1. Comparing the light speeds in sapphire and diamond, we obtain

$$\Delta v = v_s - v_d = c\left(\frac{1}{n_s} - \frac{1}{n_d}\right)$$

$$= \left(2.998\times10^8 \text{ m/s}\right)\left(\frac{1}{1.77} - \frac{1}{2.42}\right) = 4.55\times10^7 \text{ m/s}.$$

2. (a) The frequency of yellow sodium light is

$$f = \frac{c}{\lambda} = \frac{2.998 \times 10^{8} \text{ m/s}}{589 \times 10^{-9} \text{ m}} = 5.09 \times 10^{14} \text{ Hz.}$$

(b) When traveling through the glass, its wavelength is

$$\lambda_n = \frac{\lambda}{n} = \frac{589 \text{ nm}}{1.52} = 388 \text{ nm.}$$

(c) The light speed when traveling through the glass is

$$v = f\,\lambda_n = \left(5.09\times10^{14}\,\text{Hz}\right)\left(388\times10^{-9}\,\text{m}\right) = 1.97\times10^{8}\text{ m/s}.$$

3. The index of refraction is found from Eq. 35-3:

$$n = \frac{c}{v} = \frac{2.998 \times 10^8 \text{ m/s}}{1.92 \times 10^8 \text{ m/s}} = 1.56.$$

4. Note that Snell's Law (the law of refraction) leads to $\theta_1 = \theta_2$ when $n_1 = n_2$. The graph indicates that $\theta_2 = 30°$ (which is what the problem gives as the value of $\theta_1$) occurs at $n_2$ = 1.5. Thus, $n_1 = 1.5$, and the speed with which light propagates in that medium is

$$v = \frac{c}{1.5} = 2.0 \times 10^8 \text{ m/s}.$$

5. The fact that wave $W_2$ reflects two additional times has no substantive effect on the calculations, since two reflections amount to a $2(\lambda/2) = \lambda$ phase difference, which is effectively not a phase difference at all. The substantive difference between $W_2$ and $W_1$ is the extra distance $2L$ traveled by $W_2$.

(a) For wave $W_2$ to be a half-wavelength "behind" wave $W_1$, we require $2L = \lambda/2$, or $L = \lambda/4 = 155$ nm using the wavelength value given in the problem.

(b) Destructive interference will again appear if $W_2$ is $\frac{3}{2}\lambda$ "behind" the other wave. In this case, $2L' = 3\lambda/2$, and the difference is

$$L' - L = \frac{3\lambda}{4} - \frac{\lambda}{4} = \frac{\lambda}{2} = 310\,\text{nm} .$$

6. In contrast to the initial conditions of problem 30, we now consider waves $W_2$ and $W_1$ with an initial effective phase difference (in wavelengths) equal to $\frac{1}{2}$, and seek positions of the sliver which cause the wave to constructively interfere (which corresponds to an integer-valued phase difference in wavelengths). Thus, the extra distance $2L$ traveled by $W_2$ must amount to $\frac{1}{2}\lambda, \frac{3}{2}\lambda$, and so on. We may write this requirement succinctly as

$$L = \frac{2m+1}{4}\lambda \quad \text{where } m = 0, 1, 2, \ldots .$$

(a) Thus, the smallest value of $L/\lambda$ that results in the final waves being exactly in phase is when $m$ =0, which gives $L/\lambda = 1/4 = 0.25$.

(b) The second smallest value of $L/\lambda$ that results in the final waves being exactly in phase is when $m$ =1, which gives $L/\lambda = 3/4 = 0.75$.

(c) The third smallest value of $L/\lambda$ that results in the final waves being exactly in phase is when $m$ =2, which gives $L/\lambda = 5/4 = 1.25$.

7. (a) We take the phases of both waves to be zero at the front surfaces of the layers. The phase of the first wave at the back surface of the glass is given by $\phi_1 = k_1L - \omega t$, where $k_1$ ($= 2\pi/\lambda_1$) is the angular wave number and $\lambda_1$ is the wavelength in glass. Similarly, the phase of the second wave at the back surface of the plastic is given by $\phi_2 = k_2L - \omega t$, where $k_2$ ($= 2\pi/\lambda_2$) is the angular wave number and $\lambda_2$ is the wavelength in plastic. The angular frequencies are the same since the waves have the same wavelength in air and the frequency of a wave does not change when the wave enters another medium. The phase difference is

$$\phi_1 - \phi_2 = (k_1 - k_2)L = 2\pi\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)L.$$

Now, $\lambda_1 = \lambda_{air}/n_1$, where $\lambda_{air}$ is the wavelength in air and $n_1$ is the index of refraction of the glass. Similarly, $\lambda_2 = \lambda_{air}/n_2$, where $n_2$ is the index of refraction of the plastic. This means that the phase difference is

$$\phi_1 - \phi_2 = (2\pi/\lambda_{air})\,(n_1 - n_2)L.$$

The value of $L$ that makes this 5.65 rad is

$$L = \frac{(\phi_1 - \phi_2)\lambda_{air}}{2\pi(n_1 - n_2)} = \frac{5.65(400\times10^{-9}\,\text{m})}{2\pi(1.60 - 1.50)} = 3.60\times10^{-6}\,\text{m}.$$

(b) 5.65 rad is less than $2\pi$ rad = 6.28 rad, the phase difference for completely constructive interference, and greater than $\pi$ rad (= 3.14 rad), the phase difference for completely destructive interference. The interference is, therefore, intermediate, neither completely constructive nor completely destructive. It is, however, closer to completely constructive than to completely destructive.

8. (a) The time $t_2$ it takes for pulse 2 to travel through the plastic is

$$t_2 = \frac{L}{c/1.55} + \frac{L}{c/1.70} + \frac{L}{c/1.60} + \frac{L}{c/1.45} = \frac{6.30L}{c}.$$

Similarly for pulse 1:

$$t_1 = \frac{2L}{c/1.59} + \frac{L}{c/1.65} + \frac{L}{c/1.50} = \frac{6.33L}{c}.$$

Thus, pulse 2 travels through the plastic in less time.

(b) The time difference (as a multiple of $L/c$) is

$$\Delta t = t_2 - t_1 = \frac{6.33L}{c} - \frac{6.30L}{c} = \frac{0.03L}{c}.$$

Thus, the multiple is 0.03.

9. (a) We wish to set Eq. 35-11 equal to $1/2$, since a half-wavelength phase difference is equivalent to a π radians difference. Thus,

$$L_{\min} = \frac{\lambda}{2(n_2 - n_1)} = \frac{620\,\text{nm}}{2(1.65-1.45)} = 1550\,\text{nm} = 1.55\ \mu\text{m}.$$

(b) Since a phase difference of $\frac{3}{2}$ (wavelengths) is effectively the same as what we required in part (a), then

$$L = \frac{3\lambda}{2(n_2 - n_1)} = 3L_{\min} = 3(1.55\mu\text{m}) = 4.65\,\mu\text{m}.$$

10. (a) The exiting angle is 50º, the same as the incident angle, due to what one might call the "transitive" nature of Snell's law: $n_1 \sin\theta_1 = n_2 \sin\theta_2 = n_3 \sin\theta_3 = \ldots$

(b) Due to the fact that the speed (in a certain medium) is $c/n$ (where $n$ is that medium's index of refraction) and that speed is distance divided by time (while it's constant), we find

$$t = nL/c = (1.45)(25 \times 10^{-19}\ \text{m})/(3.0 \times 10^{8}\ \text{m/s}) = 1.4 \times 10^{-13}\ \text{s} = 0.14\ \text{ps}.$$

11. (a) Eq. 35-11 (in absolute value) yields

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(8.50\times10^{-6}\,\mathrm{m})}{500\times10^{-9}\,\mathrm{m}}(1.60-1.50) = 1.70.$$

(b) Similarly,

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(8.50\times10^{-6}\,\mathrm{m})}{500\times10^{-9}\,\mathrm{m}}(1.72-1.62) = 1.70.$$

(c) In this case, we obtain

$$\frac{L}{\lambda}|n_2 - n_1| = \frac{(3.25\times10^{-6}\,\mathrm{m})}{500\times10^{-9}\,\mathrm{m}}(1.79-1.59) = 1.30.$$

(d) Since their phase differences were identical, the brightness should be the same for (a) and (b). Now, the phase difference in (c) differs from an integer by 0.30, which is also true for (a) and (b). Thus, their effective phase differences are equal, and the brightness in case (c) should be the same as that in (a) and (b).

12. (a) We note that ray 1 travels an extra distance $4L$ more than ray 2. To get the least possible $L$ which will result in destructive interference, we set this extra distance equal to half of a wavelength:

$$4L = \tfrac{1}{2}\lambda \quad \Rightarrow \quad L = \frac{\lambda}{8} = 52.50 \text{ nm} .$$

(b) The next case occurs when that extra distance is set equal to $\tfrac{3}{2}\lambda$. The result is

$$L = \frac{3\lambda}{8} = 157.5 \text{ nm} .$$

13. (a) We choose a horizontal $x$ axis with its origin at the left edge of the plastic. Between $x = 0$ and $x = L_2$ the phase difference is that given by Eq. 35-11 (with $L$ in that equation replaced with $L_2$). Between $x = L_2$ and $x = L_1$ the phase difference is given by an expression similar to Eq. 35-11 but with $L$ replaced with $L_1 - L_2$ and $n_2$ replaced with 1 (since the top ray in Fig. 35-36 is now traveling through air, which has index of refraction approximately equal to 1). Thus, combining these phase differences and letting all lengths be in $\mu$m (so $\lambda = 0.600$), we have

$$\frac{L_2}{\lambda}(n_2 - n_1) + \frac{L_1 - L_2}{\lambda}(1 - n_1) = \frac{3.50}{0.600}(1.60 - 1.40) + \frac{4.00 - 3.50}{0.600}(1 - 1.40) = 0.833.$$

(b) Since the answer in part (a) is closer to an integer than to a half-integer, the interference is more nearly constructive than destructive.

14. (a) For the maximum adjacent to the central one, we set $m = 1$ in Eq. 35-14 and obtain

$$\theta_1 = \sin^{-1}\left(\frac{m\lambda}{d}\right)\Bigg|_{m=1} = \sin^{-1}\left[\frac{(1)(\lambda)}{100\lambda}\right] = 0.010\,\text{rad}.$$

(b) Since $y_1 = D \tan \theta_1$ (see Fig. 35-10(a)), we obtain

$$y_1 = (500 \text{ mm}) \tan (0.010 \text{ rad}) = 5.0 \text{ mm}.$$

The separation is $\Delta y = y_1 - y_0 = y_1 - 0 = 5.0$ mm.

15. The angular positions of the maxima of a two-slit interference pattern are given by $d\sin\theta = m\lambda$, where $d$ is the slit separation, $\lambda$ is the wavelength, and $m$ is an integer. If $\theta$ is small, sin $\theta$ may be approximated by $\theta$ in radians. Then, $\theta = m\lambda/d$ to good approximation. The angular separation of two adjacent maxima is $\Delta\theta = \lambda/d$. Let $\lambda'$ be the wavelength for which the angular separation is greater by10.0%. Then, $1.10\lambda/d = \lambda'/d$. or

$$\lambda' = 1.10\lambda = 1.10(589 \text{ nm}) = 648 \text{ nm}.$$

16. (a) We use Eq. 35-14 with $m = 3$:

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left[\frac{2\left(550\times10^{-9}\,\mathrm{m}\right)}{7.70\times10^{-6}\,\mathrm{m}}\right] = 0.216\,\mathrm{rad}.$$

(b) $\theta = (0.216)\,(180°/\pi) = 12.4°$.

17. Interference maxima occur at angles $\theta$ such that $d \sin\theta = m\lambda$, where $m$ is an integer. Since $d = 2.0$ m and $\lambda = 0.50$ m, this means that $\sin\theta = 0.25m$. We want all values of $m$ (positive and negative) for which $|0.25m| \le 1$. These are –4, –3, –2, –1, 0, +1, +2, +3, and +4. For each of these except –4 and +4, there are two different values for $\theta$. A single value of $\theta$ (–90°) is associated with $m = -4$ and a single value (+90°) is associated with $m = +4$. There are sixteen different angles in all and, therefore, sixteen maxima.

18. (a) The phase difference (in wavelengths) is

$$\phi = d\sin\theta/\lambda = (4.24\ \mu\text{m})\sin(20°)/(0.500\ \mu\text{m}) = 2.90\ .$$

(b) Multiplying this by $2\pi$ gives $\phi = 18.2$ rad.

(c) The result from part (a) is greater than $\frac{5}{2}$ (which would indicate the third minimum) and is less than 3 (which would correspond to the third side maximum).

19. The condition for a maximum in the two-slit interference pattern is $d \sin \theta = m\lambda$, where $d$ is the slit separation, $\lambda$ is the wavelength, $m$ is an integer, and $\theta$ is the angle made by the interfering rays with the forward direction. If $\theta$ is small, $\sin \theta$ may be approximated by $\theta$ in radians. Then, $\theta = m\lambda/d$, and the angular separation of adjacent maxima, one associated with the integer $m$ and the other associated with the integer $m + 1$, is given by $\Delta\theta = \lambda/d$. The separation on a screen a distance $D$ away is given by $\Delta y = D\,\Delta\theta = \lambda D/d$. Thus,

$$\Delta y = \frac{(500\times10^{-9}\,\text{m})(5.40\,\text{m})}{1.20\times10^{-3}\,\text{m}} = 2.25\times10^{-3}\,\text{m} = 2.25\ \text{mm}.$$

20. In Sample Problem 35-2, an experimentally useful relation is derived: $\Delta y = \lambda D/d$. Dividing both sides by $D$, this becomes $\Delta\theta = \lambda/d$ with $\theta$ in radians. In the steps that follow, however, we will end up with an expression where degrees may be directly used. Thus, in the present case,

$$\Delta\theta_n = \frac{\lambda_n}{d} = \frac{\lambda}{nd} = \frac{\Delta\theta}{n} = \frac{0.20°}{1.33} = 0.15°.$$

21. The maxima of a two-slit interference pattern are at angles $\theta$ given by $d \sin \theta = m\lambda$, where $d$ is the slit separation, $\lambda$ is the wavelength, and $m$ is an integer. If $\theta$ is small, $\sin \theta$ may be replaced by $\theta$ in radians. Then, $d\theta = m\lambda$. The angular separation of two maxima associated with different wavelengths but the same value of $m$ is $\Delta\theta = (m/d)(\lambda_2 - \lambda_1)$, and their separation on a screen a distance $D$ away is

$$\Delta y = D \tan \Delta\theta \approx D\,\Delta\theta = \left[\frac{mD}{d}\right](\lambda_2 - \lambda_1)$$

$$= \left[\frac{3(1.0\,\text{m})}{5.0\times10^{-3}\,\text{m}}\right]\left(600\times10^{-9}\,\text{m} - 480\times10^{-9}\,\text{m}\right) = 7.2\times10^{-5}\,\text{m}.$$

The small angle approximation $\tan \Delta\theta \approx \Delta\theta$ (in radians) is made.

22. (a) We use Eq. 35-14 to find $d$:

$$d \sin\theta = m\lambda \quad \Rightarrow \quad d = (4)(450 \text{ nm})/\sin(90°) = 1800 \text{ nm} .$$

For the third order spectrum, the wavelength that corresponds to $\theta = 90°$ is

$$\lambda = d \sin(90°)/3 = 600 \text{ nm} .$$

Any wavelength greater than this will not be seen. Thus, $600 \text{ nm} < \theta \leq 700 \text{ nm}$ are absent.

(b) The slit separation $d$ needs to be decreased.

(c) In this case, the 400 nm wavelength in the $m = 4$ diffraction is to occur at 90°. Thus

$$d_{\text{new}} \sin\theta = m\lambda \quad \Rightarrow \quad d_{\text{new}} = (4)(400 \text{ nm})/\sin(90°) = 1600 \text{ nm} .$$

This represents a change of $|\Delta d| = d - d_{\text{new}} = 200 \text{ nm} = 0.20\ \mu\text{m}$.

23. Initially, source $A$ leads source $B$ by 90°, which is equivalent to $1/4$ wavelength. However, source $A$ also lags behind source $B$ since $r_A$ is longer than $r_B$ by 100 m, which is $100\,\text{m}/400\,\text{m} = 1/4$ wavelength. So the net phase difference between $A$ and $B$ at the detector is zero.

24. Imagine a $y$ axis midway between the two sources in the figure. Thirty points of destructive interference (to be considered in the $xy$ plane of the figure) implies there are $7+1+7=15$ on each side of the $y$ axis. There is no point of destructive interference on the $y$ axis itself since the sources are in phase and any point on the $y$ axis must therefore correspond to a zero phase difference (and corresponds to $\theta = 0$ in Eq. 35-14). In other words, there are 7 "dark" points in the first quadrant, one along the $+x$ axis, and 7 in the fourth quadrant, constituting the 15 dark points on the right-hand side of the $y$ axis. Since the $y$ axis corresponds to a minimum phase difference, we can count (say, in the first quadrant) the $m$ values for the destructive interference (in the sense of Eq. 35-16) beginning with the one closest to the $y$ axis and going clockwise until we reach the $x$ axis (at any point beyond $S_2$). This leads us to assign $m = 7$ (in the sense of Eq. 35-16) to the point on the $x$ axis itself (where the path difference for waves coming from the sources is simply equal to the separation of the sources, $d$); this would correspond to $\theta = 90°$ in Eq. 35-16. Thus,

$$d = \left(7 + \tfrac{1}{2}\right)\lambda = 7.5\,\lambda \;\Rightarrow\; \frac{d}{\lambda} = 7.5\,.$$

25. Let the distance in question be $x$. The path difference (between rays originating from $S_1$ and $S_2$ and arriving at points on the $x > 0$ axis) is

$$\sqrt{d^2 + x^2} - x = \left(m + \frac{1}{2}\right)\lambda,$$

where we are requiring destructive interference (half-integer wavelength phase differences) and $m = 0, 1, 2, \cdots$. After some algebraic steps, we solve for the distance in terms of $m$:

$$x = \frac{d^2}{(2m+1)\lambda} - \frac{(2m+1)\lambda}{4}.$$

To obtain the largest value of $x$, we set $m = 0$:

$$x_0 = \frac{d^2}{\lambda} - \frac{\lambda}{4} = \frac{(3.00\lambda)^2}{\lambda} - \frac{\lambda}{4} = 8.75\lambda = 8.75(900 \text{ nm}) = 7.88 \times 10^3 \text{ nm}$$
$$= 7.88 \mu\text{m}.$$

26. (a) We note that, just as in the usual discussion of the double slit pattern, the $x = 0$ point on the screen (where that vertical line of length $D$ in the picture intersects the screen) is a bright spot with phase difference equal to zero (it would be the middle fringe in the usual double slit pattern). We are not considering $x < 0$ values here, so that negative phase differences are not relevant (and if we did wish to consider $x < 0$ values, we could limit our discussion to absolute values of the phase difference, so that – again – negative phase differences do not enter it). Thus, the $x = 0$ point is the one with the minimum phase difference.

(b) As noted in part (a), the phase difference $\phi = 0$ at $x = 0$.

(c) The path length difference is greatest at the rightmost "edge" of the screen (which is assumed to go on forever), so $\phi$ is maximum at $x = \infty$.

(d) In considering $x = \infty$, we can treat the rays from the sources as if they are essentially horizontal. In this way, we see that the difference between the path lengths is simply the distance ($2d$) between the sources. The problem specifies $2d = 6.00\,\lambda$, or $2d/\lambda = 6.00$ .

(e) Using the Pythagorean theorem, we have

$$\phi = \frac{\sqrt{D^2 + (x + d)^2}}{\lambda} - \frac{\sqrt{D^2 + (x - d)^2}}{\lambda} = 1.71$$

where we have plugged in $D = 20\lambda$, $d = 3\lambda$ and $x = 6\lambda$. Thus, the phase difference at that point is 1.71 wavelengths.

(f) We note that the answer to part (e) is closer to $\frac{3}{2}$ (destructive interference) than to 2 (constructive interference), so that the point is "intermediate" but closer to a minimum than to a maximum.

27. Consider the two waves, one from each slit, that produce the seventh bright fringe in the absence of the mica. They are in phase at the slits and travel different distances to the seventh bright fringe, where they have a phase difference of $2\pi m = 14\pi$. Now a piece of mica with thickness $x$ is placed in front of one of the slits, and an additional phase difference between the waves develops. Specifically, their phases at the slits differ by

$$\frac{2\pi x}{\lambda_m} - \frac{2\pi x}{\lambda} = \frac{2\pi x}{\lambda}(n-1)$$

where $\lambda_m$ is the wavelength in the mica and $n$ is the index of refraction of the mica. The relationship $\lambda_m = \lambda/n$ is used to substitute for $\lambda_m$. Since the waves are now in phase at the screen,

$$\frac{2\pi x}{\lambda}(n-1) = 14\pi$$

or

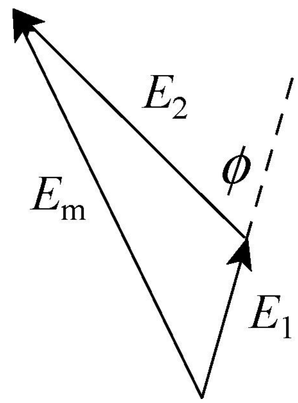
$$x = \frac{7\lambda}{n-1} = \frac{7(550\times10^{-9}\,\text{m})}{1.58-1} = 6.64\times10^{-6}\,\text{m}.$$

28. The problem asks for "the greatest value of $x$… exactly out of phase" which is to be interpreted as the value of $x$ where the curve shown in the figure passes through a phase value of $\pi$ radians. This happens as some point $P$ on the $x$ axis, which is, of course, a distance $x$ from the top source and (using Pythagoras' theorem) a distance $\sqrt{d^2+x^2}$ from the bottom source. The difference (in normal length units) is therefore $\sqrt{d^2+x^2} - x$, or (expressed in radians) is $\frac{2\pi}{\lambda}(\sqrt{d^2+x^2} - x)$. We note (looking at the leftmost point in the graph) that at $x = 0$, this latter quantity equals $6\pi$, which means $d = 3\lambda$. Using this value for $d$, we now must solve the condition

$$\frac{2\pi}{\lambda}(\sqrt{d^2+x^2} - x) = \pi.$$

Straightforward algebra then lead to $x = (35/4)\lambda$, and using $\lambda = 400$ nm we find $x = 3500$ nm, or 3.5 μm.

29. The phasor diagram is shown below.




Here $E_1$ = 1.00, $E_2$ = 2.00, and $\phi$ = 60°. The resultant amplitude $E_m$ is given by the trigonometric law of cosines:

$$E_m^2 = E_1^2 + E_2^2 - 2E_1E_2\cos(180°-\phi) .$$

Thus,

$$E_m = \sqrt{(1.00)^2 + (2.00)^2 - 2(1.00)(2.00)\cos 120°} = 2.65 .$$

30. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ (see Sample Problem 35-4) and add them as vectors:

$$y_h = 10\cos 0° + 8.0\cos 30° = 16.9$$
$$y_v = 10\sin 0° + 8.0\sin 30° = 4.0$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 17.4$$
$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 13.3° \ .$$

Thus, $y = y_1 + y_2 = y_R \sin(\omega t + \beta) = 17.4\sin(\omega t + 13.3°)$ . Quoting the answer to two significant figures, we have $y \approx 17\sin(\omega t + 13°)$.

31. In adding these with the phasor method (as opposed to, say, trig identities), we may set $t = 0$ (see Sample Problem 35-4) and add them as vectors:

$$y_h = 10\cos 0° + 15\cos 30° + 5.0\cos(-45°) = 26.5$$
$$y_v = 10\sin 0° + 15\sin 30° + 5.0\sin(-45°) = 4.0$$

so that

$$y_R = \sqrt{y_h^2 + y_v^2} = 26.8 \approx 27$$
$$\beta = \tan^{-1}\left(\frac{y_v}{y_h}\right) = 8.5°.$$

Thus, $y = y_1 + y_2 + y_3 = y_R \sin(\omega t + \beta) = 27\sin(\omega t + 8.5°)$.

32. (a) Referring to Figure 35-10(a) makes clear that

$$\theta = \tan^{-1}(y/D) = \tan^{-1}(0.205/4) = 2.93°.$$

Thus, the phase difference at point $P$ is $\phi = d\sin\theta/\lambda = 0.397$ wavelengths, which means it is between the central maximum (zero wavelength difference) and the first minimum ($\frac{1}{2}$ wavelength difference). Note that the above computation could have been simplified somewhat by avoiding the explicit use of the tangent and sine functions and making use of the small-angle approximation ($\tan\theta \approx \sin\theta$).

(b) From Eq. 35-22, we get (with $\phi = (0.397)(2\pi) = 2.495$ rad)

$$I = 4I_o(\cos(\phi/2))^2 = 0.404\ I_o$$

at point $P$ and

$$I_{cen} = 4I_o(\cos(0))^2 = 4\ I_o$$

at the center . Thus, $\frac{I}{I_{cen}} = \frac{0.404}{4} = 0.101$ .

33. With phasor techniques, this amounts to a vector addition problem $\vec{R} = \vec{A} + \vec{B} + \vec{C}$ where (in magnitude-angle notation) $\vec{A} = (10\angle 0°), \vec{B} = (5\angle 45°)$, and $\vec{C} = (5\angle -45°)$, where the magnitudes are understood to be in $\mu$V/m. We obtain the resultant (especially efficient on a vector-capable calculator in polar mode):

$$\vec{R} = (10\angle 0°) + (5\angle 45°) + (5\angle -45°) = (17.1\angle 0°)$$

which leads to

$$E_R = (17.1\,\mu\text{V/m})\sin(\omega t)$$

where $\omega = 2.0 \times 10^{14}$ rad/s.

34. (a) We can use phasor techniques or use trig identities. Here we show the latter approach. Since $\sin a + \sin(a+b) = 2\cos(b/2)\sin(a + b/2)$, we find

$$E_1 + E_2 = 2E_o \cos\left(\frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

where $E_o$ = 2.00 μV/m, ω = 1.26 × 10$^{15}$ rad/s, and $\phi$ = 39.6 rad. This shows that the electric field amplitude of the resultant wave is

$$E = 2E_o \cos(\phi/2) = 2.33 \ \mu\text{V/m} .$$

(b) Eq. 35-22 leads to

$$I = 4I_o(\cos(\phi/2))^2 = 1.35\, I_o$$

at point $P$, and

$$I_{cen} = 4I_o(\cos(0))^2 = 4\, I_o$$

at the center . Thus, $\frac{I}{I_{cen}} = \frac{1.35}{4} = 0.338$ .

(c) The phase difference ϕ (in wavelengths) is gotten from ϕ in radians by dividing by 2π. Thus, ϕ = 39.6/2π = 6.3 wavelengths. Thus, point $P$ is between the sixth side maximum (at which ϕ = 6 wavelengths) and the seventh minimum (at which ϕ = $6\frac{1}{2}$ wavelengths).

(d) The rate is given by ω = 1.26 × 10$^{15}$ rad/s.

(e) The angle between the phasors is ϕ = 39.6 rad = 2270° (which would look like about 110° when drawn in the usual way).

35. For constructive interference, we use Eq. 35-36: $2n_2 L = (m + 1/2)\lambda$ . For the smallest value of $L$, let $m$ = 0:

$$L_0 = \frac{\lambda/2}{2n_2} = \frac{624\text{nm}}{4(1.33)} = 117\text{nm} = 0.117\,\mu\text{m}.$$

(b) For the second smallest value, we set $m$ = 1 and obtain

$$L_1 = \frac{(1+1/2)\lambda}{2n_2} = \frac{3\lambda}{2n_2} = 3L_0 = 3(0.1173\,\mu\text{m}) = 0.352\,\mu\text{m}.$$

36. (a) On both sides of the soap is a medium with lower index (air) and we are examining the reflected light, so the condition for strong reflection is Eq. 35-36. With lengths in nm,

$$\lambda = \frac{2n_2L}{m+\frac{1}{2}} = \begin{cases} 3360 & \text{for } m=0 \\ 1120 & \text{for } m=1 \\ 672 & \text{for } m=2 \\ 480 & \text{for } m=3 \\ 373 & \text{for } m=4 \\ 305 & \text{for } m=5 \end{cases}$$

from which we see the latter *four* values are in the given range.

(b) We now turn to Eq. 35-37 and obtain

$$\lambda = \frac{2n_2L}{m} = \begin{cases} 1680 & \text{for } m=1 \\ 840 & \text{for } m=2 \\ 560 & \text{for } m=3 \\ 420 & \text{for } m=4 \\ 336 & \text{for } m=5 \end{cases}$$

from which we see the latter *three* values are in the given range.

37. Light reflected from the front surface of the coating suffers a phase change of $\pi$ rad while light reflected from the back surface does not change phase. If $L$ is the thickness of the coating, light reflected from the back surface travels a distance $2L$ farther than light reflected from the front surface. The difference in phase of the two waves is $2L(2\pi/\lambda_c) - \pi$, where $\lambda_c$ is the wavelength in the coating. If $\lambda$ is the wavelength in vacuum, then $\lambda_c = \lambda/n$, where $n$ is the index of refraction of the coating. Thus, the phase difference is $2nL(2\pi/\lambda) - \pi$. For fully constructive interference, this should be a multiple of $2\pi$. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) - \pi = 2m\pi$$

for $L$. Here $m$ is an integer. The solution is

$$L = \frac{(2m+1)\lambda}{4n} .$$

To find the smallest coating thickness, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{560\times10^{-9}\text{ m}}{4(2.00)} = 7.00\times10^{-8}\text{ m} .$$

38. (a) We are dealing with a thin film (material 2) in a situation where $n_1 > n_2 > n_3$, looking for strong *reflections*; the appropriate condition is the one expressed by Eq. 35-37. Therefore, with lengths in nm and $L = 500$ and $n_2 = 1.7$, we have

$$\lambda = \frac{2n_2L}{m} = \begin{cases} 1700 & \text{for } m = 1 \\ 850 & \text{for } m = 2 \\ 567 & \text{for } m = 3 \\ 425 & \text{for } m = 4 \end{cases}$$

from which we see the latter *two* values are in the given range. The longer wavelength ($m$=3) is $\lambda = 567$ nm.

(b) The shorter wavelength ($m$=4) is $\lambda = 425$ nm.

(c) We assume the temperature dependence of the refraction index is negligible. From the proportionality evident in the part (a) equation, longer $L$ means longer $\lambda$.

39. For complete destructive interference, we want the waves reflected from the front and back of the coating to differ in phase by an odd multiple of π rad. Each wave is incident on a medium of higher index of refraction from a medium of lower index, so both suffer phase changes of π rad on reflection. If $L$ is the thickness of the coating, the wave reflected from the back surface travels a distance $2L$ farther than the wave reflected from the front. The phase difference is $2L(2\pi/\lambda_c)$, where $\lambda_c$ is the wavelength in the coating. If $n$ is the index of refraction of the coating, $\lambda_c = \lambda/n$, where λ is the wavelength in vacuum, and the phase difference is $2nL(2\pi/\lambda)$. We solve

$$2nL\left(\frac{2\pi}{\lambda}\right) = (2m+1)\pi$$

for $L$. Here $m$ is an integer. The result is

$$L = \frac{(2m+1)\lambda}{4n}.$$

To find the least thickness for which destructive interference occurs, we take $m = 0$. Then,

$$L = \frac{\lambda}{4n} = \frac{600\times10^{-9}\,\text{m}}{4(1.25)} = 1.20\times10^{-7}\,\text{m}.$$

40. The situation is analogous to that treated in Sample Problem 35-6, in the sense that the incident light is in a low index medium, the thin film of acetone has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \tfrac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. This is the same as Eq. 35-36 which was developed for the opposite situation (constructive interference) regarding a thin film surrounded on both sides by air (a very different context than the one in this problem). By analogy, we expect Eq. 35-37 to apply in this problem to reflection *maxima*. A more careful analysis such as that given in §35-7 bears this out. Thus, using Eq. 35-37 with $n_2 = 1.25$ and $\lambda = 700$ nm yields

$$L = 0,\ 280\,\text{nm},\ 560\,\text{nm},\ 840\,\text{nm},\ 1120\,\text{nm}, \ldots$$

for the first several $m$ values. And the equation shown above (equivalent to Eq. 35-36) gives, with $\lambda = 600$ nm,

$$L = 120\,\text{nm}, 360\,\text{nm}, 600\,\text{nm}, 840\,\text{nm}, 1080\,\text{nm}, \ldots$$

for the first several $m$ values. The lowest number these lists have in common is $L = 840$ nm.

41. When a thin film of thickness $L$ and index of refraction $n_2$ is placed between materials 1 and 3 such that $n_1 > n_2$ and $n_3 > n_2$ where $n_1$ and $n_3$ are the indexes of refraction of the materials, the general condition for destructive interference for a thin film is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \quad \lambda = \frac{2Ln_2}{m}, \qquad m = 0, 1, 2, \ldots$$

where $\lambda$ is the wavelength of light as measured in air. Thus, we have, for $m = 1$

$$\lambda = 2Ln_2 = 2(200 \text{ nm})(1.40) = 560 \text{ nm} .$$

42. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, \ldots$$

Thus, we have,

$$\lambda = \begin{cases} 4Ln_2 = 4(285 \text{ nm})(1.60) = 1824 \text{ nm} \quad (m=0) \\ 4Ln_2/3 = 4(285 \text{ nm})(1.60)/3 = 608 \text{ nm} \quad (m=1) \end{cases}.$$

For the wavelength to be in the visible range, we choose *m*=1 with $\lambda = 608$ nm.

43. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$ , and the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,...$$

Thus, we have,

$$\lambda = \begin{cases} 4Ln_2 = 4(210 \text{ nm})(1.46) = 1226 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(210 \text{ nm})(1.46)/3 = 409 \text{ nm} \ \ (m=1) \end{cases}.$$

For the wavelength to be in the visible range, we choose $m$=1 with $\lambda = 409$ nm.

44. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,...$$

Thus, we have,

$$\lambda = \begin{cases} 4Ln_2 = 4(325 \text{ nm})(1.75) = 2275 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(325 \text{ nm})(1.75)/3 = 758 \text{ nm} \ \ (m=1) \\ 4Ln_2/5 = 4(325 \text{ nm})(1.75)/5 = 455 \text{ nm} \ \ (m=2) \end{cases} .$$

For the wavelength to be in the visible range, we choose *m*=2 with $\lambda = 455$ nm.

45. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$ , and the condition for destructive interference is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{2Ln_2}{m}, \quad m = 0,1,2,...$$

Thus, we have,

$$\lambda = \begin{cases} 2Ln_2 = 2(380 \text{ nm})(1.34) = 1018 \text{ nm} \ \ (m=1) \\ Ln_2 = (380 \text{ nm})(1.34) = 509 \text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose *m*=2 with $\lambda = 509$ nm.

46. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$ , and the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,...$$

Thus, we have,

$$\lambda = \begin{cases} 4Ln_2 = 4(415 \text{ nm})(1.59) = 2639 \text{ nm} \;\; (m=0) \\ 4Ln_2 / 3 = 4(415 \text{ nm})(1.59)/3 = 880 \text{ nm} \;\; (m=1) \\ 4Ln_2 / 5 = 4(415 \text{ nm})(1.59)/5 = 528 \text{ nm} \;\; (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose *m*=3 with $\lambda = 528$ nm.

47. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The third least thickness is ($m$=2)

$$L = \left(2 + \frac{1}{2}\right)\frac{612 \text{ nm}}{2(1.60)} = 478 \text{ nm}.$$

48. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$ , and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \ , \qquad m = 0, 1, 2, \ldots$$

The second least thickness is ($m$=1)

$$L = \left(1 + \frac{1}{2}\right)\frac{632 \text{ nm}}{2(1.40)} = 339 \text{ nm} \, .$$

49. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}\ , \qquad m = 0, 1, 2, \ldots$$

The third least thickness is ($m$=2)

$$L = \left(2 + \frac{1}{2}\right)\frac{382\ \text{nm}}{2(1.75)} = 273\ \text{nm}\ .$$

50. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$ , and the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \ , \qquad m = 0, 1, 2, \ldots$$

The second least thickness is ($m$=1)

$$L = \left(1 + \frac{1}{2}\right)\frac{482 \text{ nm}}{2(1.46)} = 248 \text{ nm} .$$

51. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$, and the condition for destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}\ , \qquad m = 0, 1, 2, \ldots$$

The second least thickness is ($m$=1)

$$L = \left(1 + \frac{1}{2}\right)\frac{342 \text{ nm}}{2(1.59)} = 161 \text{ nm}\ .$$

52. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$ , and the condition for constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \ , \qquad m = 0, 1, 2, ...$$

The second least thickness is ($m$=1)

$$L = \left(1 + \frac{1}{2}\right)\frac{587 \text{ nm}}{2(1.34)} = 329 \text{ nm}.$$

53. We solve Eq. 35-36 with $n_2$ = 1.33 and $\lambda$ = 600 nm for $m$ = 1, 2, 3,…:

$$L = 113 \text{ nm}, 338\,\text{nm}, 564\,\text{nm}, 789\,\text{nm}, \ldots$$

And, we similarly solve Eq. 35-37 with the same $n_2$ and $\lambda$ = 450 nm:

$$L = 0, 169\,\text{nm}, 338\,\text{nm}, 508\,\text{nm}, 677\,\text{nm}, \ldots$$

The lowest number these lists have in common is $L$ = 338 nm.

54. The situation is analogous to that treated in Sample Problem 35-6, in the sense that the incident light is in a low index medium, the thin film of oil has somewhat higher $n = n_2$, and the last layer (the glass plate) has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. With $\lambda = 500$ nm and $n_2 = 1.30$, the possible answers for $L$ are

$$L = 96\,\text{nm}, 288\,\text{nm}, 481\,\text{nm}, 673\,\text{nm}, 865\,\text{nm}, \ldots$$

And, with $\lambda = 700$ nm and the same value of $n_2$, the possible answers for $L$ are

$$L = 135\,\text{nm}, 404\,\text{nm}, 673\,\text{nm}, 942\,\text{nm}, \ldots$$

The lowest number these lists have in common is $L = 673$ nm.

55. The situation is analogous to that treated in Sample Problem 35-6, in the sense that the incident light is in a low index medium, the thin film has somewhat higher $n = n_2$, and the last layer has the highest refractive index. To see very little or no reflection, according to the Sample Problem, the condition

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \text{where} \quad m = 0, 1, 2, \ldots$$

must hold. The value of $L$ which corresponds to no reflection corresponds, reasonably enough, to the value which gives maximum transmission of light (into the highest index medium — which in this problem is the water).

(a) If $2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2}$ (Eq. 35-36) gives zero reflection in this type of system, then we might reasonably expect that its counterpart, Eq. 35-37, gives maximum reflection here. A more careful analysis such as that given in §35-7 bears this out. We disregard the $m = 0$ value (corresponding to $L = 0$) since there is *some* oil on the water. Thus, for $m = 1, 2, \ldots$, maximum reflection occurs for wavelengths

$$\lambda = \frac{2n_2 L}{m} = \frac{2(1.20)(460\,\text{nm})}{m} = 1104\,\text{nm}\ ,\ 552\,\text{nm},\ 368\,\text{nm}\ldots$$

We note that only the 552 nm wavelength falls within the visible light range.

(b) As remarked above, maximum transmission into the water occurs for wavelengths given by

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \Rightarrow \lambda = \frac{4n_2 L}{2m+1}$$

which yields $\lambda$ = 2208 nm, 736 nm, 442 nm … for the different values of $m$. We note that only the 442 nm wavelength (blue) is in the visible range, though we might expect some red contribution since the 736 nm is very close to the visible range.

56. For constructive interference (which is obtained for $\lambda$ = 600 nm) in this circumstance, we require

$$2L = \frac{k}{2}\lambda_n = \frac{k\lambda}{2n}$$

where $k$ = some positive odd integer and $n$ is the index of refraction of the thin film. Rearranging and plugging in $L$ = 272.7 nm and the wavelength value, this gives

$$\frac{k}{1.818} = n \; .$$

Since we expect $n > 1$, then $k = 1$ is ruled out. However, $k = 3$ seems reasonable, since it leads to $n = 1.65$, which is close to the "typical" values found in Table 34-1. Taking this to be the correct index of refraction for the thin film, we now consider the destructive interference part of the question. Now we have $2L = (\text{integer})\lambda_{dest}/n$. Thus, $\lambda_{dest}$ = (900 nm)/(integer). We note that setting the integer equal to 1 yields a $\lambda_{dest}$ value outside the range of the visible spectrum. A similar remark holds for setting the integer equal to 3. Thus, we set it equal to 2 and obtain $\lambda_{dest}$ = 450 nm.

57. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$ , and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1} \ , \quad m = 0, 1, 2, ...$$

Thus, we have,

$$\lambda = \begin{cases} 4Ln_2 = 4(285 \text{ nm})(1.60) = 1824 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(415 \text{ nm})(1.59)/3 = 608 \text{ nm} \ \ (m=1) \end{cases}.$$

For the wavelength to be in the visible range, we choose *m*=1 with $\lambda = 608$ nm.

58. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$ , and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{2Ln_2}{m}, \quad m = 0, 1, 2, \ldots$$

Thus, we have (with $m$ =1),

$$\lambda = 2Ln_2 = 2(200 \text{ nm})(1.40) = 560 \text{ nm} .$$

59. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$ , and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1} \ , \quad m = 0, 1, 2, \ldots$$

Thus, we have,

$$\lambda = \begin{cases} 4Ln_2 = 4(325 \text{ nm})(1.75) = 2275 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(415 \text{ nm})(1.59)/3 = 758 \text{ nm} \ \ (m=1) \\ 4Ln_2/5 = 4(415 \text{ nm})(1.59)/5 = 455 \text{ nm} \ \ (m=2) \end{cases} .$$

For the wavelength to be in the visible range, we choose *m*=2 with $\lambda = 455$ nm.

60. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$ , and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0,1,2,...$$

Thus, we have,

$$\lambda = \begin{cases} 4Ln_2 = 4(210 \text{ nm})(1.46) = 1226 \text{ nm} \ \ (m=0) \\ 4Ln_2/3 = 4(210 \text{ nm})(1.46)/3 = 409 \text{ nm} \ \ (m=1) \end{cases}.$$

For the wavelength to be in the visible range, we choose $m$=1 with $\lambda = 409$ nm.

61. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$ , and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{4Ln_2}{2m+1}, \quad m = 0, 1, 2, \ldots$$

Thus, we have,

$$\lambda = \begin{cases} 4Ln_2 = 4(415 \text{ nm})(1.59) = 2639 \text{ nm} \;\; (m = 0) \\ 4Ln_2/3 = 4(415 \text{ nm})(1.59)/3 = 880 \text{ nm} \;\; (m = 1) \\ 4Ln_2/5 = 4(415 \text{ nm})(1.59)/5 = 528 \text{ nm} \;\; (m = 2) \end{cases} .$$

For the wavelength to be in the visible range, we choose $m$=3 with $\lambda = 528$ nm.

62. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$ , and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = m\frac{\lambda}{n_2} \quad \Rightarrow \lambda = \frac{2Ln_2}{m}, \quad m = 0, 1, 2, \ldots$$

Thus, we have,

$$\lambda = \begin{cases} 2Ln_2 = 2(380 \text{ nm})(1.34) = 1018 \text{ nm} \ \ (m=1) \\ Ln_2 = (380 \text{ nm})(1.34) = 509 \text{ nm} \ \ (m=2) \end{cases}.$$

For the wavelength to be in the visible range, we choose $m$=2 with $\lambda = 509$ nm.

63. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$ , and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \ , \qquad m = 0,1,2,...$$

The second least thickness is (*m*=1)

$$L = \left(1 + \frac{1}{2}\right)\frac{632 \text{ nm}}{2(1.40)} = 339 \text{ nm} .$$

64. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$, and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \quad m = 0, 1, 2, \ldots$$

The third least thickness is ($m$=2)

$$L = \left(2 + \frac{1}{2}\right)\frac{612 \text{ nm}}{2(1.60)} = 478 \text{ nm}.$$

65. In this setup, we have $n_2 > n_1$ and $n_2 < n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \qquad m = 0, 1, 2, \ldots$$

The second least thickness is ($m$=1)

$$L = \left(1 + \frac{1}{2}\right)\frac{482 \text{ nm}}{2(1.46)} = 248 \text{ nm}.$$

66. In this setup, we have $n_2 > n_1$ and $n_2 > n_3$ , and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \ , \qquad m = 0,1,2,...$$

The third least thickness is ($m$=2)

$$L = \left(2 + \frac{1}{2}\right)\frac{382 \text{ nm}}{2(1.75)} = 273 \text{ nm} \, .$$

67. In this setup, we have $n_2 < n_1$ and $n_2 < n_3$ , and the condition for minimum transmission (maximum reflection) or destructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2} \; , \quad m = 0, 1, 2, \ldots$$

The second least thickness is ($m$=1)

$$L = \left(1 + \frac{1}{2}\right)\frac{587 \text{ nm}}{2(1.34)} = 329 \text{ nm}.$$

68. In this setup, we have $n_2 < n_1$ and $n_2 > n_3$, and the condition for maximum transmission (minimum reflection) or constructive interference is

$$2L = \left(m + \frac{1}{2}\right)\frac{\lambda}{n_2} \quad \Rightarrow L = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n_2}, \qquad m = 0, 1, 2, \ldots$$

The second least thickness is ($m$=1)

$$L = \left(1 + \frac{1}{2}\right)\frac{342 \text{ nm}}{2(1.59)} = 161 \text{ nm}.$$

69. Consider the interference of waves reflected from the top and bottom surfaces of the air film. The wave reflected from the upper surface does not change phase on reflection but the wave reflected from the bottom surface changes phase by $\pi$ rad. At a place where the thickness of the air film is $L$, the condition for fully constructive interference is $2L = \left(m + \frac{1}{2}\right)\lambda$ where $\lambda$ (= 683 nm) is the wavelength and $m$ is an integer. This is satisfied for $m$ = 140:

$$L = \frac{\left(m + \frac{1}{2}\right)\lambda}{2} = \frac{(140.5)\left(683 \times 10^{-9}\,\text{m}\right)}{2} = 4.80 \times 10^{-5}\,\text{m} = 0.048\,\text{mm}.$$

At the thin end of the air film, there is a bright fringe. It is associated with $m$ = 0. There are, therefore, 140 bright fringes in all.

70. By the condition $m\lambda = 2y$ where $y$ is the thickness of the air-film between the plates directly underneath the middle of a dark band), the edge of the plates (the edge where they are not touching) are $y = 8\lambda/2 = 2400$ nm apart (where we have assumed that the *middle* of the ninth dark band is at the edge). Increasing that to $y' = 3000$ nm would correspond to $m' = 2y'/\lambda = 10$ (counted as the eleventh dark band, since the first one corresponds to $m = 0$). There are thus 11 dark fringes along the top plate.

71. Assume the wedge-shaped film is in air, so the wave reflected from one surface undergoes a phase change of π rad while the wave reflected from the other surface does not. At a place where the film thickness is $L$, the condition for fully constructive interference is $2nL = \left(m + \frac{1}{2}\right)\lambda$, where $n$ is the index of refraction of the film, λ is the wavelength in vacuum, and $m$ is an integer. The ends of the film are bright. Suppose the end where the film is narrow has thickness $L_1$ and the bright fringe there corresponds to $m = m_1$. Suppose the end where the film is thick has thickness $L_2$ and the bright fringe there corresponds to $m = m_2$. Since there are ten bright fringes, $m_2 = m_1 + 9$. Subtract $2nL_1 = \left(m_1 + \frac{1}{2}\right)\lambda$ from $2nL_2 = \left(m_1 + 9 + \frac{1}{2}\right)\lambda$ to obtain $2n\,\Delta L = 9\lambda$, where $\Delta L = L_2 - L_1$ is the change in the film thickness over its length. Thus,

$$\Delta L = \frac{9\lambda}{2n} = \frac{9\left(630\times10^{-9}\ \text{m}\right)}{2(1.50)} = 1.89\times10^{-6}\ \text{m}.$$

72. We apply Eq. 35-27 to both scenarios: $m$ = 4001 and $n_2 = n_{air}$, and $m$ = 4000 and $n_2 = n_{vacuum}$ = 1.00000:

$$2L = (4001)\frac{\lambda}{n_{air}} \quad \text{and} \quad 2L = (4000)\frac{\lambda}{1.00000}.$$

Since the $2L$ factor is the same in both cases, we set the right hand sides of these expressions equal to each other and cancel the wavelength. Finally, we obtain

$$n_{air} = (1.00000)\frac{4001}{4000} = 1.00025.$$

We remark that this same result can be obtained starting with Eq. 35-43 (which is developed in the textbook for a somewhat different situation) and using Eq. 35-42 to eliminate the $2L/\lambda$ term.

73. Using the relations of §35-7, we find that the (vertical) change between the center of one dark band and the next is

$$\Delta y = \lambda 2 = 2.5 \times 10^{-4}\,\text{mm}.$$

Thus, with the (horizontal) separation of dark bands given by $\Delta x$ = 1.2 mm, we have

$$\theta \approx \tan\theta = \frac{\Delta y}{\Delta x} = 2.08 \times 10^{-4}\,\text{rad}.$$

Converting this angle into degrees, we arrive at $\theta$ = 0.012°.

74. (a) The third sentence of the problem implies $m_o = 9.5$ in $2\,d_o = m_o\lambda$ initially. Then, $\Delta t = 15$ s later, we have $m' = 9.0$ in $2d' = m'\lambda$. This means

$$|\Delta d| = d_o - d' = \tfrac{1}{2}(\,m_o\lambda - m'\lambda) = 155 \text{ nm} .$$

Thus, $|\Delta d|$ divided by $\Delta t$ gives 10.3 nm/s.

(b) In this case, $m_f = 6$ so that $d_o - d_f = \tfrac{1}{2}(m_o\lambda - m_f\lambda) = \tfrac{7}{4}\lambda = 1085$ nm = 1.09 μm.

75. Consider the interference pattern formed by waves reflected from the upper and lower surfaces of the air wedge. The wave reflected from the lower surface undergoes a π rad phase change while the wave reflected from the upper surface does not. At a place where the thickness of the wedge is *d*, the condition for a maximum in intensity is $2d = \left(m + \frac{1}{2}\right)\lambda$, where λ is the wavelength in air and *m* is an integer. Thus, $d = (2m + 1)\lambda/4$. As the geometry of Fig. 35-46 shows, $d = R - \sqrt{R^2 - r^2}$, where *R* is the radius of curvature of the lens and *r* is the radius of a Newton's ring. Thus, $\left(2m+1\right)\lambda/4 = R - \sqrt{R^2 - r^2}$. First, we rearrange the terms so the equation becomes

$$\sqrt{R^2 - r^2} = R - \frac{(2m+1)\lambda}{4}.$$

Next, we square both sides, rearrange to solve for $r^2$, then take the square root. We get

$$r = \sqrt{\frac{(2m+1)R\lambda}{2} - \frac{(2m+1)^2\lambda^2}{16}}.$$

If *R* is much larger than a wavelength, the first term dominates the second and

$$r = \sqrt{\frac{(2m+1)R\lambda}{2}}.$$

76. (a) We find $m$ from the last formula obtained in problem 75:

$$m = \frac{r^2}{R\lambda} - \frac{1}{2} = \frac{\left(10\times10^{-3}\,\text{m}\right)^2}{\left(5.0\,\text{m}\right)\left(589\times10^{-9}\,\text{m}\right)} - \frac{1}{2}$$

which (rounding down) yields $m = 33$. Since the first bright fringe corresponds to $m = 0$, $m = 33$ corresponds to the thirty-fourth bright fringe.

(b) We now replace $\lambda$ by $\lambda_n = \lambda/n_w$. Thus,

$$m_n = \frac{r^2}{R\lambda_n} - \frac{1}{2} = \frac{n_w r^2}{R\lambda} - \frac{1}{2} = \frac{(1.33)\left(10\times10^{-3}\,\text{m}\right)^2}{\left(5.0\,\text{m}\right)\left(589\times10^{-9}\,\text{m}\right)} - \frac{1}{2} = 45.$$

This corresponds to the forty-sixth bright fringe (see remark at the end of our solution in part (a)).

77. We solve for $m$ using the formula $r=\sqrt{(2m+1)R\lambda/2}$ obtained in problem 49 and find $m = r^2/R\lambda - 1/2$. Now, when $m$ is changed to $m + 20$, $r$ becomes $r'$, so

$$m + 20 = r'^2/R\lambda - 1/2.$$

Taking the difference between the two equations above, we eliminate $m$ and find

$$R=\frac{r'^2-r^2}{20\lambda}=\frac{(0.368\,\text{cm})^2-(0.162\,\text{cm})^2}{20(546\times10^{-7}\,\text{cm})}=100\,\text{cm}.$$

78. The time to change from one minimum to the next is $\Delta t$ = 12 s. This involves a change in thickness $\Delta L = \lambda/2n_2$ (see Eq. 35-37), and thus a change of volume

$$\Delta V = \pi r^2 \Delta L = \frac{\pi r^2 \lambda}{2n_2} \quad \Rightarrow \quad \frac{dV}{dt} = \frac{\pi r^2 \lambda}{2n_2\,\Delta t} = \frac{\pi(0.0180)^2\,(550 \times 10^{-9})}{2(1.40)\,(12)}$$

using SI units. Thus, the rate of change of volume is $1.67 \times 10^{-11}$ m$^3$/s.

79. A shift of one fringe corresponds to a change in the optical path length of one wavelength. When the mirror moves a distance $d$ the path length changes by $2d$ since the light traverses the mirror arm twice. Let $N$ be the number of fringes shifted. Then, $2d = N\lambda$ and

$$\lambda = \frac{2d}{N} = \frac{2(0.233\times10^{-3}\ \text{m})}{792} = 5.88\times10^{-7}\ \text{m} = 588\ \text{nm}\ .$$

80. According to Eq. 35-43, the number of fringes shifted ($\Delta N$) due to the insertion of the film of thickness $L$ is $\Delta N = (2L / \lambda)(n - 1)$. Therefore,

$$L = \frac{\lambda \Delta N}{2(n-1)} = \frac{(589\,\text{nm})(7.0)}{2(1.40-1)} = 5.2\,\mu\text{m} .$$

81. Let $\phi_1$ be the phase difference of the waves in the two arms when the tube has air in it, and let $\phi_2$ be the phase difference when the tube is evacuated. These are different because the wavelength in air is different from the wavelength in vacuum. If λ is the wavelength in vacuum, then the wavelength in air is λ/$n$, where $n$ is the index of refraction of air. This means

$$\phi_1 - \phi_2 = 2L\left[\frac{2\pi n}{\lambda} - \frac{2\pi}{\lambda}\right] = \frac{4\pi(n-1)L}{\lambda}$$

where $L$ is the length of the tube. The factor 2 arises because the light traverses the tube twice, once on the way to a mirror and once after reflection from the mirror. Each shift by one fringe corresponds to a change in phase of 2π rad, so if the interference pattern shifts by $N$ fringes as the tube is evacuated,

$$\frac{4\pi(n-1)L}{\lambda} = 2N\pi$$

and

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{60(500\times10^{-9}\text{ m})}{2(5.0\times10^{-2}\text{ m})} = 1.00030\ .$$

82. We denote the two wavelengths as λ and λ′, respectively. We apply Eq. 35-42 to both wavelengths and take the difference:

$$N' - N = \frac{2L}{\lambda'} - \frac{2L}{\lambda} = 2L\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right).$$

We now require $N' - N = 1$ and solve for $L$:

$$L = \frac{1}{2}\left(\frac{1}{\lambda} - \frac{1}{\lambda'}\right)^{-1} = \frac{1}{2}\left(\frac{1}{589.10\text{nm}} - \frac{1}{589.59\text{nm}}\right)^{-1} = 3.54\times10^{5}\ \text{nm} = 354\ \mu\text{m}.$$

83. (a) The path length difference between Rays 1 and 2 is $7d - 2d = 5d$. For this to correspond to a half-wavelength requires $5d = \lambda/2$, so that $d = 50.0$ nm.

(b) The above requirement becomes $5d = \lambda/2n$ in the presence of the solution, with $n = 1.38$. Therefore, $d = 36.2$ nm.

84. (a) Since $P_1$ is equidistant from $S_1$ and $S_2$ we conclude the sources are not in phase with each other. Their phase difference is $\Delta\phi_{source} = 0.60\ \pi$ rad, which may be expressed in terms of "wavelengths" (thinking of the $\lambda \Leftrightarrow 2\pi$ correspondence in discussing a full cycle) as $\Delta\phi_{source} = (0.60\ \pi / 2\pi)\ \lambda = 0.3\ \lambda$ (with $S_2$ "leading" as the problem states). Now $S_1$ is closer to $P_2$ than $S_2$ is. Source $S_1$ is 80 nm ($\Leftrightarrow$ 80/400 $\lambda = 0.2\ \lambda$ ) from $P_2$ while source $S_2$ is 1360 nm ($\Leftrightarrow$ 1360/400 $\lambda = 3.4\ \lambda$ ) from $P_2$. Here we find a difference of $\Delta\phi_{path} = 3.2\ \lambda$ (with $S_1$ "leading" since it is closer). Thus, the net difference is

$$\Delta\phi_{net} = \Delta\phi_{path} - \Delta\phi_{source} = 2.90\ \lambda,$$

or 2.90 wavelengths.

(b) A whole number (like 3 wavelengths) would mean fully constructive, so our result is of the following nature: intermediate, but close to fully constructive.

85. (a) Applying the law of refraction, we obtain sin $\theta_2$ / sin $\theta_1$ = sin $\theta_2$ / sin 30° = $v_s/v_d$. Consequently,

$$\theta_2 = \sin^{-1}\left(\frac{v_s \sin 30°}{v_d}\right) = \sin^{-1}\left[\frac{(3.0\,\text{m/s})\sin 30°}{4.0\,\text{m/s}}\right] = 22°.$$

(b) The angle of incidence is gradually reduced due to refraction, such as shown in the calculation above (from 30° to 22°). Eventually after many refractions, $\theta_2$ will be virtually zero. This is why most waves come in normal to a shore.

86. When the depth of the liquid ($L_{liq}$) is zero, the phase difference $\phi$ is 60 wavelengths; this must equal the difference between the number of wavelengths in length $L$ = 40 μm (since the liquid initially fills the hole) of the plastic (for ray $r_1$) and the number in that same length of the air (for ray $r_2$). That is,

$$\frac{L\,n_{plastic}}{\lambda} - \frac{L\,n_{air}}{\lambda} = 60\,.$$

(a) Since $\lambda = 400 \times 10^{-9}$ m and $n_{air} = 1$ (to good approximation), we find $n_{plastic} = 1.6$.

(b) The slope of the graph can be used to determine $n_{liq}$, but we show an approach more closely based on the above equation:

$$\frac{L\,n_{plastic}}{\lambda} - \frac{L\,n_{liq}}{\lambda} = 20$$

which makes use of the leftmost point of the graph. This readily yields $n_{liq} = 1.4$.

87. Let the $m$ = 10 bright fringe on the screen be a distance $y$ from the central maximum. Then from Fig. 35-10(a)

$$r_1 - r_2 = \sqrt{(y+d/2)^2 + D^2} - \sqrt{(y-d/2)^2 + D^2} = 10\lambda,$$

from which we may solve for $y$. To the order of $(d/D)^2$ we find

$$y = y_0 + \frac{y(y^2 + d^2/4)}{2D^2},$$

where $y_0 = 10D\lambda/d$. Thus, we find the percent error as follows:

$$\frac{y_0(y_0^2 + d^2/4)}{2y_0D^2} = \frac{1}{2}\left(\frac{10\lambda}{D}\right)^2 + \frac{1}{8}\left(\frac{d}{D}\right)^2 = \frac{1}{2}\left(\frac{5.89\,\mu\text{m}}{2000\,\mu\text{m}}\right)^2 + \frac{1}{8}\left(\frac{2.0\,\text{mm}}{40\,\text{mm}}\right)^2$$

which yields 0.032%.

88. (a) The minimum path length difference occurs when both rays are nearly vertical. This would correspond to a point as far up in the picture as possible. Treating the screen as if it extended forever, then the point is at $y = \infty$.

(b) When both rays are nearly vertical, there is no path length difference between them. Thus at $y = \infty$, the phase difference is $\phi = 0$.

(c) At $y = 0$ (where the screen crosses the $x$ axis) both rays are horizontal, with the ray from $S_1$ being longer than the one from $S_2$ by distance $d$.

(d) Since the problem specifies $d = 6.00\lambda$, then the phase difference here is $\phi = 6.00$ wavelengths and is at its maximum value.

(e) With $D = 20\lambda$, use of the Pythagorean theorem leads to

$$\phi = \frac{L_1 - L_2}{\lambda} = \frac{\sqrt{d^2 + (d+D)^2} - \sqrt{d^2 + D^2}}{\lambda} = 5.80$$

which means the rays reaching the point $y = d$ have a phase difference of roughly 5.8 wavelengths.

(f) The result of the previous part is "intermediate" – closer to 6 (constructive interference) than to $5\frac{1}{2}$ (destructive interference).

89. (a) In our solution here, we assume the reader has looked at our solution for problem 98. A light ray traveling directly along the central axis reaches the end in time

$$t_{\text{direct}} = \frac{L}{v_1} = \frac{n_1 L}{c}.$$

For the ray taking the critical zig-zag path, only its velocity component along the core axis direction contributes to reaching the other end of the fiber. That component is $v_1 \cos\theta'$, so the time of travel for this ray is

$$t_{\text{zig zag}} = \frac{L}{v_1 \cos\theta'} = \frac{n_1 L}{c\sqrt{1-\left(\frac{1}{n_1}\sin\theta\right)^2}}$$

using results from the previous solution. Plugging in $\sin\theta = \sqrt{n_1^2 - n_2^2}$ and simplifying, we obtain

$$t_{\text{zig zag}} = \frac{n_1 L}{c\left(n_2 / n_1\right)} = \frac{n_1^2 L}{n_2 c}.$$

The difference $t_{\text{zig zag}} - t_{\text{direct}}$ readily yields the result shown in the problem statement.

(b) With $n_1 = 1.58$, $n_2 = 1.53$ and $L = 300$ m, we obtain $\Delta t = 51.6$ ns.

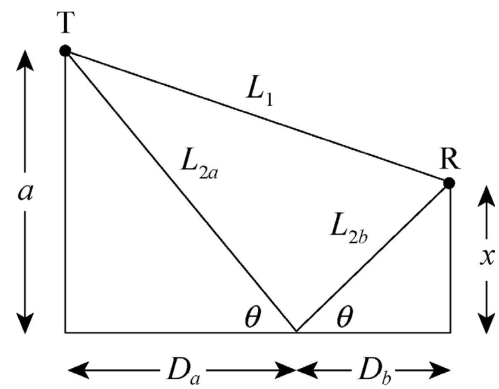
90. (a) The graph shows part of a periodic pattern of half-cycle "length" $\Delta n = 0.4$. Thus if we set $n = 1.0 + 2\,\Delta n = 1.8$ then the maximum at $n = 1.0$ should repeat itself there.

(b) Continuing the reasoning of part (a), adding another half-cycle "length" we get $1.8 + \Delta n = 2.2$ for the answer.

(c) Since $\Delta n = 0.4$ represents a half-cycle, then $\Delta n/2$ represents a quarter-cycle. To accumulate a total change of $2.0 - 1.0 = 1.0$ (see problem statement), then we need $2\Delta n + \Delta n/2 = 5/4^{\text{th}}$ of a cycle, which corresponds to 1.25 wavelengths.

91. The wave that goes directly to the receiver travels a distance $L_1$ and the reflected wave travels a distance $L_2$. Since the index of refraction of water is greater than that of air this last wave suffers a phase change on reflection of half a wavelength. To obtain constructive interference at the receiver, the difference $L_2 - L_1$ must be an odd multiple of a half wavelength. Consider the diagram below. The right triangle on the left, formed by the vertical line from the water to the transmitter T, the ray incident on the water, and the water line, gives $D_a = a/\tan\theta$. The right triangle on the right, formed by the vertical line from the water to the receiver $R$, the reflected ray, and the water line leads to $D_b = x/\tan\theta$. Since $D_a + D_b = D$,

$$\tan\theta = \frac{a+x}{D}\ .$$




We use the identity $\sin^2\theta = \tan^2\theta/(1+\tan^2\theta)$ to show that

$$\sin\theta = (a+x)/\sqrt{D^2+(a+x)^2}\ .$$

This means

$$L_{2a} = \frac{a}{\sin\theta} = \frac{a\sqrt{D^2+(a+x)^2}}{a+x}$$

and

$$L_{2b} = \frac{x}{\sin\theta} = \frac{x\sqrt{D^2+(a+x)^2}}{a+x}.$$

Therefore,

$$L_2 = L_{2a} + L_{2b} = \frac{(a+x)\sqrt{D^2 + (a+x)^2}}{a+x} = \sqrt{D^2 + (a+x)^2} \ .$$

Using the binomial theorem, with $D^2$ large and $a^2 + x^2$ small, we approximate this expression: $L_2 \approx D + (a + x)^2 / 2D$. The distance traveled by the direct wave is $L_1 = \sqrt{D^2 + (a-x)^2}$ . Using the binomial theorem, we approximate this expression: $L_1 \approx D + (a - x)^2 / 2D$. Thus,

$$L_2 - L_1 \approx D + \frac{a^2 + 2ax + x^2}{2D} - D - \frac{a^2 - 2ax + x^2}{2D} = \frac{2ax}{D} \ .$$

Setting this equal to $\left(m + \frac{1}{2}\right)\lambda$ , where $m$ is zero or a positive integer, we find $x = \left(m + \frac{1}{2}\right)(D/2a)\lambda$ .

92. (a) Looking at the figure (where a portion of a periodic pattern is shown) we see that half of the periodic pattern is of length $\Delta L$ = 750 nm (judging from the maximum at $x$ = 0 to the minimum at $x$ = 750 nm); this suggests that the wavelength (the full length of the periodic pattern) is $\lambda = 2\,\Delta L$ = 1500 nm. A maximum should be reached again at $x$ = 1500 nm (and at $x$ = 3000nm, $x$ = 4500 nm, …).

(b) From our discussion in part (b), we expect a minimum to be reached at each value $x$ = 750 nm + $n$(1500 nm), where $n$ = 1, 2, 3 … . For instance, for $n$ = 1 we would find the minimum at $x$ = 2250 nm.

(c) With $\lambda$ = 1500 nm (found in part (a)), we can express $x$ = 1200 nm as $x$ = 1200/1500 = 0.80 wavelength.

93. $v_{\min} = c/n = (2.998 \times 10^8 \text{ m/s})/1.54 = 1.95 \times 10^8 \text{ m/s}$.

94. We note that $\Delta\phi = 60° = \frac{\pi}{3}$ rad. The phasors rotate with constant angular velocity

$$\omega = \frac{\Delta\phi}{\Delta t} = \frac{\pi/3}{2.5 \times 10^{-16}} = 4.19 \times 10^{15} \text{ rad/s} .$$

Since we are working with light waves traveling in a medium (presumably air) where the wave speed is approximately $c$, then $kc = \omega$ (where $k = 2\pi/\lambda$), which leads to

$$\lambda = \frac{2\pi c}{\omega} = 450 \text{ nm} .$$

95. We infer from Sample Problem 35-2, that (with angle in radians)

$$\Delta\theta = \frac{\lambda}{d}$$

for adjacent fringes. With the wavelength change ($\lambda' = \lambda/n$ by Eq. 35-8), this equation becomes

$$\Delta\theta' = \frac{\lambda'}{d}.$$

Dividing one equation by the other, the requirement of *radians* can now be relaxed and we obtain

$$\frac{\Delta\theta'}{\Delta\theta} = \frac{\lambda'}{\lambda} = \frac{1}{n}.$$

Therefore, with $n = 1.33$ and $\Delta\theta = 0.30°$, we find $\Delta\theta' = 0.23°$.

96. We note that ray 1 travels an extra distance $4L$ more than ray 2. For constructive interference (which is obtained for $\lambda = 620$ nm) we require

$$4L = m\lambda \qquad \text{where } m = \text{some positive integer .}$$

For destructive interference (which is obtained for $\lambda' = 496$ nm) we require

$$4L = \frac{k}{2}\lambda' \qquad \text{where } k = \text{some positive \underline{odd} integer .}$$

Equating these two equations (since their left-hand sides are equal) and rearranging, we obtain

$$k = 2\,m\,\frac{\lambda}{\lambda'} = 2\,m\,\frac{620}{496} = 2.5\,m\ .$$

We note that this condition is satisfied for $k = 5$ and $m = 2$. It is satisfied for some larger values, too, but – recalling that we want the least possible value for $L$ – we choose the solution set $(k, m) = (5, 2)$. Plugging back into either of the equations above, we obtain the distance $L$:

$$4L = 2\lambda \quad \Rightarrow \quad L = \frac{\lambda}{2} = 310.0 \text{ nm .}$$

97. (a) The path length difference is 0.5 $\mu$m = 500 nm, which is represents 500/400 = 1.25 wavelengths — that is, a meaningful difference of 0.25 wavelengths. In angular measure, this corresponds to a phase difference of (0.25)2π = π/2 radians ≈ 1.6 rad.

(b) When a difference of index of refraction is involved, the approach used in Eq. 35-9 is quite useful. In this approach, we count the wavelengths between $S_1$ and the origin

$$N_1 = \frac{Ln}{\lambda} + \frac{L'n'}{\lambda}$$

where $n$ = 1 (rounding off the index of air), $L$ = 5.0 $\mu$m, $n'$ = 1.5 and $L'$ = 1.5 $\mu$m. This yields $N_1$ = 18.125 wavelengths. The number of wavelengths between $S_2$ and the origin is (with $L_2$ = 6.0 $\mu$m) given by

$$N_2 = \frac{L_2 n}{\lambda} = 15.000.$$

Thus, $N_1 - N_2$ = 3.125 wavelengths, which gives us a meaningful difference of 0.125 wavelength and which "converts" to a phase of π/4 radian ≈ 0.79 rad.

98. (a) The difference in wavelengths, with and without the $n$ = 1.4 material, is found using Eq. 35-9:

$$\Delta N = \frac{L(n - 1)}{\lambda} = 1.143.$$

The result is equal to a phase shift of (1.143)(360°) = 411.4°, or

(b) more meaningfully -- a shift of 411.4° – 360° = 51.4°.

99. Using Eq. 35-16 with the small-angle approximation (illustrated in Sample Problem 35-2), we arrive at

$$y = \frac{\left(m + \frac{1}{2}\right)\lambda D}{d}$$

for the position of the $(m + 1)^{\text{th}}$ dark band (a simple way to get this is by averaging the expressions in Eq. 35-17 and Eq. 35-18). Thus, with $m = 1$, $y = 0.012$ m and $d = 800\lambda$, we find $D = 6.4$ m.

100. (a) We are dealing with a symmetric situation (with the film index $n_2 = 1.5$ being less than that of the materials bounding it), and with reflected light, so Eqs. 35-36 and -37 apply *with* their stated applicability. Both can be written in the form

$$\frac{2n_2L}{\lambda} = \begin{cases} \text{half-integer for bright} \\ \text{integer for dark} \end{cases}$$

Thus, we find $2n_2L/\lambda = 3$, so that we find the middle of a dark band at the left edge of the figure. Since there is nothing beyond this "middle" then a more appropriate phrasing is that there is half of a dark band next to the left edge, being darkest precisely at the edge.

(b) The right edge, where they touch, satisfies the dark reflection condition for $L = 0$ (where $m = 0$), so there is (essentially half of) a dark band at the right end.

(c) Counting half-bands and whole bands alike, we find four dark bands: ($m = 0, 1, 2, 3$).

101. (a) In this case, the film has a smaller index material on one side (air) and a larger index material on the other (glass), and we are dealing (in part (a)) with strongly transmitted light, so the condition is given by Eq. 35-37 (which would give dark *reflection* in this scenario)

$$L = \frac{\lambda}{2n_2}\left(m + \frac{1}{2}\right) = 110 \text{ nm}$$

for $n_2 = 1.25$ and $m = 0$.

(b) Now, we are dealing with strongly reflected light, so the condition is given by Eq. 35-36 (which would give no *transmission* in this scenario)

$$L = \frac{m\lambda}{2n_2} = 220\,\text{nm}$$

for $n_2 = 1.25$ and $m = 1$ (the $m = 0$ option is excluded in the problem statement).

102. We adapt the result of problem 21. Now, the phase difference in radians is

$$\frac{2\pi t}{\lambda}(n_2 - n_1) = 2m\pi.$$

The problem implies $m$ = 5, so the thickness is

$$t = \frac{m\lambda}{n_2 - n_1} = \frac{5(480\,\text{nm})}{1.7 - 1.4} = 8.0\times 10^3\,\text{nm} = 8.0\,\mu\text{m}.$$

103. (a) Since $n_2 > n_3$, this case has no π-phase shift, and the condition for constructive interference is $m\lambda = 2Ln_2$. We solve for $L$:

$$L = \frac{m\lambda}{2n_2} = \frac{m(525 \text{ nm})}{2(1.55)} = (169 \text{ nm})m.$$

For the minimum value of $L$, let $m = 1$ to obtain $L_{\min} = 169$ nm.

(b) The light of wavelength λ (other than 525 nm) that would also be preferentially transmitted satisfies $m'\lambda = 2n_2L$, or

$$\lambda = \frac{2n_2L}{m'} = \frac{2(1.55)(169 \text{ nm})}{m'} = \frac{525 \text{ nm}}{m'}.$$

Here $m' = 2, 3, 4, \ldots$ (note that $m' = 1$ corresponds to the $\lambda = 525$ nm light, so it should not be included here). Since the minimum value of $m'$ is 2, one can easily verify that no $m'$ will give a value of λ which falls into the visible light range. So no other parts of the visible spectrum will be preferentially transmitted. They are, in fact, reflected.

(c) For a sharp reduction of transmission let

$$\lambda = \frac{2n_2L}{m' + 1/2} = \frac{525 \text{ nm}}{m' + 1/2},$$

where $m' = 0, 1, 2, 3, \ldots$. In the visible light range $m' = 1$ and $\lambda = 350$ nm. This corresponds to the blue-violet light.

104. (a) Straightforward application of Eq. 35-3 and $v = \Delta x/\Delta t$ yields the result: film 1.

(b) The traversal time is equal to $4.0 \times 10^{-15}$ s.

(c) Use of Eq. 35-9 leads to the number of wavelengths:

$$N = \frac{L_1 n_1 + L_2 n_2 + L_3 n_3}{\lambda} = 7.5.$$

105. (a) Following Sample Problem 35-1, we have

$$N_2 - N_1 = \frac{L}{\lambda}(n_2 - n_1) = 1.87$$

which represents a meaningful difference of 0.87 wavelength.

(b) The result in part (a) is closer to 1 wavelength (constructive interference) than it is to $1/2$ wavelength (destructive interference) so the latter choice applies.

(c) This would insert a $\pm 1/2$ wavelength into the previous result — resulting in a meaningful difference (between the two rays) equal to 0.87 – 0.50 = 0.37 wavelength.

(d) The result in part (c) is closer to the destructive interference condition. Thus, there is intermediate illumination but closer to darkness.

106. (a) With $\lambda = 0.5\ \mu$m, Eq. 35-14 leads to

$$\theta = \sin^{-1}\frac{(3)(0.5\,\mu\text{m})}{2.00\,\mu\text{m}} = 48.6°.$$

(b) Decreasing the frequency means increasing the wavelength — which implies $y$ increases, and the third side bright fringe moves away from the center of the pattern. Qualitatively, this is easily seen with Eq. 35-17. One should exercise caution in appealing to Eq. 35-17 here, due to the fact the small angle approximation is not justified in this problem.

(c) The new wavelength is 0.5/0.9 = 0.556 $\mu$m, which produces a new angle of

$$\theta = \sin^{-1}\frac{(3)(0.556\,\mu\text{m})}{2.00\,\mu\text{m}} = 56.4°.$$

Using $y = D\tan\theta$ for the old and new angles, and subtracting, we find

$$\Delta y = D\left(\tan 56.4° - \tan 48.6°\right) = 1.49\ \text{m}.$$

107. (a) A path length difference of $\lambda/2$ produces the first dark band, of $3\lambda/2$ produces the second dark band, and so on. Therefore, the fourth dark band corresponds to a path length difference of $7\lambda/2$ = 1750 nm = 1.75 $\mu$m.

(b) In the small angle approximation (which we assume holds here), the fringes are equally spaced, so that if $\Delta y$ denotes the distance from one maximum to the next, then the distance from the middle of the pattern to the fourth dark band must be 16.8 mm = 3.5 $\Delta y$. Therefore, we obtain $\Delta y$ = 16.8/3.5 = 4.8 mm.

108. In the case of a distant screen the angle $\theta$ is close to zero so sin $\theta \approx \theta$. Thus from Eq. 35-14,

$$\Delta\theta \approx \Delta \sin\theta = \Delta\left(\frac{m\lambda}{d}\right) = \frac{\lambda}{d}\Delta m = \frac{\lambda}{d},$$

or $d \approx \lambda/\Delta\theta = 589 \times 10^{-9}$ m/0.018 rad $= 3.3 \times 10^{-5}$ m = 33 $\mu$m.

109. (a) Straightforward application of Eq. 35-3 $n = c/v$ and $v = \Delta x/\Delta t$ yields the result: pistol 1 with a time equal to $\Delta t = n\Delta x/c = 42.0 \times 10^{-12}$ s.

(b) For pistol 2, the travel time is equal to $42.3 \times 10^{-12}$ s.

(c) For pistol 3, the travel time is equal to $43.2 \times 10^{-12}$ s.

(d) For pistol 4 the travel time is equal to $41.8 \times 10^{-12}$ s.

(e) We see that the blast from pistol 4 arrives first.

110. We use Eq. 35-36 for constructive interference: $2n_2L = (m + 1/2)\lambda$, or

$$\lambda = \frac{2n_2 L}{m+1/2} = \frac{2(1.50)(410\,\text{nm})}{m+1/2} = \frac{1230\ \text{nm}}{m+1/2},$$

where $m$ = 0, 1, 2, …. The only value of $m$ which, when substituted into the equation above, would yield a wavelength which falls within the visible light range is $m$ = 1. Therefore,

$$\lambda = \frac{1230\,\text{nm}}{1+1/2} = 492\ \text{nm}.$$

111. For the first maximum $m$ = 0 and for the tenth one $m$ = 9. The separation is $\Delta y = (D\lambda/d)\Delta m = 9D\lambda/d$. We solve for the wavelength:

$$\lambda = \frac{d\Delta y}{9D} = \frac{\left(0.15\times 10^{-3}\,\text{m}\right)\left(18\times 10^{-3}\,\text{m}\right)}{9\left(50\times 10^{-2}\,\text{m}\right)} = 6.0\times 10^{-7}\,\text{m} = 600\ \text{nm}.$$

112. Light reflected from the upper oil surface (in contact with air) changes phase by $\pi$ rad. Light reflected from the lower surface (in contact with glass) changes phase by $\pi$ rad if the index of refraction of the oil is less than that of the glass and does not change phase if the index of refraction of the oil is greater than that of the glass.

• First, suppose the index of refraction of the oil is greater than the index of refraction of the glass. The condition for fully destructive interference is $2n_od = m\lambda$, where $d$ is the thickness of the oil film, $n_o$ is the index of refraction of the oil, $\lambda$ is the wavelength in vacuum, and $m$ is an integer. For the shorter wavelength, $2n_od = m_1\lambda_1$ and for the longer, $2n_od = m_2\lambda_2$. Since $\lambda_1$ is less than $\lambda_2$, $m_1$ is greater than $m_2$, and since fully destructive interference does not occur for any wavelengths between, $m_1 = m_2 + 1$. Solving $(m_2 + 1)\lambda_1 = m_2\lambda_2$ for $m_2$, we obtain

$$m_2 = \frac{\lambda_1}{\lambda_2 - \lambda_1} = \frac{500 \text{ nm}}{700\,\text{nm} - 500\,\text{nm}} = 2.50.$$

Since $m_2$ must be an integer, the oil cannot have an index of refraction that is greater than that of the glass.

• Now suppose the index of refraction of the oil is less than that of the glass. The condition for fully destructive interference is then $2n_od = (2m + 1)\lambda$. For the shorter wavelength, $2m_od = (2m_1 + 1)\lambda_1$, and for the longer, $2n_od = (2m_2 + 1)\lambda_2$. Again, $m_1 = m_2 + 1$, so $(2m_2 + 3)\lambda_1 = (2m_2 + 1)\lambda_2$. This means the value of $m_2$ is

$$m_2 = \frac{3\lambda_1 - \lambda_2}{2(\lambda_2 - \lambda_1)} = \frac{3(500\,\text{nm}) - 700\,\text{nm}}{2(700\,\text{nm} - 500\,\text{nm})} = 2.00.$$

This is an integer. Thus, the index of refraction of the oil is less than that of the glass.

113. We use the formula obtained in Sample Problem 35-6:

$$L_{\min} = \frac{\lambda}{4n_2} = \frac{\lambda}{4(1.25)} = 0.200\lambda \;\Rightarrow\; \frac{L_{\min}}{\lambda} = 0.200.$$

114. We use Eq. 35-36:

$$L_{16} = \left(16 + \frac{1}{2}\right)\frac{\lambda}{2n_2}$$
$$L_6 = \left(6 + \frac{1}{2}\right)\frac{\lambda}{2n_2}$$

The difference between these, using the fact that $n_2 = n_{air} = 1.0$, is

$$L_{16} - L_6 = (10)\frac{480\,\text{nm}}{2(1.0)} = 2400\,\text{nm} = 2.4\,\mu\text{m}.$$

115. Let the position of the mirror measured from the point at which $d_1 = d_2$ be $x$. We assume the beam-splitting mechanism is such that the two waves interfere constructively for $x = 0$ (with some beam-splitters, this would not be the case). We can adapt Eq. 35-23 to this situation by incorporating a factor of 2 (since the interferometer utilizes directly reflected light in contrast to the double-slit experiment) and eliminating the sin $\theta$ factor. Thus, the phase difference between the two light paths is $\Delta\phi = 2(2\pi x/\lambda) = 4\pi x/\lambda$. Then from Eq. 35-22 (writing $4I_0$ as $I_m$) we find

$$I = I_m \cos^2\left(\frac{\Delta\phi}{2}\right) = I_m \cos^2\left(\frac{2\pi x}{\lambda}\right).$$

116. The index of refraction of fused quartz at λ = 550 nm is about 1.459, obtained from Fig. 34-19. Thus, from Eq. 35-3, we find

$$v = \frac{c}{n} = \frac{2.998\times10^{8}\ \text{m/s}}{1.459} = 2.06\times10^{8}\ \text{m/s} \approx 2.1\times10^{8}\ \text{m/s}.$$

117. (a) We use $\Delta y = D\lambda/d$ (see Sample Problem 35-2). Because of the placement of the mirror in the problem $D = 2(20.0\text{ m}) = 40.0\text{ m}$, which we express in millimeters in the calculation below:

$$d = \frac{D\lambda}{\Delta y} = \frac{(4.00\times 10^4\text{ mm})(632.8\times 10^{-6}\text{ mm})}{100\text{ mm}} = 0.253\text{ mm} .$$

(b) In this case the interference pattern will be shifted. At the location of the original central maximum, the effective phase difference is now $\frac{1}{2}$ wavelength, so there is now a minimum instead of a maximum.

118. (a) Dividing Eq. 35-12 by the wavelength, we obtain

$$N = \frac{\Delta L}{\lambda} = \frac{d}{\lambda} \sin\theta = 39.6$$

wavelengths.

(b) This is close to a half-integer value (destructive interference), so that the correct response is "intermediate illumination but closer to darkness."

119. We adapt Eq. 35-21 to the non-reflective coating on a glass lens: $I = I_{max} \cos^2 (\phi/2)$, where $\phi = (2\pi/\lambda)(2n_2L) + \pi$.

(a) At $\lambda = 450$ nm

$$\frac{I}{I_{max}} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{2\pi n_2 L}{\lambda} + \frac{\pi}{2}\right) = \cos^2\left[\frac{2\pi(1.38)(99.6\text{nm})}{450\text{nm}} + \frac{\pi}{2}\right] = 0.883 \approx 88\%.$$

(b) At $\lambda = 650$ nm

$$\frac{I}{I_{max}} = \cos^2\left[\frac{2\pi(1.38)(99.6\text{nm})}{650\text{nm}} + \frac{\pi}{2}\right] = 0.942 \approx 94\%.$$

120. (a) Every time one more destructive (constructive) fringe appears the increase in thickness of the air gap is $\lambda/2$. Now that there are 6 more destructive fringes in addition to the one at point $A$, the thickness at $B$ is $t_B = 6(\lambda/2) = 3(600 \text{ nm}) = 1.80\ \mu\text{m}$.

(b) We must now replace $\lambda$ by $\lambda' = \lambda/n_w$. Since $t_B$ is unchanged $t_B = N(\lambda'/2) = N(\lambda/2n_w)$, or

$$N = \frac{2t_B n_w}{\lambda} = \frac{2(3\lambda)n_w}{\lambda} = 6n_w = 6(1.33) = 8 \ .$$

Counting the one at point $A$, a total of nine dark fringes will be observed.

121. We take the electric field of one wave, at the screen, to be

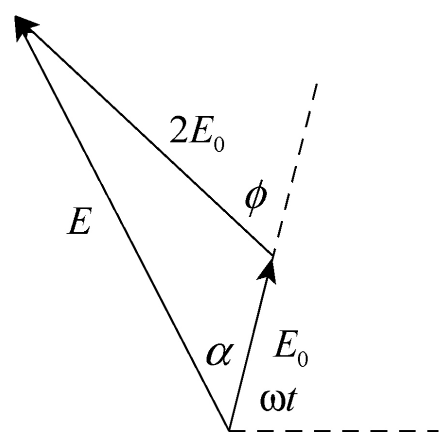
$$E_1 = E_0 \sin(\omega t)$$

and the electric field of the other to be

$$E_2 = 2E_0 \sin(\omega t + \phi) ,$$

where the phase difference is given by

$$\phi = \left(\frac{2\pi d}{\lambda}\right) \sin\theta .$$




Here $d$ is the center-to-center slit separation and $\lambda$ is the wavelength. The resultant wave can be written $E = E_1 + E_2 = E \sin(\omega t + \alpha)$, where $\alpha$ is a phase constant. The phasor diagram is shown above. The resultant amplitude $E$ is given by the trigonometric law of cosines:

$$E^2 = E_0^2 + (2E_0)^2 - 4E_0^2 \cos(180°-\phi) = E_0^2(5 + 4\cos\phi) .$$

The intensity is given by $I = I_0(5 + 4\cos\phi)$, where $I_0$ is the intensity that would be produced by the first wave if the second were not present. Since $\cos\phi = 2\cos^2(\phi/2) - 1$, this may also be written $I = I_0\left[1 + 8\cos^2(\phi/2)\right]$.

122. (a) To get to the detector, the wave from $S_1$ travels a distance $x$ and the wave from $S_2$ travels a distance $\sqrt{d^2+x^2}$ . The phase difference (in terms of wavelengths) between the two waves is

$$\sqrt{d^2+x^2}-x=m\lambda \quad m=0,1,2,\ldots$$

where we are requiring constructive interference. The solution is

$$x=\frac{d^2-m^2\lambda^2}{2m\lambda} .$$

We see that setting $m = 0$ in this expression produces $x = \infty$; hence, the phase difference between the waves when $P$ is very far away is 0.

(b) The result of part (a) implies that the waves constructively interfere at $P$.

(c) As is particularly evident from our results in part (d), the phase difference increases as $x$ decreases.

The condition for constructive interference is $\phi=2\pi m$ or $\Delta L=m\lambda$ in, and the condition for destructive interference is $\phi=2\pi(m+1/2)$ or $\Delta L=(m+1/2)\lambda$, with $m = 0, 1, 2,\ldots$

For parts (d) – (o), we can use our formula from part (a) for the 0.5λ, 1.50λ, etc. differences by allowing $m$ in our formula to take on half-integer values. The half-integer values, though, correspond to destructive interference.

(d) When the phase difference is $\phi=0$, the interference is fully constructive,

(e) and the interference occurs at $x=\infty$.

(f) When $\Delta L=0.500\lambda$ $(m=1/2)$, the interference is fully destructive.

(g) Using the values $\lambda = 0.500$ $\mu$m and $d = 2.00$ $\mu$m, we find $x = 7.88$ $\mu$m for $m=1/2$.

(h) When $\Delta L=1.00\lambda$ $(m=1)$, the interference is fully constructive.

(i) Using the formula obtained in part (a), we have $x=3.75\,\mu\text{m}$ for $m = 1$.

(j) When $\Delta L=1.500\lambda$ $(m=3/2)$, the interference is fully destructive.

(k) Using the formula obtained in part (a), we have $x = 2.29$ $\mu$m for $m=3/2$.

(l) When $\Delta L = 2.00\lambda$ $(m = 2)$, the interference is fully constructive.

(m) Using the formula obtained in part (a), we have $x = 1.50\,\mu\text{m}$ for $m = 2$.

(n) When $\Delta L = 2.500\lambda$ $(m = 5/2)$, the interference is fully destructive.

(o) Using the formula obtained in part (a), we have $x = 0.975\,\mu\text{m}$ for $m = 5/2$.

123. (a) The binomial theorem (Appendix E) allows us to write

$$\sqrt{k(1+x)} = \sqrt{k}\left(1+\frac{x}{2}+\frac{x^2}{8}+\frac{3x^3}{48}+\cdots\right) \approx \sqrt{k} + \frac{x}{2}\sqrt{k}$$

for $x \ll 1$. Thus, the end result from the solution of problem 49 yields

$$r_m = \sqrt{R\lambda m\left(1+\frac{1}{2m}\right)} \approx \sqrt{R\lambda m} + \frac{1}{4m}\sqrt{R\lambda m}$$

and

$$r_{m+1} = \sqrt{R\lambda m\left(1+\frac{3}{2m}\right)} \approx \sqrt{R\lambda m} + \frac{3}{4m}\sqrt{R\lambda m}$$

for very large values of $m$. Subtracting these, we obtain

$$\Delta r = \frac{3}{4m}\sqrt{R\lambda m} - \frac{1}{4m}\sqrt{R\lambda m} = \frac{1}{2}\sqrt{\frac{R\lambda}{m}} \; .$$

(b) We take the differential of the area: $dA = d(\pi r^2) = 2\pi r\, dr$, and replace $dr$ with $\Delta r$ in anticipation of using the result from part (a). Thus, the area between adjacent rings for large values of $m$ is

$$2\pi r_m(\Delta r) \approx 2\pi\left(\sqrt{R\lambda m} + \frac{1}{4m}\sqrt{R\lambda m}\right)\left(\frac{1}{2}\sqrt{\frac{R\lambda}{m}}\right) \approx 2\pi\left(\sqrt{R\lambda m}\right)\left(\frac{1}{2}\sqrt{\frac{R\lambda}{m}}\right)$$

which simplifies to the desired result ($\pi\lambda R$).

124. The *Hint* essentially answers the question, but we put in some algebraic details and arrive at the familiar analytic-geometry expression for a hyperbola. The distance $d/2$ is denoted $a$ and the constant value for the path length difference is denoted $\phi = r_1 - r_2$ , or

$$\sqrt{(a+x)^2 + y^2} - \sqrt{(a-x)^2 + y^2} = \phi$$

Rearranging and squaring, we have

$$\left(\sqrt{(a+x)^2 + y^2}\right)^2 = \left(\sqrt{(a-x)^2 + y^2} + \phi\right)^2$$

$$a^2 + 2ax + x^2 + y^2 = a^2 - 2ax + x^2 + y^2 + \phi^2 + 2\,\phi\sqrt{(a-x)^2 + y^2}$$

Many terms on both sides are identical and may be eliminated. This leaves us with

$$-2\,\phi\sqrt{(a-x)^2 + y^2} = \phi^2 - 4ax$$

at which point we square both sides again:

$$4\,\phi^2a^2 - 8\,\phi^2ax + 4\,\phi^2x^2 + 4\,\phi^2y^2 = \phi^4 - 8\,\phi^2ax + 16a^2x^2$$

We eliminate the $-8\,\phi^2ax$ term from both sides and plug in $a = 2d$ to get back to the original notation used in the problem statement:

$$\phi^2d^2 + 4\,\phi^2x^2 + 4\,\phi^2y^2 = \phi^4 + 4\,d^2x^2$$

Then a simple rearrangement puts it in the familiar analytic geometry format:

$$\phi^2d^2 - \phi^4 = 4(d^2 - \phi^2)x^2 - 4\,\phi^2y^2$$

which can be further simplified by dividing through by $\phi^2d^2 - \phi^4$.