Chapter 12

1. From Compton scattering $\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$, so $\Delta\lambda_{\max} = 2h/mc$. From Chapter 3 Problem 43 we know that the kinetic energy of the recoiling particle is

$$K = \frac{\Delta\lambda/\lambda}{1+\Delta\lambda/\lambda}\frac{hc}{\lambda} = 5.7\ \text{MeV}$$

Using $\Delta\lambda 2h/mc$ from above and for convenience letting $x = 2h/mc\lambda$, we have

$$K = \frac{2h/mc\lambda}{1+2h/mc\lambda}\frac{hc}{\lambda} = \frac{x}{1+x}\frac{mc^2}{2}x$$

$$K(1+x) = x^2\frac{mc^2}{2} \qquad \frac{mc^2}{2}x^2 - Kx - K = 0$$

This is a quadratic equation that can be solved numerically to find $x = 0.117 = 2h/mc\lambda$, so

$$\lambda = \frac{2h}{mcx} = \frac{2\,(1240\ \text{eV}\cdot\text{nm})}{(938.27\times10^6\ \text{eV})\,(0.117)} = 2.26\times10^{-6}\ \text{nm}$$

The photon's energy is

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{2.26\times10^{-6}\ \text{nm}} = 54.9\ \text{MeV}$$

2. a) integral: ^{6}Li, ^{18}F   b) half-integral: ^{3}He, ^{7}Li, ^{19}F

3. In each case the atomic number equals the number of protons (Z), and the atomic charge is Ze.
   $^{3}_{2}$He: $Z=2$, $N=1$, $A=3$, $m=3.02$ u
   $^{4}_{2}$He: $Z=2$, $N=2$, $A=4$, $m=4.00$ u
   $^{17}_{8}$O: $Z=8$, $N=9$, $A=17$, $m=17.0$ u
   $^{42}_{20}$Ca: $Z=20$, $N=22$, $A=42$, $m=42.0$ u
   $^{210}_{82}$Pb: $Z=82$, $N=128$, $A=210$, $m=210.0$ u
   $^{235}_{92}$U: $Z=92$, $N=143$, $A=235$, $m=235.0$ u

4. ^{6}Li: $Z=3$, $N=3$ ^{13}C: $Z=6$, $N=7$
   ^{40}K: $Z=19$, $N=21$ ^{64}Cu: $Z=29$, $N=35$

5. Isotopes ^{36}Ca through ^{51}Ca; Isobars ^{40}Cl, ^{40}Ar, ^{40}K, ^{40}Sc; Isotones ^{34}Si,^{35}P, ^{36}S, ^{37}Cl, ^{38}Ar, ^{39}K, ^{41}Sc, ^{42}Ti

6. From the text nuclear density is 2.3×10^{17} kg/m^3, which is 2.3×10^{14} times the density of water.

*7.

$$\frac{\mu_p}{\mu_e} = \frac{2.79\mu_N}{-1.00116\mu_B} = -2.786\frac{\mu_N}{\mu_B} = -2.786\frac{m_e}{m_p} = -2.786\frac{0.51100}{938.27} = -1.52 \times 10^{-3}$$

*8. From Appendix 8 the mass of the nuclide is 55.935 u or 9.29×10^{-26} kg.

$$\rho = \frac{m}{\frac{4}{3}\pi r^3} = \frac{m}{\frac{4}{3}\pi r_0^3 A} = \frac{9.29 \times 10^{-26}\ \text{kg}}{\frac{4}{3}\pi\left(1.2 \times 10^{-15}\ \text{m}\right)^3 (56)} = 2.29 \times 10^{17}\ \text{kg/m}^3$$

9. The electron binding energy is virtually the same as in hydrogen, or 13.6 eV. The deuteron rest energy is $mc^2 = 1876$ MeV. The ratio of these is

$$\frac{13.6\ \text{eV}}{1876 \times 10^6\ \text{eV}} = 7.25 \times 10^{-9}$$

"Nuclear calculations" might also refer to nuclear binding energies. For the deuteron the binding energy is 2.2 MeV, so the ratio of the electronic to nuclear binding energy is on the order of 10^{-5}. It is generally safe to ignore electronic binding energies.

10. Using Equation (12.14)

$$E_{\min} = B_d\left(1 + \frac{B_d}{2M\left({}^2\text{H}\right)c^2}\right) = 1.00059 B_d \cong 2.22\ \text{MeV}$$

From this calculation we can see that the error is 0.00059 or 0.059%.

*11. The distance equals the nuclear radius:

$$r = r_0 A^{1/3} = (1.2\ \text{fm})\left(3^{1/3}\right) = 1.73\ \text{fm}$$

$$F_g = \frac{GMm}{r^2} = \frac{\left(6.673 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.673 \times 10^{-27}\ \text{kg}\right)^2}{\left(1.73 \times 10^{-15}\ \text{m}\right)^2} = 6.2 \times 10^{-35}\ \text{N}$$

$$F_e = \frac{ke^2}{r^2} = \frac{\left(8.988 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602 \times 10^{-19}\ \text{C}\right)^2}{\left(1.73 \times 10^{-15}\ \text{m}\right)^2} = 77\ \text{N}$$

To compare with the strong force, we need the potential energy:

$$|V_g| = \frac{GMm}{r} = \frac{\left(6.673 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.673 \times 10^{-27}\ \text{kg}\right)^2}{\left(1.73 \times 10^{-15}\ \text{m}\right)\left(1.602 \times 10^{-13}\ \text{J/MeV}\right)} = 6.7 \times 10^{-37}\ \text{MeV}$$

$$|V_e| = \frac{ke^2}{r} = \frac{\left(8.988 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.602 \times 10^{-19}\ \text{C}\right)^2}{\left(1.73 \times 10^{-15}\ \text{m}\right)\left(1.602 \times 10^{-13}\ \text{J/MeV}\right)} = 0.83\ \text{MeV}$$

The electrostatic force is 50 times weaker than the strong force. The gravitational force is almost 10^{38} times weaker than the strong force.

12. The required force is

$$F_e = \frac{ke^2}{r^2} = \frac{\left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.602\times10^{-19}\ \mathrm{C}\right)^2}{\left(2\times10^{-15}\ \mathrm{m}\right)^2} = 58\ \mathrm{N}$$

We can also compare the potential energy:

$$V = \frac{ke^2}{r} = \frac{\left(8.988\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(1.602\times10^{-19}\ \mathrm{C}\right)^2}{\left(2\times10^{-15}\ \mathrm{m}\right)\left(1.602\times10^{-13}\ \mathrm{J/MeV}\right)} = 0.72\ \mathrm{MeV}$$

13. a) Think of the nucleus as the composite of ${}^{A-1}_{Z}D$ and ${}^{1}n$, so that

$$B = \left[M\left({}^{A-1}_{Z}D\right) + m_n - M\left({}^{A}_{Z}X\right)\right]c^2$$

b) ${}^{6}\mathrm{Li}$:

$$\begin{aligned} B &= \left(M\left({}^{5}\mathrm{Li}\right) + m_n - M\left({}^{6}\mathrm{Li}\right)\right)c^2 \\ &= \left(5.012540\ \mathrm{u} + 1.008665\ \mathrm{u} - 6.015122\ \mathrm{u}\right)c^2\left(931.49\ \mathrm{MeV}/\left(\mathrm{u}\cdot c^2\right)\right) = 5.67\ \mathrm{MeV} \end{aligned}$$

${}^{17}\mathrm{O}$:

$$B = \left(M\left({}^{16}\mathrm{O}\right) + m_n - M\left({}^{17}\mathrm{O}\right)\right)c^2 = 4.14\ \mathrm{MeV}$$

${}^{207}\mathrm{Pb}$:

$$B = \left(M\left({}^{206}\mathrm{Pb}\right) + m_n - M\left({}^{207}\mathrm{Pb}\right)\right)c^2 = 6.74\ \mathrm{MeV}$$

14. a) As in Problem 13, consider the nucleus as the composite of ${}^{A-1}_{Z-1}D$ and ${}^{1}\mathrm{H}$, so that

$$B = \left[M\left({}^{A-1}_{Z-1}D\right) + M\left({}^{1}\mathrm{H}\right) - M\left({}^{A}_{Z}X\right)\right]c^2$$

b) ${}^{8}\mathrm{Be}$:

$$\begin{aligned} B &= \left[M\left({}^{7}\mathrm{Li}\right) + M\left({}^{1}\mathrm{H}\right) - M\left({}^{8}\mathrm{Be}\right)\right]c^2 \\ &= \left(7.016004\ \mathrm{u} + 1.007825\ \mathrm{u} - 8.005305\ \mathrm{u}\right)c^2\left(931.49\ \mathrm{MeV}/\left(\mathrm{u}\cdot c^2\right)\right) = 17.3\ \mathrm{MeV} \end{aligned}$$

${}^{15}\mathrm{O}$:

$$B = \left[M\left({}^{14}\mathrm{N}\right) + M\left({}^{1}\mathrm{H}\right) - M\left({}^{15}\mathrm{O}\right)\right]c^2 = 7.3\ \mathrm{MeV}$$

${}^{32}\mathrm{S}$:

$$B = \left[M\left({}^{31}\mathrm{P}\right) + M\left({}^{1}\mathrm{H}\right) - M\left({}^{32}\mathrm{S}\right)\right]c^2 = 8.9\ \mathrm{MeV}$$

15. The energy release comes from the mass difference:

$$\begin{aligned}\Delta E &= \Delta mc^2 = \left(3M\left({}^4\text{He}\right) - M\left({}^{12}\text{C}\right)\right)c^2 \\ &= \left(3\,(4.002603\text{ u}) - 12.000\text{ u}\right)c^2\left(931.49\text{ MeV}/\left(\text{u}\cdot c^2\right)\right) = 7.27\text{ MeV}\end{aligned}$$

16. With an even number of nucleons the ^{4}He nucleus has integer spin. Spins of like particles tend to anti-align, so the net spin should be zero. The ^{3}He nucleus has three nucleons, so its spin is half-integer. Because the spins of the two protons tend to align, the net spin should be one-half.

17. As in Problem 14

$$B = \left[M\left({}^{106}\text{Pd}\right) + M\left({}^{1}\text{H}\right) - M\left({}^{107}\text{Ag}\right)\right]c^2 = 5.8\text{ MeV}$$

The electron binding energy is less by a factor of 5.8 MeV/25.6 keV = 227.

*18. For ^{4}He the radius is

$$r = r_0 A^{1/3} = (1.2\text{ fm})\left(4^{1/3}\right) = 1.90\text{ fm}$$

$$V = \frac{e^2}{4\pi\epsilon_0 r} = \frac{1.44\times10^{-9}\text{ eV}\cdot\text{m}}{1.90\times10^{-15}\text{ m}} = 0.76\text{ MeV}$$

For ^{40}Ca:

$$\begin{aligned}\Delta E_{\text{coul}} &= \frac{3}{5}\frac{Z\,(Z-1)\,e^2}{4\pi\epsilon_0 R} = 0.72\,Z\,(Z-1)\,A^{-1/3}\text{ MeV} \\ &= 0.72\,(20)\,(19)\,40^{-1/3}\text{ MeV} = 80\text{ MeV}\end{aligned}$$

For ^{208}Pb:

$$\Delta E_{\text{coul}} = 0.72\,(82)\,(81)\,208^{-1/3}\text{ MeV} = 807\text{ MeV}$$

There is a factor of ten between each of these three nuclides.

19. Adding another neutron to ^{18}O reduces δ from $+\Delta$ to 0, makes the symmetry term more negative, and makes the surface term more negative. The net effect of these three terms more than offsets the increase in the volume term.

20. δ drops from $+\Delta$ in ^{42}Ca to 0 in ^{41}Ca. The volume term is substantially higher in ^{42}Ca, more than offsetting the difference in the surface effect. The symmetry term is only slightly higher for ^{42}Ca. Comparing ^{42}Ca and ^{42}Ti, we see that the coulomb term is higher in ^{42}Ti, because it has two more protons.

*21.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(5.271 \text{ y})\,(3.156 \times 10^7 \text{ s/y})} = 4.167 \times 10^{-9} \text{ s}^{-1}$$

$$N = \frac{R}{\lambda} = \frac{2.4 \times 10^7 \text{ s}^{-1}}{4.167 \times 10^{-9} \text{ s}^{-1}} = 5.76 \times 10^{15}$$

$$m = \left(5.76 \times 10^{15}\right) \frac{1 \text{ mol}}{6.022 \times 10^{23}} \frac{60 \text{ g}}{\text{mol}} = 0.57\ \mu\text{g}$$

22.

$$R = R_0 e^{-\lambda t} = \frac{R_0}{5} \text{ at } t = T = 3600 \text{ s}$$

$$\lambda = \frac{\ln 5}{T}$$

$$t_{1/2} = \frac{\ln 2}{\lambda} = \frac{\ln 2}{\ln 5} T = \frac{\ln 2}{\ln 5}\,(3600 \text{ s}) = 1550 \text{ s} \ \cong 26 \text{ minutes}$$

*23. In general (using the definition of the mean value of a function)

$$\tau = \frac{\int_0^\infty t\,R(t)\,dt}{\int_0^\infty R(t)\,dt} = \frac{1}{N_0} \int_0^\infty t\,R(t)\,dt$$

because all nuclei must decay between $t = 0$ and $t = \infty$. Using $R = R_0 e^{-\lambda t}$ we have

$$\tau = \frac{R_0}{N_0} \int_0^\infty t\,e^{-\lambda t}\,dt = \frac{R_0}{N_0} \frac{1}{\lambda^2} = \frac{\lambda N_0}{\lambda^2 N_0} = \frac{1}{\lambda} = \frac{t_{1/2}}{\ln 2}$$

24. With 96% ^{238}U and 4% ^{235}U the number N of ^{238}U atoms is

$$N = (10 \text{ kg}) \frac{1 \text{ mol}}{0.235 \text{ kg}} \frac{6.022 \times 10^{23}}{\text{mol}} \left(\frac{96}{4}\right) = 6.15 \times 10^{26}$$

$$R = \lambda N = \frac{\ln 2}{t_{1/2}} N = \frac{\ln 2}{2.22 \times 10^{16} \text{ s}} \left(6.15 \times 10^{26}\right) = 1.92 \times 10^{10} \text{ Bq}$$

25.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(1.25 \times 10^9 \text{ y})\,(3.156 \times 10^7 \text{ s/y})} = 1.757 \times 10^{-17} \text{ s}^{-1}$$

$$N = (70\text{ kg})\,\frac{1\text{ mol}}{0.040\text{ kg}}\,\frac{6.022\times10^{23}}{\text{mol}}\,(0.00012)\,(0.003) = 3.794\times10^{20}$$

$$R = \lambda N = \left(1.757\times10^{-17}\text{ s}^{-1}\right)\left(3.794\times10^{20}\right) = 6.67\times10^{3}\text{ Bq}$$

*26.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(109.8\text{ min})\,(60\text{ s/min})} = 1.052\times10^{-4}\text{ s}^{-1}$$

$$R = R_0 e^{-\lambda t} = \left(10^{7}\text{ Bq}\right)\exp\left(-\left(1.052\times10^{-4}\text{ s}^{-1}\right)(48)\,(3600\text{ s})\right) = 0.127\text{ Bq}$$

27.

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(12.33\text{ y})} = 5.622\times10^{-2}\text{ y}^{-1}$$

The mass decreases by the same exponential factor as N, so

$$m = m_0 e^{-\lambda t} = (2000\text{ kg})\exp\left(-\left(5.622\times10^{-2}\text{ y}^{-1}\right)(50\text{ y})\right) = 120\text{ kg}$$

28. For convenience assume a mass of 1 kg of each material.

For ^{3}H:

$$N = (1\text{ kg})\,\frac{1\text{ mol}}{0.003\text{ kg}}\,\frac{6.022\times10^{23}}{\text{mol}} = 2.01\times10^{26}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(12.33\text{ y})} = 5.622\times10^{-2}\text{ y}^{-1}$$

$$R = \lambda N = \left(5.622\times10^{-2}\text{ y}^{-1}\right)\left(2.01\times10^{26}\right) = 1.13\times10^{25}\text{ y}^{-1}$$

For ^{222}Rn:

$$N = (1\text{ kg})\,\frac{1\text{ mol}}{0.222\text{ kg}}\,\frac{6.022\times10^{23}}{\text{mol}} = 2.71\times10^{24}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(3.82\text{ d})\,(1\text{ y}/365.25\text{ d})} = 66.28\text{ y}^{-1}$$

$$R = \lambda N = \left(66.28\text{ y}^{-1}\right)\left(2.71\times10^{24}\right) = 1.80\times10^{26}\text{ y}^{-1}$$

For ^{239}Pu:

$$N = (1\text{ kg})\,\frac{1\text{ mol}}{0.239\text{ kg}}\,\frac{6.022\times10^{23}}{\text{mol}} = 2.52\times10^{24}$$

$$\lambda = \frac{\ln 2}{t_{1/2}} = \frac{\ln 2}{(24110\text{ y})} = 2.875\times10^{-5}\text{ y}^{-1}$$

$$R = \lambda N = \left(2.875\times10^{-5}\text{ y}^{-1}\right)\left(2.52\times10^{24}\right) = 7.245\times10^{19}\text{ y}^{-1}$$

The order of activity is ^{222}Rn $>$ ^{3}H $>$ ^{239}Pu.

29. The decays in question are ^{52}Fe $\rightarrow n+$ ^{51}Fe and ^{52}Fe $\rightarrow p+$ ^{51}Mn.
neutron decay:

$$Q = (51.948117 - 50.956825 - 1.008665)\,\mathrm{u}\cdot c^2 = -16.2\text{ MeV}$$

which is not allowed because $Q < 0$.
proton decay:

$$Q = (51.948117 - 50.948216 - 1.007825)\,\mathrm{u}\cdot c^2 = -7.4\text{ MeV}$$

which is not allowed because $Q < 0$. Because ^{40}Fe has an excess of protons, it should decay.

30. The separation energies are equal to the absolute value of the Q values found: 7.7 MeV for neutron decay and 5.6 MeV for proton decay.

31. We need not consider β^- decay, because there is nothing with mass 1 and charge 2. The β^+ decay is $p \rightarrow n + \beta^+$, with

$$Q = (m_p - m_n - m_e)\,c^2 = (1.007825 - 1.008665 - 0.00054858)\,\mathrm{u}\cdot c^2 = -1.8\text{ MeV}$$

which is not allowed with $Q < 0$. For electron capture $p + \beta^- \rightarrow n$, with

$$Q = (m_p + m_e - m_n)\,c^2 = (1.007825 - 1.008665 + 0.00054858)\,\mathrm{u}\cdot c^2 = -0.78\text{ MeV}$$

which is not allowed with $Q < 0$.

32. For ^{144}Sm:

$$\begin{aligned} Q &= \left[M\left(^{144}\mathrm{Sm}\right) - M\left(^{140}\mathrm{Nd}\right) - M\left(^{4}\mathrm{He}\right)\right]c^2 \\ &= [143.911996 - 139.909317 - 4.002602]\,\mathrm{u}\cdot \mathrm{c}^2 = +0.07\text{ MeV} \end{aligned}$$

For ^{147}Sm:

$$\begin{aligned} Q &= \left[M\left(^{147}\mathrm{Sm}\right) - M\left(^{143}\mathrm{Nd}\right) - M\left(^{4}\mathrm{He}\right)\right]c^2 \\ &= [146.9149 - 142.9098 - 4.002602]\,\mathrm{u}\cdot \mathrm{c}^2 = +2.24\text{ MeV} \end{aligned}$$

With $Q > 0$ both isotopes may decay, though the lighter one is nearly stable. The ^{147}Sm abundance is greater because its decay occurs with an extremely long half-life of 10^{11} years.

33. The decay is ^{241}Am $\rightarrow$ ^{237}Np + ^{4}He.

$$\begin{aligned} Q &= \left[M\left(^{241}\mathrm{Am}\right) - M\left(^{237}\mathrm{Np}\right) - M\left(^{4}\mathrm{He}\right)\right]c^2 \\ &= [241.056824 - 237.048168 - 4.002603]\,\mathrm{u}\cdot \mathrm{c}^2 = 5.64\text{ MeV} \end{aligned}$$

By Equation (12.32) the emitted alpha particle's energy is

$$K_\alpha = \frac{A-4}{A}Q = \frac{237}{241}\,(5.64\text{ MeV}) = 5.55\text{ MeV}$$

Then by conservation of energy the daughter's kinetic energy is 0.09 MeV.

34. Let M be the mass of the decaying nucleus, E_1 and p_1 the energy and momentum of the recoiling nucleus, and let E_2 and p_2 be the energy and momentum of the β^-. Then by conservation of momentum $\vec{p}_1 + \vec{p}_2 = 0$, and by conservation of energy $Mc^2 = E_1 + E_2$. Then using the energy-momentum invariant $E^2 = p^2c^2 + E_0^2$ one can rewrite the energy conservation equation in terms of the two momenta. Then we would have two equations in two unknowns (p_1 and p_2), and by the laws of algebra there are unique solutions for p_1 and p_2. Hence there are unique solutions for E_1 and E_2.

*35. $^{80}\mathrm{Br} \rightarrow {}^{76}\mathrm{As} + {}^{4}\mathrm{He}$:

$$Q = [79.918530 - 75.922394 - 4.002603]\,\mathrm{u} \cdot \mathrm{c}^2 = -6.0 \text{ MeV (not allowed)}$$

$^{80}\mathrm{Br} \rightarrow {}^{80}\mathrm{Kr} + \beta^-$:

$$Q = (79.918530 - 79.916378)\,\mathrm{u} \cdot c^2 = 2.0 \text{ MeV (allowed)}$$

$^{80}\mathrm{Br} \rightarrow {}^{80}\mathrm{Se} + \beta^+$:

$$Q = (79.918530 - 79.916522 - 2\,(0.000549))\,\mathrm{u} \cdot c^2 = 0.85 \text{ MeV (allowed)}$$

$^{80}\mathrm{Br} + \beta^- \rightarrow {}^{80}\mathrm{Se}$

$$Q = (79.918530 - 79.916522)\,\mathrm{u} \cdot c^2 = 1.9 \text{ MeV (allowed)}$$

*36. $^{227}\mathrm{Ac} \rightarrow {}^{223}\mathrm{Fr} + {}^{4}\mathrm{He}$:

$$Q = [227.027749 - 223.019733 - 4.002603]\,\mathrm{u} \cdot \mathrm{c}^2 = 6.1 \text{ MeV (allowed)}$$

$^{227}\mathrm{Ac} \rightarrow {}^{227}\mathrm{Th} + \beta^-$:

$$Q = (227.027749 - 227.027701)\,\mathrm{u} \cdot c^2 = 0.045 \text{ MeV (allowed, barely)}$$

$^{227}\mathrm{Ac} \rightarrow {}^{227}\mathrm{Ra} + \beta^+$:

$$Q = (227.027749 - 227.029170 - 2\,(0.000549))\,\mathrm{u} \cdot c^2 = -2.3 \text{ MeV (not allowed)}$$

$^{227}\mathrm{Ac} + \beta^- \rightarrow {}^{227}\mathrm{Ra}$

$$Q = (227.027749 - 227.029170)\,\mathrm{u} \cdot c^2 = -1.3 \text{ MeV (not allowed)}$$

37. $^{230}\mathrm{Pa} \rightarrow {}^{226}\mathrm{Ac} + {}^{4}\mathrm{He}$:

$$Q = [230.034534 - 226.026090 - 4.002603]\,\mathrm{u} \cdot \mathrm{c}^2 = 5.4 \text{ MeV (allowed)}$$

$^{230}\mathrm{Pa} \rightarrow {}^{230}\mathrm{U} + \beta^-$:

$$Q = (230.034534 - 230.033924)\,\mathrm{u} \cdot c^2 = 0.57 \text{ MeV (allowed)}$$

$^{230}\mathrm{Pa} \rightarrow {}^{230}\mathrm{Th} + \beta^+$:

$$Q = (230.034534 - 230.033127 - 2\,(0.000549))\,\mathrm{u} \cdot c^2 = 0.27 \text{ MeV (allowed)}$$

$^{230}\mathrm{Pa} + \beta^- \rightarrow {}^{230}\mathrm{Th}$:

$$Q = (230.034534 - 227.033127)\,\mathrm{u} \cdot c^2 = 1.3 \text{ MeV (allowed)}$$

38. Looking at Figure 12.15 it appears the gamma ray energy can be 0.226 MeV or 0.230 MeV in a transition from E_x to the ground state. Clearly a 0.072 MeV transition is possible. There are also two possible transitions from E_x to the 0.072 MeV level. The energies of those gamma rays are 0.226 MeV -0.072 MeV $= 0.154$ MeV and 0.230 MeV -0.072 MeV $= 0.158$ MeV.

39. After six days the number of radon atoms remaining is

$$N = N_0 \exp\left(-\frac{(\ln 2)\,(6\text{ d})}{3.82\text{ d}}\right) = 0.3367 N_0$$

For ideal gases the partial pressure is proportional to N. Therefore the partial pressure of radon is 0.337 atm and the partial pressure of helium is 0.663.

40. ^{60}Co decays by β^- to an excited state of ^{60}Ni, which is unstable and emits a gamma ray.

41. There are three explanations. Looking at the Q formulas, we see that two extra electron masses are required for β^+ decay, and these are not available in "close calls." Second, the product of β^- decay should be more stable with a greater N/A ratio. Finally, successive alpha decays place daughters farther and farther below the line of stability. Only β^- decays can move back in the direction of the stability line.

42. For the reaction $^{14}\text{O} \rightarrow {}^{14}\text{N} + \beta^+$ we have

$$Q = (14.008595 - 14.003074 - 2\,(0.000549))\,\text{u} \cdot c^2 = 4.1\text{ MeV}$$

which is allowed. For the other reaction $p \rightarrow n + \beta^+$ we have

$$Q = (1.007825 - 1.008665 - 2\,(0.000549))\,\text{u} \cdot c^2 = -1.8\text{ MeV}$$

which is not allowed.

43. From Equation (12.50)

$$R' = \frac{N\left({}^{206}\text{Pb}\right)}{N\left({}^{238}\text{U}\right)} = e^{\lambda t} - 1$$

Then rearranging we have

$$t = \frac{t_{1/2}}{\ln 2} \ln\left(R' + 1\right)$$

For $R' = 0.76$:

$$t = \frac{4.47 \times 10^9\text{ y}}{\ln 2} \ln 1.76 = 3.65 \times 10^9\text{ y}$$

For $R' = 3.1$:

$$t = \frac{4.47 \times 10^9\text{ y}}{\ln 2} \ln 4.1 = 9.10 \times 10^9\text{ y}$$

The large time difference implies different origins.

44. ^{40}K, ^{87}Rb, ^{138}La, and ^{147} Sm have half-lives that are close to the age of the earth. However, of these only ^{87}Rb and ^{147}Sm have reasonable abundances.

45. As in Problem 43

$$t = \frac{4.47 \times 10^9 \text{ y}}{\ln 2} \ln 1.01 = 6.4 \times 10^7 \text{ y}$$

*46.

$$R' = \frac{N\left({}^{206}\text{Pb}\right)}{N\left({}^{238}\text{U}\right)} = e^{\lambda t} - 1 = e^{\ln 2} - 1 = 1$$

*47. From the A values it is clear that there are $28/4 = 7$ alpha decays. But seven alpha decays reduces Z from 92 to 78, so there must be four β^- decays in order to bring Z up to 92. The other possible beta decays (β^+ and electron capture) are not seen naturally. In any case the net result must be a change of four charge units. We would have to look at a table of nuclides to determine the exact chain(s).

48.

$$0.0072 = \frac{N\left({}^{235}\text{U}\right)}{N\left({}^{238}\text{U}\right)} = \frac{N_0\left({}^{235}\text{U}\right)\exp\left(-\lambda_{235}t\right)}{N_0\left({}^{238}\text{U}\right)\exp\left(-\lambda_{238}t\right)}$$

so

$$\begin{aligned} \frac{N_0\left({}^{235}\text{U}\right)}{N_0\left({}^{238}\text{U}\right)} &= 0.0072\frac{\exp\left(-\lambda_{238}t\right)}{\exp\left(-\lambda_{235}t\right)} \\ &= 0.0072\exp\left(\left(\frac{\ln 2}{0.7038 \times 10^9 \text{ y}} - \frac{\ln 2}{4.468 \times 10^9 \text{ y}}\right)\left(4.5 \times 10^9 \text{ y}\right)\right) = 0.301 \end{aligned}$$

or 30.1%.

49. a) After one day

$$N = N_0 \exp\left(-\frac{(\ln 2)(24 \text{ h})}{25.39 \text{ h}}\right) = 0.519N_0$$

so 51.9 g is still ^{252} Fm. The remainder (41.1 g) is almost all ^{248}Cf, because its half-life is 334 days.

b) After one month the amount of fermium is

$$N = N_0 \exp\left(-\frac{(\ln 2)(30)(24 \text{ h})}{25.39 \text{ h}}\right) \cong 0$$

Almost none of the fermium is left. It is more difficult to determine the remaining amount of ^{248}Cf. Of the amount created on the first day of the month the remaining amount is about

$$N_0 \exp\left(-\frac{(\ln 2)\,(29\ \mathrm{d})}{333.5\ \mathrm{d}}\right) = 0.942 N_0$$

Of the amount created on the fourth day of the month the remaining amount is about

$$N_0 \exp\left(-\frac{(\ln 2)\,(25\ \mathrm{d})}{333.5\ \mathrm{d}}\right) = 0.949 N_0$$

Because most of the fermium decayed within the first four days, we can see that the should be between 94% and 95% remaining as ^{248}Cf by the end of the month.

c) Five years is more than five half-lives of californium, so there is very little of it left. However, the half-life of curium is 18.1 years, so there is at least

$$N_0 \exp\left(-\frac{(\ln 2)\,(5\ \mathrm{y})}{18.1\ \mathrm{y}}\right) = 0.826 N_0$$

of ^{244}Cm after five years.

d) After 100 years more than five half-lives of ^{244}Cm have elapsed. The half life of its decay product ^{240}Pu is quite long, 6593 years, so most of the sample is ^{240}Pu after 100 years.

e) After ^{240}Pu decays to ^{236}U with a half life of 6563 years, the next decay is to ^{232}Th, with a half-life of 2.34×10^7 y. It takes this amount of time for "most" of the sample to be thorium.

*50. a) $t_{1/2} = 1.5 \times 10^{24}\ \mathrm{y} = 4.7 \times 10^{31}\ \mathrm{s}$

$$R = \lambda N = \frac{\ln 2}{t_{1/2}} \frac{N_A}{0.128\ \mathrm{kg}} = \frac{\ln 2}{4.7 \times 10^{31}\ \mathrm{s}} \frac{6.022 \times 10^{23}}{0.128\ \mathrm{kg}} = 6.94 \times 10^{-8}\ \mathrm{s}^{-1} \cdot \mathrm{kg}^{-1}$$

b)

$$\frac{10\ \mathrm{s}^{-1}}{6.94 \times 10^{-8}\ \mathrm{s}^{-1} \cdot \mathrm{kg}^{-1}} = 1.44 \times 10^8\ \mathrm{kg}$$

This is not a realistic sample size.

*51. Adding charge dq to a solid sphere of radius r we have an energy change

$$dE = \frac{Q\,dq}{4\pi\varepsilon_0 r}$$

where $Q = \frac{4}{3}\rho\pi r^3$ is the charge already there and ρ is the charge density. Also we know $dq = \rho dV = 4\pi\rho r^2\,dr$.

$$dE = \frac{16\pi^2\rho^2 r^4}{3\,(4\pi\varepsilon_0)} dr$$

Integrating from 0 to R we find

$$\Delta E = \int_0^R \frac{16\pi^2 \rho^2 r^4}{3\,(4\pi\varepsilon_0)} dr = \frac{16\pi^2 \rho^2 R^5}{15\,(4\pi\varepsilon_0)}$$

The charge density is $\rho = Q/V = 3Q/4\pi R^3$, so

$$\Delta E = \frac{16\pi^2 R^5}{15\,(4\pi\varepsilon_0)} \left(\frac{3Q}{4\pi R^3}\right)^2 = \frac{3Q^2}{5\,(4\pi\varepsilon_0)\,R} = \frac{3\,(Ze)^2}{5\,(4\pi\varepsilon_0)\,R}$$

52. For Δx use the diameter $2r = 2r_0 A^{1/3} = 2\,(1.2\text{ fm})\,2^{1/3} = 3.02$ fm. Then $p_{\min} = \hbar/2\Delta x$ and

$$K_{\min} = \frac{p_{\min}^2}{2m} = \frac{\hbar^2}{8m\,(\Delta x)^2} = \frac{(197.3 \times 10^{-9}\text{ eV}\cdot\text{m})^2}{8\,(938 \times 10^6\text{ eV})\,(3.02 \times 10^{-15}\text{ m})^2} = 569\text{ keV}$$

$$\lambda = \frac{h}{p} = \frac{2h\Delta x}{\hbar} = 4\pi\Delta x = 4\pi\,(3.02\text{ fm}) = 38\text{ fm}$$

These results are reasonable, because the energy is smaller than the deuteron's binding energy, and the wavelength computed is the *maximum* wavelength, so the actual wavelength may be less. Shorter wavelengths, corresponding to the size of the nucleus, are allowed by the uncertainty principle.

53. The parity term in the semi-empirical formula, along with the more negative coulomb and symmetry terms, keep larger odd-odd nuclei from being stable. The spins are significant in this case, because transitions with very large ΔJ are very forbidden.

54. With an even number of neutrons, this isotope is apparently best, judging from the semi-empirical formula. The alpha decay would be $^{165}\text{Ho} \rightarrow {}^{161}\text{Tb} + {}^{4}\text{He}$ with

$$Q = [164.930319 - 160.927566 - 4.002603]\,\text{u}\cdot\text{c}^2 = 0.14\text{ MeV}$$

With a very low Q value, this process is allowed, but barely. With such a small Q value, it is difficult to overcome the nuclear attraction.

55. a) Radon is created at a rate $\lambda_1 N_1$ and lost at a rate $\lambda_2 N_2$, so the net change is $dN_2/dt = \lambda_1 N_1 - \lambda_2 N_2$.

b) Using separation of variables

$$\frac{dN_2}{\lambda_1 N_1 - \lambda_2 N_2} = dt$$

Let $u = \lambda_1 N_1 - \lambda_2 N_2$ so $du = -\lambda_2\,dN_2$ (because we assume N_1 is constant). Then

$$\frac{du}{u} = -\lambda_2\,dt \qquad\qquad \ln u = -\lambda_2 t + \ln(\text{const.})$$

$$u = (\text{const.})e^{-\lambda_2 t} = \lambda_1 N_1 - \lambda_2 N_2$$

To determine the constant, set $N_2 = 0$ at $t = 0$, so the constant is $\lambda_1 N_1$:

$$\lambda_1 N_1 e^{-\lambda_2 t} = \lambda_1 N_1 - \lambda_2 N_2$$

or rearranging

$$N_2 = \frac{\lambda_1 N_1}{\lambda_2}\left(1 - e^{-\lambda_2 t}\right)$$

c) For large t we have $e^{-\lambda_2 t} \cong 0$, and so $N_2 \cong \lambda_1 N_1/\lambda_2$ (which is constant).

56. a) From the uncertainty principle $\Delta E \Delta t = \hbar/2$, so

$$\Delta E = \frac{\hbar}{2\Delta t} = \frac{6.582 \times 10^{-16}\ \text{eV}\cdot\text{s}}{2\,(1.9 \times 10^{-10}\ \text{s})} = 1.73 \times 10^{-6}\ \text{eV}$$

b) We have $5\Delta E/E = v/c$, which we can solve for v:

$$v = \frac{5\Delta E}{E}c = \frac{5\,(1.73 \times 10^{-6}\ \text{eV})}{129 \times 10^3\ \text{eV}}\left(2.998 \times 10^8\ \text{m/s}\right) = 2.01\ \text{cm/s}$$

57. a) Examining the mass numbers it must come from four alpha decays of ^{238}U.

b) From the table of nuclides the decays are:

$^{222}\text{Rn} \rightarrow {}^{218}\text{Po} + {}^{4}\text{He}$ $\quad$ $^{218}\text{Po} \rightarrow {}^{214}\text{Pb} + {}^{4}\text{He}$ $\quad$ $^{214}\text{Pb} \rightarrow {}^{214}\text{Bi} + \beta^-$

$^{214}\text{Bi} \rightarrow {}^{214}\text{Po} + \beta^-$ $\quad$ $^{214}\text{Po} \rightarrow {}^{210}\text{Pb} + {}^{4}\text{He}$

c) The only half life longer than an hour is the first decay, with $t_{1/2} = 3.8$ days. Therefore more than half the decays will occur in four days.