Chapter 10 – Solid State Physics

10-1. The molar volume is $\dfrac{M}{\rho} = 2N_A r_0^3$

$$r_0 = \left[\frac{M}{2N_A\rho}\right] = \left[\frac{74.55\ g/moles}{2(6.022\times 10^{23}/mole)(1.984\ g/cm^3)}\right]^{1/3} = 3.15\times 10^{-8}\ cm = 0.315\ nm$$

10-2. The molar volume is $\dfrac{M}{\rho} = 2N_A r_0^3$

$$\rho = \frac{M}{2N_A r_0^3} = \frac{42.4\ g/mole}{2(6.022\times 10^{23}/mole)(0.257\times 10^{-7}\ cm)^3} = 2.07\ g/cm^3$$

10-3. $U(r_0) = -\alpha\dfrac{ke^2}{r_0}\left(1-\dfrac{1}{n}\right)$ (Equation 10-6)

$$E_d = -U(r_0) = \alpha\frac{ke^2}{r_0}\left(1-\frac{1}{n}\right)$$

$$1-\frac{1}{n} = \frac{E_d r_0}{\alpha ke^2} = \frac{(741\ kJ/mol)(0.257\ nm)}{1.7476(1.44\ eV\cdot nm)}\times\frac{1\ eV/\ ion\ pair}{96.47\ kJ/mol} = 0.7844$$

$$n = \frac{1}{1-0.7844} = 4.64$$

10-4. (a) $U_{att} = -\alpha\dfrac{ke^2}{r_0}$ (Equation 10-1)

$$= -1.7476(1.44\ eV\cdot nm)/0.314\ nm$$

$$= -8.01\ eV$$

(*Problem 10-4 continued*)

(b)
$$E_d = -U(r_0) = \alpha \frac{ke^2}{r_0}\left(1 - \frac{1}{n}\right) \quad \text{(Equation 10-6)}$$

$$= (8.01\,eV)\left(1 - \frac{1}{9}\right) = 7.12\ eV/ion\ pair$$

$$= (7.12\,eV/ion\ pair)\left(\frac{96.47\,kJ/mol}{1\ eV/ion\ pair}\right)\left(\frac{1\ cal}{4.186J}\right) = 164\ kcal/mole$$

(c)
$$1 - \frac{1}{n} = \frac{E_d r_0}{\alpha ke^2} = \frac{(165.5\,kcal/mol)(0.314\,nm)}{1.7476\,(1.44\,eV\cdot nm)} \times \frac{4.186\,J}{1\,cal}\left(\frac{1\,eV/ion\ pair}{96.47\,kJ/mol}\right)$$

$$= 0.8960 \quad \text{Therefore } n = \frac{1}{1 - 0.8960} = 9.62$$

10-5. Cohesive energy (LiBr) = $788 \times 10^3 J/mol\left(\frac{1\,mol}{6.02\times10^{23}\ ion\ pairs}\right)\left(\frac{1\,eV}{1.60\times10^{-19}}\right)$

$$= 8.182\ eV/ion\ pair = 4.09\ eV/atom$$

This is about 32% larger than the value in Table 10-1.

10-6. (a) The cubic unit cell has side *a* = *2R*; therefore the volume of the unit cell is:

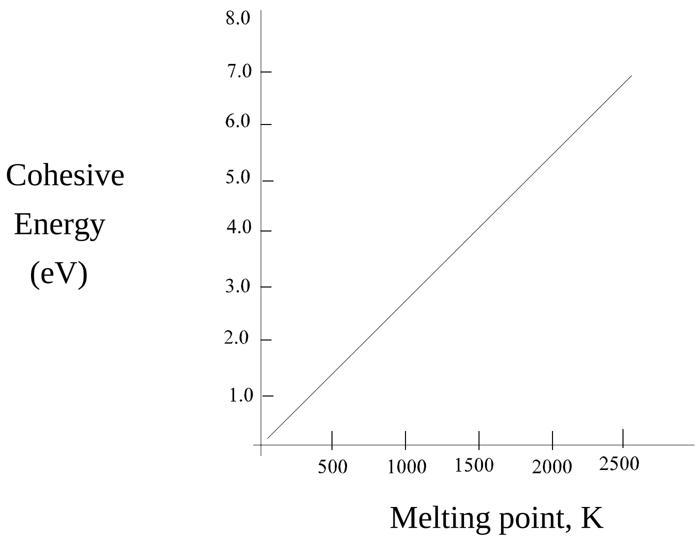
$V_{cell} = a^3 = (2R)^3 = 8R^3$

(b) The volume of the cell occupied by spheres (called the "packing fraction") is:

$$\frac{V_{\text{spheres}}}{V_{\text{cell}}} = [4/3\,\pi R^3]/(8R^3) = \pi/6 = 0.524$$

Because 1/8 of the volume of each sphere is within the cell and there are eight corners.

10-7. (a)



(b) Noting that the melting points are in kelvins on the graph,

Co melting point = 1768 K, cohesive energy = 5.15 eV

Ag melting point = 1235 K, cohesive energy = 3.65 eV

Na melting point = 371 K, cohesive energy = 1.25 eV

10-8. $$U_{\text{att}} = -ke^2\left(\frac{2}{a} + \frac{2}{2a} - \frac{2}{3a} + \frac{2}{4a} + \frac{2}{5a} - \frac{2}{6a} + \cdots\right)$$

$$U_{\text{att}} = -ke^2\left(2 + 1 - \frac{2}{3} + \frac{1}{2} + \frac{2}{5} - \frac{1}{3} + \cdots\right)$$

The quantity in parentheses is the Madelung constant α. The 35th term of the series (2/35) is approximately 1% of the total, where α = 4.18.

10-9. (a) $$\rho = \frac{m_e\langle v\rangle}{ne^2\lambda} \qquad \text{(Equation 10-17)}$$

$$= \frac{(9.11\times10^{-31}\,kg)(1.17\times10^{5}\,m/s)}{(8.47\times10^{28}\,electrons/m^3)(1.60\times10^{-19}\,C)^2(0.4\times10^{-9}\,m)}$$

$$= 1.23\times10^{-7}\,\Omega\cdot m$$

(Problem 10-9 continued)

(b) $\langle v \rangle \propto (kT/m_e)^{1/2}$ (from Equation 10-9)

$$\langle v \rangle_{100} = \left(\frac{100K}{300K}\right)^{1/2} = \frac{1}{\sqrt{3}}$$

$$\rho_{100} = \rho_{300}/\sqrt{3} = 7.00\times10^{-8}\,\Omega\cdot m$$

10-10. (a) $j = \frac{I}{A} = \frac{I}{\pi d^2/4} = \frac{4(10^{-3}A)}{\pi(1.63\times10^{-3}m)^2} = 479A/m^2$ (from Equation 10-10)

$$v_d = \frac{I}{Ane} = \frac{d}{ne} = \frac{479A/m^2}{(8.47\times10^{28}/m^3)(1.602\times10^{-19}C)} = 3.53\times10^{-8}m/s = 3.53\times10^{-6}cm/s$$

10-11. (a) There are n_a conduction electrons per unit volume, each occupying a sphere of volume $4\pi r_s^3/3$: $n_a\cdot(4\pi r_s^2//) = 1$

$$r_s^3 = \frac{3}{4\pi n_a} \rightarrow r_s = (3/4\pi n_a)^{1/3}$$

(b) $r_s = \left[\frac{3}{4\pi(8.47\times10^{28}/m^3)}\right]^{1/3} = 1.41\times10^{-10}m = 0.141\,nm$

10-12. (a) $n = \rho N_A/M$ for 1 electron/atom

$$n = \frac{(10.5\,g/cm^3)(6.022\times10^{23}/mole)}{107.9\,g/mole} = 5.86\times10^{22}/cm^3$$

(b) $n = \frac{(19.3\,g/cm^3)(6.022\times10^{23}/mole)}{196.97\,g/mole} = 5.90\times10^{22}/cm^3$

Both agree with the values given in Table 10-3.

10-13. (a) $n = 2\rho N_A / M$ for two free electrons/atom

$$n = \frac{2\,(1.74\,g/cm^3)(6.022\times10^{23}/mole)}{24.31\,g/mole} = 8.62\times10^{22}/cm^3 = 8.62\times10^{28}/m^3$$

(b) $n = \dfrac{2\,(7.1\,g/cm^3)(6.022\times10^{23}/mole)}{65.37\,g/mole} = 13.1\times10^{22}/cm^3 = 13.1\times10^{28}/m^3$

Both are in good agreement with the values in Table 10-3, $8.61\times10^{28}/m^3$ for Mg and $13.2\times10^{28}/m^3$ for Zn.

10-14. (a) $\rho = \dfrac{m_e\langle v\rangle}{ne^2\lambda}$ (Equation 10-17) $\sigma = \dfrac{1}{\rho} = \dfrac{ne^2\lambda}{m_e\langle v\rangle}$ (Equation 10-18)

$$\rho = \frac{(9.11\times10^{-31}\,kg)(1.08\times10^5\,m/s)}{(8.47\times10^{28}\,m^{-3})(1.602\times10^{-19}\,C)^2\,(0.37\times10^{-9}\,m)} = 1.22\times10^{-7}\,\Omega\cdot m$$

$$\sigma = \frac{1}{\rho} = \frac{1}{1.22\times10^{-7}\,\Omega\cdot m} = 8.17\times10^6(\Omega\cdot m)^{-1}$$

(b) $\langle v\rangle = \sqrt{\dfrac{8kT}{\pi m_e}}$ (Equation 10-9)

$$\rho(200K) = \rho(300K)\langle v(200K)\rangle/\langle v(300K)\rangle$$

$$= \rho(300K)(200K/300K)^{1/2}$$

$$= (1.22\times10^{-7}\,\Omega\cdot m)(200K/300K)^{1/2} = 9.96\times10^{-8}\,\Omega\cdot m$$

$$\sigma(200\,K) = \frac{1}{\rho(200K)} = \frac{1}{9.96\times10^{-8}\,\Omega\cdot m} = 1.00\times10^7(\Omega\cdot m)^{-1}$$

(c) $\rho(100K) = \rho(300K)\langle v(100K)\rangle/\langle v(300K)\rangle$

$$= (1.22\times10^{-7}\,\Omega\cdot m)(100K/300K)^{1/2} = 7.04\times10^{-8}\,\Omega\cdot m$$

$$\sigma(100\,K) = \frac{1}{\rho(100K)} = \frac{1}{7.04\times10^{-8}\,\Omega\cdot m} = 1.42\times10^7(\Omega\cdot m)^{-1}$$

10-15. $\langle E \rangle = \frac{3}{5} E_F$ (Equation 10 − 37)

(a) for Cu: $\langle E \rangle = \frac{3}{5}(7.06\, eV) = 4.24\, eV$

(b) for Li: $\langle E \rangle = \frac{3}{5}(4.77\, eV) = 2.86\, eV$

$$E_F = \frac{(hc)^2}{2mc^2}\left(\frac{3}{8\pi V} \cdot \frac{N}{V} \right)^{2/3}$$

$$= \frac{(1240\, eV\cdot nm)^2}{2(5.11\times 10^5\, eV)} \left[\frac{3\,(5.90\times 10^{28}\, m^{-3})}{8\pi} \left(\frac{10^{-9}\, m}{1\, nm} \right)^3 \right]^{1/3} = 5.53\, eV$$

10-16. A long, thin wire can be considered one-dimensional.

$$E_F = \frac{h^2}{32m}\left(\frac{N}{L}\right)^2 = \frac{(hc)^2}{32mc^2}\left(\frac{N}{L}\right)^2 \quad \text{(Equation 10-30)}$$

For *Mg*: $N/L = (8.61\times 10^{28}/m^2)^{1/3}$

$$E_F = \frac{(1240\, eV\cdot nm \times 10^{-9}\, m/nm)^2 (8.61\times 10^{28}/m^3)^{2/3}}{32(0.511\times 10^6\, eV)} = 1.87\, eV$$

10-17. (a) For Ag: $E_F = \frac{h^2}{2m}\left(\frac{3N}{8\pi V}\right)^{2/3} = \frac{(1240\, eV\cdot nm\times 10^{-9}\, m/nm)^2}{2(0.511\times 10^6\, eV)}\left(\frac{3\times 5.86\times 10^{28}\, m^{-3}}{8\pi}\right)^{2/3} = 5.50\, eV$

For Fe: Similarly, E_F = 11.2 eV

(b) For Ag: $E_F = kT_F$ (Equation 10 − 38)

$$T_F = \frac{E_F}{k} = \frac{5.50\, eV}{8.617\times 10^{-5}\, eV/K} = 6.38\times 10^4\, K$$

For Fe: Similarly, $T_F = 13.0 \times 10^4$ K

Both results are in close agreement with the values given in Table 10-3.

10-18. Note from Fig.10-14 that most of the excited electrons are within about 2kT above the Fermi energy E_F, i.e., $\Delta E \approx 2kT$. Note, too, that $kT << E_F$, so the number ΔN of excited electrons is: $\Delta N \approx N(E_F)n(E_F)\Delta E \approx N(E_F)(1/2)(2kT) \approx N(E_F)kT$ and

$$N = \frac{8\pi V}{3}\left(\frac{2m}{h^2}\right)^{3/2} E_F^{3/2} \quad \text{(from Equation 10-35)}$$

Differentiating Equation 10-34 gives: $N(E_F) = \dfrac{\pi V}{2}\left(\dfrac{8m}{h^2}\right)^{3/2} E_F^{1/2}$

Then, $$\frac{\Delta N}{N} = \frac{\frac{\pi V}{2}\left(\frac{8m}{h^2}\right)^{3/2} E_F^{1/2}\,kT}{\frac{8\pi v}{3}\left(\frac{2m}{h^2}\right)^{3/2} E_F^{3/2}} = \frac{3}{2}kT/E_F$$

E_F for Ag = 5.35 eV, so $\dfrac{\Delta N}{N} = \dfrac{3}{2}(8.617\times10^{-5}\,eV/K)(300K)/5.53\,eV = 0.0070 \approx 0.1\%$

10-19. $$u_F = \left(\frac{2E_F}{m_e}\right)^{1/2} = c\left(\frac{2E_F}{m_e c^2}\right)^{1/2} \quad \text{(Equation 10-39)}$$

(a) for Na: $u_F = (3.00\times10^8\,m/s)\left[\dfrac{2(3.26\,eV)}{5.11\times10^5\,eV}\right]^{1/2} = 1.07\times10^6\,m/s$

(b) for Au: $u_F = (3.00\times10^8\,m/s)\left[\dfrac{2(5.55\,eV)}{5.11\times10^5\,eV}\right]^{1/2} = 1.40\times10^6\,m/s$

(c) for Sn: $u_F = (3.00\times10^8\,m/s)\left[\dfrac{2(10.3\,eV)}{5.11\times10^5\,eV}\right]^{1/2} = 1.90\times10^6\,m/s$

10-20. $\rho = \dfrac{m_e u_F}{ne^2\lambda}$ (Equation 1-40) $\qquad \lambda = \dfrac{m_e u_F}{ne^2\rho}$

(Problem 10-20 continued)

(a) for Na: $\lambda = \dfrac{(9.11\times10^{-31}kg)(1.07\times10^{6}m/s)}{(2.65\times10^{28}m^{-3})(1.609\times10^{-19}C)^{2}(4.2\times10^{-8}\Omega\cdot m)}$

$= 3.42\times10^{-8}m = 34.2\,nm$

(b) for Au: $\lambda = \dfrac{(9.11\times10^{-31}kg)(1.40\times10^{6}m/s)}{(5.90\times10^{28}m^{-3})(1.609\times10^{-19}C)^{2}(2.04\times10^{-8}\Omega\cdot m)}$

$= 4.14\times10^{-8}m = 41.4\,nm$

(c) for Sn: $\lambda = \dfrac{(9.11\times10^{-31}kg)(1.90\times10^{6}m/s)}{(14.8\times10^{28}m^{-3})(1.609\times10^{-19}C)^{2}(10.6\times10^{-8}\Omega\cdot m)}$

$= 4.31\times10^{-8}m = 43.1\,nm$

10-21. $C_v(electrons) = \dfrac{\pi^2}{2}R\dfrac{T}{T_F}$ (Equation 10 – 45)

$C_v(electrons) = \dfrac{\pi^2}{2}R\dfrac{kT}{E_F}$ because $E_F = kT_F$. C_v due to the lattice vibrations is 3R, assuming

$T >> T_E$ (rule of Dulong and Petit): $\dfrac{\pi^2}{2}R\dfrac{kT}{E_F} = 0.10(3R)$

$$T = \frac{0.10(3)(2)E_F}{\pi^2 k} = \frac{(0.60)(7.06\,eV)}{\pi^2(8.617\times10^{-5}eV/K)} = 4.98\times10^{3}K$$

This temperature is much higher than the Einstein temperature for a metal such as copper.

10-22. $U = \dfrac{3}{5}NE_F + \alpha N\left(\dfrac{kT}{E_F}\right)kT$ (Equation 10 – 44)

Average energy/electron = $U/N = \dfrac{3}{5}E_F + \alpha\left(\dfrac{kT}{E_F}\right)kT = \dfrac{3}{5}E_F + \dfrac{\pi^2}{4}\dfrac{(kT)^2}{E_F}$

(Problem 10-22 continued)

For copper $E_F = 7.06\,eV$, so

At T = 0K: $U/N = \frac{3}{5}(7.06\,eV) = 4.236\,eV$

At T = 300K: $U/N = \frac{3}{5}(7.06\,eV) + \frac{\pi^2}{4}\frac{(8.61\times10^{-5}\,eV/K)^2\,(300\,K)^2}{7.06} = 4.236\,eV$

The average energy/electron at 300 K is only 0.0002 eV larger than at 0 K, a consequence of the fact that 300 K is very small compared to the T_F for Cu (81,600 K). The classical value of $U/N = (3/2)kT = 0.039\,eV$, is far too small.

10-23. $C_v(electrons) = \frac{\pi^2}{2}R\frac{T}{T_F}$ (Equation 10 – 45)

Melting temperature of Fe = 1811 K (from Table 10-1)

T_F for $Fe = 13\times10^4\,K$ (from Table 10-3)

The maximum C_v for the Fe electrons, which is just before Fe melts, is:

$$C_v(electrons) = \frac{\pi^2}{2}R\left(\frac{1811\,K}{13\times10^4\,K}\right) = 0.0219\,R$$

The heat capacity of solids, including Fe, is $3R$ (rule of Dulong and Petit, see Section 8-1).

$$\frac{C_V(electrons)}{C_V} = \frac{0.0219\,R}{3R} = 0.0073$$

10-24. $P = \frac{\rho_+ - \rho_-}{\rho} = \frac{M}{\mu\rho} = \frac{\mu B}{kT}$ (from Equation 10 – 50)

$$P = \frac{(9.285\times10^{-24}\,J/T)(2.0\,T)}{(1.38\times10^{-23}\,J/K)(200\,K)} = 6.7\times10^{-3}$$

10-25.

$$\chi = \frac{\mu_0 M}{B} = \frac{\mu_0 \rho \mu^2}{kT}$$

$$\chi \text{ units} = \left(\frac{N}{A^2}\right)\left(\frac{1}{m^3}\right)\left(\frac{J}{T}\right)^2\left(\frac{1}{J}\right)$$

$$= \frac{NJ^2}{A^2 m^3 T^2 J}$$

$$= \frac{NJ}{A^2 m^3 (Wb/m)^2}$$

$$= \frac{NJ}{A^2 m^3 (N/Am)^2} = \frac{NJA^2m^2}{A^2m^3N^2}$$

$$= \frac{J}{Nm} = \frac{Nm}{Nm} = 1 \quad \text{dimensionless}$$

10-26. $E = hc/\lambda$

$$\lambda = hc/E = 1240\,eV{\cdot}nm/1.14\,eV = 1.088\times10^3\,nm = 1.09\times10^{-6}m = 1.09\times10^3\,nm$$

10-27. (a) For germanium:

$$\lambda = hc/E = 1240\,eV{\cdot}nm/0.74\,eV = 1.68\times10^3\,nm = 1.68\times10^{-6}m = 1.68\times10^3\,nm$$

(b) For diamond:

$$\lambda = hc/E = 1240\,eV{\cdot}nm/7.0\,eV = 177\,nm$$

10-28. (a) $E = hc/\lambda = 1240\,eV{\cdot}nm/(3.35\,\mu m \times 10^3\,nm/\mu m) = 0.37\,eV$

(b) $E = kT = 0.37\,eV \quad \therefore\ T = 0.37\,eV/k = 0.37\,eV/8.617\times10^{-5}\,eV/K = 4300\,K$

10-29. (a) $N = \dfrac{mN_A}{M} = \dfrac{\rho V N_A}{M} = \dfrac{(2.33\,g/cm^3)(100\,nm\times10^{-7}\,cm/nm)^3(6.02\times10^{23}/mol)}{28\,g/mol}$

$= 5.01\times10^7\,Si\ atoms$

(b) $\Delta E \approx 13\,eV/(4\times5.01\times10^7) = 6.5\times10^{-8}\,eV$

10-30. (a) $E_1 = -\frac{1}{2}\left(\frac{ke^2}{\hbar}\right)^2 \frac{m^*}{\kappa^2}\frac{1}{(1)^2}$ (Equation 10-58)

$$E_1 = -\frac{1}{2}\frac{\left[(9\times10^9 N\cdot m^2/C^2)(1.60\times10^{-19}C)^2\right]^2}{(1.055\times10^{-34}J\cdot s)^2}\cdot\frac{(0.2)(9.11\times10^{-31}kg)}{(11.8)^2}$$

$$= -3.12\times10^{-21}J = -0.0195\,eV$$

Ionization energy = 0.0195 eV

(b) $\langle r_1\rangle = a_0(1)^2(m_e/m^*)\,\kappa$ (Equation 10-59)

$$= 0.0529\,nm(1/0.2)(11.8) = 3.12\,nm$$

(c) $E_g(Si) = 1.11\,eV$ at $293\,K$

$E_1/E_g = 0.0195/1.11 = 0.0176$ or about 2%

10-31. (a) $E_1 = -\frac{1}{2}\left(\frac{ke^2}{\hbar}\right)^2 \frac{m^*}{\kappa^2}\frac{1}{(1)^2}$ (Equation 10-58)

$$E_1 = -\frac{1}{2}\frac{\left[(9\times10^9 N\cdot m^2/C^2)(1.60\times10^{-19}C)^2\right]^2}{(1.055\times10^{-34}J\cdot s)^2}\cdot\frac{(0.34)(9.11\times10^{-31}kg)}{(15.9)^2}$$

$$= -2.92\times10^{-21}J = -0.0182\,eV$$

(b) $\langle r_1\rangle = a_0(1)^2(m_e/m^*)\,\kappa$ (Equation 10-59)

$$= 0.0529\,nm(1/0.34)(15.9) = 2.48\,nm$$

10-32. Electron configuration of *Si*: $1s^2 2s^2 2p^6 3s^2 3p^2$

(a) *Al* has a $3s^2$ 3p configuration outside the closed n = 2 shell (3 electrons), so a p-type semiconductor will result.

(b) *P* has a $3s^2$ $3p^3$ configuration outside the closed n = 2 shell (5 electrons), so an n-type semiconductor results.

10-33. $E = kT = 0.01\,eV \quad \therefore\ T = 0.01\,eV/8.617\times10^{-5}\,eV/K = 116\,K$

10-34. (a) $V_H = v_d Bw = \dfrac{dBw}{nq} = \dfrac{iB}{qnt}$ (Equation 10-60 and Example 10-10)

The density of charge carriers n is:

$$n = \frac{iB}{qtV_H} = \frac{(20A)(0.25T)}{(1.60\times10^{-19}C)(0.2\times10^{-3}m)(2.2\times10^{-6}V)} = 7.10\times10^{28}\ \text{carriers/m}^3$$

(b) $N = \dfrac{\rho N_A}{M} = \dfrac{(5.75\times10^3\,kg/m^3)(6.02\times10^{26}/mol)}{118.7\,kg/mol} = 2.92\times10^{28}$

Each Sn atom contributes $n/N = 7.10\times10^{28}/2.92\times10^{28} = 2.4$ charge carriers

10-35. $I_{net} = I_0(e^{eV_b/kT} - 1)$ (Equation 10-64)

(a) $e^{eV_b/kT} = 10$, so $eV_b/kT = \ln 10$. Therefore,

$$(1.609\times10^{-19}C)V_b/(1.381\times10^{-23}J/K)(300K) = \ln 10$$

$$V_b = (1.38\times10^{-23}J/K)(300K)\ln 10/1.60\times10^{-19}C = 0.0596\,V = 59.6\,mV$$

(b) $e^{eV_b/kT} = 0.1$

$$V = 0.0596\,V(\ln 0.1/\ln 10) = -0.0596\,V = -59.6\,mV$$

10-36. $I_{net} = I_0(e^{eV_b/kT} - 1)$

(a) $e^{eV_b/kT} = 5$

$$eV_b/kT = \ln(5)$$

$$V_b = \frac{kT\ln(5)}{e} = \frac{(1.38\times10^{-23}J/K)(200K)\ln(5)}{1.60\times10^{-19}C}$$

$$= -0.0278\,V = -27.8\,mV$$

(Problem 10-36 continued)

(b) $e^{eV_b/kT} = 0.5$

$eV_b/kT = \ln(0.5)$

$$V_b = \frac{kT\ln(0.5)}{e} = \frac{(1.38\times10^{-23}J/K)(200K)\ln(0.5)}{1.60\times10^{-19}C}$$

$$= -0.0120\,V = -12.0\,mV$$

10-37. $I_{net} = I_0(e^{eV_b/kT} - 1)$ (Equation 10-64)

Assuming T = 300K,

$$\frac{I(0.2V) - V(0.1V)}{I(0.1V)} = \frac{I_0(e^{e(0.2V)/kT} - 1) - I_0(e^{e(0.1V)/kT} - 1)}{I_0(e^{e(0.1V)/kT} - 1)}$$

$$= \frac{e^{e(0.2V)/kT} - e^{e(0.1V)/kT}}{e^{e(0.1V)/kT} - 1}$$

$$= 47.6/1$$

10-38. (a) From Equation 10-64, *exp*(eV_b/kT) = 10

Taking *ln* of both sides and solving for V_b,

$$V_b = (kT/e)\ln10 = \frac{(1.38\times10^{-23}J/K)(77K)\ln(10)}{(1.60\times10^{-19}C)}$$

$$V_b = 0.0153 \text{ volts} = 15.3\,mV$$

(b) Similarly, for *exp*(eV_b/kT) = 1; V_b = 0

(c) For (a): $I_{\text{net}} = I_0(e^{eV_b/kT} - 1)$

$$I_{\text{net}} = 1\,mA(10 - 1) = 9\,mA$$

For (b): $I_{net} = 0$

10-39. $M^{\alpha}T_c = constant$ (Equation 10-70)

First, we find the constant for *Pb* using the mass of natural *Pb* from Appendix A, T_c for *Pb* from Table 10-6, and α for *Pb* from Table 10-7.

$constant = (207.19u)^{0.49}(7.196K) = 98.20$

For ^{206}Pb: $T_c = constant/M^{\alpha} = 98.20/(205.974u)^{0.49} = 7.217\,K$

For ^{207}Pb: $T_c = constant/M^{\alpha} = 98.20/(206.976u)^{0.49} = 7.200\,K$

For ^{208}Pb: $T_c = constant/M^{\alpha} = 98.20/(207.977u)^{0.49} = 7.183\,K$

10-40. (a) $E_g = 3.5kT_c$ (Equation 10-71)

T_c for *I* is 3.408 K, so, $E_g = 3.5(8.617\times10^{-5}eV/K)(3.408K) = 1.028\times10^{-3}\,eV$

(b) $E_g = hc/\lambda$

$\lambda = hc/E_g = 1240eV{\cdot}nm/1.028\times10^{-3}eV$

$= 1.206\times10^{6}nm = 1.206\times10^{-3}m = 1.206\,nm$

10-41. (a) $E_g = 3.5kT_c$ For *Sn*: $T_c = 3.722$ K

$E_g = 3.5(8.617\times10^{-5}eV/K)(3.722K) = 0.0011\,eV$

This is about twice the measured value.

(b) $E_g = hc/\lambda$

$\lambda = hc/E = 1240eV{\cdot}nm/6\times10^{-4}eV$

$= 2.07\times10^{6}nm = 2.07\times10^{-3}\,m$

10-42. At $T/T_c = 0.5$ $E_g = (T)/E_g(0) = 0.95$ where $E_g(0) = 3.5kT_C$ (Equation 10-71)

So $E_g(T) = 0.95(3.5)kT_c = 3.325kT_c$

(Problem 10-42 continued)

(a) For *Sn*: $E_g(T) = 3.325(8.617\times10^{-5}\,eV/K)(3.722K) = 1.07\times10^{-3}\,eV$

(b) For *Nb*: $E_g(T) = 3.325(8.617\times10^{-5}\,eV/K)(9.25K) = 2.65\times10^{-3}\,eV$

(c) For *Al*: $E_g(T) = 3.325(8.617\times10^{-5}\,eV/K)(1.175K) = 3.37\times10^{-4}\,eV$

(d) For *Zn*: $E_g(T) = 3.325(8.617\times10^{-5}\,eV/K)(0.85K) = 2.44\times10^{-4}\,eV$

10-43. $B_C(T)/B_C(0) = 1 - (T/T_C)^2$

(a) $B_C(T)/B_C(0) = 0.1 = 1 - (T/T_C)^2$

$(T/T_C)^2 = 1 - 0.1 = 0.9$

$T/T_C = 0.95$

(b) Similarly, for $B_C(T)/B_C(0) = 0.5$

$T/T_C = 0.71$

(c) Similarly, for $B_C(T)/B_C(0) = 0.9$

$T/T_C = 0.32$

10-44. T_F for Cu is 81,700 K, so only those electrons within $E_F - kT$ of the Fermi energy could be in states above the Fermi level. The fraction *f* excited above E_F is approximately:

$f = kT/E_F = T/T_F$

(a) $f = 300K/81{,}700K = 3.7\times10^{-3}$

(b) $f = 1000K/81{,}700K = 12.2\times10^{-3}$

10-45.

−e	+e	−e	+e	−e	+e	−e	+e	−e	+e	−e	+e	−e
•	•	•	•	•	•	•	•	•	•	•	•	•
−6	−5	−4	−3	−2	−1	0	1	2	3	4	5	6

→| r_0 |←

(a) For the negative ion at the origin (position 0) the attractive potential energy is:

$$V = -\frac{2ke^2}{r_0}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots\right)$$

(b) $V = -\alpha\dfrac{ke^2}{r_0}$, so the Madelung constant is

$$\alpha = 2\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \cdots\right)$$

Noting that $\ln(1+x) = x - \dfrac{x^2}{2} + \dfrac{x^3}{3} - \dfrac{x^4}{4} + \cdots$, $\ln(2) = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \cdots$

and $\alpha = 2\ln 2 = 1.386$

10-46. $C_v = \dfrac{\pi^2}{2}R\dfrac{T}{T_F}$ (Equation 10−45)

$$T_F = \frac{\pi^2}{2}R\frac{T}{(3.74\times10^{-4}J/mol\cdot K)\,T}$$

and $$E_F = kT_F = \frac{\pi^2}{2}R\frac{kT}{(3.74\times10^{-4}J/mol\cdot K)\,T}$$

$$= \frac{\pi^2(8.314J/mol\cdot K)(1.38\times10^{-23}J/K)}{2(3.74\times10^{-4}J/mol\cdot K)}$$

$$= 1.51\times10^{-18}J(1/1.60\times10^{-19}J/eV) = 9.45\ eV$$

10-47. (a) $N = \int_0^{E_F} g(E)\,dE = \int_0^{E_F} AE^{1/2}\,dE = A(2/3)E^{3/2}\Big|_0^{E_F} = (2A/3)\,E_F^{3/2}$

(b) $N' = \int_{E_F-kT}^{E_F} AE^{1/2}\,dE = (2A/3)\left[E_F^{3/2} - (E_F - kT)^{3/2}\right]$

$= (2A/3)\left[E_F^{3/2} - E_F^{3/2}(1 - kT/E_F)^{3/2}\right]$

Because $kT << E_F$ for most metals,

$(1 - kT/E_F)^{3/2} \approx 1 - (3/2)kTE_F^{1/2}$

$N' = (2A/3)\left[E_F^{3/2} - E_F^{3/2} + (3/2)kTE_F^{1/2}\right]$

$= AkTE_F^{1/2}$

The fraction within kT of E_F is then $f = \dfrac{N'}{N} = \dfrac{AkTE_F^{1/2}}{(2A/3)E_F^{3/2}} = \dfrac{3kT}{2E_F}$

(c) For *Cu* $E_F = 7.04\ eV$; at 300 K, $f = \dfrac{3(0.02585\ eV)}{2(7.04\,eV)} = 0.0055$

10-48. (a) Number of pairs $N = (6.60\times10^3\,eV/photon)/(0.72\,eV/pair) = 9.17\times10^5\ pairs/photon$

(b) $\Delta N = \pm\sqrt{N} = \pm957$

Energy resolution $\dfrac{\Delta E}{E} \approx \dfrac{\Delta N}{N} = \dfrac{957}{9.17\times10^5} = 0.0010 = 0.10\%$

10-49. $\lambda = \dfrac{m_{\text{eff}}\langle v\rangle}{n\rho e^2}$ (Equation 10-17)

$\langle v\rangle = (3kT/m_{\text{eff}})^{1/2} = \left[\dfrac{3(1.38\times10^{-23}J/K)(300\,k)}{0.2(9.11\times10^{-31}kg)}\right]^{1/2}$

$\langle v\rangle = 2.61\times10^5\,m/s$

(Problem 10-49 continued)

Substituting into λ:

$$\lambda = \frac{0.2\,(9.11\times10^{-31}\,kg)(2.61\times10^{5}\,m/s)}{(10^{-22}\,m^{-3})(5\times10^{-3}\,\Omega m)(1.60\times10^{-19}\,C)^2}$$

$$\lambda = 3.7\times10^{-8}\,m = 37\,nm$$

For Cu: $u_F = (2E_F/m_e)^{1/2} = \left[\frac{2(7.06\,eV)}{9.11\times10^{-31}\,kg}\right]^{1/2}$

$$u_F = 1.57\times10^{6}\,m/s$$

$n = 8.47\times10^{28}\,m^{-3}$ and $\rho = 1.7\times10^{-8}\,\Omega\cdot m$ (Example 10-6)

Substituting as above, λ = 3.9 × 10^{-8} m = 39 nm

The mean free paths are approximately equal.

10-50. (a) For small V_b (from Equation 10-64)

$$e^{eV_b/kT} \approx 1 + eV_b/kT, \text{ so } I = I_0 eV_b/kT = V_b/R$$

$$R = V_b/(I_0 eV_b/kT) = kT/eI_0 = 0.025\,eV/(e\times10^{-9}A) = 25.0\,\text{M}\Omega$$

(b) For V_b = −0.5 V; $R = V_b/I = 0.5\,V/10^{-9}A = 500\,\text{M}\Omega$

(c) For V_b = +0.5 V; $I = 10^{-9}A(e^{0.5/0.025} - 1) = 0.485\,A$

Thus, $R = V_b/I = 0.5/0.485\,A = 1.03\,\Omega$

(d) $$\frac{dI}{dV_b} = \frac{eI_0}{kT}e^{eV_b/kT}$$

$$R_{ac} = \frac{dV_b}{dI} = \frac{kT}{eI_0}e^{-eV_b/kT} = 25\,\text{M}\Omega\,e^{-20} = 0.0515\,\Omega$$

10-51. $a_0 = \dfrac{\epsilon_0 h^2}{\pi m_e e^2} = \dfrac{\kappa\epsilon_0 h^2}{\pi (m_e)_{eff}\, e^2} = \dfrac{\kappa\hbar^2}{(m_e)_{eff}\, ke^2}$

silicon: $$a_0 = \frac{12(1.055\times10^{-34}J\cdot s)^2}{0.2(9.11\times10^{-31}kg)(9\times10^9 N\cdot m^2/C^2)(1.602\times10^{-19}C)^2}$$

$$= 3.17\times10^{-9}m = 3.17\ nm$$

This is about 14 times the lattice spacing in silicon (0.235 nm).

germanium: $$a_0 = \frac{16(1.055\times10^{-34}J\cdot s)^2}{0.10(9.11\times10^{-31}kg)(9\times10^9 N\cdot m^2/C^2)(1.602\times10^{-19}C)^2}$$

$$= 8.46\times10^{-9}m = 8.46\ nm$$

This is nearly 35 times the lattice spacing in germanium (0.243 nm).

10-52. (a) $\rho = \dfrac{m_e u_F}{ne^2\lambda}$ (from Equation 10-40)

So the equation in the problem can be written as $\rho = \rho_m + \rho_i$. Because the impurity increases ρ_m by $1.1\times10^{-8}\Omega\cdot m$, $\rho_i = 1.1\times10^{-8}\Omega\cdot m$ and

$\lambda_i = \dfrac{m_e u_F}{ne^2(1.18\times10^{-8}\Omega\cdot m)}$ where $n = 8.47\times10^{28} electrons/m^3$ (from Table 10-3)

and $u_F = (2E_F/m_e)^{1/2} = 1.57\times10^6\ m/s$. Therefore,

$$\lambda_i = \frac{(9.11\times10^{-31}kg)(1.157\times10^6 m/s)}{(8.47\times10^{28}/m^3)(1.60\times10^{-19}C)^2(1.1\times10^{-8}\Omega\cdot m)}$$

$$= 6.00\times10^{-8}m = 60.0\ nm$$

(Problem 10-52 continued)

(b) $\lambda = 1/n_a \pi r^2$ and $d = 2r$ (Equation 10-19)

So we have $d^2 = 4/n_i \pi \lambda_i$ where $n_i = 1\%$ of $n = 8.47\times 10^{26}/m^3$

$d^2 = 4/(8.47\times 10^{26}/m^3)\pi(6.60\times 10^{-8}m) = 2.28\times 10^{-20}m^2 \rightarrow d = 0.151\,nm$

10-53. (a) The modified Schrödinger equation is:

$$-\frac{\hbar^2}{2m^* r^2}\frac{d}{dr}\left(r^2\frac{dR(r)}{dr}\right) + \left[-\frac{ke^2}{r\kappa} + \frac{\hbar^2\ell(\ell+1)}{2m^* r^2}\right]R(r) = ER(r)$$

The solution of this equation, as indicated following Equation 7-24, leads to solutions of the form: $R_{n\ell}(r) = a_o' e^{-r/a_0' n} r^{-1}\mathfrak{L}_{n\ell}(r/a_0')$, where $a_0' = (\hbar^2\kappa^2/ke^2 m^*)$.

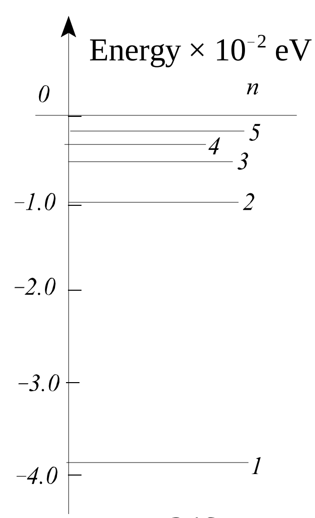
(b) By substitution into Equation 7-25, the allowed energies are:

$$E_n = -\frac{1}{2}\left(\frac{ke^2}{\hbar\kappa}\right)^2\frac{m^*}{n^2} = -\frac{E_1}{n^2} \text{ where } E_1 = \frac{1}{2}\left(\frac{ke^2}{/\kappa}\right)^2 m^*$$

(c) For *As* electrons in *Si* $m^* = 0.2m_c$ (see Problem 10-30) and $\kappa(Si) = 11.8$,

$$E_1 = -\frac{1}{2}\frac{[(9\times 10^9 N\cdot m^2/C^2)(1.60\times 10^{-19}C)^2]^2}{(1.055\times 10^{-34}J\cdot s)^2}\cdot\frac{(0.2)(9.11\times 10^{-31}kg)}{(11.8)^2}$$

$$= -3.12\times 10^{-21}J = -0.0195\,eV$$



10-54. $U = -\alpha \frac{ke^2}{r_0}\left[\frac{r_0}{r} - \frac{1}{n}\left(\frac{r_0}{r}\right)^n\right]$ (Equation 10-5)

$$F = -\frac{dU}{dr} = -Kr \text{ yields } K = \alpha \frac{(n-1)ke^2}{r_0^3}$$

(a) For *NaCl*: $\alpha = 1.7476$, n = 9.35, and $r_0 = 0.282$ nm and

$$\mu = \frac{m(Na)m(Cl)}{m(Na) + m(Cl)} = \frac{(22.99u)(35.45u)}{22.99u + 35.45u} = 13.95u$$

$$f = \frac{1}{2\pi}\sqrt{\frac{K}{M}} = \frac{1}{2\pi}\left[\frac{\alpha(n-1)ke^2}{(13.95u)r_o^3}\right]^{1/2}$$

$$= \frac{1}{2\pi}\left[\frac{(1.7476)(8.35)(9\times10^9 N\cdot m^2/C^2)(1.60\times10^{-19}C)^2}{(13.95u\times1.66\times10^{-27}kg/u)(0.282\times10^{-9}m)^3}\right)^{1/2} = 1.28\times10^{13}Hz$$

(b) $\lambda = c/f = (3.00\times10^8 m/s)/1.28\times10^{13}Hz = 23.4\mu m$

This is of the same order of magnitude as the wavelength of the infrared absorption bands in *NaCl*.

10-55. (a) Electron drift speed is reached for:

$$\frac{dv}{dt} = 0 \rightarrow v_d = -eE\tau/m \quad \text{(Equation 10-14)}$$

(b) Writing Ohm's law as $j = \sigma E$ (Equation 10-12) and $j = |v_d|ne$ (from Equation 10-13)

$j = eE\tau ne/m = E\tau ne^2/m$, which satisfies Ohm's law because $j \propto E$. Thus,

$\sigma = \tau ne^2/m$ and $\rho = 1/\sigma = m/\tau ne^2$.

10-56. (a) For r, s, t all even $(-1)^{r+s+t} = +1$ and the ion's charge at that location is:

$-1(-1.60 \times 10^{-19}\text{ C}) = 1.60 \times 10^{-19}$ C.

Similarly, for any permutation of

r, s even; t odd $(-1)^{r+s+t} = -1$, ion charge $= -1.60 \times 10^{-19}$ C.

r even; s, t odd $(-1)^{r+s+t} = +1$, ion charge $= -1.60 \times 10^{-19}$ C.

r, s, t all odd $(-1)^{r+s+t} = +1$, ion charge $= -1.60 \times 10^{-19}$ C.

(b) $$U = -\alpha \frac{ke^2}{r}$$

If the interatomic distance $r = a$, then a cube $2a$ on each side

$$U = -ke^2\left(\frac{4}{a} - \frac{4}{\sqrt{2}\,a} + \frac{2}{a} - \frac{4}{\sqrt{2}\,a} - \frac{4}{\sqrt{2}\,a} + \frac{4}{\sqrt{3}\,a} + \frac{4}{\sqrt{3}\,a}\right)$$

$$U = -\frac{ke^2}{a}(2.1335) \quad \text{where } \alpha = 2.1335.$$

Similarly, for larger cubes (using spreadsheet). The value of α is approaching 1.7476 slowly.

10-57. (a) $$M = \mu(\rho_+ - \rho_-) \Rightarrow \frac{M}{\rho} = \mu\frac{(\rho_+ - \rho_-)}{\rho}$$

$$\frac{M}{\rho} = \mu\frac{e^{\mu B/kT} - e^{-\mu B/kT}}{e^{\mu B/kT} + e^{-\mu B/kT}} = \mu\tanh(\mu B/kT)$$

and

$$M = \mu\rho\tanh(\mu B/kT)$$

(b) For $\mu B << kT$, $T>>0$ and $\tanh(\mu B/kT) \approx \mu B/kT$

$$\chi = \frac{\mu_0 M}{B} = \frac{\mu_0\mu\rho\mu B}{BkT} = \frac{\mu_0\rho\mu^2}{kT}$$