Chapter 2 – Relativity II

2-1. $u_{yB}^2 = u_0^2(1 - v^2/c^2) \qquad u_{xB}^2 = v^2$

$$\sqrt{1 - (u_{xB}^2 + u_{yB}^2)/c^2} = \sqrt{1 - v^2/c^2 - (u_0^2/c^2)(1 - v^2/c^2)}$$

$$= \sqrt{(1 - v^2/c^2)(1 - u_0^2/c^2)}$$

$$= (1 - v^2/c^2)^{1/2}(1 - u_0^2/c^2)^{1/2}$$

$$p_{yB} = \frac{mu_{yB}}{\sqrt{1 - (u_{xB}^2 + u_{yB}^2)/c^2}} = \frac{-mu_0\sqrt{1 - v^2/c^2}}{\sqrt{1 - v^2/c^2}\sqrt{1 - u_0^2/c^2}}$$

$$= -mu_0/\sqrt{1 - u_0^2/c^2} = -p_{yA}$$

2-2. $d(\gamma mu) = m(u\,d\gamma + \gamma\,du)$

$$= m\left[u\left(-\frac{1}{2}\right)\left(\frac{-2u}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-3/2} + \left(1 - \frac{u^2}{c^2}\right)^{-1/2}\right]du$$

$$= m\left[\left(\frac{u^2}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-3/2} + \left(1 - \frac{u^2}{c^2}\right)\left(1 - \frac{u^2}{c^2}\right)^{-3/2}\right]du$$

$$= m\left(1 - \frac{u^2}{c^2}\right)^{-3/2}du$$

2-3. (a) $\gamma = \dfrac{1}{\sqrt{1 - u^2/c^2}} = \dfrac{1}{\sqrt{1 - (0.6c)^2/c^2}} = \dfrac{1}{0.8} = 1.25$

(b) $p = \gamma mu = \gamma(mc^2)(u/c)/c = 1.25(0.511\,MeV)(0.6)/c = 0.383\,MeV/c$

(c) $E = \gamma mc^2 = 1.25(0.511\,MeV) = 0.639\,MeV$

(d) $E_k = (\gamma - 1)mc^2 = 0.25(0.511\,MeV) = 0.128\,MeV$

2-4. The quantity required is the kinetic energy.

$E_k = (\gamma - 1)mc^2 = [(1 - u^2/c^2)^{-½} - 1]mc^2$

(a) $E_k = [(1 - (0.5)^2)^{-½} - 1]mc^2 = 0.155mc^2$

(b) $E_k = [(1 - (0.9)^2)^{-½} - 1]mc^2 = 1.29mc^2$

(c) $E_k = [(1 - (0.99)^2)^{-½} - 1]mc^2 = 6.09mc^2$

2-5. $\Delta E = \Delta mc^2 \quad \therefore \quad \Delta m = \Delta E/c^2 = \dfrac{10J}{(3.0\times10^8 m/s)^2} = 1.1\times10^{-16} kg$

Because work is done *on* the system, the mass *increases* by this amount.

2-6. $m(u) = m/\sqrt{1-u^2/c^2}$ (Equation 2-5)

(a) $\dfrac{m(u)-m}{m} = 0.05 \rightarrow \dfrac{m/\sqrt{1-u^2/c^2} - m}{m} = 0.05$

$\dfrac{1}{\sqrt{1-u^2/c^2}} - 1 = 0.05 \rightarrow \dfrac{1}{\sqrt{1-u^2/c^2}} = 1.05 \rightarrow 1-u^2/c^2 = 1/(1.05)^2$

Thus, $u^2/c^2 = 1 - 1/(1.05)^2 = 0.0930 \rightarrow u/c = 0.305$

(b) $m(u) = 5m$

$\dfrac{m}{\sqrt{1-u^2/c^2}} = 5m \rightarrow 1-u^2/c^2 = 1/25$

Thus, $u^2/c^2 = 1-1/25 = 0.960 \rightarrow u/c = 0.980$

(c) $m(u) = 20m$

$\dfrac{m}{\sqrt{1-u^2/c^2}} = 20m \rightarrow 1-u^2/c^2 = 1/400$

Thus, $u^2/c^2 = 1-1/400 = 0.9975 \rightarrow u/c = 0.99870$

2-7. (a) $v = (\text{Earth} - \text{moon distance})/\text{time} = 3.8\times10^8 m/1.5s = 0.84c$

(b) $E_k = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$ (Equation 2-9)

$mc^2(\text{proton}) = 938.3\ MeV \qquad \gamma = 1/\sqrt{1-(0.84)^2} = 1.87$

(Problem 2-7 continued)

$$E_k = 938.\,MeV(1.87 - 1) = 813\,MeV$$

(c) $$m(u) = \frac{m}{\sqrt{1 - u^2/c^2}} = \frac{938.3\,MeV/c^2}{\sqrt{1-(0.84)^2}} = 1.730\times10^3\,MeV/c^2 = 1.730\,GeV/c^2$$

(d) Classically, $E_k = \frac{1}{2}mv^2 = \frac{1}{2}(938.\,MeV/c^2)(0.84c)^2 = 331\,MeV$

$$\%\text{ error} = \frac{813\,MeV - 331\,MeV}{813\,MeV}\times 100 = 59\%$$

2-8. $E_k = mc^2(\gamma - 1)$ (Equation 2-9)

$$E_k(u_2) - E_k(u_1) = W_{21} = mc^2(\gamma(u_2) - 1) - mc^2(\gamma(u_1) - 1)$$

Or $W_{21} = mc^2[\gamma(u_2) - (\gamma(u_1)]$

(a) $W_{21} = 938.\,MeV[(1-0.16^2)^{-1/2} - (1-0.15^2)^{-1/2}] = 1.51\,MeV$

(b) $W_{21} = 938.\,MeV[(1-0.86^2)^{-1/2} - (1-0.85^2)^{-1/2}] = 57.6\,MeV$

(c) $W_{21} = 938.\,MeV[(1-0.96^2)^{-1/2} - (1-0.95^2)^{-1/2}] = 3.35\times10^3\,MeV = 3.35\,GeV$

2-9. $E = \gamma mc^2$ (Equation 2-10)

(a) $200\,GeV\times 197 = 3.94\times10^4\,GeV = \gamma(0.938\,GeV)$ where $mc^2(\text{proton}) = 0.938\,GeV$

$$\gamma = \frac{1}{\sqrt{1-v^2/c^2}} = \frac{3.94\times10^4\,GeV}{0.938\,GeV} = 4.20\times10^4$$ Because $E >> mc^2$,

$$\frac{v}{c} \approx 1 - \frac{1}{2\gamma^2}$$ (Equation 2-40)

$$\frac{v}{c} \approx 1 - \frac{1}{2(4.20\times10^4)^2} = 1 - 0.0000000002834$$ Thus, $v \approx 0.999999999717\,c$

(b) $E \approx pc$ for $E >> mc^2$ (Equation 2-36)

$$p = E/c = 3.94\times10^4\,GeV/c$$

(c) Assuming one *Au* nucleus (system S') to be moving in the $+x$ direction of the lab (system *S*), then *u* for the second *Au* nucleus is in the $-x$ direction. The second *Au's* energy measured in the S' system is :

(Problem 2-9 continued)

$$E' = \gamma(E + vp_x) = (4.20\times10^4)(3.94\times10^4 GeV + v3.94\times10^4 GeV/c)$$

$$= (4.20\times10^4)(3.94\times10^4 GeV)(1 + v/c)$$

$$= (4.20\times10^4)(3.94\times10^4 GeV)(2)$$

$$= 3.31\times10^9\ GeV$$

$$p_x' = \gamma(p_x - vE/c^2) = (4.20\times10^4)(-3.94\times10^4 GeV - v3.94\times10^4 GeV/c^2)$$

$$= -(4.20\times10^4)(3.94\times10^4 GeV)(2)$$

$$= -3.31\times10^9\ GeV/c$$

2-10. (a) $E = mc^2 = (10^{-3}\ kg)c^2 = 9.0\times10^{13}\ J$

(b) $1\,kWh = 1\times10^3 J{\cdot}h/s(3600\,s/h) = 3.6\times10^6\ J$

so, $9.0\times10^{13} J/3.6\times10^6 J/kWh = 2.5\times10^7\ kWh$

@$0.10/kWh would sell for $2.5 × 10^6 or $2.5 million.

(c) $100\,W = 100\,J/s$, so $1g$ of dirt will light the bulb for:

$$\frac{9.0\times10^{13} J}{100\,J/s} = 9.0\times10^{11}\ s = \frac{9.0\times10^{11}\ s}{3.16\times10^7\ s/y} = 2.82\times10^4\ y$$

2-11. $E = \gamma m c^2$ (Equation 2-10)

where $\gamma = 1/\sqrt{1-u^2/c^2} = 1/\sqrt{1-(0.2)^2} = 1.0206$

$E = (1.0206)(0.511\,MeV) = 0.5215\ MeV$

$E_k = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$ (Equation 2-9)

$= (0.511\,MeV)(1.0206 - 1) = 0.01054\ MeV$

$E^2 = (pc)^2 + (mc^2)^2$ (Equation 2-32)

$$p^2 = \frac{1}{c^2}(E^2 - (mc^2)^2) = \frac{1}{c^2}[(0.5215\,MeV)^2 - (0.511\,MeV)^2]$$

$$= 0.01087/c^2 \rightarrow p = 0.104\ MeV/c$$

2-12. $E = \gamma mc^2$ (Equation 2-10)

$\gamma = E/mc^2 = 1400\,MeV/938\,MeV = 1.4925$

(a) $\gamma = 1/\sqrt{1-u^2/c^2} = 1.4925 \rightarrow 1-u^2/c^2 = 1/(1.4925)^2$

$u^2/c^2 = 1-1/(1.4925)^2 = 0.551 \rightarrow u = 0.74c$

(b) $E^2 = (pc)^2 + (mc^2)^2$ (Equation 2-32)

$p = \frac{1}{c}[E^2-(mc^2)^2]^{1/2} = \frac{1}{c}[(1400\,MeV)^2 - 938\,MeV)^2]^{1/2} = 1040\,MeV/c$

2-13. (a) $\gamma = E/mc^2 = 2 = 1/\sqrt{1-u^2/c^2} \rightarrow 1-u^2/c^2 = 1/4$

$\rightarrow u^2/c^2 = 1-1/4 = 0.75 \rightarrow u = 0.866c$

(b) $p = \frac{1}{c}[E^2-(mc^2)^2]^{1/2}$ (From Equation 2-32)

$p = \frac{1}{c}[(2mc^2)^2-(mc^2)^2]^{1/2} = \frac{1}{c}[4(mc^2)^2-(mc^2)^2]^{1/2} = \sqrt{3}\,mc$

2-14. $u = 2.2\times10^6 m/s$ and $\gamma = 1/\sqrt{1-u^2/c^2}$

(a) $E_k = 0.5110\,MeV(\gamma-1) = 0.5110\,MeV(1/\sqrt{1-u^2/c^2}-1) = 0.5110(2.689\times10^{-5})$

$= 1.3741\times10^{-5}\,MeV$

$E_k(\text{classical}) = \frac{1}{2}mu^2 = \frac{1}{2}mc^2(u^2/c^2) = (0.5110\,MeV/2)(2.2\times10^6/c^2)^2$

$= 1.374\times10^{-5}\,MeV$

$\%\text{ difference} = \frac{1\times10^{-9}}{1.3741\times10^{-5}}\times100 = 0.0073\%$

(b) $p = \frac{1}{c}\sqrt{E^2-(mc^2)^2} = \frac{1}{c}\sqrt{(\gamma mc^2)^2-(mc^2)^2}$

$= mc\sqrt{\gamma^2-1} = \frac{mc^2}{c}[(1/\sqrt{1-(2.2\times10^6/3.0\times10^8)^2})^2-1]^{1/2}$

$= 0.5110\,MeV/c(7.33\times10^{-3}) = 3.74745\times10^{-3}\,MeV/c$

(Problem 2-14 continued)

$$p(\text{classical}) = mu = \frac{mc^2}{c}\left(\frac{u}{c}\right) = (0.5110\,MeV/c)(2.2\times10^6/3.0\times10^8)$$

$$= 3.74733\times10^{-3}\,MeV/c$$

$$\%\ \text{difference} = \frac{1.2\times10^{-7}}{3.74745\times10^{-3}}\times 100 = 0.0030\%$$

2-15. (a) $60\,W = 60\,J/s(3.16\times10^7\,s/y) = 1.896\times10^9\,J$

$m = E/c^2 = 1.896\times10^9 J/(3.0\times10^8\,m/s)^2 = 2.1\times10^{-8}\,kg = 2.1\times10^{-5}\,g = 21\,\mu g$

(b) It would make no difference if the inner surface were a perfect reflector. The light energy would remains in the enclosure, but light has no rest mass, so the balance reading would still go down by 21 μg.

2-16. $^4He \rightarrow\ ^3H + p + e$

$Q = [m(^3H) + m_p + m_e - m(^4He)]c^2$

$= 2809.450\,MeV + 938.280\,MeV + 0.511\,MeV - 3728.424\,MeV = 19.817\,MeV$

2-17. $^3H \rightarrow\ ^2H + n$

Energy to remove the *n*

$= 22.014102\,u(^2H) + 1.008665\,u(n) - 3.016049\,u(^3H)$

$= 0.006718\,u \times 931.5\,MeV/u = 6.26\,MeV$

2-18. (a) $\Delta m = \dfrac{\Delta E}{c^2} = \dfrac{\Delta E\,u}{uc^2} = \dfrac{4.2\,eV}{931.5\times10^6\,eV}u = 4.5\times10^{-9}\,u$

(b) $\text{error} = \dfrac{\Delta m}{m(Na) + m(Cl)} = \dfrac{4.5\times10^{-9}\,u}{23\,u + 35.5\,u} = 7.7\times10^{-11} = 7.7\times10^{-9}\,\%$

2-19. (a) $$\Delta m = m(^4He) - 2m(^2H) = \frac{m(^4He)c^2 - 2m(^2H)c^2}{uc^2}u$$
$$= [3727.409\,MeV - 2\times 1875.628\,MeV]u/931.5\,MeV$$
$$= -0.0256\,u$$

(b) $\Delta E = |\Delta m|c^2 = (0.0256uc^2)(931.5\,MeV/uc^2) = 23.8\,MeV$

(c) $$\frac{dN}{dt} = \frac{P}{\Delta E} = \frac{1\,W}{23.847\,MeV}\times\frac{1\,eV}{1.602\times10^{-19}J} = 2.62\times10^{11}/s$$

2-20. $$\frac{dm}{dt} = \frac{1}{c^2}\frac{dE}{dt} = \frac{100\times10^6 W}{9\times10^6 J/kg}\times\frac{3600\,s}{h} = 4.0\times10^{-6}kg/h$$

2-21. Conservation of energy requires that $E_i^2 = E_f^2$, or

$(p_ic)^2 + (2m_pc^2)^2 = (p_fc)^2 + (2m_pc^2 + m_\pi c^2)^2$ and conservation of momentum requires that $p_i = p_f$, so

$$4(m_pc^2)^2 = 4(m_pc^2)^2 + 2m_pc^2\times 2m_\pi c^2 + (m_\pi c^2)^2$$
$$0 = 2m_pc^2\times 2m_\pi c^2 + (m_\pi c^2)^2$$
$$0 = m_\pi c^2\left(2 + \frac{m_\pi c^2}{2m_pc^2}\right) = m_\pi c^2\left(2 + \frac{m_\pi}{2m_p}\right)$$

Thus, $m_\pi c^2(2 + m_\pi/2m_p)$ is the minimum or threshold energy E_t that a beam proton must have to produce a π^0.

$$E_t = m_\pi c^2\left(2 + \frac{m_\pi c^2}{2m_pc^2}\right) = 135\,MeV\left(2 + \frac{135}{2(938)}\right) = 280\,MeV$$

2-22. $$\Delta E = \Delta mc^2 \quad \therefore \quad \Delta m = \Delta E/c^2 = \frac{200\times10^6 eV}{(5.61\times10^{32}eV/g)} = 3.57\times10^{-25}\,g$$

2-23. Conservation of momentum requires the pions to be emitted in opposite directions with equal momenta, hence equal kinetic energy. K^0 rest energy = 497.7 *MeV*. π^+ and π^- rest energy (each) = 139.6 *MeV*. So, total $E_K = 497.7\,MeV - 2(139.6\,MeV) = 218.5\,MeV$ and each pion will have $218.5\,MeV/2 = 109.25\ MeV$ of kinetic energy.

2-24. $1.0\,W = 1.0\,J/s \rightarrow p = E/c = (1.0\,J/s)/c$

(a) On being absorbed by your hand the momentum change is $\Delta p = (1.0\,J/s)/c$ and, from the impulse-momentum theorem,

$$F\Delta t = \Delta p \text{ where } \Delta t = 1\,s \quad \rightarrow \quad F = (1.0\,J/s)/c\,\Delta t = (1.0/c)N = 3.3\times 10^{-9}N$$

This magnitude force would be exerted by gravity on mass *m* given by:

$$m = F/g = 3.3\times 10^{-9}N/(9.8\,m/s^2) = 3.4\times 10^{-10}kg = 0.34\,\mu g$$

(b) On being reflected from your hand the momentum change is twice the amount in part (a) by conservation of momentum. Therefore, $F = 6.6\times 10^{-9}, m = 0.68\,\mu g$.

2-25. Positronium at rest: $(2mc^2)^2 = E_i^2 + (p_i c)^2$

Because $\boldsymbol{p_i} = 0$, $E_i = 2mc^2 = 2(0.511\,MeV) = 1.022\,MeV$

After photon creation; $(2mc^2)^2 = E_f^2 + (p_f c)^2$

Because $\boldsymbol{p_f} = 0$ and energy is conserved,

$(2mc^2)^2 = E_f^2 = (1.022\,MeV)^2$ or $2mc^2 = 1.022\,MeV$ for the photons.

2-26. $E^2 = (pc)^2 + (mc^2)^2$ (Equation 2-31)

$$E = [(pc)^2 + (mc^2)^2]^{1/2} = mc^2\left[1 + \left(\frac{pc}{mc^2}\right)^2\right]^{1/2}$$

$$= mc^2[1 + p^2/m^2c^2)^{1/2} \approx mc^2\left[1 + \frac{1}{2}(p/mc)^2 + \cdots\right] = mc^2\left[1 + \frac{p^2}{2m^2c^2}\right]$$

$$= mc^2 + p^2/2m$$

2-27. $E^2 = (pc)^2 + (mc^2)^2$ (Equation 2-31)

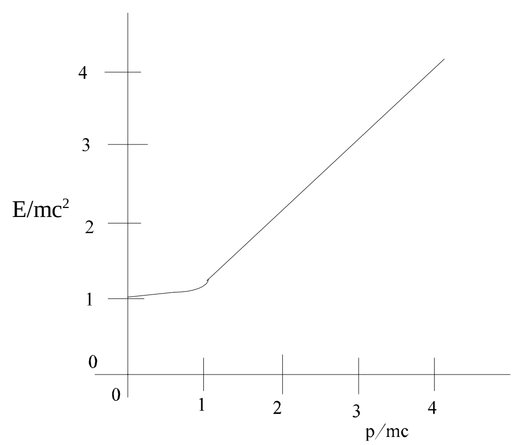
(a) $(pc)^2 = E^2 = E^2 - (mc^2)^2 = (5\,MeV)^2 - (0.511\,MeV)^2 = 24.74$

Or $p = \sqrt{24.74}/c = 4.97\,MeV/c$

(b) $E = \gamma mc^2 \rightarrow \gamma = E/mc^2 = 1/\sqrt{1-u^2/c^2} \rightarrow 1 - u^2/c^2 = (mc^2/E)^2$

$u/c = [1-(mc^2/E)^2]^{1/2} = [1-(0.511/5.0)^2]^{1/2} = 0.995$

2-28.



2-29. $E^2 = (pc)^2 + (mc^2)^2$ (Equation 2-31)

$(1746\,MeV)^2 = (500\,MeV)^2 + (mc^2)^2$

$mc^2 = [(1746\,MeV)^2 - (500\,MeV)^2]^{1/2} = 1673\,MeV \rightarrow m = 1673\,MeV/c^2$

$E = \gamma mc^2 \rightarrow \gamma = 1/\sqrt{1-u^2/c^2} = E/mc^2$

$u/c = [1-(mc^2/E)^2]^{1/2} = [1-(1673\,MeV/1746\,MeV)^2]^{1/2} = 0.286 \rightarrow u = 0.286c$

2-30. (a) $BqR = m\gamma u = p$ (Equation 2-37)

$B = \dfrac{m\gamma u}{qR}$ and $E = \gamma mc^2$ which we have written as (see Problem 2-29)

$u/c = [1-(mc^2/E)^2]^{1/2} = [1-(0.511\,MeV/4.0\,MeV)^2]^{1/2} = 0.992$

And $\gamma = 1/\sqrt{1-u^2/c^2} = 1/\sqrt{1-(0.992)^2} = 7.83$

Then, $B = \dfrac{(9.11\times10^{-31}\,kg)(7.83)(0.992c)}{(1.60\times10^{-19}\,C)(4.2\times10^{-2}\,m)} = 0.316\,T$

(b) γm exceeds m by a factor of $\gamma = 7.83$.

2-31. (a) $p = qBR = e(0.5T)(2.0) \times \dfrac{3.0\times10^8 m/s}{c} = 300 MeV/c$

(b) $E_k = E - mc^2 = [(pc)^2 + (mc^2)^2]^{½} - mc^2$

$= [(300MeV)^2 + (938.28MeV)^2]^{½} - 938.28MeV$

$= 46.8MeV$

2-32. $\alpha = 2GM/c^2R$ (Equation 2-44)

Earth radius $R = 6.37\times10^6\ m$ and mass $M = 5.98\times10^{24}\ kg$

$$\alpha = \frac{2(6.67\times10^{-11} N\cdot m^2/kg^2)(5.98\times10^{24} kg)}{(3.00\times10^8 m/s)^2(6.37\times10^6 m)} = 1.39\times10^{-9}\ \text{radians}$$

$= 2.87\times10^{-4}$ arc seconds

2-33. Because the clock furthest from Earth (where Earth's gravity is less) runs the faster, answer (c) is correct.

2-34. $$\Delta\phi = \frac{6\pi GM}{c^2(1-\epsilon^2)R} = \frac{6\pi(6.67\times10^{-11} N\cdot m^2/kg^2)(1.99\times10^{30} kg)}{(3.00\times10^8 m/s)^2(1-0.048^2)(7.80\times10^{11} m)}\ (\text{Equation } 2-51)$$

$= 3.64\times10^{-8}$ radians/century $= 7.55\times10^{-3}$ arc seconds/century

2-35. The transmission is redshifted on leaving Earth to frequency *f*, where

$f_0 - f = f_0 gh/c^2$. Synchronous satellite orbits are at 6.623R_E where

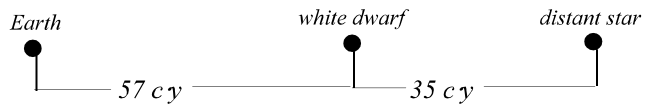
$$g = \frac{GM_E}{(6.623R_E)^2} = \frac{9.9 m/s^2}{(6.623)^2} = 0.223 m/s^2$$

$h = 6.623R_E = 6.623(6.37\times10^6 m) = 4.22\times10^7 m$

$f_0 - f = (9.375\times10^9 Hz)(0.223 m/s^2)(4.22\times10^7)/(3.00\times10^8)^2 = 0.980\ Hz$

$f = f_0 - 0.980\ Hz = 9.374999999\times10^9\ Hz$

2-36.



On passing "below" the white dwarf, light from the distant star is deflected through an angle:

$$\alpha = 2Gm/c^2R = \frac{2(6.67\times10^{-11}N\cdot m^2/kg^2)(3)(1.99\times10^{30}kg)}{(3.00\times10^8 m/s)(10^7 m)}$$

$= 8.85\times10^{-4}\ radians = 0.051°$ or the angle between the arcs is $2\alpha = 0.102°$

2-37. The speed v of the satellite is:

$v = 2\pi R/T = 2\pi(6.37\times10^6 m)/(90\,\text{min}\times60\,s/\text{min}) = 7.42\times10^3 m/s$

Special relativistic effect:

After one year the clock in orbit has recorded time $\Delta t' = \Delta t/\gamma$, and the clocks differ by:

$\Delta t - \Delta t' = \Delta t - \Delta t/\gamma = \Delta t(1-1/\gamma) \approx \Delta t(v^2/2c^2)$, because $v << c$. Thus,

$\Delta t - \Delta t' = (3.16\times10^7 s)(7.412\times10^3)^2/(2)(3.0\times10^8 m)^2 = 0.00965\,s = 9.65\,ms$

Due to special relativity time dilation the orbiting clock is behind the Earth clock by 9.65 *ms*.

General relativistic effect:

$$\frac{\Delta f}{f_0} = \frac{gh}{c^2} = \frac{(9.8\,m/s^2)(3.0\times10^5 m)}{(3.0\times10^8 m/s)^2} = 3.27\times10^{-11} s/s$$

In one year the orbiting clock gains

$(3.27\times10^{-11} s/s)(3.16\times10^7 s/y) = 1.03\,ms$

The net difference due to both effects is a slowing of the orbiting clock by 9.65 – 1.03 = 8.62 *ms*.

2-38. The rest energy of the mass m is an invariant, so observers in S' will also measure m = 4.6 *kg*, as in Example 2-9. The total energy E' is then given by:

$(mc^2)^2 = (E')^2 - (\boldsymbol{p}'c)^2$ Because, $\boldsymbol{p}' = 0$,

$E' = mc^2 = 4.6\,kg\times(3.0\times10^8 m/s)^2 = 4.14\times10^{14} J$

2-39. (a) $E = \gamma m_e c^2 \quad \rightarrow \quad \gamma = E/m_e c^2 = 50\times10^3 MeV/0.511 MeV = 9.78\times10^4$

$L = L_0/\gamma = 10^{-2}\, m$

$L_0 = 9.78\times10^4 (10^{-2} m) = 978 m$ (length of one bundle)

The width of the bundle is the same as in the lab.

(b) An observer on the bundle "sees" the accelerator shortened to 978 m from its proper length L_0, so $L_0 = \gamma(978) = 978\times10^4(978) = 9.57\times10^7\, m$.

(Note that this is about 2.5 times Earth's 40,000 km circumference at the equator.)

(c) The e^+ bundle is 10^{-2} m long in the lab frame, so in the e^- frame its length would be measured to be: $L = (10^{-2} m)/\gamma = 10^{-2} m/9.78\times10^4 = 1.02\times10^{-7}\, m$

2-40. $E_k = \gamma mc^2 - mc^2 = mc^2(\gamma - 1)$ If $E_k = mc^2 = 938 MeV$, then $\gamma = 2$.

(a) $(mc^2)^2 = E^2 - (pc)^2$ (Equation 2-32) Where $E = \gamma mc^2 = 2(938 MeV)$

$(pc)^2 = E^2 - (mc^2)^2 = (2\times938)^2 - (938)^2 = 2.46\times10^6$

$p = (2.64\times10^6)^{1/2}/c = 1.62\times10^3\, MeV/c$

(b) $p = \gamma mu \quad \rightarrow \quad u = p/\gamma m = (1.62\times10^3 MeV/c)/(2)(938 MeV/c^2) = 0.866c$

2-41. (a) The momentum p of the ejected fuel is:

$p = \gamma mu = mu/\sqrt{1 - u^2/c^2} = 10^3 kg(c/2)/\sqrt{1 - 0.5^2} = 1.73\times10^{11}\, kg\cdot m/s$

Conservation of momentum requires that this also be the momentum p_s of the spaceship: $p_s = m_s u_s/\sqrt{1 - u_s^2/c^2} = 1.73\times10^{11}\, kg\cdot m/s$

Or, $m_s u_s/\sqrt{1 - u_s^2/c^2} = (1.73\times10^{11}\, kg\cdot m/s)^2$

$m_s^2 c_s^2 = (1 - u_s^2/c^2)(1.73\times10^{11}\, kg\cdot m/s)^2 = (1.73\times10^{11} kg\cdot m/s^2) - (3.33\times10^5 kg^2)u_s^2$

$(10^6 kg)^2 u_s^2 + (3.33\times10^5 kg^2)u_s^2 = (1.73\times10^{11} kg\cdot m/s)^2$

Or, $u_s = (1.73\times10^{11} kg\cdot m/s)/10^6 kg = 1.73\times10^5 m/s = 5.77\times10^{-4} c$

(Problem 2-41 continued)

(b) In classical mechanics, the momentum of the ejected fuel is :

$mu = mc/2 = 10^3 c/2$, which must equal the magnitude of the spaceship's momentum $m_s u_s$, so

$$u_s = 10^3(c/2)/m_s = \frac{10^3 kg(3.0\times10^8 m/s)}{2(10^6 kg)} = 5.0\times10^{-4}c = 1.5\times10^5 m/s$$

(c) The initial energy E_i before the fuel was ejected is $E_i = m_s c^2$ in the ship's rest system. Following fuel ejection, the final energy E_f is:

$$E_f = \text{energy of fuel} + \text{energy of ship} = mc^2/\sqrt{1-u^2/c^2} + (m_s - m)c^2/\sqrt{1-u_s^2/c^2}$$

where $u = 0.5c$ and $u_s << c$, so

$$E_f = 1.155mc^2 + (m_s - m)c^2 = (1.155-1)mc^2 + m_s c^2$$

The change in energy ΔE is

$$\Delta E = E_f - E_i = [(0.155)(10^3 kg)c^2 + 10^6 kgc^2] - [10^6 kgc^2]$$

$\Delta E = 155 kgc^2$ or $155 kg = \Delta E/c^2$ of mass has been converted to energy.

2-42. (a) $p = 300BR(q/e)$ (Equation 2-38)

$$p = 300(1.5T)(6.37\times10^6)(1) = 2.87\times10^9 MeV/c$$

For E >>mc^2, E = E_k and E = pc (Equation 2-32) ∴ E_k = pc = 2.87×10^9 MeV

(b) For E = pc, u=c and

$$T = 2\pi R/c = 2\pi(6.37\times10^6 m)/c = 0.133 s$$

2-43. $\frac{f}{f_0} = 1 - GM/c^2R$ (Equation 2-47)

The fractional shift is: $\frac{f_o - f}{f_0} = 1 - \frac{f}{f_0} = GM/c^2R = 7\times10^{-4}$

The dwarf's radius is:

(Problem 2-43 continued)

$$R = GM/c^2(7\times10^{-4}) = \frac{6.67\times10^{-11}N\cdot m^2/kg^2(2\times10^{30}kg)}{(3.00\times10^8 m/s)^2(7\times10^{-4})} = 2.12\times10^{-6}\,m$$

Assuming the dwarf to be spherical, the density is:

$$\rho = \frac{M}{V} = \frac{2\times10^{30}\,kg}{4\pi(2.12\times10^6 m)^3/3} = 5.0\times10^{10}\,kg/m^3$$

2-44. The minimum energy photon needed to create an $e^- - e^+$ pair is $E_p = 1.022\,MeV$ (see Example 2-13). At minimum energy, the pair is created at rest, i.e., with no momentum. However, the photon's momentum must be $p = E/c = 1.022\,MeV/c$ at minimum. Thus, momentum conservation is violated unless there is an additional mass 'nearby' to absorb recoil momentum.

2-45. $$p_y' = \gamma' m u_y' = \left[\frac{\gamma(1 - vu_x/c^2)}{\sqrt{1-u^2/c^2}}\right]\times m \times\left[\frac{u_y}{\gamma(1-u_x v/c^2)}\right]$$

Canceling γ and $(1 - vu_x/c^2)$, gives: $p_y' = \dfrac{mu_y}{\sqrt{1-u^2/c^2}} = p_y$

In an exactly equivalent way, $p_z' = p_z$.

2-46. (a) $u_x' = (u_x - v)/(1 - u_x v/c^2)$ where $v = u$ and $u_x = -u$, so $u_x' = -2u/(1+u^2/c^2)$.

Thus, the speed of the particle that is moving in S' is:

$u' = 2u/(1+u^2/c^2)$ from which we see that:

$$1 - \left(\frac{u'}{c}\right) = 1 - \frac{u^2}{c^2} = 1 - \frac{4u^2}{c^2}/(1+u^2/c^2)^2$$

$$= (1 + 2u^2/c^2 + u^4/c^4 - 4u^2/c^2)/(1+u^2/c^2)$$

$$= (1-u^2/c^2)^2/(1+u^2/c^2)^2$$

(Problem 2-46 continued)

And thus, $\left[1-\left(\dfrac{u'}{c}\right)^2\right]^{1/2} = \dfrac{1-u^2/c^2}{1+u^2/c^2}$

(b) The initial momentum p_i' in S' is due to the moving particle,

$p_i' = mu'/\sqrt{1-(u'/c)^2}$ where u' and $\sqrt{1-(u'/c)^2}$ were given in (a).

$$p_i' = m\frac{2u(1+u^2/c^2)}{(1+u^2/c^2)(1-u^2/c^2)} = 2mu/(1-u^2/c^2)$$

(c) After the collision, conservation of momentum requires that:

$p_f' = Mu/(1-u^2/c^2)^{1/2} = p_i' = 2mu/(1-u^2/c^2)$ Or $M = 2m/(1-u^2/c^2)^{1/2}$

(d) In S: $E_i = 2mc^2/\sqrt{1-u^2/c^2}$ and $E_f = mc^2$ (M is at rest.) Because we saw in (c) that $M = 2m/(1-u^2/c^2)^{1/2}$, then $E_i = E_f$ in S.

In S': $E_i' = mc^2 + mc^2/\sqrt{1-(u'/c)^2}$ and substituting for the square root from (a),

$E_i' = 2mc^2/(1-u^2/c^2)$ and $E_f' = Mc^2/\sqrt{1-u^2/c^2}$. Again substituting for M from (c), we have: $E_i' = E_f'$.

2-47. (a) Each proton has $E_k = m_pc^2(\gamma-1)$, and because we want $E_k = m_pc^2$, then $\gamma = 2$ and $u = 0.866c$. (See Problem 2-40.)

(b) In the lab frame S':

$u_x' = \dfrac{u_x - v}{1-u_xv/c^2}$ where $u = v$ and $u_x = -u$ yields:

$$u_x' = \frac{-2u}{1+u^2/c^2} = \frac{-2(0.866c)}{1+(0.866)^2} = -0.990c$$

(c) For $u_x' = -0.990c$, $\gamma = 1/\sqrt{1-(0.99)^2} = 7.0$ and the necessary kinetic energy in the lab frame S is : $E_k = m_pc^2(\gamma-1) = m_pc^2(7-1) = 6m_pc^2$

2-48. (a) $p_i = 0 = E/c - Mv$ or $v = E/Mc$

(b) The box moves a distance $\Delta x = v\Delta t$, where $\Delta t = L/c$, so

$\Delta x = (E/Mc)(L/c) = EL/Mc^2$

(c) Let the center of the box be at $x = 0$. Radiation of mass m is emitted from the left end of the box (e.g.) and the center of mass is at: $x_{CM} = \dfrac{M(0) + m(L/2)}{M + m} = \dfrac{mL}{2(M + m)}$

When the radiation is absorbed at the other end the center of mass is at:

$$x_{CM} = \frac{M(EL/Mc^2) + m(L/2 - EL/Mc^2)}{M + m}$$

Equating the two values of x_{CM} (if CM is not to move) yields:

$m = (E/c^2)/(1 - E/Mc^2)$

Because $E << Mc^2$, then $m \approx E/c^2$ and the radiation has this mass.

2-49. (a) If ν mass is 0:

$E_\mu^2 = (p_\mu c)^2 + (m_\mu c)^2$ and $E_\nu^2 = (p_\nu c^2)^2 + 0$

$E_{k\mu} + E_\nu = 139.56755\,MeV - 105.65839\,MeV$

$m_\mu c^2(\gamma - 1) + E_\nu = 33.90916\,MeV$

$p_\mu c = \left(E_\mu^2 - (m_\mu c^2)^2\right)^{1/2} = 33.90916 - m_\mu c^2(\gamma - 1)$

Squaring, we have

$(m_\mu c^2)^2(\gamma^2 - 1) = (33.90916)^2 - 2(33.90916)(m_\mu c^2)(\gamma - 1) + (m_\mu c^2)^2(\gamma - 1)^2$

Collecting terms, then solving for $(\gamma - 1)$,

$\gamma - 1 = \dfrac{(33.90916)^2}{2(m_\mu c^2)^2 + 2(33.90916)m_\mu c^2}$ Substituting $m_\mu c^2 = 105.65839\,MeV$,

$\gamma - 1 = 0.0390 \rightarrow \gamma = 1.0390$ so,

$E_{k\mu} = 4.12\,MeV$ and $p_\mu = \dfrac{1}{c}[(109.78)^2 - (105.66)^2]^{1/2} = 29.8 / MeV/c$

$E_\nu = 29.8\,MeV$ and $p_\nu = 29.8\,MeV/c$

(b) If ν_μ mass $= 250\,keV$, then $E_\nu^2 = (p_\nu c)^2$ and

$E_{k\mu} + E_{k\nu} = 139.56755\,MeV - 105.65839\,MeV - 0.250\,MeV = 33.67916\,MeV$

Solving as in (a) yields

$E_\mu = 109.78\,MeV$, $p_\mu = 29.8\,MeV/c$, $E_\nu = 29.8\,MeV$, $p_\nu = 29.8\,MeV/c$

2-50. $\dfrac{f}{f_0} = 1 - Gm/c^2R$ (Equation 2-47)

Since $c = f\lambda$ and $c = f_0\lambda_0$,

$$\frac{c}{\lambda}\times\frac{\lambda_0}{c} = \frac{\lambda_0}{\lambda} = 1 - GM/c^2R = 1 - \frac{6.67\times10^{-11}N\cdot m^2/kg^2\,(1.99\times10^{30}kg)}{(3.00\times10^8 m/s)^2\,(6.96\times10^6 m)}$$

$$= 1 - 0.000212 = 0.999788$$

$$\lambda = \lambda_0/0.999788 = 720.00\,nm/0.999788 = 720.15\,nm$$

$$\Delta\lambda = \lambda - \lambda_0 = 0.15\,nm$$

2-51. $u_y' = (u_y/\gamma)(1 - vu_x/c^2)^{-1}$

$$a_y' = \frac{du_y'}{dt'} = \frac{\dfrac{du_y}{\gamma}(1 - vu_x/c^2)^{-1} + \dfrac{u_y}{\gamma}\left(\dfrac{v\,dx}{c^2}\right)(1 - vu_x/c^2)^{-2}}{\gamma(dt - v\,dx/c^2)}$$

$$a_y' = \frac{1}{\gamma^2}\left[\frac{(du_y/dt)(1 - vu_x/c^2)^{-1} + (u_yv/c^2)(du_x/dt)(1 - vu_x/c^2)^{-2}}{(1 - v(dx/dt)/c^2)}\right]$$

$$a_y' = \frac{a_y}{\gamma^2(1 - vu_x/c^2)^2} + \frac{a_xu_yv/c^2}{\gamma^2(1 - vu_x/c^2)^3}$$

2-52. (a) $F_x = \dfrac{dp_x}{dt} = \dfrac{d(\gamma mv)}{dt}$ $F_x' = ma_x'$ because $u_x' = 0$

$$F_x = \gamma m(dv/dt) + mv\,d[(1 - v^2/c^2)^{-1/2}]/dt$$

$$F_x = \frac{ma_x}{(1 - v^2/c^2)^{1/2}} + \frac{m(v^2/c^2)a_x}{(1 - v^2/c^2)^{3/2}}$$

$$F_x = \frac{ma_x(1 - v^2/c^2) + m(v^2/c^2)a_x}{(1 - v^2/c^2)^{3/2}}$$

$$F_x = \gamma^3 ma_x$$

(Problem 2-52 continued)

Because $u_x' = 0$, note from Equation 2-1 (inverse form) that $a_x = a_x'/\gamma^3$.

Therefore, $F_x = \gamma^3 m a_x'/\gamma^3 = m a_x' = F_x'$

(b) $F_y = \dfrac{dp_y}{dt} = \dfrac{d(\gamma, v_y)}{dt}$ $\quad F_y' = m a_y'$ because $u_y' = u_x' = 0$

$F_y = \gamma m a_y$ because S' moves in +*x* direction and the instantaneous transverse impulse (small) changes only the direction of **v**. From the result of Problem 2-5 (inverse form) with $u_y' = u_x' = 0$, $a_y = a_y'/\gamma^2$

Therefore, $F_y = \gamma m a_y = \gamma m a_y'/\gamma^2 = m a_y'/\gamma = F_y'/\gamma$

2-53. (a) Energy and momentum are conserved.

Initial system: $E = Mc^2$, $p = 0$

invariant mass: $(Mc^2)^2 = E^2 - (pc)^2 = (Mc^2)^2 + 0$

Final system:

invariant mass: $(2mc^2)^2 = (Mc^2)^2 + 0$

For 1 particle (from symmetry)

$(mc^2)^2 = (Mc^2/2)^2 - p^2c^2 = (Mc^2/2)^2 - (\gamma uc)^2$

Rearranging,

$$1 = \left(\frac{Mc^2}{2mc^2}\right)^2 - (\gamma u/c)^2 \Rightarrow \gamma^2 = \left(\frac{Mc^2}{2mc^2}\right)^2 = \frac{1}{1-u^2/c^2}$$

Solving for *u*,

$$u = \left[1 - \left(\frac{2mc^2}{Mc^2}\right)^2\right]^{1/2} c$$

(b) Energy and momentum are conserved.

Initial system: $E = 4mc^2$

invariant mass: $(Mc^2)^2 = (4mc^2)^2 - (pc)^2$

Final system:

invariant mass: $(2mc^2)^2 = (4mc^2)^2 - (pc)^2$

where $(pc)^2 = (4mc^2)^2 - (Mc^2)^2$

(Problem 2-53 continued)

$$\frac{u}{c} = \frac{pc}{E} = \frac{\left[(4mc^2)^2 - (Mc^2)^2\right]^{1/2}}{4mc^2}$$

$$\left(\frac{u}{c}\right)^2 = \frac{(4mc^2)^2 - (Mc^2)^2}{(4mc^2)^2} = 1 - \left(\frac{Mc^2}{4mc^2}\right)^2$$

$$u = \left[1 - \left(\frac{Mc^2}{4mc^2}\right)^2\right]^{1/2} c$$