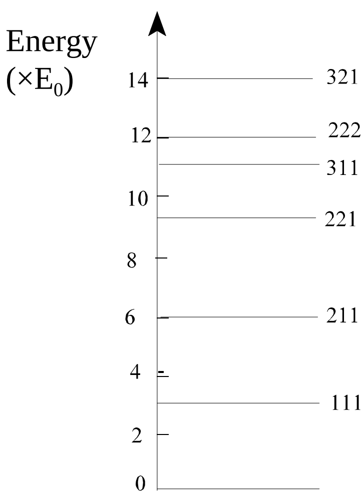
Chapter 7 – Atomic Physics

7-1. $E_{n_1 n_2 n_3} = \dfrac{\hbar^2 \pi^2}{2mL^2}(n_1^2 + n_2^2 + n_3^2)$ (Equation 7-4)

$E_{311} = \dfrac{\hbar^2 \pi^2}{2mL^2}(3^2 + 1^2 + 1^2) = 11E_0$ where $E_0 = \dfrac{\hbar^2 \pi^2}{2mL^2}$

$E_{222} = E_0(2^2 + 2^2 + 2^2) = 12E_0$ and $E_{321} = E_0(3^2 + 2^2 + 1^2) = 14E_0$

The 1st, 2nd, 3rd, and 5th excited states are degenerate.



7-2. $E_{n_1 n_2 n_3} = \dfrac{\hbar^2 \pi^2}{2m}\left(\dfrac{n_1^2}{L_1^2} + \dfrac{n_2^2}{L_2^2} + \dfrac{n_3^2}{L_3^2}\right) = \dfrac{\hbar^2 \pi^2}{2mL_1^2}\left(n_1^2 + \dfrac{n_2^2}{4} + \dfrac{n_3^2}{9}\right)$ (Equation 7-5)

$n_1 = n_2 = n_3 = 1$ is the lowest energy level.

$E_{111} = E_0(1 + 1/4 + 1/9) = 1.361E_0$ where $E_0 = \dfrac{\hbar^2 \pi^2}{2mL_1^2}$

The next nine levels are, increasing order,

(Problem 7-2 continued)

n_1	n_2	n_3	E ($\times E_0$)
1	1	2	1.694
1	2	1	2.111
1	1	3	2.250
1	2	2	2.444
1	2	3	3.000
1	1	4	3.028
1	3	1	3.360
1	3	2	3.472
1	2	4	3.778

7-3. (a) $\psi_{n_1 n_2 n_3}(x,y,z) = A\cos\frac{n_1 \pi x}{L}\ \sin\frac{n_2 \pi y}{L}\ \sin\frac{n_3 \pi z}{L}$

(b) They are identical. The location of the coordinate origin does not affect the energy level structure.

7-4. $\psi_{111}(x,y,z) = A\sin\frac{\pi x}{L_1}\ \sin\frac{\pi y}{2L_1}\ \sin\frac{\pi z}{3L_1}$

$\psi_{112}(x,y,z) = A\sin\frac{\pi x}{L_1}\ \sin\frac{\pi y}{2L_1}\ \sin\frac{2\pi z}{3L_1}$

$\psi_{121}(x,y,z) = A\sin\frac{\pi x}{L_1}\ \sin\frac{\pi y}{L_1}\ \sin\frac{\pi z}{3L_1}$

$\psi_{122}(x,y,z) = A\sin\frac{\pi x}{L_1}\ \sin\frac{\pi y}{L_1}\ \sin\frac{2\pi z}{3L_1}$

$\psi_{113}(x,y,z) = A\sin\frac{\pi x}{L_1}\ \sin\frac{\pi y}{2L_1}\ \sin\frac{\pi z}{L_1}$

7-5. $$E_{n_1 n_2 n_3} = \frac{\hbar^2 \pi^2}{2m}\left(\frac{n_1^2}{L_1^2} + \frac{n_2^2}{(2L_1)^2} + \frac{n_3^2}{(4L_1)^2}\right) = \frac{\hbar^2 \pi^2}{2mL_1^2}\left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{16}\right)$$ (from Equation 7-5)

$$E_0 = \left(n_1^2 + \frac{n_2^2}{4} + \frac{n_3^2}{16}\right) \text{ where } E_0 = \frac{\hbar^2 \pi^2}{2mL_1^2}$$

(a)

n_1	n_2	n_3	E ($\times E_0$)
1	1	1	1.313
1	1	2	1.500
1	1	3	1.813
1	2	1	2.063
1	1	4	2.250
1	2	2	2.250
1	2	3	2.563
1	1	5	2.813
1	2	4	3.000
1	1	6	3.500

(b) 1,1,4 and 1,2,2

7-6. $$\psi_{111}(x,y,z) = A \sin\frac{\pi x}{L_1} \sin\frac{\pi y}{2L_1} \sin\frac{\pi z}{4L_1}$$

$$\psi_{112}(x,y,z) = A \sin\frac{\pi x}{L_1} \sin\frac{\pi y}{2L_1} \sin\frac{\pi z}{2L_1}$$

$$\psi_{113}(x,y,z) = A \sin\frac{\pi x}{L_1} \sin\frac{\pi y}{2L_1} \sin\frac{3\pi z}{4L_1}$$

$$\psi_{121}(x,y,z) = A \sin\frac{\pi x}{L_1} \sin\frac{\pi y}{L_1} \sin\frac{\pi z}{4L_1}$$

(Problem 7-6 continued)

$$\psi_{114}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{\pi z}{L_1}$$

$$\psi_{122}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{\pi z}{2L_1}$$

$$\psi_{123}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{3\pi z}{4L_1}$$

$$\psi_{115}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{5\pi z}{4L_1}$$

$$\psi_{124}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{L_1}\sin\frac{\pi z}{L_1}$$

$$\psi_{116}(x,y,z) = A\sin\frac{\pi x}{L_1}\sin\frac{\pi y}{2L_1}\sin\frac{3\pi z}{2L_1}$$

7-7. $$E_0 = \frac{\hbar^2\pi^2}{2mL^2} = \frac{(1.055\times 10^{-34}J{\cdot}s)^2\,\pi^2}{2(9.11\times 10^{-31}kg)(0.10\times 10^{-9}m)^2(1.60\times 10^{-19}J/eV)} = 37.68\,eV$$

$$E_{311} - E_{111} = \Delta E = 11E_0 - 3E_0 = 8E_0 = 301\,eV$$

$$E_{222} - E_{111} = \Delta E = 12E_0 - 3E_0 = 9E_0 = 339\,eV$$

$$E_{321} - E_{111} = \Delta E = 14E_0 - 3E_0 = 11E_0 = 415\,eV$$

7-8. (a) Adapting Equation 7-3 to two dimensions (i.e., setting $k_3 = 0$), we have

$$\psi_{n_1 n_2} = A\sin\frac{n_1\pi x}{L}\sin\frac{n_2\pi y}{L}$$

(b) From Equation 7-5,

$$E_{n_1 n_2} = \frac{\hbar^2\pi^2}{2mL^2}(n_1^2 + n_2^2)$$

(c) The lowest energy degenerate states have quantum numbers $n_1 = 1$, $n_2 = 2$ and $n_1 = 2$, $n_2 = 1$.

7-9. (a) For $n = 3$, $\ell = 0, 1, 2$

(b) For $\ell = 0$, m = 0

$\ell = 1$, m = −1, 0, +1

$\ell = 2$, m = −2, −1, 0 +1, +2

(c) There are nine different *m*-states, each with two spin states, for a total of 18 states for $n = 3$.

7-10. (a) For $n = 2$, $\ell = 0, 1$

For $\ell = 0$, m = 0 with two spin states

For $\ell = 1$, m = −1, 0, +1, each with two spin states

The total number of states with $n = 2$ is eight.

(b) For $n = 4$, $\ell = 0, 1, 2, 3$

Adding to those states found in (a),

For $\ell = 2$, there are $2\ell + 1 = 5$ *m* states and 10 total, including spin.

For $\ell = 3$, there are $2\ell + 1 = 7$ *m* states and 14 total, including spin.

Thus, for $n = 4$ there are a total of 8 + 10 + 14 = 32 states, including spin.

(c) All $n = 2$ states have the same energy. $E_2 = -13.6\,eV/n^2 = -13.6\,eV/4 = -3.4\,eV$

All $n = 4$ states have the same energy. $E_4 = -13.6\,eV/n^2 = -13.6\,eV/16 = -0.85\,eV$

7-11. (a) $L = I\omega = (10^{-5}\,kg{\cdot}m^2)(2\pi)(735\,\text{min}^{-1})(1\,\text{min}/60\,s) = 7.7\times10^{-4}\,kg{\cdot}m^2/s$

(b) $L = \sqrt{\ell(\ell+1)}\,\hbar = 7.7\times10^{-4}\,kg{\cdot}m^2/s$

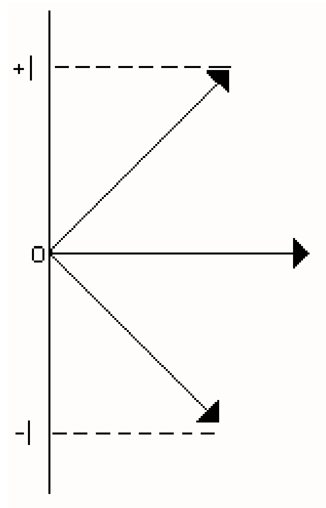
$$\ell(\ell+1) = \frac{(7.7\times10^{-4}\,kg{\cdot}m^2 s)^2}{(1.055\times10^{-34}\,J{\cdot}s)^2}$$

$$\ell \approx \frac{7.7\times10^{-4}\,kg{\cdot}m^2/s}{1.055\times10^{-34}\,J{\cdot}s} \approx 7.3\times10^{30}$$

7-12. (a)

$\ell = 1$

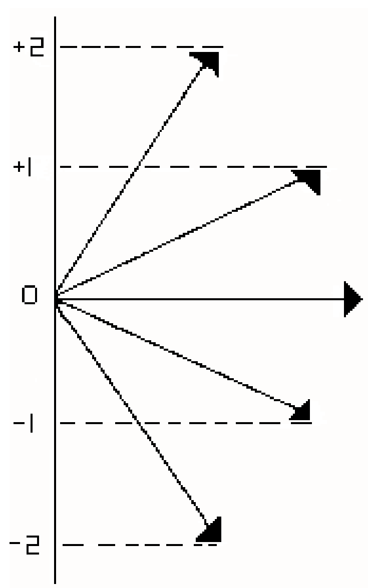
$|\mathbf{L}| = \sqrt{2}\,\hbar$



(b)

$\ell = 2$

$|\mathbf{L}| = \sqrt{6}\,\hbar$

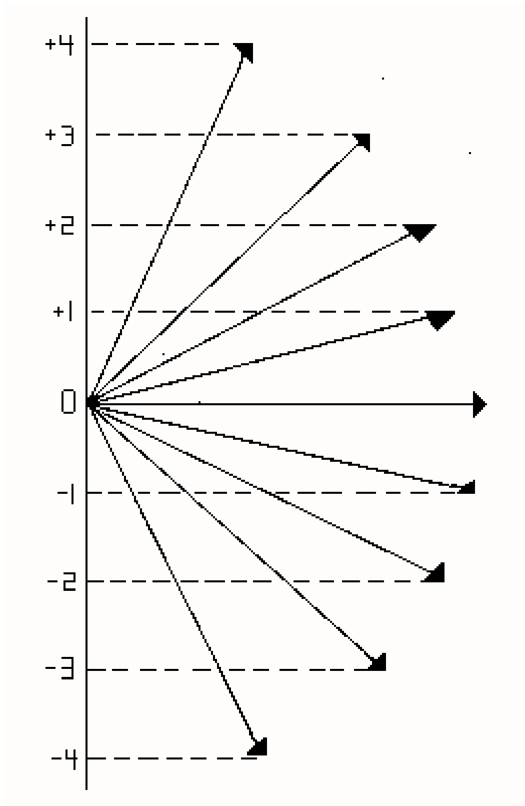


(Problem 7-12 continued)

(c)

$\ell = 4$

$|\mathbf{L}| = \sqrt{20}\,\hbar$



(d) $|\boldsymbol{L}| = \sqrt{\ell(\ell+1)}\,\hbar$ (See diagram above.)

7-13. $L^2 = L_x^2 + L_y^2 + L_z^2 \rightarrow L_x^2 + L_y^2 = L^2 - L_z^2 = \ell(\ell+1)\hbar^2 - (m\hbar)^2 = (6 - m^2)\hbar^2$

(a) $(L_x^2 + L_y^2)_{\text{min}} = (6 - 2^2)\hbar^2 = 2\hbar^2$

(b) $(L_x^2 + L_y^2)_{\text{max}} = (6 - 0^2)\hbar^2 = 6\hbar^2$

(c) $L_x^2 + L_y^2 = (6 - 1)\hbar^2 = 5\hbar^2$ L_x and L_y cannot be determined separately.

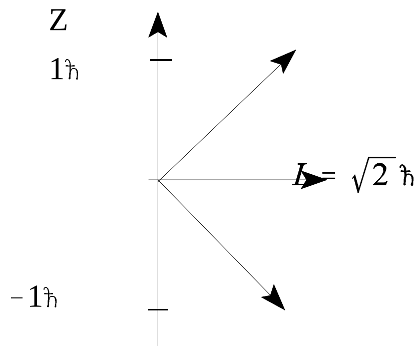
(d) n = 3

7-14. (a) For $\ell = 1$, $L = \sqrt{\ell(\ell=1)}\,\hbar = \sqrt{2}\,\hbar = 1.49\times10^{-34}\,J\cdot s$

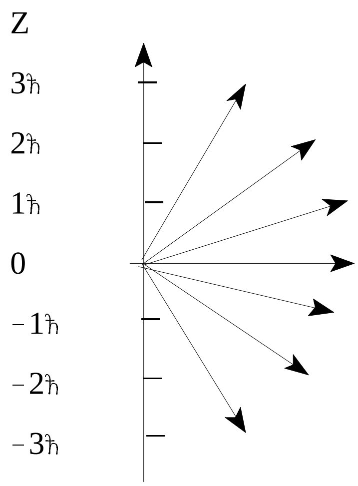
(b) For $\ell = 1$, $m = -1, 0, = 1$

(Problem 7-14 continued)

(c)



(d) For $\ell = 3$, $L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{12}\,\hbar = 3.65\times10^{-34} J\cdot s$ and $m = -3.\ -2, -1, 0, 1, 2, 3$.



7-15. $\boldsymbol{L} = \boldsymbol{r}\times\boldsymbol{p}$ $\qquad \dfrac{d\boldsymbol{L}}{dt} = \dfrac{d\boldsymbol{r}}{dt}\times\boldsymbol{p} + \boldsymbol{r}\times\dfrac{d\boldsymbol{p}}{dt}$

$\dfrac{d\boldsymbol{r}}{dt}\times\boldsymbol{p} = \boldsymbol{v}\times m\boldsymbol{v} = m\boldsymbol{v}\times\boldsymbol{v} = 0$ and $\boldsymbol{r}\times\dfrac{d\boldsymbol{p}}{dt} = \boldsymbol{r}\times\boldsymbol{F}$. Since for $V = V(\boldsymbol{r})$, i.e., central forces, **F** is parallel to **r**, then **r** × **F** = 0 and $\dfrac{d\boldsymbol{L}}{dt} = 0$

7-16. (a) For $\ell = 3$, n = 4, 5, 6, ... and m = −3, −2, −1, 0, 1, 2, 3

(b) For $\ell = 4$, n = 5, 6, 7,... and m = −4, −3, −2, −1, 0, 1, 2, 3, 4

(c) For $\ell = 0$, n = 1 and m = 0

(d) The energy depends only on *n*. The minimum in each case is:

$$E_4 = -13.6\,eV/n^2 = -13.6\,eV/4^2 = -0.85\,eV$$

$$E_5 = -13.6\,eV/5^2 = -0.54\,eV$$

$$E_1 = -13.6\,eV$$

7-17. (a) 6*f* state: n = 6, $\ell = 3$

(b) $E_6 = -13.6\,eV/n^2 = -13.6\,eV/6^2 = -0.38\,eV$

(c) $L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{3(3+1)}\,\hbar = \sqrt{12}\,\hbar = 3.65\times10^{-34}\,J\cdot s$

(d) $L_Z = m\hbar$ $L_Z = -3\hbar, -2\hbar, -1\hbar, 0, 1\hbar, 2\hbar, 3\hbar$

7-18. Referring to Table 7-2, $R_{30} = 0$ when

$$\left(1 - \frac{2r}{3a_0} + \frac{2r^2}{27a_0^2}\right) = 0$$

Letting $r/a_0 = x$, this condition becomes

$$x^2 - 9x + 13.5 = 0$$

Solving for *x* (quadratic formula or completing the square), *x* = 1.90, 7.10

$\therefore r/a_0 = 1.90,\ 7.10$ Compare with Figure 7-10(a).

7-19. (a) For the ground state n = 1, $\ell = 0$, and m = 0.

$$\psi_{100} = R_{10}Y_{00} = \frac{2}{\sqrt{a_0^3}}e^{-r/a_0}\frac{1}{\sqrt{4\pi}} = \frac{2}{\sqrt{4\pi a_0^2}}e^{-r/a_0} = \frac{2e^{-1}}{\sqrt{4\pi a_0^3}} \quad \text{at} \quad r = a_0$$

(Problem 7-19 continued)

(b) $\psi^2 = \dfrac{1}{\pi a_0^3} e^{-2r/a_0} = \dfrac{1}{\pi a_0^3} e^{-2}$ at $r = a_0$

(c) $P(r) = \psi^2 \cdot 4\pi r^2 = \dfrac{4}{a_0} e^{-2}$ at $r = a_0$

7-20. (a) For the ground state, $P(r)\Delta r = \psi^2 (4\pi r^2)\Delta r = \dfrac{4r^2}{a_0^3} e^{-2r/a_0} \Delta r$

For $\Delta r = 0.03 a_0$, at r = a_0 we have

$$P(r)\Delta r = \frac{4a_0^2}{a_0^3} e^{-2} (0.03 a_0) = 0.0162$$

(b) For Δr = 0.03a_0, at r = 2a_0 we have $P(r)\Delta r = \dfrac{4(2a_0)^2}{a_0^3} e^{-4} (0.03 a_0) = 0.0088$

7-21. $P(r) = Cr^2 e^{-2Zr/a_o}$ For P(r) to be a maximum,

$$\frac{dP}{dr} = C\left[r^2\left(-\frac{2Z}{a_o}\right) e^{-2Zr/a_o} + 2re^{-2Zr/a_o}\right] = 0 \rightarrow \quad C \cdot \frac{2Zr}{a_o}\left(\frac{a_o}{Z} - r\right) e^{-2Zr/a_o} = 0$$

This condition is satisfied when $r = 0$ or $r = a_o/Z$. For r = 0, P(r) = 0 so the maximum P(r) occurs for $r = a_o/Z$.

7-22.
$$\int \psi^2 \, d\tau = \int_0^\infty \int_0^\pi \int_0^{2\pi} \psi^2 r^2 \sin\theta \, dr \, d\theta \, d\phi = 1$$

$$= 4\pi \int_0^\infty \psi^2 r^2 \, dr = 4\pi C_{210}^2 \int_0^\infty \left(\frac{Zr}{a_0}\right)^2 r^2 e^{-Zr/a_0} \, dr = 1$$

$$= 4\pi C_{210}^2 \int_0^\infty \left(\frac{Z^2 r^4}{a_0^2}\right) e^{-Zr/a_0} \, dr = 1$$

(Problem 7-22 continued)

Letting $x = Zr/a_0$, we have that $r = a_0 x/Z$ and $dr = a_0 dx/Z$ and substituting these above,

$$\int \psi^2 \, d\tau = \frac{4\pi a_0^3 C_{210}^2}{Z^3} \int_0^\infty x^4 e^{-x} \, dx$$

Integrating on the right side

$$\int_0^\infty x^4 e^{-x} dx = 6$$

Solving for C_{210}^2 yields

$$C_{210}^2 = \frac{Z^3}{24\pi a_0^3} \rightarrow C_{210} = \left(\frac{Z^3}{24\pi a_0^3}\right)^{1/2}$$

7-23. $\psi_{200} = \frac{1}{\sqrt{32\pi}}\left(\frac{Z}{a_0}\right)^{3/2}\left(1 - \frac{r}{a_0}\right)e^{-r/2a_0}$ (Z = 1 for hydrogen)

$$P(r)\Delta r = |\psi_{200}|^2 (4\pi r^2)\Delta r = \frac{1}{32\pi}\frac{1}{a_0^3}\left(1 - \frac{r}{a_0}\right)^2 e^{-r/a_0}(4\pi r^2)\Delta r$$

(a) For $\Delta r = 0.02a_0$, at $r = a_0$ we have

$$P(r)\Delta r = \frac{4\pi}{32\pi}\frac{1}{a_0^3}(1-1)^2 e^{-1} a_0^2 (0.02 a_0) = \frac{1}{8}(0)e^{-1}(0.02) = 0$$

(b) For $\Delta r = 0.02a_0$, at $r = 2a_0$ we have

$$P(r)\Delta r = \frac{4\pi}{32\pi}\frac{1}{a_0^3}(-1)e^{-2} a_0^2 (0.02 a_0) = \frac{1}{8}(1)e^{-2}(0.02) = 3.4\times 10^{-4}$$

7-24.

$$\psi_{210} = C_{210}\frac{Zr}{a_o}e^{-Zr/2a_o}\cos\theta \quad (\text{Equation } 7-34)$$

$$P(r) = |\psi_{210}|^2\,4ptr^2 = 4\pi r^2|C_{210}|^2\frac{Z^2r^2}{a_o^2}e^{-r/a_o}\cos^2\theta$$

$$= 4\pi|c_{210}|^2(Z^2/a_o^2)r^4e^{-r/a_o}\cos^2\theta$$

$$= Ar^4e^{-r/a_o}\cos^2\theta$$

where $A = 4\pi|C_{210}|^2(Z^2/a_o^2)$, a constant.

7-25. $\psi_{200} = \frac{1}{\sqrt{32\pi}}\left(\frac{1}{a_0}\right)^{3/2}\left(2-\frac{r}{2a_0}\right)e^{-r/2a_0}$ (Z = 1 for hydrogen)

(a) At r = a_0,

$$\psi_{200} = \frac{1}{\sqrt{32\pi}}\left(\frac{1}{a_0^3}\right)(2-1)e^{-1/2} = \frac{0.606}{\sqrt{32\pi}}\left(\frac{1}{a_0}\right)^{3/2}$$

(b) At r = a_0,

$$|\psi_{200}|^2 = \frac{1}{\sqrt{32\pi}}\left(\frac{1}{a_0^3}\right)e^{-1} = \frac{0.368}{32\pi}\frac{1}{a_0^3}$$

(c) At r = a_0,

$$P(r) = |\psi_{200}|^2(4\pi r^2) = \frac{4\pi}{32\pi}\frac{0.368\,a_0^2}{a_0^3} = \frac{0.368}{8a_0}$$

7-26. For the most likely value of r, P(r) is a maximum, which requires that (see Problem 7-24)

$$\frac{dP}{dr} = A\cos^2\theta\left[r^4\left(-\frac{Z}{a_o}\right)e^{-Zr/a_o} + 4r^3e^{-Zr/a_o}\right] = 0$$

(Problem 7-26 continued)

For hydrogen Z = 1 and $A\cos^2\theta\,(r^3/a_o)(4a_o - r)\,e^{-r/a_o} = 0$

This is satisfied for r = 0 and r = $4a_o$. For r = 0 , P(r) = 0 so the maximum P(r) occurs for r = $4a_o$.

7-27.

n	1	2		3		
ℓ	0	0	1	0	1	2
m_ℓ	0	1	−1, 0, 1	0	−1,0,1	−2, −1, 0, 1, 2
number of m_ℓ states/ℓ	1	1	3	1	3	5
number of degenerate states/n	$1=1^2$	$4 = 2^2$		$9 = 3^2$		

7-28. $\psi_{100} = \dfrac{2}{\sqrt{4\pi a_0^3}}e^{-r/a_0}$

Because ψ_{100} is only a function of *r*, the angle derivatives in Equation 7-9 are all zero.

$$\frac{d\psi}{dr} = \frac{2}{\sqrt{4\pi a_0^3}}\left(-\frac{1}{a_0}\right)e^{-r/a_0}$$

$$r^2\frac{d\psi}{dr} = \frac{2}{\sqrt{4\pi a_0^3}}\left(-\frac{1}{a_0}\right)r^2 e^{-r/a_0}$$

$$\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = \frac{2}{\sqrt{4\pi a_0^3}}\left(-\frac{1}{a_0}\right)\left[r^2\left(-\frac{1}{a_0}\right) + 2r\right]e^{-r/a_0}$$

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = \frac{2}{\sqrt{4\pi a_0^3}}\left(-\frac{1}{a_0}\right)\left[\frac{2}{r} - \frac{1}{a_0}\right]e^{-r/a_0}$$ Substituting into Equation 7-9,

(Problem 7-28 continued)

$$-\frac{\hbar^2}{2\mu}\left(\frac{1}{a_0^2}-\frac{2}{a_0 r}\right)\psi_{100} + V\psi_{100} = E\psi_{100}$$

For the 100 state $r = a_0$ and $2\pi a_0 = \lambda = 2\pi/k$ or $a_0 = 1/k$, so

$$\left(\frac{1}{a_0^2}-\frac{2}{a_0 r}\right) = \left(\frac{1}{a_0^2}-\frac{2}{a_0^2}\right) = -\frac{1}{a_0^2} = -k^2$$

Thus, $-\frac{\hbar^2}{2\mu}\left(\frac{1}{a_0^2}-\frac{2}{a_0 r}\right) = \frac{\hbar^2 k^2}{2\mu}$ and we have that

$\frac{\hbar^2 k^2}{2\mu} + V = E$, satisfying the Schrödinger equation.

7-29. (a) Every increment of charge follows a circular path of radius R and encloses an area πR^2, so the magnetic moment is the total current times this area. The entire charge Q rotates with frequency $f = \omega/2\pi$, so the current is

$$i = Qf = q\omega/2\pi$$

$$\mu = iA = (Q\omega/2\pi)(\pi R^2) = Q\omega R^2/2$$

$$L = I\omega = \frac{1}{2}MR^2\omega$$

$$g = \frac{2M\mu}{QL} = \frac{2MQ\omega R^2/2}{QMR^2\omega/2} = 2$$

(b) The entire charge is on the equatorial ring, which rotates with frequency $f = \omega/2\pi$.

$$i = Qf = Q\omega/2\pi$$

$$\mu = iA = (Q\omega/2\pi)(\pi R^2) = Q\omega R^2/2$$

$$g = \frac{2M\mu}{QL} = \frac{2MQ\omega R^2/2}{QMR^2\omega/5} = 5/2 = 2.5$$

7-30. Angular momentum $S = I\omega = (2/5)mr^2(v/r)$ or

$$v = (5/2)S(1/mr) = 5S/2mr = 5(3/4)^{1/2}\hbar/2mr$$

$$= \frac{5(3/4)^{1/2}(1.055\times10^{-34}J\cdot s)}{2(9.11\times10^{-31}kg)(10^{-15}m)} = 2.51\times10^{11}m/s$$

7-31. (a) The *K* ground state is $\ell = 0$, so two lines due to the spin of the single *s* electron would be seen.

(b) The *Ca* ground state is $\ell = 0$ with two *s* electrons whose spins are opposite resulting in S=0, so there will be one line.

(c) The electron spins in the *O* ground state are coupled to zero, the orbital angular momentum is 2, so five lines would be expected.

(d) The total angular momentum of the *Sn* ground state is $j = 0$, so there will be one line.

7-32. $|F_z| = m_s g_L \mu_B (dB/dz) = m_{Ag} a_z$ (From Equation 7-51)

and $a_z = m_S g_L \mu_B (dB/dz)/m_{Ag}$

Each atom passes through the magnet's 1m length in $t = (1/250)s$ and cover the additional 1m to the collector in the same time. Within the magnet they deflect in the *z* direction an amount z_1 given by: $z_1 = (1/2)a_z t^2 = (1/2)[m_s g_L \mu_B (dB/dz)/m_{Ag}](1/250)^2$

and leave the magnet with a *z*-component of velocity given by $v_z = a_z t$. The additional *z* deflection in the field-free region is $z_2 = v_z t = a_z t^2$

The total *z* deflection is then $z_1 + z_2 = 0.5mm = 5.0\times10^{-4}m$.

$$5.0\times10^{-4}m = z_1 + z_2 = (3/2)a_z t^2 = (3/2)[m_s g_L \mu_B (dB/dz)/m_{Ag}][1/250]^2$$

or

(Problem 7-32 continued)

$$\frac{dB}{dz} = \frac{(5.0\times 10^{-4} m)(250)^2 (m_{Ag})(2)}{m_s g_L \mu_B}$$

$$= \frac{(5.0\times 10^{-4} m)(250 s^{-1})^2 (1.79\times 10^{-25} kg)(2)}{3(1/2)(1)(9.27\times 10^{-24} J/T)} = 0.805\, T/m$$

7-33. (a) There should be four lines corresponding to the four m_J values −3/2, −1/2, +1/2, +3/2.

(b) There should be three lines corresponding to the three m_ℓ values −1, 0, +1.

7-34. For $n = 2, \ell = 0, 1$ and $s = 1/2$ ➜ $2\,^2S_{1/2},\ 2\,^2P_{1/2},\ 2\,^2P_{3/2}$

For $n = 4, \ell = 0, 1, 2, 3$ and $s = 1/2$ ➜

$4\,^2S_{1/2},\ 4\,^2P_{1/2},\ 4\,^2P_{3/2},\ 4\,^2D_{3/2},\ 4\,^2D_{5/2},\ 4\,^2F_{5/2},\ 4\,^2F_{7/2}$

7-35. For $\ell = 2$, $L = \sqrt{\ell(\ell+1)}\,\hbar = \sqrt{6}\,\hbar = 2.45\hbar$, $j = \ell \pm 1/2 = 3/2, 5/2$ and $J = \sqrt{j(j+1)}\,\hbar$

For j = 3/2, $J = \sqrt{(3/2)(3/2+1)}\,\hbar = \sqrt{15/4}\,\hbar = 1.94\hbar$

For j = 5/2, $J = \sqrt{(5/2)(5/2+1)}\,\hbar = \sqrt{35/4}\,\hbar = 2.96\hbar$

7-36. (a) $j = \ell \pm 1/2 = 2 \pm 1/2 = 5/2$ *or* $3/2$

(b) $J = \sqrt{j(j+1)}\,\hbar = \sqrt{\frac{5}{2}(5/2+1)}\,\hbar = 2.96\hbar$

or $= \sqrt{\frac{3}{2}(3/2+1)}\,\hbar = 1.94\hbar$

(c) **J** = **L** + **S** and $J_Z = L_Z + S_Z = m_\ell\hbar + m_s\hbar = m_j\hbar$ where $m_j = -j, -j+1, \ldots j-1, j$. For j = 5/2 the *z*-components are −5/2, −3/2, −1/2, +1/2, +3/2, +5/2. For j = 3/2, the *z*-components are −3/2, −1/2, +1/2, +3/2.

7-37. j = $\ell \pm 1/2$. $\ell = j \pm 1/2 = 3/2 \pm 1/2 = 1$ *or* 2

7-38. If $j = 5/2,\ 7/2,\ \ell = 3$. This is an *f* state.

7-39. (a) $\mathbf{L} = \mathbf{L}_1 + \mathbf{L}_2$.

$\ell = (\ell_1 = \ell_2), (\ell_1 + \ell_2 - 1), ... |\ell_1 - \ell_2| = (1+1), (1+1-1), (1-1) = 2, 1, 0$

(b) $\mathbf{S} = \mathbf{S}_1 + \mathbf{S}_2$

$s = (s_1 = s_2), (s_1 + s_2 - 1), ... |s_1 - s_2| = (1/2 + 1/2), (1/2 - 1/2) = 1, 0$

(c) $\mathbf{J} = \mathbf{L} + \mathbf{S}$

$j = (\ell + s), (\ell + s - 1), ... |\ell - s|$

For $\ell = 2$ and s = 1, j = 3, 2, 1

$\ell = 2$ and s = 0, j = 2

For $\ell = 1$ and s = 1, j = 2, 1, 0

$\ell = 1$ and s – 0, j = 1

For $\ell = 0$ and s = 1, j = 1

$\ell = 0$ and s = 0, j = 0

(d) $\mathbf{J}_1 = \mathbf{L}_1 + \mathbf{S}_1$ $\quad j_1 = \ell_1 \pm 1/2 = 3/2,\ 1/2$

$\mathbf{J}_2 = \mathbf{L}_2 + \mathbf{S}_2$ $\quad j_2 = \ell_2 \pm 1/2 = 3/2,\ 1/2$

(e) $\mathbf{J} = \mathbf{J}_1 + \mathbf{J}_2$ $\quad j = (j_1 + j_2), (j_1 + j_2 - 1), ... |j_1 - j_2|$

For $j_1 = 3/2$ and $j_2 = 3/2$, j = 3, 2, 1, 0

$j_1 = 3/2$ and $j_2 = 1/2$, j = 2,1

For $j_1 = ½$ and $j_2 = 3/2$, j = 2, 1

$j_1 = ½$ and $j_2 = 1/2$, j = 1,0

These are the same values as found in (c).

7-40. (a) $E_{3/2} = \dfrac{hc}{\lambda}$ Using values from Figure 7-22,

$$E_{3/2} = \frac{1239.852\ eV{\cdot}nm}{588.99\ nm} = 2.10505\ eV \qquad E_{1/2} = \frac{1239.852\ eV{\cdot}nm}{589.59\ nm} = 2.10291\ eV$$

(b) $\Delta E = E_{3/2} - E_{1/2} = 2.10505\ eV - 2.10291\ eV = 2.14\times 10^{-3}\ eV$

(c) $\Delta E = 2\mu_B B \ \rightarrow\ B = \dfrac{\Delta E}{2\mu_B} = \dfrac{2.14\times 10^{-3}\ eV}{2(5.79\times 10^{-4}\ eV/T)} = 18.5\,T$

7-41. $\psi_{12} = \psi(x_1, x_2) = C\sin\dfrac{\pi x_1}{L}\sin\dfrac{2\pi x_2}{L}$ Substituting into Equation 7-57 with $V = 0$,

$$-\frac{\hbar^2}{2m}\left(\frac{\partial^2\psi_{12}}{\partial x_1^2} + \frac{\partial^2\psi_{12}}{\partial x_2^2}\right) = \left(\frac{\hbar^2}{2m}\right)(1+4)\left(\frac{\pi^2}{L^2}\right)\psi_{12} = E\psi_{12}$$

Obviously, ψ_{12} is a solution if $E = \dfrac{5\hbar^2\pi^2}{2mL^2}$

7-42. $E_n = \dfrac{n^2\hbar^2\pi^2}{2mL^2}$ Neutrons have antisymmetric wave functions, but if spin is ignored then one is in the $n = 1$ state, but the second is in the $n = 2$ state, so the minimum energy is:

$E = E_1 + E_2 = (1^2 + 2^2)E_1 = 5E_1$ where

$$E_1 = \frac{(\hbar c)^2\pi^2}{2mc^2L^2} = \frac{(197.3)^2\pi^2}{2(939.6)(2.0)^2} = 51.1\ MeV \qquad E = 5E_1 = 255\ MeV$$

7-43. (a) For electrons: Including spin, two are in the $n = 1$ state, two are in the $n = 2$ state, and one is in the $n = 3$ state. The total energy is then:

$$E = 2E_1 + 2E_2 + E_3 \quad \text{where } E_n = \frac{n^2\hbar^2\pi^2}{2mL^2} \quad E = 2E_1 + 2(2^2E_1) + (3^2E_1) = 19E_1$$

where $E_1 = \dfrac{(\hbar c)^2\pi^2}{2m_ec^2L^2} = \dfrac{(197.3)^2\pi^2}{2(0.511\times 10^6)(1.0)^2} = 0.376\,eV$

$E = 19E_1 = 7.14\ eV$

(Problem 7-43 continued)

(b) Pions are bosons and all five can be in the n = 1 state, so the total energy is:

$$E = 5E_1 \quad \text{where } E_1 = \frac{0.376\,eV}{264} = 0.00142\,eV \qquad E = 5E_1 = 0.00712\,eV$$

7-44. (a) Carbon: $Z = 6;\ 1s^2 2s^2 2p^2$

(b) Oxygen: $Z = 8;\ 1s^2 2s^2 2p^4$

(c) Argon: $Z = 18;\ 1s^2 2s^2 2p^6 3s^2 3p^6$

7-45. (a) Chlorine: $Z = 17;\ 1s^2 2s^2 2p^6 3s^2 3p^5$

(b) Calcium: $Z = 20;\ 1s^2 2s^2 2p^6 3s^2 3p^6 4s^2$

(c) Germanium : $Z = 32;\ 1s^2 2s^2 2p^6 3s^2 3p^6 3d^{10} 4s^2 4p^2$

7-46. Both *Ga* and *In* have electron configurations $(\text{ns})^2\,(\text{np})$ outside of closed shells $(\text{n-1, s})^2\,(\text{n-1, p})^6\,(\text{n-1, d})^{10}$. The last *p* electron is loosely bound and is more easily removed than one of the *s* electrons of the immediately preceding elements *Zn* and *Cd*.

7-47. The outermost electron outside of the closed shell in *Li*, *Na*, *K*, *Ag*, and *Cu* has $\ell = 0$. The ground state of these atoms is therefore not split. In *B*, *Al*, and *Ga* the only electron not in a closed shell or subshell has $\ell = 1$, so the ground state of these atoms will be split by the spin-orbit interaction.

7-48. $$E_n = -\frac{Z_{eff}^2 E_1}{n^2} \quad (\text{Equation } 7-25)$$

$$Z_{eff} = n\sqrt{\frac{-E_n}{E_1}} = 3\sqrt{\frac{5.14\,eV}{13.6\,eV}} = 1.84$$

7-49. (a) Fourteen electrons, so Z = 14. Element is silicon.

(b) Twenty electrons. So Z = 20. Element is calcium.

7-50. (a) For a *d* electron, $\ell = 2$, so $L_z = -2\hbar, -2\Downarrow, -1\hbar, 0\ 1\hbar, 2\hbar$

(b) For an *f* electron, $\ell = 3$, so $L_z = -3\hbar\ -2\hbar, -2\Downarrow, -1\hbar, 0\ 1\hbar, 2\hbar, 3\hbar$

7-51. Like *Na*, the following atoms have a single *s* electron as the outermost shell and their energy level diagrams will be similar to sodium's: *Li*, *Rb*, *Ag*, *Cs*, *Fr*.

The following have two *s* electrons as the outermost shell and will have energy level diagrams similar to mercury: *He, Ca, Ti, Cd, Mg, Ba, Ra*.

7-52. Group with 2 outer shell electrons: beryllium, magnesium, calcium, nickel, and barium.

Group with 1 outer shell electron: lithium, sodium, potassium, chromium, and cesium.

7-53. Similar to H: Li, Rb, Ag, and Fr. Similar to He: Ca, Ti, Cd, Ba, Hg, and Ra.

7-54.

n	ℓ	j
4	0	½
4	1	½
4	1	3/2
5	0	½
3	2	3/2
3	2	5/2
5	1	½
5	1	3/2
4	2	3/2
4	2	5/2
6	0	½
4	3	5/2
4	3	7/2

Energy is increasing downward in the table.

7-55. Selection rules: $\Delta\ell = \pm 1 \qquad \Delta j = \pm 1, 0$

Transition	$\Delta\ell$	Δj	Comment
$4S_{1/2} \to 3S_{1/2}$	0	0	ℓ - forbidden
$4S_{1/2} \to 3P_{3/2}$	+1	+1	allowed
$4P_{3/2} \to 3S_{1/2}$	−1	−1	allowed
$4D_{5/2} \to 3P_{1/2}$	−1	−2	j - forbidden
$4D_{3/2} \to 3P_{1/2}$	−1	−1	allowed
$4D_{3/2} \to 3S_{1/2}$	−2	−1	ℓ - forbidden

7-56. (a) $E_1 = -13.6\,eV(Z-1)^2 = -13.6\,eV(74-1)^2 = -7.25\times 10^4\,eV = -72.5\,keV$

(b) $E_1(\exp) = -69.5\,keV = -13.6\,eV(Z-\sigma)^2 = -13.6\,ev(74-1)^2$

$$74-\sigma = (69.5\times 10^3\,eV/13.6\,eV)^{1/2} = 71.49$$

$$\sigma = 74 - 71.49 = 2.51$$

7-57. $\Delta j = \pm 1, 0 \quad (\text{no } j = 0 \to j = 0) \quad (\text{Equation } 7-66)$

The four states are $^2P_{3/2}$, $^2P_{1/2}$, $^2D_{5/2}$, $^2D_{3/2}$.

Transition	$\Delta\ell$	Δj	Comment
$D_{5/2} \to P_{3/2}$	−1	−1	allowed
$D_{5/2} \to P_{1/2}$	−1	−2	j - forbidden
$D_{3/2} \to P_{3/2}$	−1	0	allowed
$D_{3/2} \to P_{1/2}$	−1	−1	allowed

7-58. (a) $\Delta E = hc/\lambda$

$$E(3P_{1/2}) - E(3S_{1/2}) = \frac{1240\, eV{\cdot}nm}{589.59\, nm} = 2.10\, eV$$

$$E(3P_{1/2}) = E(3S_{1/2}) + 2.10\, eV = -5.14\, eV + 2.10\, eV = -3.04\, eV$$

$$E(3D) - E(3P_{1/2}) = \frac{1240\, eV{\cdot}nm}{818.33\, nm} = 1.52\, eV$$

$$E(3D) = E(3P_{1/2}) + 1.52\, eV = -3.04\, eV + 1.52\, eV = -1.52\, eV$$

(b)

$$For\ \ 3P: \quad Z_{eff} = 3\sqrt{\frac{3.04\, eV}{13.6\, eV}} = 1.42$$

$$For\ \ 3D: \quad Z_{eff} = 3\sqrt{\frac{1.52\, eV}{13.6\, eV}} = 1.003$$

(c) The Bohr formula gives the energy of the 3D level quite well, but not the 3P level.

7-59. (a) $\Delta E = g m_j \mu_B B$ (Equation 7-72) Where $s = 1/2$, $\ell = 0$ gives $j = ½$ and (from Equation 7-73) $g = 2$. $m_j = \pm 1/2$.

$$\Delta E = (2)(\pm 1/2)(5.79\times 10^{-5}\, eV/T)(0.55\, T) = \pm 3.18\times 10^{-5}\, eV$$

The total splitting between the $m_j = \pm ½$ states is $6.37\times 10^{-5}\, eV$.

(b) The $m_j = ½$ (spin up) state has the higher energy.

(c) $\Delta E = hf \rightarrow f = \Delta E/h = 6.37\times 10^{-5} eV/4.14\times 10^{-15} eV{\cdot}s = 1.54\times 10^{10}\, Hz$

This is in the microwave region of the spectrum.

7-60. $$E = \frac{hc}{\lambda} \quad\rightarrow\quad \Delta E \approx \frac{dE}{d\lambda}\Delta\lambda = -\frac{hc}{\lambda^2}\Delta\lambda \quad\rightarrow\quad \Delta\lambda \approx -\frac{\lambda^2}{hc}\Delta E$$

7-61. (a) $\Delta E = \dfrac{e\hbar}{2m}B = (5.79\times10^{-4}\ eV/T)(0.005\ T) = 2.90\times10^{-5}\ eV$

(b) $|\Delta\lambda| = \dfrac{\lambda^2}{hc}\Delta E = \dfrac{(579.07\,nm)^2(2.90\times10^{-5}\ eV)}{1240\ eV{\cdot}nm} = 7.83\times10^{-3}\ nm$

(c) The smallest measurable wavelength change is larger than this by the ratio 0.01 nm / 0.00783 nm = 1.28. The magnetic field would need to be increased by this same factor because $B \propto \Delta E \propto \Delta\lambda$. The necessary field would be 0.0638 T.

7-62.

$$E_n = -13.6\,eV(Z_{eff}^2/n^2)$$

$$E_2 = -13.6\,eV(Z_{eff}^2/2^2) = -5.39\ eV$$

$$Z_{eff} = 2(5.39/13.6)^{1/2} = 1.26$$

7-63. $\psi_{100} = \dfrac{1}{\sqrt{\pi}}\left(\dfrac{Z}{a_o}\right)^{3/2} e^{-Zr/a_o}$ (Equations 7–30 and 7–31)

$$P(r) = 4\pi r^2\,\psi_{100}^*\psi_{100} \quad \text{(Equation 7–32)}$$

$$= 4\pi r^2 \frac{Z^3}{\pi a_o^3} e^{-Zr/a_o} = \frac{4Z^3}{a_o^3} r^2 e^{-2Zr/a_o}$$

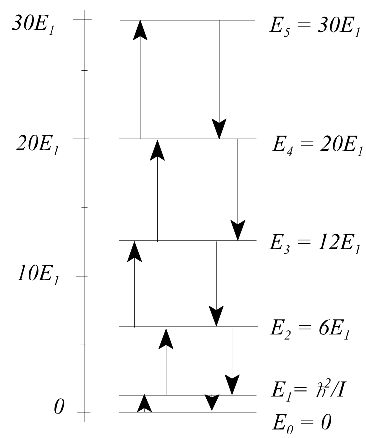
$$\langle r\rangle = \int_0^\infty r P(r)\,dr = \int_0^\infty \frac{4Z^3}{a_0^3} r^3 e^{-2Zr/a_o}\,dr$$

$$= \frac{a_o}{4Z}\int_0^\infty\left(\frac{2Zr}{a_o}\right)^3 e^{-2Zr/a_o}\,d(2Zr/a_o) = \frac{a_o}{4Z}\times 3! = \frac{3a_o}{2Z}$$

7-64. (a) $E_\ell = \dfrac{\ell(\ell+1)\hbar^2}{2I}$

ℓ:	0	1	2	3	4	5	···
$\ell+1$:	1	2	3	4	5	6	···
E_ℓ:	0	$1E_1$	$6E_1$	$12E_1$	$20E_1$	$30E_1$	···

(Problem 7-64 continued)



(b) $$E_{\ell+1+} - E_\ell = \frac{\hbar^2}{2I}[(\ell+1)(\ell+2) - \ell(\ell+1)]$$

$$= \frac{\hbar^2}{2I}[(\ell+1)(\ell+2-\ell)] = \frac{\hbar^2}{I}(\ell+1) = (\ell+1)E_1$$

The values of ℓ = 0, 1, 2, ... yield all the positive integer multiples of E_1.

(c) $$I = \frac{1}{2}m_p r^2 \rightarrow E_1 = \frac{\hbar^2}{I} = \frac{2\hbar^2}{m_p r^2} = \frac{2(\hbar c)^2}{m_p c^2 r^2}$$

$$= \frac{2(197.3\ eV{\cdot}nm)^2}{(938.28\times10^6\ eV)(0.074\,nm)^2} = 1.52\times10^{-2}\ eV$$

(d) $$\lambda = \frac{hc}{E_1} = \frac{1.24\times10^{-6}\ eV{\cdot}m}{1.52\times10^{-2}\ eV} = 8.18\times10^{-5}\,m = 81.8\ \mu m$$

7-65. (a) $|F_z| = m_s g_L \mu_B (dB/dz)$ (From Equation 7-51)

From Newton’s 2nd law,

(Problem 7-65 continued)

$$|F_z| = m_H a_z = m_s g_L \mu_B (dB/dz)$$

$$a_z = m_s g_L (dB/dz)/m_H$$

$$= (1/2)(1)(9.27\times10^{-24}\,J/T)(600\,T/m)/(1.67\times10^{-27}\,kg)$$

$$= 1.67\times10^{6}\,m/s^2$$

(b) At 14.5 km/s = $v = 1.45\times10^4\,m/s$, the atom takes $t_1 = 0.75\,m/(1.45\times10^4\,m/s) = 5.2\times10^{-5}\,s$

to traverse the magnet. In that time, its z deflection will be:

$$z_1 = (1/2)(a_z)t_1^2 = (1/2)(1.67\times10^6\,m/s^2)(5.2\times10^{-5}\,s)^2 = 2.26\times10^{-3}\,m = 2.26\,m$$

Its v_z velocity component as it leaves the magnet is $v_z = a_z t_1$ and its additional z deflection before reaching the detector 1.25 m away will be:

$$z_2 = v_z t_2 = (a_z t_1)(1.25\,m/[1.45\times10^4\,m/s])$$

$$= (1.67\times10^6\,m/s^2)(5.2\times10^{-5}\,s)(1.25)/(1.45\times10^4\,m/s)$$

$$= 7.49\times10^{-3}\,m = 7.49\,mm$$

Each line will be deflected $z_1 + z_2 = 9.75\,mm$ from the central position and, thus, separated by a total of 19.5 mm = 1.95 cm.

7-66. $\theta_{min} = \cos^{-1}[m_\ell\hbar/\sqrt{\ell(\ell+1)}\,\hbar]$ with $m_\ell = \ell$.

$\cos\theta_{min} = \ell/\sqrt{\ell(\ell+1)}$. Thus, $\cos^2\theta_{min} = \ell^2/[\ell(\ell+1)] = 1 - \sin^2\theta_{min}$

or, $\sin^2\theta_{min} = 1 - \dfrac{\ell^2}{\ell(\ell+1)} = \dfrac{\ell(\ell+1)-\ell^2}{\ell(\ell+1)} = \dfrac{\ell^2+\ell-\ell^2}{\ell(\ell+1)}$

And, $\sin\theta_{min} = \left(\dfrac{1}{\ell+1}\right)^{1/2}$ For large ℓ, θ_{min} is small. Then

$$\sin\theta_{min} \approx \theta_{min} = \left(\frac{1}{\ell+1}\right)^{1/2} \approx \frac{1}{(\ell)^{1/2}}$$

7-67. (a) $E_1 = hf = hc/\lambda_1 = 1240\,eV\cdot nm/766.41\,nm = 1.6179\,eV$

$$E_2 = hf = hc/\lambda_2 = 1240\,eV\cdot nm/769.90\,nm = 1.6106\,eV$$

(b) $\Delta E = E_1 - E_2 = 1.6179\,eV - 1.6106\,eV = 0.0073\,eV$

(c) $\Delta E/2 = gm_j\mu_B B \rightarrow B = \dfrac{\Delta E}{2gm_j\mu_B} = \dfrac{0.0073\,eV}{2(2)(1/2)(5.79\times10^{-5}\,eV/T)} = 63\,T$

7-68. $P(r) = \dfrac{4Z^3}{a_o^3} r^2 e^{-2Zr/a_o}$ (see Problem 7-63)

For hydrogen, Z = 1 and at the edge of the proton $r = R_o = 10^{-15}m$. At that point, the exponential factor in P(r) has decreased to:

$$e^{-2R_o/a_o} = e^{-2(10^{-15})/(0.529\times10^{-10}m)} = e^{-(3.78\times10^{-5})} \approx 1 - 3.78\times10^{-5} \approx 1$$

Thus, the probability of the electron in the hydrogen ground state being inside the nucleus, to better than four figures, is:

$$P(r) = \frac{4r^2}{a_o^3} \qquad P = \int_0^{r_o} P(r)\,dr = \int_0^{R_o} \frac{4r^2}{a_o^3} = \frac{4}{a_o^3}\int_0^{R_o} r^2\,dr = \frac{4}{a_o^3}\frac{r^3}{3}\Big|_0^{R_o}$$

$$= \frac{4}{a_o^3}\left(\frac{R_o^3}{3}\right) = \frac{4(10^{-15}m)^3}{3(0.529\times10^{-10}m)^3} = 9.0\times10^{-15}$$

7-69. (a) $g = 1 + \dfrac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}$ (Equation 7-73)

For $^2P_{1/2}$: j = 1/2, ℓ = 1, and s = ½

$$g = 1 + \frac{1/2(1/2+1) + 1/2(1/2+1) - 1(1+1)}{2\cdot1/2(1/2+1)} = 1 + \frac{3/4 + 3/4 - 2}{3/2} = 2/3$$

For $^2S_{1/2}$: j = 1/2, ℓ = 0, and s = ½

$$g = 1 + \frac{1/2(1/2+1) + 1/2(1/2+1) - 0}{2\cdot1/2(1/2+1)} = 1 + \frac{3/4 + 3/4}{3/2} = 2$$

(Problem 7-69 continued)

The $^2P_{1/2}$ levels shift by:

$$\Delta E = g m_j \mu_B B = \frac{2}{3}\left(\pm\frac{1}{2}\right)\mu_B B = \pm\frac{1}{3}\mu_B B \quad \text{(Equation 7-72)}$$

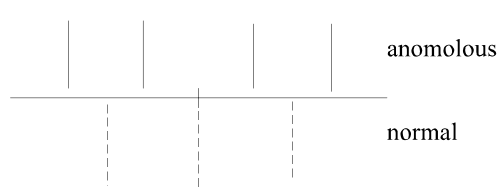
The $^2S_{1/2}$ levels shift by: $\Delta E = g m_j \mu_B B = 2\left(\pm\frac{1}{2}\right)\mu_B B = \pm\mu_B B$

To find the transition energies, tabulate the several possible transitions and the corresponding energy values (Let E_p and E_s be the B = 0 unsplit energies of the two states.):

Transition	Energy
$P_{1/2,\,1/2} \rightarrow S_{1/2,\,1/2}$	$\left(E_p + \frac{1}{3}\mu_B B\right) - (E_s + \mu_B B) = (E_p - E_s) - \frac{2}{3}\mu_B B$
$P_{1/2,\,-1/2} \rightarrow S_{1/2,\,1/2}$	$\left(E_p - \frac{1}{3}\mu_B B\right) - (E_s + \mu_B B) = (E_p - E_s) - \frac{4}{3}\mu_B B$
$P_{1/2,\,1/2} \rightarrow S_{1/2,\,-1/2}$	$\left(E_p + \frac{1}{3}\mu_B B\right) - (E_s - \mu_B B) = (E_p - E_s) + \frac{4}{3}\mu_B B$

Transition	Energy
$P_{1/2,\,-1/2} \rightarrow S_{1/2,\,-1/2}$	$\left(E_p - \frac{1}{3}\mu_B B\right) - (E_s - \mu_B B) = (E_p - E_s) + \frac{2}{3}\mu_B B$

Thus, there are four different photon energies emitted. The energy or frequency spectrum would appear as below (normal Zeeman spectrum shown for comparison).



(b) For $^2P_{3/2}$: j = 3/2, ℓ = 1, and s = ½

(Problem 7-69 continued)

$$g = 1 + \frac{3/2(3/2+1) + 1/2(1/2+1) - 1(1+1)}{2\cdot 3/2(3/2+1)} = 1 + \frac{15/4 + 3/4 - 2}{30/4} = 4/3$$

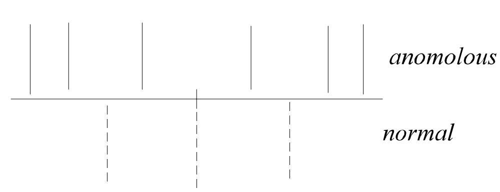
These levels shift by:

$$\Delta E = g m_j \mu_B B = \frac{4}{3}\left(\pm\frac{1}{2}\right)\mu_B B = \pm\frac{2}{3}\mu_B B \quad \Delta E = \frac{4}{3}\left(\pm\frac{3}{2}\right)\mu_B B = \pm 2\mu_B B$$

Tabulating the transitions as before:

Transition	Energy
$P_{3/2,\,3/2} \to S_{1/2,\,1/2}$	$(E_p + 2\mu_B B) - (E_s + \mu_B B) = (E_p - E_s) + \mu_B B$
$P_{3/2,\,3/2} \to S_{1/2,\,-1/2}$	forbidden, $\Delta m_j = 2$
$P_{3/2,\,1/2} \to S_{1/2,\,1/2}$	$\left(E_p - \frac{2}{3}\mu_B B\right) - (E_s + \mu_B B) = (E_p - E_s) - \frac{1}{3}\mu_B B$
$P_{3/2,\,1/2} \to S_{1/2,\,-1/2}$	$\left(E_p + \frac{2}{3}\mu_B B\right) - (E_s - \mu_B B) = (E_p - E_s) + \frac{5}{3}\mu_B B$
$P_{3/2,\,-1/2} \to S_{1/2,\,1/2}$	$\left(E_p - \frac{2}{3}\mu_B B\right) - (E_s + \mu_B B) = (E_p - E_s) - \frac{5}{3}\mu_B B$
$P_{3/2,\,-1/2} \to S_{1/2,\,-1/2}$	$\left(E_p - \frac{2}{3}\mu_B B\right) - (E_s - \mu_B B) = (E_p - E_s) + \frac{1}{3}\mu_B B$
$P_{3/2,\,-3/2} \to S_{1/2,\,1/2}$	forbidden $\Delta m_j = 2$
$P_{3/2,\,-3/2} \to S_{1/2,\,-1/2}$	$(E_p - 2\mu_B B) - (E_s - \mu_B B) = (E_p - E_s) - \mu_B B$

There are six different photon energies emitted (two transitions are forbidden); their spectrum looks as below:



7-70. (a) Substituting $\psi(r,\theta)$ into Equation 7-9 and carrying out the indicated operations yields (eventually)

$$-\frac{\hbar^2}{2\mu}\psi(r,\theta)\left[2/r^2 - 1/4a_0^2\right] - \frac{\hbar^2}{2\mu}\psi(r,\theta)\left(-2/r^2\right) + V\psi(r,\theta) = E\psi(r,\theta)$$

Canceling $\psi(r,\theta)$ and recalling that $r_2 = 4a_0$ (because ψ given is for $n = 2$) we have

$$-\frac{\hbar^2}{2\mu}\left(-1/4a_0^2\right) + V = E$$

The circumference of the $n = 2$ orbit is: $C = 2\pi(4a_0) = 2\lambda \rightarrow a_0 = \lambda/4\pi = 1/2k$.

Thus, $$\frac{\hbar^2}{2\mu}\left(-\frac{1}{4/4k^2}\right) + V = E \rightarrow \frac{\hbar^2 k^2}{2\mu} + V = E$$

(b) or $\frac{p^2}{2m} + V = E$ and Equation 7-9 is satisfied.

$$\int_0^\infty \psi^2 d\tau = \int A^2\left(\frac{r}{a_0}\right)^2 e^{-r/a_0}\cos^2\theta\, r^2 \sin\theta\, dr\, d\theta\, d\phi = 1$$

$$A^2\int_0^\infty \left(\frac{r}{a_0}\right)^2 e^{-r/a_0} r^2 dr \int_0^\pi \cos^2\theta \sin\theta\, d\theta \int_0^{2\pi} d\phi = 1$$

Integrating (see Problem 7-22),

$$A^2\left(6a_0^3\right)(2/3)(2\pi) = 1$$

$$A^2 = 1/8a_0^3\pi \Rightarrow A = \sqrt{1/8a_0^3\pi}$$

7-71. $\mu = -g_L\mu_B L/\hbar$ (Equation 7-43)

(a) The 1s state has $\ell = 0$, so it is unaffected by the external B.

The 2p state has $\ell = 1$, so it is split into three levels by the external B.

(b) The 2p → 1s spectral line will be split into three lines by the external B.

(Problem 7-71 continued)

(c) In Equation 7-43 we replace μ_B with $\mu_k = e\hbar/2m_k$, so

$$\mu_{kz} = -(1)(1)(e\hbar/2m_k) = -\mu_B(m_e/m_k) \quad \text{(From Equation 7-45)}$$

Then $\Delta E = \mu_B(m_e/m_k)B$

$$= (5.79\times10^{-5}\,eV/T)[(0.511\,MeV/c^2)/(497.7\,MeV/c^2)](1.0\,T)$$

$$= 5.94\times10^{-8}\,eV$$

$\dfrac{\Delta\lambda}{\lambda} = -\dfrac{\lambda}{hc}\Delta E$ (From Problem 7-60) Where λ for the (unsplit) 2p → 1s transition is

given by $\lambda = hc/\Delta E_k$ and $\Delta E_k = E_2 - E_1 = -13.6\,eV(m_k/m_e)(1-1/4) = 9.93\times10^3\,eV$

and $\lambda = 1240\,eV{\cdot}nm/9.93\times10^3\,eV = 0.125\,nm$

and $\dfrac{\Delta\lambda}{\lambda} = \dfrac{0.125\,nm(5.94\times10^{-8}\,eV)}{1240\,eV{\cdot}nm} = 5.98\times10^{-12}$

7-72. $\Delta E = -\boldsymbol{\mu}\cdot\boldsymbol{B} = \dfrac{ke^2}{r^3 m(mc^2)}\boldsymbol{S}\cdot\boldsymbol{L}$ where, for $n = 3$, $r = a_0 n^2 = 9a_0$

For 3P states $\boldsymbol{S}\cdot\boldsymbol{L} \approx \hbar^2$,

$$\Delta E \approx \frac{1.440\,eV{\cdot}nm\,(3.00\times10^8\,m/s\times10^9\,nm/m)^2(6.58\times10^{-16}\,eV{\cdot}s)^2}{9(0.053\,nm)^3(0.511\times10^6\,eV)^2} \approx 1.60\times10^{-4}\,eV$$

For 3D states $\boldsymbol{S}\cdot\boldsymbol{L} \approx \hbar^2/3$

$$\Delta E \approx 1.60\times10^{-4}\,eV/3 \approx 0.53\times10^{-4}\,eV$$

7-73. (a) **J** = **L** + **S** $\boldsymbol{u} = -\mu_B(\boldsymbol{L} + 2\boldsymbol{S})/\hbar$ (Equation 7-71)

$$u_J = \frac{\boldsymbol{u}\cdot\boldsymbol{J}}{J} = \frac{[-\mu_B(\boldsymbol{L} + 2\boldsymbol{S})/\hbar]\cdot[\boldsymbol{L} + \boldsymbol{S}]}{J}$$

$$\equiv -\frac{\mu_B}{\hbar J}(\boldsymbol{L}\cdot\boldsymbol{L} + 2\boldsymbol{S}\cdot\boldsymbol{S} + 3\boldsymbol{S}\cdot\boldsymbol{L})$$

$$= -\frac{\mu_B}{\hbar J}(L^2 + 2S^2 + 3\boldsymbol{S}\cdot\boldsymbol{L})$$

(b) $J^2 = \boldsymbol{J}\cdot\boldsymbol{J} = (\boldsymbol{L} + \boldsymbol{S})\cdot(\boldsymbol{L} + \boldsymbol{S}) = \boldsymbol{L}\cdot\boldsymbol{L} + \boldsymbol{S}\cdot\boldsymbol{S} + 2\boldsymbol{S}\cdot\boldsymbol{L} \quad \therefore \boldsymbol{S}\cdot\boldsymbol{L} = \frac{1}{2}(J^2 - L^2 - S^2)$

(c) $\mu_J = -\frac{\mu_B}{\hbar J}\left[L^2 + 2S^2 + \frac{3}{2}(J^2 - L^2 - S^2)\right] = -\frac{\mu_B}{2\hbar J}(3J^2 + S^2 - L^2)$

(d) $\mu_Z = \mu_J\frac{J_Z}{J} = -\frac{\mu_B}{2\hbar J}(3J^2 + S^2 - L^2)\frac{J_Z}{J} = -\frac{\mu_B}{2\hbar J}(3J^2 + S^2 - L^2)$

$$= -\mu_B\left(1 + \frac{J^2 + S^2 - L^2}{2J^2}\right)\frac{J_Z}{\hbar}$$

(e) $\Delta E = -\mu_Z B$ (Equation 7-69)

$$= +\mu_B B\left[1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}\right]m_j$$

$$= g m_j \mu_B B \quad \text{(Equation 7-72)}$$

where $g = \left[1 + \frac{j(j+1) + s(s+1) - \ell(\ell+1)}{2j(j+1)}\right]$ (Equation 7-73)

7-74. The number of steps of size unity between two integers (or half-integers) a and b is $b - a$. Including both values of a and b, the number of distinct values in this sequence is $b - a + 1$. For **F** = **I** + **J**, the largest value of f is $I+J = b$. If $I < J$, the smallest values of f is $J - I = a$. The number of different values of f is therefore $(I + J) - (J - I) + 1 = 2I + 1$. For $I > J$, the smallest value of f is $I - J = a$. In that case, the number of different values of f is $(I + J) - (I - J) + 1 = 2J + 1$. The two expressions are equal if $I = J$.

7-75. (a) $\mu_N = \dfrac{e\hbar}{2m_p} = 5.05\times10^{-27}\,J/T$

$$B \approx \frac{2k_m\mu}{r^3} = \frac{2k_m(2.8\mu_N)}{r^3} = \frac{2k_m(2.8\mu_N)}{a_o^3}$$

$$= \frac{2(10^{-7}\,H/m)(2.8)(5.05\times10^{-27}\,J/T)}{(0.529\times10^{-10}\,m)^3} = 0.0191\,T$$

(b) $\Delta E \approx 2\mu_B B = 2(5.79\times10^{-4}\,eV/T)(0.0191\,T) = 2.21\times10^{-6}\,eV$

(c) $\lambda = \dfrac{hc}{\Delta E} = \dfrac{1.24\times10^{-6}\,eV\cdot m}{2.21\times10^{-6}\,eV} = 0.561\,m = 56.1\,cm$