Chapter 12 – Nuclear Reactions and Applications

12-1. (a) $Q = M(^2H)c^2 + M(^2H)c^2 - M(^3H)c^2 - M(^1H)c^2$

$= 2(2.014102uc^2) - 3.016049uc^2 - 1.007825uc^2$

$= 0.004330uc^2(931.5MeV/uc^2) = 4.03MeV$

(b) $Q = M(^3He)c^2 + M(^2H)c^2 - M(^4He)c^2 - M(^1H)c^2$

$= 3.016029uc^2 + 2.014102uc^2 - 4.002602uc^2 - 1.007825uc^2$

$= 0.019704uc^2(931.5MeV/uc^2) = 18.35MeV$

(c) $Q = M(^6Li)c^2 + m_nc^2 - M(^3H)c^2 - M(^4He)c^2$

$= 6.01512uc^2 + 1.008665uc^2 - 3.016049uc^2 - 4.002602uc^2$

$= 0.005135uc^2(931.5MeV/uc^2) = 4.78MeV$

12-2. (a) $Q = M(^3H)c^2 + M(^1H)c^2 - M(^3He)c^2 - m_Nc^2$

$= 3.016049uc^2 + 1.007825uc^2 - 3.016029uc^2 - 1.008665uc^2$

$= -0.000820uc^2(931.5MeV/uc^2) = -0.764MeV$

(b) The threshold for this endothermic reaction is:

$$E_{th} = \frac{m + M}{M}|Q| \qquad \text{(Equation 12-4)}$$

$$= \frac{3.016049uc^2 + 1.007825uc^2}{1.007825uc^2}|0.764MeV| = 3.05\ MeV$$

(c) $E_{th} = \frac{|1.007825uc^2 + 3.016049uc^2|}{3.016049uc^2}|0.764MeV| = 1.02\ MeV$

12-3. $^{14}N + {}^{2}H \rightarrow {}^{16}O^{*}$

Possible products:

$$^{16}O^{*} \rightarrow {}^{14}N + {}^{2}H$$

$$^{16}O^{*} \rightarrow {}^{16}O + \gamma$$

$$^{16}O^{*} \rightarrow {}^{15}O + n$$

$$^{16}O^{*} \rightarrow {}^{15}N + p$$

$$^{16}O^{*} \rightarrow {}^{12}C + \alpha$$

12-4. (a) $^{12}C(\alpha,p)^{15}N$

$$Q = M(^{12}C)c^2 + M(^{4}He)c^2 - M(^{15}N)c^2 - m_p c^2$$

$$= 12.000000\,uc^2 + 4.002602\,uc^2 - 15.000108\,uc^2 - 1.007825\,uc^2$$

$$= -0.005331\,uc^2\,(931.50\,MeV/uc^2)$$

$$= -4.97\,MeV$$

(b) $^{16}O(d,p)^{17}O$

$$Q = M(^{16}O)c^2 + M(^{2}H)c^2 - M(^{17}O)c^2 - m_p c^2$$

$$= 15.994915\,uc^2 + 2.014102\,uc^2 - 16.999132\,uc^2 - 1.007825\,uc^2$$

$$= 0.002060\,uc^2\,(931.50\,MeV/uc^2)$$

$$= 1.92\,MeV$$

12-5. The number of ^{75}As atoms in sample N is:

$$N = \frac{V\rho N_A}{M} = \frac{(1\,cm \times 2\,cm \times 30\,\mu m \times 10^{-4}\,cm/\mu m)(5.73\,g/cm^3)(6.02 \times 10^{23}\,atoms/mol)}{74.9216\,g/mol}$$

$$= 2.76 \times 10^{20} \quad {}^{75}As \;\; atoms$$

The reaction rate R per second per ^{75}As atom is:

(Problem 12-5 continued)

$$R = \sigma I \qquad (\text{Equation } 12-5)$$

$$= (4.5\times10^{-24} cm^2/^{75}As)(0.95\times10^{13}\ neutrons/cm^2\cdot s)$$

$$= 4.28\times10^{-11} s^{-1}$$

$$\text{Reaction rate} = NR$$

$$= (2.76\times10^{20}\ atoms)(4.28\times10^{-11}/s\cdot atom)$$

$$= 1.18\times10^{10}/s$$

12-6. (a) $^{23}\text{Ne}(p,n)^{23}\text{Na}$ $\quad$ $^{22}\text{Ne}(d,n)^{23}\text{Na}$ $\quad$ $^{20}\text{F}(\alpha,n)^{23}\text{Na}$

(b) $^{11}\text{B}(\alpha,p)^{14}\text{C}$ $\quad$ $^{14}\text{N}(n,p)^{14}\text{C}$ $\quad$ $^{13}\text{C}(d,p)^{14}\text{C}$

(c) $^{29}\text{Si}(\alpha,d)^{31}\text{P}$ $\quad$ $^{32}\text{P}(p,d)^{31}\text{P}$ $\quad$ $^{32}\text{Si}(n,d)^{31}\text{P}$

12-7. (a) ^{14}C $\quad$ (b) n $\quad$ (c) ^{58}Ni $\quad$ (d) α

(e) ^{14}N $\quad$ (f) ^{160}Er $\quad$ (g) ^{3}H $\quad$ (h) p

12-8. $$\frac{Q}{c^2} = m_p + m_n - m_d$$

$$= 1.007276u + 1.008665u - 2.013553u$$

$$= 0.002388u \quad (\text{See Table } 11-1.)$$

$$Q = (0.002388u)(931.5\ MeV/u\cdot c^2)c^2$$

$$Q = 2.224\ MeV$$

12-9. $$P = \frac{dW}{dt} = E\frac{dN}{dt}$$

$$\frac{dN}{dt} = \frac{P}{E} = \frac{500\times10^6 J/s}{200\times10^6\ eV/fission}\left(\frac{1\ eV}{1.60\times10^{-19} J}\right)$$

$$= 1.56\times10^{19}\ fissions/s$$

12-10. The fission reaction rate is:

$$R(N) = R(0)k^N \qquad (\text{see Example } 12-7)$$

$$k^N = R(N)/R(0)$$

$$N\log k = \log[R(N)/R(0)]$$

$$N = \frac{\log[R(N)/R(0)]}{\log k}$$

(a) For the reaction rate to double $R(N) = 2R(0)$: $N = \dfrac{\log 2}{\log 1.1} = 7.27$

(b) For $R(N) = 10R(0)$: $N = \dfrac{\log 10}{\log 1.1} = 24.2$

(c) For $R(N) = 100R(0)$: $N = \dfrac{\log 100}{\log 1.1} = 48.3$

(d) Total time $t = N(1\,ms) = N\,ms$: (a) 7.27 ms (b) 24.2 ms (c) 48.3 ms

(e) Total time $t = N(100\,ms) = 100N\,ms$: (a) 0.727 s (b) 2.42 s (c) 4.83 s

12-11.

$$^{235}_{92}U_{143} + n \rightarrow \begin{cases} ^{101}_{40}Zr_{61} + {}^{134}_{52}Te_{82} + n \\ ^{101}_{41}Nb_{60} + {}^{133}_{51}Sb + 2n \\ ^{101}_{43}Tc_{58} + {}^{132}_{49}In_{83} + 3n \\ ^{101}_{45}Rh_{56} + {}^{130}_{47}Ag_{83} + 4n \end{cases}$$

12-12. $$500\,MW = \left(500\,\frac{J}{s}\right)\left(\frac{1\,MeV}{1.60\times 10^{-13}\,J}\right)\left(\frac{1\,\text{fusion}}{17.6\,MeV}\right) = 1.78\times 10^{14}\ \text{fusions/s}$$

Each fusion requires one ^{2}H atom (and one ^{3}H atom; see Equation 12-14), so ^{2}H must be provided at the rate of 1.78×10^{14} atoms/s.

12-13. The reactions per ^{238}U atom R is:

$$R = \sigma I \qquad (\text{Equation } 12-5)$$

$$= (0.02\times10^{-24}\,cm^2/atom)(5.0\times10^{11}\,n/m^2)\left(\frac{1\,m^2}{10^4\,cm^2}\right) = 1.00\times10^{-18}/atom$$

The number N of ^{238}U atoms is:

$$N = \frac{(5.0\,g)(6.02\times10^{23}\,atoms/mol)}{238.051\,g/mol} = 1.26\times10^{22} \quad {}^{238}\text{U atoms}$$

Total ^{239}U atoms produced = $R\,N$

$$= (1.00\times10^{-18}/atom)(1.26\times10^{22}\,atoms) = 1.26\times10^{4} \quad {}^{239}\text{U atoms}$$

12-14.
$$Q_1 = M(^1H)c^2 + M(^1H)c^2 - M(^2H)c^2$$

$$= 1.007825\,uc^2 + 1.007825\,uc^2 - 2.014102\,uc^2$$

$$= 0.001548\,uc^2(931.50\,MeV/uc^2)$$

$$= 1.4420\,MeV$$

$$Q_2 = M(^2H)c^2 + M(^1H)c^2 - M(^3He)c^2$$

$$= 2.014102\,uc^2 + 1.007825\,uc^2 - 3.016029\,uc^2$$

$$= 0.005898\,uc^2(931.50\,MeV/uc^2)$$

$$= 5.4940\,MeV$$

$$Q_3 = M(^3He)c^2 + M(^3He)c^2 - M(^4He)c^2 - 2m(^1H)c^2$$

$$= 2(3.016029\,uc^2) - 4.002602\,uc^2 - 2(1.007825\,uc^2)$$

$$= 0.013806\,uc^2(931.50\,MeV/uc^2)$$

$$= 12.8603\,MeV$$

$$Q = Q_1 + Q_2 + Q_3 = 1.4420\,MeV + 5.4940\,MeV + 12.8603\,MeV = 19.80\,MeV$$

12-15. Total power $= 1000\,MWe/0.30 = 3333\,MW$

$= 3.33\times10^{9} J/s(1\,MeV/1.60\times10^{-13}J)$

$= 2.08\times10^{22} MeV/s$

(a) 1 day = $8.64\times10^{4}s$

Energy/day $= 2.08\times10^{22} MeV/s\,(8.64\times10^{4}\,s/day) = 1.80\times10^{27}\,MeV$

The fission of 1kg of ^{235}U provides $4.95\times10^{26}\,MeV$ (from Example 12-4)

$1\,kg\,^{235}U/day = 1.80\times10^{27} MeV/(4.95\times10^{26}\,MeV/kg) = 3.64\,kg/day$

(b) $kg\,^{235}U/year = 3.64\,kg\,^{235}U/day\,(365\,days/year) = 1.33\times10^{3}\,kg/year$

(c) Burning coal produces $3.15\times10^{7}\,J/kg\,(1\,MeV/1.60\times10^{-13}J) = 1.97\times10^{20}\,MeV/kg\,coal$

Ratio of kg of coal needed per kg of ^{235} U is: $\dfrac{4.95\times10^{26} MeV/kg\ ^{235}U}{1.97\times10^{20}\,MeV/kg\,coal} = 2.51\times10^{6}$

For 1 day: $3.64\,kg\ ^{235}U(2.51\times10^{6}) = 9.1\times10^{6}\,kg$

This is about 10,000 tons/day, the approximate capacity of 100 railroad coal hopper cars.

For 1 year: $9.12\times10^{6}\,kg/day\,(365\,days/year) = 3.33\times10^{9}\,kg/year$

12-16. $\rho(H_2O) = 1000\,kg/m^3$, so

(a) 1000 kg: $\dfrac{10^{6}g(6.02\times10^{23}\ molecules\ H_2O/mol)(2\,H/molecule)(0.00015\ ^2H/H)}{18.02\,g/mol}$

$= 1.00\times10^{25}\ ^2H$ atoms

Each fusion releases 5.49 MeV,

Energy release $= (1.00\times10^{25})(5.49\,MeV) = 5.49\times10^{25}\,MeV$

$= (5.49\times10^{25}\,MeV)(1.60\times10^{-913}J/MeV) = 8.80\times10^{12}\,J$

(b) Energy used/person (in 1999) $= 3.58\times10^{20}\,J/5.9\times10^{9}$ people

$= 6.07\times10^{10}$ J/person·y

(Problem 12-16 continued)

Energy used per person per hour = $6.07 \times 10^{10} J/person \cdot y \times \dfrac{1y}{8760h} = 6.93 \times 10^{6} J/person \cdot h$

At that rate the deuterium fusion in 1m^3 of water would last the "typical" person

$$\frac{8.80 \times 10^{12} J}{6.93 \times 10^{6} J/person \cdot h} = 1.27 \times 10^{6} h \approx 145\, y$$

12-17. (a) $Q = M(^{235}U)c^2 + m_n c^2 - M(^{120}Cd)c^2 - M(^{110}Ru)c^2 + 5m_n c^2$

$$= 235.043924uc^2 + 1.008665uc^2 - 119.909851uc^2 - 109.913859uc^2$$

$$- 5(1.008665uc^2)$$

$$= 1.186uc^2(931.50 MeV/uc^2) = 1.10 \times 10^3 MeV$$

(b) This reaction is not likely to occur. Both product nuclei are neutron-rich and highly unstable.

12-18. The original number N_0 of ^{14}C nuclei in the sample is:

$N_0 = (15g)(6.78 \times 10^{10}\text{nuclei/g}) = 1.017 \times 10^{12}$ where the number of ^{14}C nuclei per gram of C was computed in Example 12-12. The number N of ^{14}C present after 10,000 y is:

$$N = N_0 e^{-\lambda t} = N_0 e^{-\ln 2 (t/t_{1/2})} \qquad \text{(Equation 11-18)}$$

$$= (1.017 \times 10^{12})\, e^{-\ln 2 (10000/5730)} = 3.034 \times 10^{11}$$

$$R = \lambda N = (\ln 2/t_{1/2})N \qquad \text{(from Equation 11-19)}$$

$$= (\ln 2/5730y)(1y/3.16 \times 10^{7} s)(3.034 \times 10^{11})$$

$$= 1.16 \text{ decays/s}$$

12-19. If from a living organism, the decay rate would be:

$(15.6 \text{ decays/g} \cdot \text{min})(175g) = 20{,}230 \text{ decays/min}$ (from Example 12-12)

The actual decay rate is: $(8.1 \text{ decays/s})(60\, s/\text{min}) = 486 \text{ decays/min}$

(Problem 12-19 continued)

$$\left(\frac{1}{2}\right)^n = \frac{486\ \text{decays/min}}{20{,}230\ \text{decays/min}} \quad (\text{from Example 12-12})$$

$$2^n = 20{,}230/486$$

$$n\ln 2 = \ln(20{,}230/486)$$

$$n = \ln(20{,}230/486)/\ln 2 = 5.379\ \text{half lives}$$

$$\text{Age of bone} = (5.379\ \text{half lives})(5730\,\text{y/half life}) = 30{,}800\,y$$

12-20.

$$t = \frac{t_{1/2}}{\ln 2}\ln(1 + N_D/N_P) \quad (\text{Equation 12-35})$$

$$t_{1/2}(^{87}Rb) = 4.88\times10^{10}\,y \text{ and } N_P/N_D = 36.5$$

$$t = \frac{4.88\times10^{10}y}{\ln 2}\ln[1 + (1/36.5)] = 1.90\times10^9\,y$$

12-21. The number of X rays counted during the experiment equals the number of atoms of interest in the same times the cross section for activation σ_x times the particle beam intensity, where
I = proton intensity = 650 nA × (e C/proton)$^{-1}$; σ_x = 650 b = 650×10^{-24} cm^2
m = mass = 0.35 mg/cm^2 × 0.00001; *n* = number of atoms of interest = mN_A/A
t = exposure time; detector efficiency = 0.0035 ϵ = overall efficiency = 0.60 × detector efficiency

$$N = I\sigma_x\frac{mN_A}{A}t\epsilon$$

$$N = \left(\frac{650\times10^{-9}\,C/s}{1.60\times10^{-19}\,C/proton}\right)(650\times10^{-24}\,cm^2)\left(\frac{(0.35\times10^{-3}\,g/cm^2)(0.00001)(6.02\times10^{23}\,mol^{-1})}{80\,g/mol}\right)$$

$$\times\ (15\min\times60\,s/\text{min})(0.60\times0.0035)$$

$$N = 1.31\times10^5\ \text{counts in 15 minutes}$$

12-22.
$$t = \frac{t_{1/2}}{\ln 2}\ln(1 + N_D/N_P) \qquad \text{(Equation 12-35)}$$

$$t_{1/2}(^{232}Th) = 1.40\times10^{10}\,y$$

$$N_P(^{232}Th) = \frac{4.11\,g(6.02\times10^{23}\,atoms/mol)}{232.04\,g/mol} = 1.066\times10^{22}\,atoms$$

$$N_D(^{208}Pb) = \frac{0.88\,g\,(6.02\times10^{23}\,atoms/mol)}{208\,g/mol} = 2.547\times10^{21}\,atoms$$

$$N_D/N_P = 2.547\times10^{21}/1.066\times10^{22} = 0.2389$$

$$t = \frac{1.40\times10^{10}y}{\ln 2}\ln[1 + 0.2389] = 4.33\times10^{9}y$$

12-23. $f = \dfrac{\Delta E}{h} = \dfrac{2(\mu_z)_p B}{h}$

(a) For Earth's field: $f = \dfrac{2(2.79\,\mu_N)[(3.15\times10^{-8}\,eV/T)/1\,\mu_N](0.5\times10^{-4}\,T)}{4.14\times10^{-15}\,eV\cdot s}$

$$= 2.12\times10^{3}\,Hz = 2.12\,kHz$$

(b) For B = 0.25 T:

$$f = 2.12\times10^{3}\,Hz(0.25\,T/0.5\times10^{-4}\,T) = 1.06\times10^{7}\,Hz = 10.6\,MHz$$

(c) For B = 0.5 T:

$$f = 2.12\times10^{3}\,Hz(0.5\,T/0.5\times10^{-4}\,T) = 2.12\times10^{7}\,Hz = 21.2\,MHz$$

12-24. (a)
$$N(^{12}C^{+3}) = \frac{(12\times10^{-6}\,C/s)\,(10\,\text{min})\,(60\,s/\text{min})}{3(1.60\times10^{-19}\,C)} = 1.50\times10^{16}$$

$$^{14}C/^{12}C \text{ ratio} = 1500/1.50\times10^{16} = 10^{-13}$$

(b) $\text{mass } ^{12}C = \dfrac{(1.50\times10^{15}\,atoms/\text{min})\,(75\,\text{min})\,(12)\,(1.66\times10^{-27}\,kg)}{0.015}$

$$\text{mass } ^{12}C = 1.49\times10^{-7}\,kg = 1.49\times10^{-4}\,g = 0.149\,mg$$

(Problem 12-24 continued)

(c) The $^{14}C/^{12}C$ ratio in living C is 1.35×10^{-12}.

$$\frac{\text{sample } ^{14}C/^{12}C}{\text{living } ^{14}C/^{12}C} = \frac{10^{-13}}{1.35\times10^{-12}} = \frac{0.10}{1.35} = \left(\frac{1}{2}\right)^n$$

where n = # of half-lives elapsed. Rewriting as (see Example 12-13)

$$2^n = \frac{1.35}{0.10} = 13.5$$

$$n\ln 2 = \ln 13.5 \qquad \therefore\ n = \ln 13.5/\ln 2 = 3.75$$

$$\text{age of sample} = 3.75\,t_{112} = 3.75(5730y) = 2.15\times10^4 y$$

12-25. If from live wood, the decay rate would be 15.6 disintegrations/g·min. The actual rate is 2.05 disintegrations/g·min.

$$\left(\frac{1}{2}\right)^n = \frac{2.05 \text{ decays/g}\cdot\text{min}}{15.6 \text{ decays/g}\cdot\text{min}} \qquad (\text{from Example } 12-13)$$

$$2^n = 15.6/2.05$$

$$n\ln 2 = \ln(15.6/2.05)$$

$$n = \ln(15.6/2.05)/\ln 2 = 2.928 \text{ half lives of } ^{14}C$$

$$\text{Age of spear thrower} = nt_{1/2} = (2.928)(5730y) = 16{,}800y$$

12-26. (a) $R = R_0 A^{1/3}$ where $R_0 = 1.2f$ (Equation 11 – 3)

$$R(^{141}Ba) = (1.2fm)(10^{-15}m/fm)(141)^{1/3} = 6.24\times10^{-15}m$$

$$R(^{92}Kr) = (1.2fm)(10^{-15}m/fm)(92)^{1/3} = 5.42\times10^{-15}m$$

(b) $$V = kq_1q_2/r = \frac{(8.998\times10^9 N\cdot m^2/C^2)(56)(1.60\times10^{-19}C)(36)(1.60\times10^{-19}C)}{(6.24\times10^{-15}m + 5.42\times10^{-15}m)(1.60\times10^{-19}J/eV)}$$

$$= 2.49\times10^8\,eV = 249\,MeV$$

This value is about 40% larger than the measured value.

12-27. (a) In the lab, the nucleus (at rest) is at $x = 0$ and the neutron moving at v_L is at x.

$$x_{CM} = \frac{M(0) + mx}{M + m} = \frac{mx}{M + m} \qquad v_L = \frac{dx}{dt} \quad and \quad V = \frac{dx_m}{dt} = \frac{m(dx/dt)}{M + m}$$

$$V = \frac{mv_L}{M + m}$$

(b) The nucleus at rest in the lab frame moves at speed V in the CM frame before the collision. In an elastic collision in the CM system, the particles reverse their velocities, so the speed of the nucleus is still V, but in the opposite direction.

(c) In the CM frame the nucleus velocity changes by $2V$. This is also the change in the lab system where the nucleus was initially at rest. It moves with speed $2V$ in the lab system after the collision.

(d) $$\frac{1}{2}M(2V)^{1/2} = \frac{1}{2}M\left[\frac{2mv_L}{M + m}\right]^2 = \frac{1}{2}mv_L^2\left[\frac{4mM}{(M + m)^2}\right]$$

Before collision: $E_i = \frac{1}{2}mv_L^2$

After collision: $E_f = \frac{1}{2}mv_L^2 - \frac{1}{2}mv_L^2\left[\frac{4mM}{(M + m)^2}\right] = E_i\left(1 - \frac{4mM}{(M + m)^2}\right)$

12-28. At the end of the two hour irradiation the number of ^{32}P and ^{56}Mn atoms are given by

$N = \frac{R_0}{\lambda}(1 - e^{-\lambda t})$ from Equation 11-26 where $R_0 = \sigma I$ (Equation 12-5).

For ^{32}P:

$$R_0 = (0.180 \times 10^{-24} cm^2)(10^{12}\, neutrons/cm^2 \cdot s) = 1.80 \times 10^{-13}\ {}^{32}P\ atoms/s\ per\ {}^{31}P$$

$$N_0 = \frac{R_0 t_{1/2}}{\ln 2}(1 - e^{-\ln 2(t/t_{1/2})}) = \frac{(1.80 \times 10^{-13}/s)(3600\, s/h)(342.2\, h)}{\ln 2}(1 - e^{-\ln 2(2h/342.2h)})$$

$$= 1.29 \times 10^{-9}\ {}^{32}P\ atoms/{}^{31}P\ atom$$

(Problem 12-28 continued)

For ^{56}Mn:

$$R_0 = (13.3\times10^{-24}\,cm^2)(10^{12}\,neutrons/cm^2\cdot s) = 1.33\times10^{-11}\ {}^{56}Mn\ atoms/s\ \ per\ \ {}^{55}Mn$$

$$N_0 = \frac{(1.33\times10^{-11}/s)(3600\,s/h)(2.58\,h)}{\ln 2}\left(1 - e^{-\ln 2(2h/2.58h)}\right)$$

$$= 7.42\times10^{-8}\ \ {}^{56}Mn\ atoms/{}^{55}Mn\ atom$$

(a) Two hours after the irradiation stops, the activities are:

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t} = -\frac{N_0 \ln 2}{t_{1/2}} e^{-\ln 2(t/t_{1/2})}$$

For ^{32}P: $\left|\frac{dN}{dt}\right| = \frac{(1.29\times10^{-9})\ln 2}{(14.26\,d)(8.64\times10^4\,s/d)} e^{-\ln 2(2h/342.2h)} = 7.23\times10^{-16}\,\text{decays}/{}^{31}\text{P atom}$

For ^{56}Mn: $\left|\frac{dN}{dt}\right| = \frac{(7.42\times10^{-8})\ln 2}{(2.58\,h)(3600\,s/h)} e^{-\ln 2(48h/2.58h)} = 1.39\times10^{-17}\,\text{decays}/{}^{55}\text{Mn atom}$

The total activity is the sum of these, each multiplied by the number of parent atoms initially present.

12-29. Q = 200 MeV/fission. $E = NQ = 7.0\times10^{19}J = N(200\,MeV/fission)(1.60\times10^{-13}J/MeV)$

$$N = \frac{7.0\times10^{19}J}{(200\,MeV/fission)(1.60\times10^{-13}J/MeV)} = 2.19\times10^{30}\,fissions$$

Number of moles of ^{235}U needed = $N/N_A = 2.19\times10^{30}/6.02\times10^{23} = 3.63\times10^6\,moles$

Fissioned mass/y = $(3.63\times10^6\,moles)(235\,g/mole) = 8.54\times10^8\,g = 8.54\times10^5\,kg$

This is 3% of the mass of ^{235}U atoms needed to produce the energy consumed.

Mass needed to produce $7.0\times10^{19}J = 8.54\times10^5\,kg/0.03 = 2.85\times10^7\,kg$.

Since the energy conversion system is 25% efficient:

Total mass of ^{235}U needed = $1.14\times10^8\,kg$.

12-30. The number of ^{87}Sr atoms present at any time is equal to the number of ^{87}Rb nuclei that have decayed, because ^{87}Sr is stable.

$$N(Sr) = N_0(Rb) - N(Rb) \rightarrow N(Sr)/N(Rb) = N_0(Rb)/N(Rb) - 1$$

$$N(Sr)/N(Rb) = 0.010$$

$$N_0(Rb)/N(Rb) = N(Sr)/N(Rb) + 1 = 1.010$$

and also

$$N(Rb)/N_0(Rb) = e^{-(\ln 2)t/t_{1/2}} = 1/1.010$$

$$\frac{-(\ln 2)t}{t_{1/2}} = \ln(1/1.010)$$

$$t = -t_{1/2}\ln(1/1.010)/\ln 2 = -(4.9\times 10^{10}y)\ln(1/1.010)/\ln 2$$

$$= 7.03\times 10^{8}\,y$$

12-31. (a) Average energy released/reaction is: $(3.27\,MeV + 4.03\,MeV)/2 = 3.65\,MeV$

$$P = \frac{E}{t} = 4\,W = 4\,J/s = N(3.65\,MeV)(1.60\times 10^{-13}\,J/MeV)$$

$$N = \frac{4\,J/s}{(3.65\,MeV/reaction)(1.60\times 10^{-13}\,J/MeV)} = 6.85\times 10^{12}\text{ reactions/s}$$

Half of the reactions produce neutrons, so 3.42×10^{12} neutrons/s will be released.

(b) Neutron absorption rate = $0.10(3.42\times 10^{12}) = 3.42\times 10^{11}$ neutrons/s

Energy absorption rate =

$$(0.5\,\text{MeV/neutron})(3.42\times 10^{11}\,\text{neutrons/s})(1.60\times 10^{-13}\,J/MeV) = 2.74\times 10^{-2}\,J/s$$

Radiation dose rate =

$$[(2.74\times 10^{-2}\,J/s)/(80\,kg)][100\,rad/(J/kg)] = 3.42\times 10^{-2}\,rad/s$$

$$= (3.42\times 10^{-2}\,rad/s)(4) = 0.137\,rem/s = 493\,rem/h$$

(c) 500 rem, lethal to half of those exposed, would be received in:

$$500\,rem/(492\,rem/h) = 1.02\,h$$

12-32. $R(t) = N_0 \sigma I(1 - e^{-\lambda t})$ (Equation 12 – 29)

$$\text{For Co: } N_0 = \frac{35\,Bq}{(19 \times 10^{-24}\,cm^2)(3.5 \times 10^{12}/s \cdot cm^2)(1 - e^{-1.319 \times 10^{-6}(2)})}$$

$$N_0 = 2.00 \times 10^{17} \text{ atoms}$$

$$\text{For Ti: } N_0 = \frac{115\,Bq}{(0.15 \times 10^{-24}\,cm^2)(3.5 \times 10^{12}/s \cdot cm^2)(1 - e^{-0.120(2)})}$$

$$N_0 = 1.03 \times 10^{15} \text{ atoms}$$

12-33. The net reaction is: $5\,{}^2H \rightarrow {}^3He + {}^4He + {}^1H + n + 25\,MeV$

Energy release$/{}^2$H = 5 MeV (assumes equal probabilities)

4ℓ water $\rightarrow 4000\,g/[2(1.007825) + 15.994915]\,g/mol = 222.1\ moles$

4ℓ water thus contains $2(222.1)$ moles of hydrogen, of which 1.5×10^{-4} is ^{2}H, or

Number of ^{2}H atoms = $[2(222.1)\,moles](6.02 \times 10^{23}\,atoms/mole)(1.5 \times 10^{-4}) = 4.01 \times 10^{22}$

Total energy release = $(4.01 \times 10^{22})5\,MeV = 2.01 \times 10^{23}\,MeV = 3.22 \times 10^{10}\,J$

Because the U.S. consumes about $7.0 \times 10^{19}\,J/y$, the complete fusion of the ^{2}H in 4ℓ of water would supply the nation for about $1.45 \times 10^{-2}\,s = 14.5\,ms$

12-34. (a) $\Delta\lambda \le 2hc/Mc^2$

$$\Delta E \approx \frac{hc\Delta\lambda}{\lambda^2} = \frac{(hc)^2}{\lambda^2}\frac{\Delta\lambda}{hc} = \frac{E^2\Delta\lambda}{hc}$$

$$E_p = \Delta E \le \frac{E^2}{hc}\frac{2hc}{Mc^2} = \frac{2E^2}{Mc^2}$$

$$E^2 \ge Mc^2 E_p/2 \quad \rightarrow \quad E \ge (Mc^2 E_p/2)^{1/2}$$

$$\Delta E = E_f - E_i = E_i\left(1 - \frac{4mM}{(M+m)^2}\right) - E_i = -E_i\left(\frac{4mM}{(M+m)^2}\right)$$

$$\frac{-\Delta E}{E_i} = \frac{4mM}{(M+m)^2} = \frac{4m/M}{(1+m/M)^2}$$ which is Equation 12-25.

(Problem 12-34 continued)

(b) $E = [(5.7\,MeV)(938.28\,MeV)/2]^{1/2} = 51.7\,MeV$

(c) o ⟶ vL, neutron (m) x ⟶ v, CM O ^{14}N (M)

The neutron moves at v_L in the lab, so the *CM* moves at $v = v_L m_N/(m_N + M)$ toward the right and the ^{14}N velocity in the *CM* system is *v* to the left before collision and *v* to the right after collision for an elastic collision. Thus, the energy of the nitrogen nucleus in the lab after the collision is:

$$E(^{14}N) = \frac{1}{2}M(2v)^2 = 2Mv^2 = 2M\left(\frac{mv_L}{m+M}\right)^2$$

$$= \frac{2Mm(mv_L^2)}{(m+M)^2} = \frac{4Mm}{(m+M)^2}\left(\frac{1}{2}mv_L^2\right)$$

$$= \frac{4(14.003074\,u)(1.008665\,u)}{(1.008665\,u + 14.003074\,u)^2}(5.7\,MeV)$$

$$= 1.43\,MeV$$

(d) $E \geq [(14.003074\,uc^2)(931.5\,MeV/uc^2)(1.43\,MeV)/2]^{1/2} = 96.5\,MeV$

12-35. In lab frame: photon ⟶ o deuteron, M at rest

$E = h\nu = pc$

$p = h\nu/c = E/c$

In CM frame: photon ⟶ ⟵ o deuteron, M

photon: $E \approx pc$, $p = E/c$

deuteron, M: $E_K \approx 1/2\,Mv^2 = p^2/2M$, $p = \sqrt{2ME_K}$

For $E \approx pc$ in *CM* system means that a negligible amount of photon energy goes to recoil energy of the deuteron, i.e.,

(Problem 12-35 continued)

$$\frac{p^2}{2M} << pc \approx E \qquad \text{or} \qquad \frac{(pc)^2}{2Mc^2} << pc \rightarrow pc << 2Mc^2$$

$$E \approx pc << 2Mc^2 = 2(1875.6\ MeV) = 3751.2\ MeV \quad \text{(see Table 11-1)}$$

In the lab, that incident photon energy must supply the binding energy $B = 2.22\ MeV$ plus the recoil energy E_K given by:

$$E_K = p^2/2M = (pc)^2/2Mc^2 \approx (B)^2/2Mc^2$$

$$= \frac{(2.22\ MeV)^2}{2(1875.6\ MeV)} = 0.0013\ MeV$$

So the photon energy must be $E \geq 2.22\ MeV + 0.001\ MeV = 2.221\ MeV$, which is much less than 3751 MeV.

12-36. The reactions are:

(1) $^1H + {}^1H \rightarrow {}^2H + \beta^+ + \nu$

(2) $^1H + {}^2H \rightarrow {}^3He + \gamma$

followed by

(3) $^3He + {}^3He \rightarrow {}^4He + {}^1H + {}^1H$

or

(4) $^1H + {}^3He \rightarrow {}^4He + \beta^+ + \nu$

(a) Form $2(1) + 2(2) + (3)$:

$$6\,{}^1H + 2\,{}^2H + 2\,{}^3He \rightarrow 2\,{}^2H + 2\,{}^3He + {}^4He + 2\,{}^1H + 2\beta^+ + 2\nu + 2\gamma$$

Canceling 2 ^{1}H, 2 ^{2}H, and 2 ^{3}He on both sides of the sum,

$$4\,{}^1H \rightarrow {}^4He + 2\beta^+ + 2\nu + 2\gamma$$

Form $(1) + (2) + (4)$:

$$4\,{}^1H + {}^2H + {}^3He \rightarrow {}^2H + {}^3He + {}^4He + + 2\beta^+ + 2\nu + \gamma$$

(Problem 12-36 continued)

Canceling ^{2}H and ^{3}He on both sides of the sum,

$$4\,^1H \rightarrow \,^4He + 2\beta^+ + 2\nu + \gamma$$

(b) $$Q = 4M(^1H)c^2 - M(^4He)c^2 - 2m_e c^2$$

$$= 4(938.280\,MeV) - 3727.409\,MeV - 2(0.511\,MeV)$$

$$= 24.7\,MeV$$

(c) Total energy release is 24.7 MeV plus the annihilation energy of the two β^+:

$$\text{energy release} = 24.7\,MeV + 2(2m_e c^2)$$

$$= 24.7\,MeV + 2(1.022\,MeV)$$

$$= 26.7\,MeV$$

Each cycle uses 4 protons, thus produces $26.7\,MeV/4 = 6.68\,MeV/proton$. Therefore, ^{1}H (protons) are consumed at the rate

$$\frac{dN}{dt} = \frac{P}{E} = \frac{4\times10^{26}\,J/s}{6.68\times10^6\,eV}\left(\frac{1\,eV}{1.60\times10^{-19}\,J}\right) = 3.75\times10^{38}\,protons/s$$

The number N of ^{1}H nuclei in the Sun is

$$N = \frac{M_\odot}{M(^1H)} = \frac{1/2\times2\times10^{30}\,kg}{1.673\times10^{-27}\,kg} = 5.98\times10^{56}\ protons$$

which will last at the present consumption rate for

$$t = \frac{N}{dN/dt} = \frac{5.98\times10^{56}\,protons}{3.75\times10^{38}\,protons/s} = 1.60\times10^{18}\,s$$

$$= 1.60\times10^{18}\,s\left(\frac{1\,y}{3.16\times10^7\,s}\right) = 5.05\times10^{10}\,y$$

12-37. At this energy, neither particle is relativistic, so

$$E_{He} = \frac{p_{He}^2}{2m_{He}} \quad E_n = \frac{p_n^2}{2m_n} \quad p_{He} = p_n \quad E_{He} + E_n = 17.7\,MeV$$

(Problem 12-37 continued)

$$2m_{He}E_{He} = 2m_nE_n = 2m_n(17.7\,MeV - E_{He})$$

$$(m_{He} + m_n)E_{He} = 17.7\,MeV\,m_n \qquad \text{Therefore, } E_{He} = \frac{m_n}{m_{He} + m_n}17.7\,MeV$$

$$E_{He} = \frac{1.008665\,u(17.7\,MeV)}{4.002602\,u + 1.008665\,u} = 3.56\,MeV$$

$$E_n = 17.7\,MeV - E_{He} = (17.7 - 3.56)\,MeV = 14.1\,MeV$$

12-38. (a) The number N of generations is: $N = \dfrac{5\,s}{0.08\,s/gen} = 62.5$ generations

Percentage increase in energy production = $\dfrac{R(N) - R(0)}{R(0)} \times 100$

$$= \left[\frac{R(N)}{R(0)} - 1\right] \times 100 \text{ where } R(N)/R(0) = k^N \quad (\text{from Example } 12-7)$$

$$= (k^N - 1) \times 100 = (1.005^{62.5} - 1) \times 100 = 137\%$$

(b) Because $k \propto$ neutron flux, the fractional change in flux necessary is equal to the fractional change in k: $\dfrac{k-1}{k} = \dfrac{1.005 - 1}{1.005} = 0.00498$

12-39. (a) For 5% enrichment:

$$\sigma_f = (0.05)\sigma_f(^{235}U) + (0.95)\sigma_f(^{238}U)$$

$$= (0.05)(584\,b) + (0.95)(0) = 29.2\,b$$

$$\sigma_a = 0.05\,\sigma_a(^{235}U) + 0.95\,\sigma_a(^{238}U)$$

$$= (0.05)(97\,b) + (0.95)(2.75\,b) = 7.46\,b$$

$$k = 2.4\frac{\sigma_f}{\sigma_f + \sigma_a} = \frac{2.4(29.2\,b)}{29.2\,b + 2.46\,b} = 1.91 \quad (\text{Equation } 12-11)$$

(Problem 12-39 continued)

(b) For 95% enrichment: $\sigma_f = (0.95)\sigma_f(^{235}U) + (0.05)\sigma_f(^{238}U)$

$$= (0.95)(584b) + (0.05)(0) = 554.8b$$

$$\sigma_a = 0.95\sigma_a(^{235}U) + 0.05\sigma_a(^{238}U)$$

$$= (0.95)(97b) + (0.05)(2.75b) = 92.3b$$

The reaction rate after N generations is $R(N) = R(0)k^N$.
For the rate to double $R(N) = 2R(0)$ and $2 = k^N \rightarrow N = \ln 2/\ln k$.
$N(5\%) = \ln 2/\ln 1.91 = 1.07$ generations

$N(95\%) = \ln 2/\ln 2.06 = 0.96$ generations

Assuming an average time per generation of 0.01 *s*

$t(5\%) = 1.07\times10^{-2}\,s$

$t(95\%) = 0.96\times10^{-2}\,s$

Number of generations/1s = 1/seconds/generation

In 1*s*: $N(5\%) = 93.5$ and $N(95\%) = 104$

One second after the first fission:

$$R(5\%) = R(0)k^N = (1)(1.91)^{93.5} = 1.9\times10^{26}$$

Energy rate $= (1.9\times10^{26}\,fissions/s)(200\,MeV/fission)$

$$= 3.8\times10^{28}\,MeV/s\,(1.6\times10^{-13}\,J/MeV)$$

$$= 6.1\times10^{15}\,J/s = 6.1\times10^{15}\,W$$

$$R(95\%) = R(0)k^N = (1)(2.06)^{104} = 4.4\times10^{32}$$

Energy rate $= (4.4\times10^{32}\,fissions/s)(200\,MeV/fission)$

$$= 8.8\times10^{34}\,MeV/s\,(1.6\times10^{-13}\,J/MeV)$$

$$= 1.4\times10^{22}\,J/s = 1.4\times10^{22}\,W$$