally, since $\sin\theta_1 = n\sin\theta_2$, we conclude that $\sin\theta_6 = \sin\theta_1$ and $\theta_6 = \theta_1$. The exiting ray is parallel to the incident ray.

83. We use the result of problem 33-53 to solve for ψ. Note that ϕ = 60.0° in our case. Thus, from

$$n = \frac{\sin \frac{1}{2}(\psi + \phi)}{\sin \frac{1}{2}\phi},$$

we get

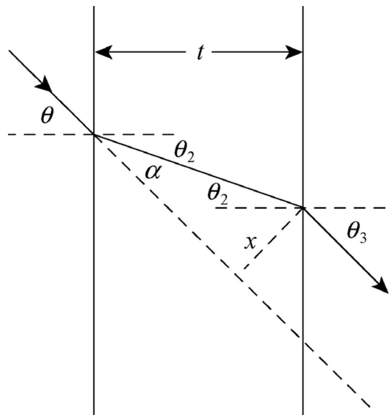
$$\sin \frac{1}{2}(\psi + \phi) = n \sin \frac{1}{2}\phi = (1.31)\sin\left(\frac{60.0°}{2}\right) = 0.655,$$

which gives $\frac{1}{2}(\psi + \phi) = \sin^{-1}(0.655) = 40.9°$. Thus,

$$\psi = 2(40.9°) - \phi \, 2(40.9°) - 60.0° = 21.8°.$$

84. The law of refraction requires that $\sin\theta_1/\sin\theta_2 = n_{\text{water}}$ = const. We can check that this is indeed valid for any given pair of θ_1 and θ_2. For example $\sin 10° / \sin 8° = 1.3$, and $\sin 20° / \sin 15°30' = 1.3$, etc. Therefore, the index of refraction of water is $n_{\text{water}} = 1.3$.

85. Let θ be the angle of incidence and θ_2 be the angle of refraction at the left face of the plate. Let n be the index of refraction of the glass. Then, the law of refraction yields $\sin\theta = n\sin\theta_2$. The angle of incidence at the right face is also θ_2. If θ_3 is the angle of emergence there, then $n\sin\theta_2 = \sin\theta_3$. Thus $\sin\theta_3 = \sin\theta$ and $\theta_3 = \theta$.



The emerging ray is parallel to the incident ray. We wish to derive an expression for x in terms of θ. If D is the length of the ray in the glass, then $D\cos\theta_2 = t$ and $D = t/\cos\theta_2$. The angle α in the diagram equals $\theta - \theta_2$ and $x = D\sin\alpha = D\sin(\theta - \theta_2)$. Thus

$$x = \frac{t\sin(\theta - \theta_2)}{\cos\theta_2}.$$

If all the angles θ, θ_2, θ_3, and $\theta - \theta_2$ are small and measured in radians, then $\sin\theta \approx \theta$, $\sin\theta_2 \approx \theta_2$, $\sin(\theta - \theta_2) \approx \theta - \theta_2$, and $\cos\theta_2 \approx 1$. Thus $x \approx t(\theta - \theta_2)$. The law of refraction applied to the point of incidence at the left face of the plate is now $\theta \approx n\theta_2$, so $\theta_2 \approx \theta/n$ and

$$x \approx t\left(\theta - \frac{\theta}{n}\right) = \frac{(n-1)t\theta}{n}.$$

86. (a) Setting v = c in the wave relation $k\text{v} = \omega = 2\pi f$, we find $f = 1.91 \times 10^8$ Hz.

(b) $E_{\text{rms}} = E_m/\sqrt{2} = B_m/c\sqrt{2} = 18.2$ V/m.

(c) $I = (E_{\text{rms}})^2/c\mu_o = 0.878$ W/m^2.

87. From Fig. 33-19 we find n_{max} = 1.470 for λ = 400 nm and n_{min} = 1.456 for λ = 700 nm.
(a) The corresponding Brewster's angles are

$$\theta_{B,max} = \tan^{-1} n_{max} = \tan^{-1} (1.470) = 55.8°,$$

(b) and $\theta_{B,min} = \tan^{-1} (1.456) = 55.5°$.

88. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2 \theta_1' \cos^2 \theta_2'$$

where $\theta_1' = (90° - \theta_1) + \theta_2 = 110°$ is the relative angle between the first and the second polarizing sheets, and $\theta_2' = 90° - \theta_2 = 50°$ is the relative angle between the second and the third polarizing sheets. Thus, we have $I/I_0 = 0.024$.

89. The time for light to travel a distance d in free space is $t = d/c$, where c is the speed of light (3.00×10^8 m/s).

(a) We take d to be 150 km = 150×10^3 m. Then,

$$t = \frac{d}{c} = \frac{150 \times 10^3 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 5.00 \times 10^{-4} \text{ s}.$$

(b) At full moon, the Moon and Sun are on opposite sides of Earth, so the distance traveled by the light is

$$d = (1.5 \times 10^8 \text{ km}) + 2\,(3.8 \times 10^5 \text{ km}) = 1.51 \times 10^8 \text{ km} = 1.51 \times 10^{11} \text{ m}.$$

The time taken by light to travel this distance is

$$t = \frac{d}{c} = \frac{1.51 \times 10^{11} \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 500 \text{ s} = 8.4 \text{ min}.$$

(c) We take d to be $2(1.3 \times 10^9$ km$) = 2.6 \times 10^{12}$ m. Then,

$$t = \frac{d}{c} = \frac{2.6 \times 10^{12} \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 8.7 \times 10^3 \text{ s} = 2.4 \text{ h}.$$

(d) We take d to be 6500 ly and the speed of light to be 1.00 ly/y. Then,

$$t = \frac{d}{c} = \frac{6500 \text{ ly}}{1.00 \text{ ly/y}} = 6500 \text{ y}.$$

The explosion took place in the year 1054 – 6500 = –5446 or 5446 b.c.

90. (a) At $r = 40$ m, the intensity is

$$I = \frac{P}{\pi d^2/4} = \frac{P}{\pi(\theta r)^2/4} = \frac{4(3.0\times10^{-3}\,\text{W})}{\pi\left[(0.17\times10^{-3}\,\text{rad})(40\,\text{m})\right]^2} = 83\,\text{W/m}^2.$$

(b) $P' = 4\pi r^2 I = 4\pi(40\text{m})^2(83\,\text{W/m}^2) = 1.7\times10^6\,\text{W}.$

91. Since intensity is power divided by area (and the area is spherical in the isotropic case), then the intensity at a distance of $r = 20$ m from the source is

$$I = \frac{P}{4\pi r^2} = 0.040\ \mathrm{W/m^2}.$$

as illustrated in Sample Problem 33-2. Now, in Eq. 33-32 for a totally absorbing area A, we note that the exposed area of the small sphere is that on a flat circle $A = \pi(0.020\ \mathrm{m})^2 = 0.0013\ \mathrm{m}^2$. Therefore,

$$F = \frac{IA}{c} = \frac{(0.040)(0.0013)}{3\times 10^{8}} = 1.7\times 10^{-13}\ \mathrm{N}.$$

92. (a) Assuming complete absorption, the radiation pressure is

$$p_r = \frac{I}{c} = \frac{1.4\times10^3\ \text{W/m}^2}{3.0\times10^8\ \text{m/s}} = 4.7\times10^{-6}\ \text{N/m}^2\,.$$

(b) We compare values by setting up a ratio:

$$\frac{p_r}{p_0} = \frac{4.7\times10^{-6}\ \text{N/m}^2}{1.0\times10^5\ \text{N/m}^2} = 4.7\times10^{-11}.$$

93. (a) From $kc = \omega$ where $k = 1.00 \times 10^6\ \text{m}^{-1}$, we obtain $\omega = 3.00 \times 10^{14}$ rad/s. The magnetic field amplitude is, from Eq. 33-5, $B = (5.00\ \text{V/m})/c = 1.67 \times 10^{-8}$ T. From the fact that $-\hat{\text{k}}$ (the direction of propagation), $\vec{E} = E_y\hat{\text{j}}$, and $\vec{B}$ are mutually perpendicular, we conclude that the only non-zero component of $\vec{B}$ is B_x, so that we have (in SI units)

$$B_x = 1.67\times10^{-8}\sin\left(\left(1.00\times10^{6}\right)z+\left(3.00\times10^{14}\right)t\right).$$

(b) The wavelength is $\lambda = 2\pi/k = 6.28 \times 10^{-6}$ m.

(c) The period is $T = 2\pi/\omega = 2.09 \times 10^{-14}$ s.

(d) The intensity is

$$I = \frac{1}{c\mu_0}\left(\frac{5.00\ \text{V/m}}{\sqrt{2}}\right)^2 = 0.0332\ \text{W/m}^2\,.$$

(e) As noted in part (a), the only nonzero component of $\vec{B}$ is B_x. The magnetic field oscillates along the x axis.

(f) The wavelength found in part (b) places this in the infrared portion of the spectrum.

94. It's useful to look back at the beginning of section 20-4 (particularly the steps leading up to Eq. 20-18) when considering "pressure due to collisions" (although using that term with light-interactions might be considered a little misleading). The v_x that occurs in that discussion in section 19-4 would correspond to the component $v\cos\theta$ in this problem because the angle is here being measured from the "normal axis" (instead of from the surface). Since it is the square of v_x that occurs in the section 20-4 discussion, we see therefore how the $\cos^2\theta$ factor comes about in this final result: $p_r(\theta) = p_{r\perp}\cos^2\theta$.

95. (a) The area of a hemisphere is $A = 2\pi r^2$, and we get $I = P/A = 3.5\ \mu\text{W/m}^2$.

(b) Our part (a) result multiplied by $0.22\ \text{m}^2$ gives $0.78\ \mu\text{W}$.

(c) The part (b) answer divided by the A of part (a) leads to $1.5 \times 10^{-17}\ \text{W/m}^2$.

(d) Then Eq. 33-26 gives $E_{\text{rms}} = 76\ \text{nV/m} \Rightarrow E_{\text{max}} = \sqrt{2}\, E_{\text{rms}} = 1.1 \times 10^{-7}\ \text{nV/m}$.

(e) $B_{\text{rms}} = E_{\text{rms}}/c = 2.5 \times 10^{-16}\ \text{T} = 0.25\ \text{fT}$.

96. (a) The electric field amplitude is $E_m = \sqrt{2}E_{\text{rms}} = 70.7$ V/m, so that the magnetic field amplitude is $B_m = 2.36\times10^{-7}$ T by Eq. 33-5. Since the direction of propagation, $\vec{E}$, and $\vec{B}$ are mutually perpendicular, we infer that the only non-zero component of $\vec{B}$ is B_x, and note that the direction of propagation being along the $-z$ axis means the spatial and temporal parts of the wave function argument are of like sign (see §16-5). Also, from λ = 250 nm, we find that $f = c/\lambda = 1.20\times10^{15}$ Hz, which leads to $\omega = 2\pi f$ = 7.53×10^{15} rad/s. Also, we note that $k = 2\pi/\lambda = 2.51\times10^{7}\ \text{m}^{-1}$. Thus, assuming some "initial condition" (that, say the field is zero, with its derivative positive, at z = 0 when t = 0), we have

$$B_x = 2.36\times10^{-7}\sin[(2.51\times10^{7})z + (7.53\times10^{15})t]$$

in SI units.

(b) The exposed area of the triangular chip is $A = \sqrt{3}\ell^2/8$, where $\ell = 2.00\times10^{-6}$ m. The intensity of the wave is

$$I = \frac{1}{c\mu_0}E_{\text{rms}}^2 = 6.64\ \text{W/m}^2.$$

Thus, Eq. 33-33 leads to

$$F = \frac{2IA}{c} = 3.83\times10^{-20}\ \text{N}.$$

97. Accounting for the “automatic” reduction (by a factor of one-half) whenever unpolarized light passes through any polarizing sheet, then our result is $\frac{1}{2}(\cos^2(30°))^3$ = 0.21.

98. The result is

$$B_z = (2.50 \times 10^{-14}\,\mathrm{T}) \sin[(1.40 \times 10^{7}\,\mathrm{m}^{-1})y + (4.19 \times 10^{15}\,\mathrm{s}^{-1})t],$$

and we briefly indicate our reasoning as follows: the amplitude B_m is equal to $E_m/c = \sqrt{2}\, E_{\mathrm{rms}}/c$. The wavenumber k is $2\pi/\lambda = 2\pi\,(450 \times 10^{-9}\,\mathrm{m})^{-1}$. The fact that it travels in the negative x direction accounts for the + sign between terms in the sine argument. Finally, $\omega = kc$ gives the angular frequency.

99. We apply Eq. 33-40 (once) and Eq. 33-42 (twice) to obtain

$$I = \frac{1}{2} I_0 \cos^2 \theta' \cos^2 \theta''.$$

With $\theta' = \theta_2 - \theta_1$ = 60° – 20° = 40° and $\theta'' = \theta_3 + (\pi/2 - \theta_2)$ = 40° + 30° = 70°, this yields $I/I_0 = 0.034$.

100. We remind ourselves that when the unpolarized light passes through the first sheet, its intensity is reduced by a factor of 2. Thus, to end up with an overall reduction of one-third, the second sheet must cause a further decrease by a factor of two-thirds (since (1/2)(2/3) = 1/3). Thus, $\cos^2\theta = 2/3 \;\Rightarrow\; \theta = 35°$.

101. (a) The magnitude of the magnetic field is

$$B = \frac{E}{c} = \frac{100\ \text{V/m}}{3.0\times10^{8}\ \text{m/s}} = 3.3\times10^{-7}\ \text{T}.$$

(b) With $\vec{E}\times\vec{B} = \mu_0\vec{S}$, where $\vec{E} = E\hat{\text{k}}$ and $\vec{S} = S(-\hat{\text{j}})$, one can verify easily that since $\hat{\text{k}}\times(-\hat{\text{i}}) = -\hat{\text{j}}$, $\vec{B}$ has to be in the negative x direction.

102. We use Eq. 33-33 for the force, where A is the area of the reflecting surface (4.0 m^2). The intensity is gotten from Eq. 33-27 where $P = P_S$ is in Appendix C (see also Sample Problem 33-2) and $r = 3.0 \times 10^{11}$ m (given in the problem statement). Our result for the force is 9.2 μN.

103. From Eq. 33-26, we have $E_{\text{rms}} = \sqrt{\mu_0 c I} = 1941\,\text{V/m}$, which implies (using Eq. 33-5) that $B_{\text{rms}} = 1941/c = 6.47 \times 10^{-6}$ T. Multiplying by $\sqrt{2}$ yields the magnetic field amplitude $B_m = 9.16 \times 10^{-6}$ T.

104. Eq. 33-5 gives $B = E/c$, which relates the field values at any instant — and so relates rms values to rms values, and amplitude values to amplitude values, as the case may be. Thus, the rms value of the magnetic field is $0.2/3 \times 10^8 = 6.67 \times 10^{-10}$ T, which (upon multiplication by $\sqrt{2}$) yields an amplitude value of magnetic field equal to 9.43×10^{-10} T.

105. (a) From Eq. 33-1,

$$\frac{\partial^2 E}{\partial t^2}=\frac{\partial^2}{\partial t^2}\left[E_m \sin(kx-\omega t)\right]=-\omega^2 E_m \sin(kx-\omega t),$$

and

$$c^2\frac{\partial^2 E}{\partial x^2}=c^2\frac{\partial^2}{\partial x^2}\left[E_m \sin(kx-\omega t)\right]=-k^2c^2 \sin(kx-\omega t)=-\omega^2 E_m \sin(kx-\omega t).$$

Consequently,

$$\frac{\partial^2 E}{\partial t^2}=c^2\frac{\partial^2 E}{\partial x^2}$$

is satisfied. Analogously, one can show that Eq. 33-2 satisfies

$$\frac{\partial^2 B}{\partial t^2}=c^2\frac{\partial^2 B}{\partial x^2}.$$

(b) From $E=E_m f(kx\pm\omega t)$,

$$\frac{\partial^2 E}{\partial t^2}=E_m\frac{\partial^2 f(kx\pm\omega t)}{\partial t^2}=\omega^2 E_m\left.\frac{d^2 f}{du^2}\right|_{u=kx\pm\omega t}$$

and

$$c^2\frac{\partial^2 E}{\partial x^2}=c^2E_m\frac{\partial^2 f(kx\pm\omega t)}{\partial t^2}=c^2E_m k^2\left.\frac{d^2 f}{du^2}\right|_{u=kx\pm\omega t}$$

Since $\omega = ck$ the right-hand sides of these two equations are equal. Therefore,

$$\frac{\partial^2 E}{\partial t^2}=c^2\frac{\partial^2 E}{\partial x^2}.$$

Changing E to B and repeating the derivation above shows that $B=B_m f(kx\pm\omega t)$ satisfies

$$\frac{\partial^2 B}{\partial t^2}=c^2\frac{\partial^2 B}{\partial x^2}.$$

106. (a) Let r be the radius and ρ be the density of the particle. Since its volume is $(4\pi/3)r^3$, its mass is $m = (4\pi/3)\rho r^3$. Let R be the distance from the Sun to the particle and let M be the mass of the Sun. Then, the gravitational force of attraction of the Sun on the particle has magnitude

$$F_g = \frac{GMm}{R^2} = \frac{4\pi GM\rho r^3}{3R^2}.$$

If P is the power output of the Sun, then at the position of the particle, the radiation intensity is $I = P/4\pi R^2$, and since the particle is perfectly absorbing, the radiation pressure on it is

$$p_r = \frac{I}{c} = \frac{P}{4\pi R^2 c}.$$

All of the radiation that passes through a circle of radius r and area $A = \pi r^2$, perpendicular to the direction of propagation, is absorbed by the particle, so the force of the radiation on the particle has magnitude

$$F_r = p_r A = \frac{\pi P r^2}{4\pi R^2 c} = \frac{P r^2}{4R^2 c}.$$

The force is radially outward from the Sun. Notice that both the force of gravity and the force of the radiation are inversely proportional to R^2. If one of these forces is larger than the other at some distance from the Sun, then that force is larger at all distances. The two forces depend on the particle radius r differently: F_g is proportional to r^3 and F_r is proportional to r^2. We expect a small radius particle to be blown away by the radiation pressure and a large radius particle with the same density to be pulled inward toward the Sun. The critical value for the radius is the value for which the two forces are equal. Equating the expressions for F_g and F_r, we solve for r:

$$r = \frac{3P}{16\pi GM\rho c}.$$

(b) According to Appendix C, $M = 1.99 \times 10^{30}$ kg and $P = 3.90 \times 10^{26}$ W. Thus,

$$r = \frac{3(3.90\times10^{26}\ \text{W})}{16\pi(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})(1.0\times10^{3}\ \text{kg}/\text{m}^3)(3.00\times10^{8}\ \text{m/s})}$$
$$= 5.8\times10^{-7}\ \text{m}.$$

107. (a) The polarization direction is defined by the electric field (which is perpendicular to the magnetic field in the wave, and also perpendicular to the direction of wave travel). The given function indicates the magnetic field is along the x axis (by the subscript on B) and the wave motion is along $-y$ axis (see the argument of the sine function). Thus, the electric field direction must be parallel to the z axis.

(b) Since k is given as 1.57×10^7/m, then $\lambda = 2\pi/k = 4.0 \times 10^{-7}$ m, which means $f = c/\lambda = 7.5 \times 10^{14}$ Hz.

(c) The magnetic field amplitude is given as $B_m = 4.0 \times 10^{-6}$ T. The electric field amplitude E_m is equal to B_m divided by the speed of light c. The rms value of the electric field is then E_m divided by $\sqrt{2}$. Eq. 33-26 then gives $I = 1.9$ kW/m^2.

108. Using Eqs. 33-40 and 33-42, we obtain

$$\frac{I_{\text{final}}}{I_0} = \frac{\left(\frac{1}{2}I_0\right)\left(\cos^2 45°\right)\left(\cos^2 45°\right)}{I_0} = \frac{1}{8} = 0.125.$$

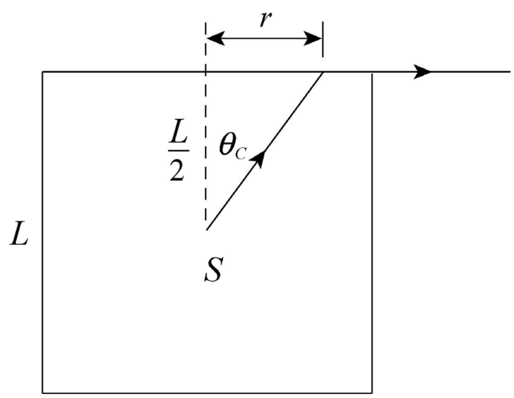
109. With the index of refraction n = 1.456 at the red end, since $\sin \theta_c = 1/n$, the critical angle is $\theta_c = 43.38°$ for red.

(a) At an angle of incidence of $\theta_1 = 42.00° < \theta_c$, the refracted light is white.

(b) At an angle of incidence of $\theta_1 = 43.10°$ which is slightly less than θ_c, the refracted light is white but dominated by red end.

(c) At an angle of incidence of $\theta_1 = 44.00° > \theta_c$, there is no refracted light.

110. (a) The diagram below shows a cross section, through the center of the cube and parallel to a face. L is the length of a cube edge and S labels the spot. A portion of a ray from the source to a cube face is also shown.



Light leaving the source at a small angle θ is refracted at the face and leaves the cube; light leaving at a sufficiently large angle is totally reflected. The light that passes through the cube face forms a circle, the radius r being associated with the critical angle for total internal reflection. If θ_c is that angle, then

$$\sin\theta_c = \frac{1}{n}$$

where n is the index of refraction for the glass. As the diagram shows, the radius of the circle is given by $r = (L/2)\tan\theta_c$. Now,

$$\tan\theta_c = \frac{\sin\theta_c}{\cos\theta_c} = \frac{\sin\theta_c}{\sqrt{1-\sin^2\theta_c}} = \frac{1/n}{\sqrt{1-(1/n)^2}} = \frac{1}{\sqrt{n^2-1}}$$

and the radius of the circle is

$$r = \frac{L}{2\sqrt{n^2-1}} = \frac{10\,\text{mm}}{2\sqrt{(1.5)^2-1}} = 4.47\,\text{mm}.$$

If an opaque circular disk with this radius is pasted at the center of each cube face, the spot will not be seen (provided internally reflected light can be ignored).

(b) There must be six opaque disks, one for each face. The total area covered by disks is $6\pi r^2$ and the total surface area of the cube is $6L^2$. The fraction of the surface area that must be covered by disks is

$$f = \frac{6\pi r^2}{6L^2} = \frac{\pi r^2}{L^2} = \frac{\pi(4.47\,\text{mm})^2}{(10\,\text{mm})^2} = 0.63.$$

111. (a) Suppose that at time t_1, the moon is starting a revolution (on the verge of going behind Jupiter, say) and that at this instant, the distance between Jupiter and Earth is ℓ_1. The time of the start of the revolution as seen on Earth is $t_1^* = t_1 + \ell_1 / c$. Suppose the moon starts the next revolution at time t_2 and at that instant, the Earth-Jupiter distance is ℓ_2. The start of the revolution as seen on Earth is $t_2^* = t_2 + \ell_2 / c$. Now, the actual period of the moon is given by $T = t_2 - t_1$ and the period as measured on Earth is

$$T^* = t_2^* - t_1^* = t_2 - t_1 + \frac{\ell_2}{c} - \frac{\ell_1}{c} = T + \frac{\ell_2 - \ell_1}{c}.$$

The period as measured on Earth is longer than the actual period. This is due to the fact that Earth moves during a revolution, and light takes a finite time to travel from Jupiter to Earth. For the situation depicted in Fig. 33-80, light emitted at the end of a revolution travels a longer distance to get to Earth than light emitted at the beginning. Suppose the position of Earth is given by the angle θ, measured from x. Let R be the radius of Earth's orbit and d be the distance from the Sun to Jupiter. The law of cosines, applied to the triangle with the Sun, Earth, and Jupiter at the vertices, yields $\ell^2 = d^2 + R^2 - 2dR\cos\theta$. This expression can be used to calculate ℓ_1 and ℓ_2. Since Earth does not move very far during one revolution of the moon, we may approximate $\ell_2 - \ell_1$ by $(d\ell / dt)T$ and T^* by $T + (d\ell / dt)(T / c)$. Now

$$\frac{d\ell}{dt} = \frac{2Rd \sin\theta}{\sqrt{d^2 + R^2 - 2dR\cos\theta}} \frac{d\theta}{dt} = \frac{2vd\sin\theta}{\sqrt{d^2 + R^2 - 2dR\cos\theta}},$$

where $v = R\,(d\theta/dt)$ is the speed of Earth in its orbit. For $\theta = 0, (d\ell / dt) = 0$ and $T^* = T$. Since Earth is then moving perpendicularly to the line from the Sun to Jupiter, its distance from the planet does not change appreciably during one revolution of the moon. On the other hand, when $\theta = 90°$, $d\ell / dt = vd / \sqrt{d^2 + R^2}$ and

$$T^* = T\left(1 + \frac{vd}{c\sqrt{d^2 + R^2}}\right).$$

The Earth is now moving parallel to the line from the Sun to Jupiter, and its distance from the planet changes during a revolution of the moon.

(b) Our notation is as follows: t is the actual time for the moon to make N revolutions, and t^* is the time for N revolutions to be observed on Earth. Then,

$$t^* = t + \frac{\ell_2 - \ell_1}{c},$$

where ℓ_1 is the Earth-Jupiter distance at the beginning of the interval and ℓ_2 is the Earth-Jupiter distance at the end. Suppose Earth is at position *x* at the beginning of the interval, and at *y* at the end. Then, $\ell_1 = d - R$ and $\ell_2 = \sqrt{d^2 + R^2}$. Thus,

$$t^* = t + \frac{\sqrt{d^2 + R^2} - (d - R)}{c}.$$

A value can be found for *t* by measuring the observed period of revolution when Earth is at *x* and multiplying by *N*. We note that the observed period is the true period when Earth is at *x*. The time interval as Earth moves from *x* to *y* is t^*. The difference is

$$t^* - t = \frac{\sqrt{d^2 + R^2} - (d - R)}{c}.$$

If the radii of the orbits of Jupiter and Earth are known, the value for $t^* - t$ can be used to compute c. Since Jupiter is much further from the Sun than Earth, $\sqrt{d^2 + R^2}$ may be approximated by *d* and $t^* - t$ may be approximated by R/c. In this approximation, only the radius of Earth's orbit need be known.