Chapter 13 – Particle Physics

13-1. (a) Because the two pions are initially at rest, the net momentum of the system is zero, both before and after annihilation. For the momentum of the system to be zero after the interaction, the momenta of the two photons must be equal in magnitude and opposite in direction, i.e., their momentum vectors must add to zero. Because the photon energy is $E = pc$, their energies are also equal.

(b) The energy of each photon equals the rest energy of a π^+ or a π^-.

$E = m_\pi c^2 = 139.6\,MeV$ (from Table 13–1)

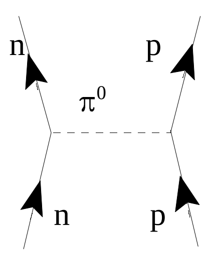
(c) $E = hf = hc/\lambda$ Thus, $\lambda = \dfrac{hc}{E} = \dfrac{1240\,MeV \cdot fm}{139.6\,MeV} = 8.88\,fm$

13-2. (a) $E_\gamma = m_\Lambda c^2 + m_\pi c^2 = 2285\,MeV + 139.6\,MeV = 2424.6\,MeV$

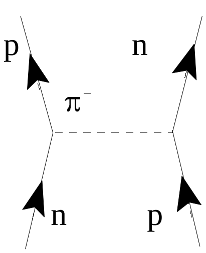
(b) $E_\gamma = 2m_p c^2 = 2(938.28\,MeV) = 1876.56\,MeV$

(c) $E_\gamma = 2m_\mu c^2 = 2(105.66\,MeV) = 211.32\,MeV$

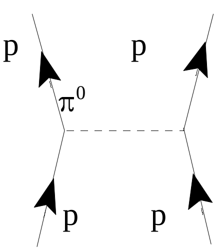
13-3. (a)



(b)



(c)



13-4. (a) ${}^{32}P \rightarrow {}^{32}S + e^-$ assuming no neutrino

$Q = M({}^{32}P)c^2 - M({}^{32}S)c^2$ (electron's mass is included in that of ^{32}S)

$= 31.973908\,uc^2 - 31.97207\,uc^2$

$= (0.001837\,uc^2)(931.5\,MeV/uc^2) = 1.711\,MeV$

To a good approximation, the electron has all of the kinetic energy $E_K \approx Q = 1.711\,MeV$

(b) In the absence of a neutrino, the ^{32}S and the electron have equal and opposite momenta.

(Problem 13-4 continued)

The momentum of the electron is given by:

$$(pc)^2 = E^2 - (m_e c^2)^2 \quad \text{(Equation 2-32)}$$

$$= (E_K + m_e c^2)^2 - (m_e c^2)^2$$

$$\approx (Q + m_e c^2)^2 - (m_e c^2)^2 = Q^2 + 2Qm_e c^2$$

The kinetic energy of the ^{32}S is then:

$$E_k = \frac{p^2}{2M} = \frac{(pc)^2}{2Mc^2} = \frac{Q^2 + 2Qm_e c^2}{2M(^{32}S)c^2}$$

$$= \frac{(1.711\,MeV)^2 + 2(1.711\,MeV)(0.511\,MeV)}{2(31.972071\,uc^2)(931.5\,MeV/uc^2)}$$

$$= 7.85\times 10^{-5}\,MeV = 78.5\,eV$$

(c) As noted above, the momenta of the electron and ^{32}S are equal in magnitude and opposite in direction.

$$(pc)^2 = Q^2 + 2Qm_e c^2 = (1.711\,MeV)^2 + 2(1.711\,MeV)(0.511\,MeV)$$

$$p = [(1.711\,MeV)^2 + 2(1.711\,MeV)(0.511\,MeV)]^{1/2}/c$$

$$= 2.16\,MeV/c$$

13-5. (a) A single photon cannot conserve both energy and momentum.

(b) To conserve momentum each photon must have equal and opposite momenta so that the total momentum is zero. Thus, they have equal energies, each equal to the rest energy of a proton: $E_\gamma = m_p c^2 = 938.28\,MeV$

(c) $E_\gamma = h\nu = hc/\lambda \quad \therefore \quad \lambda = \dfrac{hc}{E_\gamma} = \dfrac{1240\,MeV\cdot fm}{938.28\,MeV} = 1.32\,fm$

(d) $\nu = \dfrac{c}{\lambda} = \dfrac{3.00\times 10^8\,m/s}{1.32\times 10^{-15}\,m} = 2.27\times 10^{23}\,Hz$

13-6. (a) Conservation of charge: $+1+1 \rightarrow +1-1+1-1=0$. Conservation of charge is violated, so the reaction is forbidden.

(b) Conservation of charge: $+1+1 \rightarrow +1-1=0$. Conservation of charge is violated, so the reaction is forbidden.

13-7 From Table 13-6:

(a) $\Lambda(1670)$ 30 MeV

(b) $\Sigma(2030)$ 175MeV

(c) $\Delta(1232)$ 120 MeV

13-8. (a) Weak interaction

(b) Electromagnetic interaction

(c) Strong interaction

(d) Weak interaction

13-9. $\pi^0 \rightarrow \gamma + \gamma$ is caused by the electromagnetic interaction; $\pi^- \rightarrow \mu^- + \bar{\nu}_\mu$ is caused by the weak interaction. The electromagnetic interaction is the faster and stronger, so the π^0 will decay more quickly; the π^- will live longer.

13-10. (a) Electromagnetic interaction

(b) Weak interaction

(c) Electromagnetic interaction

(d) Weak interaction

(e) Strong interaction

(f) Weak interaction

13-11. For neutrino mass $m = 0$, travel time to Earth is $t = d/c$, where $d = 170{,}000\ c \cdot y$. For neutrinos with mass $m \neq 0$, $t' = d/v = d/\beta c$, where $\beta = v/c$.

(Problem 13-11 continued)

$$\Delta t = t' - t = \frac{d}{c}\left(\frac{1}{\beta} - 1\right) = \frac{d}{c}\left(\frac{1-\beta}{\beta}\right)$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}} \quad \text{(Equation 1-21)}$$

$$\gamma^2 = \frac{1}{1-\beta^2} = \frac{1}{(1-\beta)(1+\beta)}$$

$$1 - \beta = \frac{1}{\gamma^2(1+\beta)} \approx \frac{1}{2\gamma^2} \quad \text{since } \beta \approx 1$$

Substituting into Δt,

$$\Delta t \approx \frac{d}{c}\left(\frac{1}{2\gamma^2}\right) \qquad E = \gamma mc^2 \rightarrow \gamma^2 = (E/mc^2)^2 \quad \text{(Equation 2-10)}$$

$$\Delta t \approx \frac{d}{2c}\left(\frac{mc^2}{E}\right)^2$$

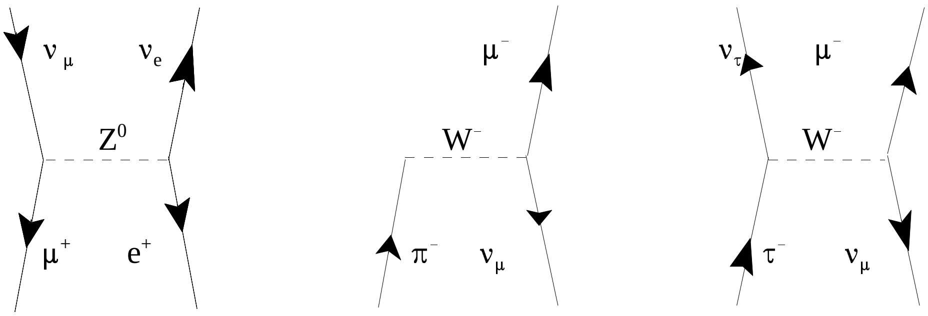
$$mc^2 = \left(\frac{(\Delta t)2cE^2}{d}\right)^{1/2} = \left(\frac{2\Delta t E^2}{d/c}\right)^{1/2}$$

$$= \left[\frac{2(12.5\,s)(10\times10^6\,eV)^2}{(170{,}000\,c\cdot y/c)(3.16\times10^7\,s/y)}\right]^{1/2} = 21.6\,eV$$

$$m \approx 22\,eV/c^2$$

13-12. The Σ^+ and Σ^- are members of an isospin multiplet, two charge states of the Σ hadron. Their mass difference is due to electromagnetic effects. The π^+ and π^- are a particle-antiparticle pair.

13-13. (a)



13-14. (a) $m_p c^2 < (m_n + m_e) c^2$ Conservation of energy and lepton number are violated.

(b) $m_n c^2 < (m_p + m_\pi) c^2$ Conservation of energy is violated.

(c) Total momentum in the center of mass system is zero, so two photons (minimum) must be emitted. Conservation of linear momentum is violated.

(d) No conservation laws are violated. This reaction, $p \cdot \bar{p}$ annihilation, occurs.

(e) Lepton number before interaction is +1; that after interaction is −1. Conservation of lepton number is violated.

(f) Baryon number is +1 before the decay; after the decay the baryon number is zero. Conservation of baryon number is violated.

13-15. (a) The strangeness of each of the particles is given in Table 13-4.

$\Delta S = +1$ The reaction can occur via the weak interaction.

(b) $\Delta S = -2$ This reaction is not allowed.

(c) $\Delta S = +1$ The reaction can occur via the weak interaction.

13-16. (a) The strangeness of each of the particles is given in Table 13-4.

$\Delta S = +2$ The reaction is not allowed.

(b) $\Delta S = +1$ This reaction can occur via the weak interaction.

(c) $\Delta S = 0$ The reaction can occur via either the strong, electromagnetic, or weak interaction.

13-17. (a) $n + n \quad T_3 = -\frac{1}{2} - \frac{1}{2} = -1 \quad T = 1$

(b) $n + p \quad T_3 = -\frac{1}{2} + \frac{1}{2} = 0 \quad T = 1 \; or \; 0$

(c) $\pi^+ + p \quad T_3 = 1 + \frac{1}{2} = \frac{3}{2} \quad T = \frac{3}{2}$

(d) $\pi^- + n \quad T_3 = -1 - \frac{1}{2} = -\frac{3}{2} \quad T = \frac{3}{2}$

(e) $\pi^+ + n \quad T_3 = 1 - \frac{1}{2} = \frac{1}{2} \quad T = \frac{1}{2} \; or \; \frac{3}{2}$

13-18. (a) $\pi^- \rightarrow e^- + \gamma$ Electron lepton number changes from 0 to 1; violates conservation of electron lepton number.

(b) $\pi^0 \rightarrow e^- + e^+ + \nu_e + \bar{\nu}_e$ Allowed by conservation laws, but decay into two photons via electromagnetic interaction is more likely.

(c) $\pi^= \rightarrow e^- + e^+ + \mu^+ + \nu_\mu$ Allowed by conservation laws but decay without the electrons is more likely.

(d) $\Lambda^0 \rightarrow \pi^+ + \pi^-$ Baryon number changes from 1 to 0; violates conservation of baryon number. Also violates conservation of angular momentum, which changes from ½ to 0.

(e) $n \rightarrow p + e^- + \bar{\nu}_e$ Allowed by conservation laws. This is the way the neutron decays.

13-19. (a) $\Omega^- \rightarrow \Lambda^0 + K^- \qquad \Omega^- \rightarrow \Xi^0 + \pi^-$

(b) $\Sigma^+ \rightarrow p + \pi^0 - \qquad \Sigma^+ \rightarrow n + \pi^+$

(c) $\Lambda^0 \rightarrow p + \pi^- \qquad \Lambda^0 \rightarrow n + \pi^0$

(d) $\pi^0 \rightarrow \gamma + \gamma \qquad \pi^0 \rightarrow e^- + e^+ + e^- + e^+$

(e) $K^= \rightarrow \mu^+ + \nu_\mu \qquad K^+ \rightarrow \pi^+ + \pi^0$

13-20. $K^- + p \rightarrow K^0 + K^+ + \Omega^-$

Because Ks have B = 0 and p has B = 1, conservation of B requires that the Ω^- to have B = 1.

$\Omega^- \rightarrow \Xi^0 + \pi^-$

The π^- has B = 0, so conservation of B requires that the Ξ^0 have B = 1.

13-21. (a) $0 + 0 \rightarrow 0 + 0$ S is conserved.

(b) $-2 \rightarrow 0 - 1$ S is not conserved.

(c) $-1 \rightarrow -1 + 0$ S is conserved.

(d) $0 + 0 \rightarrow 0 - 1$ S is not conserved.

(e) $-3 \rightarrow -2 + 0$ S is not conserved.

13-22. Listed below are the baryon number, electric charge, strangeness, and hadron identity of the various quark combinations from Table 13-8 and Figure 13-24.

	Quark Structure	Baryon Number	Electric Charge (e)	Strangeness	Hadron
(a)	uud	+1	+1	0	p
(b)	udd	+1	0	0	n
(c)	uuu	+1	+2	0	Δ^{++}
(d)	uss	+1	0	−2	Ξ^{0}
(e)	dss	+1	−1	−2	Ξ^{-}
(f)	suu	+1	+1	−1	Σ^{+}
(g)	sdd	+1	−1	−1	Σ^{-}

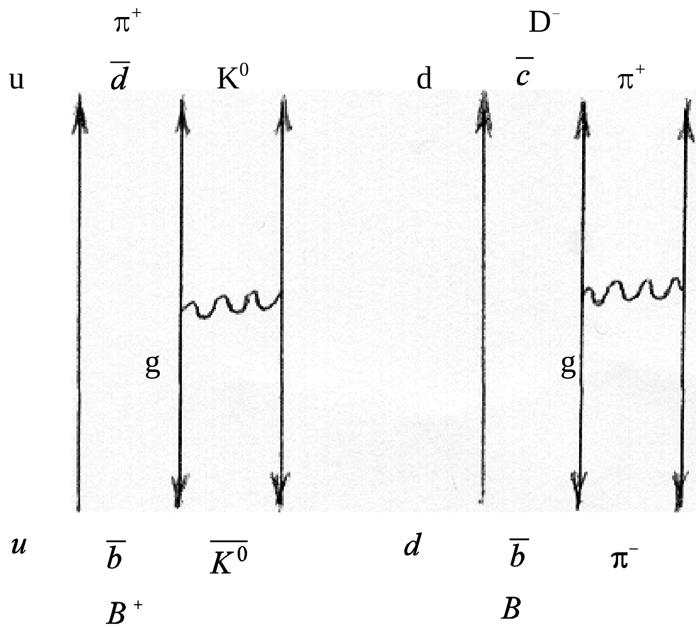
Note that 3-quark combinations are baryons.

13-23. Listed below are the baryon numbers, electric charge, strangeness, and hadron identity of the various quark combinations from Table 13-9 and Figure 13-24.

	Quark Structure	Baryon Number	Electric Charge (e)	Strangeness	Hadron
(a)	$u\bar{d}$	0	+1	0	π^{+}
(b)	$\bar{u}d$	0	−1	0	π^{-}
(c)	$u\bar{s}$	0	+2	+1	K^{+}
(d)	$s\bar{s}$	0	0	+1	0
(e)	$\bar{d}s$	0	0	−1	K^{*0}

* forms η and η$'$ along with $u\bar{u}$ and $d\bar{d}$

13-24.



13-25. (a) $T_3 = 0$ (from Figure 13-22a)

(b) $T = 1$ or 0 just as for ordinary spin.

(c) uds $\quad B = 1/3 + 1/3 + 1/3 = 1 \qquad C = 2/3 - 1/3 - 1/3 = 0$

$S = 0 + 0 + -1 = -1$ The $T = 1$ state is the Σ^0. The $T = 0$ state is the Λ^0.

13-26. The +2 charge can result from either a *uuu*, *ccc*, or *ttt* quark configuration. Of these, only the *uuu* structure also has zero strangeness, charm, topness, and bottomness. (From Table 13-7.)

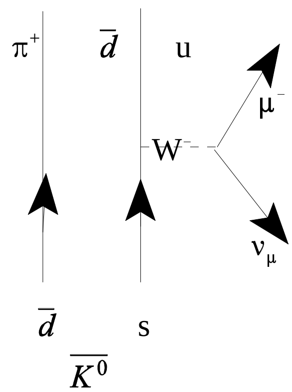
13-27. The range R is $R = \hbar c/mc^2$ (Equation 11-50). Substituting the mass of the W^+ (from Table 13-2),

$$R = \frac{(1.055\times10^{-34}J{\cdot}s)(3.00\times10^{8}m/s)}{(81\,GeV/c^2)(1.60\times10^{-10}J/GeV)} = 2.44\times10^{-18}m = 2.44\times10^{-3}fm$$

13-28.

$\overline{K^0} \rightarrow \pi^+ + \mu^- + \bar{\nu}_\mu$

$(\bar{d}s) \quad (\bar{d}u)$

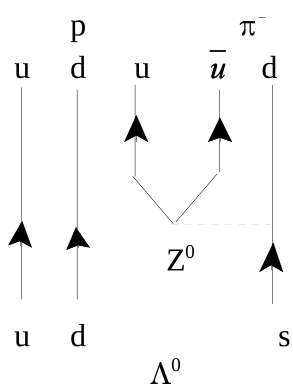


13-29.

$\Lambda^0 \rightarrow \quad p \quad + \quad \pi^-$

$(uds) \quad (uud) \quad (\bar{u}d)$

Weak decay



13-30. $n \rightarrow p + \pi^-$

$$Q = m_n c^2 - m_p c^2 - m_\pi c^2$$

$$= (939.6 - 938.3 - 139.6)\,MeV$$

$$= -138.3\ MeV$$

This decay does not conserve energy.

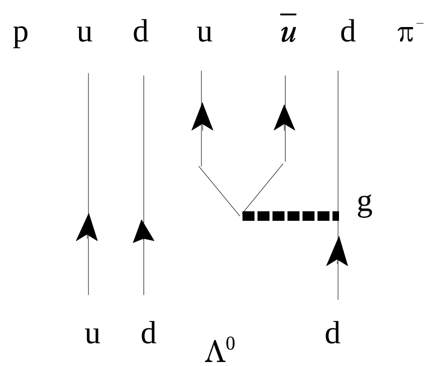
13-31.

$\Lambda^0 \rightarrow \quad p \quad + \quad \pi^-$

$(udd) \quad (uud) \quad (\bar{u}d)$

Strong decay

(See Figure 13-24e for Λ^0)



13-32. (a) The K^+ has charge +1, B = 0, and S = +1 from Table 13-4. It is a meson (quark–antiquark) structure. $u\bar{s}$ produces the correct set of quantum numbers. (From Table 13-7.)

(b) The K^0 has charge = 0, B = 0, and S = +1 from Table 13-4. The quark-antiquark structure to produce these quantum numbers is $d\bar{s}$. (From Table 13-7.)

13-33. (a) Being a meson, the D^+ is constructed of a quark-antiquark pair. The only combination with $charge = +1$, $charm = +1$, and $strangeness = 0$ is the $c\bar{d}$. (See Table 13-7.)

(b) The D^-, antiparticle of the D^+, has the quark structure $\bar{c}d$.

13-34. The Σ^0 decays via the elctromagnetic interaction whose characteristic time is $\sim 10^{-20}$ s. The Σ^+ and Σ^- both decay via the weak interaction. The difference between these two being due to their slightly different masses.

13-35. If the proton is unstable, it must decay to less massive particles, i.e., leptons. But leptons have B = 0, so $p \rightarrow e^+ + \nu_e$ would have $1 = 0 + 0 = 0$ and B is not conserved. The lepton numbers would not be conserved either; a "leptoquark" number would be conserved.

13-36. $V(H_2O) = 0.75\Delta V = 0.75(4\pi R^2 \Delta R)$, where R(Earth) = $6.37\times 10^6 m$ and

$\Delta R = 1\,km = 10^3 m$.

$$V(H_2O) = 0.75[4\pi(6.37\times 10^6 m)^2(10^3)] = 3.82\times 10^{17} m^3$$

$$M(H_2O) = V(H_2O)\rho = (3.82\times 10^{17} m^3)(1000\,kg/m^3) = 3.82\times 10^{20} kg$$

Number of moles (H_2O) = $3.82\times 10^{23} g / 18.02\,g/mole = 2.12\times 10^{22}\,moles$

Number of H_2O molecules = $N_A \times \#\text{ of moles}$

$$= (6.02\times 10^{23}\,molecules/mole)(2.12\times 10^{22}\,moles)$$

$$= 1.28\times 10^{46}\text{ molecules } H_2O$$

(Problem 13-36 continued)

Each molecule contains 10 protons (i.e., 2 in H atoms and 8 in the oxygen atom), so the number of protons in the world's oceans is $N = 1.28 \times 10^{47}$.

The decay rate is $\left|\frac{dN}{dt}\right| = \lambda N$ where $\lambda = 1/\tau = 1/10^{32}y = 10^{-32}y^{-1}$

$$= (10^{-32}y^{-1})(1.28 \times 10^{47}\, protons)$$

$$= 1.28 \times 10^{15}\, proton\ \ decays/y \approx 4 \times 10^{7}\, decays/s$$

13-37. (a) $\rho \rightarrow e^{+} + \Lambda^{0} + \nu_{e}$

$$Q = (m_p c^2 - M(\Lambda^0)c^2 - m_e c^2)MeV$$

$$= (938.3 - 1116 - 0.511)MeV = -178\,MeV$$

Energy is not conserved.

(b) $p \rightarrow \pi^{+} + \gamma$

Spin (angular momentum) $\frac{1}{2} \rightarrow 0 + 1 = 1$. Angular momentum is not conserved.

(c) $p \rightarrow \pi^{+} + K^{0}$

Spin (angular momentum) $\frac{1}{2} \rightarrow 0 + 0 = 0$. Angular momentum is not conserved.

13-38. (a) n, B = 1, Q = 0, spin = 1/2, S = 0

Quark structure	u	d	d	
B	1/3	+ 1/3	+ 1/3	=1
Q	2/3	−1/3	−1/3	=0
spin	1/2↑	1/2↑	1/2↓	= 1/2
S	0	+0	+0	=0

(Problem 13-38 continued)

$\bar{n}$, B = −1, Q = 0, spin = 1/2, S = 0

Quark structure	$\bar{u}$	$\bar{d}$	$\bar{d}$	
B	−1/3	−1/3	−1/3	= −1
Q	−2/3	+ 1/3	+ 1/3	0
spin	1/2↑	1/2↑	1/2↓	= 1/2
S	0	+0	+0	0

(b) Ξ^0, B = 1, Q = 0, spin = 1/2, S = −2

Quark structure	u	s	s	
B	1/3	+ 1/3	+ 1/3	= 1
Q	2/3	−1/3	−1/3	=0
spin	1/2↑	1/2↓	1/2↑	= 1/2
S	0	−1	−1	= −2

(c) Σ^+, B = 1, Q = 1, spin = 1/2, S = −1

Quark structure	u	u	s	
B	1/3	+ 1/3	+ 1/3	= 1
Q	2/3	2/3	−1/3	=0
spin	1/2↑	1/2↓	1/2↑	= 1/2
S	0	+0	−1	= −1

(d) Ω^-, B = 1, Q =−1, spin = 3/2, S = −3

Quark structure	s	s	s	
B	1/3	+ 1/3	+ 1/3	= 1
Q	−1/3	−1/3	−1/3	= −1
spin	1/2↑	1/2↑	1/2↑	= 3/2
S	−1	−1	−1	= −3

(Problem 13-38 continued)

(e) Ξ^-, B = 1, Q = −1, spin = 1/2, S = −2

Quark structure	u	d	d	
B	1/3	+ 1/3	+ 1/3	= 1
Q	−1/3	−1/3	−1/3	= −1
spin	1/2↑	1/2↓	1/2↑	= 1/2
S	0	−1	−1	= −2

13-39. (a)

Quark structure	d	d	d	
B	1/3	+ 1/3	+ 1/3	= 1
Q	−1/3	−1/3	−1/3	= −3/2
spin	1/2	1/2	1/2	= 3/2, 1/2
S	0	+0	+0	= 0

(b)

Quark structure	u	$\bar{c}$	
B	1/3	−1/3	=0
Q	2/3	−2/3	=0
spin	1/2	1/2	= 1 , 0
S	0	+0	=0

(c)

Quark structure	u	$\bar{b}$	
B	1/3	−1/3	=0
Q	2/3	+ 1/3	=1
spin	1/2	1/2	= 1 , 0
S	0	+0	=0

(Problem 13-39 continued)

(d)

Quark structure	$\bar{s}$	$\bar{s}$	$\bar{s}$	
B	−1/3	−1/3	−1/3	= −1
Q	1/3	+ 1/3	+ 1/3	= 1
spin	1/2	1/2	1/2	= 3/2, 1/2
S	1	+1	+1	= 3

13-40. The Z^0 has spin 1. Two identical spin −0 particles cannot have total spin 1.

13-41. (a) The final products (p, γ, e^-, neutrinos) are all stable.

(b) $\Xi^0 \rightarrow p + e^- + \bar{\nu}_e + \bar{\nu}_\mu + \nu_\mu$

(c) Conservation of charge: $0 \rightarrow +1 - 1 + 0 + 0 + 0 = 0$

Conservation of baryon number: $1 \rightarrow 1 + 0 + 0 + 0 + 0 = 1$

Conservation of lepton number:

(i) for electrons: $0 \rightarrow 0 + 1 - 1 + 0 + 0 = 0$

(ii) for muons: $0 \rightarrow 0 + 0 + 0 - 1 + 1 = 0$

Conservation of strangeness: $-2 \rightarrow 0 + 0 + 0 + 0 + 0 = 0$

Even though the chain has $\Delta S = +2$, no individual reaction in the chain exceeds $\Delta S = +1$, so they can proceed via the weak interaction.

(d) No, because energy is not conserved.

13-42. (a) $\mu^+ \rightarrow e^+ + \nu_e + \bar{\nu}_\mu$ (See Table 13-3.)

Electron lepton number: $0 = -1 + 1 + 0 = 0$

Muon lepton number: $-1 = 0 + 0 - 1 = -1$

Tau lepton number: $0 = 0 + 0 + 0 = 0$

(b) $\tau^- = \mu^- + \bar{\nu}_\mu + \nu_\tau$

Electron lepton number: $0 = 0 + 0 + 0 = 0$

Muon lepton number: $0 = 1 - 1 + 0 = 0$

(Problem 13-42 continued)

Tau lepton number: $1 = 0 + 0 + 1 = 1$

(c) $n \rightarrow p + e^{-} + \bar{\nu}_e$

Electron lepton number: $0 = 0 + 1 - 1 = 0$

Muon lepton number: $0 = 0 + 0\ 0 = 0$

Tau lepton number: $0 = 0 + 0 + 0 = 0$

(d) $\pi^{-} = \mu^{-} + \bar{\nu}_\mu$

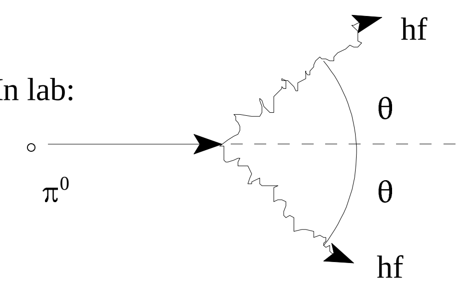
Electron lepton number: $0 = 0 + 0 = 0$

Muon lepton number: $0 = 1 - 1 = 0$

Tau lepton number: $0 = 0 + 0 = 0$

13-43. $\pi^0 \rightarrow \gamma + \gamma$

In lab:



$$E^2 = (pc)^2 + (m_\pi c^2)^2$$

Conservation of momentum requires that each carry half of the initial momentum, hence the total energy: $2(hf/c)\cos\theta = p \quad hf = E/2 = [(pc)^2 + (m_\pi c^2)^2]^{1/2}/2$

$$\cos\theta = \frac{p}{2(hf/c)} = \frac{p}{2[(pc)^2 + (m_\pi c^2)^2]^{1/2}/2c}$$

$$= \frac{pc}{[(pc)^2 + (m_\pi c^2)^2]^{1/2}} = \frac{850\,MeV}{[(850\,MeV)^2 + (135\,MeV)^2]^{1/2}} = 0.9876$$

$$\theta = \cos^{-1}0.9876 = 9.02°$$

13-44. (a) $\Lambda^0 \rightarrow p + \pi^{-}$

Energy: $1116\,MeV - (938 + 140)\,MeV = 38\,MeV$ conserved.

Electric charge: $0 \rightarrow +1 - 1 = 0$ conserved.

Baryon number: $1 \rightarrow 1 + 0 = 1$ conserved.

Lepton number: $0 \rightarrow 0 + 0 = 0$ conserved

(Problem 13-44 continued)

(b) $\Sigma^- \rightarrow n + p^-$

Energy: $1197\,MeV - (940 + 938)\,MeV = -681\,MeV$ not conserved.

Electric charge: $-1 \rightarrow 0 - 1 = -1$ conserved.

Baryon number: $1 \rightarrow 1 - 1 = 0$ not conserved.

Lepton number: $0 \rightarrow 0 + 0 = 0$ conserved.

This reaction is not allowed (energy and baryon conservation violated.)

(c) $\mu^- = e^- + \bar{\nu}_e + \nu_\mu$

Energy: $105.6\,MeV - 0.511\,MeV = 105.1\,MeV$ conserved.

Electric charge: $-1 \rightarrow -1 + 0 + 0 = -1$ conserved.

Baryon number: $0 \rightarrow 0 + 0 + 0 = 0$ conserved.

Lepton number:

(i) electrons: $0 \rightarrow 1 - 1 + 0 = 0$ conserved.

(ii) muons: $1 \rightarrow 0 + 0 + 1 = 1$ conserved.

13-45. (a) The decay products in the chain are not all stable. In particular, the neutron decays via $n \rightarrow p + e^- + \bar{\nu}_e$

(b) The net effect of the chain reaction is:

$$\Omega^- \rightarrow p + 3e^- + e^+ + 3\bar{\nu}_e + 2\bar{\nu}_\mu + 2\nu_\mu$$

(c) Charge: $-1 \rightarrow +1 - 3 + 1 = -1$ conserved

Baryon number: $1 \rightarrow 1 + 0 + 0 + 0 + 0 + 0 + 0 = 1$ conserved

Lepton number:

(i) electrons: $0 \rightarrow 0 + 3 - 1 - 3 + 1 + 0 + 0 = 0$ conserved

(ii) muons: $0 \rightarrow 0 + 0 + 0 + 0 + 0 - 2 + 2 = 0$ conserved

Strangeness: $-3 \rightarrow 0 + 0 + 0 + 0 + 0 + 0 + 0 = 0$ not conserved

Overall reaction has $\Delta S = +3$; however, none of the individual reactions exceeds $\Delta S = +1$, so they can proceed via the weak interaction.

13-46. The proton and electron are free particles. The quarks are confined, however, and cannot be separated. The gluon clouds give the *u* and *d* effective masses of about 330 MeV/c², about 1/3 of the photon's mass.

13-47. (a) $\Lambda^0 \rightarrow p + \pi^-$

$$E_{kin} = [M(\Lambda^0) - m_p - m_\pi]c^2$$

$$= [1116\,MeV/c^2 - 938.3\,MeV/c^2 - 139.6\,MeV/c^2]c^2$$

$$= 38.1\,MeV$$

(b) Because the Λ^0 decayed at rest, the *p* and π^- have momenta of equal magnitudes and opposite direction.

$$m_p v_p = m_\pi v_\pi \rightarrow m_p/m_\pi = v_\pi/v_p$$

$$\frac{E_{kin}(\pi)}{E_{kin}(p)} = \frac{\frac{1}{2}m_\pi v_\pi^2}{\frac{1}{2}m_p v_p^2} = \frac{m_\pi}{m_p}\left(\frac{m_p}{m_\pi}\right)^2 = \frac{m_p}{m_\pi} = \frac{938.3}{139.6} = 6.72$$

(c) $E_{kin} = E_{kin}(p) + E_{kin}(\pi) = E_{kin}(p) + 6.72E_{kin}(p) = 7.72E_{kin}(p) = 38.1\,MeV$

$$E_{kin}(p) = 38.1\,MeV/7.72 = 4.94\,MeV$$

$$E_{kin}(\pi) = 6.72E_{kin}(p) = 33.2\,MeV$$

13.48. $\Sigma^0 \rightarrow \Lambda^0 + \gamma$

(a) E_T for decay products is the rest energy of the Σ^0, 1193 MeV.

(b) The rest energy of $\Lambda^0 = 1116\,MeV$, so $E_\gamma = 1193\,MeV - 1116\,MeV = 77\,MeV$

and $p_\gamma = E_\gamma/c = 77\,MeV/c$

(c) The Σ^0 decays at rest, so the momentum of the Λ^0 equals in magnitude that of the photon.

$$E_{kin}(\Lambda^0) = p_\Lambda^2/2M(\Lambda) = (77\,MeV/c)^2/[2(1116\,MeV/c^2)]$$

$$= 2.66\,MeV \quad \text{small compared to } E_\gamma$$

(Problem 13-48 continued)

(d) A better estimate of E_γ and p_γ are then: $E_\gamma = 77\,MeV - 2.66\,MeV = 74.3\,MeV$ and

$p_\gamma = \; = 74.3\,MeV/c$

13-49. (a) $\Delta t = t_2 - t_1 = \dfrac{x}{u_2} - \dfrac{x}{u_1} = \dfrac{x(u_1 - u_2)}{u_1 u_2}$ Note that $u_1 u_2 \approx c^2$

$$\Delta t \approx \frac{x(u_1 - u_2)}{c^2} = \frac{x\Delta u}{c^2}$$

(b) $E = \dfrac{mc^2}{\sqrt{1 - u^2/c^2}}$ (Equation 2-10). Thus, $\dfrac{u}{c} = \left[\dfrac{1 - (m_o c^2)^2}{E^2}\right]^{1/2} \approx 1 - \dfrac{1}{2}\left(\dfrac{m_o c^2}{E}\right)^2$

(c)
$$u_1 - u_2 = c\left[1 - \frac{1}{2}\left(\frac{m_o c^2}{E_1}\right)^2 - 1 + \frac{1}{2}\left(\frac{m_o c^2}{E}\right)^2\right]$$

$$= \frac{c}{2}\left(\frac{m_o c^2}{E_2}\right)^2 - \frac{c}{2}\left(\frac{m_o c^2}{E_1}\right)^2 = \frac{c(m_o c^2)^2}{2}\left[\frac{E_1^2 - E_2^2}{E_1^2 E_2^2}\right]$$

$$= \frac{c(20\,eV)^2}{2}\left[\frac{(20\times 10^6\,eV)^2 - (5\times 10^6\,eV)^2}{(20\times 10^6\,eV)^2(5\times 10^6\,eV)^2}\right]$$

$$= \frac{c(20\,eV)^2}{2}\left[\frac{(20)^2 - (5)^2}{(20)^2(5)^2(10^6\,eV)^2}\right] = 7.5\times 10^{-12}\,c$$

$$\Delta T \approx \frac{x\,\Delta u}{c^2} = \frac{(170{,}000\,c\cdot y)(7.50\times 10^{-12}\,C)}{c^2} = 1.28\times 10^{-6}\,y = 40.3\,s$$

(d) If the neutrino rest energy is 40 eV, then $\Delta u = 3.00\times 10^{-11}\,c$ and $\Delta t \approx 161\,s$. The difference in arrival times can thus be used to set an upper limit on the neutrino's mass.

13-50. $\tau^- \rightarrow e^- + \bar{\nu}_e + \nu_\tau$

$\tau^- \rightarrow \mu^- + \bar{\nu}_\mu + \nu_\tau$

$\tau^- \rightarrow d + \bar{u} + \nu_\tau$

The last decay is the most probable (three times as likely compared to each of the others) due to the three possible quark colors.