Chapter 4 – The Nuclear Atom

4-1. $\dfrac{1}{\lambda_{mn}} = R\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$ where $R = 1.097\times10^7 m^{-1}$ (Equation 4–2)

The Lyman series ends on m = 1, the Balmer series on m = 2, and the Paschen series on m = 3. The series limits all have n = ∞, so $\dfrac{1}{n} = 0$

$$\frac{1}{\lambda_L} = R\left(\frac{1}{1^2}\right) = 1.097\times10^7 m^{-1}$$

$$\lambda_L(limit) = 1.097\times10^7 m^{-1} = 91.16\times10^{-9} m = 91.16\,nm$$

$$\frac{1}{\lambda_B} = R\left(\frac{1}{2^2}\right) = 1.097\times10^7 m^{-1}/4$$

$$\lambda_B(limit) = 4/1.097\times10^7 m^{-1} = 3.646\times10^{-7} m = 364.6\,nm$$

$$\frac{1}{\lambda_P} = R\left(\frac{1}{3^2}\right) = 1.097\times10^7 m^{-1}/9$$

$$\lambda_P(limit) = 9/1.097\times10^7 m^{-1} = 8.204\times10^{-7} m = 820.4\,nm$$

4-2. $\dfrac{1}{\lambda_{mn}} = R\left(\dfrac{1}{m^2} - \dfrac{1}{n^2}\right)$ where $m = 2$ for Balmer series (Equation 4–2)

$$\frac{1}{379.1\,nm} = \frac{1.097\times10^7 m^{-1}}{10^9\,nm/m}\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$\frac{1}{4} - \frac{1}{n^2} = \frac{10^9\,nm/m}{379.1\,nm(1.097\times10^7 m^{-1})} = 0.2405$$

$$\frac{1}{n^2} = 0.2500 - 0.2405 = 0.0095$$

$$n^2 = \frac{1}{0.0095} \rightarrow n = (1/0.0095)^{1/2} = 10.3 \rightarrow n = 10$$

4-3. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ where $m = 1$ for Lyman series (Equation 4–2)

$$\frac{1}{164.1\,nm} = \frac{1.097\times10^7 m^{-1}}{10^9 nm/m}\left(1 - \frac{1}{n^2}\right)$$

$$\frac{1}{n^2} = 1 - \frac{10^9\ nm/m}{164.1\,nm(1.097\times10^7\,m^{-1})} = 1 - 0.5555 = 0.4445$$

$$n = (1/0.4445)^{1/2} = 1.5$$

No, this is not a hydrogen Lyman series transition because n is not an integer.

4-4. $\frac{1}{\lambda_{mn}} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$ (Equation 4–2)

For the Brackett series m = 4 and the first four (i.e., longest wavelength) lines have n = 5, 6, 7, and 8.

$$\frac{1}{\lambda_{45}} = 1.097\times10^7\,m^{-1}\left(\frac{1}{4^2} - \frac{1}{5^2}\right) = 2.468\times10^5\,m^{-1}$$

$\lambda_{45} = \frac{1}{2.68\times10^5\,m^{-1}} = 4.052\times10^{-6} m = 4052\,nm$. Similarly,

$$\lambda_{46} = \frac{1}{3.809\times10^5\,m^{-1}} = 2.625\times10^{-6} m = 2625\,nm$$

$$\lambda_{47} = \frac{1}{4.617\times10^5\,m^{-1}} = 2.166\times10^{-6} m = 2166\,nm$$

$$\lambda_{48} = \frac{1}{5.142\times10^5\,m^{-1}} = 1.945\times10^{-6} m = 1945\,nm$$

These lines are all in the infrared.

4-5. None of these lines are in the Paschen series, whose limit is 820.4 nm (see Problem 4-1) and whose first line is given by: $\frac{1}{\lambda_{34}} = R\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \rightarrow \lambda_{34} = 1875\,nm$. Also, none are in the Brackett series, whose longest wavelength line is 4052 nm (see Problem 4-4). The Pfund series has $m = 5$. Its first three (i.e., longest wavelength) lines have $n = 6, 7$, and 8.

$$\frac{1}{\lambda_{56}} = R\left(\frac{1}{5^2} - \frac{1}{6^2}\right) = 1.341\times 10^5\,m^{-1}$$

$\lambda_{56} = 1/1.341\times 10^5\,m^{-1} = 7.458\times 10^{-6}m = 7458\,nm$. Similarly,

$$\lambda_{57} = 1/2.155\times 10^5\,m^{-1} = 4.653\times 10^{-6}m = 4653\,nm$$

$$\lambda_{58} = 1/2.674\times 10^5\,m^{-1} = 3.740\times 10^{-6}m = 3740\,nm$$

Thus, the line at 4103 nm is not a hydrogen spectral line.

4-6. (a) $f = \pi b^2 nt$ (Equation 4 – 5)

For Au, $n = 5.90\times 10^{28}\,atoms/m^3$ (see Example 4-2) and for this foil

$t = 2.0\,\mu m = 2.0\times 10^{-6}\,m$.

$$b = \frac{kq_\alpha Q}{m_\alpha v^2}\cot\frac{\theta}{2} = \frac{(2)(79)ke^2}{2K_\alpha}\cot\frac{90}{2} = \frac{(2)(79)(1.44\,eV\cdot nm)}{2(7.0\times 10^6\,eV)}$$

$$= 1.63\times 10^{-5}\,nm = 1.63\times 10^{-14}\,m$$

$$f = \pi(1.63\times 10^{-14}m)^2(5.90\times 10^{28}/m^3)(2.0\times 10^{-6}m) = 9.8\times 10^{-5}$$

(b) For $\theta = 45°$,

$$b(45°) = b(90°)(\cot 45°/2)/(\cot 90°/2)$$

$$= b(90°)(\tan 90°/2)/(\tan 45°/2)$$

$$= 3.92\times 10^{-5}\,nm = 3.92\times 10^{-14}\,m$$

(Problem 4-6 continued)

and $f(45°) = 5.7\times10^{-4}$

For $\theta = 75°$, $b(75°) = b(90°)(\tan 90°/2)/(\tan 75°/2)$

$= 2.12\times10^{-5}\,nm = 2.12\times10^{-14}\,m$

and $f(75°) = 1.66\times10^{-4}$

Therefore, $\Delta f(45° - 75°) = 5.7\times10^{-4} - 1.66\times10^{-4} = 4.05\times10^{-4}$

4-7. $\Delta N \propto \dfrac{1}{\sin^4(\theta/2)} = \dfrac{A}{\sin^4(\theta/2)}$ (from Equation 4-6), where A is the product of the two quantities in parentheses in Equation 4-6.

(a) $\dfrac{\Delta N(10°)}{\Delta N(1°)} = \dfrac{A/\sin^4(10°/2)}{A/\sin^4(1°/2)} = \dfrac{\sin^4(0.5°)}{\sin^4(5°)} = 1.01\times10^{-4}$

(b) $\dfrac{\Delta N(30°)}{\Delta N(1°)} = \dfrac{\sin^4(0.5°)}{\sin^4(15°)} = 1.29\times10^{-6}$

4-8. $b = \dfrac{kq_\alpha Q}{m_\alpha v^2}\cot\dfrac{\theta}{2}$ (Equation 4 − 3)

$= \dfrac{k\cdot 2e\cdot Ze}{m_\alpha v^2}\cot\dfrac{\theta}{2} = \dfrac{(1.44\,MeV\cdot fm)Z}{E_{k\alpha}}\cot\dfrac{\theta}{2}$

$= \dfrac{(1.44\,MeV\cdot fm)(79)}{7.7\,MeV}\cot\dfrac{2°}{2} = 8.5\times10^{-13}\,m$

4-9. $r_d = \dfrac{kq_\alpha Q}{½ m_\alpha v^2} = \dfrac{ke^2\cdot 2\cdot 79}{E_{k\alpha}}$ (Equation 4 − 11)

For $E_{k\alpha} = 5.0\,MeV$: $r_d = \dfrac{(1.44\,MeV\cdot fm)(2)(79)}{5.0\,MeV} = 45.5\,fm$

(Problem 4-9 continued)

For $E_{k\alpha} = 7.7\,MeV \quad r_d = 29.5\,fm$

For $E_{k\alpha} = 12\,MeV \quad r_d = 19.0\,fm$

4-10. $r_d = \dfrac{kq_\alpha Q}{\frac{1}{2}m_\alpha v^2} = \dfrac{ke^2 \cdot 2 \cdot 79}{E_{k\alpha}}$ (Equation 4 − 11)

$$E_{k\alpha} = \frac{(1.44\,MeV \cdot fm)(2)(13)}{4\,MeV} = 9.4\,MeV$$

4-11. $x_{rms} = \sqrt{N}(\delta) \quad 10° = \sqrt{N}(0.01°) \rightarrow N = (10°/0.01°)^2 = 10^6$ *collisions*

$$n = \frac{t}{\Delta t} = \frac{10^{-6}\,m}{10^{-10}\,m} = 10^4\ layers$$

10^4 atomic layers is not enough to produce a deflection of 10°, assuming 1 collision/ layer.

4-12. (a) $f = \pi b^2 nt$ (Equation 4 − 5)

For $\theta = 25°$ (refer to Problem 4-6).

$$b = \frac{(2)(79)ke^2}{2K_\alpha}\cot\frac{25}{2} = \frac{(2)(79)(1.44\,eV{\cdot}nm)}{2(7.0\times10^6 eV)}\cot\left(\frac{25°}{2}\right)$$

$$= 7.33\times10^{-5}\,nm = 7.33\times10^{-14}\,m$$

$$f = \pi(7.33\times10^{-14}\,m)^2(5.90\times10^{28}/m^3)(2.0\times10^{-6}\,m) = 1.992\times10^{-3}$$

Because $\Delta N = f{\times}N = 1000 \quad \Rightarrow \quad N = 1000/1.992\times10^{-3} = 5.02\times10^5$

For $\theta = 45°$, $b = \dfrac{(2)(79)(1.44\,eV{\cdot}nm)}{2(7.0\times10^6 eV)}\cot\left(\dfrac{45°}{2}\right) = 3.92\times10^{-14}\,m$

(Problem 4-12 continued)

$f = \pi(3.92\times10^{-14}m)^2(5.90\times10^{28}/m^3)(2.0\times10^{-6}m) = 5.70\times10^{-4}$

Because $\Delta N(\theta > 45°) = f \times N = 5.70\times10^{-4}(5.02\times10^5) = 286$

(b) $\Delta N(25° \rightarrow 45°) = 1000 - 286 = 714$

(c) For θ = 75°, $b = b(\theta{>}25°)(\tan 25°/2)/(\tan 75°/2) = 2.12\times10^{-14}m$

$f = 1.992\times10^{-3}(2.12\times10^{-14}m)^2/(7.33\times10^{-14})^2$

$= 1.992\times10^{-3}(2.12/7.33)^2 = 1.67\times10^{-4}$

For θ = 90°, $b = b(\theta{>}25°)(\tan 25°/2)/(\tan 90°/2) = 1.63\times10^{-14}m$

$f = 1.992\times10^{-3}(1.63\times10^{-14}m)^2/(7.33\times10^{-14})^2$

$= 1.992\times10^{-3}(1.63/7.33)^2 = 9.85\times10^{-5}$

$\Delta N = f \times N = 9.85\times10^{-5}(5.02\times10^5) = 49$

$\Delta N(75° \rightarrow 90°) = 84 - 49 = 35$

4-13. (a) $r_n = \dfrac{n^2 a_0}{Z}$ Equation 4-18

$r_6 = \dfrac{6^2(0.053\,nm)}{1} = 1.91\,nm$

(b) $r_6(He^+) = \dfrac{6^2(0.053\,nm)}{2} = 0.95\,nm$

$a_o = \dfrac{\hbar^2}{mke^2}$ (Equation 4-19)

4-14. $= \dfrac{\hbar\hbar c}{mcke^2} = \dfrac{\hbar c}{mc^2}\cdot\dfrac{1}{ke^2/\hbar c} = \dfrac{1}{2\pi}\cdot\dfrac{h}{mc}\cdot\dfrac{1}{ke^2/\hbar c} = \dfrac{\lambda_c}{2\pi\alpha}$

(Problem 4-14 continued)

$$E_o = \frac{mk^2e^4}{2\hbar^2} \quad \text{(from Equation 4-20)}$$

$$= \frac{mc^2(ke^2)^2}{2(\hbar c)^2} = \frac{mc^2}{2}\cdot\left(\frac{ke^2}{\hbar c}\right)^2 = \frac{1}{2}mc^2\alpha^2$$

$$a_o = \frac{\lambda_c}{2\pi\alpha} = \frac{0.00243\,nm}{2\pi(1/137)} = 0.053\,nm \qquad E_o = \frac{1}{2}mc^2\alpha^2 = \frac{5.11\times10^5 eV}{2(137)^2} = 13.6\,eV$$

4-15. $$\frac{1}{\lambda} = Z^2R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \quad \text{(Equation 4-22)}$$

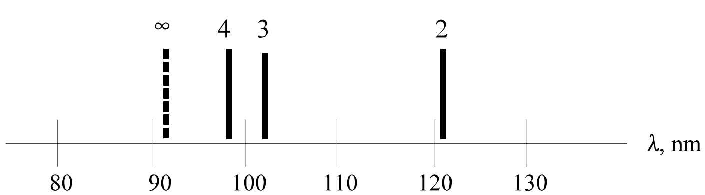
$$\frac{1}{\lambda_{ni}} = R\left(\frac{1}{1^2} - \frac{1}{n_i^2}\right) = R\left(\frac{n_i^2 - 1}{n_i^2}\right)$$

$$\lambda_{ni} = \frac{n_i^2}{R(n_i^2 - 1)} = \frac{n_i^2}{(1.0968\times10^7 m)(n_i^2 - 1)} = (91.17\,nm)\left(\frac{n_i^2}{n_i^2 - 1}\right)$$

$$\lambda_2 = \frac{4}{3}(91.17\,nm) = 121.57\,nm \qquad \lambda_3 = \frac{9}{8}(91.17\,nm) = 102.57\,nm$$

$$\lambda_4 = \frac{16}{15}(91.17\,nm) = 97.25\,nm \qquad \lambda_\infty = 91.17\,nm$$

None of these are in the visible; all are in the ultraviolet.



4-16. $L = mvr = n\hbar$ (Equation 4-17)

$$m_E = 5.98\times10^{24}\,kg \quad v_E = 2\pi r/1y = 2\pi r/3.16\times10^7 s$$

$$n = m(2\pi r/3.16\times10^7 s)r/\hbar = 2\pi m r^2/(3.16\times10^7 s)\hbar$$

$$= \frac{2\pi(5.98\times10^{24}\,kg)(1.50\times10^{11}\,m)^2}{3.16\times10^7 s(1.055\times10^{-34}\,J\cdot s)} = 2.54\times10^{74}$$

$$mv = n\hbar/r \;\rightarrow\; E = (mv)^2/2m = (n\hbar/r)^2/2m \quad \text{(from Equation 4-17)}$$

$$\Delta E = \left(\frac{\hbar}{r}\right)^2 \frac{1}{2m}(2n\Delta m) = \frac{(1.055\times10^{-34}\,J\cdot s)^2(2.54\times10^{74})(1)}{(1.50\times10^{11}\,m)^2(5.98\times10^{24}\,kg)} = 0.210\times10^{-40}\,J$$

This would not be detectable.

$$\Delta E = \frac{(n\hbar)^2}{2m}\left(-\frac{2\Delta r}{r^3}\right) = \frac{(1.055\times10^{-34}\,J\cdot s)^2(2.54\times10^{74})^2(-\Delta r)}{(1.50\times10^{11}\,m)^3(5.98\times10^{24}\,kg)} = 3.56\times10^{22}(-\Delta r)$$

or $-\Delta r = 0.210\times10^{-40} J/3.56\times10^{22}\,J/m = 5.90\times10^{-64}\,m$

The orbit radius r would still be $1.50\times10^{11}\,m$.

4-17. (a) $\lambda = 410.7\,nm$ is in the visible region of the spectrum, so this is a transition ending on $n = 2$ (see Figure 4-16).

$$\frac{hc}{\lambda} = E_n - E_2 = 13.6\,eV\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$$

$$\frac{1240\,eV\cdot nm}{410.7\,nm} = 13.6\,eV\left(\frac{1}{4} - \frac{1}{n^2}\right)$$

$$\frac{1}{n^2} = \frac{1}{4} - \frac{1240\,eV\cdot nm}{410.7\,nm(13.6\,eV)} = 0.0280$$

$$n = \left(\frac{1}{0.0280}\right)^{1/2} = 6$$

(b) This line is in the Balmer series.

4-18. The number of revolutions N in 10^{-8} s is:

$$N = 10^{-8}s/(time/revolution) = 10^{-8}s/(circumference\ \ of\ \ orbit/speed)$$

$$N = 10^{-8}s/(C/v) = 10^{-8}s/(2\pi r/v)$$

The radius of the orbit is given by

$$r = \frac{n^2 a_0}{Z} = \frac{4^2(0.0529\,nm)}{3}$$

so the circumference of the orbit C = 2πr is

$$C = 2\pi[4^2(0.0529\,nm)/3] = 1.77\,nm = 1.77\times10^{-9}m$$

The electron's speed in the orbit is given by

$$v^2 = (kZe^2/mr) = \frac{(8.99\times10^9 N\cdot m^2/C^2)(3)(1.60\times10^{-19}C)^2}{(9.11\times10^{-31}kg)(1.77\times10^{-9}m)}$$

$$v = 6.54\times10^5 m/s$$

Therefore, $N = 10^{-8}s/(C/v) = 3.70\times10^6\ revolutions$

In the planetary analogy of Earth revolving around the sun, this corresponds to 3.7 million "years".

4-19. (a) $a_\mu = \frac{\hbar^2}{\mu_\mu ke^2} = \frac{\mu_e}{\mu_\mu}\cdot\frac{\hbar^2}{\mu_e ke^2} = \frac{\mu_e}{\mu_\mu}a_o = \frac{9.11\times10^{-31}kg}{1.69\times10^{-28}kg}(0.0529\,nm) = 2.85\times10^{-4}\,nm$

(b) $E_\mu = \frac{\mu_\mu k^2e^4}{2\hbar^2} = \frac{\mu_\mu}{\mu_e}\cdot\frac{\mu_e k^2e^4}{2\hbar^2} = \frac{\mu_\mu}{\mu_e}\cdot E_o = \frac{1.69\times10^{-28}kg}{9.11\times10^{-31}kg}(13.6\,eV) = 2520\ eV$

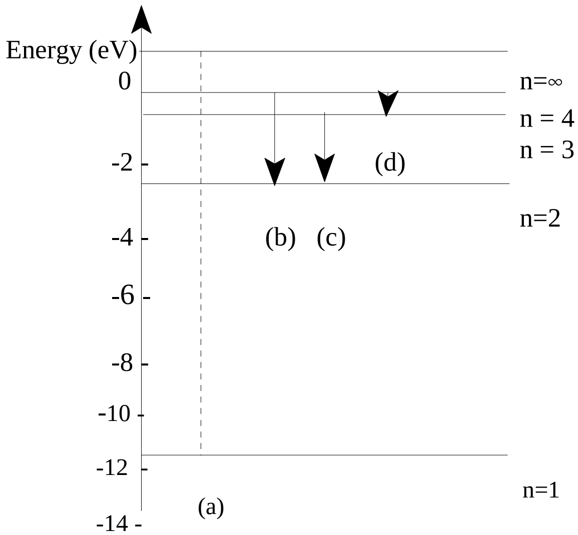
(c) The shortest wavelength in the Lyman series is the series limit ($n_i = \infty$, $n_f = 1$). The photon energy is equal in magnitude to the ground state energy $-E_\mu$.

$$\lambda_\infty = \frac{hc}{E_\mu} = \frac{1240\,eV\cdot nm}{2520\,eV} = 0.492\ nm$$

(The reduced masses have been used in this solution.)

4-20. $E = -Z'^2 E_o / n^2 \qquad Z' = \left[\dfrac{-n^2 E}{E_o}\right]^{1/2} = \left[\dfrac{-2^2(-5.39\,eV)}{13.6\,eV}\right]^{1/2} = 1.26$

4-21.



(a) Lyman limit (b) H_β line (c) H_α line (d) 1st line of Paschen series

4-22. (a) $\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_f^2} - \dfrac{1}{n_i^2}\right)$

For Lyman α: $\dfrac{1}{\lambda_L} = 1.097373 \times 10^7 m^{-1}\left(\dfrac{1}{1^2} - \dfrac{1}{2^2}\right) \rightarrow \lambda_L = 121.5023\,nm$

$E_L = \dfrac{hc}{\lambda_L} = \dfrac{1240\,eV\cdot nm}{121.5023\,nm} = 10.2056\,eV$ and $p_L = \dfrac{E_L}{c} = 10.2056\,eV/c$

Conservation of momentum requires that the recoil momentum of the H atom $p_H = p_L$ and the recoil energy E_H is:

$$E_H = (p_H)^2/2m_H = (p_H c)^2/2m_H c^2 = \frac{(10.2056\,eV/c)^2}{2(1.007825\,uc^2)(931.50\times 10^6\,eV/uc^2)}$$

$$= 5.55\times 10^{-8}\,eV$$

(Problem 4-22 continued)

(b) $$\frac{E_H}{(E_L + E_H)} \approx \frac{5.55\times 10^{-8} eV}{10.21\ eV} = 5\times 10^{-9}$$

4-23. $$r_n = \frac{n^2 a_0}{Z}$$ (Equation 4-18)

For C (Z = 6): $r_1 = a_0/6 = 0.059\,nm/6 = 8.82\times 10^{-3}\ nm$

$E_n = -E_0 Z^2/n^2$ (Equation 4-20)

$E_1 = -13.6\,eV(6)^2/(1)^2 = -490\,eV$

$$\frac{hc}{\lambda} = E_2 - E_1 \rightarrow \lambda_L = \frac{hc}{E_2 - E_1} = \frac{1240\ eV\cdot nm}{-122\,eV - (-490\,eV)} = 3.37\ nm$$

4-24. (a) The reduced mass correction to the Rydberg constant is important in this case.

$$R = R_\infty\left(\frac{1}{1 + m/M}\right) = R_\infty\left(\frac{1}{2}\right) = 5.4869\times 10^6\ m^{-1}$$ (from Equation 4-26)

$E_n = -hcR/n^2$ (from Equation 4-23)

$E_1 = -(1240\,eV\cdot nm)(5.4869\times 10^6\ m^{-1})(10^{-9} m/nm)/(1)^2 = -6.804\ eV$

Similarly, $E_2 = -1.701\ eV$ and $E_3 = -0.756\ eV$

(b) Lyman α is the $n = 2 \rightarrow n = 1$ transition.

$$\frac{hc}{\lambda_\alpha} = E_2 - E_1 \rightarrow \lambda_\alpha = \frac{hc}{E_2 - E_1} = \frac{1240\,eV\cdot nm}{-1.701\,eV - (-6.804\ eV)} = 243\ nm$$

Lyman β is the $n = 3 \rightarrow n = 1$ transition.

$$\lambda_\beta = \frac{hc}{E_3 - E_1} = \frac{1240\,eV\cdot nm}{-0.756\,eV - (-6.804\ eV)} = 205\ nm$$

4-25. (a) The result of the Bohr orbits are given by (Equation 4-18)

$r = n^2 a_0/Z$ where a_0 = 0.0529 nm and Z = 1 for hydrogen.

For n = 600, $r = (600)^2(0.0529\text{ nm}) = 1.90\times 10^4\text{ nm} = 19.0\ \mu\text{m}$

This is about the size of a tiny grain of sand.

(b) The electron's speed in a Bohr orbit is given by

$v^2 = ke^2/mr$ with Z = 1

Substituting r for the n = 600 orbit from (a), then taking the square root,

$$v^2 = (8.99\times 10^9 N\cdot m^2)(1.60\times 10^{-19}C)^2 / (9.11\times 10^{-31}kg)(19.0\times 10^{-6}\mu m)$$

$$v^2 = 1.33\times 10^7 m^2/s^2$$

$$v = 3.65\times 10^3 m/s$$

For comparison, in the n = 1 orbit, v is about 2×10^6 m/s.

4-26. (a)

$$\frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\lambda_3 = \left[(1.097\times 10^7 m^{-1})(42-1)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right)\right]^{-1} = 6.10\times 10^{-11} m = 0.0610\ nm$$

$$\lambda_4 = \left[(1.097\times 10^7 m^{-1})(42-1)^2\left(\frac{1}{1^2} - \frac{1}{4^2}\right)\right]^{-1} = 5.78\times 10^{-11} m = 0.0578\ nm$$

(b) $$\lambda_{limit} = \left[(1.097\times 10^7 m^{-1})(42-1)^2\left(\frac{1}{1^2} - 0\right)\right]^{-1} = 5.42\times 10^{-11} m = 0.0542\ nm$$

4-27. $$\frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) \quad for\ K_\alpha$$

$$Z-1 = \left[\frac{1}{\lambda R\left(1-\frac{1}{4}\right)}\right]^{1/2} = \left[\frac{1}{(0.0794\,nm)(1.097\times 10^{-2}/nm)(3/4)}\right]^{1/2}$$

$$Z = 1 + 39.1 \approx 40 \quad Zirconium$$

4-28. $$\frac{1}{\lambda} = R(Z-7.4)^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = R(Z-7.4)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \quad for\ L_\alpha$$

$$Z-7.4 = \left[\frac{1}{\lambda R\left(\frac{1}{4}-\frac{1}{9}\right)}\right]^{1/2} = \left[\frac{36}{(0.3617\times 10^{-9}m)(1.097\times 10^{7}/m)(5)}\right]^{1/2}$$

$$Z = 7.4 + 42.6 = 50 \quad Tin$$

4-29. $r_n = \dfrac{n^2 a_0}{Z}$ (Equation 4-18)

The n = 1 electrons "see" a nuclear charge of approximately Z – 1, or 78 for Au.

$$r_1 \approx 0.0529\,nm/78 = 6.8\times 10^{-4}\,nm \approx 6.8\times 10^{-4}\,nm(10^{-9}\,m/nm)(10^{15}\,fm/m) = 680\,fm$$

or about 100 times the radius of the Au nucleus.

4-30. (a) $\lambda = \left[R_\infty (Z-1)^2(1-1/n^2)\right]^{-1}$ (Equation 4-37)

The limit of the K series (n = ∞) is the energy required to remove a K electron from the tungsten (Z = 74) atom.

(Problem 4-30 continued)

$$E = hc/\lambda = hcR_\infty (Z-1)^2 (1 - 1/\infty)$$

$$= (1240\, eV\cdot nm)(1.0973\times 10^7 m^{-1})(10^{-9} m/nm)(73)^2(1)$$

$$= 7.25\times 10^4\, eV = 72.5\, keV$$

A 72.5 kV potential must be applied to produce the K_α line. In that case,

$$\lambda_{\min} = \frac{1.24\times 10^3\, nm\cdot V}{72.5\times 10^3\, V} = 0.017\, nm \quad \text{(from Equation 3-39)}$$

(b) For Cu (Z = 79) the energy required to remove a K electron from the atom is:

$$E = hc/\lambda = (1240\, eV\cdot nm)(1.0973\times 10^7 m^{-1})(10^{-9} m/nm)(28)^2(1)$$

$$= 1.067\times 10^4\, eV = 10.7\, keV$$

A 10.7 kV potential must be applied to produce the K_α line in Cu. In that case,

$$\lambda_{\min} = \frac{1.24\times 10^3\, nm\cdot V}{10.7\times 10^3\, V} = 0.116\, nm \quad \text{(from Equation 3-39)}$$

(c) For Cu (Z=29) the energy require to remove an L electron from the atom is

$$E = hcR_\infty (Z-7.4)^2 \left(\frac{1}{2^2} - \frac{1}{\infty}\right) \quad \text{(from Equation 4-38)}$$

$$= (1240\, eV\cdot nm)(1.0973\times 10^7 m^{-1})(10^{-9} m/nm)(29-7.4)^2(1/2^2)$$

$$= 1.59\times 10^3\, eV = 1.59\, keV$$

or 1.59 kV must be applied to produce the Lα line in Cu. In that case,

$$\lambda_{\min} = \frac{1.24\times 10^3\, nm\cdot V}{1.59\times 10^3\, V} = 0.782\, nm$$

4-31. $$E = \gamma m_e c^2 = \frac{511\, keV}{\sqrt{1-(2.25\times 10^8/3.00\times 10^8)^2}} = 772.6\, keV$$

After emitting a 32.5 keV photon, the total energy is:

(Problem 4-31 continued)

$$E = 740.1\,keV = \frac{511\,keV}{\sqrt{1-\beta^2}} \rightarrow \beta^2 = v^2/c^2 = 1-(511/740)^2$$

$$v = [1-(511/740)^2]^{1/2}c = 2.17\times 10^8 m/s$$

4-32. (a) $-E_1 = E_0 Z^2/n^2$ (Equation 4-20)

$$= 13.6\,eV(74-1)^2/(1)^2 = 7.25\times 10^4 eV = 72.5\ keV$$

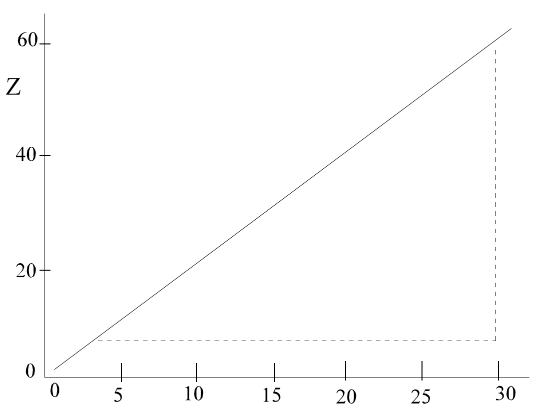
(b) $-E_1 = E_0(Z-\sigma)^2/n^2 = 69.5\times 10^3\ eV = 13.6\ eV(74-\sigma)^2/(1)^2$

$$(74-\sigma)^2 = 69.5\times 10^3\ eV/13.6\ eV$$

$$\sigma = 74-(69.5\times 10^3/13.6\,eV)^{1/2} = 2.5$$

4-33.

Element	Al	Ar	Sc	Fe	Ge	Kr	Zr	Ba
Z	13	18	21	26	32	36	40	56
E (keV)	1.56	3.19	4.46	7.06	10.98	14.10	17066	36.35
$f^{1/2}$ (10^8 $Hz^{1/2}$)	6.14	8.77	10.37	13.05	16.28	18.45	20.64	29.62



$f^{1/2}$ $(10^8\ Hz^{1/2})$

(Problem 4-33 continued)

$$\text{slope} = \frac{58 - 10}{(30 - 4.8)\times 10^8} = 1.90\times 10^{-8}\,Hz^{-1/2}$$

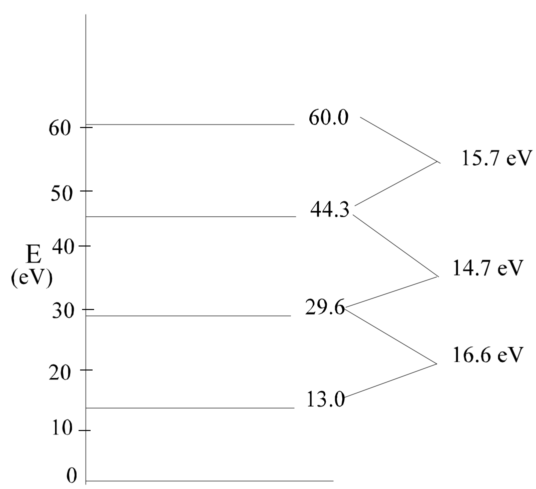
$$\text{slope}(\text{Fig. }4-18) = \frac{30 - 13}{(15 - 7)\times 10^8} = 2.13\times 10^{-8}\,Hz^{-1/2}$$

The two values are in good agreement.

4-34. (a) The available energy is not sufficient to raise ground state electrons to the n=5 level which requires $13.6 - 0.54 = 13.1\,eV$. The shortest wavelength (i.e., highest energy) spectral line that will be emitted is the 3rd line of the Lyman series, the $n = 4 \rightarrow n = 1$ transition. (See Figure 4-16.)

(b) The emitted lines will be for those transitions that begin on the n = 4, n = 3, or n = 2 levels. These are the first three lines of the Lyman series, the first two lines of the Balmer series, and the first line of the Paschen series.

4-34.



Average transition energy = 15.7 eV

4-36. $\Delta E = \dfrac{hc}{\lambda} = \dfrac{1240\,eV\cdot nm}{790\,nm} = 1.610\,eV$. The first decrease in current will occur when the voltage reaches 1.61 eV.

4-37. Using the results from Problem 4-24, the energy of the positronium Lyman α line is $\Delta E = E_2 - E_1 = -1.701\,eV - (-6.804\,eV) = 5.10\,eV$. The first Franck-Hertz current decrease would occur at 5.10 V, the second at 10.2 V.

4-38. In an elastic collision, both momentum and kinetic energy are conserved. Introductory physics texts derive the following expression when the second object (the Hg atom here) is initially at rest: $v_{1f} = \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right) v_{1i}$. The fraction of the initial kinetic energy lost by the incident electron in a head-on collision is:

$$f = \frac{KE_{ei} - KE_{ef}}{KE_{ei}} = \frac{v_{1i}^2 - v_{1f}^2}{v_{1i}^2} = \frac{v_{1i}^2 - \left(\dfrac{m_1 - m_2}{m_1 + m_2}\right)^2 v_{1i}^2}{v_{1i}^2}$$

$$= 1 - \left(\frac{m_1 - m_2}{m_1 + m_2}\right)^2 = 1 - \left(\frac{0.511\,MeV - 200\,uc^2(931.5\,MeV/uc^2}{0.511\,MeV + 200\,uc^2(931.5\,MeV/uc^2}\right)^2$$

$$= 1.10 \times 10^{-5}$$

If the collision is not head-on, the fractional loss will be less.

4-39. $N = I_0(2\pi b)\,db$ where $b = \dfrac{kq_\alpha Q}{m_\alpha v^2}\cot\dfrac{\theta}{2}$ (Equation 4-3)

and $db = \dfrac{kq_\alpha Q}{2m_\alpha v^2}\left(-csc\dfrac{\theta}{2}\right)d\theta$

$$N = I_0 2\pi \left(\frac{kq_\alpha Q}{m_\alpha v^2}\right)^2 \left(\frac{1}{2}\cot\frac{\theta}{2}\right)\left(csc^2\frac{\theta}{2}\right)d\theta$$

Using the trigonometric identities: $csc^2 = \dfrac{1}{\sin^2\theta/2}$ and

(Problem 4-39 continued)

$$\cot\frac{\theta}{2} = \frac{\sin\theta}{1-\cos\theta} = \frac{\sin\theta}{1-\cos^2(\theta/2)+\sin^2(\theta/2)} = \frac{\sin\theta}{2\sin^2\theta/2}$$

$$N = I_0 2\pi\left(\frac{kq_\alpha Q}{m_\alpha v^2}\right)^2\left(\frac{1}{2}\right)\left(\frac{\sin\theta}{2\sin^2(\theta/2)}\right)\left(\frac{1}{\sin^2\theta/2}\right)d\theta$$

and inserting $2e = q_\alpha$ and $Ze = Q$,

$$N = I_0 2\pi\left(\frac{kZe^2}{m_\alpha v^2}\right)^2\frac{\sin\theta\, d\theta}{\sin^4(\theta/2)}$$

4-40. Those scattered at $\theta = 180°$ obeyed the Rutherford formula. This is a head-on collision where the α comes instantaneously to rest before reversing direction. At that point its kinetic energy has been converted entirely to electrostatic potential energy, so

$\frac{1}{2}m_\alpha v^2 = 7.7\,MeV = \frac{k(2e)(79e)}{r}$ where r = upper limit of the nuclear radius.

$$r = \frac{k(2)(79)e^2}{7.7\,MeV} = \frac{2(79)(1.440\,MeV\cdot fm)}{7.7\,MeV} = 29.5\,fm$$

4-41. (a) $$i = q f_{rev} = e\frac{Z^2mk^2e^4}{2\pi\hbar^3n^3} \quad \text{(from Equation 4-28)}$$

$$= e\frac{mc^2(ke^2)^2(1)^2}{2\pi\hbar(\hbar c)^2(1)^3} = \frac{ec}{(h/mc)}\left(\frac{ke^2}{\hbar c}\right)^2 = \frac{ec\alpha^2}{\lambda_c}$$

$$= \frac{(1.602\times10^{-19}C)(3.00\times10^{17}nm/s)}{0.00243\,nm}\left(\frac{1}{137}\right)^2 = 1.054\times10^{-3}A$$

(b) $$\mu = iA = i\pi a_o^2 = \left(\frac{emk^2e^4}{2\pi\hbar^3}\right)\pi\left(\frac{\hbar^2}{mke^2}\right) = \frac{e\hbar}{2m}$$

$$= \frac{(1.602\times10^{-19}C)(1.055\times10^{-34}J\cdot s)}{2(9.11\times10^{-31}kg)} = 9.28\times10^{-24}A\cdot m^2$$

or

$$= (1.054\times10^{-3}A)\,\pi\,(0.529\times10^{-10}m)^2 = 9.27\times10^{-24}A\cdot m^2$$

4-42. Using the Rydberg-Ritz equation (Equation 4-2), set up the columns of the spreadsheet to carry out the computation of λ as in this example.

m	*n*	m^2	n^2	*1/C–1/D*	*1/λ*	*λ(nm)*
1	5	1	25	0.96	10534572	94.92
1	4	1	16	0.9375	10287844	97.20
1	3	1	9	0.888889	9754400	102.52
1	2	1	4	0.75	8230275	121.50
2	6	4	36	0.222222	2438600	410.07
2	5	4	25	0.21	2304477	433.94
2	4	4	16	0.1875	2057569	486.01
2	3	4	9	0.138889	1524125	656.11
3	7	9	49	0.090703	995346.9	1004.67
3	6	9	36	0.083333	914475	1093.52
3	5	9	25	0.071111	780352	1281.47
3	4	9	16	0.048611	533443.8	1874.61

4-43. $\lambda = \left[R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)\right]^{-1}$ $\quad\quad$ $\Delta\lambda \approx \frac{d\lambda}{d\mu}\Delta\mu = (-R^{-2})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)^{-1}\frac{dR}{d\mu}\Delta\mu$

Because $R \propto \mu$, $dR/d\mu = R/\mu$. $\Delta\lambda \approx (-R^{-2})\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)^{-1}(R/\mu)\Delta\mu = -\lambda(\Delta\mu/\mu)$

$\mu_H = \frac{m_e m_p}{m_e + m_p}$ $\quad\quad$ $m_D = \frac{m_e m_d}{m_e + m_d}$

$$\frac{\Delta\mu}{\mu} = \frac{\mu_D - \mu_H}{\mu_H} = \frac{\mu_D}{\mu_H} - 1 = \frac{m_e m_d/(m_e + m_d)}{m_e m_p/(m_e + m_p)} - 1 = \frac{m_d/(m_e + m_d)}{m_p/(m_e + m_d)} - 1 = \frac{m_e(m_d - m_p)}{m_p(m_e + m_d)}$$

If we approximate $m_d = 2m_p$ and $m_e << m_d$, then $\frac{\Delta\mu}{\mu} \approx \frac{m_e}{2m_p}$ and

$$\Delta\lambda = -\lambda(\Delta\mu/\mu) = -(656.3\,nm)\frac{0.511\,MeV}{2(938.28\,MeV)} = -0.179\,nm$$

4-44. $f = n\pi b^2 t$ (Equation 4-5) $n = \dfrac{\rho N_A}{m} = \dfrac{(2.70\,g/cm^3)(6.02\times10^{23}/mole)}{27.0\,g/mole} = 6.02\times10^{22}/cm^3$

$$b = \frac{kqQ}{mv^2}\cot\frac{\theta}{2} \quad \text{(Equation 4-3)}$$

$$= \frac{keZe}{2E_k}\cos\frac{\theta}{2} = \frac{(1.44\times10^{-7}eV\cdot cm)(13)}{2(10^7 eV)}\cot\frac{\theta}{2}$$

$$= (9.36\times10^{-14}cm)\cot\frac{\theta}{2}$$

$$f = (6.02\times10^{22}/cm^3)\,\pi\left[(9.36\times10^{-14}cm)\cot\frac{\theta}{2}\right]^2(10^{-4}cm) = (1.66\times10^{-7})\cot^2\frac{\theta}{2}$$

(a) $\theta = 10°$ $\quad \cot\frac{\theta}{2} = 11.43 \quad f = (1.66\times10^{-7})(11.43)^2 = 2.17\times10^{-5}$

(b) $\theta = 90°$ $\quad \cot\frac{\theta}{2} = 1 \quad f = (1.66\times10^{-7})(1)^2 = 1.66\times10^{-7}$

4-45. (a) $E_n = -E_0 Z^2/n^2$ (Equation 4-20)

For Li^{++}, Z = 3 and $E_n = -13.6eV(9)/n^2 = -122.4/n^2\,eV$

The first three Li^{++} levels that have the same (nearly) energy as H are:

n = 3, $E_3 = -13.6$ eV $\quad$ n = 4, $E_6 = -3.4$ eV $\quad$ n = 9, $E_9 = -1.51$ eV

Lyman α corresponds to the $n = 6 \rightarrow n = 3\;Li^{++}$ transition. Lyman β corresponds to the $n = 9 \rightarrow n = 3\;Li^{++}$ transition.

(b) $R(H) = R_\infty(1/(1 + 0.511\,MeV/938.8\,MeV)) = 1.096776\times10^7\,m^{-1}$

$R(Li) = R_\infty(1/(1 + 0.511\,MeV/6535\,MeV)) = 1.097287\times10^7\,m^{-1}$

For Lyman α:

$$\frac{1}{\lambda} = R(H)\left(1 - \frac{1}{2^2}\right) = 1.096776\times10^7 m^{-1}(10^{-9}m/nm)(3/4) \rightarrow \lambda = 121.568\,nm$$

For Li^{++} equivalent:

$$\frac{1}{\lambda} = R(Li)\left(\frac{1}{3^2} - \frac{1}{6^2}\right)Z^2 = 1.097287\times10^7 m^{-1}(10^{-9}m/nm)\left(\frac{1}{9} - \frac{1}{36}\right)(3)^2$$

$\lambda = 121.512\,nm \qquad \Delta\lambda = 0.056\,nm$

4-46. $\Delta N = \left(\frac{I_0 A_{SC}\, nt}{r^2}\right)\left(\frac{kZe^2}{2E_K}\right)^2 \frac{1}{\sin^4\frac{\theta}{2}}$ (Equation 4-6)

where $A_{SC} = 0.50\, cm^2$ $\quad r = 10\, cm$ $\quad t = 10^{-6} m$

$$n(Ag) = \frac{(10.5\, g/cm^3)(6.02\times 10^{23}\, atoms/mol)}{107.5\, g.mol}$$

$$= 5.88\times 10^{22}\, atoms/cm^3 = 5.88\times 10^{28}\, atoms/m^3$$

$$E_K = 6.0\, MeV \qquad I_0 = 1.0\, nA = (10^{-9}\, C/s)\left(\frac{1}{2(1.60\times 10^{-19})C}\right) = 3.13\times 10^9\, alphas/s$$

(a) At $\theta = 60°$

$$\Delta N = \left(\frac{(3.13\times 10^9\, \alpha/s)(0.50\, cm^2)(5.88\times 10^{28}/m^3)(10^{-6})}{10^2\, cm^2}\right)$$

$$= \left(\frac{(9\times 10^9 N{\cdot}m^2/C^2)(1.60\times 10^{-19}C)^2(47)}{2(6.0\, MeV)(1.60\times 10^{-13}\, J/MeV)}\right)\left(\frac{1}{\sin^4\frac{60°}{2}}\right) = 468\, \alpha/s$$

(b) At $\theta = 120°$: $\Delta N = \Delta N(60°)\left(\sin^4\frac{60°}{2}\right)\Big/\left(\sin^4\frac{120°}{2}\right) = 52\, \alpha/s$

4-47. $E_n = -E_0 Z^2/n^2$ (Equation 4-20)

For Ca, Z = 20 and $E_1 = -13.6\, eV(20)^2/1^2 = -5.440\, keV$

The fact that E_1 computed this way is only approximate is not a serious problem because the measured X ray energies provide us the correct *spacings* between the levels.

$$E_2 = E_1 + 3.69\, keV = -5.440 + 3.69 = -1.750\, keV$$

$$E_3 = E_2 + 0.341\, keV = -1.750 + 0.341 = -1.409\, keV$$

$$E_4 = E_3 + 0.024\, keV = -1.409 + 0.024 = -1.385\, keV$$

These are the ionization energies for the levels. Auger electron energies $\Delta E - |E_n|$

(Problem 4-47 continued)

where $\Delta E = 3.69\ keV$.

Auger L electron : $3.69\ keV - 1.750\ keV = 1.94\ keV$

Auger M electron : $3.69\ keV - 1.409\ keV = 2.28\ keV$

Auger N electron : $3.69\ keV - 1.385\ keV = 2.31\ keV$

4-48. (a) $E_\alpha = hc/\lambda = 1240\,eV{\cdot}nm/0.071\,nm = 17.465\ keV$

$E_\beta = hc/\lambda = 1240\,eV{\cdot}nm/0.063\,nm = 19.683\ keV$

(b) Select Nb (Z = 41)

The Kβ Mo X rays have enough energy to eject photoelectrons, producing 0.693 keV electrons. The Kα Mo X rays could not produce photoelectrons in Nb.

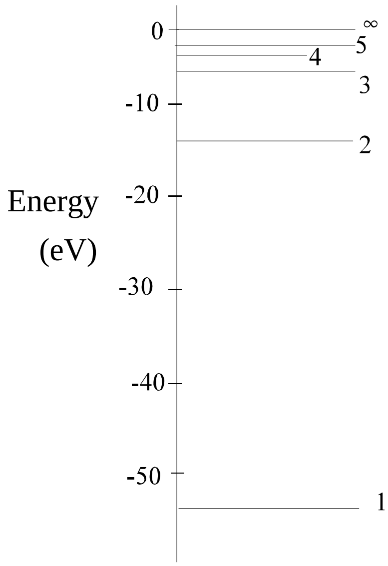
4-49. (a) $b = R\sin\beta = R\sin\left(\dfrac{180° - \theta}{2}\right) = R\cos\dfrac{\theta}{2}$

(b) Scattering through an angle larger than θ corresponds to an impact parameter smaller than b. Thus, the shot must hit within a circle of radius *b* and area πb^2. The rate at which this occurs is $I_o \pi b^2 = I_o R^2 \cos^2\dfrac{\theta}{2}$

(c) $\sigma = \pi b_o^2 = \pi\left(R\cos\dfrac{0}{2}\right)^2 = \pi R^2$

(d) An α particle with an arbitrarily large impact parameter still feels a force and is scattered.

4-50. For He: $E_n = -13.6\,eV\,Z^2/n^2 = -54.4\,eV/n^2$ (from Equation 4-20)



$E_1 = -54.4\,eV$

$E_2 = -13.6\,eV$

$E_3 = -6.04\,eV$

$E_4 = -3.04\,eV$

$E_5 = -2.18\,eV$

$E_\infty = 0$

(b) Ionization energy is 54.4 eV.

(c) H Lyman α $\lambda = hc/\Delta E = 1240\,eV{\cdot}nm/(13.6\,eV - 3.4\,eV) = 121.6\,nm$

H Lyman β $\lambda = hc/\Delta E = 1240\,eV{\cdot}nm/(13.6\,eV - 1.51\,eV) = 102.6\,nm$

He^+ Balmer α $\lambda = 1240\,eV{\cdot}nm/(13.6\,eV - 6.04\,eV) = 164.0\,nm$

He^+ Balmer β $\lambda = 1240\,eV{\cdot}nm/(13.6\,eV - 3.40\,eV) = 121.6\,nm$

$\Delta\alpha = 42.4\,nm \qquad \Delta\beta = 19.0\,nm$

(The reduced mass correction factor does not change the energies calculated above to three significant figures.)

(d) $E_n = -13.6\,eV\,Z^2/n^2$ Because for He^+ Z=2, then $Z^2 = 2^2$. Every time n is an even number a 2^2 can be factored out of n^2 and canceled with the $Z^2 = 2^2$ in the numerator; e.g., for He^+,

$$E_2 = -13.6\,eV{\cdot}2^2/2^2 = -13.6\,eV \quad \text{(H ground state)}$$

$$E_4 = -13.6\,eV{\cdot}2^2/4^2 = -13.6\,eV/2^2 \quad \text{(H – 1}^{\text{st}}\text{ excited state)}$$

$$E_6 = -13.6\,eV{\cdot}2^2/6^2 = -13.6\,eV/3^2 \quad \text{(H – 2}^{\text{nd}}\text{ excited state)}$$

⋮

etc.

(Problem 4-50 continued)

Thus, all of the H energy level values are to be found within the He^+ energy levels, so He^+ will have within its spectrum lines that match (nearly) a line in the H spectrum.

4-51.

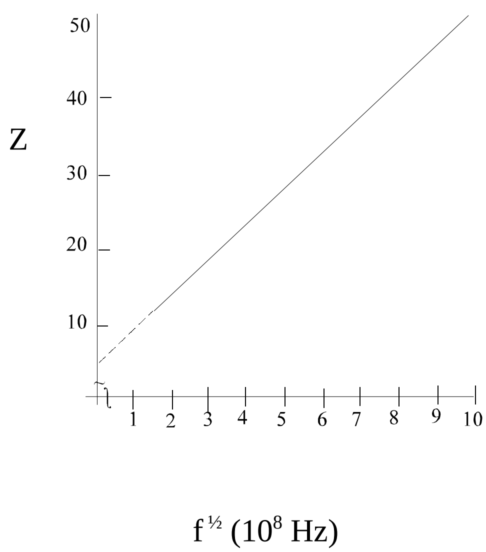
Element	P	Ca	Co	Kr	Mo	I
Z	15	20	27	36	42	53
Lα λ (nm)	10.41	4.05	1.79	0.73	0.51	0.33
$f^{1/2}$ (10^8 Hz)	1.70	2.72	4.09	6.41	7.67	9.53

where $f^{1/2} = \left[(3.00\times10^8\,m/s)(10^9\,nm/m)/\lambda\right]^{1/2}$

$$\text{Slope} = \frac{50-15}{(9.15-1.58)\times10^8\,Hz} = 4.62\times10^{-8}\,Hz^{-1}$$

$$\text{Slope (Fig. 4-18)} = \frac{74-46}{(14-8)\times10^8\,Hz} = 4.67\times10^{-8}\,Hz^{-1}$$

The agreement is very good.



The $f^{1/2} = 0$ intercept on the Z axis is the minimum Z for which an Lα Xray could be emitted. It is about Z = 8.

4-52. (a) $E_n = -\dfrac{ke^2}{2r_n} = -\dfrac{ke^2}{2n^2 r_o}$ $\qquad E_{n-1} = -\dfrac{ke^2}{2(n-1)^2 r_o}$

$$hf = E_n - E_{n-1} = -\frac{ke^2}{2n^2 r_o} - \left(-\frac{ke^2}{2(n-1)^2 r_o}\right)$$

$$f = \frac{ke^2}{2hr_o}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{ke^2}{2hr_o}\frac{n^2 - (n^2 - 2n + 1)}{n^2(n-1)^2}$$

$$= \frac{ke^2}{2hr_o}\frac{2n-1}{n^2(n-1)^2} \approx \frac{ke^2}{r_o hn^3} \quad \text{for } n >> 1$$

(b) $f_{rev} = \dfrac{v}{2\pi r} \rightarrow f_{rev}^2 = \dfrac{v^2}{4\pi^2 r^2} = \dfrac{1}{4\pi^2 mr}\dfrac{mv^2}{r} = \dfrac{1}{4\pi^2 mr}\dfrac{ke^2}{r^2} = \dfrac{ke^2}{4\pi^2 mr_o^3 n^6}$

(c) The correspondence principle implies that the frequencies of radiation and revolution are equal.

$$f^2 = \left(\frac{ke^2}{r_o hn^3}\right)^2 = \frac{ke^2}{4\pi^2 mr_o^3 n^6} = f_{rev}^2 \qquad r_o = \frac{ke^2}{4\pi^2 mn^6}\left(\frac{hn^3}{ke^2}\right)^2 = \frac{h^2}{4\pi^2 mke^2} = \frac{\hbar^2}{mke^2}$$

which is the same as a_o in Equation 4-19.

4-53. $\dfrac{kZe^2}{r} = \dfrac{mv^2}{r} \rightarrow \dfrac{kZe^2}{r^2} = \dfrac{(\gamma mv)^2}{mr}$ (from Equation 4-12)

$$\gamma v = \left(\frac{kZe^2}{mr}\right)^{1/2} = \frac{v}{\sqrt{1-\beta^2}}$$

$$\frac{c^2\beta^2}{1-\beta^2} = \left(\frac{kZe^2}{mr}\right) \quad \text{Therefore,} \quad \beta^2\left[c^2 + \left(\frac{kZe^2}{mr}\right)\right] = \left(\frac{kZe^2}{mr}\right)$$

$$\beta^2 \approx \frac{1}{c^2}\left(\frac{kZe^2}{ma_o}\right) \rightarrow \beta = 0.0075Z^{1/2} \rightarrow v = 0.0075cZ^{1/2} = 2.25\times10^6 m/s\times Z^{1/2}$$

(Problem 4-53 continued)

$$E = KE - kZe^2/r = mc^2(\gamma - 1) - \frac{kZe^2}{r} = mc^2\left[\frac{1}{\sqrt{1-\beta^2}} - 1\right] - \frac{kZe^2}{r}$$

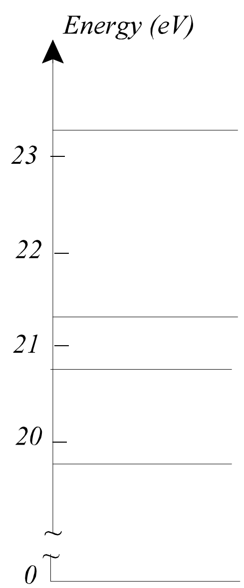
and substituting $\beta = 0.0075$ and $r = a_0$

$$E = 511 \times 10^3\, eV\left[\frac{1}{\sqrt{1-(0.0075)^2}} - 1\right] - 28.8ZeV$$

$$= 14.4eV - 28.8ZeV = -14.4ZeV$$

4-54. (The solution to this problem depends on the kind of calculator or computer you use and the program you write.)

4-55.



Levels constructed from Figure 4-25.

4-56. Centripetal acceleration would be provided by the gravitational force:

$$F_G = G\frac{Mm}{r^2} = \frac{mv^2}{r}$$ M = proton mass and m = electron mass, so $v = \left(\frac{GM}{r}\right)^{1/2}$

(Problem 4-56 continued)

$L = mvr = n\hbar \rightarrow r = n\hbar/mv$ or

$$r_n = \frac{n\hbar}{m(GM/r_n)^{1/2}} \rightarrow r_n^2 = \frac{n^2\hbar^2 r_n}{m^2 GM} \text{ and, } r_n = \frac{n^2\hbar^2}{GMm^2} \rightarrow a_0 = \frac{\hbar^2}{GMm^2}$$

The total energy is: $E = \frac{1}{2}mv^2 + \left(-\frac{GMm}{r}\right) = \frac{1}{2}m\left(\frac{GM}{r}\right) - -\frac{GMm}{2r}$

$$E_n = -\frac{GMm}{2r_n} = -\frac{(GMm)(GMm^2)}{2n^2\hbar^2} = -\frac{G^2M^2m^3}{2n^2\hbar^2}$$

The gravitational Hα line is: $\Delta E = E_2 - E_3 = \frac{G^2M^2m^3}{2\hbar^2}\left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

$$\Delta E = \frac{(6.67\times10^{-11}N\cdot m^2/kg^2)^2(1.67\times10^{-27}kg)^2(9.11\times10^{-31}kg)^3(0.1389)}{2(1.055\times10^{-34})^2}$$

$$= 5.85\times10^{-98}J = 3.66\times10^{-79}eV$$

$$f = \frac{\Delta E}{h} = \frac{5.85\times10^{-98}J}{6.63\times10^{-34}J\cdot s} = 8.28\times10^{-65}Hz$$

For the Balmer limit in each case,

$\Delta E = 3.66\times10^{-79}eV(0.250/0.1389) = 6.58\times10^{-79}eV$

$f = 6.58\times10^{-79}eV/h = 1.59\times10^{-64}Hz$

These values are immeasurably small. They do not compare with the actual H values.

4-57. Refer to Figure 4-16. All possible transitions starting at $n = 5$ occur.

$n = 5$ to $n = 4, 3, 2, 1$

$n = 4$ to $n = 3, 2, 1$

$n = 3$ to $n = 2, 1$

$n = 2$ to $n = 1$

Thus, there are 10 different photon energies emitted.

(Problem 4-57 continued)

n_i	n_f	*fraction*	*no. of photons*
5	4	1/4	125
5	3	1/4	125
5	2	1/4	125
5	1	1/4	125
4	3	1/4 × 1/3	42
4	2	1/4 × 1/3	42
4	1	1/4 × 1/3	42
3	2	$1/2\,[1/4 + 1/4(1/3)]$	83
3	1	$1/2\,[1/4 + 1/4(1/3)]$	83
2	1	$[1/2\,(1/4 + 1/4)(1/3)) + 1/4(1/3) + 1/4]$	250

Total = 1,042

Note that the number of electrons arriving at the $n = 1$ level (125 + 42 + 83 + 250) is 500, as it should be.