1. The image is 10 cm behind the mirror and you are 30 cm in front of the mirror. You must focus your eyes for a distance of 10 cm + 30 cm = 40 cm.

2. The bird is a distance d_2 in front of the mirror; the plane of its image is that same distance d_2 behind the mirror. The lateral distance between you and the bird is $d_3 = 5.00$ m. We denote the distance from the camera to the mirror as d_1, and we construct a right triangle out of d_3 and the distance between the camera and the image plane ($d_1 + d_2$). Thus, the focus distance is

$$d = \sqrt{\left(d_1 + d_2\right)^2 + d_3^2} = \sqrt{\left(4.30\text{m} + 3.30\text{m}\right)^2 + \left(5.00\text{m}\right)^2} = 9.10\text{m}.$$

3. The intensity of light from a point source varies as the inverse of the square of the distance from the source. Before the mirror is in place, the intensity at the center of the screen is given by $I_P = A/d^2$, where A is a constant of proportionality. After the mirror is in place, the light that goes directly to the screen contributes intensity I_P, as before. Reflected light also reaches the screen. This light appears to come from the image of the source, a distance d behind the mirror and a distance $3d$ from the screen. Its contribution to the intensity at the center of the screen is

$$I_r = \frac{A}{(3d)^2} = \frac{A}{9d^2} = \frac{I_P}{9}.$$

The total intensity at the center of the screen is

$$I = I_P + I_r = I_P + \frac{I_P}{9} = \frac{10}{9} I_P.$$

The ratio of the new intensity to the original intensity is $I/I_P = 10/9 = 1.11$.

4. When S is barely able to see B the light rays from B must reflect to S off the edge of the mirror. The angle of reflection in this case is 45°, since a line drawn from S to the mirror's edge makes a 45° angle relative to the wall. By the law of reflection, we find

$$\frac{x}{d/2} = \tan 45° \Rightarrow x = \frac{d}{2} = \frac{3.0\,\text{m}}{2} = 1.5\,\text{m}.$$

5. We apply the law of refraction, assuming all angles are in radians:

$$\frac{\sin\theta}{\sin\theta'} = \frac{n_w}{n_{air}},$$

which in our case reduces to $\theta' \approx \theta/n_w$ (since both θ and θ' are small, and $n_{air} \approx 1$). We refer to our figure below.
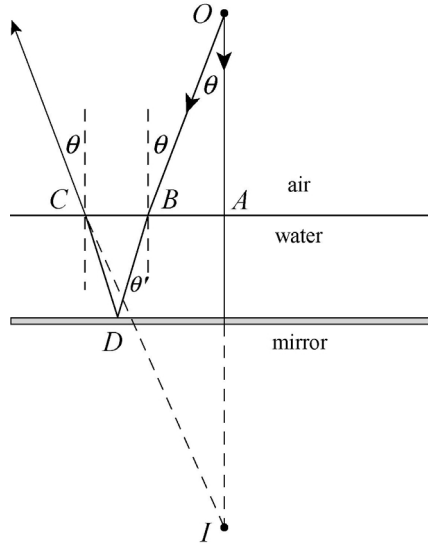
The object O is a vertical distance d_1 above the water, and the water surface is a vertical distance d_2 above the mirror. We are looking for a distance d (treated as a positive number) below the mirror where the image I of the object is formed. In the triangle $O\,AB$

$$|AB| = d_1 \tan\theta \approx d_1\theta,$$

and in the triangle CBD

$$|BC| = 2d_2 \tan\theta' \approx 2d_2\theta' \approx \frac{2d_2\theta}{n_w}.$$

Finally, in the triangle ACI, we have $|AI| = d + d_2$. Therefore,

$$d = |AI| - d_2 = \frac{|AC|}{\tan\theta} - d_2 \approx \frac{|AB| + |BC|}{\theta} - d_2 = \left(\frac{d_1}{\theta} + \frac{2d_2\theta}{n_w}\right)\frac{1}{\theta} - d_2 = d_1 + \frac{2d_2}{n_w} - d_2$$

$$= 250\,\text{cm} + \frac{2(200\,\text{cm})}{1.33} - 200\,\text{cm} = 351\,\text{cm}.$$

6. We note from Fig. 34-32 that $m = \frac{1}{2}$ when $p = 5$ cm. Thus Eq. 34-7 (the magnification equation) gives us $i = -10$ cm in that case. Then, by Eq. 34-9 (which applies to mirrors and thin-lenses) we find the focal length of the mirror is $f = 10$ cm. Next, the problem asks us to consider $p = 14$ cm. With the focal length value already determined, then Eq. 34-9 yields $i = 35$ cm for this new value of object distance. Then, using Eq. 34-7 again, we find $m = i/p = -2.5$.

7. We use Eqs. 34-3 and 34-4, and note that $m = -i/p$. Thus,

$$\frac{1}{p} - \frac{1}{pm} = \frac{1}{f} = \frac{2}{r}.$$

We solve for p:

$$p = \frac{r}{2}\left(1 - \frac{1}{m}\right) = \frac{35.0\,\text{cm}}{2}\left(1 - \frac{1}{2.50}\right) = 10.5\text{ cm.}$$

8. The graph in Fig. 34-33 implies that $f = 20$ cm, which we can plug into Eq. 34-9 (with $p = 70$ cm) to obtain $i = +28$ cm.

9. We recall that a concave mirror has a positive value of focal length.

(a) Then (with $f = +12$ cm and $p = +18$ cm), the radius of curvature is $r = 2f = +24$ cm .

(b) Eq. 34-9 yields $i = pf/(p-f) = +36$ cm.

(c)Then, by Eq. 34-7, the lateral magnification is $m = -i/p = -2.0$.

(d) Since the image distance computation produced a positive value, the image is real (R).

(e) The magnification computation produced a negative value, so it is inverted (I).

(f) For a mirror, the side where a real image forms is the <u>same</u> as the side where the object is.

10. A concave mirror has a positive value of focal length.

(a) Then (with $f = +10$ cm and $p = +15$ cm), the radius of curvature is $r = 2f = +20$ cm .

(b) Eq. 34-9 yields $i = pf/(p-f) = +30$ cm.

(c)Then, by Eq. 34-7, $m = -i/p = -2.0$.

(d) Since the image distance computation produced a positive value, the image is real (R).

(e) The magnification computation produced a negative value, so it is inverted (I).

(f) For a mirror, the side where a real image forms is <u>the same</u> as the side where the object is.

11. A concave mirror has a positive value of focal length.

(a) Then (with $f = +18$ cm and $p = +12$ cm) , the radius of curvature is $r = 2f = +36$ cm.

(b) Eq. 34-9 yields $i = pf/(p{-}f) = -36$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +3.0$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

12. A concave mirror has a positive value of focal length.

(a) Then (with $f = +36$ cm and $p = +24$ cm), the radius of curvature is $r = 2f = +72$ cm.

(b) Eq. 34-9 yields $i = pf/(p-f) = -72$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +3.0$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

13. A convex mirror has a negative value of focal length.

(a) Then (with $f = -10$ cm and $p = +8$ cm), the radius of curvature is $r = 2f = -20$ cm.

(b) Eq. 34-9 yields $i = pf /(p-f) = -4.4$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +0.56$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

14. A convex mirror has a negative value of focal length.

(a) Then (with $f = -35$ cm and $p = +22$ cm), the radius of curvature is $r = 2f = -70$ cm.

(b) Eq. 34-9 yields $i = pf/(p-f) = -14$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +0.61$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

15. A convex mirror has a negative value of focal length.

(a) Then (with $f = -8$ cm and $p = +10$ cm), the radius of curvature is $r = 2f = -16$ cm.

(b) Eq. 34-9 yields $i = pf/(p-f) = -4.4$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +0.44$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

16. A convex mirror has a negative value of focal length.

(a) Then (with $f = -14$ cm and $p = +17$ cm), the radius of curvature is $r = 2f = -28$ cm.

(b) Eq. 34-9 yields $i = pf/(p-f) = -7.7$ cm.

(c) Then, by Eq. 34-7, $m = -i/p = +0.45$.

(d) Since the image distance is negative, the image is virtual (V).

(e) The magnification computation produced a positive value, so it is upright [not inverted] (NI).

(f) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

17. (a) From Eqs. 34-3 and 34-4, we obtain

$$i = pf/(p-f) = pr/(2p-r).$$

Differentiating both sides with respect to time and using $v_O = -dp/dt$, we find

$$v_I = \frac{di}{dt} = \frac{d}{dt}\left(\frac{pr}{2p-r}\right) = \frac{-rv_O(2p-r)+2v_O pr}{(2p-r)^2} = \left(\frac{r}{2p-r}\right)^2 v_O.$$

(b) If $p = 30$ cm, we obtain

$$v_I = \left[\frac{15 \text{ cm}}{2(30 \text{ cm})-15 \text{ cm}}\right]^2 (5.0 \text{ cm/s}) = 0.56 \text{ cm/s}.$$

(c) If $p = 8.0$ cm, we obtain

$$v_I = \left[\frac{15 \text{ cm}}{2(8.0 \text{ cm})-15 \text{ cm}}\right]^2 (5.0 \text{ cm/s}) = 1.1 \times 10^3 \text{ cm/s}.$$

(d) If $p = 1.0$ cm, we obtain

$$v_I = \left[\frac{15 \text{cm}}{2(1.0 \text{cm})-15 \text{cm}}\right]^2 (5.0 \text{cm/s}) = 6.7 \text{cm/s}.$$

18. We note that there is "singularity" in this graph (Fig. 34-34) like there was in Fig. 34-33), which tells us that there is no point where $p = f$ (which causes Eq. 34-9 to "blow up"). Since $p > 0$, as usual, then this means that the focal length is <u>not</u> positive. We know it is not a flat mirror since the curve shown does decrease with p, so we conclude it is a convex mirror. We examine the point where $m = 0.50$ and $p = 10$ cm. Combining Eq. 34-7 and Eq. 34-9 we obtain $m = -i/p = -f/(p - f)$. This yields $f = -10$ cm (verifying our expectation that the mirror is convex). Now, for $p = 21$ cm, we find $m = -f/(p - f)$ = +0.32.

19. (a) The mirror is concave.

(b) $f = +20$ cm (positive, because the mirror is concave).

(c) $r = 2f = 2(+20 \text{ cm}) = +40$ cm.

(d) The object distance $p = +10$ cm, as given in the Table.

(e) The image distance is $i = (1/f - 1/p)^{-1} = (1/20 \text{ cm} - 1/10 \text{ cm})^{-1} = -20$ cm.

(f) $m = -i/p = -(-20 \text{ cm}/10 \text{ cm}) = +2.0$.

(g) The image is virtual (V).

(h) The image is upright or not inverted (NI).

(i) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

20. (a) The fact that the magnification is 1 means that the mirror is flat (plane).

(b) Flat mirrors (and flat "lenses" such as a window pane) have $f = \infty$ (or $f = -\infty$ since the sign does not matter in this extreme case).

(c) The radius of curvature is $r = 2f = \infty$ (or $r = -\infty$) by Eq. 34-3.

(d) $p = + 10$ cm, as given in the Table.

(e) Eq. 34-4 readily yields $i = pf/(p-f) = -10$ cm.

(f) The magnification is $m = -i/p = +1.0$.

(g) The image is virtual since $i < 0$.

(h) The image is upright, or not inverted (NI).

(i) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

21. (a) Since $f > 0$, the mirror is concave.

(b) $f = +20$ cm, as given in the Table.

(c) Using Eq. 34-3, we obtain $r = 2f = +40$ cm.

(d) $p = +10$ cm, as given in the Table.

(e) Eq. 34-4 readily yields $i = pf/(p-f) = +60$ cm.

(f) Eq. 34-6 gives $m = -i/p = -2.0$.

(g) Since $i > 0$, the image is real (R).

(h) Since $m < 0$, the image is inverted (I).

(i) For a mirror, the side where a real image forms is the <u>same</u> as the side where the object is.

22. (a) Since $m = -1/2 < 0$, the image is inverted. With that in mind, we examine the various possibilities in Figs. 34-7, 34-9 and 34-10, and note that an inverted image (for reflections from a single mirror) can only occur if the mirror is concave (and if $p > f$).

(b) Next, we find i from Eq. 34-6 (which yields $i = mp = 30$ cm) and then use this value (and Eq. 34-4) to compute the focal length; we obtain $f = +20$ cm.

(c) Then, Eq. 34-3 gives $r = 2f = +40$ cm.

(d) $p = 60$ cm, as given in the Table.

(e) As already noted, $i = +30$ cm.

(f) $m = -1/2$, as given.

(g) Since $i > 0$, the image is real (R).

(h) As already noted, the image is inverted (I).

(i) For a mirror, the side where a real image forms is the <u>same</u> as the side where the object is.

23. (a) Since $r < 0$ then (by Eq. 34-3) $f < 0$, which means the mirror is convex.

(b) The focal length is $f = r/2 = -20$ cm.

(c) $r = -40$ cm, as given in the Table.

(d) Eq. 34-4 leads to $p = +20$ cm.

(e) $i = -10$ cm, as given in the Table.

(f) Eq. 34-6 gives $m = +0.50$.

(g) The image is virtual (V).

(h) The image is upright, or not inverted (NI).

(i) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

24. (a) Since $0 < m < 1$, the image is upright but smaller than the object. With that in mind, we examine the various possibilities in Figs. 34-7, 34-9 and 34-10, and note that such an image (for reflections from a single mirror) can only occur if the mirror is convex.

(b) Thus, we must put a minus sign in front of the "20" value given for f, i.e., $f = -20$ cm.

(c) Eq. 34-3 then gives $r = 2f = -40$ cm.

(d) To solve for i and p we must set up Eq. 34-4 and Eq. 34-6 as a simultaneous set and solve for the two unknowns. The results are $p = +180$ cm $= +1.8$ m, and

(e) $i = -18$ cm.

(f) $m = 0.10$, as given in the Table.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI), as already noted.

(i) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

25. (a) The mirror is convex, as given.

(b) Knowing the mirror is convex means we must put a minus sign in front of the "40" value given for r. Then, Eq. 34-3 yields $f = r/2 = -20$ cm.

(c) $r = -40$ cm.

(d) The fact that the mirror is convex also means that we need to insert a minus sign in front of the "4.0" value given for i, since the image in this case must be virtual (see Figs. 34-7, 34-9 and 34-10). Eq. 34-4 leads to $p = +5.0$ cm.

(e) As noted above, $i = -4.0$.

(f) Eq. 34-6 gives $m = +0.8$.

(g) The image is virtual (V) since $i < 0$.

(h) The image is upright, or not inverted (NI).

(i) For a mirror, the side where a virtual image forms is <u>opposite</u> from the side where the object is.

26. (a) Since the image is inverted, we can scan Figs. 34-7, 34-9 and 34-10 in the textbook and find that the mirror must be concave.

(b) This also implies that we must put a minus sign in front of the "0.50" value given for m. To solve for f, we first find $i = -pm = +12$ cm from Eq. 34-6 and plug into Eq. 34-4; the result is $f = +8$ cm.

(c) Thus, $r = 2f = +16$ cm.

(d) $p = +24$ cm, as given in the Table.

(e) As shown above, $i = -pm = +12$ cm.

(f) $m = -0.50$, with a minus sign.

(g) The image is real (R) since $i > 0$.

(h) The image is inverted (I), as noted above.

(i) For a mirror, the side where a real image forms is the same as the side where the object is.

27. (a) The fact that the focal length is given as a negative value means the mirror is convex.

(b) $f = -30$ cm, as given in the Table.

(c) The radius of curvature is $r = 2f = -60$ cm.

(d) Eq. 34-9 gives $p = if/(i - f) = +30$ cm.

(e) $i = -15$, as given in the Table.

(f) From Eq. 34-7, we get $m = +1/2 = 0.50$.

(g) The image distance is given as a negative value (as it would have to be, since the mirror is convex), which means the image is virtual (V).

(h) Since $m > 0$, the image is upright (not inverted: NI).

(i) The image is on the side of the mirror <u>opposite</u> to the object.

28. (a) We are told that the image is on the same side as the object; this means the image is real (R) and further implies that the mirror is concave.

(b) The focal distance is $f = +20$ cm.

(c) The radius of curvature is $r = 2f = +40$ cm.

(d) $p = +60$ cm, as given in the Table.

(e) Eq. 34-9 gives $i = pf/(p - f) = +30$ cm.

(f) Eq. 34-7 gives $m = -i/p = -0.50$.

(g) As noted above, the image is real (R).

(h) The image is inverted (I) since $m < 0$.

(i) For a mirror, the side where a real image forms is the same as the side where the object is.

29. (a) As stated in the problem, the image is inverted (I) which implies that it is real (R). It also (more directly) tells us that the magnification is equal to a negative value: $m = -0.40$. By Eq. 34-7, the image distance is consequently found to be $i = +12$ cm. Real images don't arise (under normal circumstances) from convex mirrors, so we conclude that this mirror is concave.

(b) The focal length is $f = +8.6$ cm, using Eq. 34-9 $f = +8.6$ cm.

(c) The radius of curvature is $r = 2f = +17.2$ cm ≈ 17 cm.

(d) $p = +30$ cm, as given in the Table.

(e) As noted above, $i = +12$ cm.

(f) Similarly, $m = -0.40$, with a minus sign.

(g) The image is real (R).

(h) The image is inverted (I).

(i) For a mirror, the side where a real image forms is the <u>same</u> as the side where the object is.

30. (a) From Eq. 34-7, we get $i = -mp = +28$ cm, which implies the image is real (R) and on the same side as the object. Since $m < 0$, we know it was inverted (I). From Eq. 34-9, we obtain $f = ip/(i + p) = +16$ cm, which tells us (among other things) that the mirror is concave.

(b) $f = ip/(i + p) = +16$ cm.

(c) $r = 2f = +32$ cm.

(d) $p = +40$ cm, as given in the Table.

(e) $i = -mp = +28$ cm.

(f) $m = -0.70$, as given in the Table.

(g) The image is real (R).

(h) The image is inverted (I).

(i) For a mirror, the side where a real image forms is <u>the same</u> as the side where the object is.

31. (a) The fact that the magnification is equal to a positive value means that the image is upright (not inverted: NI), and further implies (by Eq. 34-7) that the image distance (i) is equal to a negative value $\Rightarrow$ the image is virtual (V). Looking at the discussion of mirrors in sections 34-3 and 34-4, we see that a positive magnification of magnitude less than unity is only possible for convex mirrors.

(b) For $0 < m < 1$ this will only give a positive value for $p = f/(1 - 1/m)$ if $f < 0$. Thus, with a minus sign, we have $f = -30$ cm.

(c) $r = 2f = -60$ cm.

(d) $p = f/(1 - 1/m) = + 120$ cm $= 1.2$ m.

(e) $i = -mp = -24$ cm.

(f) $m = +0.20$, as given in the Table.

(g) The image is virtual (V).

(h) The image is upright, or not inverted (NI).

(i) For a mirror, the side where a virtual image forms is opposite from the side where the object is.

32. (a) We use Eq. 34-8 and note that $n_1 = n_{air} = 1.00$, $n_2 = n$, $p = \infty$, and $i = 2r$:

$$\frac{1.00}{\infty} + \frac{n}{2r} = \frac{n-1}{r}.$$

We solve for the unknown index: $n = 2.00$.

(b) Now $i = r$ so Eq. 34-8 becomes

$$\frac{n}{r} = \frac{n-1}{r},$$

which is not valid unless $n \to \infty$ or $r \to \infty$. It is impossible to focus at the center of the sphere.

33. We use Eq. 34-8 (and Fig. 34-10(d) is useful), with $n_1 = 1.6$ and $n_2 = 1$ (using the rounded-off value for air).

$$\frac{1.6}{p} + \frac{1}{i} = \frac{1-1.6}{r}$$

Using the sign convention for r stated in the paragraph following Eq. 34-8 (so that $r = -5.0$ cm), we obtain $i = -2.4$ cm for objects at $p = 3.0$ cm. Returning to Fig. 34-36 (and noting the location of the observer), we conclude that the tabletop seems 7.4 cm away.

34. In addition to $n_1 = 1.0$, we are given (a) $n_2 = 1.5$, (b) $p = +10$ and (c) $r = +30$.

(d) Eq. 34-8 yields

$$i = n_2 \left(\frac{n_2 - n_1}{r} - \frac{n_1}{p} \right)^{-1} = 1.5 \left(\frac{1.5 - 1.0}{30 \text{ cm}} - \frac{1.0}{10 \text{ cm}} \right) = -18 \text{ cm.}$$

(e) The image is virtual (V) and upright since $i < 0$.

(f) The object and its image are in the same side. The ray diagram would be similar to Fig. 34-11(c) in the textbook.

35. In addition to $n_1 = 1.0$, we are given (a) $n_2 = 1.5$, (b) $p = +10$ and (d) $i = -13$.

(c) Eq. 34-8 yields

$$r = (n_2 - n_1)\left(\frac{n_1}{p} + \frac{n_2}{i}\right)^{-1} = (1.5-1.0)\left(\frac{1.0}{10} + \frac{1.5}{-13}\right)^{-1} = -32.5\,\text{cm} \approx -33\ \text{cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are in the same side. The ray diagram would be similar to Fig. 34-11(e).

36. In addition to $n_1 = 1.0$, we are given (a) $n_2 = 1.5$, (c) $r = +30$ and (d) $i = +600$.

(b) Eq. 34-8 gives

$$p = \frac{n_1}{\dfrac{n_2 - n_1}{r} - \dfrac{n_2}{i}} = \frac{1.0}{\dfrac{1.5 - 1.0}{30} - \dfrac{1.5}{600}} = 71 \, \text{cm}.$$

(d) With $i > 0$, the image is real (R) and inverted.

(e) The object and its image are in the opposite side. The ray diagram would be similar to Fig. 34-11(a) in the textbook.

37. In addition to $n_1 = 1.5$, we are given (a) $n_2 = 1.0$, (b) $p = +10$ and (d) $i = -6.0$.

(c) We manipulate Eq. 34-8 to find r:

$$r = \left(n_2 - n_1\right)\left(\frac{n_1}{p} + \frac{n_2}{i}\right)^{-1} = \left(1.0 - 1.5\right)\left(\frac{1.5}{10} + \frac{1.0}{-6.0}\right)^{-1} = 30 \text{ cm.}$$

(e) The image is virtual (V) and upright.

(f) The object and its image are in the same side. The ray diagram would be similar to Fig. 34-11(f) in the textbook, but with the object and the image located closer to the surface.

38. In addition to $n_1 = 1.5$, we are given (a) $n_2=1.0$, (c) $r=-30$ and (d) $i=-7.5$.

(b)We manipulate Eq. 34-8 to find p:

$$p = \frac{n_1}{\dfrac{n_2 - n_1}{r} - \dfrac{n_2}{i}} = \frac{1.5}{\dfrac{1.0 - 1.5}{-30} - \dfrac{1.0}{-7.5}} = 10\,\text{cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are in the same side. The ray diagram would be similar to Fig. 34-11(d) in the textbook.

39. In addition to $n_1 = 1.5$, we are given (a) $n_2 = 1.0$, (b) $p = +70$ and (c) $r = +30$.

(d) We manipulate Eq. 34-8 to find the image distance:

$$i = n_2 \left(\frac{n_2 - n_1}{r} - \frac{n_1}{p} \right)^{-1} = 1.0 \left(\frac{1.0 - 1.5}{30 \text{ cm}} - \frac{1.5}{70 \text{ cm}} \right)^{-1} = -26 \text{ cm}.$$

(e) The image is virtual (V) and upright.

(f) The object and its image are in the same side. The ray diagram would be similar to Fig. 34-11(f) in the textbook.

40. In addition to $n_1 = 1.5$, we are given (b) $p = +100$, (c) $r = -30$ and (d) $i = +600$.

(a) We manipulate Eq. 34-8 to separate the indices:

$$n_2\left(\frac{1}{r}-\frac{1}{i}\right)=\left(\frac{n_1}{p}+\frac{n_1}{r}\right) \;\Rightarrow\; n_2\left(\frac{1}{-30}-\frac{1}{600}\right)=\left(\frac{1.5}{100}+\frac{1.5}{-30}\right) \;\Rightarrow\; n_2\left(-0.035\right)=-0.035$$

which implies $n_2 = 1.0$.

(e) The image is real (R) and inverted.

(f) The object and its image are in the opposite side. The ray diagram would be similar to Fig. 34-11(b) in the textbook.

41. Let the diameter of the Sun be d_s and that of the image be d_i. Then, Eq. 34-5 leads to

$$d_i = |m| d_s = \left(\frac{i}{p}\right) d_s \approx \left(\frac{f}{p}\right) d_s = \frac{(20.0 \times 10^{-2}\,\text{m})(2)(6.96 \times 10^8\,\text{m})}{1.50 \times 10^{11}\,\text{m}} = 1.86 \times 10^{-3}\,\text{m}$$

$$= 1.86\,\text{mm}.$$

42. The singularity the graph (where the curve goes to $\pm\infty$) is at $p = 30$ cm, which implies (by Eq. 34-9) that $f = 30$ cm > 0 (converging type lens). For $p = 100$ cm, Eq. 34-9 leads to $i = +43$ cm.

43. We use the lens maker's equation, Eq. 34-10:

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where f is the focal length, n is the index of refraction, r_1 is the radius of curvature of the first surface encountered by the light and r_2 is the radius of curvature of the second surface. Since one surface has twice the radius of the other and since one surface is convex to the incoming light while the other is concave, set $r_2 = -2r_1$ to obtain

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} + \frac{1}{2r_1}\right) = \frac{3(n-1)}{2r_1}.$$

(a) We solve for the smaller radius r_1:

$$r_1 = \frac{3(n-1)f}{2} = \frac{3(1.5-1)(60 \text{ mm})}{2} = 45 \text{ mm.}$$

(b) The magnitude of the larger radius is $|r_2| = 2r_1 = 90$ mm .

44. Since the focal length is a constant for the whole graph, then $1/p + 1/i =$ constant. Consider the value of the graph at $p = 20$ cm; we estimate its value there to be -10 cm. Therefore, $1/20 + 1/(-10) = 1/70 + 1/i_{new}$. Thus, $i_{new} = -16$ cm.

45. (a) We use Eq. 34-10:

$$f = \left[(n-1) \left(\frac{1}{r_1} - \frac{1}{r_2} \right) \right]^{-1} = \left[(1.5-1) \left(\frac{1}{\infty} - \frac{1}{-20 \text{ cm}} \right) \right]^{-1} = +40 \text{ cm}.$$

(b) From Eq. 34-9,

$$i = \left(\frac{1}{f} - \frac{1}{p} \right)^{-1} = \left(\frac{1}{40 \text{ cm}} - \frac{1}{40 \text{ cm}} \right)^{-1} = \infty.$$

46. Combining Eq. 34-7 and Eq. 34-9, we have $m(\,p - f\,) = -f$. The graph in Fig. 34-39 indicates that $m = 2$ where $p = 5$ cm, so our expression yields $f = 10$ cm. Plugging this back into our expression and evaluating at $p = 14$ cm yields $m = -2.5$.

47. We solve Eq. 34-9 for the image distance:

$$i = \left(\frac{1}{f} - \frac{1}{p} \right)^{-1} = \frac{fp}{p-f}.$$

The height of the image is thus

$$h_i = m h_p = \left(\frac{i}{p} \right) h_p = \frac{f h_p}{p-f} = \frac{(75 \text{ mm})(1.80 \text{ m})}{27 \text{ m} - 0.075 \text{ m}} = 5.0 \text{ mm}.$$

48. Combining Eq. 34-7 and Eq. 34-9, we have $m(p - f) = -f$. The graph in Fig. 34-40 indicates that $m = 0.5$ where $p = 15$ cm, so our expression yields $f = -15$ cm. Plugging this back into our expression and evaluating at $p = 35$ cm yields $m = +0.30$.

49. Using Eq. 34-9 and noting that $p + i = d = 44$ cm, we obtain $p^2 - dp + df = 0$. Therefore,

$$p = \frac{1}{2}(d \pm \sqrt{d^2 - 4df}) = 22 \text{ cm} \pm \frac{1}{2}\sqrt{(44 \text{ cm})^2 - 4(44 \text{ cm})(11 \text{ cm})} = 22 \text{ cm}.$$

50. We recall that for a converging (C) lens, the focal length value should be positive ($f = +4$ cm).

(a) Eq. 34-9 gives $i = pf/(p-f) = +5.3$ cm.

(b) Eq. 34-7 give $m = -i/p = -0.33$.

(c) The fact that the image distance i is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (see Fig. 34-14).

51. We recall that for a converging (C) lens, the focal length value should be positive ($f =$ +16 cm).

(a) Eq. 34-9 gives $i = pf/(p-f) = -48$ cm.

(b) Eq. 34-7 give $m = -i/p = +4.0$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-14).

52. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -6$ cm).

(a) Eq. 34-9 gives $i = pf/(p-f) = -3.8$ cm.

(b) Eq. 34-7 give $m = -i/p = +0.38$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-14).

53. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -12$ cm).

(a) Eq. 34-9 gives $i = pf/(p-f) = -4.8$ cm.

(b) Eq. 34-7 give $m = -i/p = +0.60$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-14).

54. We recall that for a converging (C) lens, the focal length value should be positive ($f =$ +35 cm).

(a) Eq. 34-9 gives $i = pf/(p-f) = -88$ cm.

(b) Eq. 34-7 give $m = -i/p = +3.5$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-14).

55. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -14$ cm).

(a) Eq. 34-9 gives $i = pf/(p-f) = -8.6$ cm.

(b) Eq. 34-7 give $m = -i/p = +0.39$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-14).

56. We recall that for a diverging (D) lens, the focal length value should be negative ($f = -31$ cm).

(a) Eq. 34-9 gives $i = pf/(p-f) = -8.7$ cm.

(b) Eq. 34-7 give $m = -i/p = +0.72$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image is on the same side as the object (see Fig. 34-14).

57. We recall that for a converging (C) lens, the focal length value should be positive ($f =$ +20 cm).

(a) Eq. 34-9 gives $i = pf/(p-f) = +36$ cm.

(b) Eq. 34-7 give $m = -i/p = -0.80$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) A negative value of magnification means the image is inverted (I).

(e) The image is on the opposite side of the object (see Fig. 34-14).

58. (a) A convex (converging) lens, since a real image is formed.

(b) Since $i = d - p$ and $i/p = 1/2$,

$$p = \frac{2d}{3} = \frac{2(40.0 \text{ cm})}{3} = 26.7 \text{ cm.}$$

(c) The focal length is

$$f = \left(\frac{1}{i} + \frac{1}{p}\right)^{-1} = \left(\frac{1}{d/3} + \frac{1}{2d/3}\right)^{-1} = \frac{2d}{9} = \frac{2(40.0 \text{ cm})}{9} = 8.89 \text{ cm.}$$

59. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = +84$ cm.

(b) Eq. 34-7 give $m = -i / p = -1.4$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (see Fig. 34-14).

60. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -26$ cm.

(b) Eq. 34-7 give $m = -i/p = +4.3$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image it is on the same side as the object (see Fig. 34-14).

61. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -18$ cm.

(b) Eq. 34-7 give $m = -i/p = +0.76$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image it is on the same side as the object (see Fig. 34-14).

62. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -9.7$ cm.

(b) Eq. 34-7 give $m = -i/p = +0.54$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image it is on the same side as the object (see Fig. 34-14).

63. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -30$ cm.

(b) Eq. 34-7 give $m = -i/p = +0.86$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image it is on the same side as the object (see Fig. 34-14).

64. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = -63$ cm.

(b) Eq. 34-7 give $m = -i/p = +2.2$.

(c) The fact that the image distance is a negative value means the image is virtual (V).

(d) A positive value of magnification means the image is not inverted (NI).

(e) The image it is on the same side as the object (see Fig. 34-14).

65. (a) Combining Eq. 34-9 and Eq. 34-10 gives $i = +55$ cm.

(b) Eq. 34-7 give $m = -i/p = -0.74$.

(c) The fact that the image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image is on the side opposite from the object (see Fig. 34-14).

66. (a) Eq. 34-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = +30$ cm. Since $f > 0$, this must be a converging ("C") lens. From Eq. 34-9, we obtain

$$i = \frac{1}{\dfrac{1}{f} - \dfrac{1}{p}} = \frac{1}{\dfrac{1}{30} - \dfrac{1}{10}} = -15\,\text{cm}.$$

(b) Eq. 34-6 yields $m = -i/p = -(-15)/10 = +1.5$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image it is on the same side as the object. The ray diagram would be similar to Fig. 34-15(b) in the textbook.

67. (a) Eq. 34-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = -30$ cm. Since $f < 0$, this must be a diverging ("D") lens. From Eq. 34-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-30} - \frac{1}{10}} = -7.5 \, \text{cm}.$$

(b) Eq. 34-6 yields $m = -i/p = -(-7.5)/10 = +0.75$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image it is on the same side as the object. The ray diagram would be similar to Fig. 34-15(c) in the textbook.

68. (a) Eq. 34-10 yields $f = \frac{1}{n-1}(1/r_1 - 1/r_2)^{-1} = -120$ cm. Since $f < 0$, this must be a diverging ("D") lens. From Eq. 34-9, we obtain

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{-120} - \frac{1}{10}} = -9.2 \, \text{cm}.$$

(b) Eq. 34-6 yields $m = -i/p = -(-9.2)/10 = +0.92$.

(c) Since $i < 0$, the image is virtual (V).

(d) Since $m > 0$, the image is upright, or not inverted (NI).

(e) The image it is on the same side as the object. The ray diagram would be similar to Fig. 34-15(c) in the textbook.

69. (a) The fact that $m > 1$ means the lens is of the converging type (C) (it may help to look at Fig. 34-14 to illustrate this).

(b) A converging lens implies $f = +20$ cm, with a plus sign.

(d) Eq. 34-9 then gives $i = -13$ cm.

(e) Eq. 34-7 gives $m = -i/p = +1.7$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image it is on the same side as the object (see Fig. 34-14).

70. (a) The fact that $m < 1$ and that the image is upright (not inverted: NI) means the lens is of the diverging type (D) (it may help to look at Fig. 34-14 to illustrate this).

(b) A diverging lens implies that $f = -20$ cm, with a minus sign.

(d) Eq. 34-9 gives $i = -5.7$ cm.

(e) Eq. 34-7 gives $m = -i / p = +0.71$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(h) The image it is on the same side as the object (see Fig. 34-14).

71. (a) Eq. 34-7 yields $i = -mp = -(0.25)(16) = -4.0$ cm. Eq. 34-9 gives $f = -5.3$ cm, which implies the lens is of the diverging type (D).

(b) From (a), we have $f = -5.3$ cm.

(d) Similarly, $i = -4.0$ cm.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image it is on the same side as the object (see Fig. 34-14).

72. (a) Eq. 34-7 readily yields $i = +4.0$ cm. Then Eq. 34-9 gives $f = +3.2$ cm, which implies the lens is of the converging type (C).

(b) From (a), we have $f = +3.2$ cm.

(d) Similarly, $i = +4.0$ cm.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object.

73. (a) Eq. 34-7 readily yields $i = -20$ cm. Then Eq. 34-9 gives $f = +80$ cm, which implies the lens is of the converging type (C).

(b) From (a), we have $f = +80$ cm.

(d) Similarly, $i = -20$ cm.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image it is on the same side as the object (see Fig. 34-14).

74. (b) Since this is a converging lens ("C") then $f > 0$, so we should put a plus sign in front of the "10" value given for the focal length.

(d) Eq. 34-9 gives

$$i = \frac{1}{\dfrac{1}{f} - \dfrac{1}{p}} = \frac{1}{\dfrac{1}{10} - \dfrac{1}{20}} = +20\,\text{cm}.$$

(e) From Eq. 34-6, $m = -20/20 = -1.0$.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object.

75. (a) Since $f > 0$, this is a converging lens ("C").

(d) Eq. 34-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{5}} = -10 \, \text{cm}.$$

(e) From Eq. 34-6, $m = -(-10)/5 = +2.0$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image it is on the same side as the object (see Fig. 34-14).

76. (a) We are told the magnification is positive and greater than 1. Scanning the single-lens-image figures in the textbook (Figs. 34-14, 34-15 and 34-17), we see that such a magnification (which implies an upright image larger than the object) is only possible if the lens is of the converging ("C") type (and if $p < f$).

(b) We should put a plus sign in front of the "10" value given for the focal length.

(d) Eq. 34-9 gives

$$i = \frac{1}{\frac{1}{f} - \frac{1}{p}} = \frac{1}{\frac{1}{10} - \frac{1}{5}} = -10\,\text{cm}.$$

(e) $m = -i/p = +2.0$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

(h) The image it is on the same side as the object (see Fig. 34-14).

77. (a) We are told the magnification is less than 1, and we note that $p < |f|$). Scanning Figs. 34-14, 34-15 and 34-17, we see that such a magnification (which implies an image smaller than the object) and object position (being fairly close to the lens) are simultaneously possible only if the lens is of the diverging ("D") type.

(b) Thus, we should put a minus sign in front of the "10" value given for the focal length.

(d) Eq. 34-9 gives

$$i = \frac{1}{\dfrac{1}{f} - \dfrac{1}{p}} = \frac{1}{\dfrac{1}{-10} - \dfrac{1}{5}} = -3.3\,\text{cm}.$$

(e) $m = -i/p = +0.67$.

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(g) A positive value of magnification means the image is not inverted (NI).

78. (a) We are told the absolute value of the magnification is 0.5 and that the image was upright (NI). Thus, $m = +0.5$. Using Eq. 34-6 and the given value of p, we find $i = -5.0$ cm; it is a virtual image. Eq. 34-9 then yields the focal length: $f = -10$ cm. Therefore, the lens is of the diverging ("D") type.

(b) From (a), we have $f = -10$ cm.

(d) Similarly, $i = -5.0$ cm.

(e) $m = +0.5$, with a plus sign

(f) The fact that the image distance i is a negative value means the image is virtual (V).

(h) The image it is on the same side as the object (see Fig. 34-14).

79. (a) Using Eq. 34-6 (which implies the image is inverted) and the given value of p, we find $i = -mp = +5.0$ cm; it is a real image. Eq. 34-9 then yields the focal length: $f = +3.3$ cm. Therefore, the lens is of the converging ("C") type.

(b) From (a), we have $f = +3.3$ cm.

(d) Similarly, $i = -mp = +5.0$ cm.

(f) The fact that the image distance is a positive value means the image is real (R).

(g) The fact that the magnification is a negative value means the image is inverted (I).

(h) The image is on the side opposite from the object. The ray diagram would be similar to Fig. 34-15(a) in the textbook.

80. (a) The image from lens 1 (which has f_1 = +15 cm) is at i_1 = –30 cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has f_2 = +8 cm) with $p_2 = d - i_1$ = 40 cm. Then Eq. 34-9 (applied to lens 2) yields i_2 = +10 cm.

(b) Eq. 34-11 yields $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = -0.75$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image it is on the side opposite from the object (relative to lens 2).

81. (a) The image from lens 1 (which has $f_1 = +8$ cm) is at $i_1 = 24$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = +6$ cm) with $p_2 = d - i_1 = 8$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +24$ cm.

(b) Eq. 34-11 yields $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = +6.0$.

(c)The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image it is on the side opposite from the object (relative to lens 2).

82. (a) The image from lens 1 (which has $f_1 = +12$ cm) is at $i_1 = +60$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = +10$ cm) with $p_2 = d - i_1 = 7$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -23$ cm.

(b) Eq. 34-11 yields $M = m_1 m_2 = (-i_1/p_1)(-i_2/p_2) = i_1 i_2 / p_1 p_2 = -13$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image it is on the same side as the object (relative to lens 2).

83. (a) The image from lens 1 (which has $f_1 = +9$ cm) is at $i_1 = 16.4$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = +5$ cm) with $p_2 = d - i_1 = -8.4$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +3.1$ cm.

(b) Eq. 34-11 yields $M = m_1 m_2 = (-i_1 / p_1)(-i_2 / p_2) = i_1 i_2 / p_1 p_2 = -0.31$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image it is on the side opposite from the object (relative to lens 2). Since this result involves a negative value for p_2 (and perhaps other "non-intuitive" features), we offer a few words of explanation: lens 1 is converging the rays towards an image (that never gets a chance to form due to the intervening presence of lens 2) that would be real and inverted (and 8.4 cm beyond lens 2's location). Lens 2, in a sense, just causes these rays to converge a little more rapidly, and causes the image to form a little closer (to the lens system) than if lens 2 were not present.

84. (a) The image from lens 1 (which has $f_1 = -6$ cm) is at $i_1 = -3.4$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = +6$ cm) with $p_2 = d - i_1 = 15.4$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = +9.8$ cm.

(b) Eq. 34-11 yields $M = -0.27$.

(c) The fact that the (final) image distance is a positive value means the image is real (R).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image it is on the side opposite from the object (relative to lens 2).

85. (a) The image from lens 1 (which has $f_1 = +6$ cm) is at $i_1 = -12$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = -6$ cm) with $p_2 = d - i_1 = 20$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -4.6$ cm.

(b) Eq. 34-11 yields $M = +0.69$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image it is on the same side as the object (relative to lens 2).

86. (a) The image from lens 1 (which has $f_1 = +8$ cm) is at $i_1 = +24$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = -8$ cm) with $p_2 = d - i_1 = 6$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -3.4$ cm.

(b) Eq. 34-11 yields $M = -1.1$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is a negative value means the image is inverted (I).

(e) The image it is on the same side as the object (relative to lens 2).

87. (a) The image from lens 1 (which has $f_1 = -12$ cm) is at $i_1 = -7.5$ cm (by Eq. 34-9). This serves as an "object" for lens 2 (which has $f_2 = -8$ cm) with $p_2 = d - i_1 = 17.5$ cm. Then Eq. 34-9 (applied to lens 2) yields $i_2 = -5.5$ cm.

(b) Eq. 34-11 yields $M = +0.12$.

(c) The fact that the (final) image distance is negative means the image is virtual (V).

(d) The fact that the magnification is positive means the image is not inverted (NI).

(e) The image it is on the same side as the object (relative to lens 2).

88. The minimum diameter of the eyepiece is given by

$$d_{ey} = \frac{d_{ob}}{m_\theta} = \frac{75 \text{ mm}}{36} = 2.1 \text{ mm.}$$

89. (a) If L is the distance between the lenses, then according to Fig. 34-18, the tube length is

$$s = L - f_{ob} - f_{ey} = 25.0 \text{ cm} - 4.00 \text{ cm} - 8.00 \text{ cm} = 13.0 \text{ cm}.$$

(b) We solve $(1/p) + (1/i) = (1/f_{ob})$ for p. The image distance is

$$i = f_{ob} + s = 4.00 \text{ cm} + 13.0 \text{ cm} = 17.0 \text{ cm},$$

so

$$p = \frac{i f_{ob}}{i - f_{ob}} = \frac{(17.0 \text{ cm})(4.00 \text{ cm})}{17.0 \text{ cm} - 4.00 \text{ cm}} = 5.23 \text{ cm}.$$

(c) The magnification of the objective is

$$m = -\frac{i}{p} = -\frac{17.0 \text{ cm}}{5.23 \text{ cm}} = -3.25.$$

(d) The angular magnification of the eyepiece is

$$m_\theta = \frac{25 \text{ cm}}{f_{ey}} = \frac{25 \text{ cm}}{8.00 \text{ cm}} = 3.13.$$

(e) The overall magnification of the microscope is

$$M = m m_\theta = (-3.25)(3.13) = -10.2.$$

90. (a) Without the magnifier, $\theta = h/P_n$ (see Fig. 34-17). With the magnifier, letting $i = -|i| = -P_n$, we obtain

$$\frac{1}{p} = \frac{1}{f} - \frac{1}{i} = \frac{1}{f} + \frac{1}{|i|} = \frac{1}{f} + \frac{1}{P_n}.$$

Consequently,

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f + 1/P_n}{1/P_n} = 1 + \frac{P_n}{f} = 1 + \frac{25 \text{ cm}}{f}.$$

With $f = 10$ cm, $m_\theta = 1 + \dfrac{25\text{cm}}{10\text{cm}} = 3.5$.

(b) In the case where the image appears at infinity, let $i = -|i| \to -\infty$, so that $1/p + 1/i = 1/p = 1/f$, we have

$$m_\theta = \frac{\theta'}{\theta} = \frac{h/p}{h/P_n} = \frac{1/f}{1/P_n} = \frac{P_n}{f} = \frac{25 \text{ cm}}{f}.$$

With $f = 10$ cm,

$$m_\theta = \frac{25\text{cm}}{10\text{cm}} = 2.5.$$

91. (a) When the eye is relaxed, its lens focuses far-away objects on the retina, a distance i behind the lens. We set $p = \infty$ in the thin lens equation to obtain $1/i = 1/f$, where f is the focal length of the relaxed effective lens. Thus, $i = f = 2.50$ cm. When the eye focuses on closer objects, the image distance i remains the same but the object distance and focal length change. If p is the new object distance and f' is the new focal length, then

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f'}.$$

We substitute $i = f$ and solve for f':

$$f' = \frac{pf}{f+p} = \frac{(40.0\ \text{cm})(2.50\ \text{cm})}{40.0\ \text{cm} + 2.50\ \text{cm}} = 2.35\ \text{cm}.$$

(b) Consider the lens maker's equation

$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where r_1 and r_2 are the radii of curvature of the two surfaces of the lens and n is the index of refraction of the lens material. For the lens pictured in Fig. 34-43, r_1 and r_2 have about the same magnitude, r_1 is positive, and r_2 is negative. Since the focal length decreases, the combination $(1/r_1) - (1/r_2)$ must increase. This can be accomplished by decreasing the magnitudes of both radii.

92. We refer to Fig. 34-18. For the intermediate image $p = 10$ mm and

$$i = (f_{ob} + s + f_{ey}) - f_{ey} = 300 \text{ m} - 50 \text{ mm} = 250 \text{ mm},$$

so

$$\frac{1}{f_{ob}} = \frac{1}{i} + \frac{1}{p} = \frac{1}{250 \text{ mm}} + \frac{1}{10 \text{ mm}} \Rightarrow f_{ob} = 9.62 \text{ mm},$$

and

$$s = (f_{ob} + s + f_{ey}) - f_{ob} - f_{ey} = 300 \text{ mm} - 9.62 \text{ mm} - 50 \text{ mm} = 240 \text{ mm}.$$

Then from Eq. 34-14,

$$M = -\frac{s}{f_{ob}} \frac{25 \text{ cm}}{f_{ey}} = -\left(\frac{240 \text{ mm}}{9.62 \text{ mm}} \right)\left(\frac{150 \text{ mm}}{50 \text{ mm}} \right) = -125.$$

93. (a) Now, the lens-film distance is

$$i = \left(\frac{1}{f} - \frac{1}{p} \right)^{-1} = \left(\frac{1}{5.0 \text{ cm}} - \frac{1}{100 \text{ cm}} \right)^{-1} = 5.3 \text{ cm}.$$

(b) The change in the lens-film distance is 5.3 cm – 5.0 cm = 0.30 cm.

94. (a) In the closest mirror M_1, the "first" image I_1 is 10 cm behind M_1 and therefore 20 cm from the object O. This is the smallest distance between the object and an image of the object.

(b) There are images from both O and I_1 in the more distant mirror, M_2: an image I_2 located at 30 cm behind M_2. Since O is 30 cm in front of it, I_2 is 60 cm from O. This is the second smallest distance between the object and an image of the object.

(c) There is also an image I_3 which is 50 cm behind M_2 (since I_1 is 50 cm in front of it). Thus, I_3 is 80 cm from O. In addition, we have another image I_4 which is 70 cm behind M_1 (since I_2 is 70 cm in front of it). The distance from I_4 to O for is 80 cm.

(d) Returning to the closer mirror M_1, there is an image I_5 which is 90 cm behind the mirror (since I_3 is 90 cm in front of it). The distances (measured from O) for I_5 is 100 cm = 1.0 m.

95. (a) Parallel rays are bent by positive-f lenses to their focal points F_1, and rays that come from the focal point positions F_2 in front of positive-f lenses are made to emerge parallel. The key, then, to this type of beam expander is to have the rear focal point F_1 of the first lens coincide with the front focal point F_2 of the second lens. Since the triangles that meet at the coincident focal point are similar (they share the same angle; they are vertex angles), then $W_f/f_2 = W_i/f_1$ follows immediately. Substituting the values given, we have

$$W_f = \frac{f_2}{f_1}W_i = \frac{30.0\ \text{cm}}{12.5\ \text{cm}}(2.5\ \text{mm}) = 6.0\ \text{mm}.$$

(b) The area is proportional to W^2. Since intensity is defined as power P divided by area, we have

$$\frac{I_f}{I_i} = \frac{P/W_f^2}{P/W_i^2} = \frac{W_i^2}{W_f^2} = \frac{f_1^2}{f_2^2} \quad \Rightarrow \quad I_f = \left(\frac{f_1}{f_2}\right)^2 I_i = 1.6\ \text{kW/m}^2.$$

(c) The previous argument can be adapted to the first lens in the expanding pair being of the diverging type, by ensuring that the front focal point of the first lens coincides with the front focal point of the second lens. The distance between the lenses in this case is

$$f_2 - |f_1| = 30.0\ \text{cm} - 26.0\ \text{cm} = 4.0\ \text{cm}.$$

96. By Eq. 34-9, $1/i + 1/p$ is equal to constant $(1/f)$. Thus,

$$1/(-10) + 1/(15) = 1/i_{new} + 1/(70).$$

This leads to $i_{new} = -21$ cm.

97. (a) The "object" for the mirror which results in that box-image is equally in front of the mirror (4 cm). This object is actually the first image formed by the system (produced by the first transmission through the lens); in those terms, it corresponds to $i_1 = 10 - 4 = 6$ cm. Thus, with $f_1 = 2$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow p_1 = 3.00 \text{ cm.}$$

(b) The previously mentioned box-image (4 cm behind the mirror) serves as an "object" (at $p_3 = 14$ cm) for the return trip of light through the lens ($f_3 = f_1 = 2$ cm). This time, Eq. 34-9 leads to

$$\frac{1}{p_3} + \frac{1}{i_3} = \frac{1}{f_3} \Rightarrow i_3 = 2.33 \text{ cm.}$$

98. (a) First, the lens forms a real image of the object located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1$$

to the right of the lens, or at

$$p_2 = 2(f_1 + f_2) - 2f_1 = 2f_2$$

in front of the mirror. The subsequent image formed by the mirror is located at a distance

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{f_2} - \frac{1}{2f_2}\right)^{-1} = 2f_2$$

to the left of the mirror, or at

$$p'_1 = 2(f_1 + f_2) - 2f_2 = 2f_1$$

to the right of the lens. The final image formed by the lens is at a distance i'_1 to the left of the lens, where

$$i'_1 = \left(\frac{1}{f_1} - \frac{1}{p'_1}\right)^{-1} = \left(\frac{1}{f_1} - \frac{1}{2f_1}\right)^{-1} = 2f_1.$$

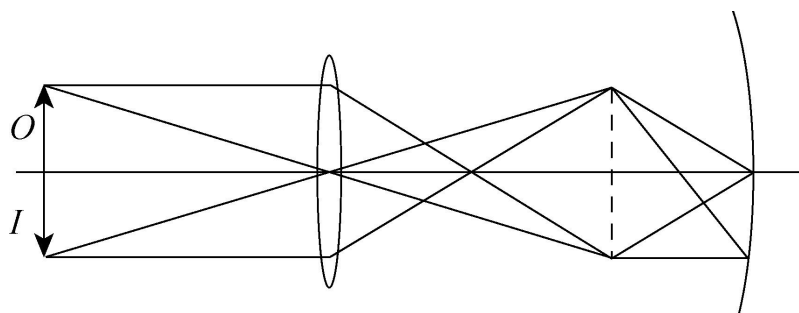This turns out to be the same as the location of the original object.

(b) The lateral magnification is

$$m = \left(-\frac{i_1}{p_1}\right)\left(-\frac{i_2}{p_2}\right)\left(-\frac{i'_1}{p'_1}\right) = \left(-\frac{2f_1}{2f_1}\right)\left(-\frac{2f_2}{2f_2}\right)\left(-\frac{2f_1}{2f_1}\right) = -1.0.$$

(c) The final image is real (R).

(d) It is at a distance i'_1 to the left of the lens,

(e) and inverted (I), as shown in the figure below.

99. We refer to Fig. 34-2 in the textbook. Consider the two light rays, r and r', which are closest to and on either side of the normal ray (the ray that reverses when it reflects). Each of these rays has an angle of incidence equal to θ when they reach the mirror. Consider that these two rays reach the top and bottom edges of the pupil after they have reflected. If ray r strikes the mirror at point A and ray r' strikes the mirror at B, the distance between A and B (call it x) is

$$x = 2d_o \tan \theta$$

where d_o is the distance from the mirror to the object. We can construct a right triangle starting with the image point of the object (a distance d_o behind the mirror; see I in Fig. 34-2). One side of the triangle follows the extended normal axis (which would reach from I to the middle of the pupil), and the hypotenuse is along the extension of ray r (after reflection). The distance from the pupil to I is $d_{ey} + d_o$, and the small angle in this triangle is again θ. Thus,

$$\tan \theta = \frac{R}{d_{ey} + d_o}$$

where R is the pupil radius (2.5 mm). Combining these relations, we find

$$x = 2d_o \frac{R}{d_{ey} + d_o} = 2(100\,\text{mm}) \frac{2.5\,\text{mm}}{300\,\text{mm} + 100\,\text{mm}}$$

which yields $x = 1.67$ mm. Now, x serves as the diameter of a circular area A on the mirror, in which all rays that reflect will reach the eye. Therefore,

$$A = \frac{1}{4} \pi x^2 = \frac{\pi}{4} (1.67\,\text{mm})^2 = 2.2\,\text{mm}^2 \ .$$

100. We use Eq. 34-10, with the conventions for signs discussed in §34-6 and §34-7.

(a) For lens 1, the bi-convex (or double convex) case, we have

$$f = \left[(n-1)\left(\frac{1}{r_1} - \frac{1}{r_2} \right) \right]^{-1} = \left[(1.5-1)\left(\frac{1}{40\ cm} - \frac{1}{-40\ cm} \right) \right]^{-1} = 40\ cm.$$

(b) Since $f > 0$ the lens forms a real image of the Sun.

(c) For lens 2, of the planar convex type, we find

$$f = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{-40cm} \right) \right]^{-1} = 80cm.$$

(d) The image formed is real (since $f > 0$).

(e) Now for lens 3, of the meniscus convex type, we have

$$f = \left[(1.5-1)\left(\frac{1}{40cm} - \frac{1}{60cm} \right) \right]^{-1} = 240cm = 2.4\ m.$$

(f) The image formed is real (since $f > 0$).

(g) For lens 4, of the bi-concave type, the focal length is

$$f = \left[(1.5-1)\left(\frac{1}{-40cm} - \frac{1}{40cm} \right) \right]^{-1} = -40cm.$$

(h) The image formed is virtual (since $f < 0$).

(i) For lens 5 (plane-concave), we have

$$f = \left[(1.5-1)\left(\frac{1}{\infty} - \frac{1}{40cm} \right) \right]^{-1} = -80cm.$$

(j) The image formed is virtual (since $f < 0$).

(k) For lens 6 (meniscus concave),

$$f = \left[(1.5-1) \left(\frac{1}{60\,\text{cm}} - \frac{1}{40\,\text{cm}} \right) \right]^{-1} = -240\,\text{cm} = -2.4 \text{ m.}$$

(l) The image formed is virtual (since $f < 0$).

101. (a) The first image is figured using Eq. 34-8, with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 8/5$.

$$\frac{1}{p} + \frac{8}{5i} = \frac{1.6 - 1}{r}$$

For a "flat lens" $r = \infty$, so we obtain

$$i = -8p/5 = -64/5$$

(with the unit cm understood) for that object at $p = 10$ cm. Relative to the second surface, this image is at a distance of $3 + 64/5 = 79/5$. This serves as an object in order to find the final image, using Eq. 34-8 again (and $r = \infty$) but with $n_1 = 8/5$ and $n_2 = 4/3$.

$$\frac{8}{5p'} + \frac{4}{3i'} = 0$$

which produces (for $p' = 79/5$)

$$i' = -5p/6 = -79/6 \approx -13.2.$$

This means the observer appears $13.2 + 6.8 = 20$ cm from the fish.

(b) It is straightforward to "reverse" the above reasoning, the result being that the final fish-image is 7.0 cm to the right of the air-wall interface, and thus 15 cm from the observer.

102. (a) There are three images. Two are formed by single reflections from each of the mirrors and the third is formed by successive reflections from both mirrors. The positions of the images are shown on the two diagrams that follow.
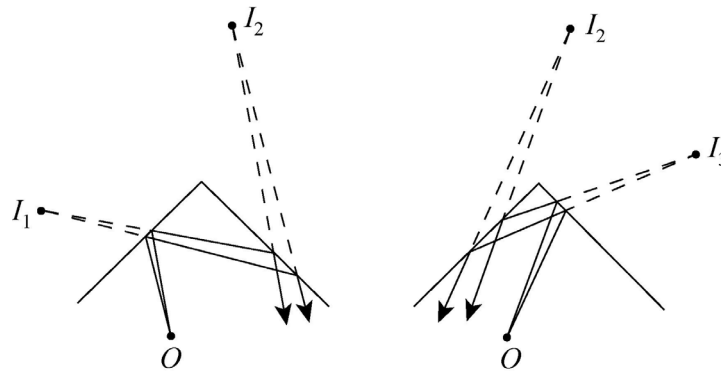
The diagram on the left shows the image I_1, formed by reflections from the left-hand mirror. It is the same distance behind the mirror as the object O is in front, and lies on the line perpendicular to the mirror and through the object. Image I_2 is formed by light that is reflected from both mirrors. We may consider I_2 to be the image of I_1 formed by the right-hand mirror, extended. I_2 is the same distance behind the line of the right-hand mirror as I_1 is in front and it is on the line that is perpendicular to the line of the mirror. The diagram on the right shows image I_3, formed by reflections from the right-hand mirror. It is the same distance behind the mirror as the object is in front, and lies on the line perpendicular to the mirror and through the object. As the diagram shows, light that is first reflected from the right-hand mirror and then from the left-hand mirror forms an image at I_2.

(b) For $\theta = 45°$, we have two images in the second mirror caused by the object and its "first" image, and from these one can construct two new images I and I' behind the first mirror plane. Extending the second mirror plane, we can find two further images of I and I' which are on equal sides of the extension of the first mirror plane. This circumstance implies there are no further images, since these final images are each other's "twins." We show this construction in the figure below. Summarizing, we find $1 + 2 + 2 + 2 = 7$ images in this case.

O

first image

I' I
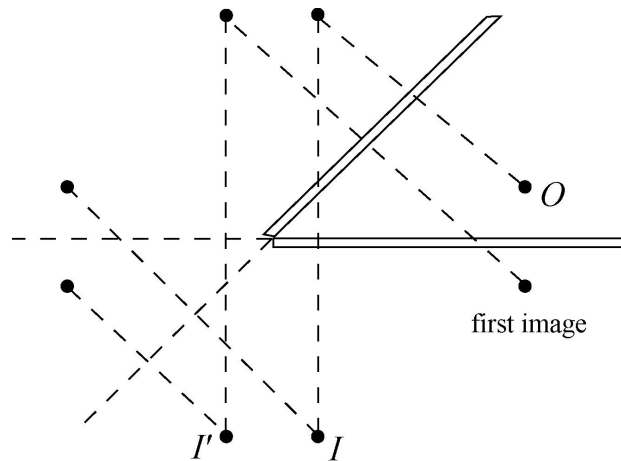
(c) For $\theta = 60°$, we have two images in the second mirror caused by the object and its "first" image, and from these one can construct two new images I and I' behind the first mirror plane. The images I and I' are each other's "twins" in the sense that they are each other's reflections about the extension of the second mirror plane; there are no further images. Summarizing, we find $1 + 2 + 2 = 5$ images in this case.

For $\theta = 120°$, we have two images I'_1 and I_2 behind the extension of the second mirror plane, caused by the object and its "first" image (which we refer to here as I_1). No further images can be constructed from I'_1 and I_2, since the method indicated above would place any further possibilities in front of the mirrors. This construction has the disadvantage of deemphasizing the actual ray-tracing, and thus any dependence on where the observer of these images is actually placing his or her eyes. It turns out in this case that the number of images that can be seen ranges from 1 to 3, depending on the locations of both the object and the observer.

(d) Thus, the smallest number of images that can be seen is 1. For example, if the observer's eye is collinear with I_1 and I'_1, then the observer can only see one image (I_1 and not the one behind it). Note that an observer who stands close to the second mirror would probably be able to see two images, I_1 and I_2.

(e) Similarly, the largest number would be 3. This happens if the observer moves further back from the vertex of the two mirrors. He or she should also be able to see the third image, I'_1, which is essentially the "twin" image formed from I_1 relative to the extension of the second mirror plane.

103. For a thin lens, $(1/p) + (1/i) = (1/f)$, where p is the object distance, i is the image distance, and f is the focal length. We solve for i:

$$i = \frac{fp}{p-f}.$$

Let $p = f + x$, where x is positive if the object is outside the focal point and negative if it is inside. Then,

$$i = \frac{f(f+x)}{x}.$$

Now let $i = f + x'$, where x' is positive if the image is outside the focal point and negative if it is inside. Then,

$$x' = i - f = \frac{f(f+x)}{x} - f = \frac{f^2}{x}$$

and $xx' = f^2$.

104. For an object in front of a thin lens, the object distance p and the image distance i are related by $(1/p) + (1/i) = (1/f)$, where f is the focal length of the lens. For the situation described by the problem, all quantities are positive, so the distance x between the object and image is $x = p + i$. We substitute $i = x - p$ into the thin lens equation and solve for x:

$$x = \frac{p^2}{p - f}.$$

To find the minimum value of x, we set $dx/dp = 0$ and solve for p. Since

$$\frac{dx}{dp} = \frac{p(p - 2f)}{(p - f)^2},$$

the result is $p = 2f$. The minimum distance is

$$x_{min} = \frac{p^2}{p - f} = \frac{(2f)^2}{2f - f} = 4f.$$

This is a minimum, rather than a maximum, since the image distance i becomes large without bound as the object approaches the focal point.

105. We place an object far away from the composite lens and find the image distance i. Since the image is at a focal point, $i = f$, where f equals the effective focal length of the composite. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens, $(1/p_1) + (1/i_1) = (1/f_1)$, where f_1 is the focal length of this lens and i_1 is the image distance for the image it forms. Since $p_1 = \infty$, $i_1 = f_1$. The thin lens equation, applied to the second lens, is $(1/p_2) + (1/i_2) = (1/f_2)$, where p_2 is the object distance, i_2 is the image distance, and f_2 is the focal length. If the thicknesses of the lenses can be ignored, the object distance for the second lens is $p_2 = -i_1$. The negative sign must be used since the image formed by the first lens is beyond the second lens if i_1 is positive. This means the object for the second lens is virtual and the object distance is negative. If i_1 is negative, the image formed by the first lens is in front of the second lens and p_2 is positive. In the thin lens equation, we replace p_2 with $-f_1$ and i_2 with f to obtain

$$-\frac{1}{f_1} + \frac{1}{f} = \frac{1}{f_2}$$

or

$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} = \frac{f_1 + f_2}{f_1 f_2}.$$

Thus,

$$f = \frac{f_1 f_2}{f_1 + f_2}.$$

106. (a) If the object distance is x, then the image distance is $D - x$ and the thin lens equation becomes

$$\frac{1}{x} + \frac{1}{D-x} = \frac{1}{f}.$$

We multiply each term in the equation by $fx(D - x)$ and obtain $x^2 - Dx + Df = 0$. Solving for x, we find that the two object distances for which images are formed on the screen are

$$x_1 = \frac{D - \sqrt{D(D-4f)}}{2} \quad \text{and} \quad x_2 = \frac{D + \sqrt{D(D-4f)}}{2}.$$

The distance between the two object positions is

$$d = x_2 - x_1 = \sqrt{D(D-4f)}.$$

(b) The ratio of the image sizes is the same as the ratio of the lateral magnifications. If the object is at $p = x_1$, the magnitude of the lateral magnification is

$$|m_1| = \frac{i_1}{p_1} = \frac{D - x_1}{x_1}.$$

Now $x_1 = \frac{1}{2}(D - d)$, where $d = \sqrt{D(D - f)}$, so

$$|m_1| = \frac{D - (D-d)/2}{(D-d)/2} = \frac{D+d}{D-d}.$$

Similarly, when the object is at x_2, the magnitude of the lateral magnification is

$$|m_2| = \frac{I_2}{p_2} = \frac{D - x_2}{x_2} = \frac{D - (D+d)/2}{(D+d)/2} = \frac{D-d}{D+d}.$$

The ratio of the magnifications is

$$\frac{m_2}{m_1} = \frac{(D-d)/(D+d)}{(D+d)/(D-d)} = \left(\frac{D-d}{D+d}\right)^2.$$

107. The sphere (of radius 0.35 m) is a convex mirror with focal length $f = -0.175$ m. We adopt the approximation that the rays are close enough to the central axis for Eq. 34-4 to be applicable.

(a) With $p = 1.0$ m, the equation $1/p + 1/i = 1/f$ yields $i = -0.15$ m, which means the image is 0.15 m from the front surface, appearing to be *inside* the sphere.

(b) The lateral magnification is $m = -i/p$ which yields $m = 0.15$. Therefore, the image distance is $(0.15)(2.0 \text{ m}) = 0.30$ m.

(c) Since $m > 0$, the image is upright, or not inverted (NI).

108. (a) We use Eq. 34-8 (and Fig. 34-11(b) is useful), with $n_1 = 1$ (using the rounded-off value for air) and $n_2 = 1.5$.

$$\frac{1}{p} + \frac{1.5}{i} = \frac{1.5 - 1}{r}$$

Using the sign convention for r stated in the paragraph following Eq. 34-8 (so that $r = +6.0$ cm), we obtain $i = -90$ cm for objects at $p = 10$ cm. Thus, the object and image are 80 cm apart.

(b) The image distance i is negative with increasing magnitude as p increases from very small values to some value p_0 at which point $i \rightarrow -\infty$. Since $1/(-\infty) = 0$, the above equation yields

$$\frac{1}{p_0} = \frac{1.5 - 1}{r} \Rightarrow p_0 = 2r.$$

Thus, the range for producing virtual images is $0 < p \le 12$ cm.

109. (a) In this case $i < 0$ so $i = -|i|$, and Eq. 34-9 becomes $1/f = 1/p - 1/|i|$. We differentiate this with respect to time (t) to obtain

$$\frac{d|i|}{dt} = \left(\frac{|i|}{p}\right)^2 \frac{dp}{dt}.$$

As the object is moved toward the lens, p is decreasing, so $dp/dt < 0$. Consequently, the above expression shows that $d|i|/dt < 0$; that is, the image moves in from infinity. The angular magnification $m_\theta = \theta'/\theta$ also increases as the following graph shows ("read" the graph from left to right since we are considering <u>decreasing</u> p from near the focal length to near 0). To obtain this graph of m_θ, we chose $f = 30$ cm and $h = 2$ cm.
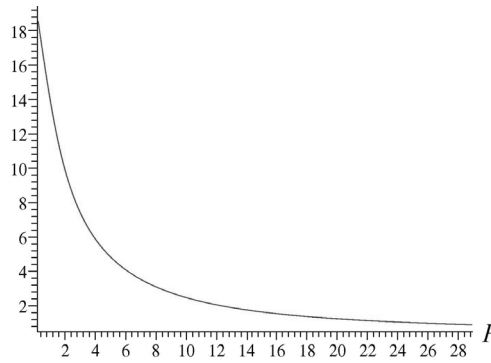
(b) When the image appears to be at the near point (that is, $|i| = P_n$), m_θ is at its maximum usable value. The textbook states in section 34-8 that it generally takes P_n to be equal to 25 cm (this value, too, was used in making the above graph).

(c) In this case,

$$p = if/(i - f) = |i|f/(|i| + f) = P_n f/(P_n + f).$$

If we use the small angle approximation, we have $\theta' \approx h'/|i|$ and $\theta \approx h/P_n$ (note: this approximation was <u>not</u> used in obtaining the graph, above). We therefore find $m_\theta \approx (h'/|i|)/(h/P_n)$ which (using Eq. 34-7 relating the ratio of heights to the ratio of distances) becomes

$$m_\theta \approx (h'/h)(P_n/|i|) = (|i|/p)(P_n/|i|) = (P_n/p) = [P_n/(P_n f/(P_n + f))] = \frac{P_n + f}{f}$$

which readily simplifies to the desired result.

(d) The linear magnification (Eq. 34-7) is given by $(h'/h) \approx m_\theta(|i|/P_n)$ (see the first in the chain of equalities, above). Once we set $|i| = P_n$ (see part (b)) then this shows the equality in the magnifications.

110. (a) Suppose one end of the object is a distance p from the mirror and the other end is a distance $p + L$. The position i_1 of the image of the first end is given by

$$\frac{1}{p} + \frac{1}{i_1} = \frac{1}{f}$$

where f is the focal length of the mirror. Thus,

$$i_1 = \frac{f_p}{p - f}.$$

The image of the other end is located at

$$i_2 = \frac{f(p + L)}{p + L - f},$$

so the length of the image is

$$L' = i_1 - i_2 = \frac{fp}{p - f} - \frac{f(p + L)}{p + L - f} = \frac{f^2 L}{(p - f)(p + L - f)}.$$

Since the object is short compared to $p - f$, we may neglect the L in the denominator and write

$$L' = L \left(\frac{f}{p - f} \right)^2.$$

(b) The lateral magnification is $m = -i/p$ and since $i = fp/(p - f)$, this can be written $m = -f/(p - f)$. The longitudinal magnification is

$$m' = \frac{L'}{L} = \left(\frac{f}{p - f} \right)^2 = m^2.$$

111. (a) In this case $m > +1$ and we know that lens 1 is converging (producing a virtual image), so that our result for focal length should be positive. Since $|P + i_1| = 20$ cm and $i_1 = -2p_1$, we find $p_1 = 20$ cm and $i_1 = -40$ cm. Substituting these into Eq. 34-9,

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1}$$

leads to $f_1 = +40$ cm, which is positive as we expected.

(b) The object distance is $p_1 = 20$ cm, as shown in part (a).

(c) In this case $0 < m < 1$ and we know that lens 2 is diverging (producing a virtual image), so that our result for focal length should be negative. Since $|p + i_2| = 20$ cm and $i_2 = -p_2/2$, we find $p_2 = 40$ cm and $i_2 = -20$ cm. Substituting these into Eq. 34-9 leads to $f_2 = -40$ cm, which is negative as we expected.

(d) $p_2 = 40$ cm, as shown in part (c).

112. The water is medium 1, so $n_1 = n_w$ which we simply write as n. The air is medium 2, for which $n_2 \approx 1$. We refer to points where the light rays strike the water surface as A (on the left side of Fig. 34-52) and B (on the right side of the picture). The point midway between A and B (the center point in the picture) is C. The penny P is directly below C, and the location of the "apparent" or Virtual penny is V. We note that the angle $\angle CVB$ (the same as $\angle CVA$) is equal to θ_2, and the angle $\angle CPB$ (the same as $\angle CPA$) is equal to θ_1. The triangles CVB and CPB share a common side, the horizontal distance from C to B (which we refer to as x). Therefore,

$$\tan \theta_2 = \frac{x}{d_a} \qquad \text{and} \qquad \tan \theta_1 = \frac{x}{d}.$$

Using the small angle approximation (so a ratio of tangents is nearly equal to a ratio of sines) and the law of refraction, we obtain

$$\frac{\tan \theta_2}{\tan \theta_1} \approx \frac{\sin \theta_2}{\sin \theta_1} \quad \Rightarrow \quad \frac{\frac{x}{d_a}}{\frac{x}{d}} \approx \frac{n_1}{n_2} \quad \Rightarrow \quad \frac{d}{d_a} \approx n$$

which yields the desired relation: $d_a = d/n$.

113. A converging lens has a positive-valued focal length, so $f_1 = +6$ cm, $f_2 = +3$ cm, and $f_3 = +3$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 etc), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = 9$ cm and $i_2 = 6$ cm. Our final results are as follows:

(a) $i_3 = +7.5$ cm.

(b) $m = -0.75$.

(c) The image is real (R).

(d) The image is inverted (I).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

114. A converging lens has a positive-valued focal length, so $f_1 = +6$ cm, $f_2 = +6$ cm, and $f_3 = +5$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 etc), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -3$ cm and $i_2 = 9$ cm. Our final results are as follows:

(a) $i_3 = +10$ cm.

(b) $m = +0.75$.

(c) The image is real (R).

(d) The image is not inverted (NI).

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

115. A converging lens has a positive-valued focal length, so $f_1 = +8$ cm, $f_2 = +6$ cm, and $f_3 = +6$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 etc), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = 24$ cm and $i_2 = -12$ cm. Our final results are as follows:

(a) $i_3 = +8.6$ cm.

(b) $m = +2.6$.

(c) The image is real (R).

(d) The image is not inverted (NI)

(e) It is on the opposite side of lens 3 from the object (which is expected for a real final image).

116. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = -6$ cm, $f_2 = +6$ cm, and $f_3 = +4$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 etc), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -2.4$ cm and $i_2 = 12$ cm. Our final results are as follows:

(a) $i_3 = -4.0$ cm.

(b) $m = -1.2$.

(c) The image is virtual (V).

(d) The image is inverted (I).

(e) It is on the same side as the object (relative to lens 3) as expected for a virtual image.

117. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = -8.0$ cm, $f_2 = -16$ cm, and $f_3 = +8.0$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 etc), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -4.0$ cm and $i_2 = -6.86$ cm. Our final results are as follows:

(a) $i_3 = +24.2$ cm.

(b) $m = -0.58$.

(c) The image is real (R).

(d) The image is inverted (I).

(e) It is on the opposite side of lens 3 from the object (as expected for a real image).

118. A converging lens has a positive-valued focal length, and a diverging lens has a negative-valued focal length. Therefore, $f_1 = +6$ cm, $f_2 = -4$ cm, and $f_3 = -12$ cm. We use Eq. 34-9 for each lens separately, "bridging the gap" between the results of one calculation and the next with $p_2 = d_{12} - i_1$ and $p_3 = d_{23} - i_2$. We also use Eq. 34-7 for each magnification (m_1 etc), and $m = m_1 m_2 m_3$ (a generalized version of Eq. 34-11) for the net magnification of the system. Our intermediate results for image distances are $i_1 = -12$ cm and $i_2 = -3.33$ cm. Our final results are as follows:

(a) $i_3 = -5.15$ cm ≈ -5.2 cm .

(b) $m = +0.285 \approx +0.29$.

(c) The image is virtual (V).

(d) The image is not inverted (NI).

(e) It is on the same side as the object (relative to lens 3) as expected for a virtual image.

119. (a) The discussion in the textbook of the refracting telescope (a subsection of §34-8) applies to the Newtonian arrangement if we replace the objective lens of Fig. 34-19 with an objective mirror (with the light incident on it from the right). This might suggest that the incident light would be blocked by the person's head in Fig. 34-19, which is why Newton added the mirror M' in his design (to move the head and eyepiece out of the way of the incoming light). The beauty of the idea of characterizing both lenses and mirrors by focal lengths is that it is easy, in a case like this, to simply carry over the results of the objective-lens telescope to the objective-mirror telescope, so long as we replace a positive f device with another positive f device. Thus, the converging lens serving as the objective of Fig. 34-19 must be replaced (as Newton has done in Fig. 34-54) with a concave mirror. With this change of language, the discussion in the textbook leading up to Eq. 34-15 applies equally as well to the Newtonian telescope: $m_\theta = -f_{ob}/f_{ey}$.

(b) A meter stick (held perpendicular to the line of sight) at a distance of 2000 m subtends an angle of

$$\theta_{stick} \approx \frac{1\ m}{2000\ m} = 0.0005\ rad.$$

multiplying this by the mirror focal length gives (16.8 m) (0.0005) = 8.4 mm for the size of the image.

(c) With $r = 10$ m, Eq. 34-3 gives $f_{ob} = 5$ m. Plugging this into (the absolute value of) Eq. 34-15 leads to $f_{ey} = 5/200 = 2.5$ cm.

120. Consider a single ray from the source to the mirror and let θ be the angle of incidence. The angle of reflection is also θ and the reflected ray makes an angle of 2θ with the incident ray.
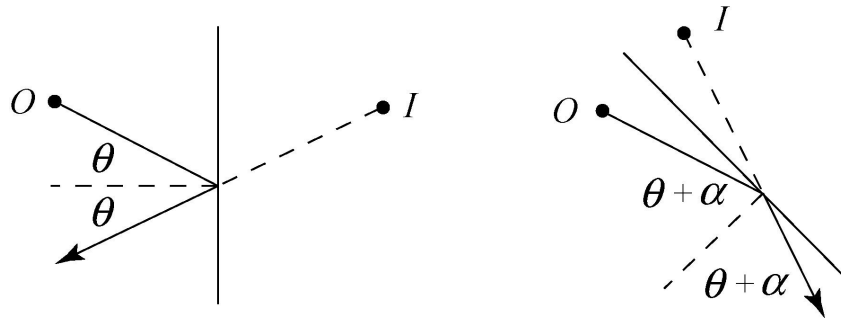
Now we rotate the mirror through the angle α so that the angle of incidence increases to $\theta + \alpha$. The reflected ray now makes an angle of $2(\theta + \alpha)$ with the incident ray. The reflected ray has been rotated through an angle of 2α. If the mirror is rotated so the angle of incidence is decreased by α, then the reflected ray makes an angle of $2(\theta - \alpha)$ with the incident ray. Again it has been rotated through 2α. The diagrams below show the situation for $\alpha = 45°$. The ray from the object to the mirror is the same in both cases and the reflected rays are 90° apart.

121. (a) If we let $p \rightarrow \infty$ in Eq. 34-8, we get $i = n_2 r / (n_2 - n_1)$. If we set $n_1 = 1$ (for air) and restrict n_2 so that $1 < n_2 < 2$, then this suggests that $i > 2r$ (so this image does form before the rays strike the opposite side of the sphere). We can still consider this as a sort of "virtual" object for the second imaging event, where this "virtual" object distance is $2r - i = (n - 2) r / (n - 1)$, where we have simplified the notation by writing $n_2 = n$. Putting this in for p in Eq. 34-8 and being careful with the sign convention for r in that equation, we arrive at the final image location: $i' = (0.5)(2 - n)r/(n - 1)$.

(b) The image is to the right of the right side of the sphere.

122. Setting $n_{air} = 1$, $n_{water} = n$, and $p = r/2$ in Eq. 34-8 (and being careful with the sign convention for r in that equation), we obtain $i = -r/(1 + n)$, or $|i| = r/(1 + n)$. Then we use similar triangles (where h is the size of the fish and h' is that of the "virtual fish") to set up the ratio

$$\frac{h'}{r - |i|} = \frac{h}{r/2} \ .$$

Using our previous result for $|i|$, this gives $h'/h = 2(1 - 1/(1 + n)) = 1.14$.

123. (a) Our first step is to form the image from the first lens. With $p_1 = 10$ cm and $f_1 = -15$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow i_1 = -6.0\,\text{cm.}$$

The corresponding magnification is $m_1 = -i_1/p_1 = 0.60$. This image serves the role of "object" for the second lens, with $p_2 = 12 + 6.0 = 18$ cm, and $f_2 = 12$ cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \Rightarrow i_2 = 36 \text{ cm}.$$

(b) The corresponding magnification is $m_2 = -i_2/p_2 = -2.0$, which results in a net magnification of $m = m_1 m_2 = -1.2$. The height of the final image is (in absolute value) $(1.2)(1.0 \text{ cm}) = 1.2$ cm.

(c) The fact that i_2 is positive means that the final image is real.

(d) The fact that m is negative means that the orientation of the final image is inverted with respect to the (original) object.

124. (a) Without the diverging lens (lens 2), the real image formed by the converging lens (lens 1) is located at a distance

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1} \right)^{-1} = \left(\frac{1}{20 \text{ cm}} - \frac{1}{40 \text{ cm}} \right)^{-1} = 40 \text{ cm}$$

to the right of lens 1. This image now serves as an object for lens 2, with $p_2 = -(40 \text{ cm} - 10 \text{ cm}) = -30 \text{ cm}$. So

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2} \right)^{-1} = \left(\frac{1}{-15 \text{ cm}} - \frac{1}{-30 \text{ cm}} \right)^{-1} = -30 \text{ cm}.$$

Thus, the image formed by lens 2 is located 30 cm to the left of lens 2.

(b) The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1.0 > 0$, so the image is not inverted.

(c) The image is virtual since $i_2 < 0$.

(d) The magnification is $m = (-i_1/p_1) \times (-i_2/p_2) = +1.0$, so the image has the same size as the object.

125. (a) For the image formed by the first lens

$$i_1 = \left(\frac{1}{f_1} - \frac{1}{p_1}\right)^{-1} = \left(\frac{1}{10\,\text{cm}} - \frac{1}{20\,\text{cm}}\right)^{-1} = 20\,\text{cm}.$$

For the subsequent image formed by the second lens $p_2 = 30$ cm $-$ 20 cm $=$ 10 cm, so

$$i_2 = \left(\frac{1}{f_2} - \frac{1}{p_2}\right)^{-1} = \left(\frac{1}{12.5\,\text{cm}} - \frac{1}{10\,\text{cm}}\right)^{-1} = -50\,\text{cm}.$$

Thus, the final image is 50 cm to the left of the second lens, which means that it coincides with the object.

(b) The magnification is

$$m = \left(\frac{i_1}{p_1}\right)\left(\frac{i_2}{p_2}\right) = \left(\frac{20\,\text{cm}}{20\,\text{cm}}\right)\left(\frac{-50\,\text{cm}}{10\,\text{cm}}\right) = -5.0,$$

which means that the final image is five times larger than the original object.

(c) The image is virtual since $i_2 < 0$.

(d) The image is inverted since $m < 0$.

126. (a) We solve Eq. 34-9 for the image distance i: $i = pf/(p - f)$. The lens is diverging, so its focal length is $f = -30$ cm. The object distance is $p = 20$ cm. Thus,

$$i = \frac{(20 \text{ cm})(-30 \text{ cm})}{(20 \text{ cm}) - (-30 \text{ cm})} = -12 \text{ cm.}$$

The negative sign indicates that the image is virtual and is on the same side of the lens as the object.

(b) The ray diagram, drawn to scale, is shown below.

lens
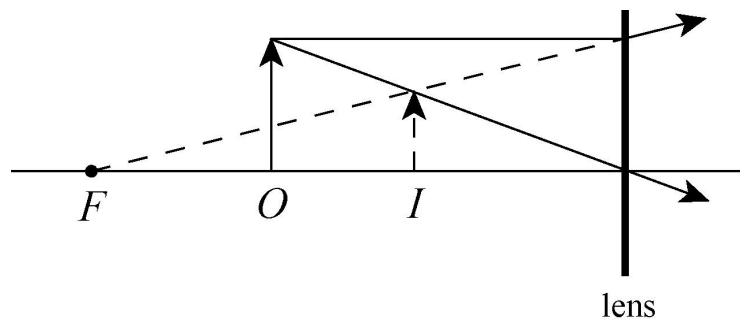
127. We set up an *xyz* coordinate system where the individual planes (*xy, yz, xz*) serve as the mirror surfaces. Suppose an incident ray of light *A* first strikes the mirror in the *xy* plane. If the unit vector denoting the direction of *A* is given by

$$\cos(\alpha)\hat{i} + \cos(\beta)\hat{j} + \cos(\gamma)\hat{k}$$

where α, β, γ are the angles *A* makes with the axes, then after reflection off the *xy* plane the unit vector becomes $\cos(\alpha)\hat{i} + \cos(\beta)\hat{j} - \cos(\gamma)\hat{k}$ (one way to rationalize this is to think of the reflection as causing the angle γ to become $\pi - \gamma$). Next suppose it strikes the mirror in the *xz* plane. The unit vector of the reflected ray is now $\cos(\alpha)\hat{i} - \cos(\beta)\hat{j} - \cos(\gamma)\hat{k}$. Finally as it reflects off the mirror in the *yz* plane α becomes $\pi - \alpha$, so the unit vector in the direction of the reflected ray is given by $-\cos(\alpha)\hat{i} - \cos(\beta)\hat{j} - \cos(\gamma)\hat{k}$, exactly reversed from *A*'s original direction. A further observation may be made: this argument would fail if the ray could strike any given surface twice and some consideration (perhaps an illustration) should convince the student that such an occurrence is not possible.

128. The fact that it is inverted implies $m < 0$. Therefore, with $m = -1/2$, we have $i = p/2$, which we substitute into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{2}{p} = \frac{1}{f}$$

$$\frac{3}{30.0} = \frac{1}{f}$$

with the unit cm understood. Consequently, we find $f = 30/3 = 10.0$ cm. The fact that $f > 0$ implies the mirror is concave.

129. Since $m = -2$ and $p = 4.00$ cm, then $i = 8.00$ cm (and is real). Eq. 34-9 is

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

and leads to $f = 2.67$ cm (which is positive, as it must be for a converging lens).

130. (a) The mirror has focal length $f = 12.0$ cm. With $m = +3$, we have $i = -3p$. We substitute this into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{-3p} = \frac{1}{12}$$

$$\frac{2}{3p} = \frac{1}{12}$$

with the unit cm understood. Consequently, we find $p = 2(12)/3 = 8.0$ cm.

(b) With $m = -3$, we have $i = +3p$, which we substitute into Eq. 34-4:

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{1}{3p} = \frac{1}{12}$$

$$\frac{4}{3p} = \frac{1}{12}$$

with the unit cm understood. Consequently, we find $p = 4(12)/3 = 16$ cm.

(c) With $m = -1/3$, we have $i = p/3$. Thus, Eq. 34-4 leads to

$$\frac{1}{p} + \frac{1}{i} = \frac{1}{f}$$

$$\frac{1}{p} + \frac{3}{p} = \frac{1}{12}$$

$$\frac{4}{p} = \frac{1}{12}$$

with the unit cm understood. Consequently, we find $p = 4(12) = 48$ cm.

131. (a) Since $m = +0.200$, we have $i = -0.2p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex (and that $f = -40.0$ cm).

(b) Substituting $i = -p/5$ into Eq. 34-4 produces

$$\frac{1}{p} - \frac{5}{p} = -\frac{4}{p} = \frac{1}{f}.$$

Therefore, we find $p = 160$ cm.

132. Since $0 < m < 1$, we conclude the lens is of the diverging type (so $f = -40$ cm). Thus, substituting $i = -3p/10$ into Eq. 34-9 produces

$$\frac{1}{p} - \frac{10}{3p} = -\frac{7}{3p} = \frac{1}{f}.$$

Therefore, we find $p = 93.3$ cm and $i = -28.0$ cm, or $|i| = 28.0$ cm.

133. (a) Our first step is to form the image from the first lens. With $p_1 = 3.00$ cm and $f_1 = +4.00$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow i_1 = -12.0 \text{cm.}$$

The corresponding magnification is $m_1 = -i_1/p_1 = 4$. This image serves the role of "object" for the second lens, with $p_2 = 8.00 + 12.0 = 20.0$ cm, and $f_2 = -4.00$ cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \Rightarrow i_2 = -3.33 \text{ cm .}$$

(b) The fact that i_2 is negative means that the final image is virtual (and therefore to the left of the second lens).

(c) The image is virtual.

(d) With $m_2 = -i_2/p_2 = 1/6$, the net magnification is $m = m_1 m_2 = 2/3 > 0$. The fact that m is positive means that the orientation of the final image is the same as the (original) object. Therefore, the image is not inverted.

134. (a) Our first step is to form the image from the first lens. With $p_1 = 4.00$ cm and $f_1 = -4.00$ cm, Eq. 34-9 leads to

$$\frac{1}{p_1} + \frac{1}{i_1} = \frac{1}{f_1} \Rightarrow i_1 = -2.00\,\text{cm}.$$

The corresponding magnification is $m_1 = -i_1/p_1 = 1/2$. This image serves the role of "object" for the second lens, with $p_2 = 10.0 + 2.00 = 12.0$ cm, and $f_2 = -4.00$ cm. Now, Eq. 34-9 leads to

$$\frac{1}{p_2} + \frac{1}{i_2} = \frac{1}{f_2} \Rightarrow i_2 = -3.00\,\text{cm},$$

or $|i_2| = 3.00$ cm .

(b) The fact that i_2 is negative means that the final image is virtual (and therefore to the left of the second lens).

(c) The image is virtual.

(d) With $m_2 = -i_2/p_2 = 1/4$, the net magnification is $m = m_1 m_2 = 1/8 > 0$. The fact that m is positive means that the orientation of the final image is the same as the (original) object. Therefore, the image is not inverted.

135. Of course, the shortest possible path between A and B is the straight line path which does not go to the mirror at all. In this problem, we are concerned with only those paths which do strike the mirror. The problem statement suggests that we turn our attention to the mirror-image point of A (call it A') and requests that we construct a proof without calculus. We can see that the length of any line segment AP drawn from A to the mirror (at point P on the mirror surface) is the same as the length of its "mirror segment" $A'P$ drawn from A' to that point P. Thus, the total length of the light path from A to P to B is the same as the total length of segments drawn from A' to P to B. Now, we dismissed (in the first sentence of this solution) the possibility of a straight line path directly from A to B because it does not strike the mirror. However, we *can* construct a straight line path from A' to B which does intersect the mirror surface! Any other pair of segments ($A'P$ and PB) would give greater total length than the straight path (with $A'P$ and PB collinear), so if the straight path $A'B$ obeys the law of reflection, then we have our proof. Now, since $A'P$ is the mirror-twin of AP, then they both approach the mirror surface with the same angle α (one from the front side and the other from the back side). And since $A'P$ is collinear with PB, then PB also makes the same angle α with respect to the mirror surface (by vertex angles). If AP and PB are each α degrees away from the front of the mirror, then they are each θ degrees (where θ is the complement of α) measured from the normal axis. Thus, the law of reflection is consistent with the concept of the shortest light path.

136. (a) Since a beam of parallel light will be focused at a distance f from the (converging) lens, then the shorter the focal length f the greater the ability for the lens to bend the light. A window pane is an example of a "lens" with $f = \infty$, yet it has essentially zero bending ability. Therefore, $P = 1/f$ is a reasonable definition.

(b) First we must consider the two-lens situation in the limit that d (their separation) becomes vanishingly small. We place an object far away from the composite lens and find the image distance i. Since the image is at a focal point $i = f$, the effective focal length of the composite can be determined in this way. The final image is produced by two lenses, with the image of the first lens being the object for the second. For the first lens we have $1/p_1 + 1/i_1 = 1/f_1$, where f_1 is the focal length of the first lens. Since $p_1 = \infty$, we find $i_1 = f_1$. The thin lens equation, applied to the second lens, gives

$$i_2 = i = p_2 f_2 / (p_2 - f_2),$$

where $p_2 = d - i_1 = -f_1$ in this situation. Therefore, i (thought of as f for the equivalent single lens [equivalent to the 2 lens system] as explained above) is equal to

$$-f_1 f_2 / (-f_1 - f_2) \quad \text{or} \quad \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} \; .$$

Next, using the definition for P, we readily get the desired result.

137. (a) Suppose that the lens is placed to the left of the mirror. The image formed by the converging lens is located at a distance

$$i = \left(\frac{1}{f} - \frac{1}{p}\right)^{-1} = \left(\frac{1}{0.50\,\mathrm{m}} - \frac{1}{1.0\,\mathrm{m}}\right)^{-1} = 1.0\,\mathrm{m}$$

to the right of the lens, or 2.0 m – 1.0 m = 1.0 m in front of the mirror. The image formed by the mirror for this real image is then at 1.0 m to the right of the the mirror, or 2.0 m + 1.0 m = 3.0 m to the right of the lens. This image then results in another image formed by the lens, located at a distance

$$i' = \left(\frac{1}{f} - \frac{1}{p'}\right)^{-1} = \left(\frac{1}{0.50\mathrm{m}} - \frac{1}{3.0\mathrm{m}}\right)^{-1} = 0.60\mathrm{m}$$

to the left of the lens (that is, 2.6 cm from the mirror).

(b) The lateral magnification is

$$m = \left(-\frac{i}{p}\right)\left(-\frac{i'}{p'}\right) = \left(-\frac{1.0\,\mathrm{m}}{1.0\,\mathrm{m}}\right)\left(-\frac{0.60\,\mathrm{m}}{3.0\,\mathrm{m}}\right) = +0.20 \ .$$

(c) The final image is real since $i' > 0$.

(d) The image is to the left of the lens.

(e) It also has the same orientation as the object since $m > 0$. Therefore, the image is not inverted.

138. (a) Since $m = +0.250$, we have $i = -0.25p$ which indicates that the image is virtual (as well as being diminished in size). We conclude from this that the mirror is convex and that $f < 0$; in fact, $f = -2.00$ cm. Substituting $i = -p/4$ into Eq. 34-4 produces

$$\frac{1}{p} - \frac{4}{p} = -\frac{3}{p} = \frac{1}{f}$$

Therefore, we find $p = 6.00$ cm and $i = -1.50$ cm, or $|i| = 1.50$ cm.

(b) The focal length is negative.

(c) As shown in (a), the image is virtual.

139. First, we note that — *relative to the water* — the index of refraction of the carbon tetrachloride should be thought of as $n = 1.46/1.33 = 1.1$ (this notation is chosen to be consistent with problem 15). Now, if the observer were in the water, directly above the 40 mm deep carbon tetrachloride layer, then the apparent depth of the penny as measured below the surface of the carbon tetrachloride is $d_a = 40$ mm/1.1 = 36.4 mm. This "apparent penny" serves as an "object" for the rays propagating upward through the 20 mm layer of water, where this "object" should be thought of as being 20 mm + 36.4 mm = 56.4 mm from the top surface. Using the result of problem 15 again, we find the perceived location of the penny, for a person at the normal viewing position above the water, to be 56.4 mm/1.33 = 42 mm below the water surface.

140. (a) We show the $\alpha = 0.500$ rad, $r = 12$ cm, $p = 20$ cm calculation in detail. The understood length unit is the centimeter:

The distance from the object to point x: $\quad d = p - r + x = 8 + x$

$$y = d \tan \alpha = 4.3704 + 0.54630x$$

From the solution of $x^2 + y^2 = r^2$ we get $x = 8.1398$.

$$\beta = \tan^{-1}(y/x) = 0.8253 \text{ rad}$$
$$\gamma = 2\beta - \alpha = 1.151 \text{ rad}$$

From the solution of $\tan(\gamma) = y/(x + i - r)$ we get $i = 7.799$. The other results are shown without the intermediate steps:

For $\alpha = 0.100$ rad, we get $i = 8.544$ cm; for $\alpha = 0.0100$ rad, we get $i = 8.571$ cm. Eq. 34-3 and Eq. 34-4 (the mirror equation) yield $i = 8.571$ cm.

(b) Here the results are: ($\alpha = 0.500$ rad, $i = -13.56$ cm), ($\alpha = 0.100$ rad, $i = -12.05$ cm), ($\alpha = 0.0100$ rad, $i = -12.00$ cm). The mirror equation gives $i = -12.00$ cm.