1. From the time dilation equation $\Delta t = \gamma \Delta t_0$ (where Δt_0 is the proper time interval, $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = v/c$), we obtain

$$\beta = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2}.$$

The proper time interval is measured by a clock at rest relative to the muon. Specifically, $\Delta t_0 = 2.2000 \ \mu$s. We are also told that Earth observers (measuring the decays of moving muons) find $\Delta t = 16.000 \ \mu$s. Therefore,

$$\beta = \sqrt{1 - \left(\frac{2.2000 \ \mu s}{16.000 \ \mu s}\right)^2} = 0.99050.$$

2. (a) We find β from $\gamma = 1/\sqrt{1-\beta^2}$:

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.0100000)^2}} = 0.14037076.$$

(b) Similarly, $\beta = \sqrt{1 - (10.000000)^{-2}} = 0.99498744$.

(c) In this case, $\beta = \sqrt{1 - (100.00000)^{-2}} = 0.99995000$.

(d) The result is $\beta = \sqrt{1 - (1000.0000)^{-2}} = 0.99999950$.

3. We solve the time dilation equation for the time elapsed (as measured by Earth observers):

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-(0.9990)^2}}$$

where $\Delta t_0 = 120$ y. This yields $\Delta t = 2684$ y $\approx 2.68 \times 10^3$ y.

4. Due to the time-dilation effect, the time between initial and final ages for the daughter is longer than the four years experienced by her father:

$$t_{f\,\text{daughter}} - t_{i\,\text{daughter}} = \gamma(4.000 \text{ y})$$

where γ is Lorentz factor (Eq. 37-8). Letting T denote the age of the father, then the conditions of the problem require

$$T_i = t_{i\,\text{daughter}} + 20.00 \text{ y} \quad \text{and} \quad T_f = t_{f\,\text{daughter}} - 20.00 \text{ y} .$$

Since $T_f - T_i = 4.000$ y, then these three equations combine to give a single condition from which γ can be determined (and consequently v):

$$44 = \gamma 4 \implies \gamma = 11 \implies \beta = \frac{2\sqrt{30}}{11} = 0.9959.$$

5. In the laboratory, it travels a distance $d = 0.00105$ m $= vt$, where $v = 0.992c$ and t is the time measured on the laboratory clocks. We can use Eq. 37-7 to relate t to the proper lifetime of the particle t_0:

$$t = \frac{t_0}{\sqrt{1-(v/c)^2}} \quad \Rightarrow \quad t_0 = t\sqrt{1-\left(\frac{v}{c}\right)^2} = \frac{d}{0.992c}\sqrt{1-0.992^2}$$

which yields $t_0 = 4.46 \times 10^{-13}$ s $= 0.446$ ps.

6. From the value of Δt in the graph when $\beta = 0$, we infer than Δt_o in Eq. 37-9 is 8.0 s. Thus, that equation (which describes the curve in Fig. 37-23) becomes

$$\Delta t = \frac{\Delta t_o}{\sqrt{1 - (v/c)^2}} = \frac{8.0 \text{ s}}{\sqrt{1 - \beta^2}}$$

If we set $\beta = 0.98$ in this expression, we obtain approximately 40 s for Δt.

7. (a) The round-trip (discounting the time needed to "turn around") should be one year according to the clock you are carrying (this is your proper time interval Δt_0) and 1000 years according to the clocks on Earth which measure Δt. We solve Eq. 37-7 for β:

$$\beta = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = \sqrt{1 - \left(\frac{1\text{y}}{1000\text{y}}\right)^2} = 0.99999950.$$

(b) The equations do not show a dependence on acceleration (or on the direction of the velocity vector), which suggests that a circular journey (with its constant magnitude centripetal acceleration) would give the same result (if the speed is the same) as the one described in the problem. A more careful argument can be given to support this, but it should be admitted that this is a fairly subtle question which has occasionally precipitated debates among professional physicists.

8. The contracted length of the tube would be

$$L = L_0 \sqrt{1 - \beta^2} = (3.00\,\text{m})\sqrt{1 - 0.999987^2} = 0.0153\,\text{m}.$$

9. (a) The rest length $L_0 = 130$ m of the spaceship and its length L as measured by the timing station are related by Eq. 37-13. Therefore, $L = (130\,\mathrm{m})\sqrt{1-(0.740)^2} = 87.4\,\mathrm{m}.$

(b) The time interval for the passage of the spaceship is

$$\Delta t = \frac{L}{v} = \frac{87.4\,\mathrm{m}}{(0.740)(3.00\times10^8\,\mathrm{m/s})} = 3.94\times10^{-7}\,\mathrm{s}.$$

10. Only the "component" of the length in the x direction contracts, so its y component stays

$$\ell'_y = \ell_y = \ell \sin 30° = 0.5000\,\text{m}$$

while its x component becomes

$$\ell'_x = \ell_x \sqrt{1 - \beta^2} = \ell \cos 30° \sqrt{1 - 0.90^2} = 0.3775\,\text{m}.$$

Therefore, using the Pythagorean theorem, the length measured from S' is

$$\ell' = \sqrt{\left(\ell'_x\right)^2 + \left(\ell'_y\right)^2} = 0.63\,\text{m}.$$

11. The length L of the rod, as measured in a frame in which it is moving with speed v parallel to its length, is related to its rest length L_0 by $L = L_0/\gamma$, where $\gamma = 1/\sqrt{1-\beta^2}$ and $\beta = v/c$. Since γ must be greater than 1, L is less than L_0. For this problem, $L_0 = 1.70$ m and $\beta = 0.630$, so $L = (1.70\,\text{m})\sqrt{1-(0.630)^2} = 1.32\,\text{m}$.

12. (a) We solve Eq. 37-13 for v and then plug in:

$$\beta = \sqrt{1-\left(\frac{L}{L_0}\right)^2} = \sqrt{1-\left(\frac{1}{2}\right)^2} = 0.866.$$

(b) The Lorentz factor in this case is $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}} = 2.00$.

13. (a) The speed of the traveler is $v = 0.99c$, which may be equivalently expressed as 0.99 ly/y. Let d be the distance traveled. Then, the time for the trip as measured in the frame of Earth is

$$\Delta t = d/v = (26 \text{ ly})/(0.99 \text{ ly/y}) = 26.3 \text{ y} \approx 26 \text{ y}.$$

(b) The signal, presumed to be a radio wave, travels with speed c and so takes 26.0 y to reach Earth. The total time elapsed, in the frame of Earth, is

$$26.3 \text{ y} + 26.0 \text{ y} = 52.3 \text{ y} \approx 52 \text{ y}.$$

(c) The proper time interval is measured by a clock in the spaceship, so $\Delta t_0 = \Delta t/\gamma$. Now

$$\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.99)^2} = 7.09.$$

Thus, $\Delta t_0 = (26.3 \text{ y})/(7.09) = 3.7 \text{ y}$.

14. From the value of L in the graph when $\beta = 0$, we infer than L_o in Eq. 37-13 is 0.80 m. Thus, that equation (which describes the curve in Fig. 37-24) with SI units understood becomes

$$L = L_o\sqrt{1 - (v/c)^2} = 0.80\sqrt{1 - \beta^2}\ .$$

If we set $\beta = 0.95$ in this expression, we obtain approximately 0.25 m for L.

15. (a) Let $d = 23000$ ly $= 23000$ c y, which would give the distance in meters if we included a conversion factor for years $\rightarrow$ seconds. With $\Delta t_0 = 30$ y and $\Delta t = d/v$ (see Eq. 37-10), we wish to solve for v from Eq. 37-7. Our first step is as follows:

$$\Delta t = \frac{d}{v} = \frac{\Delta t_0}{\sqrt{1-\beta^2}} \quad \Rightarrow \quad \frac{23000 \text{ y}}{\beta} = \frac{30 \text{ y}}{\sqrt{1-\beta^2}},$$

at which point we can cancel the unit year and manipulate the equation to solve for the speed parameter β. This yields

$$\beta = \frac{1}{\sqrt{1+(30/23000)^2}} = 0.99999915.$$

(b) The Lorentz factor is $\gamma = 1/\sqrt{1-\beta^2} = 766.6680752$. Thus, the length of the galaxy measured in the traveler's frame is

$$L = \frac{L_0}{\gamma} = \frac{23000 \text{ ly}}{766.6680752} = 29.99999 \text{ ly} \approx 30 \text{ ly}.$$

16. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 37-21 to apply without additional terms. In part (a), we apply these equations directly with $v = +0.400c = 1.199 \times 10^8$ m/s, and in part (c) we simply change $v \to -v$ and recalculate the primed values.

(a) The position coordinate measured in the S' frame is

$$x' = \gamma(x - vt) = \frac{x - vt}{\sqrt{1 - \beta^2}} = \frac{3.00 \times 10^8 \, \text{m} - \left(1.199 \times 10^8 \, \text{m/s}\right)\left(2.50 \, \text{s}\right)}{\sqrt{1 - \left(0.400\right)^2}}$$

$$= 2.7 \times 10^5 \, \text{m/s} \approx 0,$$

where we conclude that the numerical result (2.7×10^5 or 2.3×10^5 depending on how precise a value of v is used) is not meaningful (in the significant figures sense) and should be set equal to zero (that is, it is "consistent with zero" in view of the statistical uncertainties involved).

(b) The time coordinate measured in the S' frame is

$$t' = \gamma\left(t - \frac{vx}{c^2}\right) = \frac{t - \beta x / c}{\sqrt{1 - \beta^2}} = \frac{2.50 \, \text{s} - \left(0.400\right)\left(3.00 \times 10^8 \, \text{m}\right) / 2.998 \times 10^8 \, \text{m/s}}{\sqrt{1 - \left(0.400\right)^2}} = 2.29 \, \text{s}.$$

(c) Now, we obtain

$$x' = \frac{x + vt}{\sqrt{1 - \beta^2}} = \frac{3.00 \times 10^8 \, \text{m} + \left(1.199 \times 10^8 \, \text{m/s}\right)\left(2.50 \, \text{s}\right)}{\sqrt{1 - \left(0.400\right)^2}} = 6.54 \times 10^8 \, \text{m}.$$

(d) Similarly,

$$t' = \gamma\left(t + \frac{vx}{c^2}\right) = \frac{2.50 \, \text{s} + \left(0.400\right)\left(3.00 \times 10^8 \, \text{m}\right) / 2.998 \times 10^8 \, \text{m/s}}{\sqrt{1 - \left(0.400\right)^2}} = 3.16 \, \text{s}.$$

17. The proper time is not measured by clocks in either frame S or frame S' since a single clock at rest in either frame cannot be present at the origin and at the event. The full Lorentz transformation must be used:

$$x' = \gamma(x - vt) \quad \text{and} \quad t' = \gamma(t - \beta x/c)$$

where $\beta = v/c = 0.950$ and $\gamma = 1\sqrt{1-\beta^2} = 1/\sqrt{1-(0.950)^2} = 3.20256$. Thus,

$$x' = (3.20256)\left(100 \times 10^3\,\text{m} - (0.950)(2.998 \times 10^8\ \text{m/s})(200 \times 10^{-6}\text{s})\right)$$
$$= 1.38 \times 10^5\ \text{m} = 138\,\text{km}.$$

(b) The temporal coordinate in S' is

$$t' = (3.20256)\left[200 \times 10^{-6}\text{s} - \frac{(0.950)(100 \times 10^3\ \text{m})}{2.998 \times 10^8\ \text{m/s}}\right] = -3.74 \times 10^{-4}\ \text{s} = -374\,\mu\text{s}\ .$$

18. The "coincidence" of $x = x' = 0$ at $t = t' = 0$ is important for Eq. 37-21 to apply without additional terms. We label the event coordinates with subscripts: $(x_1, t_1) = (0, 0)$ and $(x_2, t_2) = (3000, 4.0 \times 10^{-6})$ with SI units understood.

(a) We expect $(x'_1, t'_1) = (0, 0)$, and this may be verified using Eq. 37-21.

(b) We now compute (x'_2, t'_2), assuming $v = +0.60c = +1.799 \times 10^8$ m/s (the sign of v is not made clear in the problem statement, but the Figure referred to, Fig. 37-9, shows the motion in the positive x direction).

$$x'_2 = \frac{x - vt}{\sqrt{1-\beta^2}} = \frac{3000 - (1.799\times10^8)(4.0\times10^{-6})}{\sqrt{1-(0.60)^2}} = 2.85\times10^3$$

$$t'_2 = \frac{t - \beta x/c}{\sqrt{1-\beta^2}} = \frac{4.0\times10^{-6} - (0.60)(3000)/(2.998\times10^8)}{\sqrt{1-(0.60)^2}} = -2.5\times10^{-6}$$

(c) The two events in frame S occur in the order: first 1, then 2. However, in frame S' where $t'_2 < 0$, they occur in the reverse order: first 2, then 1. So the two observers see the two events in the reverse sequence.

We note that the distances $x_2 - x_1$ and $x'_2 - x'_1$ are larger than how far light can travel during the respective times $(c(t_2 - t_1) = 1.2$ km and $c|t'_2 - t'_1| \approx 750\,\text{m})$, so that no inconsistencies arise as a result of the order reversal (that is, no signal from event 1 could arrive at event 2 or vice versa).

19. (a) We take the flashbulbs to be at rest in frame S, and let frame S' be the rest frame of the second observer. Clocks in neither frame measure the proper time interval between the flashes, so the full Lorentz transformation (Eq. 37-21) must be used. Let t_s be the time and x_s be the coordinate of the small flash, as measured in frame S. Then, the time of the small flash, as measured in frame S', is

$$t_s' = \gamma\left(t_s - \frac{\beta x_s}{c}\right)$$

where $\beta = v/c = 0.250$ and

$$\gamma = 1/\sqrt{1-\beta^2} = 1/\sqrt{1-(0.250)^2} = 1.0328.$$

Similarly, let t_b be the time and x_b be the coordinate of the big flash, as measured in frame S. Then, the time of the big flash, as measured in frame S', is

$$t_b' = \gamma\left(t_b - \frac{\beta x_b}{c}\right).$$

Subtracting the second Lorentz transformation equation from the first and recognizing that $t_s = t_b$ (since the flashes are simultaneous in S), we find

$$\Delta t' = \frac{\gamma\beta(x_s - x_b)}{c} = \frac{(1.0328)(0.250)(30\times10^3\text{ m})}{3.00\times10^8\text{ m/s}} = 2.58\times10^{-5}\text{ s}$$

where $\Delta t' = t_b' - t_s'$.

(b) Since $\Delta t'$ is negative, t_b' is greater than t_s' . The small flash occurs first in S'.

20. We refer to the solution of problem 18. We wish to adjust Δt so that

$$\Delta x' = 0 = \gamma(-720\,\text{m} - v\Delta t)$$

in the limiting case of $|v| \to c$. Thus,

$$\Delta t = \frac{720\,\text{m}}{2.998 \times 10^8\,\text{m/s}} = 2.40 \times 10^{-6}\,\text{s} \ .$$

21. (a) The Lorentz factor is

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} = \frac{1}{\sqrt{1-(0.600)^2}} = 1.25 \,.$$

(b) In the unprimed frame, the time for the clock to travel from the origin to $x = 180$ m is

$$t = \frac{x}{v} = \frac{180\,\text{m}}{(0.600)(3.00\times10^8\,\text{m/s})} = 1.00\times10^{-6}\,\text{s} \,.$$

The proper time interval between the two events (at the origin and at $x = 180$ m) is measured by the clock itself. The reading on the clock at the beginning of the interval is zero, so the reading at the end is

$$t' = \frac{t}{\gamma} = \frac{1.00\times10^{-6}\,\text{s}}{1.25} = 8.00\times10^{-7}\,\text{s} \,.$$

22. The time-dilation information in the problem (particularly, the 15 s on "his wristwatch… which takes 30.0 s according to you") reveals Lorentz factor is $\gamma = 2.00$ (see Eq. 37-9), which implies his speed is $v = 0.866c$.

(a) With $\gamma = 2.00$, Eq. 37-13 implies the contracted length is 0.500 m.

(b) There is no contraction along direction perpendicular to the direction of motion (or "boost" direction), so meter stick 2 still measures 1.00 m long.

(c) As in part (b), the answer is 1.00 m.

(d) Eq. 1′ in Table 37-2 gives

$$x_2' - x_1' = \gamma(\Delta x' - v\Delta t') = (2.00)[20.0 - (0.866)(2.998 \times 10^8)(40.0 \times 10^{-9})] = 19.2 \text{ m}.$$

(e) Eq. 2′ in Table 37-2 gives

$$t_2' - t_1' = \gamma(\Delta t' - v\Delta x'/c^2) = (2.00)[40.0 \times 10^{-9} - (0.866)(20.0)/(2.998 \times 10^8)^2]$$

which yields −35.5 ns. In absolute value, the two events are separated by 35.5 ns.

(f) The negative sign obtained in part (e) implies event 2 occurred before event 1.

23. (a) In frame S, our coordinates are such that $x_1 = +1200$ m for the big flash, and $x_2 = 1200 - 720 = 480$ m for the small flash (which occurred later). Thus,

$$\Delta x = x_2 - x_1 = -720 \text{ m}.$$

If we set $\Delta x' = 0$ in Eq. 37-25, we find

$$0 = \gamma(\Delta x - v\Delta t) = \gamma\left(-720\,\text{m} - v(5.00 \times 10^{-6}\,\text{s})\right)$$

which yields $v = -1.44 \times 10^8$ m/s, or $\beta = v/c = 0.480$.

(b) The negative sign in part (a) implies that frame S' must be moving in the $-x$ direction.

(c) Eq. 37-28 leads to

$$\Delta t' = \gamma\left(\Delta t - \frac{v\Delta x}{c^2}\right) = \gamma\left(5.00 \times 10^{-6}\,\text{s} - \frac{(-1.44 \times 10^8\,\text{m}/\text{s})(-720\,\text{m})}{(2.998 \times 10^8\,\text{m}/\text{s})^2}\right)$$

which turns out to be positive (regardless of the specific value of γ). Thus, the order of the flashes is the same in the S' frame as it is in the S frame (where Δt is also positive). Thus, the big flash occurs first, and the small flash occurs later.

(d) Finishing the computation begun in part (c), we obtain

$$\Delta t' = \frac{5.00 \times 10^{-6}\,\text{s} - (-1.44 \times 10^8\,\text{m/s})(-720\,\text{m})/(2.998 \times 10^8\,\text{m/s})^2}{\sqrt{1 - 0.480^2}} = 4.39 \times 10^{-6}\,\text{s}.$$

24. From Eq. 2 in Table 37-2, we have $\Delta t = v\,\gamma\,\Delta x'/c^2 + \gamma\,\Delta t'$. The coefficient of $\Delta x'$ is the slope (4.0 μs / 400 m) of the graph, and the last term involving $\Delta t'$ is the "y-intercept" of the graph. From the first observation, we can solve for $\beta = v/c = 0.949$ and consequently $\gamma = 3.16$. Then, from the second observation, we find
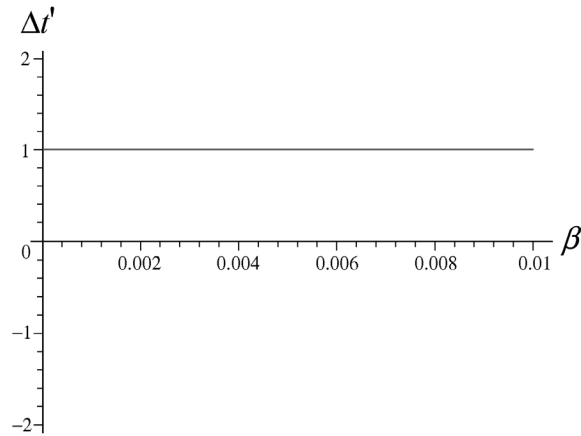
$$\Delta t' = (2.0\ \mu s)/\gamma = 0.63\ \mu s.$$

25. (a) Eq. 2′ of Table 37-2, with time in microseconds, becomes

$$\Delta t' = \gamma(\Delta t - \beta \Delta x/c) = \gamma[1.00 - \beta(400/299.8)]$$

where the Lorentz factor is itself a function of β (see Eq. 37-8).

(b) A plot of $\Delta t'$ as a function of β in the range $0 < \beta < 0.01$ is shown below:

Note the limits of the vertical axis are +2 μs and −2 μs. We note how "flat" the curve is in this graph; the reason is that for low values of β, Bullwinkle's measure of the temporal separation between the two events is approximately our measure, namely +1.0 μs. There are no non-intuitive relativistic effects in this case.

(c) A plot of $\Delta t'$ as a function of β in the range $0.1 < \beta < 1$ is shown below:
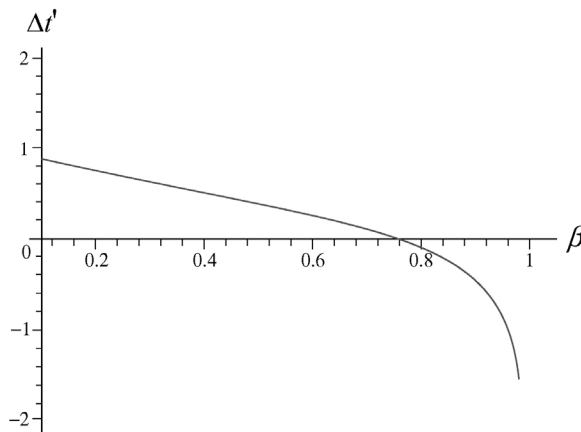
(d) Setting

$$\Delta t' = \gamma(\Delta t - \beta \Delta x/c) = \gamma[1.00 - \beta(400/299.8)] = 0,$$

leads to $\beta = 299.8/400 \approx 0.750$.

(e) For the graph shown in part (c), that as we increase the speed, the temporal separation according to Bullwinkle is positive for the lower values and then goes to zero and finally (as the speed approaches that of light) becomes progressively more negative. For the lower speeds with $\Delta t' > 0 \Rightarrow t_A' < t_B'$, or $0 < \beta < 0.750$, according to Bullwinkle event A occurs before event B just as we observe.

(f) For the higher speeds with $\Delta t' < 0 \Rightarrow t_A' > t_B'$, or $0.750 < \beta < 1$, according to Bullwinkle event B occurs before event A (the opposite of what we observe).

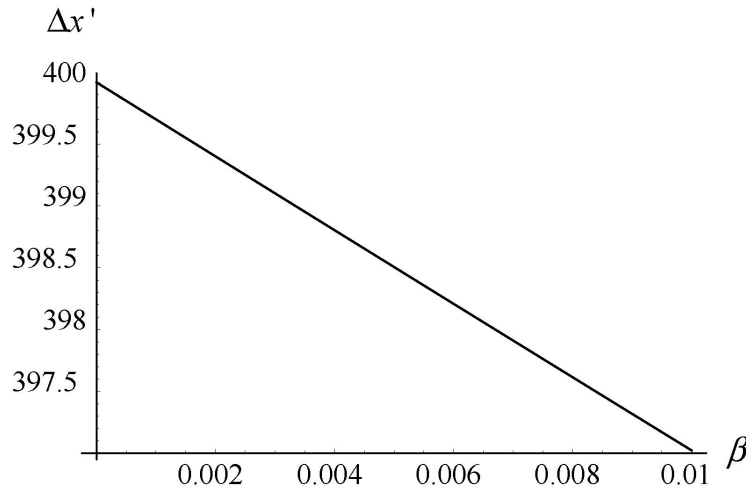(g) No, event A cannot cause event B or vice versa. We note that

$$\Delta x/\Delta t = (400 \text{ m})/(1.00 \ \mu s) = 4.00 \times 10^8 \text{ m/s} > c.$$

A signal cannot travel from event A to event B without exceeding c, so causal influences cannot originate at A and thus affect what happens at B, or vice versa.
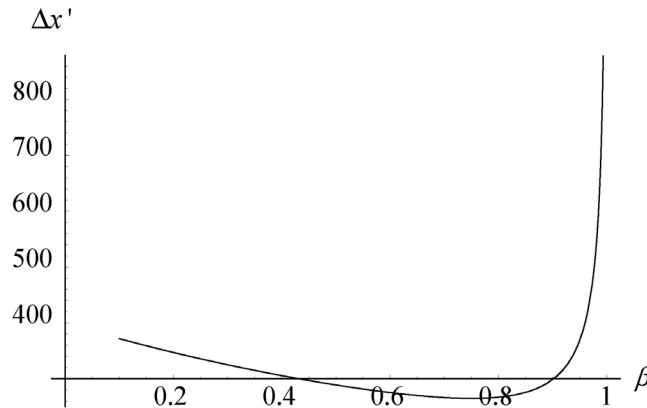
26. (a) From Table 37-2, we find

$$\Delta x' = \gamma(\Delta x - \beta c\Delta t) = \gamma[400 \text{ m} - \beta c(1.00 \text{ } \mu s)] = \frac{400 \text{ m} - (299.8 \text{ m})\beta}{\sqrt{1-\beta^2}}$$

(b) A plot of $\Delta x'$ as a function of β with $0 < \beta < 0.01$ is shown below:

(c) A plot of $\Delta x'$ as a function of β with $0.1 < \beta < 1$ is shown below:

(d) To find the minimum, we can take a derivative of $\Delta x'$ with respect to β, simplify, and then set equal to zero:

$$\frac{d \Delta x'}{d \beta} = \gamma^3(\beta\Delta x - c\Delta t) = 0 \quad \Rightarrow \quad \beta = \frac{c\Delta t}{\Delta x} = 0.7495 \approx 0.750.$$

(e) Substituting this value of β into the part (a) expression yields $\Delta x' = 264.8$ m ≈ 265 m for its minimum value.

27. We assume S' is moving in the $+x$ direction. With $u' = +0.40c$ and $v = +0.60c$, Eq. 37-29 yields

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.40c + 0.60c}{1 + (0.40c)(+0.60c)/c^2} = 0.81c \ .$$

28. Using the notation of Eq. 37-29 and taking "away" (from us) as the positive direction, the problem indicates $v = +0.4c$ and $u = +0.8c$ (with 3 significant figures understood). We solve for the velocity of Q_2 relative to Q_1 (in multiple of c):

$$\frac{u'}{c} = \frac{u/c - v/c}{1 - uv/c^2} = \frac{0.8 - 0.4}{1 - (0.8)(0.4)} = 0.588$$

in a direction away from Earth.

29. (a) One thing Einstein's relativity has in common with the more familiar (Galilean) relativity is the reciprocity of relative velocity. If Joe sees Fred moving at 20 m/s eastward away from him (Joe), then Fred should see Joe moving at 20 m/s westward away from him (Fred). Similarly, if we see Galaxy A moving away from us at $0.35c$ then an observer in Galaxy A should see our galaxy move away from him at $0.35c$, or 0.35 in multiple of c.

(b) We take the positive axis to be in the direction of motion of Galaxy A, as seen by us. Using the notation of Eq. 37-29, the problem indicates $v = +0.35c$ (velocity of Galaxy A relative to Earth) and $u = -0.35c$ (velocity of Galaxy B relative to Earth). We solve for the velocity of B relative to A:

$$\frac{u'}{c} = \frac{u/c - v/c}{1 - uv/c^2} = \frac{(-0.35) - 0.35}{1 - (-0.35)(0.35)} = -0.62 \, ,$$

or $|u'/c| = 0.62$.

30. (a) We use Eq. 37-29:

$$v = \frac{v'+u}{1+uv'/c^2} = \frac{0.47c+0.62c}{1+(0.47)(0.62)} = 0.84c \ ,$$

in the direction of increasing x (since $v > 0$). In unit-vector notation, we have $\vec{v} = (0.84c)\hat{i}$.

(b) The classical theory predicts that $v = 0.47c + 0.62c = 1.1c$, or $\vec{v} = (1.1c)\hat{i}$

(c) Now $v' = -0.47c\,\hat{i}$ so

$$v = \frac{v'+u}{1+uv'/c^2} = \frac{-0.47c+0.62c}{1+(-0.47)(0.62)} = 0.21c \ ,$$

or $\vec{v} = (0.21c)\hat{i}$

(d) By contrast, the classical prediction is $v = 0.62c - 0.47c = 0.15c$, or $\vec{v} = (0.15c)\hat{i}$

31. Using the notation of Eq. 37-29 and taking the micrometeorite motion as the positive direction, the problem indicates $v = -0.82c$ (spaceship velocity) and $u = +0.82c$ (micrometeorite velocity). We solve for the velocity of the micrometeorite relative to the spaceship:

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{0.82c - (-0.82c)}{1 - (0.82)(-0.82)} = 0.98c$$

or 2.94×10^8 m/s. Using Eq. 37-10, we conclude that observers on the ship measure a transit time for the micrometeorite (as it passes along the length of the ship) equal to

$$\Delta t = \frac{d}{u'} = \frac{350\,\text{m}}{2.94 \times 10^8\,\text{m/s}} = 1.2 \times 10^{-6}\,\text{s}\,.$$

32. The Figure shows that $u' = 0.80c$ when $v = 0$. We therefore infer (using the notation of Eq. 37-29) that $u = 0.80c$. Now, u is a fixed value and v is variable, so u' as a function of v is given by

$$u' = \frac{0.80c - v}{1 - 0.80\, v/c}$$

which is Eq. 37-29 rearranged so that u' is isolated on the left-hand side. We use this expression to answer parts (a) and (b).

(a) Substituting $v = 0.90c$ in the expression above leads to $u' = -0.357c \approx -0.36c$.

(b) Substituting $v = c$ in the expression above leads to $u' = -c$ (regardless of the value of u).

33. (a) In the messenger's rest system (called S_m), the velocity of the armada is

$$v' = \frac{v - v_m}{1 - vv_m / c^2} = \frac{0.80c - 0.95c}{1 - (0.80c)(0.95c)/c^2} = -0.625c .$$

The length of the armada as measured in S_m is

$$L_1 = \frac{L_0}{\gamma_{v'}} = (1.0\,\text{ly})\sqrt{1 - (-0.625)^2} = 0.781 \text{ ly} .$$

Thus, the length of the trip is

$$t' = \frac{L'}{|v'|} = \frac{0.781\,\text{ly}}{0.625c} = 1.25 \text{ y} .$$

(b) In the armada's rest frame (called S_a), the velocity of the messenger is

$$v' = \frac{v - v_a}{1 - vv_a / c^2} = \frac{0.95c - 0.80c}{1 - (0.95c)(0.80c)/c^2} = 0.625c .$$

Now, the length of the trip is

$$t' = \frac{L_0}{v'} = \frac{1.0\,\text{ly}}{0.625c} = 1.60 \text{ y} .$$

(c) Measured in system S, the length of the armada is

$$L = \frac{L_0}{\gamma} = 1.0\,\text{ly}\sqrt{1 - (0.80)^2} = 0.60 \text{ ly} ,$$

so the length of the trip is

$$t = \frac{L}{v_m - v_a} = \frac{0.60\,\text{ly}}{0.95c - 0.80c} = 4.00 \text{ y} .$$

34. (a) Eq. 37-34 leads to

$$v = \frac{\Delta\lambda}{\lambda}c = \frac{12.00\,\text{nm}}{513.0\,\text{nm}}(2.998\times10^8\,\text{m/s}) = 7.000\times10^6\,\text{m/s}.$$

(b) The line is shifted to a larger wavelength, which means shorter frequency. Recalling Eq. 37-31 and the discussion that follows it, this means galaxy NGC is moving away from Earth.

35. The spaceship is moving away from Earth, so the frequency received is given directly by Eq. 37-31. Thus,

$$f = f_0 \sqrt{\frac{1-\beta}{1+\beta}} = (100 \, \text{MHz}) \sqrt{\frac{1-0.9000}{1+0.9000}} = 22.9 \, \text{MHz} \ .$$

36. (a) Eq. 37-34 leads to a speed of

$$v = \frac{\Delta\lambda}{\lambda}c = (0.004)(3.0\times10^8\,\text{m/s}) = 1.2\times10^6\,\text{m/s} \approx 1\times10^6\,\text{m/s}.$$

(b) The galaxy is receding.

37. We obtain

$$v = \frac{\Delta\lambda}{\lambda} c = \left(\frac{620 - 540}{620} \right) c = 0.13c.$$

38. We use the transverse Doppler shift formula, Eq. 37-37: $f = f_0\sqrt{1-\beta^2}$, or

$$\frac{1}{\lambda} = \frac{1}{\lambda_0}\sqrt{1-\beta^2}\,.$$

We solve for $\lambda - \lambda_0$:

$$\lambda - \lambda_0 = \lambda_0\left(\frac{1}{\sqrt{1-\beta^2}} - 1\right) = (589.00\,\text{mm})\left[\frac{1}{\sqrt{1-(0.100)^2}} - 1\right] = +2.97\,\text{nm}\,.$$

39. (a) The frequency received is given by

$$f = f_0 \sqrt{\frac{1-\beta}{1+\beta}} \quad \Rightarrow \quad \frac{c}{\lambda} = \frac{c}{\lambda_0}\sqrt{\frac{1-0.20}{1+0.20}}$$

which implies

$$\lambda = (450\,\text{nm})\sqrt{\frac{1+0.20}{1-0.20}} = 550\ \text{nm} \ .$$

(b) This is in the yellow portion of the visible spectrum.

40. (a) The work-kinetic energy theorem applies as well to Einsteinian physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K = m_e c^2 (\gamma - 1)$ (Eq. 37-52) and $m_e c^2 = 511$ keV $= 0.511$ MeV (Table 37-3), and obtain

$$W = m_e c^2 \left(\frac{1}{\sqrt{1 - \beta^2}} - 1 \right) = (511 \,\text{keV}) \left[\frac{1}{\sqrt{1 - (0.500)^2}} - 1 \right] = 79.1 \text{ keV} .$$

(b) $W = (0.511 \,\text{MeV}) \left(\dfrac{1}{\sqrt{1 - (0.990)^2}} - 1 \right) = 3.11$ MeV.

(c) $W = (0.511 \,\text{MeV}) \left(\dfrac{1}{\sqrt{1 - (0.990)^2}} - 1 \right) = 10.9$ MeV.

41. (a) From Eq. 37-52, $\gamma = (K/mc^2) + 1$, and from Eq. 37-8, the speed parameter is $\beta = \sqrt{1-(1/\gamma)^2}$. Table 37-3 gives $m_e c^2 = 511$ keV $= 0.511$ MeV, so the Lorentz factor is

$$\gamma = \frac{100\,\text{MeV}}{0.511\text{MeV}} + 1 = 196.695.$$

(b) The speed parameter is

$$\beta = \sqrt{1 - \frac{1}{(196.695)^2}} = 0.999987.$$

Thus, the speed of the electron is $0.999987c$, or 99.9987% of the speed of light.

42. The mass change is

$$\Delta M = \left(4.002603\,\text{u} + 15.994915\,\text{u}\right) - \left(1.007825\,\text{u} + 18.998405\text{u}\right) = -0.008712\text{u}.$$

Using Eq. 37-50 and Eq. 37-46, this leads to

$$Q = -\Delta M c^2 = -\left(-0.008712\,\text{u}\right)\left(931.5\,\text{MeV}/\text{u}\right) = 8.12\,\text{MeV}.$$

43. (a) The work-kinetic energy theorem applies as well to Einsteinian physics as to Newtonian; the only difference is the specific formula for kinetic energy. Thus, we use $W = \Delta K$ where $K = m_e c^2 (\gamma - 1)$ (Eq. 37-52), and $m_e c^2 = 511$ keV $= 0.511$ MeV (Table 37-3). Noting that $\Delta K = m_e c^2 (\gamma_f - \gamma_i)$, we obtain

$$W = m_e c^2 \left(\frac{1}{\sqrt{1-\beta_f^2}} - \frac{1}{\sqrt{1-\beta_i^2}} \right) = (511 \,\text{keV}) \left(\frac{1}{\sqrt{1-(0.19)^2}} - \frac{1}{\sqrt{1-(0.18)^2}} \right)$$

$$= 0.996 \ \text{keV} \approx 1.0 \ \text{keV}.$$

(b) Similarly,

$$W = (511 \,\text{keV}) \left(\frac{1}{\sqrt{1-(0.99)^2}} - \frac{1}{\sqrt{1-(0.98)^2}} \right) = 1055 \,\text{keV} \approx 1.1 \ \text{MeV}.$$

We see the dramatic increase in difficulty in trying to accelerate a particle when its initial speed is very close to the speed of light.

44. From Eq. 28-37, we have

$$Q = -\Delta Mc^2 = -(3(4.00151\text{u}) - 11.99671\text{u})c^2 = -(0.00782\text{u})(931.5\text{MeV/u})$$
$$= -7.28\text{Mev}.$$

Thus, it takes a minimum of 7.28 MeV supplied to the system to cause this reaction. We note that the masses given in this problem are strictly for the nuclei involved; they are not the "atomic" masses which are quoted in several of the other problems in this chapter.

45. (a) The strategy is to find the γ factor from $E = 14.24 \times 10^{-9}$ J and $m_p c^2 = 1.5033 \times 10^{-10}$ J and from that find the contracted length. From the energy relation (Eq. 37-48), we obtain

$$\gamma = \frac{E}{mc^2} = 94.73.$$

Consequently, Eq. 37-13 yields

$$L = \frac{L_0}{\gamma} = 0.222 \text{ cm} = 2.22 \times 10^{-3} \text{ m}.$$

(b) The time dilation formula (Eq. 37-7) leads to

$$\Delta t = \gamma \Delta t_0 = 7.01 \times 10^{-10} \text{ s}$$

which can be checked using $\Delta t = L_0/v$ in our frame of reference.

(c) From the γ factor, we find the speed:

$$v = c\sqrt{1 - \left(\frac{1}{\gamma}\right)^2} = 0.99994c.$$

Therefore, the trip (according to the proton) took

$$\Delta t_0 = 2.22 \times 10^{-3}/0.99994c = 7.40 \times 10^{-12} \text{ s}.$$

46. (a) From the information in the problem, we see that each kilogram of TNT releases $(3.40 \times 10^6$ J/mol$)/(0.227$ kg/mol$) = 1.50 \times 10^7$ J. Thus,

$$(1.80 \times 10^{14} \text{ J})/(1.50 \times 10^7 \text{ J/kg}) = 1.20 \times 10^7 \text{ kg}$$

of TNT are needed. This is equivalent to a weight of $\approx 1.2 \times 10^8$ N.

(b) This is certainly more than can be carried in a backpack. Presumably, a train would be required.

(c) We have $0.00080mc^2 = 1.80 \times 10^{14}$ J, and find $m = 2.50$ kg of fissionable material is needed. This is equivalent to a weight of about 25 N, or 5.5 pounds.

(d) This can be carried in a backpack.

47. We set Eq. 37-55 equal to $(3.00mc^2)^2$, as required by the problem, and solve for the speed. Thus,

$$\left(pc\right)^2 + \left(mc^2\right)^2 = 9.00\left(mc^2\right)^2$$

leads to $p = mc\sqrt{8} \approx 2.83mc$.

48. (a) Using $K = m_ec^2(\gamma - 1)$ (Eq. 37-52) and

$$m_ec^2 = 510.9989 \text{ keV} = 0.5109989 \text{ MeV},$$

we obtain

$$\gamma = \frac{K}{m_ec^2} + 1 = \frac{1.0000000 \text{ keV}}{510.9989 \text{ keV}} + 1 = 1.00195695 \approx 1.0019570.$$

(b) Therefore, the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = \sqrt{1 - \frac{1}{(1.0019570)^2}} = 0.062469542.$$

(c) For $K = 1.0000000 \text{ MeV}$, we have

$$\gamma = \frac{K}{m_ec^2} + 1 = \frac{1.0000000 \text{ MeV}}{0.5109989 \text{ MeV}} + 1 = 2.956951375 \approx 2.9569514.$$

(d) The corresponding speed parameter is

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.941079236 \approx 0.94107924.$$

(e) For $K = 1.0000000 \text{ GeV}$, we have

$$\gamma = \frac{K}{m_ec^2} + 1 = \frac{1000.0000 \text{ MeV}}{0.5109989 \text{ MeV}} + 1 = 1957.951375 \approx 1957.9514.$$

(f) The corresponding speed parameter is

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.99999987$$

49. Since the rest energy E_0 and the mass m of the quasar are related by $E_0 = mc^2$, the rate P of energy radiation and the rate of mass loss are related by $P = dE_0/dt = (dm/dt)c^2$. Thus,

$$\frac{dm}{dt} = \frac{P}{c^2} = \frac{1 \times 10^{41}\ \text{W}}{\left(2.998 \times 10^8\ \text{m/s}\right)^2} = 1.11 \times 10^{24}\ \text{kg/s}.$$

Since a solar mass is 2.0×10^{30} kg and a year is 3.156×10^7 s,

$$\frac{dm}{dt} = \left(1.11 \times 10^{24}\ \text{kg/s}\right)\left(\frac{3.156 \times 10^7\ \text{s/y}}{2.0 \times 10^{30}\ \text{kg/smu}}\right) \approx 18\,\text{smu/y}.$$

50. From Eq. 37-52, $\gamma = (K/mc^2) + 1$, and from Eq. 37-8, the speed parameter is $\beta = \sqrt{1 - (1/\gamma)^2}$.

(a) Table 37-3 gives $m_e c^2 = 511$ keV $= 0.511$ MeV, so the Lorentz factor is

$$\gamma = \frac{10.00\,\text{MeV}}{0.5110\,\text{MeV}} + 1 = 20.57,$$

(b) and the speed parameter is

$$\beta = \sqrt{1 - \frac{1}{(20.57)^2}} = 0.9988.$$

(c) Using $m_p c^2 = 938.272$ MeV, the Lorentz factor is

$$\gamma = 1 + 10.00 \text{ MeV}/938.272 \text{ MeV} = 1.01065 \approx 1.011.$$

(d) The speed parameter is

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.144844 \approx 0.1448.$$

(e) With $m_\alpha c^2 = 3727.40$ MeV, we obtain $\gamma = 10.00/3727.4 + 1 = 1.00268 \approx 1.003$.

(f) The speed parameter is

$$\beta = \sqrt{1 - \gamma^{-2}} = 0.0731037 \approx 0.07310.$$

51. (a) We set Eq. 37-41 equal to mc, as required by the problem, and solve for the speed. Thus,

$$\frac{mv}{\sqrt{1-v^2/c^2}} = mc$$

leads to $\beta = 1/\sqrt{2} = 0.707$.

(b) Substituting $\beta = 1/\sqrt{2}$ into the definition of γ, we obtain

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{1}{\sqrt{1-(1/2)}} = \sqrt{2} \approx 1.41.$$

(c) The kinetic energy is

$$K = (\gamma-1)mc^2 = \left(\sqrt{2}-1\right)mc^2 = 0.414mc^2 = 0.414E_0.$$

which implies $K/E_0 = 0.414$.

52. (a) We set Eq. 37-52 equal to $2mc^2$, as required by the problem, and solve for the speed. Thus,

$$mc^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right) = 2mc^2$$

leads to $\beta = 2\sqrt{2}/3 \approx 0.943$.

(b) We now set Eq. 37-48 equal to $2mc^2$ and solve for the speed. In this case,

$$\frac{mc^2}{\sqrt{1-\beta^2}} = 2mc^2$$

leads to $\beta = \sqrt{3}/2 \approx 0.866$.

53. The energy equivalent of one tablet is

$$mc^2 = (320 \times 10^{-6} \text{ kg}) (3.00 \times 10^8 \text{ m/s})^2 = 2.88 \times 10^{13} \text{ J}.$$

This provides the same energy as

$$(2.88 \times 10^{13} \text{ J})/(3.65 \times 10^7 \text{ J/L}) = 7.89 \times 10^5 \text{ L}$$

of gasoline. The distance the car can go is

$$d = (7.89 \times 10^5 \text{ L}) (12.75 \text{ km/L}) = 1.01 \times 10^7 \text{ km}.$$

This is roughly 250 times larger than the circumference of Earth (see Appendix C).

54. (a) Squaring Eq. 37-47 gives

$$E^2 = \left(mc^2\right)^2 + 2mc^2 K + K^2$$

which we set equal to Eq. 37-55. Thus,

$$\left(mc^2\right)^2 + 2mc^2 K + K^2 = \left(pc\right)^2 + \left(mc^2\right)^2 \Rightarrow m = \frac{\left(pc\right)^2 - K^2}{2Kc^2}.$$

(b) At low speeds, the pre-Einsteinian expressions $p = mv$ and $K = \frac{1}{2}mv^2$ apply. We note that $pc \gg K$ at low speeds since $c \gg v$ in this regime. Thus,

$$m \rightarrow \frac{\left(mvc\right)^2 - \left(\frac{1}{2}mv^2\right)^2}{2\left(\frac{1}{2}mv^2\right)c^2} \approx \frac{\left(mvc\right)^2}{2\left(\frac{1}{2}mv^2\right)c^2} = m.$$

(c) Here, $pc = 121$ MeV, so

$$m = \frac{121^2 - 55^2}{2(55)c^2} = 105.6\,\text{MeV}/c^2.$$

Now, the mass of the electron (see Table 37-3) is $m_e = 0.511$ MeV/c^2, so our result is roughly 207 times bigger than an electron mass, i.e., $m/m_e \approx 207$. The particle is a muon.

55. The distance traveled by the pion in the frame of Earth is (using Eq. 37-12) $d = v\Delta t$. The proper lifetime Δt_0 is related to Δt by the time-dilation formula: $\Delta t = \gamma \Delta t_0$. To use this equation, we must first find the Lorentz factor γ (using Eq. 37-48). Since the total energy of the pion is given by $E = 1.35 \times 10^5$ MeV and its mc^2 value is 139.6 MeV, then

$$\gamma = \frac{E}{mc^2} = \frac{1.35 \times 10^5 \text{ MeV}}{139.6 \text{ MeV}} = 967.05.$$

Therefore, the lifetime of the moving pion as measured by Earth observers is

$$\Delta t = \gamma \Delta t_0 = (967.1)(35.0 \times 10^{-9} \text{ s}) = 3.385 \times 10^{-5} \text{ s},$$

and the distance it travels is

$$d \approx c\Delta t = (2.998 \times 10^8 \text{ m/s})(3.385 \times 10^{-5} \text{ s}) = 1.015 \times 10^4 \text{ m} = 10.15 \text{ km}$$

where we have approximated its speed as c (note: its speed can be found by solving Eq. 37-8, which gives $v = 0.9999995c$; this more precise value for v would not significantly alter our final result). Thus, the altitude at which the pion decays is 120 km – 10.15 km = 110 km.

56. (a) The binomial theorem tells us that, for x small,

$$(1 + x)^{\nu} \approx 1 + \nu x + \tfrac{1}{2}\, \nu(\nu - 1)\, x^2$$

if we ignore terms involving x^3 and higher powers (this is reasonable since if x is small, say $x = 0.1$, then x^3 is much smaller: $x^3 = 0.001$). The relativistic kinetic energy formula, when the speed v is much smaller than c, has a term that we can apply the binomial theorem to; identifying $-\beta^2$ as x and $-1/2$ as ν, we have

$$\gamma = \left(1 - \beta^2\right)^{-1/2} \approx 1 + (-\tfrac{1}{2})(-\beta^2) + \tfrac{1}{2}\,(-\tfrac{1}{2})((-\tfrac{1}{2}) - 1)(-\beta^2)^2.$$

Substituting this into Eq. 37-52 leads to

$$K = mc^2(\gamma - 1) \approx mc^2\!\left((-\tfrac{1}{2})(-\beta^2) + \tfrac{1}{2}\,(-\tfrac{1}{2})((-\tfrac{1}{2}) - 1)(-\beta^2)^2\right)$$

which simplifies to

$$K \approx \tfrac{1}{2} mc^2 \beta^2 + \tfrac{3}{8} mc^2 \beta^4 = \tfrac{1}{2} mv^2 + \tfrac{3}{8} mv^4/c^2 .$$

(b) If we use the mc^2 value for the electron found in Table 37-3, then for $\beta = 1/20$, the classical expression for kinetic energy gives

$$K_{\text{classical}} = \tfrac{1}{2} mv^2 = \tfrac{1}{2} mc^2 \beta^2 = \tfrac{1}{2}(8.19 \times 10^{-14}\,\text{J})(1/20)^2 = 1.0 \times 10^{-16}\,\text{J} .$$

(c) The first-order correction becomes

$$K_{\text{first-order}} = \tfrac{3}{8} mv^4/c^2 = \tfrac{3}{8} mc^2 \beta^4 = \tfrac{3}{8}(8.19 \times 10^{-14}\,\text{J})(1/20)^4 = 1.9 \times 10^{-19}\,\text{J}$$

which we note is much smaller than the classical result.

(d) In this case, $\beta = 0.80 = 4/5$, and the classical expression yields

$$K_{\text{classical}} = \tfrac{1}{2} mv^2 = \tfrac{1}{2} mc^2 \beta^2 = \tfrac{1}{2}(8.19 \times 10^{-14}\,\text{J})(4/5)^2 = 2.6 \times 10^{-14}\,\text{J} .$$

(e) And the first-order correction is

$$K_{\text{first-order}} = \tfrac{3}{8} mv^4/c^2 = \tfrac{3}{8} mc^2 \beta^4 = \tfrac{3}{8}(8.19 \times 10^{-14}\,\text{J})(4/5)^4 = 1.3 \times 10^{-14}\,\text{J}$$

which is comparable to the classical result. This is a signal that ignoring the higher order terms in the binomial expansion becomes less reliable the closer the speed gets to c.

(f) We set the first-order term equal to one-tenth of the classical term and solve for β:

$$\tfrac{3}{8}mc^2 \beta^4 = \tfrac{1}{10}(\tfrac{1}{2}mc^2 \beta^2)$$

and obtain $\beta = \sqrt{2/15} \approx 0.37$.

57. Using the classical orbital radius formula $r_0 = mv/|q|B$, the period is $T_0 = 2\pi r_0/v = 2\pi m/|q|B$. In the relativistic limit, we must use

$$r = \frac{p}{|q|B} = \frac{\gamma mv}{|q|B} = \gamma r_0$$

which yields

$$T = \frac{2\pi r}{v} = \gamma \frac{2\pi m}{|q|B} = \gamma T_0$$

(b) The period T is not independent of v.

(c) We interpret the given 10.0 MeV to be the kinetic energy of the electron. In order to make use of the mc^2 value for the electron given in Table 37-3 (511 keV = 0.511 MeV) we write the classical kinetic energy formula as

$$K_{\text{classical}} = \frac{1}{2}mv^2 = \frac{1}{2}(mc^2)\left(\frac{v^2}{c^2}\right) = \frac{1}{2}(mc^2)\beta^2.$$

If $K_{\text{classical}} = 10.0$ MeV, then

$$\beta = \sqrt{\frac{2K_{\text{classical}}}{mc^2}} = \sqrt{\frac{2(10.0\,\text{MeV})}{0.511\,\text{MeV}}} = 6.256,$$

which, of course, is impossible (see the Ultimate Speed subsection of §37-2). If we use this value anyway, then the classical orbital radius formula yields

$$r = \frac{mv}{|q|B} = \frac{m\beta c}{eB} = \frac{(9.11\times10^{-31}\,\text{kg})\,(6.256)\,(2.998\times10^8\,\text{m/s})}{(1.6\times10^{-19}\,\text{C})\,(2.20\text{T})} = 4.85\times10^{-3}\,\text{m}.$$

(d) Before using the relativistically correct orbital radius formula, we must compute β in a relativistically correct way:

$$K = mc^2(\gamma-1) \Rightarrow \gamma = \frac{10.0\,\text{MeV}}{0.511\,\text{MeV}} + 1 = 20.57$$

which implies (from Eq. 37-8)

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 0.99882.$$

Therefore,

$$r = \frac{\gamma mv}{|q|B} = \frac{\gamma m\beta c}{eB} = \frac{(20.57)(9.11\times10^{-31}\,\text{kg})(0.99882)(2.998\times10^8\,\text{m/s})}{(1.6\times10^{-19}\,\text{C})(2.20\text{T})}$$
$$= 1.59\times10^{-2}\,\text{m}.$$

(e) The classical period is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(4.85\times10^{-3}\,\text{m})}{(6.256)(2.998\times10^8\,\text{m/s})} = 1.63\times10^{-11}\,\text{s}.$$

(f) The period obtained with relativistic correction is

$$T = \frac{2\pi r}{\beta c} = \frac{2\pi(0.0159\,\text{m})}{(0.99882)(2.998\times10^8\,\text{m/s})} = 3.34\times10^{-10}\,\text{s}.$$

58. (a) The proper lifetime Δt_0 is 2.20 μs, and the lifetime measured by clocks in the laboratory (through which the muon is moving at high speed) is $\Delta t = 6.90$ μs. We use Eq. 37-7 to solve for the speed parameter:

$$\beta = \sqrt{1 - \left(\frac{\Delta t_0}{\Delta t}\right)^2} = 0.948 .$$

(b) From the answer to part (a), we find $\gamma = 3.136$. Thus, with (see Table 37-3)

$$m_\mu c^2 = 207 m_e c^2 = 105.8 \text{ MeV},$$

Eq. 37-52 yields

$$K = m_\mu c^2 (\gamma - 1) = 226 \text{ MeV}.$$

(c) We write $m_\mu c = 105.8$ MeV/c and apply Eq. 37-41:

$$p = \gamma m_\mu v = \gamma m_\mu c \beta = (3.136)(105.8 \text{ MeV} / c)(0.9478) = 314 \text{ MeV} / c$$

which can also be expressed in SI units ($p = 1.7 \times 10^{-19}$ kg·m/s).

59. (a) Before looking at our solution to part (a) (which uses momentum conservation), it might be advisable to look at our solution (and accompanying remarks) for part (b) (where a very different approach is used). Since momentum is a vector, its conservation involves two equations (along the original direction of alpha particle motion, the x direction, as well as along the final proton direction of motion, the y direction). The problem states that all speeds are much less than the speed of light, which allows us to use the classical formulas for kinetic energy and momentum ($K = \frac{1}{2}mv^2$ and $\vec{p} = m\vec{v}$, respectively). Along the x and y axes, momentum conservation gives (for the components of $\vec{v}_{\text{oxy}}$):

$$m_\alpha v_\alpha = m_{\text{oxy}} v_{\text{oxy},x} \Rightarrow v_{\text{oxy},x} = \frac{m_\alpha}{m_{\text{oxy}}} v_\alpha \approx \frac{4}{17} v_\alpha$$

$$0 = m_{\text{oxy}} v_{\text{oxy},y} + m_p v_p \Rightarrow v_{\text{oxy},y} = -\frac{m_p}{m_{\text{oxy}}} v_p \approx -\frac{1}{17} v_p.$$

To complete these determinations, we need values (inferred from the kinetic energies given in the problem) for the initial speed of the alpha particle (v_α) and the final speed of the proton (v_p). One way to do this is to rewrite the classical kinetic energy expression as $K = \frac{1}{2}(mc^2)\beta^2$ and solve for β (using Table 37-3 and/or Eq. 37-46). Thus, for the proton, we obtain

$$\beta_p = \sqrt{\frac{2K_p}{m_p c^2}} = \sqrt{\frac{2(4.44 \text{ MeV})}{938 \text{ MeV}}} = 0.0973.$$

This is almost 10% the speed of light, so one might worry that the relativistic expression (Eq. 37-52) should be used. If one does so, one finds $\beta_p = 0.969$, which is reasonably close to our previous result based on the classical formula. For the alpha particle, we write

$$m_\alpha c^2 = (4.0026 \text{ u})(931.5 \text{ MeV/u}) = 3728 \text{ MeV}$$

(which is actually an overestimate due to the use of the "atomic mass" value in our calculation, but this does not cause significant error in our result), and obtain

$$\beta_\alpha = \sqrt{\frac{2K_\alpha}{m_\alpha c^2}} = \sqrt{\frac{2(7.70 \text{ MeV})}{3728 \text{ MeV}}} = 0.064.$$

Returning to our oxygen nucleus velocity components, we are now able to conclude:

$$v_{oxy,x} \approx \frac{4}{17}v_\alpha \Rightarrow \beta_{oxy,x} \approx \frac{4}{17}\beta_\alpha = \frac{4}{17}(0.064) = 0.015$$

$$|v_{oxy,y}| \approx \frac{1}{17}v_p \Rightarrow \beta_{oxy,y} \approx \frac{1}{17}\beta_p = \frac{1}{17}(0.097) = 0.0057$$

Consequently, with $m_{oxy}c^2 \approx (17\text{ u})(931.5\text{ MeV/u}) = 1.58 \times 10^4$ MeV, we obtain

$$K_{oxy} = \frac{1}{2}(m_{oxy}c^2)(\beta^2_{oxy,x} + \beta^2_{oxy,y}) = \frac{1}{2}(1.58\times10^4 \text{ MeV})(0.015^2 + 0.0057^2) \approx 2.08 \text{ MeV}.$$

(b) Using Eq. 37-50 and Eq. 37-46,

$$Q = -(1.007825\,\text{u} + 16.99914\,\text{u} - 4.00260\,\text{u} - 14.00307\,\text{u})c^2$$
$$= -(0.001295\,\text{u})(931.5\,\text{MeV/u})$$

which yields $Q = -1.206$ MeV ≈ -1.21 MeV. Incidentally, this provides an alternate way to obtain the answer (and a more accurate one at that!) to part (a). Eq. 37-49 leads to

$$K_{oxy} = K_\alpha + Q - K_p = 7.70\,\text{MeV} - 1206\,\text{MeV} - 4.44\,\text{MeV}$$
$$= 2.05\,\text{MeV}.$$

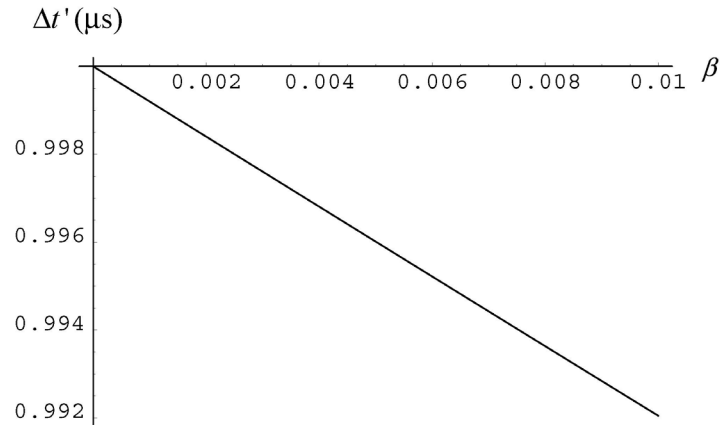This approach to finding K_{oxy} avoids the many computational steps and approximations made in part (a).
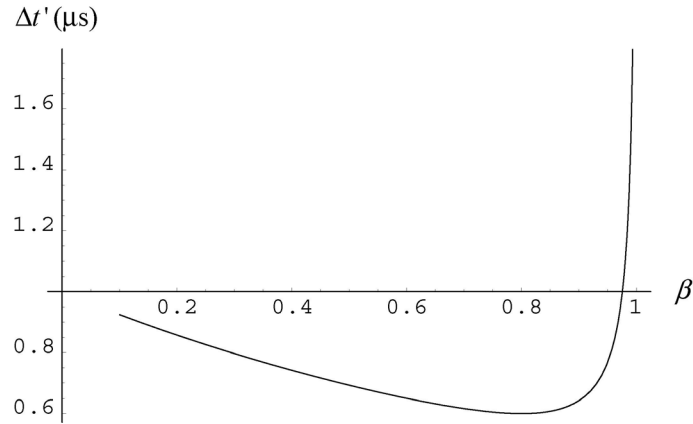
60. (a) Eq. 2′ of Table 37-2, becomes

$$\Delta t' = \gamma(\Delta t - \beta \Delta x/c) = \gamma[1.00 \ \mu s - \beta(240 \ m)/(2.998 \times 10^2 \ m/\mu s \ )]$$
$$= \gamma(1.00 - 0.800\beta) \ \mu s$$

where the Lorentz factor is itself a function of β (see Eq. 37-8).

(b) A plot of $\Delta t'$ is shown for the range $0 < \beta < 0.01$:

(c) A plot of $\Delta t'$ is shown for the range $0.1 < \beta < 1$:

(d) The minimum for the $\Delta t'$ curve can be found from by taking the derivative and simplifying and then setting equal to zero:

$$\frac{d \Delta t'}{d\beta} = \gamma^3(\beta\Delta t - \Delta x/c) = 0 \ .$$

Thus, the value of β for which the curve is minimum is $\beta = \Delta x/c\Delta t = 240/299.8$, or $\beta = 0.801$.

(e) Substituting the value of β from part (d) into the part (a) expression yields the minimum value $\Delta t' = 0.599 \ \mu s$.

(f) Yes. We note that $\Delta x/\Delta t = 2.4 \times 10^8 \ m/s < c$. A signal can indeed travel from event A to event B without exceeding c, so causal influences can originate at A and thus affect what happens at B. Such events are often described as being "time-like separated" – and we see in this problem that it is (always) possible in such a situation for us to find a frame of reference (here with $\beta \approx 0.801$) where the two events will seem to be at the same location (though at different times).

61. (a) Eq. 1′ of Table 37-2 becomes $\Delta x' = \gamma(\Delta x - \beta c \Delta t) = \gamma[(240 \text{ m}) - \beta(299.8 \text{ m})]$.

(b) A plot of $\Delta x'$ for $0 < \beta < 0.01$ is shown below:
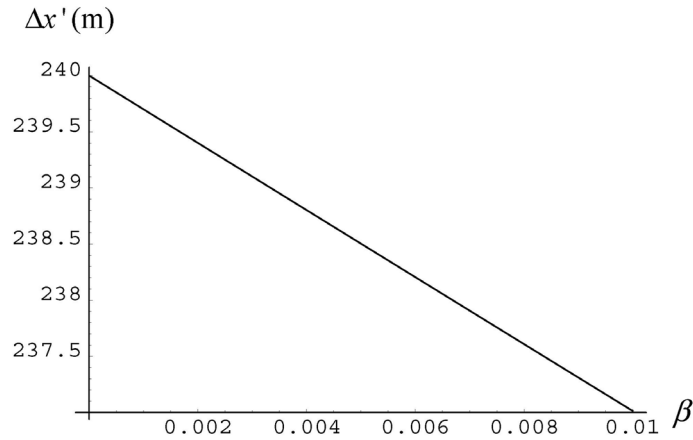
(c) A plot of $\Delta x'$ for $0.1 < \beta < 1$ is shown below:
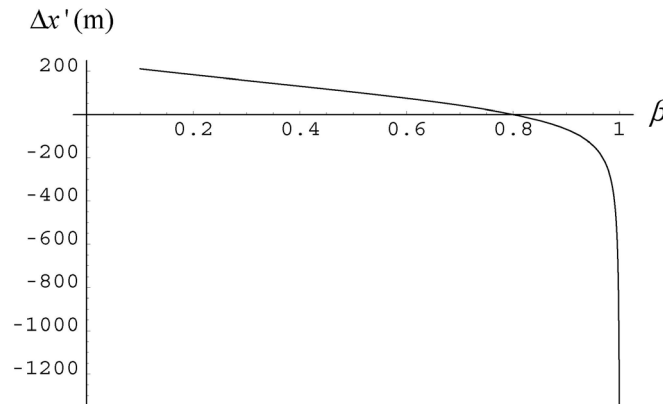
We see that $\Delta x'$ decreases from its $\beta = 0$ value (where it is equal to $\Delta x = 240$ m) to its zero value (at $\beta \approx 0.8$), and continues (without bound) downward in the graph (where it is negative – implying event B has a *smaller* value of x' than event A!).

(d) The zero value for $\Delta x'$ is easily seen (from the expression in part (b)) to come from the condition $\Delta x - \beta c \Delta t = 0$. Thus $\beta = 0.801$ provides the zero value of $\Delta x'$.

62. The line in the graph is described by Eq. 1 in Table 37-2:

$$\Delta x = v\gamma\Delta t' + \gamma\Delta x' = (\text{"slope"})\Delta t' + \text{"y-intercept"}$$

where the "slope" is 7.0×10^8 m/s. Setting this value equal to $v\gamma$ leads to $v = 2.8 \times 10^8$ m/s and $\gamma = 2.54$. Since the "y-intercept" is 2.0 m, we see that dividing this by γ leads to $\Delta x' = 0.79$ m.

63. (a) The spatial separation between the two bursts is vt. We project this length onto the direction perpendicular to the light rays headed to Earth and obtain $D_{app} = vt \sin \theta$.

(b) Burst 1 is emitted a time t ahead of burst 2. Also, burst 1 has to travel an extra distance L more than burst 2 before reaching the Earth, where $L = vt \cos \theta$ (see Fig. 37-30); this requires an additional time $t' = L/c$. Thus, the apparent time is given by

$$T_{app} = t - t' = t - \frac{vt \cos \theta}{c} = t\left[1 - \left(\frac{v}{c}\right)\cos \theta\right].$$

(c) We obtain

$$V_{app} = \frac{D_{app}}{T_{app}} = \left[\frac{(v/c)\sin \theta}{1 - (v/c)\cos \theta}\right]c = \left[\frac{(0.980)\sin 30.0°}{1 - (0.980)\cos 30.0°}\right]c = 3.24\,c.$$

64. By examining the value of u' when $v = 0$ on the graph, we infer $u = -0.20c$. Solving Eq. 37-29 for u' and inserting this value for u, we obtain

$$u' = \frac{u - v}{1 - uv/c^2} = \frac{-0.20c - v}{1 + 0.20v/c}$$

for the equation of the curve shown in the figure.

(a) With $v = 0.80c$, the above expression yields $u' = -0.86c$.

(b) As expected, setting $v = c$ in this expression leads to $u' = -c$.

65. (a) From the length contraction equation, the length L_c' of the car according to Garageman is

$$L_c' = \frac{L_c}{\gamma} = L_c\sqrt{1-\beta^2} = (30.5 \text{ m})\sqrt{1-(0.9980)^2} = 1.93 \text{ m}.$$

(b) Since the x_g axis is fixed to the garage $x_{g2} = L_g = 6.00$ m.

(c) As for t_{g2}, note from Fig. 37-32 (b) that, at $t_g = t_{g1} = 0$ the coordinate of the front bumper of the limo in the x_g frame is L_c', meaning that the front of the limo is still a distance $L_g - L_c'$ from the back door of the garage. Since the limo travels at a speed v, the time it takes for the front of the limo to reach the back door of the garage is given by

$$\Delta t_g = t_{g2} - t_{g1} = \frac{L_g - L_c'}{v} = \frac{6.00 \text{ m} - 1.93 \text{ m}}{0.9980(2.998 \times 10^8 \text{ m/s})} = 1.36 \times 10^{-8} \text{ s}.$$

Thus $t_{g2} = t_{g1} + \Delta t_g = 0 + 1.36 \times 10^{-8}$ s $= 1.36 \times 10^{-8}$ s.

(d) The limo is inside the garage between times t_{g1} and t_{g2}, so the time duration is $t_{g2} - t_{g1}$ $= 1.36 \times 10^{-8}$ s.

(e) Again from Eq. 37-13, the length L_g' of the garage according to Carman is

$$L_g' = \frac{L_g}{\gamma} = L_g\sqrt{1-\beta^2} = (6.00 \text{ m})\sqrt{1-(0.9980)^2} = 0.379 \text{ m}.$$

(f) Again, since the x_c axis is fixed to the limo $x_{c2} = L_c = 30.5$ m.

(g) Now, from the two diagrams described in part (h) below, we know that at $t_c = t_{c2}$ (when event 2 takes place), the distance between the rear bumper of the limo and the back door of the garage is given by $L_c - L_g'$. Since the garage travels at a speed v, the front door of the garage will reach the rear bumper of the limo a time Δt_c later, where Δt_c satisfies

$$\Delta t_c = t_{c1} - t_{c2} = \frac{L_c - L_g'}{v} = \frac{30.5 \text{ m} - 0.379 \text{ m}}{0.9980(2.998 \times 10^8 \text{ m/s})} = 1.01 \times 10^{-7} \text{ s}.$$

Thus $t_{c2} = t_{c1} - \Delta t_c = 0 - 1.01 \times 10^{-7}$ s $= -1.01 \times 10^{-7}$ s.

(h) From Carman's point of view, the answer is clearly no.

(i) Event 2 occurs first according to Carman, since $t_{c2} < t_{c1}$.

(j) We describe the essential features of the two pictures. For event 2, the front of the limo coincides with the back door, and the garage itself seems very short (perhaps failing to reach as far as the front window of the limo). For event 1, the rear of the car coincides with the front door and the front of the limo has traveled a significant distance beyond the back door. In this picture, as in the other, the garage seems very short compared to the limo.

(k) No, the limo cannot be in the garage with both doors shut.

(l) Both Carman and Garageman are correct in their respective reference frames. But, in a sense, Carman should lose the bet since he dropped his physics course before reaching the Theory of Special Relativity!

66. (a) According to ship observers, the duration of proton flight is $\Delta t' = (760 \text{ m})/0.980c$ = 2.59 μs (assuming it travels the entire length of the ship).

(b) To transform to our point of view, we use Eq. 2 in Table 37-2. Thus, with $\Delta x' = -750$ m, we have

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 0.572\mu\text{s}.$$

(c) For the ship observers, firing the proton from back to front makes no difference, and $\Delta t' = 2.59$ μs as before.

(d) For us, the fact that now $\Delta x' = +750$ m is a significant change.

$$\Delta t = \gamma\left(\Delta t' + (0.950c)\Delta x'/c^2\right) = 16.0\mu\text{s}.$$

67. Interpreting v_{AB} as the x-component of the velocity of A relative to B, and defining the corresponding speed parameter $\beta_{AB} = v_{AB}/c$, then the result of part (a) is a straightforward rewriting of Eq. 37-29 (after dividing both sides by c). To make the correspondence with Fig. 37-11 clear, the particle in that picture can be labeled A, frame S' (or an observer at rest in that frame) can be labeled B, and frame S (or an observer at rest in it) can be labeled C. The result of part (b) is less obvious, and we show here some of the algebra steps:

$$M_{AC} = M_{AB}\ M_{BC}$$

$$\frac{1 - \beta_{AC}}{1 + \beta_{AC}} = \frac{1 - \beta_{AB}}{1 + \beta_{AB}}\ \frac{1 - \beta_{BC}}{1 + \beta_{BC}}$$

We multiply both sides by factors to get rid of the denominators

$$(1 - \beta_{AC})\ (1 + \beta_{AB})\ (1 + \beta_{BC}) = (1 - \beta_{AB})\ (1 - \beta_{BC})\ (1 + \beta_{AC})$$

and expand:

$$1 - \beta_{AC} + \beta_{AB} + \beta_{BC} - \beta_{AC}\ \beta_{AB} - \beta_{AC}\ \beta_{BC} + \beta_{AB}\ \beta_{BC} - \beta_{AB}\ \beta_{BC}\ \beta_{AC} =$$
$$1 + \beta_{AC} - \beta_{AB} - \beta_{BC} - \beta_{AC}\ \beta_{AB} - \beta_{AC}\ \beta_{BC} + \beta_{AB}\ \beta_{BC} + \beta_{AB}\ \beta_{BC}\ \beta_{AC}$$

We note that several terms are identical on both sides of the equals sign, and thus cancel, which leaves us with

$$-\beta_{AC} + \beta_{AB} + \beta_{BC} - \beta_{AB}\ \beta_{BC}\ \beta_{AC} = \beta_{AC} - \beta_{AB} - \beta_{BC} + \beta_{AB}\ \beta_{BC}\ \beta_{AC}$$

which can be rearranged to produce

$$2\beta_{AB} + 2\beta_{BC} = 2\beta_{AC} + 2\beta_{AB}\ \beta_{BC}\ \beta_{AC}$$

The left-hand side can be written as $2\beta_{AC}\ (1 + \beta_{AB}\ \beta_{BC})$ in which case it becomes clear how to obtain the result from part (a) [just divide both sides by $2(1 + \beta_{AB}\ \beta_{BC})$].

68. We note, because it is a pretty symmetry and because it makes the part (b) computation move along more quickly, that

$$M = \frac{1-\beta}{1+\beta} \quad \Rightarrow \quad \beta = \frac{1-M}{1+M} \quad .$$

Here, with β_{AB} given as 1/2 (see problem statement), then M_{AB} is seen to be 1/3 (which is $(1-1/2)$ divided by $(1+1/2)$). Similarly for β_{BC} .

(a) Thus,

$$M_{AC} = M_{AB} \ M_{BC} = \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{9} \quad .$$

(b) Consequently,

$$\beta_{AC} = \frac{1-M_{AC}}{1+M_{AC}} = \frac{1-1/9}{1+1/9} = \frac{8}{10} = \frac{4}{5} = 0.80.$$

(c) By the definition of the speed parameter, we finally obtain $v_{AC} = 0.80c$.

69. We note, for use later in the problem, that

$$M = \frac{1-\beta}{1+\beta} \implies \beta = \frac{1-M}{1+M} \quad .$$

Now, with β_{AB} given as 1/5 (see problem statement), then M_{AB} is seen to be 2/3 (which is $(1-1/5)$ divided by $(1+1/5)$). With $\beta_{BC} = -2/5$ we similarly find $M_{BC} = 7/3$, and for $\beta_{CD} = 3/5$ we get $M_{CD} = 1/4$. Thus,

$$M_{AD} = M_{AB}\, M_{BC}\, M_{CD} = \frac{2}{3} \cdot \frac{7}{3} \cdot \frac{1}{4} = \frac{7}{18} \quad .$$

Consequently,

$$\beta_{AD} = \frac{1-M_{AD}}{1+M_{AD}} = \frac{1-7/18}{1+7/18} = \frac{11}{25} = 0.44.$$

By the definition of the speed parameter, we obtain $v_{AD} = 0.44c$.

70. We are asked to solve Eq. 37-48 for the speed v. Algebraically, we find

$$\beta = \sqrt{1 - \left(\frac{mc^2}{E}\right)^2} \; .$$

Using $E = 10.611 \times 10^{-9}$ J and the very accurate values for c and m (in SI units) found in Appendix B, we obtain $\beta = 0.99990$.

71. Using Appendix C, we find that the contraction is

$$|\Delta L| = L_0 - L = L_0\left(1 - \frac{1}{\gamma}\right) = L_0\left(1 - \sqrt{1 - \beta^2}\right)$$

$$= 2(6.370 \times 10^6 \text{ m})\left(1 - \sqrt{1 - \left(\frac{3.0 \times 10^4 \text{ m/s}}{2.998 \times 10^8 \text{ m/s}}\right)^2}\right)$$

$$= 0.064 \text{ m.}$$

72. The speed of the spaceship after the first increment is $v_1 = 0.5c$. After the second one, it becomes

$$v_2 = \frac{v' + v_1}{1 + v'v_1/c^2} = \frac{0.50c + 0.50c}{1 + (0.50c)^2/c^2} = 0.80c,$$

and after the third one, the speed is

$$v_3 = \frac{v' + v_2}{1 + v'v_2/c^2} = \frac{0.50c + 0.50c}{1 + (0.50c)(0.80c)/c^2} = 0.929c.$$

Continuing with this process, we get $v_4 = 0.976c$, $v_5 = 0.992c$, $v_6 = 0.997c$ and $v_7 = 0.999c$. Thus, seven increments are needed.

73. The mean lifetime of a pion measured by observers on the Earth is $\Delta t = \gamma \Delta t_0$, so the distance it can travel (using Eq. 37-12) is

$$d = v\Delta t = \gamma v \Delta t_0 = \frac{(0.99)(2.998 \times 10^8 \text{ m/s})(26 \times 10^{-9} \text{ s})}{\sqrt{1 - (0.99)^2}} = 55 \text{ m} \ .$$

74. (a) For a proton (using Table 37-3), we have

$$E = \gamma m_p c^2 = \frac{938\,\text{MeV}}{\sqrt{1-(0.990)^2}} = 6.65\,\text{GeV}$$

which gives

$$K = E - m_p c^2 = 6.65\,\text{GeV} - 938\,\text{MeV} = 5.71\,\text{GeV}\,.$$

(b) From part (a), $E = 6.65\,\text{GeV}$.

(c) Similarly, we have $p = \gamma m_p v = \gamma (m_p c^2) \beta / c = \dfrac{(938\,\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}} = 6.58\,\text{GeV}/c$

(d) For an electron, we have

$$E = \gamma m_e c^2 = \frac{0.511\,\text{MeV}}{\sqrt{1-(0.990)^2}} = 3.62\,\text{MeV}$$

which yields

$$K = E - m_e c^2 = 3.625\,\text{MeV} - 0.511\,\text{MeV} = 3.11\,\text{MeV}\,.$$

(e) From part (d), $E = 3.62\,\text{MeV}$.

(f) $p = \gamma m_e v = \gamma(m_e c^2)\beta / c = \dfrac{(0.511\,\text{MeV})(0.990)/c}{\sqrt{1-(0.990)^2}} = 3.59\,\text{MeV}/c\,.$

75. The strategy is to find the speed from $E = 1533$ MeV and $mc^2 = 0.511$ MeV (see Table 37-3) and from that find the time. From the energy relation (Eq. 37-48), we obtain

$$v = c\sqrt{1 - \left(\frac{mc^2}{E}\right)^2} = 0.99999994c \approx c$$

so that we conclude it took the electron 26 y to reach us. In order to transform to its own "clock" it's useful to compute γ directly from Eq. 37-48:

$$\gamma = \frac{E}{mc^2} = 3000$$

though if one is careful one can also get this result from $\gamma = 1 / \sqrt{1 - (v/c)^2}$. Then, Eq. 37-7 leads to

$$\Delta t_0 = \frac{26\,\text{y}}{\gamma} = 0.0087\,\text{y}$$

so that the electron "concludes" the distance he traveled is 0.0087 light-years (stated differently, the Earth, which is rushing towards him at very nearly the speed of light, seemed to start its journey from a distance of 0.0087 light-years away).

76. (a) Using Eq. 37-7, we expect the dilated time intervals to be

$$\tau = \gamma \tau_0 = \frac{\tau_0}{\sqrt{1-(v/c)^2}} \; .$$

(b) We rewrite Eq. 37-31 using the fact that period is the reciprocal of frequency $(f_R = \tau_R^{-1}$ and $f_0 = \tau_0^{-1})$:

$$\tau_R = \frac{1}{f_R} = \left(f_0 \sqrt{\frac{1-\beta}{1+\beta}} \right)^{-1} = \tau_0 \sqrt{\frac{1+\beta}{1-\beta}} = \tau_0 \sqrt{\frac{c+v}{c-v}} \; .$$

(c) The Doppler shift combines two physical effects: the time dilation of the moving source *and* the travel-time differences involved in periodic emission (like a sine wave or a series of pulses) from a traveling source to a "stationary" receiver). To isolate the purely time-dilation effect, it's useful to consider "local" measurements (say, comparing the readings on a moving clock to those of two of your clocks, spaced some distance apart, such that the moving clock and each of your clocks can make a close-comparison of readings at the moment of passage).

77. We use the relative velocity formula (Eq. 37-29) with the primed measurements being those of the scout ship. We note that $v = -0.900c$ since the velocity of the scout ship relative to the cruiser is opposite to that of the cruiser relative to the scout ship.

$$u = \frac{u'+v}{1+u'v/c^2} = \frac{0.980c - 0.900c}{1-(0.980)(0.900)} = 0.678c \ .$$

78. (a) The relative contraction is

$$\frac{|\Delta L|}{L_0} = \frac{L_0(1-\gamma^{-1})}{L_0} = 1 - \sqrt{1-\beta^2} \approx 1 - \left(1 - \frac{1}{2}\beta^2\right) = \frac{1}{2}\beta^2 = \frac{1}{2}\left(\frac{630\,\text{m/s}}{3.00\times10^8\,\text{m/s}}\right)^2$$
$$= 2.21\times10^{-12}.$$

(b) Letting $|\Delta t - \Delta t_0| = \Delta t_0(\gamma - 1) = \tau = 1.00\,\mu\text{s}$, we solve for Δt_0:

$$\Delta t_0 = \frac{\tau}{\gamma - 1} = \frac{\tau}{(1-\beta^2)^{-1/2} - 1} \approx \frac{\tau}{1 + \frac{1}{2}\beta^2 - 1} = \frac{2\tau}{\beta^2}$$
$$= \frac{2(1.00\times10^{-6}\,\text{s})(1\,\text{d}/86400\,\text{s})}{[(630\,\text{m/s})/(2.998\times10^8\,\text{m/s})]^2}$$
$$= 5.25\,\text{d}.$$

79. Let the reference frame be S in which the particle (approaching the South Pole) is at rest, and let the frame that is fixed on Earth be S'. Then $v = 0.60c$ and $u' = 0.80c$ (calling "downwards" [in the sense of Fig. 37-35] positive). The relative speed is now the speed of the other particle as measured in S:

$$u = \frac{u' + v}{1 + u'v/c^2} = \frac{0.80c + 0.60c}{1 + (0.80c)(0.60c)/c^2} = 0.95c \ .$$

80. We refer to the particle in the first sentence of the problem statement as particle 2. Since the total momentum of the two particles is zero in S', it must be that the velocities of these two particles are equal in magnitude and opposite in direction in S'. Letting the velocity of the S' frame be v relative to S, then the particle which is at rest in S must have a velocity of $u'_1 = -v$ as measured in S', while the velocity of the other particle is given by solving Eq. 37-29 for u':

$$u'_2 = \frac{u_2 - v}{1 - u_2 v / c^2} = \frac{(c/2) - v}{1 - (c/2)(v/c^2)} .$$

Letting $u'_2 = -u'_1 = v$, we obtain

$$\frac{(c/2) - v}{1 - (c/2)(v/c^2)} = v \quad \Rightarrow \quad v = c(2 \pm \sqrt{3}) \approx 0.27c$$

where the quadratic formula has been used (with the smaller of the two roots chosen so that $v \le c$).

81. We use Eq. 37-54 with $mc^2 = 0.511$ MeV (see Table 37-3):

$$pc = \sqrt{K^2 + 2Kmc^2} = \sqrt{(2.00)^2 + 2(2.00)(0.511)}$$

This readily yields $p = 2.46$ MeV/c.

82. (a) Our lab-based measurement of its lifetime is figured simply from

$$t = L/v = 7.99 \times 10^{-13} \text{ s.}$$

Use of the time-dilation relation (Eq. 37-7) leads to

$$\Delta t_0 = (7.99 \times 10^{-13} \text{ s})\sqrt{1-(0.960)^2} = 2.24 \times 10^{-13} \text{ s.}$$

(b) The length contraction formula can be used, or we can use the simple speed-distance relation (from the point of view of the particle, who watches the lab and all its meter sticks rushing past him at $0.960c$ until he expires): $L = v\Delta t_0 = 6.44 \times 10^{-5}$ m.

83. When $\beta = 0.9860$, we have $\gamma = 5.9972$, and when $\beta = 0.9850$, we have $\gamma = 5.7953$. Thus, $\Delta\gamma = 0.202$ and the change in kinetic energy (equal to the work) becomes (using Eq. 37-52)

$$W = \Delta K = mc^2 \Delta \gamma = 189 \text{ MeV}$$

where $mc^2 = 938$ MeV has been used (see Table 37-3).

84. (a) Eq. 37-37 yields

$$\frac{\lambda_0}{\lambda} = \sqrt{\frac{1-\beta}{1+\beta}} \quad \Rightarrow \quad \beta = \frac{1-(\lambda_0/\lambda)^2}{1+(\lambda_0/\lambda)^2} \ .$$

With $\lambda_0/\lambda = 434/462$, we obtain $\beta = 0.062439$, or $v = 1.87 \times 10^7$ m/s.

(b) Since it is shifted "towards the red" (towards longer wavelengths) then the galaxy is moving away from us (receding).

85. (a) $\Delta E = \Delta mc^2 = (3.0 \text{ kg})(0.0010)(2.998 \times 10^8 \text{ m/s})^2 = 2.7 \times 10^{14} \text{ J}$.

(b) The mass of TNT is

$$m_{TNT} = \frac{(2.7 \times 10^{14} \text{ J})(0.227 \text{ kg/mol})}{3.4 \times 10^6 \text{ J}} = 1.8 \times 10^7 \text{ kg}.$$

(c) The fraction of mass converted in the TNT case is

$$\frac{\Delta m_{TNT}}{m_{TNT}} = \frac{(3.0 \text{ kg})(0.0010)}{1.8 \times 10^7 \text{ kg}} = 1.6 \times 10^{-9},$$

Therefore, the fraction is $0.0010/1.6 \times 10^{-9} = 6.0 \times 10^6$.

86. (a) We assume the electron starts from rest. The classical formula for kinetic energy is Eq. 37-51, so if $v = c$ then this (for an electron) would be $\frac{1}{2}mc^2 = \frac{1}{2}(511\text{ ke V}) =$ 255.5 ke V (using Table 37-3). Setting this equal to the potential energy loss (which is responsible for its acceleration), we find (using Eq. 25-7)

$$V = \frac{255.5 \text{ keV}}{|q|} = \frac{255 \text{ keV}}{e} = 255.5 \text{ kV} \approx 256 \text{ keV}.$$

(b) Setting this amount of potential energy loss ($|\Delta U| = 255.5$ keV) equal to the correct relativistic kinetic energy, we obtain (using Eq. 37-52)

$$mc^2\left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) = |\Delta U| \Rightarrow v = c\sqrt{1 + \left(\frac{1}{1 - \Delta U/mc^2}\right)^2}$$

which yields $v = 0.745c = 2.23 \times 10^8$ m/s.

87. (a) $v_r = 2v = 2(27000 \text{ km/h}) = 5.4 \times 10^4 \text{ km/h}$.

(b) We can express c in these units by multiplying by 3.6: $c = 1.08 \times 10^9 \text{ km/h}$. The correct formula for v_r is $v_r = 2v/(1 + v^2/c^2)$, so the fractional error is

$$1 - \frac{1}{1+v^2/c^2} = 1 - \frac{1}{1+\left[(27000 \text{ km/h})/(1.08\times10^9 \text{ km/h})\right]^2} = 6.3\times10^{-10}.$$

88. Using Eq. 37-10,

$$\beta = \frac{v}{c} = \frac{d/c}{t} = \frac{6.0\,\text{y}}{2.0\,\text{y} + 6.0\,\text{y}} = 0.75.$$