Chapter 11 – Nuclear Physics

11-1.

Isotope	Protons	Neutrons
^{18}F	9	9
^{25}Na	11	14
^{51}V	23	28
^{84}Kr	36	48
^{120}Te	52	68
^{148}Dy	66	82
^{175}W	74	101
^{222}Rn	86	136

11-2. The momentum of an electron confined within the nucleus is:

$$\Delta p \approx \hbar/\Delta x = 1.055\times10^{-34}\,J{\cdot}s/10^{-14}\,m$$

$$\approx 1.055\times10^{-20}\,J{\cdot}s/m \times (1/1.602\times10^{-13}\,J/MeV)$$

$$\approx 6.59\times10^{-8}\,MeV{\cdot}s/m$$

The momentum must be at least as large as Δp, so $p_{\min} \geq 6.59\times10^{-8}\,MeV{\cdot}s/m$ and the electron's kinetic energy is $E_{\min} = p_{\min}c = (6.59\times10^{-8}\,MeV{\cdot}m)(3.00\times10^{8}\,m/s) = 19.8\,MeV$. This is twenty times the observed maximum beta decay energy, precluding the existence of electrons in the nucleus.

11-3. A proton-electron model of ^{6}Li would consist of 6 protons and 3 electrons. Protons and electrons are spin-1/2 (Fermi-Dirac) particles. The minimum spin for these particles in the lowest available energy states is $1/2\hbar$, so ^{6}Li (S=0) cannot have such a structure.

11-4. A proton-electron model of ^{14}N would have 14 protons and 7 electrons. All are Fermi-Dirac spin-1/2 particles. In the ground state the proton magnetic moments would add to a small fraction of the proton magnetic moment of 2.8 μ_N, but the unpaired electron would give the system a magnetic moment of the order of that of an electron, about 1 μ_B. Because μ_B is approximately 2000 times larger than μ_N, the ^{14}N magnetic moment would be about 1000 times the observed value, arguing against the existence of electrons in the nucleus.

11-5. The two proton spins would be antiparallel in the ground state with $S = 1/2 - 1/2 = 0$. So the deuteron spin would be due to the electron and equal to $1/2\hbar$. Similarly, the proton magnetic moments would add to zero and the deuteron's magnetic moment would be 1 μ_B. From Table 11-1, the observed deuteron spin is $1\hbar$ (rather than $1/2\hbar$ found above) and the magnetic moment is 0.857 μ_N, about 2000 times smaller than the value predicted by the proton-electron model.

11-6.

	Isotopes			Isotones	
(a)	^{18}F	^{17}F	^{19}F	^{16}N	^{17}O
(b)	^{208}Pb	^{206}Pb	^{210}Pb	^{207}Tl	^{209}Bi
(c)	^{120}Sn	^{119}Sn	^{118}Sn	^{121}Sb	^{122}Te

11-7.

	Nuclide	Isobars		Isotopes
(a)	$^{14}_{8}O_6$	$^{14}_{6}C_8$	$^{14}_{7}N_7$	$^{16}_{8}O_8$
(b)	$^{63}_{28}Ni_{35}$	$^{63}_{29}Cu_{34}$	$^{63}_{30}Zn_{33}$	$^{60}_{28}Ni_{32}$
(c)	$^{236}_{93}Np_{143}$	$^{236}_{92}U_{144}$	$^{236}_{94}Pu_{142}$	$^{235}_{93}Np_{142}$

11-8. $\text{mass} = A \times u = A(1.66\times10^{-27}kg)$

$\text{volume} = (4/3)\pi R^3 = (4/3)\pi(R_0 A^{1/3})^3$

where R_0 = 1.2 fm = 1.2×10^{-15} m

$$\text{density} = \frac{\text{mass}}{\text{volume}} = \frac{A(1.66\times10^{-27}kg)}{(4/3)\pi(1.2\times10^{-15}m)^3 A}$$

$$\text{density} = 2.29\times10^{17} kg/m^3$$

11-9. $B = ZM_H c^2 + Nm_N c^2 - M_A c^2$ (Equation 11 − 11)

(a) ${}^9_4Be_5$

$$B = 4(1.007825\,uc^2) + 5(1.008665\,uc^2) - 9.012182\,uc^2$$
$$= 0.062443\,uc^2 = (0.062443\,uc^2)(931.5\,MeV/uc^2)$$
$$= 58.2\,MeV$$
$$B/A = 58.2\,MeV/9 nucleons = 6.46\,MeV/nucleon$$

(b) ${}^{13}_6C_7$

$$B = 6(1.007825\,uc^2) + 7(1.008665\,uc^2) - 13.003355\,uc^2$$
$$= 0.104250\,uc^2 = (0.104250\,uc^2)(931.5\,MeV/uc^2)$$
$$= 91.1\,MeV$$
$$B/A = 91.1\,MeV/13 nucleons = 7.47\,MeV/nucleon$$

(c) ${}^{57}_{26}Fe_{31}$

$$B = 26(1.007825\,uc^2) + 31(1.008665\,uc^2) - 56.935396\,uc^2$$
$$= 0.536669\,uc^2 = (0.536669\,uc^2)(931.5\,MeV/uc^2)$$
$$= 499.9\,MeV$$
$$B/A = 499.9\,MeV/57 nucleons = 8.77\,MeV/nucleon$$

11-10. $R = R_o A^{1/3}$ where $R_o = 1.2\,fm$ (Equation 11 − 3)

(a) ${}^{16}O \rightarrow \quad R = 1.2\,fm\,(16)^{1/3} = 3.02\,fm$

(b) ${}^{56}Fe \rightarrow \quad R = 1.2\,fm\,(56)^{1/3} = 4.58\,fm$

(c) ${}^{197}Au \rightarrow \quad R = 1.2\,fm\,(197)^{1/3} = 6.97\,fm$

(d) ${}^{238}U \rightarrow \quad R = 1.2\,fm\,(238)^{1/3} = 7.42\,fm$

11-11. (a) $B = M({}^3He)c^2 + m_n c^2 - M({}^4He)c^2$

$$= 3.016029\,uc^2 + 1.008665\,uc^2 - 4.002602\,uc^2$$
$$= 0.022092\,uc^2(931.5\,MeV/uc^2) = 20.6\,MeV$$

(Problem 11-11 continued)

(b) $B = M({}^{6}Li)c^2 + m_n c^2 - M({}^{7}Li)c^2$

$= 6.015121\,uc^2 + 1.008665\,uc^2 - 7.016003\,uc^2$

$= 0.007783\,uc^2(931.5\,MeV/uc^2) = 7.25\,MeV$

(c) $B = M({}^{13}N)c^2 + m_n c^2 - M({}^{14}N)c^2$

$= 13.005738\,uc^2 + 1.008665\,uc^2 - 14.003074\,uc^2$

$= 0.011329\,uc^2(931.5\,MeV/uc^2) = 10.6\,MeV$

11-12. $B = [+a_1 A - a_2 A^{2/3} - a_3 Z^2 A^{-1/3} - a_4(A-2Z)^2 A^{-1} \pm a_5 A^{-1/2}]c^2$ (This is Equation 11-13 on the Web page www.whfreeman.com/modphysics4e.) The values of the a_i in MeV/c^2 are given in Table 11-3 (also on the Web page).

For ^{23}Na: $B = [15.67(23) - 17.23(23)^{2/3} - 0.75(11)^2(23)^{-1/3} - 93.2(23-2\times 11)^2(23)^{-1}$

$+ 0(23)^{-1/2}]c^2 = 184.9\,MeV$

$M({}^{23}Na)c^2 = 11m_p c^2 + 12m_n c^2 - B$ (Equation 11 – 14 on the Web page)

$= [11(1.007825\,uc^2) + 12(1.008665\,uc^2)] - 184.9\,MeV$

$M({}^{23}Na) = 23.279678\,u - 184.9\,MeV/c^2(1/931.5\,MeV/c^2)$

$= 23.279678\,u - 0.198497\,u = 23.081181\,u$

This result differs from the measured value of 31.972071u by only 0.009%.

11-13. $R = (1.07 \pm 0.02)A^{1/3} fm$ (Equation 11 – 5) $R = 1.4A^{1/3} fm$ (Equation 11 – 6)

(a) ${}^{16}O$: $R = 1.07A^{1/3} = 2.70\,fm$ and $R = 1.4A^{1/3} = 3.53\,fm$

(b) ${}^{63}Cu$: $R = 1.07A^{1/3} = 4.26\,fm$ and $R = 1.4A^{1/3} = 5.57\,fm$

(c) ${}^{208}Pb$: $R = 1.07A^{1/3} = 6.34\,fm$ and $R = 1.4A^{1/3} = 8.30\,fm$

11-14. $\Delta U = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{e^2}{R}(Z^2 - (Z-1)^2)$ (Equation 11-2)

where Z = 20 for Ca and ΔU = 5.49 MeV from a table of isotopes (e.g., Table of Isotopes 8th ed., Firestone, et al., Wiley 1998).

$$R = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{e^2}{R\Delta U}(Z^2 - (Z-1)^2)$$

$$R = 0.6(8.99\times10^9 N\cdot m^2/C^2)(1.60\times10^{-19}C)(2.0^2 - 19^2)/(5.49\times10^6 eV)$$

$$R = 6.13\times10^{-15}m = 6.13 fm$$

11-15. (a) $R = R_0 e^{-\lambda t} = R_0 e^{-(\ln 2)t/t_{1/2}}$ (Equation 11-19)

at $t = 0:\ R = R_0 = 4000\ counts/s$

at $t = 10s:\ R = R_0 e^{-(\ln 2)(10s)/t_{1/2}}$

$$1000 = 4000\,e^{-(\ln 2)(10s)/t_{1/2}}$$

$$1/4 = e^{-(\ln 2)(10s)/t_{1/2}}$$

$$\ln(1/4) = -(\ln 2)(10s)/\ln(1/4) = 5.0s$$

(b) at $t = 20s:\ R = (4000\,counts/s)\,e^{-(\ln 2)(20s)/5s} = 200\,counts/s$

11-16. $R = R_0 e^{-(\ln 2)t/2\min}$ at $t = 0:\ R = R_0 = 2000\,counts/s$

(a) at $t = 4\min:\ R = (2000\,counts/s)\,e^{-(\ln 2)4\min/2\min} = 500\ counts/s$

(b) at $t = 6\min:\ R = (2000\,counts/s)\,e^{-(\ln 2)6\min/2\min} = 250\ counts/s$

(c) at $t = 8\min:\ R = (2000\,counts/s)\,e^{-(\ln 2)8\min/2\min} = 125\ counts/s$

11-17. $R = R_0 e^{-\lambda t} = R_0 e^{-(\ln 2)t/t_{1/2}}$ (Equation 11-19)

(a) At $t = 0:\ R = R_0 = 115.0\ decays/s$

At $t = 2.25h:\ R = 85.2\ decays/s$

(Problem 11-17 continued)

$$85.2\,decays/s = (115.0\,decays/s)\,e^{-\lambda(2.25h)}$$

$$(85.2/115.0) = e^{-\lambda(2.25)}$$

$$\ln(85.2/115.0) = -\lambda(2.25h)$$

$$\lambda = -\ln(85.2/115.0)/2.25h = 0.133\,h^{-1}$$

$$t_{1/2} = \ln 2/\lambda = \ln 2/0.133\,h^{-1} = 5.21\,h$$

(b)
$$\left|\frac{dN}{dt}\right| = \lambda N \quad\rightarrow\quad \left|\frac{dN_0}{dt}\right| = R_0 = \lambda N_0 \qquad (\text{from Equation } 11-17)$$

$$N_0 = R_0/\lambda = (115.0\,atoms/s)/(0.133\,h^{-1})(1\,h/3600\,s)$$

$$= 3.11\times10^{6}\,atoms$$

11-18. (a) $^{226}Ra \qquad t_{1/2} = 1620\,y$

$$R = -\frac{dN}{dt} = \lambda N = \frac{\ln 2}{t_{1/2}}N = \frac{\ln 2}{t_{1/2}}\frac{N_A m}{M}$$

$$= \frac{\ln 2\,(6.022\times10^{23}/mole)\,(1\,g)}{(1620\,y)(3.16\times10^{7}\,s/y)(226.025\,g/mole)} = 3.61\times10^{10}\,s^{-1}$$

$1\,Ci = 3.7\times10^{10}\,s^{-1}$, or nearly the same.

(b)
$$Q = M(^{226}Ra)c^2 - [M(^{222}Rn)c^2 + M(^{4}He)c^2]$$

$$= 226.025402\,uc^2 - [222.017571\,uc^2 + 4.002602\,uc^2]$$

$$= 0.005229\,uc^2 = (0.005229\,uc^2)(931.5\,MeV/uc^2)$$

$$= 4.87\,MeV$$

11-19. (a) $R = -\dfrac{dN}{dt} = R_0 e^{-t\ln 2/t_{1/2}}$ (from Equation 11 – 19)

when $t = 0$, $R = R_0 = 8000\ count/s$

when $t = 10\,\text{min}$, $R = 1000\,counts/s = 8000\,counts/s \cdot e^{-10\ln 2/t_{1/2}}$

$$e^{-10\ln 2/t_{1/2}} = 1000/8000 = 1/8$$

$$-10\ln 2/t_{1/2} = \ln(1/8)$$

$$t_{1/2} = \frac{-10\ln 2}{\ln(1/8)} = 3.33\ \text{min}$$

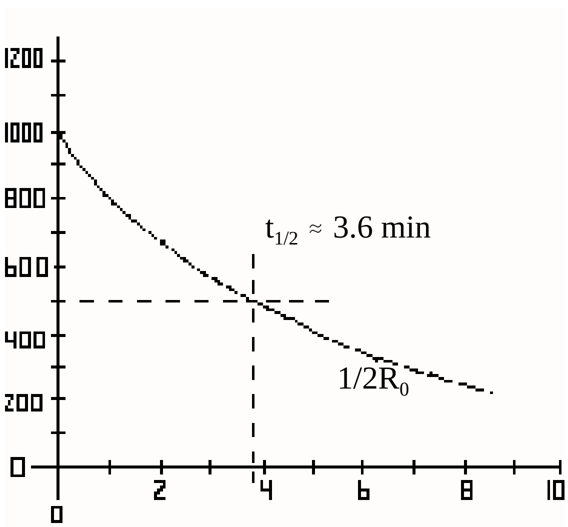
Notice that this time interval equals three half-lives.

(b) $\lambda = \ln 2/t_{1/2} = \ln 2/3.33\,\text{min} = 0.208\,\text{min}^{-1}$

(c) $R = R_0 e^{-t\ln 2/t_{1/2}} = R_0 e^{-t}$ Thus, $R = (8000\,counts/s)\,e^{-0.208(1)} = 6500\ counts/s$

11-20. (a) and (b)

counts/s



t, min

(c) Estimating from the graph, the next count (at 8 min) will be approximately 230 counts/s.

11-21. ^{62}Cu is produced at a constant rate R_0, so the number of ^{62}Cu atoms present is:

$N = R_0/\lambda(1 - e^{-\lambda t})$ (from Equation 11-26). Assuming there were no ^{62}Cu atoms initially

(Problem 11-21 continued)

present. The maximum value N can have is $R_0/\lambda = N_0$,

$$N = N_0(1 - e^{-\lambda t})$$

$$0.90N_0 = N_0(1 - e^{-t\ln 2/t_{1/2}})$$

$$e^{-t\ln 2/t_{1/2}} = 1 - 0.90 = 0.10$$

$$-t\ln 2/t_{1/2} = \ln(0.10)$$

$$t = -10\ln(0.10)/\ln 2 = 33.2\ \text{min}$$

11-22. (a) $t_{1/2} = \ln 2/\lambda = \ln 2/9.8\times 10^{-10}y^{-1} = 7.07\times 10^{8}y$ (Equation 11-22)

(b) Number of ^{235}U atoms present is:

$$N = \frac{1.0\,\mu g\,N_A}{M} = \frac{(10^{-6}g)(6.02\times 10^{23}\,atoms/mol)}{235\,g/mol} = 2.56\times 10^{15}\,atoms$$

$$-\frac{dN}{dt} = \lambda N = 9.8\times 10^{-10}y^{-1}(1/3.16\times 10^{7}\,s/y)(2.56\times 10^{15}\,atoms)\ \ (\text{Equation } 11-17)$$

$$= 0.079\ \text{decays/s}$$

(c) $N = N_0 e^{-\lambda t}$ (Equation 11-18)

$$N = (2.56\times 10^{15})\,e^{-(9.8\times 10^{-10}y^{-1})(10^{6}y)} = 2.558\times 10^{15}$$

11-23. (a) $t_{1/2} = \ln 2/\lambda = \ln 2/0.266\,y^{-1} = 2.61\,y$ (Equation 11-22)

(b) Number of N atoms in 1g is:

$$N = \frac{1.0\,g\,N_A}{M} = \frac{(1\,g)(6.02\times 10^{23}\,atoms/mol)}{22\,g/mol} = 2.74\times 10^{22}\,atoms$$

$$-\frac{dN}{dt} = \lambda N = (0.266y^{-1})(1/3.16\times 10^{7}\,s/y)(2.74\times 10^{22}\,atoms)$$

$$= 2.3\times 10^{14}\ \text{decays/s} = 2.3\times 10^{14}\,Bq$$

(Problem 11-23continued)

(c) $-\dfrac{dN}{dt} = \lambda N_0 e^{-\lambda t}$ (Equation 11 – 19)

$$= (0.266y^{-1})(1/3.16\times10^7\ s/y)(2.74\times10^{22})\,e^{-(0.266y)(3.5y)}$$

$$= 9.1\times10^{13}\ decays/s = 9.1\times10^{13}\ Bq$$

(d) $N = N_0 e^{-\lambda t}$ (Equation 11-18)

$$N = (2.74\times10^{22})\,e^{-(0.266y^{-1})(3.5y)} = 1.08\times10^{22}$$

11-24. (a) ^{22}Na has an excess of protons compared with ^{23}Na and would be expected to decay by β^+ emission and/or electron capture. (It does both.)

(b) ^{24}Na has an excess of neutrons compared with ^{23}Na and would be expected to decay by β^- emission. (It does.)

11-25. $\log t_{1/2} = AE_\alpha^{-12/} + B$ (Equation 11 – 18)

for $t_{1/2} = 10^{10}\,s,\ E_\alpha = 5.4\ MeV$

for $t_{1/2} = 1\,s,\ E_\alpha = 7.0\,MeV$ ⟵ from Figure 11 – 16

$$\log 10^{10} = A(5.4)^{-1/2} + B$$

(i) $10 = 0.4303A + B$

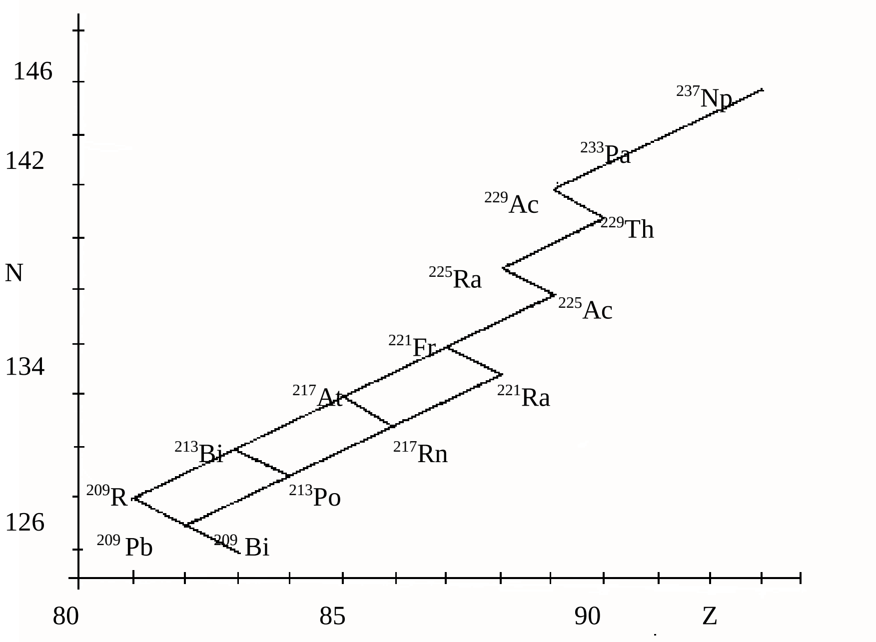
$$\log 1 = A(7.0)^{-1/2} + B$$

(ii) $0 = 0.3780A + B \rightarrow B = -0.3780A$

Substituting (ii) into (i),

$$10 = 0.4303A - 0.3780A - 0.0523A,\ A = 191,\ B = -0.3780A = -72.2$$

11-26.



11-27. $^{232}_{90}Th \rightarrow \; ^{228}_{88}Ra + \alpha$

$$Q = M(^{232}Th)c^2 - M(^{228}Ra)c^2 - M(^4He)c^2$$

$$= 232.038051\,uc^2 - 228.031064\,uc^2 - 4.002602\,uc^2$$

$$= 0.004385\,uc^2\,(931.50\,MeV/uc^2) = 4.085\,MeV$$

The decay is a 2-particle decay so the *Ra* nucleus and the α have equal and opposite momenta.

$$\rho_\alpha = \sqrt{2m_\alpha E_\alpha} = \rho_{Ra} = \sqrt{2M_{Ra}E_{Ra}} \text{ where } E_\alpha + E_{Ra} = 4.085\,MeV$$

$$2m_\alpha E_\alpha = 2M_{Ra}E_{Ra} = 2M_{Ra}(4.085 - E_\alpha)$$

$$E_\alpha = \frac{M_{Ra}}{M_{Ra} + m_\alpha}(4.085\,MeV)$$

$$= \frac{228.031064\,(4.085\,MeV)}{228.031064 + 4.002602} = (0.983)(4.085\,MeV) = 4.01\,MeV$$

11-28. $^7_4Be_3 \rightarrow \; ^7_3Li_4 + \nu_e$

(a) Yes, the decay would be altered. Under very high pressure the electrons are "squeezed" closer to the nucleus. The probability density of the electrons, particularly

(Problem 11-28 continued)

the K electrons, is increased near the nucleus making electron capture more likely, thus decreasing the half-life.

(b) Yes, the decay would be altered. Stripping all four electrons from the atom renders electron capture impossible, lengthening the half-life to infinity.

11-29. $^{67}Ga \xrightarrow{E.C.} {}^{67}Zn + \nu_e$

$$Q = M(^{67}Ga)c^2 - M(^{67}Zn)c^2$$

$$= 66.9282\,uc^2 - 66.927129\,uc^2$$

$$= 0.001075\,uc^2(931.50\,MeV/uc^2) = 1.00\,MeV$$

11-30. $^{72}Zn \rightarrow {}^{72}Ga + \beta^- + \bar{\nu}_e$

$$Q = M(^{72}Zn)c^2 - M(^{72}Ga)c^2$$

$$= 71.926858\,uc^2 - 71.926367\,uc^2$$

$$= 0.000491\,uc^2(931.50\,MeV/uc^2) = 0.457\,MeV = 457\,keV$$

This is the maximum possible β particle energy.

11-31. $^{233}Np \rightarrow {}^{232}Np + n$ and $^{233}Np \rightarrow {}^{232}U + p$

For n emission: $Q = M(^{233}Np)c^2 - M(^{232}Np)c^2 - m_n c^2$

$$= 233.040805\,uc^2 - 232.040022\,uc^2 - 1.008665\,uc^2$$

$$= -0.007882\,uc^2$$

{Q < 0 means $M(products) > M(^{233}Np)$; prohibited by conservation of energy.}

(Problem 11-31 continued)

For p emission: $Q = M(^{233}Np)c^2 - M(^{232}U)c^2 - m_n c^2$

$= 233.040805\,uc^2 - 232.037131\,uc^2 - 1.008665\,uc^2$

$= -0.004991\,uc^2$

{Q < 0 means $M(products) > M(^{233}Np)$; prohibited by conservation of energy.}

11-32.

	286	280	247	235	174	124	80	61	30	0
286	–									
280	6	–								
247	39	33	–							
235	51	45	12	–						
174	112	106	73	61	–					
124	162	156	123	111	50	–				
80	206	200	167	155	94	44	–			
61	225	219	186	174	113	63	19	–		
30	256	250	217	205	144	94	50	31	–	
0	286	280	247	235	174	124	80	61	30	–

Tabulated γ energies are in keV. Higher energy α levels in Figure 11-19 would add additional columns of γ rays.

11-33. $^8Be \rightarrow 2\alpha$

$Q = M(^8Be)c^2 - M(^4He)c^2$

$= 8.005304\,uc^2 - 2(4.002602)\,uc^2$

$= 0.000100\,uc^2\,(931.50\,MeV/uc^2) = 0.093\,MeV = 93\,keV$

Thus, the lower energy configuration for 4 protons and 4 neutrons is two α particles rather than one ^{8}Be.

11-34. (a) $^{80}Br \rightarrow {}^{80}Kr + \beta^- + \bar{\nu}_e$ and $^{80}Br \rightarrow {}^{80}Se + \beta^+ + \nu_e$ and $^{80}Br \xrightarrow{E.C.} {}^{80}Se + \nu_e$

(b) For β^- decay: $Q = M(^{80}Br)c^2 - M(^{80}Kr)c^2$

$= 79.918528uc^2 - 79.916377)uc^2$

$= 0.002151uc^2(931.50MeV/uc^2) = 2.00\,MeV$

For β^+ decay: $Q = M(^{80}Br)c^2 - M(^{80}Se)c^2 - 2m_ec^2$

$= 79.918528uc^2 - 79.916519)uc^2 - 2(0.511MeV)$

$= 0.002009uc^2(931.50MeV/uc^2) - 1.022\,MeV = 0.85\,MeV$

For E.C.: $Q = M(^{80}Br)c^2 - M(^{80}Se)c^2$

$= 79.918528uc^2 - 79.916519)uc^2$

$= 0.002009uc^2(931.50MeV/uc^2) = 1.87\,MeV$

11-35. $R = R_0A^{1/3}$ where $R_0 = 1.2fm$ (Equation 11 − 3)

For ^{12}C: $R = 1.2(12)^{1/3} = 2.745fm = 2.745\times10^{-15}\,m$ and the diameter = 5.490 × 10^{-15} m

$$\text{Coulomb force: } F_C = \frac{ke^2}{r^2} = \frac{(9.00\times10^9)(1.6\times10^{-19}C)^2}{(5.490\times10^{-15}m)^2} = 7.65\,N$$

$$\text{Gravitational force: } F_G = G\frac{m_p^2}{r^2} = \frac{6.67\times10^{-11}(1.67\times10^{-27}kg)^2}{(5.490\times10^{-15}m)^2} = 6.18\times10^{-36}\,N$$

The corresponding Coulomb potential is: $U_C = F_C\times r = 7.65N(5.490\times10^{-15}m)$

$= 4.20\times10^{-14}J\,/\,1.60\times10^{-13}J/MeV$

$= 0.26\,MeV$

The corresponding gravitational potential is: $U_G = F_G\times r = (6.18\times10^{-36}N)(5.490\times10^{-15}m)$

$= 3.39\times10^{-50}J(1/1.60\times10^{-13}J/MeV)$

$= 2.12\times10^{-37}\,MeV$

The nuclear attractive potential is exceeded by the Coulomb repulsive potential at this separation. The gravitational potential is not a factor in nuclear structure.

11-36. The range R of a force mediated by an exchange particle of mass m is:

$$R = \hbar / mc \quad \text{(Equation 11-50)}$$

$$mc^2 = \hbar c / R = 197.3\,MeV{\cdot}fm / 5fm = 39.5\,MeV$$

$$m = 39.5\,MeV/c^2$$

11-37. The range R of a force mediated by an exchange particle of mass m is:

$$R = \hbar / mc \quad \text{(Equation 11-50)}$$

$$mc^2 = \hbar c / R = 197.3\,MeV{\cdot}fm / 0.25fm = 789\,MeV$$

$$m = 789\,MeV/c^2$$

11-38.

Nuclide	last proton(s)	last neutron(s)	ℓ	j
${}^{29}_{14}Si_{15}$	$\cdots(1d_{5/2})^6$	$\cdots(2s_{1/2})$	0	½
${}^{37}_{17}Cl_{20}$	$\cdots(1d_{3/2})$	$\cdots(1d_{3/2})^4$	2	3/2
${}^{71}_{31}Ga_{40}$	$\cdots(2p_{3/2})^3$	$\cdots(2p_{1/2})^2$	1	3/2
${}^{59}_{27}Co_{32}$	$\cdots(1f_{7/2})^7$	$\cdots(2p_{3/2})^4$	3	7/2
${}^{73}_{32}Ge_{41}$	$\cdots(2p_{3/2})^4$	$\cdots(1g_{9/2})$	4	9/2
${}^{33}_{16}S_{17}$	$\cdots(2s_{1/2})^2$	$\cdots(1d_{3/2})$	2	3/2
${}^{81}_{38}Sr_{49}$	$\cdots(2p_{3/2})^6$	$\cdots(1g_{9/2})^9$	4	9/2

The nucleon configurations are taken directly from Figure 11-39, and the ℓ and j values are those of the unpaired nucleon.

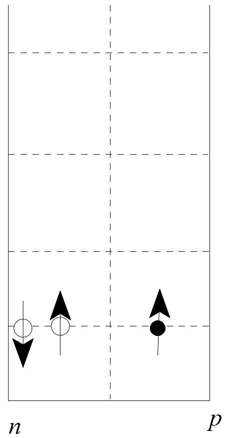
11-39.

Isotope	Odd nucleon	Predicted μ/μ_N
${}^{29}_{14}Si$	neutron	−1.91
${}^{27}_{17}Cl$	proton	+2.29
${}^{71}_{31}Ga$	proton	+2.29
${}^{59}_{27}Co$	proton	+2.29
${}^{73}_{32}Ge$	neutron	−1.91
${}^{33}_{16}S$	neutron	−1.91
${}^{87}_{38}Sr$	neutron	−1.91

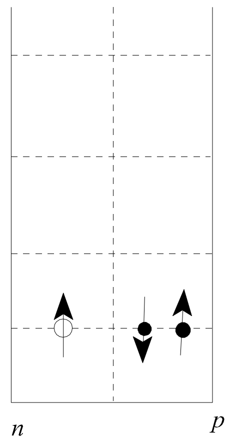
11-40. Nuclear spin of ${}^{14}N$ must be $1\hbar$ because there are 3 m_I states, +1, 0, and 1.

11-41. ${}^{36}S$, ${}^{53}Mn$, ${}^{82}Ge$, ${}^{88}Sr$, ${}^{94}Ru$, ${}^{131}In$, ${}^{145}Eu$

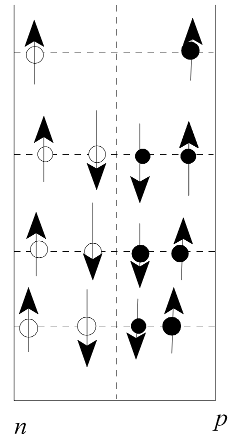
11-42.



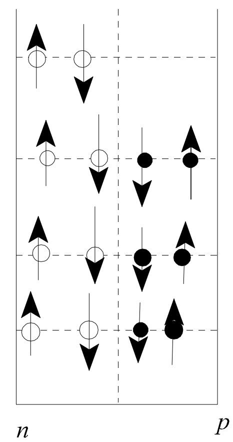
^{3}H



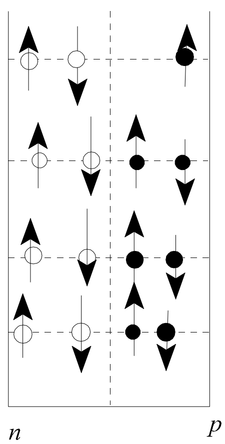
^{3}He



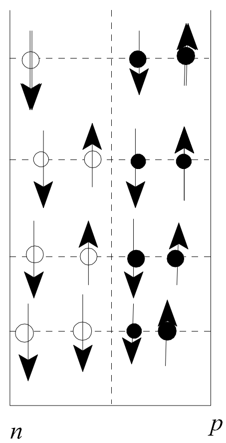
^{14}N



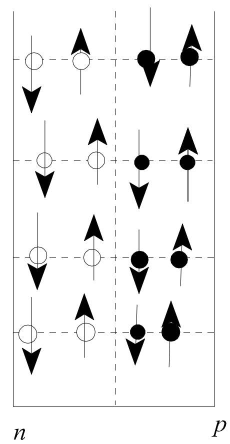
^{14}C



^{15}N



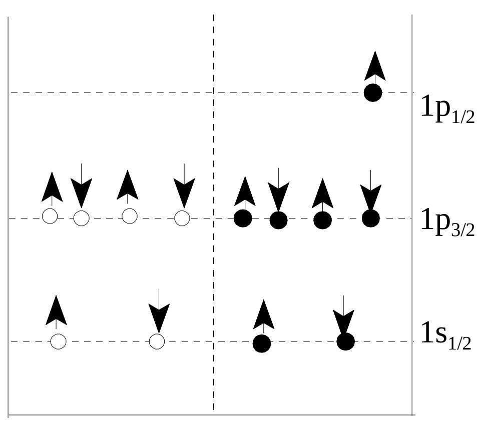
^{15}O



^{16}O

11-43. ${}^{3}_{2}He$, ${}^{40}_{20}Ca$, ${}^{60}_{28}Ni$, ${}^{124}_{50}Sn$, ${}^{204}_{82}Pb$

11-44. (a)



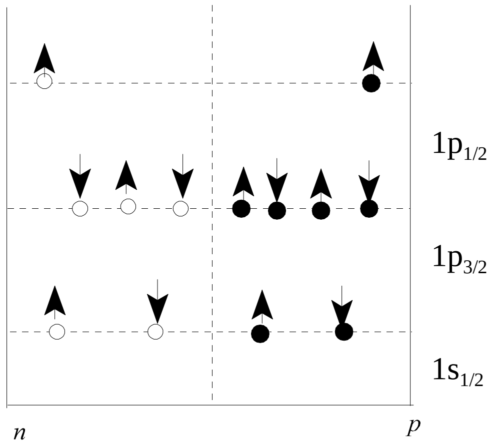
^{13}N

(Problem 11-44 continued)

(b) j = ½ due to the single unpaired proton.

(c) The first excited state will likely be the jump of a neutron to the empty neutron level, because it is slightly lower than the corresponding proton level. The j = ½ or 3/2, depending on the relative orientations of the unpaired nucleon spins.

(d)



First excited state. There are several diagrams possible.

11-45.

${}^{30}_{14}Si$	j = 0
${}^{37}_{17}Cl$	j = 3/2
${}^{55}_{27}Co$	j = 7/2
${}^{90}_{40}Zr$	j = 0
${}^{107}_{49}In$	j = 9/2

11-46. Writing Equation 11-14 as:

$$M(Z,A)c^2 = Zm_p^2 + (A-Z)m_n c^2$$

$$-[a_1 A - a_2 A^{2/3} - a_3 A^{-1/3} z^2 - a_4 (A-2Z)^2 A^{-1} + a_5 A^{-1/2}]c^2$$

and differentiating,

(Problem 11-46 continued)

$$\frac{\partial M}{\partial Z} = m_p - m_n - [-2a_3 A^{-1/3} Z - 2a_4 (A - 2Z)(-2) A^{-1}]$$

$$0 = (m_p - m_n) + 2a_3 A^{-1/3} Z - 4a_4 + 8a_4 A^{-1} Z$$

$$0 = (m_p - m_n) - 4a_4 + (2a_3 A^{-1/3} + 8a_4 A^{-1})Z$$

$$Z = \frac{(m_n - m_p) + 4a_4}{2a_3 A^{-1/3} + 8a_4 A^{-1}} \quad \text{where } a_3 = 0.75,\ a_4 = 93.2$$

(a) For A = 27: $Z = \dfrac{(1.008665 - 1.007825)(931.5\,MeV/uc^2) + 4(93.2)}{2(0.75)(27)^{-1/3} + 8(93.2)(27)^{-1}}$

$Z = 13.2 \quad \text{Minimum } Z = 13$

(b) For A = 65: Computing as in (a) with A = 65 yields Z = 31.5. Minimum Z = 29.

(c) For A =139: Computing as in (a) with A = 139 yields Z = 66. Minimum Z = 57.

11-47. (a) $R = (0.31)E^{3/2} = (0.31)(5\,MeV)^{3/2} = 3.47\,cm$

(b) $R(g/cm^2) = R(cm)\rho = (3.47\,cm)(1.29\times10^{-3}\,g/cm^3) = 4.47\times10^{-3}\,g/cm^3$

(c) $R(cm) = R(g/cm^2)/\rho = (4.47\times10^{-3}\,g/cm^2)/(2.70\,g/cm^3) = 1.66\times10^{-3}\,cm$

11-48. For one proton, consider the nucleus as a sphere of charge *e* and charge density $\rho_c = 3e/4\pi R^3$. The work done in assembling the sphere, i.e., bringing charged shell *dq* up to *r*, is: $dU_c = k\rho_c \frac{4\pi r^3}{3}(\rho_c 4\pi r^2 dr)\frac{1}{r}$ and integrating from 0 to R yields:

$$U_c = \frac{k\rho_c^2 16\pi^2 R^5}{15} = \frac{3}{5}\frac{ke^2}{R}$$

For two protons, the coulomb repulsive energy is twice U_c, or $6ke^2/5R$.

11-49. The number N of ^{144}Nd atoms is:

$$N = \frac{53.94\,g(6.02\times10^{23}\ atoms/mol)}{144\ g/mol} = 2.25\times10^{23}\ atoms$$

$$-\frac{dN}{dt} = \lambda N \quad\rightarrow\quad \lambda = (-dN/dt)/N$$

$$= (2.36\,s^{-1})/(2.25\times10^{23})$$

$$= 1.05\times10^{-23}\,s^{-1}$$

$$t_{1/2} = \ln 2/\lambda = \ln 2/1.05\times10^{-23}\,s^{-1} = 6.61\times10^{22}\,s = 2.09\times10^{15}\,y$$

11-50. $R = R_0 e^{-(\ln 2)t/t_{1/2}}$ (Equation 11-19)

(a) At $t = 0$: $R = R_0 = 115.0\ decays/\text{min}$

At $t = 4d\,5h = 4.21\,d$: $R = 73.5\ decays/\text{min}$

$$73.5\,decays/\text{min} = (115.0\,decays/\text{min})\,e^{-(\ln 2)(4.21\,d)/t_{1/2}}$$

$$(73.5/115.0) = e^{-(\ln 2)(4.21\,d)/t_{1/2}}$$

$$\ln(73.5/115.0) = -(\ln)(4.21\,d)/t_{1/2}$$

$$t_{1/2} = -(\ln 2)(2.41\,d)/\ln(73.5/115.0) = 6.52\,d$$

(b) $R = 10\,decays/\text{min} = (115.0\,decays/\text{min})\,e^{-(\ln 2)t/6.52\,d}$

$$t = -\ln(10/115.0)(6.52\,d)/\ln 2 = 23.0\,d$$

(c) $R = 2.5\,decays/\text{min} = (115.0\,decays/\text{min})\,e^{-(\ln 2)t/6.52\,d}$

$$t = -\ln(2.5/115.0)(6.52\,d)/\ln 2 = 36.0\,d\ (\text{because t} = 0)$$

This time is 13 days (= $2t_{1/2}$) after the time in (b).

11-51. For ^{227}Th: $t_{1/2} = 18.72\,d$ (nucleus A)

For ^{223}Ra: $t_{1/2} = 11.43\,d$ (nucleus B)

At t = 0 there are 10^6 *Th* atoms and 0 *Ra* atoms

(a) $N_A = N_{0A}e^{-\lambda_A t}$ (Equation 11-18)

$$N_B = \frac{N_{0A}\lambda_A}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{\lambda_B t}) + N_{0B}e^{-\lambda_B t} \quad \text{(Equation 11-26 on the Web page)}$$

$$N_A = 10^6 e^{-\ln 2(15d)/18.72d} = 5.74\times 10^5$$

$$N_B = \frac{10^6 \ln 2/18.72d}{\ln 2(1/11.43d - 1/18.72d)}(e^{-\ln 2(15)/18.72} - e^{-\ln 2(15)/11.43}) + 0 = 2.68\times 10^5$$

(b) $N_A = N_B$ means $N_{0A}e^{-\lambda_A t} = \frac{N_{0A}\lambda_A}{\lambda_B - \lambda_A}(e^{-\lambda_A t} - e^{-\lambda_B t})$

Canceling N_{0A} and rearranging,

$$-\frac{\lambda_B - \lambda_A}{\lambda_A} + 1 = e^{(\lambda_A - \lambda_B)t}$$

$$\ln\left(-\frac{\lambda_B - \lambda_A}{\lambda_A} + 1\right) = (\lambda_B - \lambda_A)\,t \qquad \lambda_A = \frac{\ln 2}{18.82\,d} = 0.0370\,d^{-1}$$

$$t = \ln\left(-\frac{\lambda_B - \lambda_A}{\lambda_A} + 1\right)/(\lambda_A - \lambda_B) \qquad \lambda_B = \frac{\ln 2}{11.43\,d} = 0.0606\,d^{-1}$$

$$= \ln\left(-\frac{0.0606 - 0.0370}{0.0370} + 1\right)/(0.00370 - 0.0606)$$

$$= 43.0\,d$$

11-52. (a) $\Gamma = \hbar/\tau = 6.582\times 10^{-16}\,eV{\cdot}s/0.13\times 10^{-9}\,s = 5.06\times 10^{-6}\,eV$

(b)

$$E_r = \frac{(hf)^2}{2Mc^2} = \frac{(0.12939\,MeV)^2}{2M(^{191}I)c^2} \quad \text{(Equation 11-47)}$$

$$= \frac{(0.12939\,MeV)^2}{2(190.960585\,uc^2)(931.5\,MeV/\mu c^2)}$$

$$= 4.71\times 10^{-8}\,MeV = 4.71\times 10^{-2}\,eV$$

(Problem 11-52 continued)

(c) (See Section 1-5)

The relativistic Doppler shift Δf for either receding or approaching is:

$$\frac{\Delta f}{f_0} \approx \beta = \frac{v}{c} \qquad h\Delta f = \Gamma \quad \text{and} \quad hf_0 = E$$

$$\frac{\Gamma}{E} = \frac{v}{c} \rightarrow v = \frac{\Gamma_c}{E} = (5.06 \times 10^{-6} eV)(3.00 \times 10^8 m/s) = 0.0117 m/s = 1.17 cm/s$$

11-53. $B = ZM(^1H)c^2 + Nm_n c^2 - M_A c^2$

For ^{3}He: $B = 2(1.007825uc^2) + 1.008665uc^2 - 3.016029uc^2$

$= 0.008826uc^2(931.50 MeV/uc^2) = 7.72 MeV$

For ^{3}H: $B = 1.007825uc^2 + 2(1.008665uc^2) - 3.016049uc^2$

$= 0.009106uc^2(931.50 MeV/uc^2) = 8.48 MeV$

$R = R_0 A^{1/3} = 1.2 fm 3^{1/3} = 1.730 fm = 1.730 \times 10^{-15} m$

$u_c = ke^2/R = ke/R(eV) = 8.32 \times 10^5 eV = 0.832 MeV$ or about 1/10 of the binding energy.

11-54. For ^{47}Ca:

$B = M(^{46}Ca)c^2 + m_n c^2 - M(^{47}Ca)c^2$

$= 45.953687uc^2 + 1.008665uc^2 - 46.954541uc^2$

$= 0.007811uc^2(931.50 MeV/uc^2) = 7.28 MeV$

For ^{48}Ca:

$B = M(^{48}Ca)c^2 + m_n c^2 - M(^{48}Ca)c^2$

$= 46.954541uc^2 + 1.008665uc^2 - 47.952534uc^2$

$= 0.010672uc^2(931.50 MeV/uc^2) = 9.94 MeV$

(Problem 11-54 continued)

Assuming the even-odd ^{47}Ca to be the "no correction" nuclide, the average magnitude of the correction needed to go to either of the even-even nuclides ^{46}Ca or ^{48}Ca is approximately

$B - \textit{average binding energy of the odd neutron} (9.94\,MeV + 7.28\,MeV)/2 = 8.61\,MeV$

So the correction for ^{46}Ca is $8.16 - 7.28 = 0.88\,MeV$ and for ^{48}Ca is $9.94 - 8.16 = 1.78\,MeV$, an "average" of about 1.33 MeV. The estimate for a_5 is then:

$a_5 A^{-1/2} = 1.33\,MeV \rightarrow a_5 = 1.33/48^{-1/2} = 9.2$

This value is about 30% below the accepted empirical value of $a_5 = 12$.

11-55. For a nucleus with $I > 0$ the α feels a centripetal force $F_c = mv^2/r = -dV/dr$ where r = distance of the α from the nuclear center. The corresponding potential energy $V \propto -ln\,r$ and becomes larger (i.e., more negative) as r increases. This lowers the total energy of the α near the nuclear boundary and results in a wider barrier, hence lower decay probability.

11-56. (a) $B = ZM(^1H)c^2 + Nm_n c^2 - M_A c^2$ (Equation 11-11)

For ^7Li: $B = 3(1.007825\,uc^2) + 4(1.008665\,uc^2) - 7.016003\,uc^2$

$= 0.042132\,uc^2(931.50\,MeV/uc^2) = 39.25\,MeV$

For ^7Be: $B = 4(1.007825\,uc^2) + 3(1.008665\,uc^2) - 7.016928\,uc^2$

$= 0.040367\,uc^2(931.50\,MeV/uc^2) = 37.60\,MeV$

$\Delta B = 1.65\,MeV$

For ^{11}B: $B = 5(1.007825\,uc^2) + 6(1.008665\,uc^2) - 11.009305\,uc^2$

$= 0.0081810\,uc^2(931.50\,MeV/uc^2) = 76.21\,MeV$

For ^{11}C: $B = 6(1.007825\,uc^2) + 5(1.008665\,uc^2) - 11.011433\,uc^2$

$= 0.078842\,uc^2(931.50\,MeV/uc^2) = 73.44\,MeV$

(Problem 11-56 continued)

$\Delta B = 2.77\ MeV$

For ^{15}N: $B = 7(1.007825uc^2) + 8(1.008665uc^2) - 15.000108uc^2$

$= 0.123987uc^2(931.50\ MeV/uc^2) = 115.5\ MeV$

For ^{15}O: $B = 8(1.007825uc^2) + 7(1.008665uc^2) - 15.003065uc^2$

$= 0.120190uc^2(931.50\ MeV/uc^2) = 112.0\ MeV$

$\Delta B = 5.54\ MeV$

(b) $\Delta B = a_3 Z^2 A^{-1/3} \rightarrow a_3 = \Delta B/Z^2 A^{-1/3}$

For A = 7; Z = 4: $a_3 = 1.65\ MeV/4^2(7)^{-1/3} = 0.20\ MeV$

For A = 11; Z = 6: $a_3 = 2.77\ MeV/6^2(11)^{-1/3} = 0.17\ MeV$

For A = 15; Z = 8: $a_3 = 3.54\ MeV/8^2(15)^{-1/3} = 0.14\ MeV$

$\langle a_3 \rangle = 0.17\ MeV$

These values differ significantly from the empirical value of a_3 = 0.75 MeV.

11-57. (a) Using $\partial M/\partial Z = 0$ from Problem 11-44,

$$Z = \frac{(m_n - m_p) + 4a_4}{2a_3 A^{-1/3} + 8a_4 A^{-1}} \quad \text{where } a_3 = 0.75\ MeV/c^2,\ a_4 = 93.2\ MeV/c^2$$

$$= \frac{1 + (m_n - m_p)/4a_4}{2A^{-1} + (a_3 A^{-1/3}/2a_4)} = \frac{A}{2}\frac{[1 + (m_n - m_p)/4a_4]}{[1 + a_3 A^{2/3}/4a_4]}$$

(b) & (c) For A = 29: $Z = \dfrac{29}{2}\dfrac{[1 + (1.008665 - 1.007276)(931.5)/(4)(93.2)]}{[1 + 0.75(29)^{2/3}/(4)(93.2)]} = 14$

The only stable isotope with A = 29 is □$^{29}_{14}Si$

For A = 59: Computing as above with A = 59 yields Z = 29. The only stable isotope with A = 59 is □$^{59}_{27}Co$

(Problem 11-57 continued)

For A = 78: Computing as above with A = 78 yields Z = 38. $^{78}_{38}Sr$ is not stable. Stable isotopes with A = 78 are $^{78}_{34}Se$ and $^{78}_{36}Kr$.

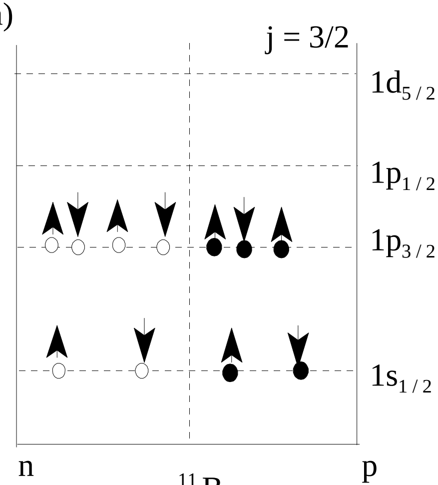
For A = 119: Computing as above with A = 119 yields Z = 59. $^{119}_{57}Pr$ is not stable.

The only stable isotope with A= 119 is $^{119}_{50}Sn$

For A = 140: Computing as above with A=140 yields Z = 69. $^{140}_{69}Tm$ is not stable. The only stable isotope with A = 140 is $^{140}_{58}Ce$

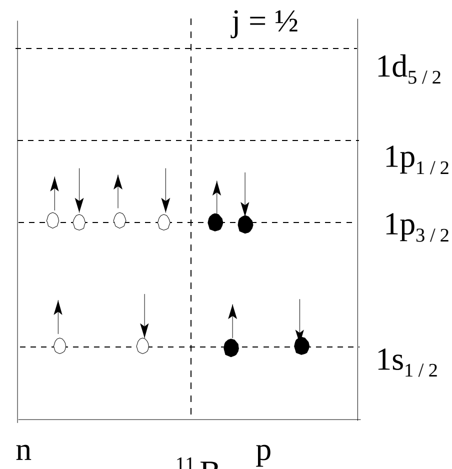
The method of finding the minimum Z for each A works well for $A \le 60$, but deviates increasingly at higher A values.

11-58. (a)



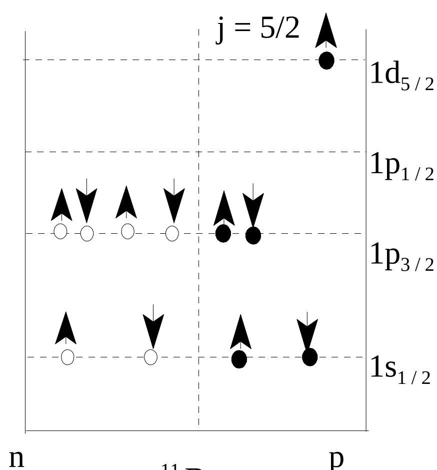
^{11}B
Ground state

(b)



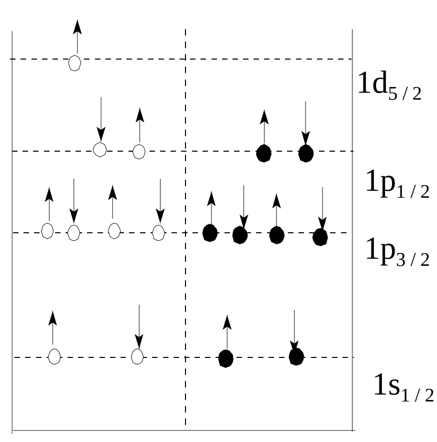
^{11}B
1st excited state

(c)

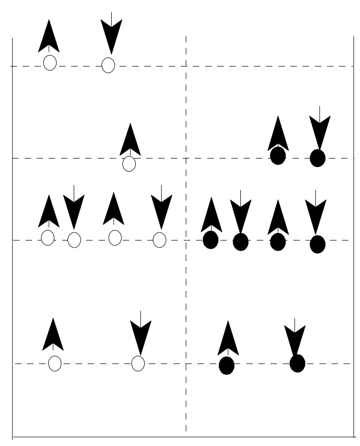


^{11}B
2nd excited state

(d)



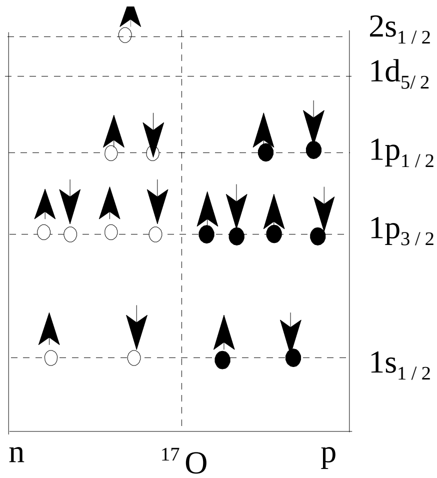
n ^{17}O p
Ground state (j = 5/2)



n ^{17}O p
1st excited state (j = 1/2)

(Problem 11-58 continued)

(e)



2^{nd} excited state (j = 1/2)

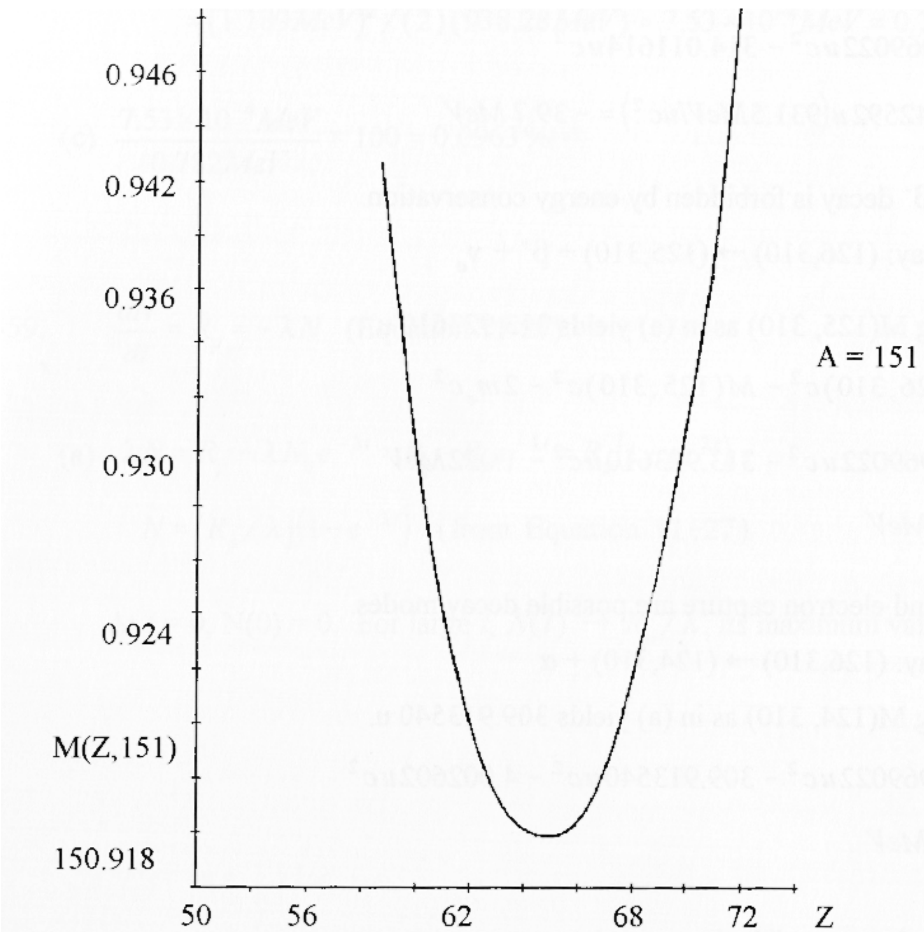
11-59. (a) Data from Appendix A are plotted on the graph. For those isotopes not listed in Appendix A, data for ones that have been discovered can be found in the reference sources, e.g., *Table of Isotopes*, R.B. Firestone, Wiley – Interscience (1998). Masses for those not yet discovered or not in Appendix A are computed from Equation 11-14 (on the Web). Values of M(Z,151) computed from Equation 11-14 are listed below. Because values found from Equation 11-14 tend to overestimate the mass in the higher A regions, the calculated value was adjusted to the measured value for Z = 56, the lowest Z known for A = 151 and the lower Z values were corrected by a corresponding amount. The error introduced by this correction is not serious because the side of the parabola is nearly a straight line in this region. On the high Z side of the A = 151 parabola, all isotopes through Z = 70 have been discovered and are in the reference cited.

Z	N	M(Z,151) [Eq. 11-14]	M(Z,151) [adjusted]
50	101	152.352638	151.565515
51	100	152.234612	151.447490
52	99	152.122188	151.335066
53	98	152.015365	151.228243

(Problem 11-59 continued)

Z	N	M(Z,151) [Eq. 11-14]	M(Z,151) [adjusted]
54	97	151.914414	151.127292
55	96	151.818525	151.031403
56	95	151.728507	150.941385*

* This value has been measured.



(b) The drip lines occur for:

protons: M(Z, 151) – [M(Z–1), 150) – m_p] = 0

neutrons: M(Z, 151) – [M(Z, 150) – m_n] = 0

Write a calculator or computer program for each using Equation 11-14 (on Web page) and solve for Z.

11-60. (a) $M(Z,A) = Zm_p + Nm_n - [a_1 A - a_2 A^{2/3} - a_3 A^{-1/3} - a_4 (A - 2Z)^2 A^{-1} + a_5 A^{-1/2}]$

from Equation 11-14 on the Web page.

For Z = 126, A = 310 $(= Z + N)$:

$$M(126,310) = 126m_p + 184m_n - [15.67(310) - 17.23(310)^{2/3} - 0.75(126)^2(310)^{-1/3} - 93.2(310 - 2\times 126)^2(310) + 12(310)^{-1} + 12(310)^{-1/2}]$$

M(126, 310) = 313.969022 u

(b) For β^- decay: (126,310) → (127,310) + β^- + $\bar{\nu}_e$

Computing M(127, 310) as in (a) yields 314.011614 u.

$$Q = M(126,310)c^2 - M(127,310)c^2$$

$$= 313.969022uc^2 - 314.011614uc^2$$

$$= -0.042592u(931.5MeV/uc^2) = -39.7\ MeV$$

Q < 0, so β^- decay is forbidden by energy conservation.

(c) For β^+ decay: (126,310) → (125,310) + β^+ + ν_e

Computing M(125, 310) as in (a) yields 313.923610 u.

$$Q = M(126,310)c^2 - M(125,310)c^2 - 2m_e c^2$$

$$= 313.969022uc^2 - 313.923610uc^2 - 1.022MeV$$

$$= 41.3\ MeV$$

β^+ decay and electron capture are possible decay modes.

For α decay: (126,310) → (124,310) + α

Computing M(124, 310) as in (a) yields 309.913540 u.

$$Q = 313.969022uc^2 - 309.913540uc^2 - 4.002602uc^2$$

$$= 49.3\ MeV$$

α decay is also a possible decay mode.

11-61. (a) If the electron's kinetic energy is 0.782 MeV, then its total energy is:

$$E = 0.782\,MeV + m_e c^2 = 0.782\,MeV + 0.511\,MeV = 1.293\,MeV$$

$$E^2 = (pc)^2 + (m_e c^2)^2 \quad \text{(Equation 2-32)}$$

$$p = \left(E^2 - (m_e c^2)^2\right)^{1/2}/c$$

$$= \left[(1.293\,MeV)^2 - (0.511\,MeV)^2\right]^{1/2}/c$$

$$= 1.189\,MeV/c$$

(b) For the proton $p = 1.189\,MeV/c$ also, so

$$E_{kin} = p^2/2m = (pc)^2/2mc^2$$

$$= (1.189\,MeV)^2/(2)(938.28\,MeV) = 7.53\times10^{-4}\,MeV = 0.753\,keV$$

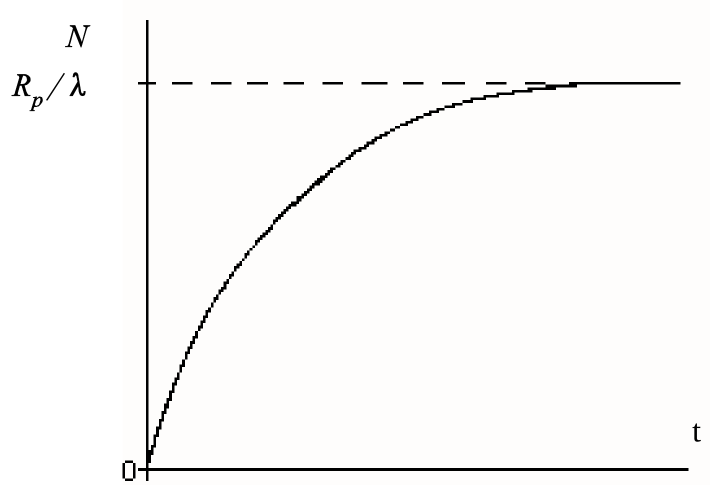
(c) $\dfrac{7.53\times10^{-4}\,MeV}{0.782\,MeV}\times 100 = 0.0963\%$

11-62. $\dfrac{dN}{dt} = R_p = -\lambda N$ (Equation 11-27)

(a) $\lambda N = R_p - \lambda N_0 e^{-\lambda t} = R_p - R_p e^{-\lambda t} = R_p(1 - e^{-\lambda t})$

$$N = (R_p/\lambda)(1 - e^{-\lambda t}) \quad \text{(from Equation 11-27)}$$

At $t = 0$, N(0) = 0. For large t, $N(t) \rightarrow R_p/\lambda$, its maximum value



(Problem 11-62 continued)

(b) For $dN/dt \approx 0$

$$R_p = \lambda N \rightarrow N = R_p/\lambda = R_p/(\ln 2/t_{1/2})$$

$$N = 100s^{-1}/(\ln 2/10\min) = (100s^{-1})(60s/\min)/(\ln 2/10\min)$$

$$= 8.66\times 10^4 \ {}^{62}Cu \ nuclei$$

11-63. (a) $4n + 3$ decay chain ${}^{235}_{92}U_{143} \rightarrow {}^{207}_{82}Pb_{125}$ There are 12 α decays in the chain. (See graph below.)

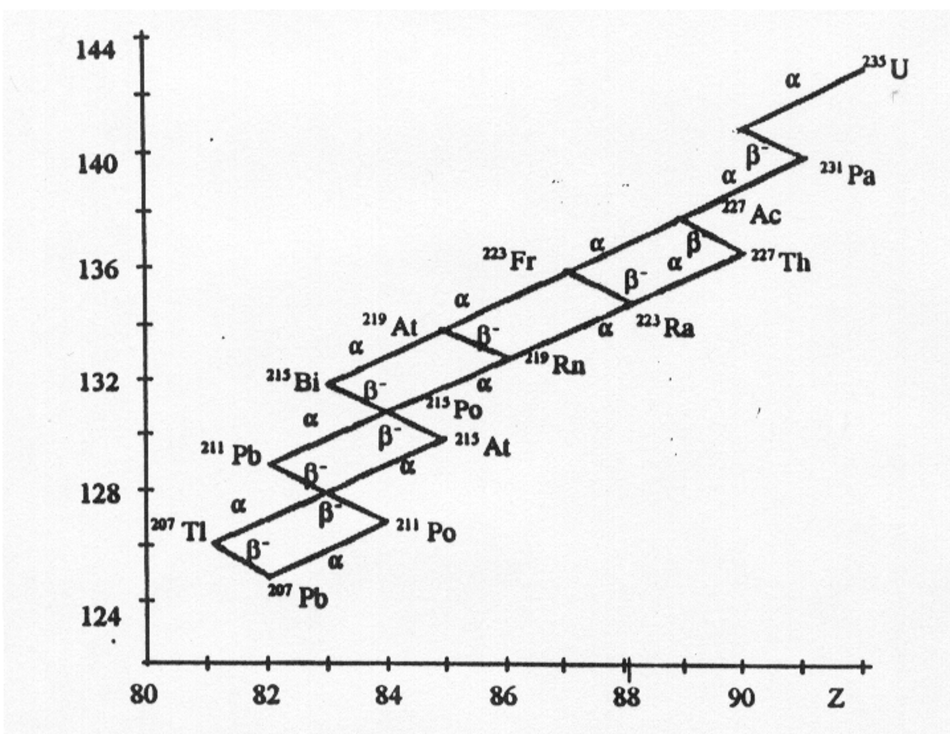
(b) There are 9 β^- decays in the chain. (See graph below.)

(c) $Q = M({}^{235}U)c^2 - M({}^{207}Pb)c^2 - 7M({}^{4}He)c^2$

$$= 235.043924uc^2 - 206.975871uc^2 - 7(4.002602)uc^2$$

$$= 0.049839uc^2(931.50MeV/uc^2)$$

$$= 46.43\ MeV$$



(Problem 11-63 continued)

(d) The number of decays in one year is

$$-\frac{dN}{dt} = \lambda N_0 e^{-\lambda t} \quad \text{where } \lambda = \ln 2/t_{1/2} = \ln 2/7.04\times10^{8}y = 9.85\times10^{-10}y^{-1}$$

$$N_0 = \frac{1\,kg(1000\,g/kg)(6.02\times10^{23}\,atoms/mol)}{235\,g/mol} = 2.56\times10^{24}\,atoms$$

$$-\frac{dN}{dt} = (9.85\times10^{-10}y^{-1})(2.56\times10^{24})e^{-\lambda(1y)} = 2.52\times10^{15}\,decays/y$$

Each decay results in the eventual release of 46.43 MeV, so the energy release per year Q is:

$$Q = 2.52\times10^{15}\,decays/y\,(46.43\,MeV/decay)$$

$$= 1.17\times10^{17}\,MeV/y\,(1.60\times10^{-13}\,J/MeV)$$

$$= 1.87\times10^{4}\,J/y\,(1\,cal/4.186J) = 4.48\times10^{3}\,cal/y$$

The temperature change ΔT is given by: $Q = cm\Delta T$ or $\Delta T = Q/cm$ where $m = 1\,kg = 1000\,g$ and the specific heat of U is $c = 0.0276\,cal/g\cdot°C$.

$$\Delta T = (4.48\times10^{3}\,cal/y)/(0.0276\,cal/g\cdot°C)(1000\,g) = 162\,°C$$