1. The number of atoms per unit volume is given by $n = d/M$, where d is the mass density of copper and M is the mass of a single copper atom. Since each atom contributes one conduction electron, n is also the number of conduction electrons per unit volume. Since the molar mass of copper is $A = 63.54\,\text{g/mol}$,

$$M = A/N_A = (63.54\,\text{g/mol})/(6.022\times10^{23}\,\text{mol}^{-1}) = 1.055\times10^{-22}\,\text{g}.$$

Thus,

$$n = \frac{8.96\,\text{g/cm}^3}{1.055\times10^{-22}\,\text{g}} = 8.49\times10^{22}\,\text{cm}^{-3} = 8.49\times10^{28}\,\text{m}^{-3}\ .$$

2. We note that $n = 8.43 \times 10^{28}\ \text{m}^{-3} = 84.3\ \text{nm}^{-3}$. From Eq. 41-9,

$$E_F = \frac{0.121(hc)^2}{m_e c^2} n^{2/3} = \frac{0.121(1240\,\text{eV}\cdot\text{nm})^2}{511\times 10^3\ \text{eV}} (84.3\,\text{nm}^{-3})^{2/3} = 7.0\ \text{eV}$$

where the result of problem 83 in Chapter 38 is used.

3. (a) Eq. 41-5 gives

$$N(E) = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} E^{1/2}$$

for the density of states associated with the conduction electrons of a metal. This can be written

$$n(E) = CE^{1/2}$$

where

$$C = \frac{8\sqrt{2}\pi m^{3/2}}{h^3} = \frac{8\sqrt{2}\pi (9.109\times 10^{-31}\,\text{kg})^{3/2}}{(6.626\times 10^{-34}\,\text{J}\cdot\text{s})^3} = 1.062\times 10^{56}\,\text{kg}^{3/2}/\text{J}^3\cdot\text{s}^3 .$$

(b) Now, $1\,\text{J} = 1\text{kg}\cdot\text{m}^2/\text{s}^2$ (think of the equation for kinetic energy $K = \frac{1}{2}mv^2$), so 1 kg = 1 J·s^2·m^{-2}. Thus, the units of C can be written $(\text{J}\cdot\text{s}^2)^{3/2}\cdot(\text{m}^{-2})^{3/2}\cdot\text{J}^{-3}\cdot\text{s}^{-3} = \text{J}^{-3/2}\cdot\text{m}^{-3}$. This means

$$C = (1.062\times 10^{56}\,\text{J}^{-3/2}\cdot\text{m}^{-3})(1.602\times 10^{-19}\,\text{J}/\text{eV})^{3/2} = 6.81\times 10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2} .$$

(c) If E = 5.00 eV, then

$$n(E) = (6.81\times 10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2})(5.00\,\text{eV})^{1/2} = 1.52\times 10^{28}\,\text{eV}^{-1}\cdot\text{m}^{-3} .$$

4. We note that there is one conduction electron per atom and that the molar mass of gold is $197\,\text{g/mol}$. Therefore, combining Eqs. 41-2, 41-3 and 41-4 leads to

$$n = \frac{(19.3\,\text{g/cm}^3)(10^6\,\text{cm}^3/\text{m}^3)}{(197\,\text{g/mol})/(6.02\times10^{23}\,\text{mol}^{-1})} = 5.90\times10^{28}\,\text{m}^{-3}\ .$$

5. (a) At absolute temperature $T = 0$, the probability is zero that any state with energy above the Fermi energy is occupied.

(b) The probability that a state with energy E is occupied at temperature T is given by

$$P(E) = \frac{1}{e^{(E-E_F)/kT} + 1}$$

where k is the Boltzmann constant and E_F is the Fermi energy. Now, $E - E_F = 0.0620$ eV and

$$(E - E_F)/kT = (0.0620\,\mathrm{eV})/(8.62\times10^{-5}\,\mathrm{eV/K})(320\,\mathrm{K}) = 2.248\,,$$

so

$$P(E) = \frac{1}{e^{2.248} + 1} = 0.0955\,.$$

See Appendix B or Sample Problem 41-1 for the value of k.

6. We use the result of problem 3:

$$n(E) = CE^{1/2} = \left[6.81\times10^{27}\,\mathrm{m}^{-3}\cdot(\mathrm{eV})^{-2/3}\right](8.0\,\mathrm{eV})^{1/2} = 1.9\times10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}\ .$$

This is consistent with Fig. 41-5.

7. According to Eq. 41-9, the Fermi energy is given by

$$E_F = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} n^{2/3}$$

where n is the number of conduction electrons per unit volume, m is the mass of an electron, and h is the Planck constant. This can be written $E_F = An^{2/3}$, where

$$A = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{h^2}{m} = \left(\frac{3}{16\sqrt{2}\pi}\right)^{2/3} \frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})^2}{9.109\times10^{-31}\ \mathrm{kg}} = 5.842\times10^{-38}\ \mathrm{J^2\cdot s^2/kg}\ .$$

Since $1\,\mathrm{J} = 1\,\mathrm{kg\cdot m^2/s^2}$, the units of A can be taken to be $\mathrm{m^2\cdot J}$. Dividing by $1.602\times10^{-19}\ \mathrm{J/eV}$, we obtain $A = 3.65\times10^{-19}\ \mathrm{m^2\cdot eV}$.

8. Let E_1 = 63 meV + E_F and E_2 = – 63 meV + E_F. Then according to Eq. 41-6,

$$P_1 = \frac{1}{e^{(E_1 - E_F)/kT} + 1} = \frac{1}{e^x + 1}$$

where $x = (E_1 - E_F)/kT$. We solve for e^x:

$$e^x = \frac{1}{P_1} - 1 = \frac{1}{0.090} - 1 = \frac{91}{9} .$$

Thus,

$$P_2 = \frac{1}{e^{(E_2 - E_F)/kT} + 1} = \frac{1}{e^{-(E_1 - E_F)/kT} + 1} = \frac{1}{e^{-x} + 1} = \frac{1}{(91/9)^{-1} + 1} = 0.91,$$

where we use $E_2 - E_F = -63$ meV $= E_F - E_1 = -(E_1 - E_F)$.

9. The Fermi-Dirac occupation probability is given by $P_{\mathrm{FD}} = 1/\left(e^{\Delta E/kT}+1\right)$, and the Boltzmann occupation probability is given by $P_B = e^{-\Delta E/kT}$. Let f be the fractional difference. Then

$$f = \frac{P_{\mathrm{B}} - P_{\mathrm{FD}}}{P_{\mathrm{B}}} = \frac{e^{-\Delta E/kT} - \frac{1}{e^{\Delta E/kT}+1}}{e^{-\Delta E/kT}} .$$

Using a common denominator and a little algebra yields

$$f = \frac{e^{-\Delta E/kT}}{e^{-\Delta E/kT}+1} .$$

The solution for $e^{-\Delta E/kT}$ is

$$e^{-\Delta E/kT} = \frac{f}{1-f} .$$

We take the natural logarithm of both sides and solve for T. The result is

$$T = \frac{\Delta E}{k\ln\left(\dfrac{f}{1-f}\right)} .$$

(a) Letting f equal 0.01, we evaluate the expression for T:

$$T = \frac{(1.00\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}{(1.38\times10^{-23}\,\mathrm{J/K})\ln\left(\dfrac{0.010}{1-0.010}\right)} = 2.50\times10^{3}\,\mathrm{K}.$$

(b) We set f equal to 0.10 and evaluate the expression for T:

$$T = \frac{(1.00\,\mathrm{eV})(1.60\times10^{-19}\,\mathrm{J/eV})}{(1.38\times10^{-23}\,\mathrm{J/K})\ln\left(\dfrac{0.10}{1-0.10}\right)} = 5.30\times10^{3}\,\mathrm{K}.$$

10. We reproduce the calculation of Problem 4: Combining Eqs. 41-2, 41-3 and 41-4, the number density of conduction electrons in gold is

$$n = \frac{(19.3\,\mathrm{g/cm^3})(6.02\times 10^{23}\,/\,\mathrm{mol})}{(197\,\mathrm{g/mol})} = 5.90\times 10^{22}\ \mathrm{cm^{-3}} = 59.0\,\mathrm{nm^{-3}}\ .$$

Now, using the result of Problem 83 in Chapter 38, Eq. 41-9 leads to

$$E_F = \frac{0.121(hc)^2}{(m_e c^2)} n^{2/3} = \frac{0.121(1240\,\mathrm{eV\cdot nm})^2}{511\times 10^3\ \mathrm{eV}}(59.0\,\mathrm{nm^{-3}})^{2/3} = 5.52\,\mathrm{eV}\ .$$

11. (a) Eq. 41-6 leads to

$$E = E_F + kT \ln (P^{-1} - 1) = 7.00\,\text{eV} + (8.62\times10^{-5}\,\text{eV/K})(1000\text{K})\ln\left(\frac{1}{0.900} - 1\right)$$

$$= 6.81\text{eV}.$$

(b) $n(E) = CE^{1/2} = \left(6.81\times10^{27}\,\text{m}^{-3}\cdot\text{eV}^{-3/2}\right)(6.81\text{eV})^{1/2} = 1.77\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$

(c) $n_0(E) = P(E)n(E) = (0.900)(1.77\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}) = 1.59\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$

12. (a) The volume per cubic meter of sodium occupied by the sodium ions is

$$V_{\text{Na}} = \frac{(971\,\text{kg})(6.022\times10^{23}/\text{mol})(4\pi/3)(98.0\times10^{-12}\,\text{m})^3}{(23.0\,\text{g}/\text{mol})} = 0.100\,\text{m}^3,$$

so the fraction available for conduction electrons is $1-(V_{\text{Na}}/1.00\,\text{m}^3) = 1-0.100 = 0.900$, or 90.0%.

(b) For copper,

$$V_{\text{Cu}} = \frac{(8960\,\text{kg})(6.022\times10^{23}/\text{mol})(4\pi/3)(135\times10^{-12}\,\text{m})^3}{(63.5\,\text{g}/\text{mol})} = 0.1876\,\text{m}^{-3}\ .$$

Thus, the fraction is $1-(V_{\text{Cu}}/1.00\,\text{m}^3) = 1-0.876 = 0.124$, or 12.4%.

(c) Sodium, because the electrons occupy a greater portion of the space available.

13. We use

$$N_0 = N(E)P(E) = CE^{1/2}\left[e^{(E-E_F)/kT}+1\right]^{-1},$$

where C is given in problem 3(b).

(a) At $E = 4.00$ eV,

$$n_0 = \frac{\left(6.81\times10^{27}\,\mathrm{m}^{-3}\cdot(\mathrm{eV})^{-3/2}\right)(4.00\,\mathrm{eV})^{1/2}}{e^{(4.00\,\mathrm{eV}-7.00\,\mathrm{eV})/[(8.62\times10^{-5}\,\mathrm{eV/K})(1000\,\mathrm{K})]}+1} = 1.36\times10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}.$$

(b) At $E = 6.75$ eV,

$$n_0 = \frac{\left(6.81\times10^{27}\,\mathrm{m}^{-3}\cdot(\mathrm{eV})^{-3/2}\right)(6.75\,\mathrm{eV})^{1/2}}{e^{(6.75\,\mathrm{eV}-7.00\,\mathrm{eV})/[(8.62\times10^{-5}\,\mathrm{eV/K})(1000\,\mathrm{K})]}+1} = 1.68\times10^{28}\,\mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}\ .$$

(c) Similarly, at $E = 7.00$ eV, the value of $n_0(E)$ is $9.01 \times 10^{27}\ \mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}$.

(d) At $E = 7.25$ eV, the value of $n_0(E)$ is $9.56 \times 10^{26}\ \mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}$.

(e) At $E = 9.00$ eV, the value of $n_0(E)$ is $1.71 \times 10^{18}\ \mathrm{m}^{-3}\cdot\mathrm{eV}^{-1}$.

14. The probability P_h that a state is occupied by a hole is the same as the probability the state is *unoccupied* by an electron. Since the total probability that a state is either occupied or unoccupied is 1, we have $P_h + P = 1$. Thus,

$$P_h = 1 - \frac{1}{e^{(E-E_F)/kT}+1} = \frac{e^{(E-E_F)/kT}}{1+e^{(E-E_F)/kT}} = \frac{1}{e^{-(E-E_F)/kT}+1} \; .$$

15. (a) We evaluate $P(E) = 1\big/\left(e^{(E-E_F)/kT}+1\right)$ for the given value of E, using

$$kT = \frac{(1.381\times10^{-23}\ \text{J / K})(273\,\text{K})}{1.602\times10^{-19}\ \text{J / eV}} = 0.02353\,\text{eV}\ .$$

For E = 4.4 eV, $(E - E_F)/kT$ = (4.4 eV – 5.5 eV)/(0.02353 eV) = – 46.25 and

$$P(E) = \frac{1}{e^{-46.25}+1} = 1.0 .$$

(b) Similarly, for E = 5.4 eV, $P(E) = 0.986 \approx 0.99$.

(c) For E = 5.5 eV, $P(E)$ = 0.50.

(d) For E = 5.6 eV, $P(E)$ = 0.014.

(e) For E = 6.4 eV, $P(E) = 2.447 \times 10^{-17} \approx 2.4 \times 10^{-17}$.

(f) Solving $P = 1/(e^{\Delta E/kT} + 1)$ for $e^{\Delta E/kT}$, we get

$$e^{\Delta E/kT} = \frac{1}{P} - 1\ .$$

Now, we take the natural logarithm of both sides and solve for T. The result is

$$T = \frac{\Delta E}{k\ln\left(\frac{1}{P}-1\right)} = \frac{(5.6\,\text{eV} - 5.5\,\text{eV})(1.602\times10^{-19}\ \text{J/eV})}{(1.381\times10^{-23}\ \text{J/K})\ln\left(\frac{1}{0.16}-1\right)} = 699\,\text{K} \approx 7.0\times10^{2}\,\text{K}.$$

16. The molar mass of carbon is $m = 12.01115$ g/mol and the mass of the Earth is $M_e = 5.98 \times 10^{24}$ kg. Thus, the number of carbon atoms in a diamond as massive as the Earth is $N = (M_e/m)N_{\mathrm{A}}$, where N_{A} is the Avogadro constant. From the result of Sample Problem 41-1, the probability in question is given by

$$P = N_e^{-E_g/kT} = \left(\frac{M_e}{m}\right) N_{\mathrm{A}} e^{-E_g/kT} = \left(\frac{5.98\times10^{24}\ \mathrm{kg}}{12.01115\,\mathrm{g/mol}}\right)(6.02\times10^{23}\ /\ \mathrm{mol})(3\times10^{-93})$$
$$= 9\times10^{-43} \approx 10^{-42}\ .$$

17. Let N be the number of atoms per unit volume and n be the number of free electrons per unit volume. Then, the number of free electrons per atom is n/N. We use the result of Exercise 11 to find n: $E_F = An^{2/3}$, where $A = 3.65 \times 10^{-19}\ \text{m}^2 \cdot \text{eV}$. Thus,

$$n = \left(\frac{E_F}{A}\right)^{3/2} = \left(\frac{11.6\,\text{eV}}{3.65\times10^{-19}\,\text{m}^2\cdot\text{eV}}\right)^{3/2} = 1.79\times10^{29}\,\text{m}^{-3}\ .$$

If M is the mass of a single aluminum atom and d is the mass density of aluminum, then $N = d/M$. Now,

$$M = (27.0\ \text{g/mol})/(6.022 \times 10^{23}\ \text{mol}^{-1}) = 4.48 \times 10^{-23}\ \text{g},$$

so

$$N = (2.70\ \text{g/cm}^3)/(4.48 \times 10^{-23}\ \text{g}) = 6.03 \times 10^{22}\ \text{cm}^{-3} = 6.03 \times 10^{28}\ \text{m}^{-3}.$$

Thus, the number of free electrons per atom is

$$\frac{n}{N} = \frac{1.79\times10^{29}\,\text{m}^{-3}}{6.03\times10^{28}\,\text{m}^{-3}} = 2.97 \approx 3.$$

18. (a) The ideal gas law in the form of Eq. 20-9 leads to $p = NkT/V = nkT$. Thus, we solve for the molecules per cubic meter:

$$n = \frac{p}{kT} = \frac{(1.0\,\text{atm})(1.0\times10^{5}\,\text{Pa}/\text{atm})}{(1.38\times10^{-23}\,\text{J}/\text{K})(273\,\text{K})} = 2.7\times10^{25}\,\text{m}^{-3} .$$

(b) Combining Eqs. 41-2, 41-3 and 41-4 leads to the conduction electrons per cubic meter in copper:

$$n = \frac{8.96\times10^{3}\,\text{kg/m}^{3}}{(63.54)(1.67\times10^{-27}\,\text{kg})} = 8.43\times10^{28}\,\text{m}^{-3} .$$

(c) The ratio is $(8.43 \times 10^{28}\,\text{m}^{-3})/(2.7 \times 10^{25}\,\text{m}^{-3}) = 3.1 \times 10^{3}$.

(d) We use $d_{\text{avg}} = n^{-1/3}$. For case (a), $d_{\text{avg}} = (2.7 \times 10^{25}\,\text{m}^{-3})^{-1/3}$ which equals 3.3 nm.

(e) For case (b), $d_{\text{avg}} = (8.43 \times 10^{28}\,\text{m}^{-3})^{-1/3} = 0.23$ nm.

19. (a) According to Appendix F the molar mass of silver is 107.870 g/mol and the density is 10.49 g/cm^3. The mass of a silver atom is

$$\frac{107.870\times10^{-3}\text{ kg / mol}}{6.022\times10^{23}\text{ mol}^{-1}}=1.791\times10^{-25}\text{ kg}\ .$$

We note that silver is monovalent, so there is one valence electron per atom (see Eq. 41-2). Thus, Eqs. 41-4 and 41-3 lead to

$$n=\frac{\rho}{M}=\frac{10.49\times10^{-3}\text{ kg/m}^3}{1.791\times10^{25}\text{ kg}}=5.86\times10^{28}\text{ m}^{-3}\ .$$

(b) The Fermi energy is

$$\begin{aligned}E_F&=\frac{0.121h^2}{m}n^{2/3}=\frac{(0.121)(6.626\times10^{-34}\text{ J}\cdot\text{s})^2}{9.109\times10^{-31}\text{ kg}}=(5.86\times10^{28}\text{ m}^{-3})^{2/3}\\&=8.80\times10^{-19}\text{ J}=5.49\text{eV}.\end{aligned}$$

(c) Since $E_F=\frac{1}{2}mv_F^2$,

$$v_F=\sqrt{\frac{2E_F}{m}}=\sqrt{\frac{2(8.80\times10^{-19}\text{ J})}{9.109\times10^{-31}\text{ kg}}}=1.39\times10^{6}\text{ m / s}\ .$$

(d) The de Broglie wavelength is

$$\lambda=\frac{h}{mv_F}=\frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(9.109\times10^{-31}\text{ kg})(1.39\times10^{6}\text{ m/s})}=5.22\times10^{-10}\text{ m}.$$

20. Let the energy of the state in question be an amount ΔE above the Fermi energy E_F. Then, Eq. 41-6 gives the occupancy probability of the state as

$$P=\frac{1}{e^{(E_\mathrm{F}+\Delta E-E_\mathrm{F})/kT}+1}=\frac{1}{e^{\Delta E/kT}+1}\,.$$

We solve for ΔE to obtain

$$\Delta E = kT\ln\left(\frac{1}{P}-1\right)=(1.38\times10^{23}\ \mathrm{J/K})(300\,\mathrm{K})\ln\left(\frac{1}{0.10}-1\right)=9.1\times10^{-21}\ \mathrm{J}\ ,$$

which is equivalent to 5.7×10^{-2} eV = 57 meV.

21. The average energy of the conduction electrons is given by

$$E_{\text{avg}} = \frac{1}{n}\int_0^\infty EN(E)P(E)dE$$

where n is the number of free electrons per unit volume, $N(E)$ is the density of states, and $P(E)$ is the occupation probability. The density of states is proportional to $E^{1/2}$, so we may write $N(E) = CE^{1/2}$, where C is a constant of proportionality. The occupation probability is one for energies below the Fermi energy and zero for energies above. Thus,

$$E_{\text{avg}} = \frac{C}{n}\int_0^{E_F} E^{3/2}dE = \frac{2C}{5n}E_F^{5/2} .$$

Now

$$n = \int_0^\infty N(E)P(E)dE = C\int_0^{E_F} E^{1/2}dE = \frac{2C}{3}E_F^{3/2} .$$

We substitute this expression into the formula for the average energy and obtain

$$E_{\text{avg}} = \left(\frac{2C}{5}\right)E_F^{5/2}\left(\frac{3}{2CE_F^{3/2}}\right) = \frac{3}{5}E_F .$$

22. (a) Combining Eqs. 41-2, 41-3 and 41-4 leads to the conduction electrons per cubic meter in zinc:

$$n = \frac{2(7.133\,\mathrm{g/cm^3})}{(65.37\,\mathrm{g/mol})/(6.02\times10^{23}\,\mathrm{mol})} = 1.31\times10^{23}\,\mathrm{cm^{-3}} = 1.31\times10^{29}\,\mathrm{m^{-3}}\ .$$

(b) From Eq. 41-9,

$$E_F = \frac{0.121h^2}{m_e}n^{2/3} = \frac{0.121(6.63\times10^{-34}\,\mathrm{J\cdot s})^2(1.31\times10^{29}\,\mathrm{m^{-3}})^{2/3}}{(9.11\times10^{-31}\,\mathrm{kg})(1.60\times10^{-19}\,\mathrm{J/eV})} = 9.43\,\mathrm{eV}.$$

(c) Equating the Fermi energy to $\frac{1}{2}m_e v_F^2$ we find (using the m_ec^2 value in Table 37-3)

$$v_F = \sqrt{\frac{2E_Fc^2}{m_ec^2}} = \sqrt{\frac{2(9.43\,\mathrm{eV})(2.998\times10^8\,\mathrm{m/s})^2}{511\times10^3\,\mathrm{eV}}} = 1.82\times10^6\,\mathrm{m/s}\ .$$

(d) The de Broglie wavelength is

$$\lambda = \frac{h}{m_e v_F} = \frac{6.63\times10^{-34}\,\mathrm{J\cdot s}}{(9.11\times10^{-31}\,\mathrm{kg})(1.82\times10^6\,\mathrm{m/s})} = 0.40\,\mathrm{nm}\ .$$

23. Let the volume be $v = 1.00 \times 10^{-6}$ m^3. Then,

$$K_{\text{total}} = NE_{\text{avg}} = nvE_{\text{avg}} = (8.43\times10^{28}\ \text{m}^{-3})(1.00\times10^{-6}\ \text{m}^3)\left(\frac{3}{5}\right)(7.00\,\text{eV})(1.60\times10^{-19}\ \text{J/eV})$$

$$= 5.71\times10^{4}\,\text{J} = 57.1\ \text{kJ}.$$

24. (a) At $T = 300$ K

$$f = \frac{3kT}{2E_F} = \frac{3(8.62\times10^{-5}\ \text{eV/K})(300\ \text{K})}{2(7.0\ \text{eV})} = 5.5\times10^{-3}\ .$$

(b) At $T = 1000$ K,

$$f = \frac{3kT}{2E_F} = \frac{3(8.62\times10^{-5}\ \text{eV/K})(1000\ \text{K})}{2(7.0\ \text{eV})} = 1.8\times10^{-2}\ .$$

(c) Many calculators and most math software packages (here we use MAPLE) have built-in numerical integration routines. Setting up ratios of integrals of Eq. 41-7 and canceling common factors, we obtain

$$frac = \frac{\int_{E_F}^{\infty} \sqrt{E}\,/\,(e^{(E-E_F)/kT}+1)dE}{\int_{0}^{\infty} \sqrt{E}\,/\,(e^{(E-E_F)/kT}+1)dE}$$

where $k = 8.62 \times 10^{-5}$ eV/K. We use the Fermi energy value for copper ($E_F = 7.0$ eV) and evaluate this for $T = 300$ K and $T = 1000$ K; we find $frac = 0.00385$ and $frac = 0.0129$, respectively.

25. The fraction f of electrons with energies greater than the Fermi energy is (approximately) given in Problem 41-24:

$$f = \frac{3kT/2}{E_F}$$

where T is the temperature on the Kelvin scale, k is the Boltzmann constant, and E_F is the Fermi energy. We solve for T:

$$T = \frac{2fE_F}{3k} = \frac{2(0.013)(4.70\,\text{eV})}{3(8.62\times10^{-5}\,\text{eV/K})} = 472\ \text{K}.$$

26. (a) Using Eq. 41-4, the energy released would be

$$E = NE_{\text{avg}} = \frac{(3.1\,\text{g})}{(63.54\,\text{g}/\text{mol})/(6.02\times10^{23}/\text{mol})}\left(\frac{3}{5}\right)(7.0\,\text{eV})(1.6\times10^{-19}\,\text{J/eV})$$
$$= 1.97\times10^{4}\,\text{J}.$$

(b) Keeping in mind that a Watt is a Joule per second, we have

$$\frac{1.97\times10^{4}\,\text{J}}{100\,\text{J/s}} = 197\,\text{s}.$$

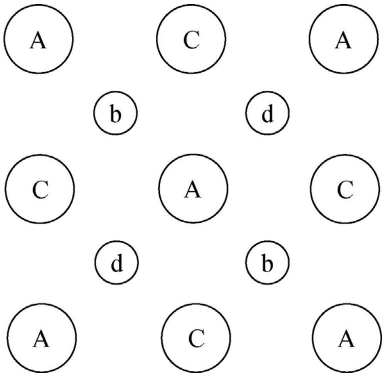
27. (a) Since the electron jumps from the conduction band to the valence band, the energy of the photon equals the energy gap between those two bands. The photon energy is given by $hf = hc/\lambda$, where f is the frequency of the electromagnetic wave and λ is its wavelength. Thus, $E_g = hc/\lambda$ and

$$\lambda = \frac{hc}{E_g} = \frac{(6.63\times 10^{-34}\,\mathrm{J\cdot s})(2.998\times 10^{8}\,\mathrm{m/s})}{(5.5\,\mathrm{eV})(1.60\times 10^{-19}\,\mathrm{J/eV})} = 2.26\times 10^{-7}\ \mathrm{m} = 226\,\mathrm{nm}\ .$$

Photons from other transitions have a greater energy, so their waves have shorter wavelengths.

(b) These photons are in the ultraviolet portion of the electromagnetic spectrum.

28. Each Arsenic atom is connected (by covalent bonding) to four Gallium atoms, and each Gallium atom is similarly connected to four Arsenic atoms.



The "depth" of their very non-trivial lattice structure is, of course, not evident in a flattened-out representation such as shown for Silicon in Fig. 41-9. Still we try to convey some sense of this (in the [1, 0, 0] view shown — for those who might be familiar with Miller indices) by using letters to indicate the depth: A for the closest atoms (to the observer), b for the next layer deep, C for further into the page, d for the last layer seen, and E (not shown) for the atoms that are at the deepest layer (and are behind the A's) needed for our description of the structure. The capital letters are used for the Gallium atoms, and the small letters for the Arsenic.

Consider the Arsenic atom (with the letter b) near the upper left; it has covalent bonds with the two A's and the two C's near it. Now consider the Arsenic atom (with the letter d) near the upper right; it has covalent bonds with the two C's which are near it and with the two E's (which are behind the A's which are near :+).

(a) The 3p, 3d and 4s subshells of both Arsenic and Gallium are filled. They both have partially filled 4p subshells. An isolated, neutral Arsenic atom has three electrons in the 4p subshell, and an isolated, neutral Gallium atom has one electron in the 4p subshell. To supply the total of eight shared electrons (for the four bonds connected to each ion in the lattice), not only the electrons from 4p must be shared but also the electrons from 4s. The core of the Gallium ion has charge $q = +3e$ (due to the "loss" of its single 4p and two 4s electrons).

(b) The core of the Arsenic ion has charge $q = +5e$ (due to the "loss" of the three 4p and two 4s electrons).

(c) As remarked in part (a), there are two electrons shared in each of the covalent bonds. This is the same situation that one finds for Silicon (see Fig. 41-9).

29. (a) At the bottom of the conduction band E = 0.67 eV. Also E_F = 0.67 eV/2 = 0.335 eV. So the probability that the bottom of the conduction band is occupied is

$$P(E)=\frac{1}{e^{(E-E_{\mathrm{F}})/kT}+1}=\frac{1}{e^{(0.67\,\mathrm{eV}-0.335\,\mathrm{eV})/\left[(8.62\times10^{-5}\,\mathrm{eV/K})(290\,\mathrm{K})\right]}+1}=1.5\times10^{-6}\,.$$

(b) At the top of the valence band E = 0, so the probability that the state is *unoccupied* is given by

$$1-P(E)=1-\frac{1}{e^{(E-E_{\mathrm{F}})/kT}+1}=\frac{1}{e^{-(E-E_{\mathrm{F}})/kT}+1}=\frac{1}{e^{-(0-0.335\,\mathrm{eV})/\left[(8.62\times10^{-5}\,\mathrm{eV/K})(290\,\mathrm{K})\right]}+1}$$

$$=1.5\times10^{-6}\,.$$

30. (a) The number of electrons in the valence band is

$$N_{\text{ev}} = N_v P(E_v) = \frac{N_v}{e^{(E_v - E_\text{F})/kT} + 1} .$$

Since there are a total of N_v states in the valence band, the number of holes in the valence band is

$$N_{\text{hv}} = N_v - N_{\text{ev}} = N_v \left[1 - \frac{1}{e^{(E_v - E_\text{F})/kT} + 1} \right] = \frac{N_v}{e^{-(E_v - E_\text{F})/kT} + 1} .$$

Now, the number of electrons in the conduction band is

$$N_{\text{ec}} = N_c P(E_c) = \frac{N_c}{e^{(E_c - E_\text{F})/kT} + 1} ,$$

Hence, from $N_{\text{ev}} = N_{\text{hc}}$, we get

$$\frac{N_v}{e^{-(E_v - E_\text{F})/kT} + 1} = \frac{N_c}{e^{(E_c - E_\text{F})/kT} + 1} .$$

(b) In this case, $e^{(E_c - E_\text{F})/kT} \gg 1$ and $e^{-(E_v - E_\text{F})/kT} \gg 1$. Thus, from the result of part (a),

$$\frac{N_c}{e^{(E_c - E_F)/kT}} \approx \frac{N_v}{e^{-(E_v - E_F)/kT}} ,$$

or $e^{(E_v - E_c + 2E_F)/kT} \approx N_v / N_c$. We solve for E_F:

$$E_F \approx \frac{1}{2}(E_c + E_v) + \frac{1}{2} kT \ln\left(\frac{N_v}{N_c}\right).$$

31. Sample Problem 41-6 gives the fraction of silicon atoms that must be replaced by phosphorus atoms. We find the number the silicon atoms in 1.0 g, then the number that must be replaced, and finally the mass of the replacement phosphorus atoms. The molar mass of silicon is 28.086 g/mol, so the mass of one silicon atom is

$$(28.086 \text{ g/mol})/(6.022 \times 10^{23} \text{ mol}^{-1}) = 4.66 \times 10^{-23} \text{ g}$$

and the number of atoms in 1.0 g is $(1.0 \text{ g})/(4.66 \times 10^{-23} \text{ g}) = 2.14 \times 10^{22}$. According to Sample Problem 41-6 one of every 5×10^{6} silicon atoms is replaced with a phosphorus atom. This means there will be $(2.14 \times 10^{22})/(5 \times 10^{6}) = 4.29 \times 10^{15}$ phosphorus atoms in 1.0 g of silicon. The molar mass of phosphorus is 30.9758 g/mol so the mass of a phosphorus atom is

$$(30.9758 \text{ g/mol})/(6.022 \times 10^{-23} \text{ mol}^{-1}) = 5.14 \times 10^{-23} \text{ g}.$$

The mass of phosphorus that must be added to 1.0 g of silicon is

$$(4.29 \times 10^{15})(5.14 \times 10^{-23} \text{ g}) = 2.2 \times 10^{-7} \text{ g}.$$

32. (a) *n*-type, since each phosphorus atom has one more valence electron than a silicon atom.

(b) The added charge carrier density is

$$n_{\text{P}} = 10^{-7}\, n_{\text{Si}} = 10^{-7}\,(5 \times 10^{28}\ \text{m}^{-3}) = 5 \times 10^{21}\ \text{m}^{-3}.$$

(c) The ratio is $(5 \times 10^{21}\ \text{m}^{-3})/[2(5 \times 10^{15}\ \text{m}^{-3})] = 5 \times 10^{5}$. Here the factor of 2 in the denominator reflects the contribution to the charge carrier density from *both* the electrons in the conduction band *and* the holes in the valence band.

33. (a) The probability that a state with energy E is occupied is given by

$$P(E)=\frac{1}{e^{(E-E_F)/kT}+1}$$

where E_F is the Fermi energy, T is the temperature on the Kelvin scale, and k is the Boltzmann constant. If energies are measured from the top of the valence band, then the energy associated with a state at the bottom of the conduction band is E = 1.11 eV. Furthermore,

$$kT = (8.62\times10^{-5}\text{ eV/K})(300\text{ K}) = 0.02586\text{ eV}.$$

For pure silicon,

$$E_F = 0.555\text{ eV and } (E - E_F)/kT = (0.555\text{ eV})/(0.02586\text{ eV}) = 21.46.$$

Thus,

$$P(E)=\frac{1}{e^{21.46}+1}=4.79\times10^{-10}.$$

(b) For the doped semiconductor, $(E - E_F)/kT$ = (0.11 eV)/(0.02586 eV) = 4.254 and

$$P(E)=\frac{1}{e^{4.254}+1}=1.40\times10^{-2}.$$

(c) The energy of the donor state, relative to the top of the valence band, is 1.11 eV – 0.15 eV = 0.96 eV. The Fermi energy is 1.11 eV – 0.11 eV = 1.00 eV. Hence,

$$(E - E_F)/kT = (0.96\text{ eV} - 1.00\text{ eV})/(0.02586\text{ eV}) = -1.547$$

and

$$P(E)=\frac{1}{e^{-1.547}+1}=0.824.$$

34. (a) Measured from the top of the valence band, the energy of the donor state is E = 1.11 eV – 0.11 eV = 1.0 eV. We solve E_F from Eq. 41-6:

$$E_F = E - kT\ln\left[P^{-1}-1\right] = 1.0\,\text{eV} - \left(8.62\times10^{-5}\ \text{eV/K}\right)\left(300\text{K}\right)\ln\left[\left(5.00\times10^{-5}\right)^{-1}-1\right]$$
$$= 0.744\,\text{eV}.$$

(b) Now E = 1.11 eV, so

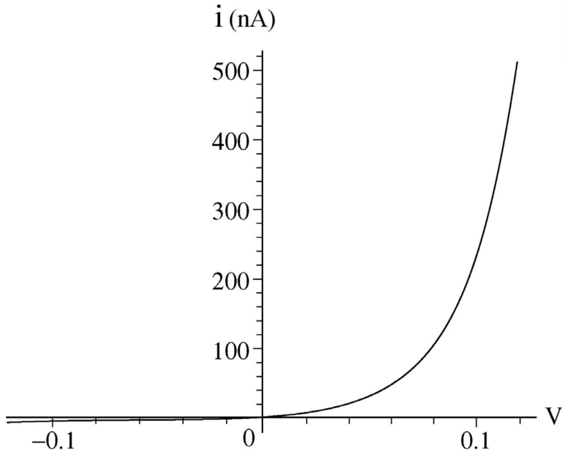
$$P(E) = \frac{1}{e^{(E-E_F)/kT}+1} = \frac{1}{e^{(1.11\text{eV}-0.744\text{eV})/\left[\left(8.62\times10^{-5}\ \text{eV/K}\right)(300\text{K})\right]}+1} = 7.13\times10^{-7}.$$

35. The energy received by each electron is exactly the difference in energy between the bottom of the conduction band and the top of the valence band (1.1 eV). The number of electrons that can be excited across the gap by a single 662-keV photon is

$$N = (662 \times 10^3 \text{ eV})/(1.1 \text{ eV}) = 6.0 \times 10^5.$$

Since each electron that jumps the gap leaves a hole behind, this is also the number of electron-hole pairs that can be created.

36. (a) The vertical axis in the graph below is the current in nanoamperes:



(b) The ratio is

$$\frac{i\Big|_{v=+0.50\,\mathrm{V}}}{i\Big|_{v=-0.50\,\mathrm{V}}} = \frac{i_0\left[e^{+0.50\,\mathrm{eV}\big/\left[\left(8.62\times10^{-5}\,\mathrm{eV/K}\right)\left(300\,\mathrm{K}\right)\right]}-1\right]}{i_0\left[e^{-0.50\,\mathrm{eV}\big/\left[\left(8.62\times10^{-5}\,\mathrm{eV/K}\right)\left(300\,\mathrm{K}\right)\right]}-1\right]} = 2.5\times10^{8}.$$

37. The valence band is essentially filled and the conduction band is essentially empty. If an electron in the valence band is to absorb a photon, the energy it receives must be sufficient to excite it across the band gap. Photons with energies less than the gap width are not absorbed and the semiconductor is transparent to this radiation. Photons with energies greater than the gap width are absorbed and the semiconductor is opaque to this radiation. Thus, the width of the band gap is the same as the energy of a photon associated with a wavelength of 295 nm. We use the result of Problem 83 of Chapter 38 to obtain

$$E_{\text{gap}} = \frac{1240\,\text{eV}\cdot\text{nm}}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{295\,\text{nm}} = 4.20\,\text{eV}.$$

38. Since (using the result of problem 83 in Chapter 38)

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{1240\,\text{eV}\cdot\text{nm}}{140\,\text{nm}} = 8.86\,\text{eV} > 7.6\,\text{eV},$$

the light will be absorbed by the KCI crystal. Thus, the crystal is opaque to this light.

39. We denote the maximum dimension (side length) of each transistor as $\ell_{\max}$, the size of the chip as A, and the number of transistors on the chip as N. Then $A = N\ell_{\max}^2$. Therefore,

$$\ell_{\max} = \sqrt{\frac{A}{N}} = \sqrt{\frac{(1.0\,\text{in.} \times 0.875\,\text{in.})\left(2.54\times10^{-2}\ \text{m/in.}\right)^2}{3.5\times10^6}} = 1.3\times10^{-5}\ \text{m} = 13\,\mu\text{m}.$$

40. (a) According to Chapter 25, the capacitance is $C = \kappa\varepsilon_0 A/d$. In our case $\kappa = 4.5$, $A = (0.50\ \mu\text{m})^2$, and $d = 0.20\ \mu\text{m}$, so

$$C = \frac{\kappa\varepsilon_0 A}{d} = \frac{(4.5)\left(8.85\times10^{-12}\ \text{F/m}\right)\left(0.50\ \mu\text{m}\right)^2}{0.20\ \mu\text{m}} = 5.0\times10^{-17}\ \text{F}.$$

(b) Let the number of elementary charges in question be N. Then, the total amount of charges that appear in the gate is $q = Ne$. Thus, $q = Ne = CV$, which gives

$$N = \frac{CV}{e} = \frac{\left(5.0\times10^{-17}\ \text{F}\right)\left(1.0\ \text{V}\right)}{1.6\times10^{-19}\ \text{C}} = 3.1\times10^{2}.$$

41. (a) Setting $E = E_F$ (see Eq. 41-9), Eq. 41-5 becomes

$$N(E_F) = \frac{8\pi m\sqrt{2m}}{h^3}\left(\frac{3}{16\pi\sqrt{2}}\right)^{1/3}\frac{h}{\sqrt{m}}n^{1/3} .$$

Noting that $16\sqrt{2} = 2^4 2^{1/2} = 2^{9/2}$ so that the cube root of this is $2^{3/2} = 2\sqrt{2}$, we are able to simplify the above expression and obtain

$$N(E_F) = \frac{4m}{h^2}\sqrt[3]{3\pi^2 n}$$

which is equivalent to the result shown in the problem statement. Since the desired numerical answer uses eV units, we multiply numerator and denominator of our result by c^2 and make use of the mc^2 value for an electron in Table 38-3 as well as the hc value found in problem 83 of Chapter 38:

$$N(E_F) = \left(\frac{4mc^2}{(hc)^2}\sqrt[3]{3\pi^2}\right)n^{1/3} = \left(\frac{4(511\times10^3\text{ eV})}{(1240\,\text{eV}\cdot\text{nm})^2}\sqrt[3]{3\pi^2}\right)n^{1/3} = (4.11\,\text{nm}^{-2}\cdot\text{eV}^{-1})\,n^{1/3}$$

which is equivalent to the value indicated in the problem statement.

(b) Since there are 10^{27} cubic nanometers in a cubic meter, then the result of problem 1 may be written

$$n = 8.49\times10^{28}\,\text{m}^{-3} = 84.9\,\text{nm}^{-3} .$$

The cube root of this is $n^{1/3} \approx 4.4/\text{nm}$. Hence, the expression in part (a) leads to

$$N(E_F) = (4.11\text{nm}^{-2}\cdot\text{eV}^{-1})(4.4\text{nm}^{-1}) = 18\text{nm}^{-3}\cdot\text{eV}^{-1} = 1.8\times10^{28}\,\text{m}^{-3}\cdot\text{eV}^{-1}.$$

If we multiply this by 10^{27} m^3/nm^3, we see this compares very well with the curve in Fig. 41-5 evaluated at 7.0 eV.

42. If we use the approximate formula discussed in problem 41-24, we obtain

$$frac = \frac{3(8.62 \times 10^{-5}\ \text{eV/K})(961 + 273\,\text{K})}{2(5.5\,\text{eV})} \approx 0.03\ .$$

The numerical approach is briefly discussed in part (c) of problem 32. Although the problem does not ask for it here, we remark that numerical integration leads to a fraction closer to 0.02.

43. The description in the problem statement implies that an atom is at the center point C of the regular tetrahedron, since its four *neighbors* are at the four vertices. The side length for the tetrahedron is given as $a = 388$ pm. Since each face is an equilateral triangle, the "altitude" of each of those triangles (which is not to be confused with the altitude of the tetrahedron itself) is $h' = \frac{1}{2}a\sqrt{3}$ (this is generally referred to as the "slant height" in the solid geometry literature). At a certain location along the line segment representing "slant height" of each face is the center C' of the face. Imagine this line segment starting at atom A and ending at the midpoint of one of the sides. Knowing that this line segment bisects the 60° angle of the equilateral face, then it is easy to see that C' is a distance $AC' = a/\sqrt{3}$. If we draw a line from C' all the way to the farthest point on the tetrahedron (this will land on an atom we label B), then this new line is the altitude h of the tetrahedron. Using the Pythagorean theorem,

$$h = \sqrt{a^2 - (AC')^2} = \sqrt{a^2 - \left(\frac{a}{\sqrt{3}}\right)^2} = a\sqrt{\frac{2}{3}} .$$

Now we include coordinates: imagine atom B is on the $+y$ axis at $y_b = h = a\sqrt{2/3}$, and atom A is on the $+x$ axis at $x_a = AC' = a/\sqrt{3}$. Then point C' is the origin. The tetrahedron center point C is on the y axis at some value y_c which we find as follows: C must be equidistant from A and B, so

$$y_b - y_c = \sqrt{x_a^2 + y_c^2}$$

$$a\sqrt{\frac{2}{3}} - y_c = \sqrt{\left(\frac{a}{\sqrt{3}}\right)^2 + y_c^2}$$

which yields $y_c = a/2\sqrt{6}$.

(a) In unit vector notation, using the information found above, we express the vector starting at C and going to A as

$$\vec{r}_{ac} = x_a\hat{\text{i}} + (-y_c)\hat{\text{j}} = \frac{\text{a}}{\sqrt{3}}\hat{\text{i}} - \frac{a}{2\sqrt{6}}\hat{\text{j}} .$$

Similarly, the vector starting at C and going to B is

$$\vec{r}_{bc} = (y_b - y_c)\hat{\text{j}} = \tfrac{a}{2}\sqrt{3/2}\,\hat{\text{j}} .$$

Therefore, using Eq. 3-20,

$$\theta = \cos^{-1}\left(\frac{\vec{r}_{ac}\cdot\vec{r}_{bc}}{|\vec{r}_{ac}||\vec{r}_{bc}|}\right) = \cos^{-1}\left(-\frac{1}{3}\right)$$

which yields θ= 109.5° for the angle between adjacent bonds.

(b) The length of vector $\vec{r}_{bc}$ (which is, of course, the same as the length of $\vec{r}_{ac}$) is

$$|\vec{r}_{bc}| = \frac{a}{2}\sqrt{\frac{3}{2}} = \frac{388\text{pm}}{2}\sqrt{\frac{3}{2}} = 237.6 \text{ pm} \approx 238 \text{ pm}.$$

We note that in the solid geometry literature, the distance $\frac{a}{2}\sqrt{\frac{3}{2}}$ is known as the circumradius of the regular tetrahedron.

44. According to Eq. 41-6,

$$P(E_F+\Delta E)=\frac{1}{e^{(E_F+\Delta E-E_F)/kT}+1}=\frac{1}{e^{\Delta E/kT}+1}=\frac{1}{e^{x}+1}$$

where $x=\Delta E/kT$. Also,

$$P(E_F+\Delta E)=\frac{1}{e^{(E_F-\Delta E-E_F)/kT}+1}=\frac{1}{e^{-\Delta E/kT}+1}=\frac{1}{e^{-x}+1}\ .$$

Thus,

$$P(E_F+\Delta E)+P(E_F-\Delta E)=\frac{1}{e^{x}+1}+\frac{1}{e^{-x}+1}=\frac{e^{x}+1+e^{-x}+1}{(e^{-x}+1)(e^{x}+1)}=1\ .$$

A special case of this general result can be found in problem 13, where ΔE = 63 meV and

$$P(E_F + 63\ \text{meV}) + P(E_F - 63\ \text{meV}) = 0.090 + 0.91 = 1.0.$$

45. (a) The derivative of $P(E)$ is

$$\left(\frac{-1}{\left(e^{(E-E_F)/kT}+1\right)^2}\right)\frac{d}{dE}e^{(E-E_F)/kT}=\left(\frac{-1}{\left(e^{(E-E_F)/kT}+1\right)^2}\right)\frac{1}{kT}e^{(E-E_F)/kT} .$$

Evaluating this at $E = E_F$ we readily obtain the desired result.

(b) The equation of a line may be written $y = m(x - x_o)$ where m is the slope (here: equal to $-1/kT$, from part (a)) and x_o is the x-intercept (which is what we are asked to solve for). It is clear that $P(E_F) = 2$, so our equation of the line, evaluated at $x = E_F$, becomes

$$2 = (-1/kT)(E_F - x_o),$$

which leads to $x_o = E_F + 2kT$.

46. (a) For copper, Eq. 41-10 leads to

$$\frac{d\rho}{dT}=[\rho\alpha]_{\rm Cu}=(2\times10^{-8}\,\Omega\cdot{\rm m})(4\times10^{-3}\,{\rm K}^{-1})=8\times10^{-11}\,\Omega\cdot{\rm m/K}\ .$$

(b) For silicon,

$$\frac{d\rho}{dT}=[\rho\alpha]_{\rm Si}=(3\times10^{3}\,\Omega\cdot{\rm m})(-70\times10^{-3}\,{\rm K}^{-1})=-2.1\times10^{2}\,\Omega\cdot{\rm m/K}\ .$$

47. We use the ideal gas law in the form of Eq. 20-9:

$$p = nkT = (8.43\times10^{28}\ \text{m}^{-3})(1.38\times10^{-23}\ \text{J/K})(300\ \text{K}) = 3.49\times10^{8}\,\text{Pa} = 3.49\times10^{3}\ \text{atm}\ .$$

48. We equate E_F with $\frac{1}{2}m_e v_F^2$ and write our expressions in such a way that we can make use of the electron mc^2 value found in Table 37-3:

$$v_F = \sqrt{\frac{2E_F}{m}} = c\sqrt{\frac{2E_F}{mc^2}} = (3.0\times10^5\ \text{km/s})\sqrt{\frac{2(7.0\,\text{eV})}{5.11\times10^5\ \text{eV}}} = 1.6\times10^3\ \text{km/s}\,.$$

49. We compute $\left(\frac{3}{16\sqrt{2\pi}}\right)^{2/3} \approx 0.121$.