Chapter 5 – The Wavelike Properties of Particles

5-1. (a) $\lambda = \dfrac{h}{p} = \dfrac{h}{mv} = \dfrac{(6.63\times10^{-34}J\cdot s)(3.16\times10^{7}s/y)}{(10^{-3}kg)(1\,m/y)} = 2.1\times10^{-23}m$

(b) $v = \dfrac{h}{m\lambda} = \dfrac{6.63\times10^{-34}J\cdot s}{(10^{-3}kg)(10^{-2}m)} = 6.6\times10^{-29}m/s = 2.1\times10^{-21}m/y$

5-2. $\lambda = \dfrac{h}{p} \approx \dfrac{h}{E/c} = \dfrac{hc}{E} = \dfrac{1240\,MeV\cdot fm}{100\,MeV} = 12.4\,fm$

5-3. $E_k = eV_o = \dfrac{p^2}{2m} = \dfrac{h^2}{2m\lambda^2} = \dfrac{(hc)^2}{2mc^2\lambda^2}$ $\quad V_o = \dfrac{1}{e}\cdot\dfrac{(1240\,eV\cdot nm)^2}{2(5.11\times10^5\,eV)(0.04\,nm)^2} = 940\,V$

5-4. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2mE_k}} = \dfrac{hc}{\sqrt{2mc^2E_k}}$ (from Equation 5-2)

(a) For an electron: $\lambda = \dfrac{1240\,eV\cdot nm}{[(2)(0.511\times10^6\,eV)(4.5\times10^3\,eV)]^{1/2}} = 0.0183\,nm$

(b) For a proton: $\lambda = \dfrac{1240\,eV\cdot nm}{[(2)(938.3\times10^6\,eV)(4.5\times10^3\,eV)]^{1/2}} = 4.27\times10^{-4}\,nm$

(c) For an alpha particle: $\lambda = \dfrac{1240\,eV\cdot nm}{[(2)(3.728\times10^9\,eV)(4.5\times10^3\,eV)]^{1/2}} = 2.14\times10^{-4}\,nm$

5-5. $\lambda = h/p = h/\sqrt{2mE_k} = hc/[2mc^2(1.5kT)]^{1/2}$ (from Equation 5-2)

Mass of N_2 molecule $= 2\times14.0031\,u(931.5\,MeV/uc^2) = 2.609\times10^4\,MeV/c^2 = 2.609\times10^{10}\,eV/c^2$

$\lambda = \dfrac{1240\,eV\cdot nm}{[(2)(2.609\times10^{10}eV)(1.5)(8.617\times10^{-5}\,eV/K)(300\,K)]^{1/2}} = 0.0276\,nm$

5-6. $$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} = \frac{hc}{\sqrt{2mc^2E_k}} = \frac{1240\,eV\cdot nm}{\left[2(939.57\times10^6\,eV)(0.02\,eV)\right]^{1/2}} = 0.202\,nm$$

5-7. (a) If there is a node at each wall, then $n(\lambda/2) = L$ where $n = 1, 2, 3, \ldots$ or $\lambda = 2L/n$.

(b) $p = h/\lambda = hm/2L \qquad E = p^2/2m = (hn/2L)^2/2m = h^2n^2/8mL^2$

$$E_n = \frac{(hc)^2 n^2}{8mc^2L^2}$$

For n = 1: $$E_1 = \frac{(1240\,eV\cdot nm)^2\,(1)^2}{8\,(938\times10^6\,eV)(0.01\,nm)^2} = 2.05\,eV$$

For n = 2: $E_2 = 2.05\,eV(2)^2 = 8.20\,eV$

5-8. (a) $\lambda/\lambda_c = 10^2$ is a nonrelativistic situation, so

$$\lambda/\lambda_c = \left[(hc/\sqrt{2mc^2E_k})/(hc/mc^2)\right] = (mc^2/2E_k)^{1/2}$$

$$E_k = \frac{mc^2}{2(\lambda/\lambda_c)^2} = \frac{0.511\times10^6\,eV}{2(10^2)^2} = 25.6\,eV$$

(b) $\lambda/\lambda_c = 0.2$ is relativistic for an electron, so $\lambda = h/\gamma mu \rightarrow \gamma u = h/\lambda m$

$$\frac{u/c}{\sqrt{1-(u/c)^2}} = \frac{h}{mc\lambda} = \frac{\lambda_c}{\lambda}$$

$$\frac{(u/c)^2}{1-(u/c)^2} = \left(\frac{\lambda_c}{\lambda}\right)^2 \rightarrow u/c = \frac{\lambda_c/\lambda}{\left[1+(\lambda_c/\lambda)^2\right]^{1/2}}$$

$$u/c = \frac{(1/0.2)}{\left[1+(1/0.2)^2\right]^{1/2}} = 0.981 \rightarrow \gamma = 5.10$$

$$E_k = mc^2(\gamma-1) = 0.511\,MeV(\gamma-1) = 2.10\,MeV$$

(Problem 5-8 continued)

(c) $\lambda/\lambda_c = 10^{-3}$

$$u/c = \frac{(1/10^{-3})}{[1 + (1/10^{-3})^2]^{1/2}} = 0.9999 \rightarrow \gamma = 1000$$

$$E_k = mc^2(\gamma - 1) = 0.511\,MeV(999) = 510\,MeV$$

5-9. $E_k = mc^2(\gamma - 1) \qquad p = \gamma m u$

(a) $E_k = 2\,GeV \qquad mc^2 = 0.938\,GeV$

$\gamma - 1 = E_k/mc^2 = 2\,GeV/0.938\,GeV = 2.132$ Thus, $\gamma = 3.132$

Because, $\gamma = 1/\sqrt{1-(u/c)^2}$ where $u/c = 0.948$

$$\lambda = \frac{h}{p} = \frac{h}{\gamma mc(u/c)} = \frac{hc}{\gamma mc^2(u/c)}$$

$$= \frac{1240\,eV\cdot nm}{(3.132)(938\times10^6 eV)(0.948)} = 4.45\times10^{-7}\,nm = 0.445\,fm$$

(b) $E_k = 200\,GeV$

$\gamma - 1 = E_k/mc^2 = 200\,GeV/0.938\,GeV = 213.$ Thus, $\gamma = 214$ and $u/c = 0.9999$

$$\lambda = \frac{1240\,MeV\cdot fm}{(214)(938\,MeV)(0.9999)} = 6.18\times10^{-3}\,fm$$

5-10. $n\lambda = D\sin\phi \qquad$ (Equation 5 – 5)

$$\sin\phi = \frac{n\lambda}{D} = \frac{n}{D}\frac{hc}{\sqrt{2mc^2E_k}} \qquad \text{(see Problem 5 – 6)}$$

$$= \frac{1}{0.215\,nm}\cdot\frac{1240\,eV\cdot nm}{[2(5.11\times10^5 eV)]^{1/2}\sqrt{E_k}} = \frac{(5.705\,eV)^{1/2}}{\sqrt{E_k}}$$

(Problem 5-10 continued)

(a) $\sin\phi = \dfrac{(5.705\,eV)^{1/2}}{\sqrt{75\,eV}} = 0.659 \qquad \phi = \sin^{-1}(0.659) = 41.2°$

(b) $\sin\phi = \dfrac{(5.705\,eV)^{1/2}}{\sqrt{100\,eV}} = 0.570 \qquad \phi = \sin^{-1}(0.570) = 34.8°$

5-11. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2m_p E_k}} = 0.25\,nm$

Squaring and rearranging,

$$E_k = \frac{h^2}{2m_p\lambda^2} = \frac{(hc)^2}{2(m_p c^2)\lambda^2} = \frac{(1240\,eV\cdot nm)^2}{2(938\times10^6\,eV)(0.25\,nm)^2}$$

$E_k = 0.013\,eV$

$n\lambda = D\sin\phi \rightarrow \sin\phi = n\lambda/D = (1)(0.25\,nm)/0.304\,nm$

$\sin\phi = 0.822 \rightarrow \phi = 55°$

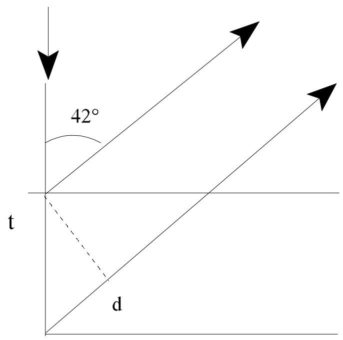
5-12. (a) $n\lambda = D\sin\phi \quad \therefore \quad D = \dfrac{n\lambda}{\sin\phi} = \dfrac{nhc}{\sin\phi\sqrt{2mc^2E_k}}$

$$= \frac{(1)(1240\,eV\cdot nm)}{(\sin 55.6°)[2(5.11\times10^5\,eV)(50\,eV)]^{1/2}} = 0.210\,nm$$

(b) $\sin\phi = \dfrac{n\lambda}{D} = \dfrac{(1)(1240\,eV\cdot nm)}{(0.210\,nm)[2(5.11\times10^5\,eV)(100\,eV)]^{1/2}}$

$= 0.584 \qquad \phi = \sin^{-1}(0.584) = 35.7°$

5-13.



$d = t\cos 42°$

$n\lambda = t + d = t(1 + \cos 42°) = 0.30\,nm(1 + \cos 42°)$

For the first maximum n = 1, so $\lambda = 0.523\,nm$

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE_k}} \rightarrow E_k = \frac{h^2}{2m\lambda_2} = \frac{(hc)^2}{2mc^2\lambda^2}$$

$$E_k = \frac{(1240\,eV{\cdot}nm)^2}{2(939.6\times 10^6\,eV)(0.523\,nm)^2} = 3.0\times 10^{-3}\,eV$$

5-14. $\lambda = \dfrac{D\sin\phi}{n}$ (Equation 5-6)

For 54 eV electrons λ = 0.165 nm and $\sin\phi = (0.165\,nm)n/0.215\,nm = 0.767n$

For n = 2 and larger $\sin\phi > 1$, so no values of *n* larger than one are possible.

5-15. $\sin\phi = n\lambda/D$ (Equation 5-6)

$$\lambda = h/p = h/\sqrt{2mE_k} = hc/\sqrt{2mc^2E_k}$$

$$= \frac{1240\,eV{\cdot}nm}{[2(0.511\times 10^6\,eV)(350\,eV)]^{1/2}} = 0.0656\,nm$$

$\sin\phi = n(0.0656\,nm)/0.315\,nm = 0.208\,n$

For $n = 1$, $\phi = 12°$ For $n = 2$, $\phi = 24.6°$ For $n = 3$, $\phi = 38.6°$

For $n = 4$, $\phi = 56.4°$ This is the largest possible φ. All larger *n* values have $\sin\phi > 1$.

5-16. (a) $\Delta t < \dfrac{1}{f} = \dfrac{1}{100{,}000\, s^{-1}} = 10^{-5}\, s = 10\,\mu s$

(b) $\Delta f\,\Delta t \approx \dfrac{1}{2\pi} \quad \therefore \quad \Delta f \approx \dfrac{1}{2\pi\,\Delta t} = \dfrac{1}{2\pi \times 10^{-5} s} = 1.59 \times 10^{4}\, Hz$

5-17. (a)

$$\begin{aligned} y &= y_1 + y_2 \\ &= 0.002m\cos(8.0x/m - 400t/s) + 0.002m\cos(7.6x/m - 380t/s) \\ &= 2(0.002m)\cos\left[\frac{1}{2}(8.0x/m - 7.6x/m) - \frac{1}{2}(400t/s - 380t/s)\right] \\ &\qquad \times \cos\left[\frac{1}{2}(8.0x/m + 7.6x/m) - \frac{1}{2}(400t/s + 380t/s)\right] \\ &= 0.004m\cos(0.2x/m - 10t/s) \times \cos(7.8x/m - 390t/s) \end{aligned}$$

(b) $v = \dfrac{\overline{\omega}}{\overline{k}} = \dfrac{390/s}{7.8/m} = 50\, m/s$

(c) $v_g = \dfrac{\Delta\omega}{\Delta k} = \dfrac{20/s}{0.4/m} = 50\, m/s$

(d) Successive zeros of the envelope requires that $0.2\,\Delta x/m = \pi$, thus $\Delta x = \dfrac{\pi}{0.2} = 5\pi m$ with

$\Delta k = k_1 - k_2 = 0.4m^{-1}$ and $\Delta x = \dfrac{2\pi}{\Delta k}$.

5-18. (a) $v = f\lambda$ Thus, $\dfrac{dv}{d\lambda} = f + \lambda\dfrac{df}{d\lambda}$, multiplying by λ, $\lambda\dfrac{dv}{d\lambda} = \lambda f + \lambda^2\dfrac{df}{d\lambda} = v + \dfrac{\lambda^2}{2\pi}\dfrac{d\omega}{d\lambda}$

$-\dfrac{\lambda^2}{2\pi}\dfrac{d\omega}{d\lambda} = v - \lambda\dfrac{dv}{d\lambda}$ Because $k = 2\pi/\lambda$, $dk = -(2\pi/\lambda^2)d\lambda$ and

$\dfrac{d\omega}{dk} = v_g = v - \lambda\dfrac{dv}{d\lambda}$

(b) v decreases as λ decreases. $dv/d\lambda$ is positive.

5-19. (a) $c = f\lambda = \lambda/T \rightarrow T = \lambda/c = 2\times10^{-2}m/3\times10^{8}m/s = 6.7\times10^{-11}\ s/wave$

The number of waves = $0.25\,\mu s/(6.7\times10^{-11}\ s/wave) = 3.73\times10^{3}$

Length of the packet = (# waves)(λ) = $2\times10^{-2}m\,(3.73\times10^{3}) = 74.6\ m$

(b) $f = c/\lambda = (3\times10^{8}m/s)/2\times10^{-2}m = 1.50\times10^{10}\ Hz$

(c) $\Delta\omega\Delta t \approx 1 \rightarrow \Delta\omega \approx 1/\Delta t = 1/0.25\times10^{-6}s = 4.0\times10^{6}\ rad/s = 637\ kHz$

5-20. $\Delta\omega\Delta t \approx 1 \rightarrow \Delta\omega \approx 1/\Delta t = 1/0.25\,s = 4.0\ rad/s$ or $\Delta f \approx 0.6\ Hz$

5-21. $\Delta\omega\Delta t \approx 1 \rightarrow (2\pi\Delta f)\,\Delta t = 1$ Thus, $\Delta t \approx 1/(2\pi\times5000) = 3.2\times10^{-5}s$

5-22. (a)

$$\lambda = h/p = h/\sqrt{2mE_k} = hc/\sqrt{2mc^2E_k}$$

$$= \frac{1240\ eV\cdot nm}{[2(0.511\times10^{6}\ eV)(5\,eV)]^{1/2}} = 0.549\ nm$$

$d\sin\theta = \lambda/2$ For first minimum (see Figure 5-16).

$$d = \frac{\lambda}{2\sin\theta} = \frac{0.549\ nm}{2\sin5°} = 3.15\,nm \text{ slit separation}$$

(b) $\sin 5° = 0.5\,cm/L$ where L = distance to detector plane $L = \dfrac{0.5\ cm}{2\sin5°} = 5.74\ cm$

5-23. (a) The particle is found with equal probability in any interval in a force-free region. Therefore, the probability of finding the particle in any interval Δx is proportional to Δx. Thus, the probability of finding the sphere *exactly* in the middle, i.e., with Δx = 0 is zero.

(b) The probability of finding the sphere somewhere within 24.9cm to 25.1 cm is proportional to Δx = 0.2 cm. Because there is a force free length L = 48 cm available to the sphere and the probability of finding it somewhere in L is unity, then the probability that it will be found in Δx = 0.2 cm between 24.9 cm and 25.1 cm (or any interval of equal size) is: $P\Delta x = (1/48)(0.2\,cm) = 0.00417$.

5-24. Because the particle must be in the box, $\int_0^L \psi^* \psi dx = 1 = \int_0^L A^2 \sin^2(\pi x/L) dx = 1$

Let $u = \pi x/L$; $x = 0 \to u = 0$; $x = L \to u = \pi$ and $dx = (L/\pi)du$, so we have

$$\int_0^\pi A^2(L/\pi)\sin^2 u\,du = A^2(L/\pi)\int_0^\pi \sin^2 u\,du = 1$$

$$(L/\pi)A^2\int_0^\pi \sin^2 u\,du = (L/\pi)A^2\left[\frac{u}{2} - \frac{\sin 2u}{4}\right]_0^\pi = (L/\pi)A^2(\pi/2) = (LA^2)/2 = 1$$

$$\therefore\ A^2 = 2/L \to A = (2/L)^{1/2}$$

5-25. (a) At $x = 0$: $P dx = |\psi(0,0)|^2 dx = |Ae^0|^2 dx = A^2 dx$

(b) At $x = \sigma$: $P dx = |Ae^{-\sigma^2/4\sigma^2}|^2 dx = |Ae^{-1/4}|^2 dx = 0.61A^2 dx$

(c) At $x = 2\sigma$: $P dx = |Ae^{-4\sigma^2/4\sigma^2}|^2 dx = |Ae^{-1}|^2 dx = 0.14A^2 dx$

(d) The electron will most likely be found at x = 0, where *Pdx* is largest.

5-26. (a) One does not know at which oscillation of small amplitude to start or stop counting.

$$f = \frac{N}{\Delta t} \qquad \Delta f = \frac{\Delta N}{\Delta t} \approx \frac{1}{\Delta t}$$

(b) $\lambda = \dfrac{\Delta x}{N}$ and $k = \dfrac{2\pi}{\lambda} = \dfrac{2\pi N}{\Delta x}$, so $\Delta k = \dfrac{2\pi\,\Delta n}{\Delta x} \approx \dfrac{2\pi}{\Delta x}$

5-27. $\Delta E\,\Delta t \approx \hbar \to \Delta E \approx \hbar/\Delta t = \dfrac{1.055\times10^{-34} J{\cdot}s}{10^{-7} s(1.60\times10^{-19} J/eV)} \approx 6.6\times10^{-9}\, eV$

5-28. $\Delta x\,\Delta p \approx \hbar \quad \therefore \quad \Delta x \approx \dfrac{\hbar}{\Delta p} = \dfrac{\hbar}{m\,\Delta v} = \dfrac{1.055\times10^{-34}J{\cdot}s}{(10^{-6}\times10^{-3}kg)(0.01\times0.01\,m/s)} \approx 10^{-21}m$

5-29. $\Delta E\,\Delta t \approx \hbar \rightarrow \Delta E \approx \hbar/\Delta t = \dfrac{6.58\times10^{-16}eV{\cdot}s}{3.823\,d\,(8.64\times10^{4}s/d)} \approx 1.99\times10^{-21}\,eV$

The energy uncertainty of the excited state is ΔE, so the α energy can be no sharper than ΔE.

5-30. $\Delta x\Delta p \approx \hbar \rightarrow \lambda\Delta p \approx h \rightarrow \Delta p \approx h/\lambda$. Because $\lambda - h/p$, $p = h/\lambda$; thus $\Delta p = p$.

5-31. For the cheetah $p = mv = 30\,kg(40\,m/s) = 1200\,kg{\cdot}m/s$. Because $\Delta p = p$ (see Problem 5-30),

$\Delta x \approx \hbar/\Delta p = 50\,J{\cdot}s/1200\,kg{\cdot}m/s \approx 4.2\times10^{-2}\,m = 4.2\,cm$

5-32. Because $c = f\lambda$ for photon, $\lambda = cf = hc/hf = hc/E$, so

$$E = hc/\lambda = \frac{1240\,eV{\cdot}nm}{5.0\times10^{-3}nm} = 2.48\times10^{5}eV$$

and $p = E/c = \dfrac{2.48\times10^{5}eV}{3.0\times10^{8}m/s} = 8.3\times10^{-7}eV{\cdot}s/m$

For electron:

$$\Delta p = h/\Delta x = \frac{4.14\times10^{-15}eV{\cdot}s}{5.0\times10^{-12}m} = 8.3\times10^{-4}eV{\cdot}s/m$$

Notice that Δp for the electron is 1000 times larger than λ for the photon.

5-33. (a) For ^{48}Ti: $\Delta E_u(upper\ state) \approx \hbar/\Delta t = \dfrac{1.055\times10^{-34}J\cdot s}{1.4\times10^{-14}s\,(1.60\times10^{-13}J/MeV)} \approx 4.71\times10^{-10}\,MeV$

$$\Delta E_L(lower\ state) \approx \hbar/\Delta t = \frac{1.055\times10^{-34}J\cdot s}{3.0\times10^{-12}s\,(1.60\times10^{-13}J/MeV)} \approx 2.20\times10^{-10}\,MeV$$

$$\Delta E\,(total) = \Delta E_u + \Delta E_L = 6.91\times10^{-10}\,MeV$$

$$\frac{\Delta E_T}{E} = \frac{6.91\times10^{-10}MeV}{1.312\,MeV} = 5.3\times10^{-10}$$

(b) For Hα: $\Delta E_u \approx \dfrac{1.055\times10^{-34}J\cdot s}{10^{-8}s\,(1.60\times10^{-19}J/eV)} \approx 6.59\times10^{-8}\,eV$

and $\Delta E_L \approx 6.59\times10^{-8}\,eV$ also.

$\Delta E_T = 1.32\times10^{-7}\,eV$ is the uncertainty in the Hα transition energy of 1.9 eV.

5-34. The size of the object needs to be of the order of the wavelength of the 10 MeV neutron. $\lambda = h/p = h/\gamma mu$. γ and u are found from:

$$E_k = m_n c^2(\gamma - 1) \quad \text{or} \quad \gamma - 1 = 10\,MeV/939\,MeV$$

$$\gamma = 1 + 10/939 = 1.0106 = 1/(1 - u^2/c^2)^{1/2} \quad \text{or} \quad u = 0.14\,c$$

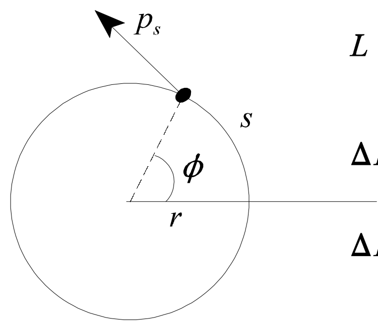
Then, $\lambda = h/\gamma mu = hc/[\gamma mc^2(u/c)]$

$$= \frac{1240\,eV\cdot nm}{[(1.0106)(939\times10^6 eV)(0.14)]} = 9.33\times10^{-6}\,nm = 9.33\,fm$$

Nuclei are of this order of size and could be used to show the wave character of 10 MeV neutrons.

5-35. $\Delta E\,\Delta t \approx \hbar \quad \therefore \quad \tau \approx \Delta t \approx \dfrac{\hbar}{\Delta E} = \dfrac{1.055\times10^{-34}J\cdot s}{(1\,eV)(1.602\times10^{-19}J/eV)} = 6.6\times10^{-16}s$

5-36.



$$L = rp_s \qquad \phi = \frac{s}{r}$$

$$\Delta L = r\,\Delta p_s \qquad \Delta\phi = \frac{\Delta s}{r}$$

$$\Delta L\,\Delta\phi = (r\,\Delta p_s)(\Delta s/r) = \Delta p_s\,\Delta s > \hbar$$

In the Bohr model, $L = n\hbar$ and may be known as well as $\Delta L \approx 0.1\,\hbar$. Then $\Delta\phi > \hbar/(0.1\,\hbar) = 10\,rad$. This exceeds one revolution, so that ϕ is completely unknown.

5-37. $E = hf \rightarrow \Delta E = h\Delta f$

$\Delta E \Delta t \approx h \rightarrow \Delta f \Delta t \approx 1$ where $\Delta t = 0.85$ ms

$\Delta f = 1/0.85$ ms = 1.

For $\lambda = 0.01$ nm $\quad f = c/\lambda = \dfrac{3.00\times10^{8}\,m/s \times 10^{9}\,nm/s}{0.01\,nm}$ 18×10^{9} Hz

$f = 3.00\times10^{19}$ Hz

$$\frac{\Delta f}{f} = \frac{1.18\times10^{9}\,Hz}{3.00\times10^{19}\,Hz} = 3.9\times10^{-11}$$

5-38. $\Delta p\,\Delta x \approx \hbar \rightarrow \Delta p \approx \hbar/\Delta x = \hbar/1fm$ Thus $(\Delta p)^2 = \overline{p^2} = \hbar^2/(1fm)^2$

For neutron:

$$E = \frac{p^2}{2m_n} = \frac{\hbar^2}{(1fm)^2}\cdot\frac{1}{2m_n} = \frac{(\hbar c)^2}{2m_n c^2(1fm)^2} = \frac{(197.3\,MeV\cdot fm)^2}{2(939\,MeV)(1fm)^2} = 20.7\,MeV$$

For electron: The electron is relativistic, because classical kinetic energy $p^2/2m$ is much larger than the rest energy. Therefore,

(Problem 5-38 continued)

$$E^2 = (pc)^2 + (m_e c^2)^2$$

$$E^2 = (\hbar c)^2/(fm)^2 + (m_e c^2)^2$$

$$E^2 = (197.30\,MeV\cdot fm)^2/(fm)^2 + (0.511\,MeV)^2$$

$$E^2 = 3.8928\times 10^4\,MeV^2$$

$$\therefore E = 197.30\,MeV$$

$$\text{and } E_\kappa = 197.30\,MeV - 0.511\,MeV = 196.79\,MeV$$

5-39. (a) $E^2 = p^2c^2 + m^2c^4$, $E = hf = \hbar\omega$, $p = h/\lambda = \hbar/k$, $\hbar^2\omega^2 = \hbar^2k^2c^2 + m^2c^4$

$$v = \frac{\omega}{k} = \frac{\hbar\omega}{\hbar k} = \frac{\sqrt{\hbar^2k^2c^2 + m^2c^4}}{\hbar k} = c\sqrt{1 + m^2c^2/\hbar^2k^2} > c$$

(b)
$$v_g = \frac{d\omega}{dk} = \frac{d}{dk}\sqrt{\frac{k^2c^2 + m^2c^4}{\hbar k}}$$

$$= \frac{c^2k}{\sqrt{\dfrac{k^2c^2 + m^2c^4}{\hbar^2}}} = \frac{c^2k}{\omega} = \frac{c^2\hbar k}{\hbar\omega} = \frac{c^2p}{E} = u \qquad \text{(by Equation 2-41)}$$

5-40. $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$ (Equation 5-11) $\qquad y_3 = c_1y_1 + c_2y_2$

$$\frac{\partial^2 y_3}{\partial x^2} = C_1\frac{\partial^2 y_1}{\partial x^2} + C_2\frac{\partial^2 y_2}{\partial x^2}$$

$$= C_1\left(\frac{1}{v^2}\frac{\partial^2 y_1}{\partial t^2}\right) + C_2 1\left(\frac{1}{v^2}\frac{\partial^2 y_2}{\partial t^2}\right)$$

$$= \frac{1}{v^2}\frac{\partial^2}{\partial t^2}(C_1y_1 + C_2\,y_2) = \frac{1}{v^2}\frac{\partial^2 y_3}{\partial t^2}$$

5-41. $E = \overline{E} = \dfrac{\overline{p^2}}{2m} + \dfrac{1}{2}m\omega^2\,\overline{x^2} = \dfrac{(\Delta p)^2}{2m} + \dfrac{1}{2}m\omega^2(\Delta x)^2.$ Substitute $\Delta p = \dfrac{\hbar}{2\Delta x}$

$$E = \frac{\hbar^2}{8m(\Delta x)^2} + \frac{1}{2}m\omega^2(\Delta x)^2.$$ To minimize E, set $dE/d(\Delta x) = 0$

$$0 = \frac{dE}{d(\Delta x)} = \frac{\hbar^2}{8m}\frac{-2}{(\Delta x)^3} + m\omega^2\Delta x = \frac{m\omega^2}{(\Delta x)^3}\left[-\frac{\hbar^2}{4m^2\omega^2} + (\Delta x)^4\right]$$

$$\therefore \quad (\Delta x)^2 = \frac{\hbar}{2m\omega}$$

$$E_{\min} = \frac{\hbar^2}{8m}\cdot\frac{2m\omega}{\hbar} + \frac{m\omega^2}{2}\frac{\hbar}{2m\omega} = \frac{1}{4}\hbar\omega + \frac{1}{4}\hbar\omega = \frac{1}{2}\hbar\omega$$

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{(1\,N/m)}{10^{-2}\,kg}} = 10/s$$

$$E_{\min} = \frac{1}{2}\hbar\omega = \frac{1}{2}(1.055\times10^{-34}\,J\cdot s)(10\,/s) = 5.27\times10^{-34}\,J$$

5-42. (a) $n(\lambda/2) = L \rightarrow \lambda = 2L/n$. Because $\lambda = h/p = h/\sqrt{2mE}$, then:

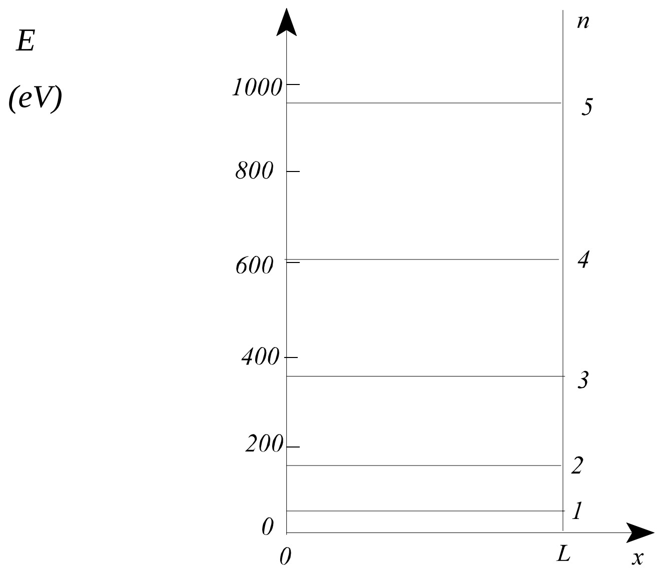
$$E = \frac{h^2}{2m\lambda^2} = \frac{h^2}{2m(2L/n)^2} = \frac{h^2n^2}{8mL^2}$$ If $E_1 = h^2/8mL^2$, then:

$$E_n = \frac{h^2n^2}{8mL^2} = n^2E_1$$

(b) For L = 0.1 nm, $E_1 = \dfrac{h^2}{8mL^2} = \dfrac{(hc)^2}{8mc^2L^2} = \dfrac{(1240\,eV\cdot nm)^2}{8(0.511\times10^6\,eV)(0.1\,nm)^2}$

$E_1 = 17.6\,eV$ and $E_n = 37.6n^2\,eV$

(Problem 5-42 continued)



(c) $f = \Delta E/h \rightarrow c/\lambda = \Delta E/h \rightarrow \lambda = \dfrac{hc}{\Delta E}$

For $n = 2 \rightarrow n = 1$ transition, $\Delta E = 112.8\,eV$ and $\lambda = \dfrac{1240\ eV{\cdot}nm}{112.8\,eV} = 11.0\,nm$

(d) For $n = 3 \rightarrow n = 2$ transition, $\Delta E = 188\,eV$ and $\lambda = \dfrac{1240\ eV{\cdot}nm}{188\,eV} = 6.6\,nm$

(e) For $n = 5 \rightarrow n = 1$ transition, $\Delta E = 903\,eV$ and $\lambda = \dfrac{1240\ eV{\cdot}nm}{903\,eV} = 1.4\,nm$

5-43. (a) For proton: $E_1 = \dfrac{(hc)^2}{8m_p c^2 L^2}$ from Problem 5-42.

$$E_1 = \frac{(1240\,MeV{\cdot}fm)^2}{8(938\,MeV)(1\,fm)^2} = 205\,MeV \text{ and } E_n = 205\,n^2\,MeV$$

$$\therefore E_2 = 820\,MeV \text{ and } E_3 = 1840\,MeV$$

(Problem 5-43 continued)

(b) For $n = 2 \rightarrow n = 1$ transition,

$$\lambda = \frac{hc}{\Delta E} = \frac{1240\,MeV{\cdot}fm}{615\,MeV} = 2.02\,fm$$

(c) For $n = 3 \rightarrow n = 2$ transition,

$$\lambda = \frac{1240\,MeV{\cdot}fm}{1020\,MeV} = 1.22\,fm$$

(d) For $n = 3 \rightarrow n = 1$ transition,

$$\lambda = \frac{1240\,MeV{\cdot}fm}{1635\,MeV} = 0.76\,fm$$

5-44. (a) $E \geq \hbar^2/2mL^2$ (Equation 5 − 31) And $E = \hbar^2/2mA^2$

(b) For electron with $A = 10^{-10}\,m$:

$$E = \frac{(\hbar c)^2}{2mc^2A^2} = \frac{(197.3\,eV{\cdot}nm)^2}{2(0.511\times10^6 eV)(10^{-1}nm)^2} = 3.81\,eV$$

For electron with A = 1 cm or A = 10^{-2}

$$E = 3.81\,eV(10^{-1})^2/(10^7\,nm)^2 = 3.81\times10^{-16}\,eV$$

(c) $$E = \frac{\hbar^2}{2mL^2} = \frac{(1.055\times10^{-34}\,J{\cdot}s)^2}{2(100\times10^{-3}g\times10^{-3}kg/g)(2\times10^{-2})^2} = 1.39\times10^{-61}J = 8.7\times10^{-43}\,eV$$

5-45. $\Delta p = m\Delta v = m(0.0001)(500\,m/s) = 0.05\,m$

For proton: $\Delta x\,\Delta p \approx \hbar$

$$\Delta x \approx \hbar/\Delta p = (6.58\times10^{-16}eV{\cdot}s)/(0.05\,m/s)(938\times10^6 eV)$$

$$\approx 1.40\times10^{-23}m = 1.40\times10^{-8}fm$$

For bullet: $\Delta x \approx = (1.055\times10^{-34}J{\cdot}s)/(0.05\,m/s)(10\times10^{-3}kg) \approx 2.1\times10^{-31}m$

5-46. $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\dfrac{\partial^2 y}{\partial t^2}$ (Equation 5-11) where $y = f(\phi)$ and $\phi = x - vt$

$$\frac{\partial y}{\partial x} = \frac{\partial f}{\partial \phi}\cdot\frac{\partial \phi}{\partial x} \quad \text{and} \quad \frac{\partial^2 y}{\partial x^2} = \frac{\partial f}{\partial \phi}\cdot\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial \phi}{\partial x}\cdot\frac{\partial^2 f}{\partial \phi^2}\cdot\frac{\partial \phi}{\partial x}$$

$$\frac{\partial y}{\partial t} = \frac{\partial f}{\partial \phi}\cdot\frac{\partial \phi}{\partial t} \quad \text{and} \quad \frac{\partial^2 y}{\partial t^2} = \frac{\partial f}{\partial \phi}\cdot\frac{\partial^2 \phi}{\partial t^2} + \frac{\partial \phi}{\partial t}\cdot\frac{\partial^2 f}{\partial \phi^2}\cdot\frac{\partial \phi}{\partial t}$$

Noting that $\partial^2\phi/\partial x^2 = 0$, $\partial\phi/\partial x = 1$, $\partial^2\phi/\partial t^2 = 0$, and $\partial\phi/\partial t = -v$, we then have:

$$\frac{\partial f}{\partial \phi}\cdot 0 + 1\cdot\frac{\partial^2 f}{\partial \phi^2}\cdot 1 = \frac{1}{v^2}\left(\frac{\partial f}{\partial \phi}\cdot 0 + (-v)\cdot\frac{\partial^2 f}{\partial \phi^2}\cdot(-v)\right)$$

$$\frac{\partial^2 f}{\partial \phi^2} = \frac{\partial^2 f}{\partial \phi^2}$$

5-47. (a) $\lambda = h/p$ The electrons are not moving at relativistic speeds, so

$$\lambda = h/mv = 6.63\times10^{-34} J{\cdot}s/(9.11\times10^{-31} kg)(3\times10^6 m/s)$$

$$= 2.43\times10^{-10} m = 0.243\ nm$$

(b) The energy, momentum, and wavelength of the two photons are equal.

$$E = \frac{1}{2}mv^2 + mc^2 = \frac{1}{2}mc^2(v^2/c^2) + mc^2$$

$$= mc^2\left[\frac{1}{2}(v^2/c^2) + 1\right]$$

$$= 0.511\times10^6\ eV\left[\frac{1}{2}(3\times10^6/3\times10^8)^2 + 1\right]$$

$$\approx 0.511\ MeV$$

(c) $p = E/c = 0.511\ MeV/c$

(d) $\lambda = hc/E = 1240\, eV{\cdot}nm/0.511\times10^6\ eV = 2.43\times10^{-3}\ nm$

5.48. (a) $$Q = m_p c^2 - m_n c^2 - m_\pi c^2$$

$$= 1.007825 u c^2 - 1.008665 u c^2 - 139.6\ MeV$$

$$= 938.8 MeV - 939.6 MeV - 139.6 MeV$$

$$= -140.4\ MeV$$

$$\Delta E = 140.4\ MeV$$

(b) $\Delta E\, \Delta t \approx \ \rightarrow\ \Delta t \approx \hbar / \Delta E = 6.58 \times 10^{-16} eV{\cdot}s / 140.4 \times 10^6\, eV \approx 4.7 \times 10^{-24} s$

(c) $d = c\Delta t = 3.0 \times 10^8 m/s (4.7 \times 10^{-24} s) = 1.4 \times 10^{-15} m = 1.4\, fm$

5-49. $$hf = \gamma m c^2 \ \rightarrow\ \gamma = \frac{hf}{mc^2} = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$1 - v^2/c^2 = \left(\frac{mc^2}{hf} \right)^2$$

$$\frac{v}{c} = \left[1 - \left(\frac{mc^2}{hf} \right)^2 \right]^{1/2}$$

Expanding the right side, assuming $mc^2 << hf$,

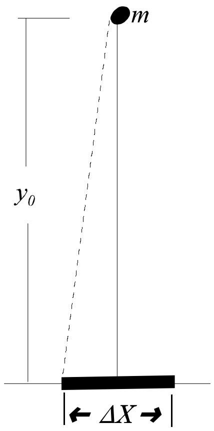
$\frac{v}{c} = 1 - \frac{1}{2}\left(\frac{mc^2}{hf} \right)^2 - \frac{1}{8}\left(\frac{mc^2}{hf} \right)^4 + \cdots$ and neglecting all but the first two terms,

$\frac{v}{c} = 1 - \frac{1}{2}\left(\frac{mc^2}{hf} \right)^2$ Solving this for *m* and inserting deBroglie's assumptions that

$\frac{v}{c} \geq 0.99$ and $\lambda = 30m$, *m* is then :

$$m = \frac{[(1 - 0.99)2]^{1/2} (6.63 \times 10^{-34} J{\cdot}s)}{(3.00 \times 10^8\, m/s)(30m)} = 1.04 \times 10^{-44}\, kg$$

5-50. (a)



$$\Delta x\,\Delta p \approx \hbar \rightarrow m\,\Delta x\,\Delta v_x \approx \hbar \rightarrow \Delta v_x \approx \hbar/m\,\Delta x$$

$$y_0 = \frac{1}{2}gt^2 \rightarrow t = \left(\frac{2y_0}{g}\right)^{1/2} \qquad \frac{1}{2}\Delta X = \Delta v_x \cdot t = \Delta v_x\left(\frac{2y_0}{g}\right)^{1/2}$$

$$\Delta X = 2\,\Delta v_x\left(\frac{2y_0}{g}\right)^{1/2} = \frac{2\hbar\left(\frac{2y_0}{g}\right)^{1/2}}{m\,\Delta x}$$

(b) If also $\Delta y\,\Delta p_y \approx \hbar \rightarrow \Delta v_y \approx \hbar/my$ and $\frac{1}{2}\Delta X = \Delta x(t + \Delta t)$ where $\Delta v_y = g\Delta t$ or

$$\Delta t = \Delta v_y/g = \hbar/mg\Delta y$$

so, $\Delta X = \dfrac{2\hbar}{m\Delta x}\left[(2y_0/g)^{1/2} + \hbar/mg\Delta y\right]$

5-51. $\frac{1}{2}m\,\overline{v^2} = \frac{3}{2}kT$

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \left[\frac{3(1.381\times10^{-23}J/K)(300K)}{56u(1.66\times10^{-27}kg/u)}\right]^{½} = 366\,m/s$$

$$f' = f_o(1 + v/c) \rightarrow hf' = hf_o(1 + v/c)$$

$$\Delta E = hf' - hf_o = hf_o v/c = \frac{(1\,eV)(366\,m/s)}{3.0\times10^8\,m/s} = 1.2\times10^{-6}\,eV$$

This is about 12 times the natural line width.

$$\Delta E = hf_o v/c = \frac{(10^6\,eV)(366\,m/s)}{3.0\times10^8\,m/s} = 1.2\,eV$$

This is over 10^7 times the natural line width.

5-52. $\rho_{recoil} = \rho_{\gamma} = E_{\gamma}/c$ $\qquad E_{recoil} = \dfrac{(\rho_{recoil})^2}{2m} = \dfrac{E_{\gamma}^2}{2mc^2}$

(a) $E_{recoil} = \dfrac{(1\,eV)^2}{2(56uc^2)} \dfrac{uc^2}{931.5\times 10^6 eV} = 9.6\times 10^{-12}\,eV$

This is about 10^{-4} times the natural line width estimated at 10^{-7} eV.

(b) $E_{recoil} = \dfrac{(1\,MeV)^2}{2(56uc^2)} \dfrac{uc^2}{931.5\times 10^6 eV} = 9.6\,eV$

This is about 10^{8} times the natural line width.