Chapter 8 – Statistical Physics

8-1. (a) $v_{rms} = \sqrt{\dfrac{3RT}{M}} = \left[\dfrac{3(8.31\,J/mole\cdot K)(300\,K)}{2(1.0079\times 10^{-3}\,kg/mole)}\right]^{1/2} = 1930\,m/s$

(b) $T = \dfrac{Mv_{rms}^2}{3R} = \dfrac{2(1.0079\times 10^{-3}\,kg/mole)(11.2\times 10^{3}\,m/s)^2}{3(8.31\,J/mole\cdot K)} = 1.01\times 10^{4}\,K$

8-2. (a) $\overline{E_K} = \dfrac{3}{2}kT \quad \therefore\; T = \dfrac{2\overline{E_K}}{3k} = \dfrac{2(13.6\,eV)}{3(8.617\times 10^{-5}\,eV/K)} = 1.05\times 10^{5}\,K$

(b) $\overline{E_K} = \dfrac{3}{2}kT = \dfrac{3}{2}(8.617\times 10^{-5}\,eV/K)(10^{7}\,K) = 1.29\,keV$

8-3. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ (Equation 8-12)

(a) For O_2: $v_{rms} = \sqrt{\dfrac{3(8.31\,J/K\cdot mol)(273\,K)}{32\times 10^{-3}\,kg/mol}} = 461\,m/s$

(b) For H_2: $v_{rms} = \sqrt{\dfrac{3(8.31\,J/K\cdot mol)(273\,K)}{2\times 10^{-3}\,kg/mol}} = 1840\,m/s$

8-4. $\left[\dfrac{3RT}{M}\right]^{1/2} = \left[\dfrac{(J/mole\cdot K)(K)}{kg/mole}\right]^{1/2} = \left[\dfrac{kg\cdot m^2/s^2}{kg}\right]^{1/2} = m/s$

8-5. (a) $E_K = n \cdot \frac{3}{2}RT = (1\,mole)\frac{3}{2}(8.31\,J/mole{\cdot}K)(273) = 3400\,J$

(b) One mole of any gas has the same translational kinetic energy at the same temperature.

8-6. $\langle v^2 \rangle = \frac{1}{N}\int_0^\infty v^2 n(v)dv = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\int_0^\infty v^4 e^{-\lambda v^2}dv$ where $\lambda = m/2kT$

$\langle v^2 \rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2} I_4$ where I_4 is given in Table B1-1.

$$I_4 = \frac{3}{8}\pi^{1/2}\lambda^{-5/2} = \frac{3}{8}\pi^{1/2}(m/2kT)^{-5/2}$$

$$\langle v^2 \rangle = 4\pi\left(\frac{m}{2\pi kT}\right)^{3/2}\left(\frac{3}{8}\right)\pi^{1/2}\left(\frac{2kT}{m}\right)^{5/2} = \frac{3kT}{m} = \frac{3RT}{mN_A} = \frac{3RT}{M}$$

$$v_{rms} = \sqrt{\langle v^2 \rangle} = \sqrt{\frac{3RT}{M}}$$

8-7. $\langle v \rangle = \sqrt{\frac{8kT}{\pi m}} = \left[\frac{8(1.381\times10^{-23}J/K)(300K)}{\pi(1.009u)(1.66\times10^{-27}kg/u)}\right]^{1/2} = 2510\,m/s$

$$v_m = \sqrt{\frac{2kT}{m}} = \left[\frac{2(1.381\times10^{-23}J/K)(300K)}{\pi(1.009u)(1.66\times10^{-27}kg/u)}\right]^{1/2} = 2220\,m/s$$

$n(v) = 4\pi N(m/2\pi kT)^{3/2}v^2 e^{-mv^2/kT}$ (Equation 8-28)

(Problem 8-7 continued)

At the maximum: $\frac{dn}{dv} = 0 = 4\pi N(m/2\pi kT)^{3/2}\{2v + v^2(-mv/kT)\}e^{-mv^2/2kT}$

$$0 = ve^{-mv^2/2kT}(2 - mv^2/kT)$$

The maximum corresponds to the vanishing of the last factor. (The other two factors give minima at v = 0 and v = ∞.) So $2 - mv^2/kT = 0$ and $v_m = (2kT/m)^{1/2}$.

8-8. (a) $f(v_x) = (m/2\pi kT)^{1/2}e^{-mv_x^2/2kT}$ (Equation 8 − 20)

$$= \frac{1}{\sqrt{2\pi}}\sqrt{\frac{m}{kT}}\,e^{-(m/kT)(v_x^2/2)}$$

$$= (2\pi)^{-1/2}v_0^{-1}e^{-v_x^2/2v_0^2} \quad \textit{where } v_0 = v_{x,\,rms} = (kT/m)^{1/2}$$

(b) $\Delta N = Nf(v_x)\Delta v_x = N_A f(v_x)(0.01v_0)$

$$= (6.02\times10^{23})(2\pi)^{-1/2}v_0^{-1}e^{-0}(0.01v_0) = 2.40\times10^{21}$$

(c) $\Delta N = 6.02\times10^{23}(2\pi)^{-1/2}v_o^{-1}e^{-1/2}(0.01v_0) = 1.46\times10^{21}$

(d) $\Delta N = 6.02\times10^{23}(2\pi)^{-1/2}v_o^{-1}e^{-2}(0.01v_0) = 3.25\times10^{20}$

(e) $\Delta N = 6.02\times10^{23}(2\pi)^{-1/2}v_o^{-1}e^{-32}(0.01v_0) = 3.04\times10^{7}$

8-9. $m(v)dv = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}v^2e^{-mv^2/2kT}dv$ (Equation 8-28)

$\frac{du}{dv} = A\left[v^2\left(-\frac{2vm}{2kT}\right) + 2v\right]e^{-mv^2/2kT}$ The v for which $dn/dv = 0$ is v_m.

(Problem 8-9 continued)

$$A\left[-\frac{2mv^3}{2kT} + 2v\right]e^{-mv^2/2kT} = 0$$

Because A = constant and the exponential term is only zero for $v = \infty$, only the quantity in [] can be zero, so $-\frac{2mv^3}{2kT} + 2v = 0$

or $v^2 = \frac{2kT}{m} \rightarrow v_m = \sqrt{\frac{2kT}{m}}$ (Equation 8-29)

8-10. The number of molecules N in 1 liter at 1 atm, 20°C is:

$$N = 1\ell(1\,g\cdot mol/22.4\,\ell)(N_A\ molecules/g\cdot mol)$$

Each molecule has, on the average, $3kT/2$ kinetic energy, so the total translational kinetic energy in one liter is: $KE = \frac{6.02\times 10^{23}}{22.4}\left[\frac{3(1.381\times 10^{-23}J/K)(293\,K)}{2}\right] = 163\,J$

8-11.

$$\frac{n_2}{n_1} = \frac{g_2\,e^{-E_2/kT}}{g_1\,e^{-E_1/kT}} = \frac{g_2}{g_1}\,e^{-(E_2-E_1)/kT}$$

$$e^{(E_2-E_1)/kT} = \frac{g_2}{g_1}\cdot\frac{n_1}{n_2} = (E_2 - E_1)/kT = \ln\left(\frac{g_2}{g_1}\cdot\frac{n_1}{n_2}\right)$$

$$T = \frac{E_2 - E_1}{k\ln[(g_2/g_1)(n_1/n_2)]} = \frac{10.2\,eV}{(8.617\times 10^{-5}eV/K)\ln(4\times 10^6)} = 7790\,K$$

8-12. $\frac{n_2}{n_1} = \frac{g_2}{g_1}e^{-(E_1-E_2)/kT} = \frac{3}{1}\,e^{-\left[\frac{4\times 10^{-3}eV}{(8.617\times 10^{-5}eV/K)(300K)}\right]} = 0.155$

8-13. There are two degrees of freedom; therefore, $C_v = 2(R/2) = R$, $C_p = R + R = 2R$, and $\gamma = 2R/R = 2$.

8-14. $C_v = 3R/M$ (a) Al: $C_v = \dfrac{3(1.99\ cal/mole{\cdot}K)}{27.0\ g/mole} = 0.221\ cal/g{\cdot}K \quad \{0.215\ cal/g{\cdot}K\}$

(b) Cu: $C_v = \dfrac{3(1.99\ cal/mole{\cdot}K)}{62.5\ g/mole} = 0.0955\ cal/g{\cdot}K \quad \{0.0920\ cal/g{\cdot}K\}$

(c) Pb: $C_v = \dfrac{3(1.99\ cal/mole{\cdot}K)}{207\ g/mole} = 0.0288\ cal/g{\cdot}K \quad \{0.0305\ cal/g{\cdot}K\}$

The values for each element shown in brackets are taken from the *Handbook of Chemistry and Physics* and apply at 25° C.

8-15. $n(E) = \dfrac{2\pi N}{(\pi kT)^{3/2}} E^{1/2} e^{-E/kT}$ (Equation 8 – 35).

At the maximum:

$$\frac{dn}{dE} = 0 = \frac{2\pi N}{(\pi kT)^{3/2}}\left\{\frac{1}{2}E^{-1/2} + E^{1/2}\left(-\frac{1}{kT}\right)\right\}e^{-E/kT}$$

$$= E^{-1/2}e^{-E/kT}\left(\frac{1}{2} - E/kT\right)$$

The maximum corresponds to the vanishing of the last factor. (The vanishing of the other two factors correspond to minima at E = 0 and E = ∞.) $1/2 - E/kT = 0 \quad \rightarrow \quad E = 1/2kT$.

8-16. (a) $n(v) = 4\pi N(m/2\pi kT)^{3/2} v^2 e^{-mv^2/2kT}$ (Equation 8 – 28)

$$= \frac{4\pi N}{\pi^{3/2}}\left(\frac{m}{2kT}\right)^{3/2} v^2 e^{-v^2(m/2kT)}$$

$$= \frac{4N}{\sqrt{\pi}}\frac{1}{v_m^3} v^2 e^{-v^2/2v_m^2} \quad where\ v_m = \sqrt{2kT/m}$$

$$= \frac{4N}{\sqrt{\pi}}\frac{1}{v_m}\left(\frac{v}{v_m}\right)^2 e^{-(v/v_m)^2}$$

$$\Delta N = n(v)\Delta v = \frac{4N_Z}{\sqrt{\pi}}\frac{1}{v_m}\left(\frac{v}{v_m}\right)^2 e^{-(v/v_m)^2}(0.01\,v_m)$$

$$= 1.36\times10^{22}(v/v_m)^2 e^{-(v/v_m)^2}$$

(b) $\Delta N = 1.36\times10^{22}(0)^2 e^{-0} = 0$

(c) $\Delta N = 1.36\times10^{22}(1)^2 e^{-1} = 5.00\times10^{21}$

(d) $\Delta N = 1.36\times10^{22}(2)^2 e^{-(2)^2} = 9.96\times10^{20}$

(e) $\Delta N = 1.36\times10^{22}(8)^2 e^{-(8)^2} = 1.369\times10^{-4}$ (or no molecules most of the time)

8-17. $n(E) = Ag(E)e^{-E/kT}$ (Equation 8-14)

The degeneracies of the lowest four hydrogen states are:

n = 1, $g(E_1) = 2$; n = 2, $g(E_2) = 8$; n = 3, $g(E_3) = 18$; n = 4, $g(E_4) = 32$

For the Sun $kT = 0.500\,eV$.

$$\frac{n(E_2)}{n(E_1)} = \frac{Ag_2e^{-E_2/kT}}{Ag_1e^{-E_1/kT}} = \frac{8}{2}e^{-(E_2-E_1)/kT} = 4e^{-(10.2)/(0.500)} = 4e^{-20.4} = 5.5\times10^{-9}$$

$$\frac{n(E_3)}{n(E_1)} = \frac{18}{2}e^{-(12.1)/(0.500)} = 9e^{-24.2} = 2.8\times10^{-10}$$

$$\frac{n(E_4)}{n(E_1)} = \frac{32}{2}e^{-(12.8)/(0.500)} = 16e^{-25.5} = 1.3\times10^{-10}$$

8-18. (a) $e^{-\alpha} = \frac{N}{V}\frac{h^3}{2(2\pi m_e kT)^{3/2}}$ (Equation 8–68)

$$\frac{N}{V} = e^{-\alpha}\,\frac{2(2\pi m_e kT)^{3/2}}{h^3} = e^{-\alpha}\,\frac{2(2\pi m_e c^2 kT)^{3/2}}{(hc)^3}$$

$$\cong 1\cdot 2\,\frac{[2\pi(5.11\times10^5\,eV)(2.585\times10^{-2}\,eV)]^{1/2}}{(1240\,eV\cdot nm)^3}\left(\frac{10^7\,nm}{1\,cm}\right)^3 = 2.51\times10^{19}/cm^3$$

8-19. (a) $e^{-\alpha}(O_2) = \frac{N}{V}\frac{h^3}{(2\pi MkT)^{3/2}}$ (Equation 8–68)

$$= \frac{N_A}{V_M}\frac{(hc)^3}{(2\pi Mc^2 kT)^{3/2}}$$

$$= \frac{(6.022\times10^{23}/mole)}{(22.4\times10^3\,cm^3/mole)}\frac{(1.24\times10^{-4}\,eV\cdot cm)^3}{[2\pi(32\,uc^2)(931.5\times10^6\,eV/u)(8.617\times10^{-5}\,eV/K)(273\,K)]}$$

$$= 1.75\times10^{-7}$$

(b) At temperature T, $e^{-\alpha}(O_2) = 1.75\times10^{-7}\,(273)^{3/2}/T^{3/2}$

$$1 = (1.75\times10^{-7})(273)^{3/2}/T^{3/2} \;\rightarrow\; T^{3/2} = (1.75\times10^{-7})(273)^{3/2}$$

$$\therefore\; T = (1.75\times10^{-7})^{2/3}\,(273) = 8.5\,mK$$

8-20. Assuming the gases are ideal gases, the pressure is given by: $P = \frac{2}{3}\frac{N\langle E\rangle}{V}$

for classical, FD, and BE particles. P_{FD} will be highest due to the exclusion principle, which, in effect, limits the volume available to each particle so that each strikes the walls more frequently than the classical particles. On the other hand, P_{BE} will be lowest, because the particles tend to be in the same state, which in effect, is like classical particles with a mutual attraction, so they strike the walls less frequently.

8-21. (a) $f_{BE} = \dfrac{1}{e^{\alpha} e^{E/kT} - 1}$ For α = 0 and $f_{BE} = 1$, at T = 5800 K

$$\frac{1}{e^{E/5800k} - 1} = 1 \rightarrow e^{E/5800k} = 2$$

$$E/(5800K)\left(8.62\times10^{-5}\,eV/K\right) = \ln 2$$

E = 0.347 eV

(b) For E = 0.35 V, α = 0, and f_{BE} = 0.5,

$$\frac{1}{e^{0.35/kT} - 1} = 0.5 \rightarrow e^{0.35/kT} = 3$$

$$0.35\,eV/\left(8.67\times10^{-5\,eV/K}\right)T = \ln 3$$

$$T = \frac{0.35\,eV}{\ln 3\left(8.67\times10^{-5}\,eV/K\right)} = 3660\,K$$

8-22. $\lambda = \dfrac{h}{p} = \dfrac{h}{\sqrt{2m\langle E\rangle}} = \dfrac{h}{\sqrt{2m(3kT/2)}} = \dfrac{h}{(3mkT)^{1/2}}$

The distance between molecules in an ideal gas $(V/N)^{1/3}$ is found from

$PV = nRT = nRT\left(N_A/N_A\right) = NkT \rightarrow \left(V/N\right)^{1/3} = \left(kT/P\right)^{1/3}$ and equating this to λ above,

$$\left(kT/P\right)^{1/3} = \frac{h}{(3mkT)^{1/2}}$$

$\dfrac{kT}{P} = \dfrac{h^3}{(3mkT)^{3/2}}$ and solving for T, yields: $T^{5/2} = \dfrac{P}{k}\dfrac{h^3}{(3mk)^{3/2}}$

$$T = \left[\frac{P h^3}{k(3mk)^{3/2}}\right]^{5/2} = \left[\frac{(101\,kPa)\left(6.63\times10^{-34}J\cdot s\right)^3}{3\left(2\times1.67\times10^{-27}kg\right)\left(1.38\times10^{-23}J/K\right)^{5/2}}\right]^{2/5} = 4.4\,K$$

8-23. $\dfrac{N_0}{N} \approx 1 - \left(\dfrac{T}{T_C}\right)^{3/2}$ (Equation 8-76)

(a) For $T = 3T_C/4 \rightarrow \dfrac{N_0}{N} \approx 1 - \left(\dfrac{3T}{4T_C}\right)^{3/2} = 0.351$

(b) For $T = T_C/2 \rightarrow \dfrac{N_0}{N} \approx 1 - \left(\dfrac{T}{2T_C}\right)^{3/2} = 0.646$

(c) For $T = T_C/4 \rightarrow \dfrac{N_0}{N} \approx 1 - \left(\dfrac{T}{4T_C}\right)^{3/2} = 0.875$

(d) For $T = T_C/8 \rightarrow \dfrac{N_0}{N} \approx 1 - \left(\dfrac{T}{8T_C}\right)^{3/2} = 0.956$

8-24. For small values of α, $e^{\alpha} = 1 + \alpha + (\alpha^2/2!) + \cdots$ and $N_0 = \dfrac{1}{e^{\alpha} - 1} \rightarrow N_0(e^{\alpha} - 1) = 1$

which for small α values becomes: $N_0(1 + \alpha + \cdots - 1) = N_0\alpha = 1$ or $N_0 = \dfrac{1}{\alpha}$

8-25. $T_C = \dfrac{h^2}{2mk}\left[\dfrac{N}{2\pi(2.315)V}\right]^{2/3}$ (Equation 8-72)

The density of liquid *Ne* is 1.207 g/cm^3, so

$$\frac{N}{V} = \frac{(1.207\,g/cm^3)(6.022\times10^{23}\,molecules/mol)(10^6\,cm^2/m^3)}{20.18\,g/mol} = 3.601\times10^{28}\,/m^3$$

$$T_C = \frac{(6.626\times10^{-34}\,J\cdot s)^2}{2(20u\times1.66\times10^{-27}\,kg/u)(1.381\times10^{-23}\,J/K)}\left[\frac{3.601\times10^{28}\,m^3}{2\pi(2.315)}\right]^{2/3} = 0.895\,K$$

Thus, T_C at which ^{20}Ne would become a superfluid is much lower than its freezing temperature of 24.5 K.

8-26. Power per unit area R arriving at Earth is given by the Stefan-Boltzmann law: $R = \sigma T^4$ where σ is Stefan's constant. For a 5% decrease in the Sun's temperature,

$$\frac{R(T) - R(0.95T)}{R(T)} = \frac{\sigma T^4 - \sigma(0.95T)^4}{\sigma T^4} = 1 - (0.95)^4 = 0.186$$, or a decrease of 18.6%.

8-27. $\langle E\rangle = \dfrac{hf}{e^{hf/kT} - 1}$ (Equation 8-84)

(a) For $T = 10hf/k;\ hf = kT/10 \rightarrow \langle E\rangle = \dfrac{hf}{e^{1/10} - 1} = \dfrac{kT/10}{0.1051} = 0.951kT$

(b) For $T = hf/k;\ hf = kT \rightarrow \langle E\rangle = \dfrac{hf}{e^{1} - 1} = \dfrac{kT}{1.718} = 0.582kT$

(c) For $T = 0.1hf/k;\ hf = 10kT \rightarrow \langle E\rangle = \dfrac{hf}{e^{10} - 1} = \dfrac{10kT}{2.20\times 10^4} = 4.54\times 10^{-4}kT$

According to equipartition $\langle E\rangle = kT$ in each case.

8-28. $C_V = 3N_A k\left(\dfrac{hf}{kT}\right)^2 \dfrac{e^{hf/kT}}{\left(e^{hf/kT} - 1\right)^2}$ As T $\rightarrow \infty$, hf/kT gets small and

$e^{hf/kT} \approx 1 + hf/kT + \cdots$

$$C_V = 3N_A k\left(\frac{hf}{kT}\right)^2 \frac{(1 + hf/kT + \cdots)}{(hf/kT)^2} \approx 3N_A k = 3N_A(R/N_A) = 3R$$

The rule of Dulong and Petit.

8-29. $C_V = 3R\left(\dfrac{hf}{kT}\right)^2 \dfrac{e^{hf/kT}}{\left(e^{hf/kT} - 1\right)^2}$ (Equation 8-86)

Writing $hf/kT = Af$ where $A = h/kT = 2.40\times 10^{-13}$ when $T = 200K$,

$$C_V = 3R(Af)^2 \frac{e^{Af}}{\left(e^{Af}\right)^2 - 2e^{Af} + 1} = eR(Af)^2 \frac{1}{e^{Af} - 2 + \left(1/e^{Af}\right)}$$

(Problem 8-29 continued)

Because Af is "large", $1/e^{Af} \approx 0$ and e^{Af} dominates $(Af)^2$, so

$$C_V \approx 3R/e^{Af} \rightarrow e^{Af} \approx 3R/C_V \rightarrow f \approx \ln(3R/C_V)(1/A)$$

For *Al*, $C_V(200K) = 20.1\,J/K\cdot mol$ (From Figure 8-14)

$$f = \ln\left(\frac{3(8.31)}{20.1}\right)(1/2.40\times10^{-13}) = 8.97\times10^{11}\,Hz$$

For *Is*, $C_V(200K) = 13.8\,J/K\cdot mol$ (From Figure 8-14)

$$f = \ln\left(\frac{3(8.31)}{13.8}\right)(1/2.40\times10^{-13}) = 2.46\times10^{12}\,Hz$$

8-30. $$C_V = 3R\left(\frac{hf}{kT}\right)^2 \frac{e^{hf/kT}}{(e^{hf/kT}-1)^2}$$ (Equation 8-86)

At the Einstein temperature $T_E = hf/k$,

$$C_V = 3R(1)^2\frac{e^1}{(e^1-1)^2} = 3R(0.9207) = 3(8.31\,J/K\cdot mol)(0.9207)$$

$$= 22.95\,J/K\cdot mol = 5.48\,cal/K\cdot mol$$

8-31. Rewriting Equation 8-93 as

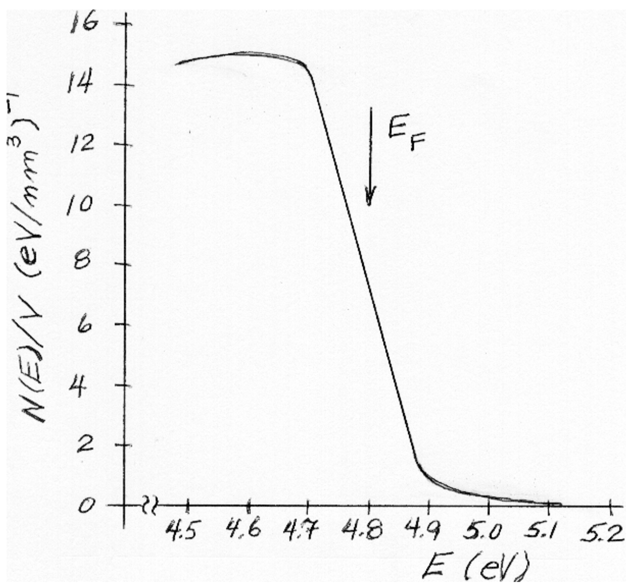
$$\frac{n(E)}{V} = \frac{\pi}{2}\left[\frac{8mc^2}{(hc)^2}\right]^{3/2}\frac{E^{1/2}}{e^{(E-E_F)/kT}+1}$$

Set up the equation on a spreadsheet whose initial and final columns are E(eV) and n(E)/V $(\text{eV}\cdot\text{nm}^3)^{-1}$, respectively.

(Problem 8-31 continued)

E (eV)	n(E)/V (eV·nm^3)$^{-1}$
4.5	14.4
4.6	14.6
4.7	14.5
4.8 (= E_F)	7.46
4.9	0.306
5	0.0065
5.1	0.00014

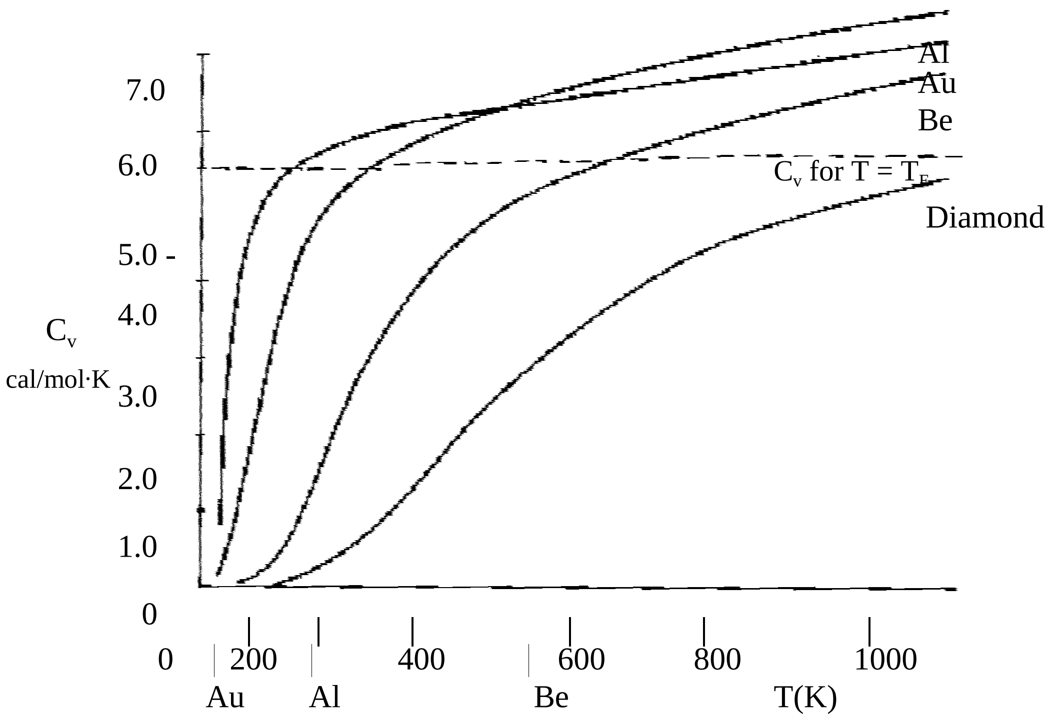
The graph of these values is below.



From the graph, about 0.37 electrons/nm^3 or 3.7×10^{26} electrons/m^3 within 0.1 eV below E_F have been excited to levels above E_F.

8-32. From the graph. T_E (Au) = 136 K T_E (Al) = 243 K

T_E (Diamond) = off the graph (well over 1000 K) T_E (Be) = 575 K



8-33. Approximating the nuclear potential with an infinite square well and ignoring the Coulomb repulsion of the protons, the energy levels for both protons and neutrons are given by $E_n = (n^2 h^2)/(8mL^2)$ and six levels will be occupied in ^{22}Ne, five levels with 10 protons and six levels with 12 neutrons.

$$E_F(\text{protons}) = \frac{(5)^2(1240\,MeV\cdot fm)^2}{8(1.0078\,u \times 931.5\,MeV/u)(3.15\,fm)^2} = 516\,MeV$$

$$E_F(\text{neutrons}) = \frac{(6)^2(1240\,MeV\cdot fm)^2}{8(1.0087\,u \times 931.5\,MeV/u)(3.15\,fm)^2} = 742\,MeV$$

$$\langle E\rangle(\text{protons}) = (3/5)E_F = 310\,MeV$$

$$\langle E\rangle(\text{neutrons}) = (3/5)E_F = 445\,MeV$$

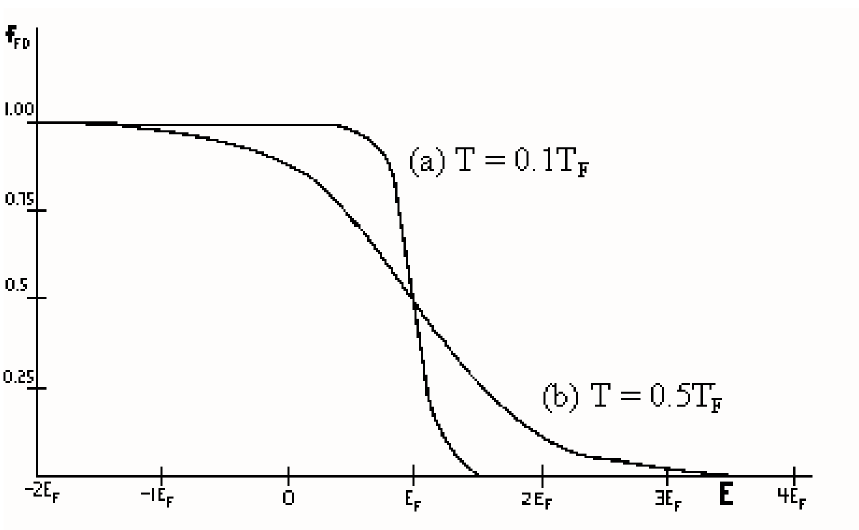
As we will discover in Chapter 11, these estimates are nearly an order of magnitude too large. The number of particles is not a large sample.

8-34. $E_1 = h^2/8mL^2$. All 10 bosons can be in this level, so $E_1\,(\text{total}) = 10h^2/8mL^2$.

8-35. (a) $$f_{FD}(E) = \frac{1}{e^{(E-E_F)/kT}+1} \quad (\text{Equation } 8-92)$$

$$= \frac{1}{e^{(E-E_F)/0.1E_F}+1} = \frac{1}{e^{10(E-E_F)/E_F}+1}$$

(b) $$f_{FD}(E) = \frac{1}{e^{(E-E_F)/0.5E_F}+1} = \frac{1}{e^{2(E-E_F)/E_F}+1}$$



8-36. $$\frac{N_o}{N} \approx 1 - \left(\frac{T}{T_c}\right)^{3/2} \quad (\text{Equation } 8-76)$$

(a) $$\frac{N_o}{N} \approx 1 - \left(\frac{T_c/2}{T_c}\right)^{3/2} = 1 - \left(\frac{1}{2}\right)^{3/2} = 0.646$$

(b) $$\frac{N_o}{N} \approx 1 - \left(\frac{T_c/4}{T_c}\right)^{3/2} = 1 - \left(\frac{1}{4}\right)^{3/2} = 0.875$$

8-37. For a one-dimensional well approximation, $E_n = (n^2h^2)/(8mL^2)$. At the Fermi level E_F, $n = N/2$, where N = number of electrons.

$$E_F = \frac{(N/2)^2h^2}{8mL^2} = \frac{h^2}{32m}\left(\frac{N}{L}\right)^2$$ where N/L = number of electrons/unit length,

i.e., the density of electrons. Assuming 1 free electron/*Au* atom,

$$\frac{N}{L} = \left[\frac{N_A\rho}{M}\right]^{1/3} = \left[\frac{6.02\times10^{23}\,electrons/mol\,(19.32\,g/cm^3)(10^2\,cm/m)^3}{197\,g/mol}\right]^{1/3} = 3.81\times10^9\,m^{-1}$$

$$E_F = \frac{(6.63\times10^{-34}J\cdot s)^2(3.81\times10^9m^{-1})^2}{8(9.11\times10^{-31}kg)(1.60\times10^{-19}J/eV)} = 647\,eV$$

This is the energy of an electron in the Fermi level above the bottom of the well. Adding the work function to such an electron just removes it from the metal, so the well is $5.47\,eV + 4.8\,eV = 10.3\,eV$ deep.

8-38.

$$\langle E_{K\,(escape)}\rangle = \frac{1}{2}\langle v^2_{escape}\rangle = \int_0^2\left(\frac{1}{2}mv^2\right)F(v)\,dv$$

$$= \frac{\int_0^\infty\left(\frac{1}{2}mv^2\right)v^3e^{-mv^2/2kT}\,dv}{\int_0^\infty v^3e^{-mv^2/2kT}\,dv} = \frac{1}{2}m\frac{I_5}{I_3}$$

$$= \frac{1}{2}m\frac{\lambda^{-3}}{\lambda^{-2}/2} = \frac{m}{\lambda} = m\frac{2kT}{m} = 2kT \qquad where\ \lambda = \frac{m}{2kT}$$

8-39. The amounts of superfluid liquid helium and normal liquid helium will be equal when $N_0 = N/2$ in Equation 8-76. For helium we use T_C equal to the temperature at the lambda point, 2.71 K.

$$\frac{N_0}{N} = \frac{N/2}{N} \approx 1 - \left(\frac{T}{T_C}\right)^{3/2} \rightarrow \left(\frac{T}{2.17K}\right)^{3/2} \approx 1 - 1/2 = 1/2$$

$$T = 2.17K(1/2)^{2/3} = 1.37\,K$$

8-40. The contribution to C_V of the H_2 molecular vibration is approximately R, so the vibrational energy at about 2000 K (where vibration begins) is:

$$E \approx 2000K(R) = 2000K(8.31\,J/mol\cdot K) = 1.66\times 10^4\,J/mol$$

$$E \approx (1.66\times 10^4\,J/mol)/N_A$$

$$\approx \frac{1.66\times 10^4\,J/mol}{6.02\times 10^{23}\,molecules/mol} = 2.76\times 10^{-20}\,J/molecule$$

$$\approx \frac{2.76\times 10^{-20}\,J/molecule}{1.60\times 10^{-19}\,J/eV} = 0.173\,eV/molecule$$

The vibrational frequency f is then: $f = \dfrac{E}{h} = \dfrac{0.173\,eV}{4.14\times 10^{-15}\,eV\cdot s} = 4.17\times 10^{13}\,Hz$

8-41. (a) $f(u)du = Ce^{-E/kT}du = Ce^{-Au^2/kT}du$ (from Equation 8-17)

$$1 = \int_{-\infty}^{+\infty} f(u)du = \int_{-\infty}^{+\infty} Ce^{-Au^2/kT}du = 2C\int_0^{\infty} e^{-Au^2/kT}du$$

$$= 2CI_0 = 2C\sqrt{\pi}\,\lambda^{-1/2}/2 \quad where\ \lambda = A/kT$$

$$= C\sqrt{\pi}\sqrt{kT/A} \quad \rightarrow \quad C = \sqrt{A/\pi kT}$$

(b) $$\langle E\rangle = \langle Au^2\rangle = \int_{-\infty}^{+\infty} Au^2 f(u)du = \int_{-\infty}^{+\infty} Au^2\sqrt{A/\pi kT}\, e^{-Au^2/kT}du$$

$$= A\sqrt{A/\pi kT}\,(2I_2) = A\sqrt{A/\pi kT}\,2\cdot(\sqrt{\pi}/4)\lambda^{-3/2} \quad where\ \lambda = A/kT$$

$$= \frac{1}{2}A\sqrt{A/kT}\,(kT/A)^{3/2} = \frac{1}{2}kT$$

8-42. $f(v_x) = (m/2\pi kT)^{1/2} e^{-mv_x^2/2kT}$ (Equation 8–20)

$$\langle |v_x| \rangle = \int_{-\infty}^{+\infty} |v_x| f(v_x)\, dv_x$$

$$= \int_{-\infty}^{0} (-v_x)(m/2\pi kT)^{1/2} e^{-mv_x^2/2kT} dv_x + \int_{0}^{+\infty} v_x (m/2\pi kT)^{1/2} e^{-mv_x^2/2kT} dv_x$$

$$= 2\int_0^{\infty} (m/2\pi kT)^{1/2} v_x e^{-mv_x^2/2kT} dv_x$$

$$= 2(m/2\pi kT)^{1/2} I_1 \quad with\ \lambda = m/2kT$$

$$= 2(m/2\pi kT)^{1/2} \cdot 1/2 \cdot \frac{2kT}{m} = \sqrt{\frac{2kT}{\pi m}}$$

8-43. $f_{FD} = \dfrac{1}{e^{\alpha} e^{E/kT} + 1} = \dfrac{1}{e^{(E-E_F)/kT} + 1}$ where $\alpha = -\dfrac{E_F}{kT}$

For $E >> E_F$, $e^{(E-E_F)/kT} >> 1$ and

$$f_{FD} \approx \frac{1}{e^{(E-E_F)/kT}} = \frac{1}{e^{-E_F/kT} \cdot e^{E/kT}} = \frac{1}{e^{\alpha} e^{E/kT}} = f_B$$

8-44. $N = e^{-\alpha} \dfrac{4\pi (2m_e)^{3/2} V}{h^3} \displaystyle\int_0^{\infty} E^{1/2} e^{-E/kT} dE$ (Equation 8-67)

Considering the integral, we change the variable: $E/kT = u^2$, then

$E = kTu^2$, $E^{1/2} = (kT)^{1/2} u$, and $dE = kT(2u)du$. So,

(Problem 8-44 continued)

$$\int_0^\infty E^{1/2} e^{-E/kT} dE = 2(kT)^{3/2} \int_0^\infty u^2 e^{-u^2} du$$

The value of the integral (from tables) is $\sqrt{\pi}/4$, so

$$N = e^{-\alpha} \frac{4\pi(2m_e)^{3/2} V}{h^3} \frac{2(kT)^{3/2}\sqrt{\pi}}{4} \quad \text{or} \quad e^{\alpha} = \frac{2(2m_e \pi kT)^{3/2} V}{Nh^3}$$, which is Equation 8-68.

8-45. (a) $N = \sum_i n_i = f_0(E_0) + f_1(E_1)$ (with $g_0 = g_1 = 1$)

$$= Ce^0 + Ce^{-\epsilon/kT} = C(1 + e^{-\epsilon/kT})$$

So, $C = \dfrac{N}{1 + e^{-\epsilon/kT}}$

(b) $$\langle E \rangle = \frac{0 \cdot n_0 + \epsilon n_1}{N} = \frac{\epsilon C e^{-\epsilon/kT}}{N} = \frac{N\epsilon e^{-\epsilon/kT}}{(1 + e^{-\epsilon/kT})N} = \frac{\epsilon e^{-\epsilon/kT}}{1 + e^{-\epsilon/kT}}$$

As T → 0, $e^{-\epsilon/kT} = 1/e^{-\epsilon/kT} \rightarrow 0$, so $\langle E \rangle \rightarrow 0$

As T → ∞, $e^{-\epsilon/kT} = 1/e^{-\epsilon/kT} \rightarrow 1$, so $\langle E \rangle \rightarrow \epsilon/2$

(c) $$C_V = \frac{dE}{dT} = \frac{d(N\langle E \rangle)}{dT} = \frac{d}{dT}\left(\frac{N\epsilon e^{-\epsilon/kT}}{1 + e^{-\epsilon/kT}}\right)$$

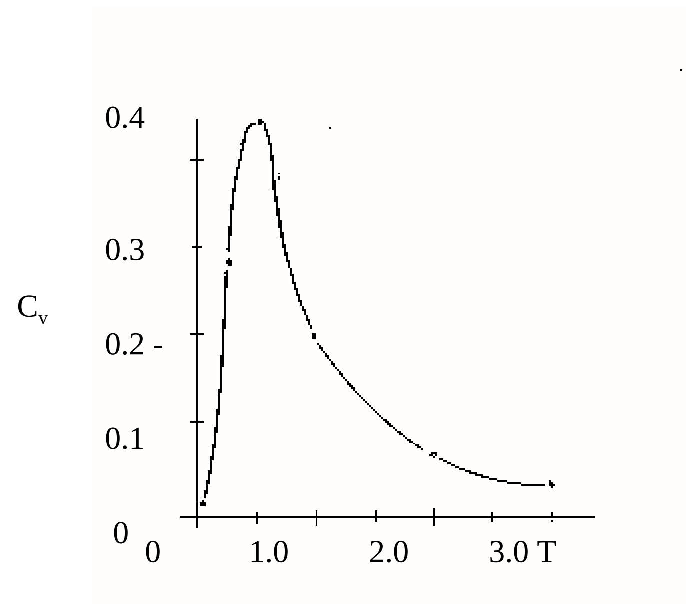
$$= \frac{N\epsilon^2}{kT^2}\left[\frac{-(e^{-\epsilon/kT})^2}{(1 + e^{-\epsilon/kT})^2} + \frac{e^{-\epsilon/kT}}{(1 + e^{-\epsilon/kT})}\right]$$

$$= Nk\left(\frac{\epsilon}{kT}\right)^2 \frac{e^{-\epsilon/kT}}{(1 + e^{-\epsilon/kT})^2}$$

(Problem 8-45 continued)

(d)

T (× ϵ/k)	0.1	0.25	0.5	1.0	2.0	3.0
C_v (× Nk)	0.005	0.28	0.42	0.20	0.06	0.03



8-46. (a) $$p = \hbar\left[k_x^2 + k_y^2 + k_z^2\right]^{1/2} = \hbar\left[\left(\frac{n_1\pi}{L}\right)^2 + \left(\frac{n_2\pi}{L}\right)^2 + \left(\frac{n_3\pi}{L}\right)^2\right]^{1/2}$$

$$= \frac{\hbar\pi}{L}\left[n_1^2 + n_2^2 + n_3^2\right]^{1/2} = \frac{\hbar\pi N}{L}$$

$$E = pc = \frac{\hbar c\pi N}{L}$$

(b) Consider the space whose axes are n_1, n_2, and n_3. The points in space correspond to all possible integer values of n_1, n_2, and n_3, all of which are located in the all positive octant. Each state has unit volume associated with it. Those states between N and $N + dN$ lie in a spherical shell of the octant whose radius is N and whose thickness is dN. Its volume is $(1/8)4\pi N^2 dN$. Because photons can have two polarizations (spin directions), the number of possible states is $2\cdot(1/8)4\pi N^2 dN = \pi N^2 dN$.

(Problem 8-46 continued)

(c) This number of photon states has energy between E and $E+dE$, where $N = EL/\pi\hbar c$.

The density of states *g(E)* is thus:

$$g(E)dE = \text{number of photon states at } E \in dE$$
$$= \text{number of photon states at } N \in dN$$
$$= \pi N^2 dN$$
$$= \pi(EL/\pi\hbar c)^2 (L/\pi\hbar c)\, dE$$
$$= \frac{8\pi L^3}{(2\pi\hbar c)^3} E^2\, dE = \frac{8\pi L^3}{(hc)^3} E^2\, dE$$

The probability that a photon exists in a state is given by:

$$f_{BE}(E) = \frac{1}{e^{\alpha} e^{E/kT} - 1} = \frac{1}{e^{E/kT} - 1} \quad \text{(Equation 8-48)}$$

The number of photons with energy between E and $E+dE$ is then:

$$n(E)dE = f_{BE}(E)g(E)dE = \frac{8\pi(L/hc)^3 E^2\, dE}{e^{E/kT} - 1}$$

(d) The number of photons per unit volume within this energy range is $n(E)dE/L^3$.

Because each photon has energy E, the energy density for photons is:

$$u(E)dE = E\cdot n(E)dE/L^3 = \frac{8\pi E^3\, dE}{(hc)^3(e^{E/kT} - 1)}$$

which is also the density of photons with wavelength between λ and λ+dλ, where $\lambda = hc/E \;\rightarrow\; E = hc/\lambda$. So

$$d\lambda = \left|\frac{d\lambda}{dE}\right| dE = \frac{hc}{E^2} dE = \frac{\lambda^2}{hc} dE \;\rightarrow\; dE = \frac{hc}{\lambda^2} d\lambda$$

$$u(\lambda)d\lambda = u(E)dE = \frac{8\pi(hc/\lambda)^3(hc/\lambda^2)d\lambda}{(hc)^3(e^{hc/\lambda kT} - 1)} = \frac{8\pi hc\lambda^{-5} d\lambda}{e^{hc/\lambda kT} - 1}$$