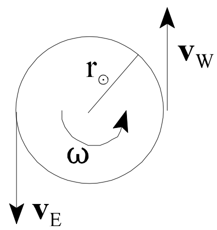
Chapter 14 – Astrophysics and Cosmology

Chapter 14 of Modern Physics 4/e is available through the Freeman Physics Web site at www.whfreeman.com/modphysics4e.

14-1.



$|\boldsymbol{v}_W - \boldsymbol{v}_E| = 4$ km/s. Assuming Sun's rotation to be uniform, so that $\boldsymbol{v}_W = -\boldsymbol{v}_E$, then $|\boldsymbol{v}_W| = |\boldsymbol{v}_E| = 2$ km/s.

Because v = 2πr/T, $v_E = 2\pi r_\odot / T$ or

$$T = \frac{2\pi r_\odot}{v_E} = \frac{2\pi(6.96\times10^5\ km)}{2\ km/s} = 2.19\times10^6\ s = 25.3\ days$$

14-2. $$|U| = 2GM_\odot^2/R_\odot = \frac{2(6.67\times10^{-11})(1.99\times10^{30})^2}{6.96\times10^8} J = 7.59\times10^{41}\ J$$

The Sun's luminosity $L_\odot = 3.85\times10^{26}\ W$

$$\therefore\ t_L = \frac{|U|}{L_\odot} = \frac{7.59\times10^{41}\ J}{3.85\times10^{26}\ J/s} = 1.97\times10^{15}\ s = 6.26\times10^7\ years$$

14-3. The fusion of ^{1}H to ^{4}He proceeds via the proton-proton cycle. The binding energy of ^{4}He is so high that the binding energy of two ^{4}He nuclei exceeds that of ^{8}Be produced in the fusion reaction: $^4\text{He} + {}^4\text{He} \rightarrow {}^8\text{Be} + \gamma$ and the ^{8}Be nucleus fissions quickly to two ^{4}He nuclei via an electromagnetic decay. However, at high pressures and temperatures a very small amount is always present, enough for the fusion reaction: $^8\text{Be} + {}^4\text{He} \rightarrow {}^{12}\text{C} + \gamma$ to proceed. This 3-^{4}He fusion to ^{12}C produces no net ^{8}Be and bypasses both Li and B, so their concentration in the cosmos is low.

14-4. The Sun is 30,000 c·y from Galactic center = radius of orbit

$$\therefore\ time\ for\ 1\ orbit = \frac{2\pi r}{v} = \frac{2\pi(30{,}000\ c{\cdot}y \times 9.45\times10^{15}\ m/c{\cdot}y)}{2.5\times10^5\ m/s} = 7.13\times10^{15}\ s = 2.26\times10^8\ yr$$

$$Age\ of\ Sun \approx 10^{10}\ yr,\ \therefore\ \#\ of\ orbits = \frac{10^{10}\ yr}{2.26\times10^8\ y/orbit} \approx 44$$

14-5. Observed mass (average) ≈ 1 H atom/m^3 = 1.67 × 10^{-27} kg/m^3 = 10% of total mass

$$\therefore\ missing\ mass = 9\times1.67\times10^{-27}kg/m^3 = 1.50\times10^{-26}kg/m^3$$

500 ν/cm^3 = 500 × 10^6 ν/m^3,

so the mass of each ν would be = $\dfrac{1.50\times10^{-26}kg/m^3}{500\times10^6\,\nu/m^3} = 3.01\times10^{-35}kg$

or $m_\nu = \dfrac{3.01\times10^{-35}kg}{1.60\times10^{-19}J/eV}\times c^2\left(\dfrac{m^2}{s^2}\right) = 16.9\,eV$

14-6. $1\,c\cdot s:\quad c\times1\,s = 3.00\times10^8 m/s\times1\,s = 3.00\times10^8 m = 3.00\times10^5 km$

$1\,c\cdot\min:\quad c\times1\min\times60\,s/\min = 3.00\times10^5 km\times60\,s = 1.80\times10^7 km$

$1\,c\cdot h:\quad c\times1\,h\times3600\,s/h = 1.08\times10^9 km$

$1\,c\cdot day:\quad c\times24\,h\times3600\,s/h = 2.59\times10^{10} km$

14-7. (a) See Fig. 14-14. 1 AU = 1.496 × 10^{11} m. R = 1 pc when θ = 1", so $R = \dfrac{1\,AU}{1''}$ or

$$R = \frac{1\,AU}{1''}\times\frac{3600''}{1^\circ}\times\frac{180^\circ}{\pi rad} = 3.086\times10^{16}m = 1\,pc$$

$$1\,pc = \frac{3.086\times10^{16}m}{9.45\times10^{15}m/c{\cdot}y} = 3.26\,c{\cdot}y$$

(b) When θ = 0.01", R = 100 pc and the volume of a sphere with that radius is

$V = \dfrac{4}{3}\pi R^3 = 4.19\times10^6 pc^3$. If the density of stars is 0.08/pc^3, then the number of stars

in the sphere is equal to $0.08/pc^3\times4.19\times10^6 pc^3 = 3.4\times10^5$ stars.

14-8. $L = 4\pi r^2 f \qquad m_1 - m_2 = 2.5\log_{(}f_1/f_{2)}$

Thus, $L_p = 4\pi r_p^2 f_p\ and\ L_B = 4\pi r_B^2 f_B\ and\ L_P = L_B\ \therefore\ r_p^2 f_p = f_B^2 f_B \ \rightarrow\ r_B^2 = r_p^2(f_p/f_B)$

(Problem 14-8 continued)

$$\log(f_p/f_B) = \frac{1.16 - 0.41}{2.5} = 0.30 \;\rightarrow\; f_p/f_B = 2.00$$

Because $r_p = 12\,pc, r_B = r_p(f_p/f_B)^{1/2} = 12\sqrt{2} = 17.0\,pc$

14-9. (a) $M = 0.3\,M_\odot \quad T_e = 3300\,K \quad L = 5\times10^{-2}L_\odot = 1.93\times10^{25}\,W$

(b) $M = 3.0\,M_\odot \quad T_e = 13{,}500\,K \quad L = 10^2 L_\odot = 3.85\times10^{28}\,W$

(c) R ~ M ➜ R = αM ➜ $\alpha = R_\odot/M_\odot$

$$R_{0.3} = \alpha(0.3\,M_\odot) = \frac{R_\odot}{M_\odot}(0.3\,M_\odot) = 0.3\,R_\odot = 2.09\times10^8\,m \quad \text{Similarly, } R_{3.0} = 3.0\,R_\odot = 2.09\times10^9\,m$$

$$t_L \sim M^{-3} \rightarrow t_L = \beta M^{-3} \rightarrow \beta = t_{L\odot}/M_\odot^{-3} = t_{L\odot}M_\odot^3$$

(d)

$$t_L(0.3) = \beta(0.3\,M_\odot)^{-3} = t_{L\odot}M_\odot^3(0.3\,M_\odot)^{-3} = (0.3)^{-3}\,t_{L\odot}$$

$$or \quad t_L(0.3) = 37\,t_{L\odot}. \quad \text{Similarly, } t_L(3.0) = 0.04\,t_{L\odot}$$

14-10. Angular separation $\theta = \frac{S}{R} = \frac{\text{distance between binaries}}{\text{distance Earth}}$

$$\theta = \frac{100\times10^6\,km}{100\,c\cdot y} = \frac{10^{11}m}{100\,c\cdot y(3.15\times10^7\,s/y)} = 1.057\times10^{-7}\,\text{rad}$$

$$\theta = 6.06\times10^{-6}\,\text{degrees} = 1.68\times10^{-9}\,\text{arcseconds}$$

14-11. Equation 14-18: ${}^{56}_{26}Fe \rightarrow 13\,{}^{4}_{2}He + 4n.\; m_{56_{Fe}} = 55.939395\,u,\; m_{4_{He}} = 4.002603\,u,$

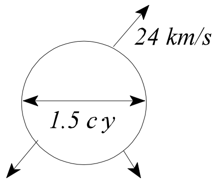
$m_n = 1.008665\,u$. Energy required: $13\,m_{4_{He}} + 4\,m_n - m_{56_{Fe}} = 0.129104\,u.$

$1\,u = 931.49432\,MeV/c^2 \;\therefore 0.129104\,u \rightarrow 120\,MeV$

Equation 14-19: ${}^{4}_{2}He \rightarrow 2\,{}^{1}H + 2n \qquad m_{1_H} = 1.007825$

Energy required: $2m_{1_H} + 2m_n - m_{4_{He}} = 0.030377\,u = 28.3\,MeV$

14-12.



(a) $r = \dfrac{1.5\ c \cdot y}{2}$; assuming constant expansion rate,

Age of shell = $\dfrac{1.5\ c \cdot y/2}{2.4 \times 10^4 m/s} = 2.95 \times 10^{11}\ s = 9400 y$

(b) $L_{star} = 12 L_{\odot} \qquad T_{e\ star} = 1.4\ T_{e\odot}$

$$R \propto M \ \rightarrow\ R = \alpha M \qquad T_e \propto M^{1/2} \ \rightarrow\ T_e = \beta M^{1/2} \qquad L \propto M^4 \ \rightarrow\ L = \gamma M^4$$

$$\therefore\ \ \alpha = R_{\odot}/M_{\odot}, \quad \beta = T_{e\odot}/M_{\odot}^{1/2}, \quad \gamma = L_{\odot}/M_{\odot}^4$$

$$R_{star} = \frac{R_{\odot}}{M_{\odot}} m_{star}, \quad T_{e\ star} = \frac{T_{e\odot}}{M_{\odot}^{1/2}} M_{star}^{1/2}, \quad L_{star} = \frac{L_{\odot}}{M_{\odot}^4} M_{star}^4$$

Using either the T_e or L relations, $R_{star} = \dfrac{M_{star}}{M_{\odot}} R_{\odot} = \left(\dfrac{T_{e\ star}}{T_{e\odot}}\right)^2 R_{\odot} = (1.4)^2 R_{\odot} = 1.96\ R_{\odot}$

or $R_{star} = \left(\dfrac{L_{star}}{L_{\odot}}\right)^{1/2} = 1.86 R_{\odot}$

14-13. $R_S = 2GM/c^2$ (Equation 14-24)

(a) Sun $R_S = 2 \times 6.67 \times 10^{-11} \times 1.99 \times 10^{30} / c^2 = 2.9 \times 10^3\ m \approx 3 km$

(b) Jupiter $(m_J = 318 m_E) \qquad R_S = 2.8\ m$

(c) Earth $R_S = 8.86 \times 10^{-3}\ m$ (≈ 9 mm!)

14-14. $M = 2M_{\odot}$

(a) (Equation 14-22) $R = 1.6 \times 10^{14} M^{-1/3} = 1.6 \times 10^{14} (2M_{\odot})^{-1/3} = 1.01 \times 10^4\ m$

(b) $0.5 rev/s = \pi rad/s = \omega$

$$K = \frac{1}{2} I \omega^2 \ \ where\ for\ a\ sphere$$

$$I = \frac{2}{5} MR^2 = \frac{1}{2}\left(\frac{2}{5} \times 2M_{\odot} \times (1.01 \times 10^4)^2\right) = 8.0 \times 10^{38} J$$

(Problem 14-14 continued)

(c) $dK = I\omega d\omega = I\omega^2\left(\frac{d\omega}{\omega}\right)$ *where* $\frac{d\omega}{\omega} = \frac{1}{10^8}/\,day$

$$= 2K\left(\frac{d\omega}{\omega}\right) = \frac{(2)(8.0\times10^{38}\,J)}{(10^8\,d)(8.64\times10^5\,s/d)} = 1.85\times10^{25}\,J/s \rightarrow L = 1.85\times10^{25}\,W$$

14-15. Milky Way contains $\approx 10^{11}$ stars of average mass $M_\odot$, therefore the visible mass $= 1.99\times10^{30}\times10^{11} = 1.99\times10^{41}\,kg \approx 10\%$ *of total*

(a) Mass of a central black hole $= 9\times1.99\times10^{41} = 1.8\times10^{42}\,kg$

(b) Its radius would be $R_S = 2GM/c^2$ (Equation 14-24).

$$R_S = 2\times6.67\times10^{-11}\times1.8\times10^{42}/c^2 = 2.6\times10^{15}\,m \approx 17{,}000\,AU$$

14-16. v = 72,000 km/s. (a) $v = Hr \rightarrow r = \frac{v}{H} = \frac{72{,}000\,km/s}{20\,km/s/10^6\,c{\cdot}y} = 3.60\times10^9\,c{\cdot}y$

(b) From Equation 14-28 the maximum age of the galaxy is: $1/H = 4.74\times10^{17}\,s = 1.5\times10^{10}\,y$

$$1/H = r/v \rightarrow \Delta(1/H) = \Delta r/v \quad \therefore \quad \frac{\Delta(1/H)}{(1/H)} = \frac{\Delta r}{r} = 10\%$$

so the maximum age will also be in error by 10%.

14-17. The process that generated the increase could propagate across the core at a maximum rate of *c*, thus the core can be at most

$1.5\,y\times3.15\times10^7\,s/y\times3.0\times10^8\,m/s = 1.42\times10^{16}\,m = 9.5\times10^4\,AU$ in diameter. The Milky Way diameter is $\approx 60{,}000\,c{\cdot}y = 3.8\times10^9\,AU$.

14-18. Equation 14-30: $\rho_c = \dfrac{3H^2}{8\pi G} = \dfrac{3}{8\pi(1/H)^2 G}$

$$\rho_c = \frac{3}{8\pi\left(1.5\times10^{10}y\times3.15\times10^{7}s/y\right)^2\left(6.67\times10^{-11}Nm^2/kg^2\right)} = 8.02\times10^{-27}\,kg/m^3$$

(This is about 5 hydrogen atoms/m^3 !)

14-19. Present size $\approx 10^{10}c{\cdot}y = S_p \approx \dfrac{1}{T} \Rightarrow S_p = \alpha\dfrac{1}{T}$ *with* $T = 2.7K \quad \therefore\ \alpha = 2.7\times10^{10}\,c{\cdot}y\,K$

(See Figure 14-25.)

(a) 2000 years ago, $S \approx S_p$ (b) 10^6 years ago $S \approx S_p$

(c) 10 seconds after the Big Bang $S \approx \left(2.7\times10^{10}\,c{\cdot}y\,K\right)/\left(10^9\,K\right)2.7\times10^{-9}S_p \approx 25\,c{\cdot}y$

(d) 1 second after the Big Bang $S \approx \left(2.7\times10^{10}\,c{\cdot}y\,K\right)/\left(5\times10^9\,K\right)5.4\times10^{-10}S_p \approx 5\,c{\cdot}y$

(e) 10^{-6} seconds after the Big Bang

$$S \approx \left(2.7\times10^{10}\,c{\cdot}y\,K\right)/\left(5\times10^{12}\,K\right)5.4\times10^{-13}S_p \approx 0.005\,c{\cdot}y \approx 6.4\times10^4\,AU$$

14-20.

$$\rho(Planck\ time) = \frac{m_{pl}}{\ell_{pl}^3} = \frac{5.5\times10^{-8}kg}{\left(10^{-35}\right)^3 m^3} = 5.5\times10^{97}\,kg/m^3$$

$$\rho(proton) = \frac{1.67\times10^{-27}kg}{\left(10^{-15}\right)^3 m^3} = 1.67\times10^{18}\,kg/m^3$$

$$\rho(osmium) = 2.45\times10^4\,kg/m^3$$

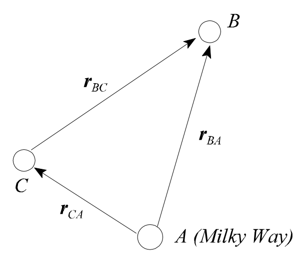
14-21. Wien's law (Equation 3-11): $\lambda_{max} = \dfrac{2.898\,mm\cdot K}{T} = \dfrac{2.898\,mm\cdot K}{2.728\,K} = 1.062\,mm$

(this is in the microwave region of the EM spectrum)

14-22. Muon rest energy = 208 m_e = 106 MeV/c^2. The universe cooled to this energy (average) at about 10^{-3} s (see Figure 14-25). 2.728K corresponds to average energy ≈ 10^{-3} eV. Therefore,

$$m = \frac{10^{-3}\,eV \times 1.6\times 10^{-19}\,J/eV}{c^2} = 1.8\times 10^{-39}\,kg$$

14-23.



If Hubble's law applies in A, then $\boldsymbol{v}_{BA} = H\boldsymbol{r}_{BA}$, $\boldsymbol{v}_{CA} = H\boldsymbol{r}_{CA}$. From mechanics, $\boldsymbol{v}_{BC} = \boldsymbol{v}_{BA} - \boldsymbol{v}_{CA} = H(\boldsymbol{r}_{BA} - \boldsymbol{r}_{CA}) = H\boldsymbol{r}_{BC}$ and

Hubble's law applies in C, as well, and by extension in all other galaxies.

14-24. (a) H available for fusion $= M_\odot \times 0.75\times 0.13\,s$

$$= 2.0\times 10^{30}\,kg \times 0.75 \times 0.13$$

$$= 2.0\times 10^{29}\,kg$$

(b) Lifetime of H fuel $= \dfrac{2.0\times 10^{29}\,kg}{6.00\times 10^{11}\,kg/s} = 3.3\times 10^{17}\,s$

$$= 3.3\times 10^{17}\,s / 3.15\times 10^{7}\,s/y = 1.03\times 10^{10}\,y$$

(c) Start being concerned in $1.03\times 10^{10}\,y - 0.46\times 10^{10}\,y = 5.7\times 10^{9}\,y$

14-25. SN1987A is in the Large Magellanic cloud, which is 170,000 c·y away; therefore (a) supernova occurred 170,000 years BP.

(b) $$E = K + m_o c^2 = \frac{m_o c^2}{\sqrt{1 - \dfrac{v^2}{c^2}}}$$

(Problem 14-25 continued)

K = 10^9 eV, $m_o^2 = 9.38\times10^8\,eV$ ➜ $10^9 + 9.38\times10^8$

$$= \frac{9.38\times10^8}{\sqrt{1-\frac{v^2}{c^2}}} \quad or \quad v = 0.875\,c$$

Therefore, the distance protons have traveled in $170{,}000y = v\times170{,}000y = 149{,}000\,c{\cdot}y$. No, they are not here yet.

14-26. $M_\odot = 1.99\times10^{30}\,kg$. (a) When first formed, mass of H = 0.7 $M_\odot$,

$m(^1H) = 1.007825\,u\times1.66\times10^{-27}\,kg/u$, thus

$$number\ of\ H\ atoms = \frac{0.7\times M_\odot}{1.007825\,u\times1.66\times10^{-27}\,kg/u} = 8.33\times10^{56}$$

(b) If all H → He; $4\,^1H \rightarrow\ ^4He + 26.72\,eV$. The number of He atoms produced = .

$$\frac{8.33\times10^{56}}{4}$$

Total energy produced = $\dfrac{8.33\times10^{56}}{4}\times26.72\,MeV = 5.56\times10^{57}\,MeV = 8.89\times10^{44}\,J$

(c) 23% of max possible = $0.23\times8.89\times10^{44}\,J$

$$t_L = \frac{0.23\times8.89\times10^{44}}{L_\odot} = 5.53\times10^{17}\,s = 1.7\times10^{10}\,y \qquad (L_\odot = 3.85\times10^{26}\,W)$$

14-27. (a) $F = Gm_1m_2/r^2 = a_cm_2 = (v^2/r)m_2$

$v^2/r = Gm_1/r^2$ and orbital frequency $f = v/2\pi r$

Substituting for f and noting that the period $T = 1/f$, $4\pi^2f^2 = Gm_1/r^3$

or, $T^2 = 4\pi^2r^3/Gm_1$ which is Kepler's third law.

(Problem 14-27 continued)

(b) Rearranging Kepler's third law in part (a),

$$m_E = 4\pi^2 r_{\text{moon}}^3 / GT^2$$

$$m_E = \frac{4\pi^2 (3.84\times 10^8 m)^3}{(6.67\times 10^{-11} Nm^2/kg^2)(27.3\,d \times 8.64\times 10^4 s/d)^2}$$

$$m_E = 6.02\times 10^{24} kg$$

(c) $$T = 2\pi\left[\frac{r_{sh}^3}{Gm_E}\right]^{1/2} = 2\pi\left[\frac{(6.67\times 10^6)^3}{(6.67\times 10^{-11})(6.02\times 10^{24})}\right]^{1/2}$$

$$T = 5.44\times 10^3 s = 1.5\,h$$

(d) $$m_{\text{comb}} = \frac{4\pi^2 (1.97\times 10^7)^3}{(6.67\times 10^{-11})(6.46\,d \times 3.1\times 10^4 s/d)^2} = 1.48\times 10^{22}\,kg$$

14-28. (a) $T = 12d \;\rightarrow\; \omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{12\times 24\times 3600} = 6.06\times 10^{-6}/s$

(b) for $m_1 > m_2$: reduced mass $\mu = \dfrac{m_1 m_2}{m_1 + m_2}$, then $\mu\dfrac{v^2}{r} = G\dfrac{m_1 m_2}{r^2}$ and

$$\frac{m_1 m_2}{m_1 + m_2}\left(\frac{\omega^2 r^2}{r}\right) = \frac{G\, m_1 m_2}{r^2} \quad or \quad m_1 + m_2 = \frac{\omega^2 r^3}{G}$$

(c) $v_1 = r_1\omega_1$, $v_2 = r_2\omega_2$, *and* $\omega_1 = \omega_2$ from the graph $v_1 = 200\,km/s$ *and* $v_2 = 100\,km/s$

$$\therefore \quad r_1 = \frac{200\times 10^3\,m/s}{6.06\times 10^{-6}/s} = 3.3\times 10^{10}\,m$$ and, similarly, $r_2 = 1.6\times 10^{10}\,m$

$$\therefore \quad r = r_1 = r_2 = 4.9\times 10^{10}\,m$$

(Problem 14-28 continued)

Assuming circular orbits, $\dfrac{m_1 v_1^2}{r_1} = \dfrac{m_2 v_2^2}{r_2}$ *and* $m_1 = \dfrac{r_1 v_2^2}{r_2 v_1^2} m_2$ Substituting yields,

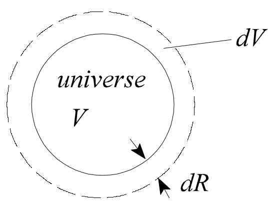
$m_1 = 6.63\times 10^{30}\ kg$ *and* $m_2 = 1.37\times 10^{31}\ kg$

14-29. $E = \dfrac{1}{2}mv^2 + (-GmM_\odot/r) \qquad F_G = GM_\odot m/r^2 = mv^2/r$

$$or \quad GM_\odot m/r = mv^2 \Rightarrow \frac{1}{2}mv^2 = \frac{1}{2}GM_\odot m/r$$

$$\therefore \quad E = \frac{1}{2}\frac{GM_\odot m}{r} + \left(-\frac{GM_\odot m}{r}\right) = \frac{1}{2}\left(-\frac{GM_\odot m}{r}\right)$$

14-30.



$H = \dfrac{20\ km/s}{10^6\ c{\cdot}y}$ Current average density = 1 H atom/m^3

$$V = \frac{4}{3}\pi R^3 \rightarrow dV = 4\pi R^2 dR$$

The current expansion rate at R is

$$v = HR = \frac{20\ km/s}{10^6\ c{\cdot}y}\times 10^{10}\ c{\cdot}y = 20\times 10^4\ km/s = 20\times 10^7\ m/s$$

$$\therefore \quad dR = 20\times 10^7\ m/s \times 3.16\times 10^7\ s/y \times \frac{10^6\ y}{10^6\ y}$$

$$dV = 4\pi R^2 dR = 4\pi\times(10^{10})^2(9.45\times 10^{15}\ m/c{\cdot}y)^2 \times 20\times 10^7\ m/s \times 3.16\times 10^7\ s/y \times \frac{10^6\ y}{10^6\ y}$$

$$= \frac{7.07\times 10^{74}\ m^3}{10^6\ c{\cdot}y} = \frac{\#\ of\ H\ atoms}{10^6\ c{\cdot}y}\ to\ be\ added$$

Current volume $V = \dfrac{4}{3}\pi(10^{10})^3 = 8.4\times 10^{77}\ m^3$

(Problem 14-30 continued)

$$\therefore \text{“}new\text{” } H \text{ } atoms = \frac{7.07\times10^{74}\ atoms/10^6\ c{\cdot}y}{8.4\times10^{77}\ m^3} \approx 0.001 \text{ “}new\text{” } H \text{ } atoms/m^3\cdot10^6\ c{\cdot}y\text{; no}$$

14-31. (a) Equation 8-12: $v_{rms} = \sqrt{3RT/M}$ is used to compute v_{rms} vs T for each gas ® = gas constant.

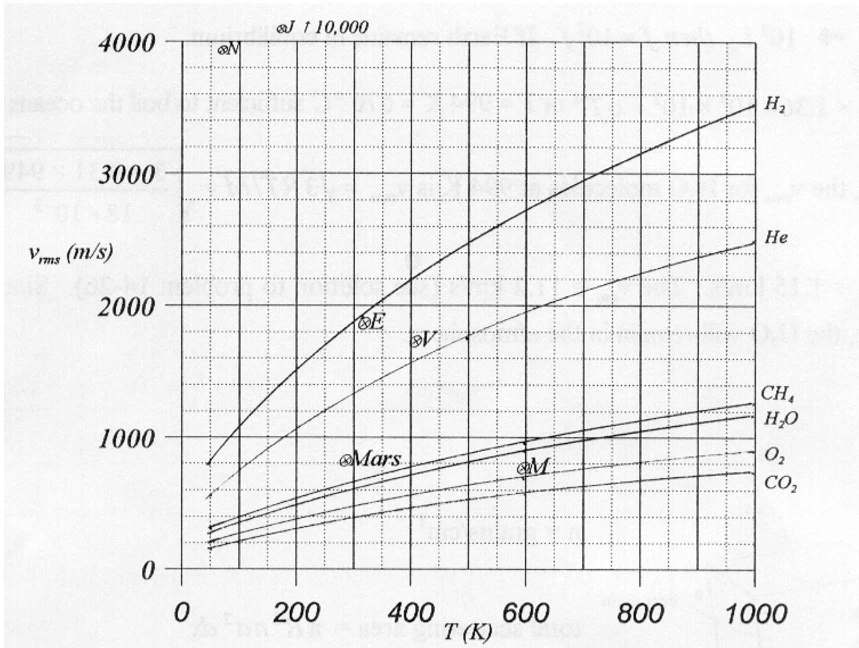
Gas	M (x10^{-3} kg)	$\sqrt{3R/M}$	v_{rms} (m/s) at T = :				
			50K	200K	500K	750K	1000K
H_2O	18	37.2	263	526	832	1020	1180
CO_2	44	23.8	168	337	532	652	753
O_2	32	27.9	197	395	624	764	883
CH_4	16	39.5	279	558	883	1080	1250
H_2	2	111.6	789	1580	2500	3060	3530
He	4	78.9	558	1120	1770	2160	2500

The escape velocity $v_{esc} = \sqrt{2gR} = \sqrt{2GM/R}$, where the planet masses M and radii R are given in Table 14-4.

Planet	Earth	Venus	Mercury	Jupiter	Neptune	Mars
v_{esc} (km/s)	11.2	10.3	4.5	60.2	23.4	5.1
$v_{esc}/6$ (m/s)	1870	1720	750	10,000	3900	850

On the graph of v_{rms} vs. T the $v_{esc}/6$ points are shown for each planet.

(Problem 14-31 continued)



(b) $v_{esc} = \sqrt{2GM/R} \qquad v_{Pl} = \sqrt{2GM_{Pl}/R_{Pl}} \qquad v_E = \sqrt{2GM_E/R_E}$

$$\therefore \frac{v_{Pl}}{v_E} = \sqrt{\frac{M_{Pl}/R_{Pl}}{M_E/R_E}} = \sqrt{\frac{\alpha M_E/\beta R_E}{M_E/R_E}} = \sqrt{\frac{\alpha}{\beta}} \rightarrow v_{Pl} = \sqrt{\frac{\alpha}{\beta}}\, v_E = \sqrt{\frac{M_{Pl}/M_E}{R_{Pl}/R_E}}$$

(c) All six gases will still be in Jupiter's atmosphere and Neptune's atmosphere, because v_{esc} for these is so high. H_2 will be gone from Earth; H_2 and probably He will be gone from Venus; H_2 and He are gone from Mars. Only CO_2 and probably O_2 remain in Mercury's atmosphere.

14-32. (a) α Centauri $d(in\, pc) = \dfrac{\text{Earth's orbit radius (in AU)}}{\sin\theta_p}$

$$d = \frac{1\,AU}{\sin 0.742''} = 2.78\times10^5\, pc = 9.06\times10^5\, c\cdot y$$

(Problem 14-32 continued)

(b) Procyon $d = \dfrac{1\,AU}{\sin 0.286''} = 7.21\times 10^5\,pc = 2.35\times 10^6\,c\cdot y$

14-33. Earth is currently in thermal equilibrium with surface temperature ≈300 K. Assuming Earth radiates as a blackbody $I = \sigma T^4$ and $I = \sigma(300)^4 = 459\ W/m^2$. The solar constant $f = 1.36\times 10^3\ W/m^2$ currently, so Earth absorbs 459/1360 = 0.338 of incident solar energy.

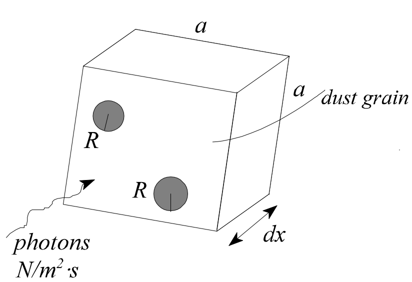
When $L_\odot \rightarrow 10^2 L_\odot$ *then* $f = 10^2 f$. If the Earth remains in equilibrium.

$I = 0.338\times 1.36\times 10^3\times 10^2 = \sigma T^4$ or $T = 994\,K = 676\,°C$ sufficient to boil the oceans away.

However, the v_{rms} for H_2O molecules at 994 K is $v_{rms} = \sqrt{3RT/M} = \sqrt{\dfrac{3\times 8.31\times 949}{18\times 10^{-3}}} =$

1146 m/s = 1.15 km/s. The v_{esc} = 11.2 km/s (see solution to problem 14-26). Because $v_{rms} \approx 0.1\,v_{esc}$, the H_2O will remain in the atmosphere.

14-34. (a)



n = grains/cm^3

total scattering area = $\pi R^2 n a^2\,dx$

which is $\dfrac{\pi R^2 a^2 n\,dx}{a^2} = \pi R^2 n\,dx$ of the total area =

fraction scattered = dN/N

$$\int_{N_o}^{N}\frac{dN}{N} = -n\pi R^2\int_0^d dx \quad or \quad N = N_o e^{-n\pi R^2 d}$$

From those photons that scatter at x = 0 (N_o), those that have <u>not</u> scattered again after traveling some distance x = L is $N_L = N_o e^{-n\pi R^2 L}$. The average value of L (= d_o) is given by

(Problem 14-34 continued)

$$d_o = \frac{\int_0^\infty L \frac{dN_L}{dL} dL}{\int_0^\infty \frac{dN_L}{dL} dL} = \frac{1}{n\pi R^2} \qquad \left(Note: \frac{dN_L}{dL} = -n\pi R^2 N_o e^{-n\pi R^2 L} \right)$$

(b) $I = I_o e^{-d/d_o}$ near the Sun $d_o \approx 3000\,c{\cdot}y \quad R = 10^{-5}\,cm$

$$\therefore \quad 3000\,c{\cdot}y \times 9.45\times10^{17}\,cm/c{\cdot}y = \frac{1}{n\pi(10^{-5})^2} \qquad \therefore \quad n = 1.1\times10^{-12}/cm^3$$

(c) $\rho_{grains} = 2\,gm/cm^3$

$$\therefore \quad \frac{m_{grains}}{cm^3 \ of \ space} = 2\times\frac{4}{3}\pi(10^{-5})^3 \times 1.1\times10^{-12}/cm^3 = 9.41\times10^{-27}\,gm/cm^3$$

$$\therefore \quad mass \ in \ 300\,c{\cdot}y = \frac{9.41\times10^{-27}\,gm/cm^3}{M_\odot} \times (9.45\times10^{17}\,cm/c{\cdot}y)^3 \times 300 = 0.0012 \qquad (\approx 0.1\% M_\odot)$$

14-35. $56\,{}^1_1H \rightarrow 14\,{}^{14}_{2}He \rightarrow {}^{56}_{26}Fe + 2\beta^+ (+2e^-) \qquad (2\beta + (+2e^-) = 2.04\,MeV/c^2)$

$$14\,m_{4_{He}} = 14\times4.002603 - m_{56_{Fe}} = 55.939395\,u = \Delta$$
$$= 56.036442\,u$$

Net energy difference (<u>release</u>) = $\begin{cases} 14\times26.72\,MeV \\ 2.04\,MeV \\ 90.40\,MeV \ (=\Delta) \\ \hline 466.5\,MeV \end{cases}$

$$2\,{}^{56}_{26}Fe \rightarrow {}^{112}_{48}Cd + 4\beta^+ (+4e^-) \qquad (4\beta + (4e^-) = 4.08\,MeV/c^2)$$

$$2m({}^{56}Fe) = 2\times55.939395\,u \qquad m({}^{112}Cd) = 111.902762\,u$$

Net energy required = $2m_{56_{Fe}} - m_{112_{Cd}} = -0.023972\,u + 4.08\,MeV = -18.25\,MeV$

14-36. (a) $dt = \dfrac{1.024 \times 10^4 \pi^2 G^2 M\, dM}{hc^4}$

rearranging, the mass rate of change is

$$\frac{dM}{dt} = \frac{hc^4}{(1.024 \times 10^{24})\pi^2 G^2 M}$$

Clearly, the larger the mass *M*, the lower the rate at which the black hole loses mass.

(b) $t = \dfrac{(1.024 \times 10^4)\,\pi^2\,(6.62 \times 10^{-11})^2\,(2.0 \times 10^{30})^2}{hc^4}$

$$t = 3.35 \times 10^{44}\, s = 1.06 \times 10^{37}\, y$$

far larger than the present age of the universe.

(c)

$$t = \frac{(1.024 \times 10^4)\,\pi^4\,(6.67 \times 10^{-11})^2\,(2.0 \times 10^{30} \times 10^{12})^2}{6.63 \times 10^{-34}(3.00 \times 10^8)^4}$$

$$t = 3.35 \times 10^{68}\, s = 1.06 \times 10^{61}\, y$$