Chapter 1 – Relativity I

1-1. Once airborne, the plane's motion is relative to still air. In 10 min the air mass has moved $18\, m/s \times 60\, s/\text{min} \times 10\,\text{min} = 10.8\ km$ toward the east. The north and up coordinates relative to the ground (and perpendicular to the wind direction) are unaffected. The 25 km point has moved 10.8 km east and is, after 10 min, at $25 - 10.8 = 14.2\ km$ west of where the plane left the ground (0, 0, 0) after 10 min the plane is at (14.2 km, 16 km, 0.5 km).

1-2. (a) $$t = \frac{2L}{c} = \frac{2\left(2.74\times10^4\ m\right)}{3.00\times10^8\ m/s} = 1.83\times10^{-4}\ s$$

(b) From Equation 1-7 the correction $\delta t \approx \frac{2L}{c}\times\frac{v^2}{c^2}$

$$\delta t = \left(1.83\times10^{-4} s\right)\left(10^{-4}\right)^2 = 1.83\times10^{-12}\ s$$

(c) From experimental measurements $$\frac{\delta c}{c} = \frac{4\ km/s}{299{,}796\ km/s} = 1.3\times10^{-5}$$

No, the relativistic correction of order 10^{-8} is three orders of magnitude smaller than the experimental uncertainty.

1-3. $$\frac{0.4\ \text{fringe}}{\left(29.8\ km/s\right)^2} = \frac{1.0\ \text{fringe}}{\left(v\ km/s\right)^2} \rightarrow v^2 = \frac{1.0}{0.4}\left(29.8\ km/s\right)^2 = 2.22\times10^3 \rightarrow v = 47.1\ km/s$$

1-4. (a) This is an exact analog of Example 1-3 with L = 12.5 m, c = 130 mph, and v = 20 mph. Calling the plane flying perpendicular to the wind #1 and the one flying

(Problem 1-4 continued)

parallel to the wind #2, #1 will win by Δt where

$$\Delta t = \frac{Lv^2}{c^3} = \frac{(12.5\,mi)(20\,mi/h)^2}{(130\,mi/h)^3} = 0.0023\,h = 8.2\,s$$

(b) Pilot #1 must use a heading $\theta = \sin^{-1}(20/130) = 8.8°$ relative to his course on both legs. Pilot #2 must use a heading of 0° relative to the course on both legs.

1-5. (a) In this case, the situation is analogous to Example 1-3 with $L = 3 \times 10^8\,m$, $v = 3\times 10^4\,m/s$, and $c = 3\times 10^8\,m/s$. If the flash occurs at $t = 0$, the interior is dark until $t = 2$ s at which time a bright circle of light reflected from the circumference of the great circle plane perpendicular to the direction of motion reaches the center, the circle splits in two, one moving toward the front and the other toward the rear, their radii decreasing to just a point when they reach the axis 10^{-8} s after arrival of the first reflected light ring. Then the interior is again dark.

(b) In the frame of the seated observer, the spherical wave expands outward at c in all directions. The interior is dark until t = 2s, at which time the spherical wave (that reflected from the inner surface at t = 1s) returns to the center showing the entire inner surface of the sphere in reflected light, following which the interior is again dark.

1-6. Yes, you will see your image and it will look as it does now. The reason is the second postulate: All observers have the same light speed. In particular, you and the mirror are in the same frame. Light reflects from you to the mirror at speed c relative to you and the mirror and reflects from the mirror back to you also at speed c, independent of your motion.

1-7. $\Delta N = \dfrac{2Lv^2}{\lambda c^2}$ (Equation 1-12) Where $\lambda = 590\,nm$, $L = 11\,m$, and $\Delta N = 0.01\,fringe$

(Problem 1-7 continued)

$$v^2 = \frac{\Delta N \lambda c^2}{2L} = (0.01\,fringe)(590\times10^{-9}\,m)(3.00\times10^{8}\,m/s)^2\;2(11\,m)$$

$$v = 4.91\times10^{3}\,m/s \approx 5\,km/s$$

1-8. (a) No. Result depends on the relative motion of the frames.

(b) No. Results will depend on the speed of the proton relative to the frames. (This answer anticipates a discussion in Chapter 2. If by "mass," the "rest mass" is implied, then the answer is "yes," because that is a fundamental property of protons.)

(c) Yes. This is guaranteed by the 2nd postulate.

(d) No. The result depends on the relative motion of the frames.

(e) No. The result depends on the speeds involved.

(f) Yes. Result is independent of motion.

(g) Yes. The charge is an intrinsic property of the electron, a fundamental constant.

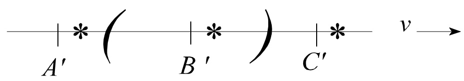
1-9. The wave from the front travels 500 m at speed $c + (150/3.6)\,m/s$ and the wave from the rear travels at $c - (150/3.6)\,m/s$. As seen in Figure 1-15, the travel time is longer for the wave from the rear.

$$\Delta t = t_r - t_f = \frac{500\,m}{3.00\times10^{8}\,m/s - (150/3.6)\,m/s} - \frac{500\,m}{3.00\times10^{8}\,m/s + (150/3.6)\,m/s}$$

$$= 500\left[\frac{3\times10^{8} + (150/3.6) - 3\times10^{8} + (150/3.6)}{(3\times10^{8})^2 - 2(150/3.6)(3\times10^{8}) - (150/3.6)^2}\right]$$

$$\approx 500\,\frac{2(150/3.6)}{(3\times10^{8})^2} \approx 4.63\times10^{-13}\,s$$

1-10.



While the wavefront is expanding to the position shown, the original positions of A', B', and C' have moved to *-marks, according to the observer in S.

(a) According to an S' observer, the wavefronts arrive simultaneously at A' and B'.

(b) According to an S observer, the wavefronts do not arrive at A' and C' simultaneously.

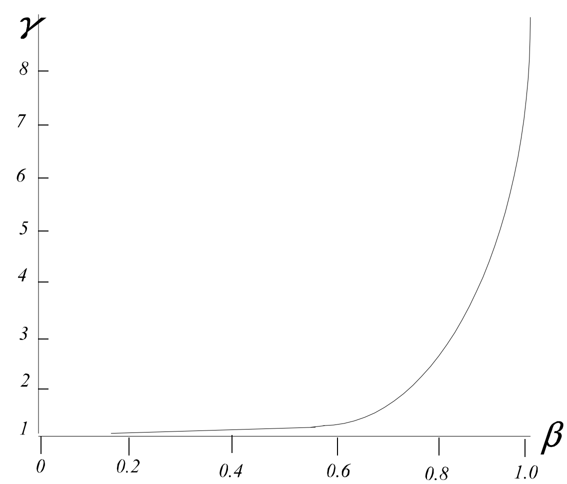
(c) The wavefront arrives at A' first, according to the S observer, an amount Δt before arrival at C', where

$$\Delta t = \frac{B'C'}{c - v} - \frac{B'A'}{c + v} \quad \text{since } B'C' = B'A' = L' \text{, Thus}$$

$$\Delta t = L'\left[\frac{c + v - c + v}{c^2 - v^2}\right] = L'\left[\frac{2v}{c^2 - v^2}\right]$$

1-11.

β	$\gamma = 1/(1-\beta^2)^{1/2}$
0	1
0.2	1.0206
0.4	1.0911
0.6	1.2500
0.8	1.6667
0.85	1.8983
0.90	2.2942
0.925	2.6318
0.950	3.2026
0.975	4.5004
0.985	5.7953
0.990	7.0888
0.995	10.0125



1-12. $t_1 = \gamma\left(t_1' + \frac{vx_o'}{c^2}\right) \qquad t_2 = \gamma\left(t_2' + \frac{vx_o'}{c^2}\right)$ (from Equation 1-21)

(a) $t_2 - t_1 = \gamma\left(t_2' + \frac{vx_o'}{c^2} - t_1' - \frac{vx_o'}{c^2}\right) = \gamma(t_2' - t_1')$

(b) The quantities x_1' and x_2' in Equation 1-21 are each equal to x_o', but x_1 and x_2 in Equation 1-20 are different and unknown.

1-13. (a) $\gamma = 1/(1 - v^2/c^2)^{1/2} = 1/[1 - (0.85c)^2/c^2]^{1/2} = 1.898$

$x' = \gamma(x - vt) = 1.898[75\,m - (0.85c)(2.0\times10^{-5}\,s)] = -9.537\times10^3\,m$

$y' = y = 18\,m$

$z' = z = 4.0\,m$

$t' = \gamma(t - vx/c^2) = 1.898[2.0\times10^{-5}\,s - (0.85c)(75\,m)/c^2] = 3.756\times10^{-5}\,m$

(b) $x = \gamma(x' + vt') = 1.898[-9.537\times10^3 m + (0.85c)(3.756\times10^{-5}s)] = 75.8$

(difference is due to rounding of γ, x', and t'.

$y = y' = 18\,m$

$z = z' = 4.0\,m$

$t = \gamma(t' + vx'/c^2) = 1.898[3.756\times10^{-5}\,s + (0.85c)(-9.537\times10^3\,m)/c^2]$

$= 2.0\times10^{-5}\,s$

1-14. To show that $\Delta t = 0$ (refer to Figure 1-9 and Example 1-3).

$$t_1 = \frac{L}{\sqrt{c^2 - v^2}} + \frac{L}{\sqrt{c^2 - v^2}} = \frac{2L}{\sqrt{c^2 - v^2}} = \frac{2L}{c}\frac{1}{\sqrt{1 - v^2/c^2}}$$

t_2, because length parallel to motion is shortened, is given by:

(Problem 1-14 continued)

$$t_2 = \frac{L\sqrt{1-v^2/c^2}}{c+v} + \frac{L\sqrt{1-v^2/c^2}}{c-v} = \frac{2Lc\sqrt{1-v^2/c^2}}{c^2(1-v^2/c^2)}$$

$$t_2 = \frac{2L}{c}\frac{\sqrt{1-v^2/c^2}}{\left(\sqrt{1-v^2/c^2}\right)^2} = \frac{2L}{c}\frac{1}{\sqrt{1-v^2/c^2}} = t_1$$

Therefore, $t_2 - t_1 = 0$ and no fringe shift is expected.

1-15 (a) Let frame S be the rest frame of Earth and frame S$^/$ be the spaceship moving at speed *v* to the right relative to Earth. The other spaceship moving to the left relative to Earth at speed *u* is the "particle". Then $v = 0.9c$ and $u_x = -0.9c$.

$$u_x^/ = \frac{u_x - v}{1 - u_x v/c^2} \quad \text{(Equation 1-24)}$$

$$= \frac{-0.9c - 0.9c}{1-(-0.9c)(0.9c)/c^2} = \frac{-1.8c}{1.81} = -0.9945\,c$$

(b) Calculating as above with $v = 3.0\times10^4\,m/s = -u_x$

$$u_x^/ = \frac{-3.0\times10^4 m/s - 3.0\times10^4 m/s}{1 - \frac{(-3.0\times10^4 m/s)(3.0\times10^4 m/s)}{(3.0\times10^8 m/s)}} = \frac{-6.0\times10^4 m/s}{1 + 10^{-8}} \approx -6.0\times10^4\,m/s$$

1-16. $a_x^/ = \dfrac{du_x^/}{dt^/}$ where $u_x^/ = \dfrac{u_x - v}{1-u_x v/c^2}$ (Equation 1-24)

And $t^/ = \gamma(t - vx/c^2)$ (Equation 1-20)

$$du_x^/ = (u_x - v)(vd_x/c^2)(1-u_x v/c^2)^{-2} + (1-u_x v/c^2)^{-1}du_x$$

(Problem 1-16 continued)

$$= \frac{\left(\frac{v}{c^2}\right)(u_x - v)du_x + (1 - u_x v/c^2)du_x}{(1 - u_x v/c^2)^2}$$

$$dt' = \gamma(dt - v\,dx/c^2)$$

$$a_x' = \frac{du_x'}{dt'} = \frac{\left(\frac{v}{c^2}\right)(u_x - v)(du_x/dt) + (1 - u_x v/c^2)(du_x/dt)}{\gamma(1 - u_x v/c^2)^3}$$

$$= \frac{(du_x/dt)(1 - v^2/c^2)}{\gamma(1 - u_x v/c^2)^3} = \frac{a_x}{\gamma^3(1 - u_x v/c^2)^3}$$

$$a_y' = \frac{du_y'}{dt'} \text{ where } u_y' = \frac{u_y - v}{1 - u_x v/c^2} \quad \text{(Equation 1-24)}$$

$$du_y' = (du_y/\gamma)(1 - u_x v/c^2)^{-1} + (u_y/\gamma)(1 - u_x c^2/c^2)^{-2}\,du_x$$

$$= \frac{(du_y)(1 - u_x v/c^2) + (u_y v/c^2)\,du_x}{\gamma(1 - u_x v/c^2)^2}$$

$$a_y' = \frac{du_y'}{dt'} = \frac{(du_y/dt)(1 - u_x v/c^2) + (u_y v/c^2)(du_x/dt)}{\gamma(1 - u_x v/c^2)^2\,\gamma(1 - u_x v/c^2)}$$

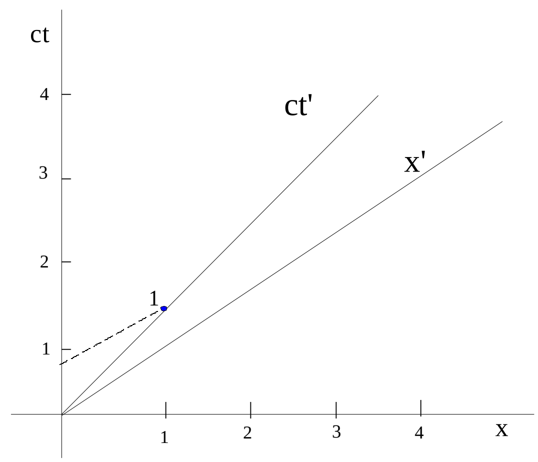
$$= \frac{a_y(1 - u_x v/c^2) + a_x(u_y v/c^2)}{\gamma^2(1 - u_x v/c^2)^3}$$

a_z' is found in the same manner and is given by:

(Problem 1-16 continued)

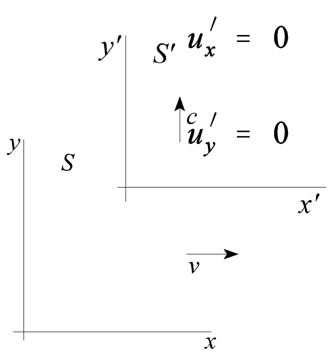
$$a_z' = \frac{a_z\left(1 - u_x v/c^2\right) + a_x\left(u_z v/c^2\right)}{\gamma^2\left(1 - u_x v/c^2\right)^3}$$

1-17. (a) As seen from the diagram, when the observer in the rocket (S') system sees $1\ c\cdot s$ tick by on the rocket's clock, only 0.6 $c\cdot s$ have ticked by on the laboratory clock.



(b) When 10 seconds have passed on the rocket's clock, only 6 seconds have passed on the laboratory clock.

1-18. (a)



(Problem 1-18 continued)

$$u_x = \frac{u_x' + v}{1 + vu_x'/c^2} = \frac{0 + v}{1 + 0} = v \qquad \text{(Equation 1-25)}$$

$$u_y = \frac{u_y'}{\gamma\left(1 + vu_x'/c^2\right)} = \frac{c}{\gamma(1 + 0)} = \frac{c}{\gamma}$$

(b) $u = \sqrt{u_x^2 + u_y^2} = \sqrt{v^2 + c^2/\gamma^2} = \sqrt{v^2 + c^2\left(1 - v^2/c^2\right)} = c$

1-19. By analogy with Equation 1-25,

(a) $u_x' = \dfrac{u_x'' + v}{1 + vu_x''/c^2} = \dfrac{0.9c + 0.9c}{1 + (0.9c)(0.9c)/c^2} = \dfrac{1.8c}{1.81} = 0.9945\,c$

(b)

$$u_x = \frac{u_x' + v}{1 + vu_x'/c^2} = \frac{\left(1.8c/1.81\right) + 0.9c}{1 + \left(1.8c/1.81\right)(0.9c)/c^2} = \frac{1.8 + (0.9)(1.81)}{1.81 + (1.8)(0.9)}c$$

$$= \frac{3.429}{3.430}c = 0.9997\,c$$

1-20. (a) $\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$ (Equation 1 − 18)

$$= \left(1 - v^2/c^2\right)^{-1/2} = 1 + \left(-\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\frac{1}{2!}\left(-\frac{v^2}{c^2}\right)^2 + \cdots$$

$$= 1 + \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^2} + \cdots \approx 1 + \frac{1}{2}\frac{v^2}{c^2}$$

(Problem 1-20 continued)

(b)

$$\frac{1}{\gamma} = \sqrt{1 - v^2/c^2} = (1 - v^2/c^2)^{½}$$

$$= 1 + \left(\frac{1}{2}\right)\left(-\frac{v^2}{c^2}\right) + \left(\frac{1}{2}\right)\left(-\frac{1}{2}\right)\frac{1}{2!}\left(-\frac{v^2}{c^2}\right)^2 + \cdots$$

$$= 1 - \frac{1}{2}\frac{v^2}{c^2} - \frac{1}{8}\frac{v^4}{c^4} + \cdots = 1 - \frac{1}{2}\frac{v^2}{c^2}$$

(c)

$$\gamma - 1 = \frac{1}{2}\frac{v^2}{c^2} + \frac{3}{8}\frac{v^4}{c^4} + \cdots \qquad 1 - \frac{1}{\gamma} = \frac{1}{2}\frac{v^2}{c^2} + \frac{1}{8}\frac{v^4}{c^4} + \cdot\cdot$$

$$\therefore \quad \gamma - 1 \approx 1 - \frac{1}{\gamma} \approx \frac{1}{2}\frac{v^2}{c^2}$$

1-21. $\Delta t = \gamma\,\Delta t' \quad \text{(Equation 1-28)}$

$$\frac{\Delta t - \Delta t'}{\Delta t'} = \frac{\gamma\Delta t' = \Delta t'}{\Delta t'} = \gamma - 1 \approx \frac{1}{2}\frac{v^2}{c^2} = \gamma - 1 \approx \frac{1}{2}\frac{v^2}{c^2}$$

$$v^2 = 2c^2\frac{\Delta t - \Delta t'}{\Delta t'} \qquad v = c\left(2 \times \frac{\Delta t - \Delta t'}{\Delta t'}\right)^{½} = c(2 \times 0.01)^{½} = 0.14\,c$$

1-22. (a) $L = L_p/\gamma \quad \text{(Equation 1-20)}$

$$\frac{L_p - L}{L_p} = \frac{L_p - (1/\gamma)L_p}{L_p} = 1 - \frac{1}{\gamma} \approx \frac{1}{2}\frac{v^2}{c^2} = \frac{1}{2}\frac{(3\times10^{-6})^2}{c^2} = 4.5\times10^{-12} = 4.5\times10^{-10}\,\%$$

(b) $$\Delta t = \Delta t'/\gamma \approx \Delta t'\left[1 - \frac{1}{2}\frac{v^2}{c^2}\right] = 3.16\times10^7\,s\left[1 - \frac{1}{2}(3\times10^{-6})^2\right]$$

$\approx 3.16\times10^7\,s$ elapses on the pilot's clock also. The pilot's clock loses:

(Problem 1-22 continued)

$$\Delta t' - \Delta t = \Delta t'(1 - 1/\gamma) \approx \Delta t' \frac{1}{2}\frac{v^2}{c^2}$$

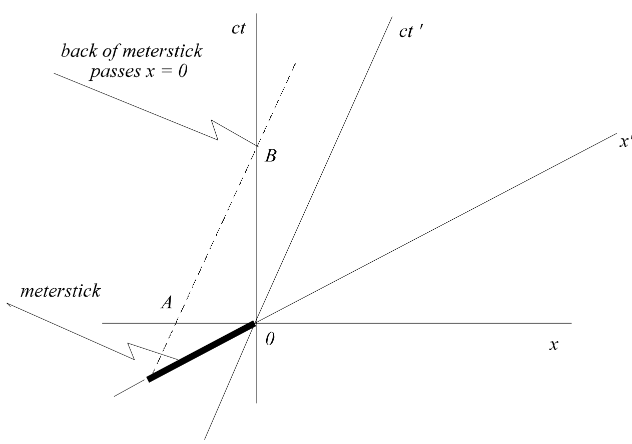
$$= (3.16\times 10^7 s)(1/2)(3\times 10^{-6})^2 = 1.42\times 10^{-4} s\,(1\,\text{min}/60 s) = 2.37\times 10^{-6}\,\text{min}$$

1-23. (a) $L = L_p/\gamma \quad (\text{Equation } 1-20)$

$$= L_p\sqrt{1 - v^2/c^2} = 1.0m[1 - (0.6c)^2/c^2]^{1/2} = 0.80\,m$$

(b) $t = L/v = 0.80m/0.6c = 4.4\times 10^{-9} s$

(c)



The projection $\overline{OA}$ on the *x* axis is L.

The length $\overline{OB}$ on the *ct* axis yields *t*.

1-24. (a)

$$\Delta t = \gamma\,\Delta t' = \frac{\Delta t'}{\sqrt{1 - v^2/c^2}} \quad (\text{Equation } 1-28)$$

$$= \frac{2.6\times 10^{-8} s}{[1-(0.9c)^2/c^2]^{1/2}} = \frac{2.6\times 10^{-8} s}{\sqrt{0.19}} = 5.96\times 10^{-8}\,s$$

(b) $s = v\Delta t = (0.9)(3.0\times 10^8 m/s)(6.0\times 10^{-8} s) = 16.1\,m$

(Problem 1-24 continued)

(c) $s = v\Delta t = (0.9)(3.0\times10^{8}m/s)(2.6\times10^{-8}s) = 7.0\,m$

(d) $(\Delta s)^2 = (c\Delta t)^2 - (\Delta x)^2 \quad (\text{Equation } 1-33)$

$$= [c(6.0\times10^{-8})]^2 - (16.1\,m)^2 = 324 - 259 = 65 \quad \rightarrow \quad \Delta s = 7.8m$$

1-25. From Equation 1-30, $L = L_p/\gamma = L_p\sqrt{1 - v^2/c^2}$

where L = 85m and L_p = 100m

$$\sqrt{1 - v^2/c^2} = L/L_p = 85/100$$

$$\textit{Squaring} \quad 1 - v^2/c^2 = (85/100)^2$$

$$\therefore \quad v^2 = [1 - (85/100)^2]c^2 = 0.2775c^2 \;\; \textit{and} \;\; v = 0.527c = 1.58\times10^{8}m/s$$

1-26. (a) t = distance to Alpha Centauri/spaceship speed = $4c\cdot y/0.75c = (4/0.75)y = 5.33y$

(b) For a passenger on the spaceship, the distance is:

$$L = L_0\sqrt{1-v^2/c^2} = 4c\cdot y\sqrt{1-(0.75)^2} = 4c\cdot y(0.661)/0.75c = 2.65c\cdot y$$

and $t = 2.65c\cdot y/0.75c = 3.53\,y$

1-27. Using Equation 1-30, with L_{A_p} *and* L_{B_p} equal to the proper lengths of A and B and L_A = length of A measured by B and L_B = length of B measured by A.

$$L_A = L_{A_p}/\gamma = 100m\sqrt{1 - (0.92c)^2} = 39.2$$

$$L_{B_p} = \gamma L_B = 36/\sqrt{1 - (0.92c)^2/c^2} = 91.9m$$

1-28. In S': $\Delta x' = 1.0\,m\cos 30° = 0.866\,m$

$$\Delta y' = 1.0\,m\sin 30° = 0.500\,m \qquad \text{Where } \theta' = 30°$$

In S: $\Delta x = \Delta x'\sqrt{1-\beta^2} = 0.866\,m\sqrt{1-(0.8)^2} = 0.520\,m$

$$\Delta y = \Delta y' = 0.500\,m$$

Where $\theta = \tan^{-1}\dfrac{0.500}{0.520} = 43.9°$

$$L = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(0.520\,m)^2 + (0.500)^2} = 0.721\,m$$

1-29. (a) In S': $V' = a' \times b' \times c' = (2m)(2m)(4m) = 16\,m^3$

In S : Both a' and c' have components in the x' direction.

$a'_x = a'\sin 25° = (2m)\sin 25° = 0.84\,m$ and $c'_x = c'\cos 25° = (4m)\cos 25° = 3.63\,m$

$$a_x = a'_x\sqrt{1-\beta^2} = 0.84\sqrt{1-(0.65)^2} = 0.64\,m$$

$$c_x = c'_x\sqrt{1-\beta^2} = 3.634\sqrt{1-(0.65)^2} = 2.76\,m$$

$a_y = a'_y = a'\cos 25° = 2\cos 25° = 1.81\,m$ and $c_y = c'_y = c'\sin 25° = 4\sin 25° = 1.69\,m$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(0.64)^2 + (1.81)^2} = 1.92\,m$$

$$c = \sqrt{c_x^2 + c_y^2} = \sqrt{(2.76)^2 + (1.69)^2} = 3.24\,m$$

b' (in z direction) is unchanged, so $b = b' = 2\,m$

θ (between c and xy-plane) = $\tan^{-1}(1.69/2.76) = 31.5°$

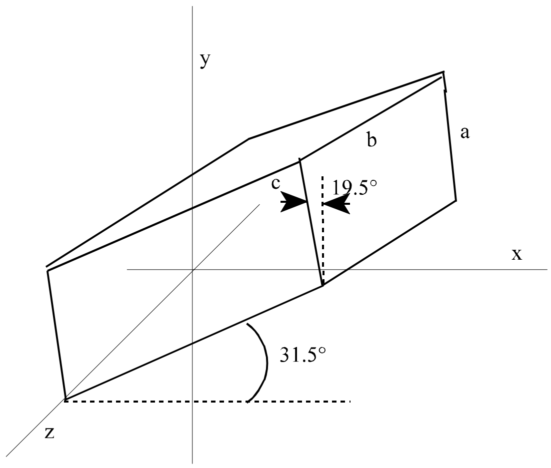
ϕ (between a and yz-plane) = $\tan^{-1}(0.64/1.81) = 19.5°$

V = (area of ay face) · b (see part [b])

$$V = (c \times a\sin 78°) \times b = (3.24\,m)(1.92\,m\sin 78°)(2\,m) = 12.2\,m^3$$

(Problem 1-29 continued)

(b)



1-30. $$\lambda' = \frac{c}{f'} = \frac{c}{\sqrt{\frac{1+\beta}{1-\beta}}} = \sqrt{\frac{1-v/c}{1+v/c}}\lambda_o \quad \text{(Equation 1-38)}$$

$$\left(\frac{\lambda'}{\lambda_o}\right)^2 = \frac{1-v/c}{1+v/c} \rightarrow \left(\frac{\lambda'}{\lambda_o}\right)^2 (1+v/c) = 1-v/c$$

Solving for v/c, $\frac{v}{c}\left[\left(\frac{\lambda'}{\lambda_o}\right)^2 + 1\right] = 1 - \left(\frac{\lambda'}{\lambda_o}\right)^2 \quad \therefore \quad \frac{v}{c} = \frac{1-(\lambda'/\lambda_o)^2}{1+(\lambda'/\lambda_o)^2}$

$\lambda_o = 650\,nm$. For yellow $\lambda' = 590\,nm$. $\frac{v}{c} = \frac{1-(590\,nm/650\,nm)^2}{1+(590\,nm/650\,nm)^2} = 0.097$

Similarly, for green $\lambda' = 525\,nm \rightarrow \frac{v}{c} = 0.210$

and for blue $\lambda' = 460\,nm \rightarrow \frac{v}{c} = 0.333$

1-31.

$$\lambda' = \frac{c}{f'} = \frac{c}{\frac{\sqrt{1-\beta}}{1+\beta} f_o} = \sqrt{\frac{1+v/c}{1-v/c}}\,\lambda_o \quad \text{(Equation 1-39)}$$

$$\frac{\lambda' - \lambda_o}{\lambda_o} = \frac{\lambda'}{\lambda_o} - 1 = \sqrt{\frac{1+v/c}{1-v/c}} - 1 = \left[\frac{1+(1.85\times10^7\,m/s)/(3.00\times10^8\,m/s)}{1-(1.85\times10^7\,m/s)/(3.00\times10^8\,m/s)}\right]^{1/2} - 1$$

$$= 0.064$$

1-32. Because the shift is a blue shift, the star is moving toward Earth.

$$f = \sqrt{\frac{1+\beta}{1-\beta}}\, f_0 \quad \text{where } f = 1.02 f_0$$

$$(1.02)^2 = \frac{1+\beta}{1-\beta} \quad \rightarrow \quad \beta = \frac{(1.02)^2 - 1}{(1.02)^2 + 1} = 0.0198$$

$$v = 0.0198\,c = 5.94\times10^6\ m/s$$

1-33. $f = \sqrt{\frac{1-\beta}{1+\beta}}\, f_0 \quad \rightarrow \quad \lambda = \sqrt{\frac{1-\beta}{1+\beta}}\, \lambda_0 = \sqrt{\frac{1-\beta}{1+\beta}}\,(656.3\ nm)$

For $\beta = 10^{-3}$: $\lambda = (656.3\ nm)\sqrt{\frac{1+10^{-3}}{1-10^{-3}}} = 657.0\ nm$

For $\beta = 10^{-2}$: $\lambda = (656.3\ nm)\sqrt{\frac{1+10^{-2}}{1-10^{-2}}} = 662.9\ nm$

For $\beta = 10^{-1}$: $\lambda = (656.3\ nm)\sqrt{\frac{1+10^{-1}}{1-10^{-1}}} = 725.6\ nm$

1-34. (a) Time to star: $t = d/v = 15c\cdot y/0.999c = 15.015\,y$

Time of visit = 10 y

Time to return to Earth: $t = d/v = 15c\cdot y/0.999c = 15.015\,y$

Total time away = $40.030\,y$

(b) Distance to star: $L = L_0\sqrt{1/\beta^2} = 15c\cdot y\sqrt{1-(0.999)^2} = 0.6707\,c\cdot y$

Time till star "arrives": $t = d/v = 0.6707\,c\cdot y/0.999c = 0.671\,y$

Time of visit = 10 y

Time till Earth "arrives" = 0.671 y

Total time away = 11.34 y

1-35. Distance to moon = $3.85\times10^8\,m = R$

Angular velocity ω needed for $v = c$:

$$\omega = v/R = C/R = (3.00\times10^8\,m/s)/(3.85\times10^8\,m) = 0.78\,rad/s$$

Information could only be transmitted by modulating the beam's frequency or intensity, but the modulation could move along the beam only at speed c, thus arriving at the moon only at that rate.

1-36. (a) Using Equation 1-28 and Problem 1-20(b),

$$\Delta t' = \Delta t/\gamma = \Delta t(1 - v^2/2c^2) = \Delta t - \Delta t v^2/2c^2$$

where $\Delta t = 3.15\times10^7 s/y$ and

$$v = 2\pi R_E/T = (2\pi)(6.37\times10^6 m)/(108\,\text{min})(60 s/\text{min})$$

$$v = 6.177\times10^3 m/s = 2.06\times10^{-5} c$$

Time lost by satellite clock = $\Delta t v^2/c^2$

(Problem 1-36 continued)

$$= (3.15\times10^{7}s)(2.06\times10^{-5})^{2}/2 = 0.00668s = 6.68ms$$

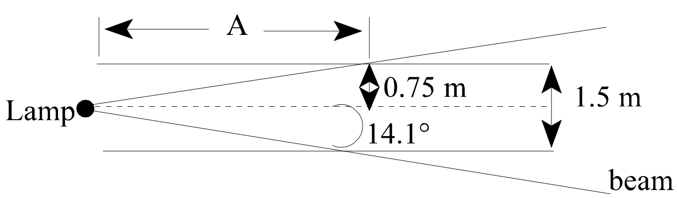
(b) $1s = \Delta t(v^2/2c^2)$

$$\Delta t = 2/(v^2/c^2) = 2/(2.06\times10^{-5})^{2} = 4.71\times10^{9}s = 150y$$

1-37. $\cos\theta = \dfrac{\cos\theta' + \beta}{1+\beta\cos\theta'}$ (Equation 1-43)

where θ' = half-angle of the beam in $S' = 30°$

For $\beta = 0.65$, $\cos\theta = \dfrac{\cos 30° + 0.65}{1 + (0.65)\cos 30°} = 0.97$ or $\theta = 1.41°$



The train is A from you when the headlight disappears, where

$$A = \frac{0.75m}{\tan 14.1°} = 3.0m$$

1-38. (a) $\Delta t = \gamma\Delta t_0$ For the time difference to be 1 s, $\Delta t - \Delta t_0 = 1s$

$$\Delta t - \Delta t/\gamma = 1 \rightarrow \Delta t\left(1-\frac{1}{\gamma}\right) = 1$$

Substituting $\dfrac{1}{\gamma} \approx 1 - \dfrac{1}{2}\dfrac{v^2}{c^2}$ (From Problem 1-20)

$$\Delta t\left(1-1+\frac{1}{2}\frac{v^2}{c^2}\right) \approx 1 \rightarrow \Delta t \approx 2c^2/v^2 = 2\frac{(3.0\times10^{8})^{2}}{(1.5\times10^{6}/3.6\times10^{3})^{2}}$$

$$\approx 1.04\times10^{12}s \approx 32{,}000y$$

(Problem 1-38 continued)

(b) $\Delta t - \Delta t_0 = 273 \times 10^{-9}\,s$. Using the same substitution as in (a),

$\Delta t(1 - 1/\gamma) = 273 \times 10^{-9}$ and the circumference of Earth $C = 40{,}000\,km$, so,

$4.0 \times 10^7 m = v\Delta t$ or $\Delta t = 4.0 \times 10^7 / v$, and

$4.0 \times 10^7 / v \approx (2c^2/v^2)(273 \times 10^{-9})$, or $v = \dfrac{2c^2(273 \times 10^{-9})}{4.0 \times 10^7} = 1230\,m/s$

Where v is the relative speed of the planes flying opposite directions. The speed of each plane was $(1230\,m/s)/2 = 615\,m/s = 2210\,km/h = 1380\,mph$.

1-39. Simplifying the interval to $(ct') - (x')^2$, we substitute the Lorentz transformation:

$t' = \gamma(t - vx/c^2)$ and $x' = \gamma(x - vt)$

$ct' = \gamma(ct - vx/c) \quad \rightarrow \quad (ct')^2 = \gamma^2(ct - vx/c)^2$

$$\begin{aligned}(ct')^2 - (x')^2 &= \gamma^2[(ct - vx/c)^2 - (x - vt)^2] \\ &= \gamma^2[c^2t^2 - 2vxt + v^2x^2/c^2 - x^2 + 2vxt - v^2t^2] \\ &= \gamma^2[(c^2 - v^2)t^2 - (1 - v^2/c^2)x^2] \\ &= (ct)^2 - (x)^2\end{aligned}$$

1-40. (a) Alpha Centauri is 4 $c \cdot$y away, so the traveler went $L = \sqrt{1 - \beta^2}\,(8c\cdot y)$ in 6 y, or

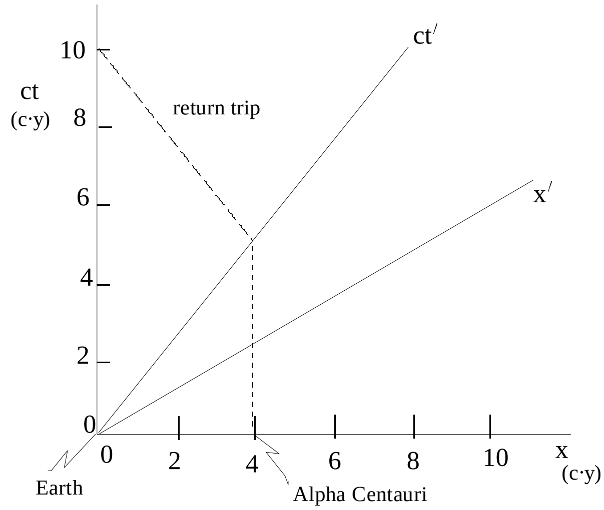
$$\begin{aligned}8c\cdot y\sqrt{1 - v^2/c^2} &= v(6y) \\ \sqrt{1 - v^2/c^2} &= v(6/8c) = (3/4)(v/c) \\ &= 1 - \beta^2 = (3/4)^2\beta^2 \\ (3/4)^2\beta^2 + \beta^2 &= 1 \\ \beta^2 &= 1/(1 + 0.5625) \\ v &= 0.8c\end{aligned}$$

(Problem 1-40 continued)

(b) $\Delta t = \gamma \Delta t_0 = \gamma(6y)$ and $\gamma = 1/\sqrt{1-\beta^2} = 1.667$

$\Delta t = 1.667(6y) = 10y$ or $4y$ older than the other traveler.

(c)



1-41. Orbit circumference $\approx 4.0 \times 10^7\, m$.

Satellite speed $v = 4.0 \times 10^7 m/(90 \min \times 60 s/\min) = 7.41 \times 10^3 m/s$

$$\Delta t - \Delta t_0 = t_{diff}$$

$$\Delta t - \Delta t/\gamma = t_{diff} = \Delta t(1 - 1/\gamma) \approx \Delta t\left(\frac{1}{2}\beta^2\right) \text{ (Problem 1-20)}$$

$$t_{diff} = (3.16 \times 10^7 m/s)(1/2)(7.41 \times 10^3/3.0 \times 10^8)^2$$

$$= 0.0096 s = 9.6 ms$$

1-42. (a) $\Delta t' = \gamma \Delta t - \frac{\gamma v}{c^2} \Delta x$ (Equation 1-22)

For events to be simultaneous in S', $\Delta t' = 0$.

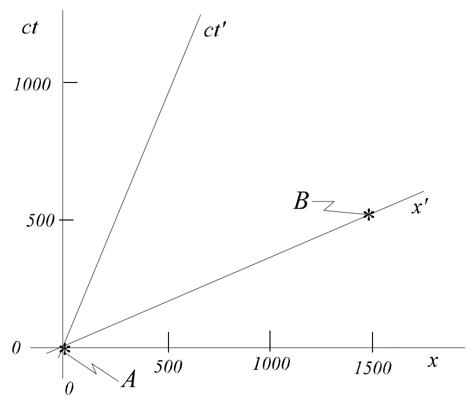
$$\gamma \Delta t = \frac{\gamma v}{c^2} \Delta x \rightarrow 2 \times 10^{-6} s = \frac{v}{c^2}(1.5 \times 10^3 m)$$

$$v = (2 \times 10^{-6} s)(3 \times 10^8 m/s) / 1.5 \times 10^3 m$$

$$= 1.2 \times 10^8 m/s = 0.4c$$

(b) Yes.

(c)



Note: B is on the x' axis, i.e., where $ct' = 0$, as is A. For any x' slope greater than 0.4 the order of B and A is reversed.

(d) $(\Delta s)^2 = (\Delta x)^2 - (c \Delta t)^2$ (Equation 1 – 35)

$$= (1.5 \times 10^3)^2 - [(3 \times 10^8 m/s)(2 \times 10^{-6} s)]^2$$

$$= 2.25 \times 10^6 - 3.6 \times 10^5 = 1.89 \times 10^6 m^2$$

$$\Delta s = 1370 \, m$$

$$L_p = \Delta s = 1370 m$$

1-43. (a) $\gamma = 1/\sqrt{1 - \beta^2} = 1/\sqrt{1 - (0.92)^2} = 2.55$

(b) $\tau = 2.6 \times 10^{-8} s$ $\Delta t(lab) = \gamma \tau = 2.55(2.6 \times 10^{-8} s) = 6.63 \times 10^{-8} s$

(c) $N(t) = N_0 e^{-t/\tau}$ (Equation 1-31) $L = \sqrt{1 - \beta^2} L_0 = \sqrt{1 - (0.92)^2}(50m) = 19.6m$

(Problem 1-43 continued)

Where L is the distance in the pion system. At 0.92c, the time to cover 19.6m is:

$t = 19.6m/0.92c = 7.10\times10^{-8}s$. So for $N_0 = 50{,}000$ pions initially, at the end of 50m in the

lab, $N = (5.0\times10^4)e^{-7.1/2.6} = 3{,}260$

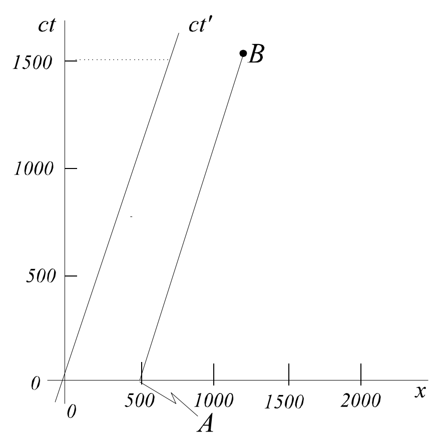
(d) Ignoring relativity, the time required to cover 50m at 0.92c is $1.81\times10^{-7}s$ and N would then be: $N = (5.0\times10^4)e^{-18.1/2.6} = 47$

1-44. $\Delta L = L_p - L = L_p - L_p(1/\gamma) = L_p(1-1/\gamma) = L_p\left(\frac{1}{2}\frac{v^2}{c^2}\right)$ (See Problem 1-20)

For L_p = 11 m and $v = 3\times10^4\ m/s$ $\Delta L = 11(0.5)(10^{-8} = 5.5\times10^{-8}\ m$

$$\text{"Shrinkage"} = \frac{5.5\times10^{-8}m}{10^{-10}m/atomic\ \ diameter} = 550\ \text{atomic diameter}$$

1-45. (a)



(b) Slope of ct' axis = 2.08 = $1/\beta$, so β = 0.48 and $v = 1.44\times10^8\ m/s$

(c) $ct' = \gamma ct$ and $\gamma = 1/\sqrt{1-\beta^2}$ so $ct'\sqrt{1-\beta^2} = ct$

For $ct' = 1000m$ and $\beta = 0.48$ $ct = 877\ m$

(d) $t = \gamma\Delta t' = 1.14\Delta t' \rightarrow \Delta t' = 5\mu s/1.14 = 4.39$

1-46. (a) $L = L_p/\gamma = L_p\sqrt{1 - u^2/c^2} = 100m\sqrt{1-(0.85)^2} = 52.7m$

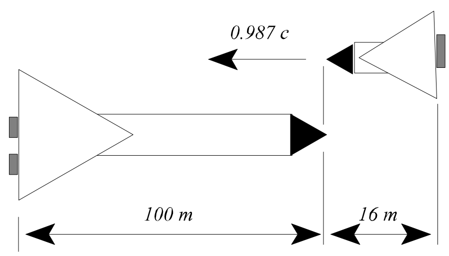
(b) $u' = \dfrac{u + u}{1 + uu/c^2} = \dfrac{0.85c + 0.85c}{1 + (0.85)^2} = \dfrac{1.70c}{1.72} = 0.987c$

(c) $L' = L_p/\gamma' = L_p\sqrt{1 - u'^2/c^2} = 100m\sqrt{1-(1.70/1.72)^2} = 16.1m$

(d) As viewed from Earth, the ships pass in the time required for one ship to move its own contracted length.

$$\Delta t = \frac{L}{u} = \frac{52.7m}{0.85 \times 3.00 \times 10^8 m/s} = 2.1 \times 10^{-7}s$$

(e)



1-47. In Doppler radar, the frequency received at the (approaching) aircraft is shifted by approximately $\Delta f/f_o \approx v/c$. Another frequency shift in the same direction occurs at the receiver, so the total shift $\Delta f/f_o \approx 2v/c$. $v = (c/2)(8\times10^{-7}) = 120m/s$

1-48. $f = \dfrac{f_0}{1-\beta} \times \dfrac{1}{\gamma}$ (Equation 1-37)

$$f = \frac{f_0\sqrt{1-\beta^2}}{1-\beta} = \frac{f_0\sqrt{(1-\beta)(1+\beta)}}{\sqrt{(1-\beta)^2}} = f_0\left(\frac{\sqrt{(1-\beta)(1+\beta)}}{\sqrt{(1-\beta)^2}}\right) \rightarrow f = \sqrt{\frac{1+\beta}{1-\beta}}f_0$$

Which is Equation 1-38.

1-49. $t_2' - t_1' = \gamma(t_2 - t_1) - \dfrac{\gamma v}{c^2}(x_b - x_a)$ (Equation 1-22)

(a) $t_2' - t_1' = 0 \rightarrow t_2 - t_1 = (v/c^2)(x_b - x_a) \rightarrow (0.5 - 1.0)y = (v/c^2)(2.0 - 1.0)\, c\cdot y$

(Problem 1-49 continued)

Thus, $-0.5 = (v/c) \rightarrow v = 0.5c$ in the $-x$ direction.

(b) $t' = \gamma(t - vx/c^2)$

Using the first event to calculate t' (because t' is the same for both events),

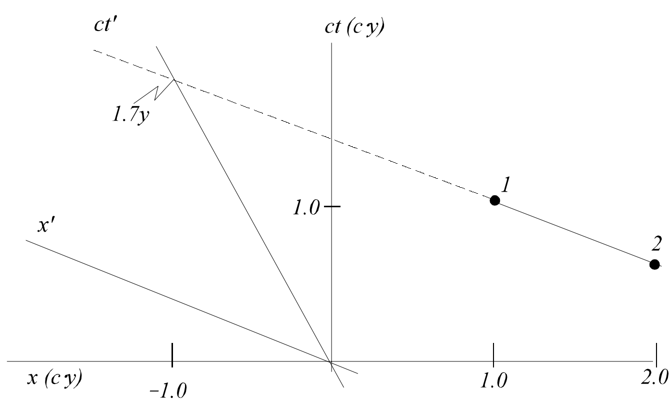
$t = (1/\sqrt{1-(0.5)^2})[1y - (0.5c)(1c\cdot y)/c^2] = 1.155(0.5) = 0.58y$

(c) $(\Delta s)^2 = (\Delta x)^2 - (c\Delta t)^2 = (1c\cdot y)^2 - (0.5c\cdot y)^2 = 0.75(c\cdot y)^2 \rightarrow \Delta s = 0.866c\cdot y$

(d) The interval is spacelike.

(e) $L_p = \Delta s = 0.866c\cdot y$

1-50. (a)



Because events are simultaneous in S', line between 1 and 2 is parallel to x' axis. Its slope is $-0.5 = -\beta$. $v = -0.5c$.

(b) From diagram $t' = 1.7\,y$.

1-51. $x_b' - x_r' = \gamma[(x_b - x_r) - v(t_b - t_r)]$ (1)

$t_b' - t_r' = \gamma[(t_b - t_r) - v(x_b - x_r)/c^2]$ (2)

Where $x_b - x_r = 2400m$ $t_b - t_r = 5\mu s$ $x_b' - x_r' = 2400m$ $t_r' - t_r' = -5\mu s$

Dividing (1) by (2) and inserting the values,

(Problem 1-51 continued)

$$\frac{2400}{-5\times10^{-6}} = \frac{2400 - v(5\times10^{-6})}{5\times10^{-6} - v(2400)/c^2} = -2400 + v(2400)^2/5\times10^{-6}c^2 = 2400 - 5\times10^{-6}v$$

$$v\left[\frac{(2400)^2}{5\times10^{-6}c^2} + 5\times10^{-6}\right] = 4800 \rightarrow v = 2.69\times10^8 m/s$$ in $+x$ direction.

1-52. $u_x = 0.85c\cos50°$ $\quad u_y = 0.85c\sin50°$

$\beta = 0.72 \rightarrow \gamma 1/\sqrt{1-\beta^2} = 1.441 \qquad v = 0.72c$

$$u_x' = \frac{u_x - v}{1 - vu_x/c^2} \qquad u_y' = \frac{u_y}{\gamma(1 - vu_x/c^2)}$$

$$u_x' = \frac{0.85c\cos50° - 0.72c}{1-(0.72c)(0.85c\cos50°/c^2)} = \frac{-0.1736c}{1-0.3934} = -0.286c$$

$$u_y' = \frac{0.85c\sin50°}{1.441[1-(0.72c)(0.85c\cos50°/c^2)]} = 0.745c$$

$$u' = \sqrt{u_x'^2 + u_y'^2} = 0.798c$$

$\tan\theta' = u_y'/u_x' = 0.745/(-0.286)$ $\theta' = 111°$ with respect to the $+x'$ axis.

1-53. This is easier to do in the *xy* and $x'y'$ planes. Let the center of the meterstick, which is parallel to the x axis and moves upward with speed v_y in *S*, at $x = y = x' = y' = 0$ at $t = t' = 0$. The right hand end of the stick, e.g., will not be at $t' = 0$ in S' because the clocks in S' are not synchronized with those in *S*. In S' the components of the sticks velocity are:

$$u_y' = \frac{u_y}{\gamma(1 - vu_x/c^2)} = \frac{v_y}{\gamma}$$ because $u_y = v_y$ and $u_x = 0$

$$u_x' = \frac{u_x - v}{1 - vu_x/c^2} = -v$$ because $u_x = 0$

(Problem 1-53 continued)

When the center of the stick is located as noted above, the right end in S' will be at:

$x' = \gamma(x - vt) = 0.5\gamma$ because $t = 0$. The S' clock there will read:

$t' = \gamma(t - vx/c^2) = -0.5\gamma v/c^2$ because $t = 0$. Therefore, when $t' = 0$ at the center, the right end is at $x'y'$ given by:

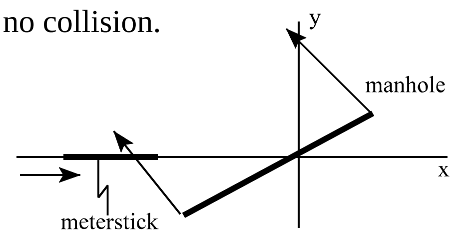
$$x' = 0.5\gamma \qquad y' = u_y' t' = \frac{v_y}{y}\left(\frac{0.5\gamma v}{c^2}\right)$$

$$\text{and } \theta' = \tan^{-1}\frac{y'}{x'} = \frac{v_y}{\gamma}\left(\frac{0.5\gamma v}{c^2}\right)/0.5\gamma = (v_y v/c^2)\sqrt{1-\beta^2}$$

For $\beta = 0.65 \quad \theta' = (0.494 v_y/c)$

1-54. $$\begin{aligned} 0 &= x^2 + y^2 + z^2 - (ct)^2 \\ &= [\gamma(x' + vt')]^2 + y'^2 + z'^2 - [c\gamma(t' + vx'/c^2)]^2 \\ &= x'^2(\gamma^2 - c^2\gamma^2 v^2/c^4 + y'^2 + z'^2 + t'^2(\gamma^2 v^2 - c^2\gamma^2) + x't'(2\gamma^2 v - 2vc^2\gamma^2/c^2) \\ &= x'^2 + y'^2 + z'^2 - (ct')^2 \end{aligned}$$

1-55. The solution to this problem is essentially the same as Problem 1-53, with the manhole taking the place of the meterstick and with the addition of the meterstick moving to the right along the x-axis. Following from Problem 1-53, the manhole is tilted up on the right and so the meterstick passes through it; there is no collision.



1-56. (a) $t_2^{/} = \gamma(t_2 - vx_2/c^2)$ and $t_1^{/} = \gamma(t_1 - vx_1/c^2)$

$$t_2^{/} - t_1^{/} = \gamma[t_2 - t_1 - v(x_2 - x_1)/c^2] = \gamma[T - vD/c^2]$$

(b) For simultaneity in S$^{/}$, $t_2^{/} = t_1^{/}$, or $T - vD/c^2 \rightarrow v/c = cT/D$. Because $v/c < 1, cT/D$ is also <1 or $D > cT$.

(c) If D < cT, then $(T - vD/c^2) > (T - vcT/c^2) = T(1 - v/c)$. For T > 0 this is always positive because $v/c < 1$. Thus, $t_2^{/} - t_1^{/} = \gamma(T - vD/c^2)$ is always positive.

(d) Assume $T = D/c^{/}$ with $c^{/} > c$. Then $T - vD/c^2 = (D/c^{/}) - (vD/c^2) = (D/c)\left(\frac{c}{c^{/}} - \frac{v}{c}\right)$

This changes sign at $v/c = c/c^{/}$ which is still smaller than 1. For any larger *v* (still smaller than c) $t_2^{/} - t_1^{/} = \gamma(T - vD/c^2) < 0$ or $t_1^{/} > t_2^{/}$

1-57. $v = 0.6c$ $\qquad \gamma = \dfrac{1}{\sqrt{1 - (0.6)^2}} = 1.25$

(a) The clock in S reads $\gamma \times 60\,\text{min} = 75\,\text{min}$ when the S$^{/}$ clock reads 60 min and the first signal from S$^{/}$ is sent. At that time, the S$^{/}$ observer is at $v \times 75\,\text{min} = 0.6c \times 75\,\text{min} = 45\,c\cdot\text{min}$. The signal travels for 45 min to reach the S observer and arrives at 75 min + 45 min = 120 min on the S clock.

(b) The observer in S sends his first signal at 60 min and its subsequent wavefront is found at $x = c(t - 60\,\text{min})$. The S$^{/}$ observer is at $x = vt = 0.6ct$ and receives the wavefront when these *x* positions coincide, i.e., when

$$\begin{aligned} c(t - 60\,\text{min}) &= 0.6ct \\ 0.4ct &= 60\,c\cdot\text{min} \\ t &= (60\,c\cdot\text{min})/0.4c = 150\,\text{min} \\ x &= 0.6c(150\,\text{min}) = 90\,c\cdot\text{min} \end{aligned}$$

The confirmation signal sent by the S$^{/}$ observer is sent at that time and place, taking 90 min to reach the observer in S. It arrives at 150 min + 90 min = 240 min.

(Problem 1-57 continued)

(c) Observer in S:

Sends first signal	60 min
Receives first signal	120 min
Receives confirmation	240 min

The S$^{/}$ observer makes identical observations.

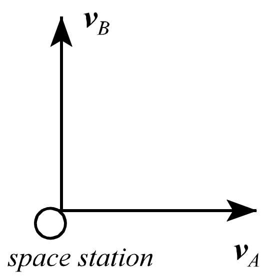
1-58. Clock at *r* moves with speed $u = r\omega$, so time dilation at that clock's location is:

$$\Delta t_0 = \gamma \Delta t_r \rightarrow \Delta t_r = \Delta t_0 \sqrt{1 - u^2/c^2} = \Delta t_0 \sqrt{1 - r^2\omega^2/c^2}$$

Or, for $r\omega << c$, $\Delta t_r \approx \Delta t_0 \left(1 - \frac{1}{2} r^2\omega^2/c^2\right)$

And, $\dfrac{\Delta t_r - \Delta t_0}{\Delta t_0} = \dfrac{\Delta t_0 \left(1 - \frac{1}{2} r^2\omega^2/c^2\right) - \Delta t_0}{\Delta t_0} \approx -\dfrac{r^2\omega^2}{2C^2}$

1-59.



$$u_x^{/} = \frac{u_x - v}{1 - u_x v/c^2}$$

$$u_y^{/} = \frac{u_y}{\gamma(1 - u_x v/c^2)}$$

(a) For $\boldsymbol{v}_{BA}$: $v = v_B$, $v_{Ax} = 0$, $v_{Ay} = -v_A$. So, $v_{Ax}^{/} = \dfrac{v_{Ax} - v}{1 - v_{Ax} v/c^2} = -v_B$

$$v_{Ay}^{/} = \frac{v_{Ay}}{\gamma_B (1 - v_{Ax} v/c^2)} = \frac{-v_A}{\gamma_B} \quad \text{where } \gamma_B = \frac{1}{\sqrt{1 - v_B^2/c^2}}$$

(Problem 1-59 continued)

$$v_{BA} = \sqrt{v_{Ax}'^2 + v_{Ay}'^2} = \sqrt{v_\beta^2 + (v_A/\gamma_B)^2}$$

$$\tan\theta_{BA}' = \frac{v_{Ay}'}{v_{Ax}'} = \frac{-v_A/\gamma_B}{-v_B} = \frac{v_A}{\gamma_B v_B}$$

(b) For $\mathbf{v}_{AB}$: $v = v_A$, $v_{By} = v_B$, $v_{Bx} = 0$. So, $v_{Bx}' = \dfrac{v_{Bx} - v}{1 - v_{Bx} v/c^2} = -v_A$

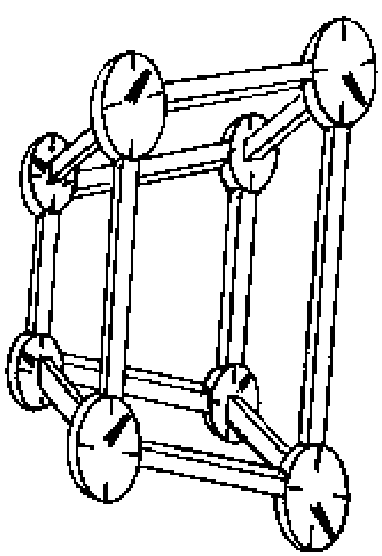
$$v_{By}' = \frac{v_{By}}{\gamma_A(1 - v_{Bx} v/c^2)} = \frac{v_B}{\gamma_A} \quad \text{where } \gamma_A = \frac{1}{\sqrt{1 - v_A^2/c^2}}$$

$$v_{BA} = \sqrt{(-v_A)^2 + (v_B/\gamma_A)^2}$$

$$\tan\theta_A' = \frac{v_B}{\gamma_A(-v_A)} = -\frac{v_B}{\gamma_A v_A}$$

(c) The situations are not symmetric. B viewed from A moves in the $+y$ direction, and A viewed from B moves in the $-y$ direction, so $\tan\theta_A' = -\tan\theta_B' = 45°$ only if $v_A = v_B$ and $\gamma_A = \gamma_B = 1$, which requires $v_A = v_B = 0$.

1-60.



1-61. (a) Apparent time $A \rightarrow B = T/2 - t_A + t_B$ and apparent time $B \rightarrow A = T/2 + t_A - t_B$ where t_A = light travel time from point A to Earth and t_B = light travel time from point B to Earth.

$$A \rightarrow B = \frac{T}{2} - \frac{L}{c+v} + \frac{L}{c-v} = \frac{T}{2} + \frac{2vL}{c^2 - v^2}$$

$$B \rightarrow A = \frac{T}{2} - \frac{L}{c+v} - \frac{L}{c-v} = \frac{T}{2} - \frac{2vL}{c^2 - v^2}$$

(b) Star will appear at A and B simultaneously when $t_B = T/2 + t_A$ or when the period is:

$$T = 2[t_B - t_A] = 2\left[\frac{L}{c-v} - \frac{L}{c+v}\right] = \frac{4vL}{c^2 - v^2}$$

1-62. The angle of u' with the x′ axis is:

$$\tan\theta' = \frac{u'_y}{u'_x} = \frac{u_y}{\gamma\left(1 - \frac{vu_x}{c^2}\right)} \frac{(1 - vu_x/c^2)}{u_x - v}$$

$$\tan\theta' = \frac{u_y}{\gamma(u_x - v)} = \frac{u\sin\theta}{\gamma(u\cos\theta - v)} = \frac{\sin\theta}{\gamma(\cos\theta - v/u)}$$