

# Chapter 13

1. a)  ${}^4_2\text{He}$       b)  ${}^1_1\text{H}$       c)  ${}^{16}_8\text{O}$       d)  ${}^{73}_{32}\text{Ge}$       e)  ${}^{108}_{48}\text{Cd}$       f)  ${}^3_2\text{He}$

2. a)  ${}^{16}\text{O} (d, n) {}^{17}\text{F}$       b)  ${}^7\text{Li} (p, \alpha) {}^4\text{He}$       c)  ${}^{13}\text{C} (\alpha, p) {}^{16}\text{N}$   
d)  ${}^{73}\text{Ge} (d, n) {}^{74}\text{As}$       e)  ${}^{107}\text{Ag} ({}^3\text{He}, {}^4\text{He}) {}^{106}\text{Ag}$       f)  ${}^{162}\text{Dy} ({}^3\text{He}, p) {}^{164}\text{Ho}$

\*3. Probability is equal to  $nt\sigma$ , or

$$nt\sigma = \frac{6.022 \times 10^{23}}{238 \text{ g}} \frac{19 \text{ g}}{\text{cm}^3} (3 \text{ cm}) (0.6 \times 10^{-24} \text{ cm}^2) = 0.087$$

4.  $p+{}^{21}\text{Ne}$ ,  ${}^3\text{He} + {}^{19}\text{F}$ , and  ${}^6\text{Li} + {}^{16}\text{O}$

5. The probability is  $nt\sigma$ .

$$nt\sigma = \frac{6.022 \times 10^{23}}{108 \text{ g}} \frac{10.5 \text{ g}}{\text{cm}^3} (0.2 \text{ cm}) (0.17 \times 10^{-24} \text{ cm}^2) = 0.199$$

6. We can write the probability of landing in a solid angle  $\Delta\Omega$  as

$$\frac{d\sigma}{d\Omega} (\Delta\Omega) nt = (0.2 \times 10^{-27} \text{ cm}^2/\text{sr}) (3 \times 10^{-3} \text{ sr}) \left( \frac{6.022 \times 10^{23}}{12 \text{ g}} \right) \left( \frac{10^{-4} \text{ g}}{\text{cm}^2} \right) = 3.01 \times 10^{-12}$$

The rate of incident particles is

$$(0.2 \times 10^{-6} \text{ C/s}) \left( \frac{1}{2(1.602 \times 10^{-19} \text{ C})} \right) = 6.24 \times 10^{11} \text{ s}^{-1}$$

so the rate of detected particles is

$$(6.24 \times 10^{11} \text{ s}^{-1}) (3.01 \times 10^{-12}) = 1.88 \text{ s}^{-1}$$

and the number detected in one hour is

$$1.88 \text{ s}^{-1} (3600 \text{ s}) = 6770$$

7. a)  ${}^{16}\text{O} (n, \alpha) {}^{13}\text{C}$       b)  ${}^{16}\text{O} (d, n) {}^{17}\text{F}$       c)  ${}^{16}\text{O} (\gamma, p) {}^{15}\text{N}$   
d)  ${}^{16}\text{O} (\alpha, p) {}^{19}\text{F}$       e)  ${}^{16}\text{O} (d, {}^3\text{He}) {}^{15}\text{N}$       f)  ${}^{16}\text{O} ({}^7\text{Li}, p) {}^{22}\text{Ne}$

All the products listed above are stable except the one in part (b).

\*8. a)

$$Q = [M(^{16}\text{O}) + M(^2\text{H}) - M(^4\text{He}) - M(^{14}\text{N})] \text{ u} \cdot c^2 = 3.11 \text{ MeV (exothermic)}$$

b)

$$Q = [M(^{12}\text{C}) + M(^{12}\text{C}) - M(^2\text{H}) - M(^{22}\text{Na})] \text{ u} \cdot c^2 = -7.95 \text{ MeV (endothermic)}$$

c)

$$Q = [M(^{23}\text{Na}) + M(^1\text{H}) - M(^{12}\text{C}) - M(^{12}\text{C})] \text{ u} \cdot c^2 = -2.24 \text{ MeV (endothermic)}$$

9. b)

$$K_{\text{th}} = (7.95 \text{ MeV}) \left( \frac{24}{12} \right) = 15.9 \text{ MeV}$$

c)

$$K_{\text{th}} = (2.24 \text{ MeV}) \left( \frac{24}{23} \right) = 2.34 \text{ MeV}$$

10. In the center of mass system we require

$$9.63 \text{ MeV} + M(^{16}\text{O}) c^2 = 9.63 \text{ MeV} + 14899.17 \text{ MeV} = 14908.80 \text{ MeV}$$

We also have

$$M(^4\text{He}) c^2 + M(^{12}\text{C}) c^2 = 3728.40 \text{ MeV} + 11177.93 \text{ MeV} = 14906.33 \text{ MeV}$$

Therefore we need

$$14908.80 \text{ MeV} - 14906.33 \text{ MeV} = 2.47 \text{ MeV}$$

in bombarding energy. In the lab frame

$$\frac{16}{12} (2.47 \text{ MeV}) = 3.29 \text{ MeV}$$

The lifetime of the state can be computed using the uncertainty principle:  $\Gamma\tau = \hbar/2$  so

$$\tau = \frac{\hbar}{2\Gamma} = \frac{6.582 \times 10^{-16} \text{ eV} \cdot \text{s}}{2(510 \times 10^3 \text{ eV})} = 6.45 \times 10^{-22} \text{ s}$$

\*11. a)

$$Q = K_y + K_Y - K_x = 1.1 \text{ MeV} + 6.4 \text{ MeV} - 5.5 \text{ MeV} = 2.0 \text{ MeV}$$

b) The  $Q$  value does not change for a particular reaction.

12.

$$Q = [M(^{20}\text{Ne}) + M(^4\text{He}) - M(^{12}\text{C}) - M(^{12}\text{C})] \text{ u} \cdot c^2 = -4.62 \text{ MeV}$$

$$K_{\text{th}} = \frac{24}{20} (4.62 \text{ MeV}) = 5.54 \text{ MeV}$$

The sum of the carbon kinetic energies will be

$$K_y + K_Y = Q + K_x = -4.62 \text{ MeV} + 45 \text{ MeV} = 40.4 \text{ MeV}$$

13. Letting  $M = m_b + m_B$ ,  $\gamma = (1 - v^2/c^2)^{-1/2}$ , and  $\gamma' = (1 - v_{\text{cm}}^2/c^2)^{-1/2}$ , we have from conservation of energy and conservation of momentum:

$$\gamma m_A c^2 + m_a c^2 = \gamma' M c^2$$

$$\gamma m_a v = \gamma' M v_{\text{cm}}$$

or, squaring:

$$\begin{aligned} \frac{\gamma^2 m_a^2 v^2}{c^2} &= \frac{\gamma'^2 M^2 v_{\text{cm}}^2}{c^2} \\ \gamma^2 m_a^2 (1 - \gamma^{-2}) &= \gamma'^2 M^2 (1 - \gamma'^{-2}) \\ \gamma^2 m_a^2 - m_a^2 &= \gamma'^2 M^2 - M^2 \end{aligned}$$

From the energy equation  $\gamma' M = \gamma m_A + m_a$ , so

$$\gamma^2 m_a^2 - m_a^2 = (\gamma m_A + m_a)^2 - M^2$$

After multiplying the binomial and rearranging we have

$$\begin{aligned} 2\gamma m_a m_A &= M^2 - m_a^2 - m_A^2 \\ 2(\gamma - 1) m_a m_A &= M^2 - (m_a^2 + 2m_A m_a + m_A^2) = M^2 - (m_a + m_A)^2 \\ K_{\text{th}} &= (\gamma - 1) m_a c^2 = \frac{M^2 - (m_a + m_A)^2}{2m_A} = \frac{(M - (m_a + m_A))(M + (m_a + m_A))}{2m_A} \\ &= -Q \frac{m_A + m_a + m_B + m_b}{2m_A} \end{aligned}$$

14. The energy available for the gamma ray is

$$Q = [M(^{10}\text{B}) + M(n) - M(^{11}\text{B})] \text{ u} \cdot c^2 = 11.5 \text{ MeV}$$

15. The kinetic energy is  $Q + 5.1 \text{ MeV}$ , or

$$K = [M(^9\text{Be}) + M(^4\text{He}) - M(n) - M(^{12}\text{C})] \text{ u} \cdot c^2 + 5.1 \text{ MeV} = 5.70 \text{ MeV} + 5.1 \text{ MeV} = 10.8 \text{ MeV}$$

\*16. a)

$$Q = [M(^{16}\text{O}) + M(^4\text{He}) - M(^1\text{H}) - M(^{19}\text{F})] \text{ u} \cdot c^2 = -8.12 \text{ MeV}$$

$$K_{\text{th}} = \frac{20}{16} (8.12 \text{ MeV}) = 10.15 \text{ MeV}$$

b)

$$Q = [M(^{12}\text{C}) + M(^2\text{H}) - M(^3\text{He}) - M(^{11}\text{B})] \text{ u} \cdot c^2 = -10.46 \text{ MeV}$$

$$K_{\text{th}} = \frac{14}{12} (10.46 \text{ MeV}) = 12.20 \text{ MeV}$$

17. a)

$$N = \frac{6.022 \times 10^{23}}{59 \text{ g}} (40 \times 10^{-3} \text{ g}) = 4.083 \times 10^{20}$$

$$\text{activation rate} = n\sigma\phi = (4.083 \times 10^{20}) (20 \times 10^{-28} \text{ m}^2) (10^{18} \text{ m}^{-2} \cdot \text{s}^{-1}) = 8.166 \times 10^{11} \text{ s}^{-1}$$

Then in one week the number of  $^{60}\text{Co}$  produced is

$$8.166 \times 10^{11} \text{ s}^{-1} (7 (86400 \text{ s})) = 4.94 \times 10^{17}$$

b) Because the half-life of  $^{60}\text{Co}$  is much longer than one week, we can assume almost all the nuclei produced are still present. The activity is

$$R = \lambda N = \frac{\ln 2}{t_{1/2}} N = \frac{\ln 2}{1.66 \times 10^8 \text{ s}} (4.94 \times 10^{17}) = 2.06 \times 10^9 \text{ Bq}$$

c)

$$10^{14} \text{ Bq} \frac{40 \times 10^{-3} \text{ g}}{2.06 \times 10^9 \text{ Bq}} = 1942 \text{ g}$$

We would put 1.94 kg of  $^{59}\text{Co}$  into the reactor for one week.

\*18. The equation for  $K_{\text{cm}}$  is correct because we know from classical mechanics that the system is equivalent to a mass  $M_x + M_X$  moving with a speed  $v_{\text{cm}}^2$ , so the center of mass kinetic energy is

$$K_{\text{cm}} = \frac{1}{2} (M_x + M_X) v_{\text{cm}}^2$$

Letting  $M = M_x + M_X$  we have by conservation of momentum  $M v_{\text{cm}} = M_x v_x$ , or  $v_{\text{cm}} = M_x v_x / M$ . Thus

$$K_{\text{cm}} = \frac{1}{2} M v_{\text{cm}}^2 = \frac{1}{2} M \frac{M_x^2}{M^2} v_x^2 = \frac{1}{2} \frac{M_x^2}{M} v_x^2$$

$$K'_{\text{cm}} = K_{\text{lab}} - K_{\text{cm}} = \frac{M_x v_x^2}{2} - \frac{1}{2} \frac{M_x^2}{M} v_x^2 = \frac{M_x v_x^2}{2} \left( 1 - \frac{M_x}{M} \right) = \frac{M_x v_x^2}{2} \left( \frac{M - M_x}{M} \right)$$

$$K'_{\text{cm}} = K_{\text{lab}} \left( \frac{M_X}{M_x + M_X} \right)$$

\*19.

$$K'_{\text{cm}} = \frac{M_X}{M_x + M_X} K_{\text{lab}} = \frac{14}{18} (7.7 \text{ MeV}) = 5.99 \text{ MeV}$$

$$E^* = [M(^{14}\text{N}) + M(^4\text{He}) - M(^{18}\text{F})] \text{ u} \cdot c^2 = 4.41 \text{ MeV}$$

$$E_x = E^* + K'_{\text{cm}} = 10.40 \text{ MeV}$$

20.

$$E^* = [M(^{208}\text{Pb}) + M(n) - M(^{209}\text{Pb})] \text{ u} \cdot c^2 = 3.94 \text{ MeV}$$

Because of the excited nucleus, we expect gamma decay.  $^{209}\text{Pb}$  also undergoes  $\beta^-$  decay.

21. From Chapter 12 the radius of the nucleus is

$$r(^{14}\text{N}) = (1.2 \text{ fm}) 14^{1/3} = 2.89 \text{ fm}$$

At that radius the potential energy is

$$V = \frac{q_1 q_2}{4\pi\epsilon_0 r} = \frac{(2)(7)(1.44 \times 10^{-9} \text{ eV} \cdot \text{m})}{2.89 \times 10^{-15} \text{ m}} = 6.98 \text{ MeV}$$

$$Q = [M(^{14}\text{N}) + M(^4\text{He}) - M(^1\text{H}) - M(^{17}\text{O})] \text{ u} \cdot c^2 = -1.19 \text{ MeV}$$

$$K_{\text{exit}} = Q + K_\alpha = -1.19 \text{ MeV} + 7.7 \text{ MeV} = 6.51 \text{ MeV} = K(p) + K(^{17}\text{O})$$

Most of the energy at the forward angle will be in the proton, so there should be sufficient energy to overcome the 6.98 MeV barrier in a tunneling process (though it appears to be forbidden classically).

\*22. From Chapter 12 Problem 23 we see that the mean lifetime is

$$\tau = \frac{t_{1/2}}{\ln 2} = \frac{109 \text{ ms}}{\ln 2} = 157 \text{ ms}$$

The from Equation (13.13)

$$\Gamma = \frac{\hbar}{2\tau} = \frac{6.582 \times 10^{-16} \text{ eV} \cdot \text{s}}{2(0.157 \text{ s})} = 2.10 \times 10^{-15} \text{ eV}$$

$^{17}\text{Ne}$  can decay by positron decay or electron capture.

23.

$$E^* = [M(^1\text{H}) + M(^{16}\text{O}) - M(^{17}\text{F})] \text{ u} \cdot c^2 = 0.60 \text{ MeV}$$

No, this energy is greater than 0.495 MeV.

24. a)

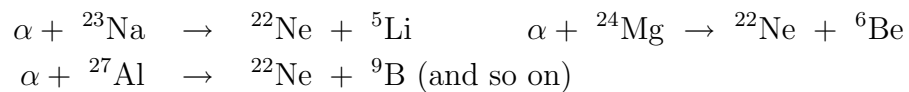
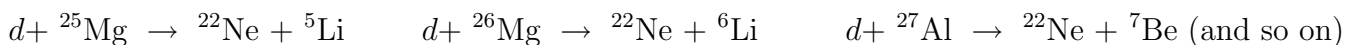
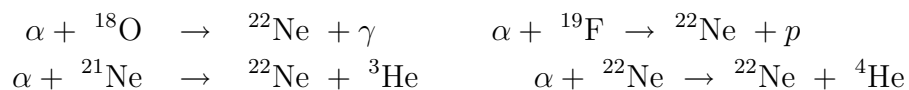
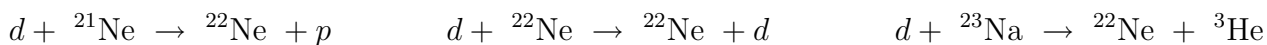
$$E^* = [M(^{239}\text{Pu}) + M(n) - M(^{240}\text{Pu})] \text{ u} \cdot c^2 = 6.53 \text{ MeV}$$

b)

$$K_{\text{cm}} = \frac{239}{240} (1.00 \text{ MeV}) = 1.00 \text{ MeV}$$

$$E = 6.53 \text{ MeV} + 1.00 \text{ MeV} = 7.53 \text{ MeV}$$

25.



26.

$$\Delta E = [M(^{239}\text{Pu}) + M(n) - M(^{95}\text{Zr}) - M(^{142}\text{Xe}) - 3M(n)] \text{ u} \cdot c^2 = 183.6 \text{ MeV}$$

\*27. a)

$$m = (4 \times 10^{-4}) (10^6 \text{ kg}) = 400 \text{ kg}$$

b)

$$(400 \text{ kg}) \frac{6.022 \times 10^{23}}{0.238 \text{ kg}} = 1.01 \times 10^{27} \text{ atoms}$$

c)

$$R = (0.69 \text{ kg}^{-1} \cdot \text{s}^{-1}) (400 \text{ kg}) = 276 \text{ Bq}$$

d)

$$(276 \text{ s}^{-1}) \frac{86400 \text{ s}}{d} = 2.38 \times 10^7 \text{ d}^{-1}$$

\*28.

$$\frac{N(^{235}\text{U})}{N(^{238}\text{U})} = \frac{7}{993} = \frac{N_0(^{235}\text{U})e^{-\lambda_1 t}}{N_0(^{238}\text{U})e^{-\lambda_2 t}}$$

where the subscripts 1 and 2 refer to the 235 and 238 isotopes, respectively.

$$\frac{N_0(^{235}\text{U})}{N_0(^{238}\text{U})} = \frac{7}{993} \exp((\lambda_1 - \lambda_2)t)$$

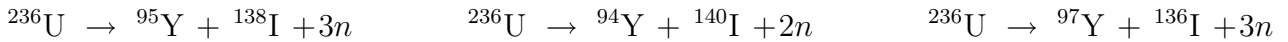
$$\begin{aligned} (\lambda_1 - \lambda_2)t &= \ln 2 \left( \frac{1}{t_{1/2}(235)} - \frac{1}{t_{1/2}(238)} \right) t \\ &= \ln 2 \left( \frac{1}{7.04 \times 10^8 \text{ y}} - \frac{1}{4.47 \times 10^9 \text{ y}} \right) (2 \times 10^9 \text{ y}) = 1.566 \end{aligned}$$

Then

$$\frac{N_0(^{235}\text{U})}{N_0(^{238}\text{U})} = \frac{7}{993} \exp((\lambda_1 - \lambda_2)t) = \frac{7}{993} \exp(1.566) = 0.0337$$

which is about five times higher than today. Natural fission reactors cannot operate because the relatively low abundance of  $^{235}\text{U}$  today prohibits a critical mass being reached.

29. Here are three possibilities, out of many:



30.

$$10^3 \text{ MWe} = (10^9 \text{ J/s}) \frac{86400 \text{ s/d}}{1.602 \times 10^{-13} \text{ J/MeV}} = 5.39 \times 10^{26} \text{ MeV/d}$$

$$(5.39 \times 10^{26} \text{ MeV/d}) \left( \frac{1 \text{ (fission)}}{200 \text{ MeV}} \right) \frac{0.235 \text{ kg}}{6.022 \times 10^{23}} = 1.05 \text{ kg/d}$$

\*31. For uranium we assume as in the previous problem that each fission produces 200 MeV of energy.

$$(1 \text{ kg}) \left( \frac{6.022 \times 10^{23} \text{ atoms}}{0.235 \text{ kg}} \right) \frac{200 \text{ MeV}}{\text{atom}} = 5.13 \times 10^{26} \text{ MeV}$$

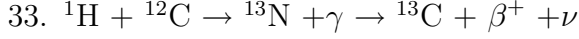
Converting to kWh we find  $2.30 \times 10^7$  kWh. On the other hand, for coal the conversion of 29000 Btu is 8.50 kWh. Therefore we see that fission produces over one million times more energy per kilogram of fuel.

\*32. a)

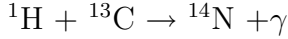
$$\frac{3}{2}kT = \frac{3}{2} \left( 8.617 \times 10^{-5} \text{ eV/K} \right) (300 \text{ K}) = 3.88 \times 10^{-2} \text{ eV}$$

b)

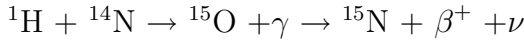
$$\frac{3}{2}kT = \frac{3}{2} \left( 8.617 \times 10^{-5} \text{ eV/K} \right) \left( 15 \times 10^6 \text{ K} \right) = 1.94 \text{ keV}$$



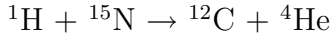
$$Q = \left[ M({}^1\text{H}) + M({}^{12}\text{C}) - M({}^{13}\text{C}) \right] \text{u} \cdot c^2 = 4.16 \text{ MeV}$$



$$Q = \left[ M({}^1\text{H}) + M({}^{13}\text{C}) - M({}^{14}\text{N}) \right] \text{u} \cdot c^2 = 7.55 \text{ MeV}$$



$$Q = \left[ M({}^1\text{H}) + M({}^{14}\text{N}) - M({}^{15}\text{N}) \right] \text{u} \cdot c^2 = 10.05 \text{ MeV}$$



$$Q = \left[ M({}^1\text{H}) + M({}^{15}\text{N}) - M({}^{12}\text{C}) - M({}^4\text{He}) \right] \text{u} \cdot c^2 = 4.97 \text{ MeV}$$

The total  $Q$  is

$$Q = 4.16 \text{ MeV} + 7.55 \text{ MeV} + 10.05 \text{ MeV} + 4.97 \text{ MeV} = 26.73 \text{ MeV}$$

34. The if the depth is  $d$  and the radius of the earth is  $r$ , volume of the oceans is

$$V = \frac{2}{3} \left( 4\pi r^2 \right) d = \frac{8}{3} \pi \left( 6.37 \times 10^6 \text{ m} \right)^2 (3000 \text{ m}) = 1.02 \times 10^{18} \text{ m}^3$$

With a density of  $1000 \text{ kg/m}^3$ , the mass is  $1.02 \times 10^{21} \text{ kg}$ , so the number of water molecules is

$$N = \left( 1.02 \times 10^{21} \text{ kg} \right) \frac{6.022 \times 10^{23}}{0.018 \text{ kg}} = 3.41 \times 10^{46}$$

With a 0.015% abundance, the number of deuterium atoms (in  $\text{D}_2\text{O}$  molecules) is

$$(2) 3.41 \times 10^{46} (0.00015) = 1.02 \times 10^{43}$$

There will be at most half this number of fusions, or  $5.1 \times 10^{42}$ . The energy released is

$$\left( 5.1 \times 10^{42} \right) (4.0 \text{ MeV}) \left( 1.602 \times 10^{-13} \text{ J/MeV} \right) = 3.27 \times 10^{30} \text{ J}$$



35. a) We use temperatures because that is what we strive for experimentally. Kinetic energy is useful for comparison with the Coulomb barrier or  $Q$  values.

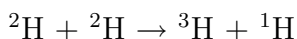
b)  $K = \frac{3}{2}kT$

c)

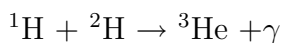
$$T = \frac{2K}{3k} = \frac{2(6000 \text{ eV})}{3(8.617 \times 10^{-5} \text{ eV/K})} = 4.64 \times 10^7 \text{ K}$$



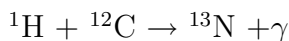
$$Q = [M({}^4\text{H}) + M({}^3\text{He}) - M({}^7\text{Be})] \text{ u} \cdot c^2 = 1.59 \text{ MeV}$$



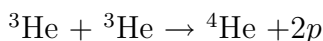
$$Q = [M({}^2\text{H}) + M({}^2\text{H}) - M({}^3\text{H}) - M({}^1\text{H})] \text{ u} \cdot c^2 = 4.03 \text{ MeV}$$



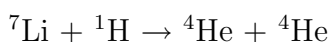
$$Q = [M({}^1\text{H}) + M({}^2\text{H}) - M({}^3\text{He})] \text{ u} \cdot c^2 = 5.48 \text{ MeV}$$



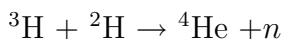
$$Q = [M({}^1\text{H}) + M({}^{12}\text{C}) - M({}^{13}\text{N})] \text{ u} \cdot c^2 = 1.94 \text{ MeV}$$



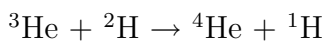
$$Q = [M({}^3\text{He}) + M({}^3\text{He}) - M({}^4\text{He}) - 2M({}^1\text{H})] \text{ u} \cdot c^2 = 12.9 \text{ MeV}$$



$$Q = [M({}^1\text{H}) + M({}^7\text{Li}) - 2M({}^4\text{He})] \text{ u} \cdot c^2 = 17.3 \text{ MeV}$$



$$Q = [M({}^3\text{H}) + M({}^2\text{H}) - M({}^4\text{He}) - M(n)] \text{ u} \cdot c^2 = 17.6 \text{ MeV}$$



$$Q = [M({}^3\text{He}) + M({}^2\text{H}) - M({}^4\text{He}) - M({}^1\text{H})] \text{ u} \cdot c^2 = 18.4 \text{ MeV}$$

37. The reaction is  ${}^1\text{H} + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma$ .

$$Q = [M({}^{12}\text{C}) + M({}^1\text{H}) - M({}^{13}\text{N})] \text{ u} \cdot c^2 = 1.94 \text{ MeV}$$

Then

$$K_{\text{th}} = -Q \left( \frac{M({}^1\text{H}) + M({}^{12}\text{C})}{M({}^{12}\text{C})} \right)$$

which will be negative, so the threshold energy is not a concern. However, the Coulomb barrier is

$$V = \frac{q_1 q_2}{4\pi\epsilon_0 r} = \frac{6e^2}{4\pi\epsilon_0 (r_p + r_0 A^{1/3})} = \frac{6 (1.44 \times 10^{-9} \text{ eV} \cdot \text{m})}{(1.2 \times 10^{-15} \text{ m} + (1.2 \times 10^{-15} \text{ m}) (12)^{1/3})} = 2.19 \text{ MeV}$$

Setting this coulomb barrier energy equal to the thermal energy  $\frac{3}{2}kT$ , we have

$$T = \frac{2K}{3k} = \frac{2 (2.19 \times 10^6 \text{ eV})}{3 (8.617 \times 10^{-5} \text{ eV/K})} = 1.69 \times 10^{10} \text{ K}$$

38.

$$Q = [3M(^4\text{He}) - M(^{12}\text{C})] \text{ u} \cdot c^2 = 7.27 \text{ MeV}$$

39. a)

$$K = \frac{3}{2}kT = \frac{3}{2} (8.617 \times 10^{-5} \text{ eV/K}) (300 \text{ K}) = 3.88 \times 10^{-2} \text{ eV}$$

b)

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 (3.88 \times 10^{-2} \text{ eV}) (1.602 \times 10^{-19} \text{ J/eV})}{1.675 \times 10^{-27} \text{ kg}}} = 2720 \text{ m/s}$$

c)

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.675 \times 10^{-27} \text{ kg}) (2720 \text{ m/s})} = 1.45 \times 10^{-10} \text{ m}$$

40.  $R = 512/60 \text{ Bq} = 8.533 \text{ Bq}$ . The initial rate was  $R_0 = 4 \times 10^5 \text{ Bq}$ . Over 60 days the activity has decreased to

$$R'_0 = R_0 e^{-\lambda t} = (4 \times 10^5 \text{ Bq}) \exp\left(-\frac{\ln 2}{44.5 \text{ d}} (60 \text{ d})\right) = 1.571 \times 10^5 \text{ Bq}$$

Therefore the fraction  $f$  of the ring that has worn off into the oil is

$$f = \frac{R}{R'_0} = \frac{8.533 \text{ Bq}}{1.571 \times 10^5 \text{ Bq}} = 5.43 \times 10^{-5}$$

\*41. a) With a half life of 66 h, not much is left after one week.

b)

$$R = R_0 e^{-\lambda t} = (10^{11} \text{ Bq}) \exp\left(-\frac{\ln 2}{66 \text{ h}} (216 \text{ h})\right) = 1.035 \times 10^{10} \text{ Bq}$$

c)

$$R = R_0 e^{-\lambda t} = (10^{11} \text{ Bq}) \exp\left(-\frac{\ln 2}{66 \text{ h}} (96 \text{ h})\right) = 3.649 \times 10^{10} \text{ Bq}$$

42. a) We need 1000 Bq at  $t = 30$  minutes with  $t_{1/2} = 83.1$  minutes.

$$R = R'_0 e^{-\lambda t} = (R'_0) \exp \left( -\frac{\ln 2}{83.1 \text{ min}} (30 \text{ min}) \right) = 1000 \text{ Bq}$$

$$R'_0 = (1000 \text{ Bq}) \exp \left( \frac{\ln 2}{83.1 \text{ min}} (30 \text{ min}) \right) = 1284 \text{ Bq}$$

Because only 27% go to the first excited state, we need an activity  $R_0 = R'_0/0.27 = 4756 \text{ Bq}$ . Then  $R_0 = \lambda N = N \ln 2/t_{1/2}$ , so solving for  $N$ :

$$N = \frac{R_0 t_{1/2}}{\ln 2} = \frac{(4756 \text{ s}^{-1}) (83.1 \text{ min}) (60 \text{ s/min})}{\ln 2} = 3.42 \times 10^7$$

b)

$$m = (3.42 \times 10^7) \frac{139 \text{ g}}{6.022 \times 10^{23}} = 7.89 \times 10^{-15} \text{ g}$$

The fraction  $f$  activated is

$$f = \frac{7.89 \times 10^{-15} \text{ g}}{(0.717) (10^{-8} \text{ g})} = 1.10 \times 10^{-6}$$

43. a)

$$I = (0.15) (4 \times 10^5 \text{ s}^{-1}) (2 \times 1.602 \times 10^{-19} \text{ C}) = 1.92 \times 10^{-14} \text{ A}$$

b) One-tenth of the above value is  $1.92 \times 10^{-15} \text{ A}$ .

44. a)

$$R = (5000 \text{ J/s}) \frac{1 \text{ MeV}}{1.602 \times 10^{-13} \text{ J}} \frac{1 \text{ particle}}{5.3 \text{ MeV}} = 5.89 \times 10^{15} \text{ Bq}$$

$$N = \frac{R}{\lambda} = \frac{(5.89 \times 10^{15} \text{ s}^{-1}) (138 \text{ d}) (86400 \text{ s/d})}{\ln 2} = 1.013 \times 10^{23}$$

$$m = (1.013 \times 10^{23}) \left( \frac{0.210 \text{ kg}}{6.022 \times 10^{23}} \right) = 3.53 \times 10^{-2} \text{ kg}$$

b) For 7 kW we need a mass of  $(7/5) (3.53 \times 10^{-2} \text{ kg}) = 0.0494 \text{ kg}$ . Letting the initial mass be  $m_0$ , we have the usual exponential decay and so

$$m_0 = m \exp(\lambda t) = (0.0494 \text{ kg}) \exp \left( \frac{(\ln 2) (730 \text{ d})}{138 \text{ d}} \right) = 1.93 \text{ kg}$$

45. The fraction incident is the patient's cross-sectional area divided by  $4\pi (\text{distance})^2$ . Note that there are two gammas per decay.

$$R = 2 (3 \times 10^{14} \text{ Bq}) \frac{0.3 \text{ m}^2}{4\pi (4 \text{ m})^2} = 8.95 \times 10^{11} \text{ Bq}$$

46. From Chapter 12 the diameter of the uranium nucleus is

$$2r(^{238}\text{U}) = 2(1.2 \text{ fm}) 238^{1/3} = 14.87 \text{ fm}$$

The momentum of the neutron is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{14.87 \times 10^{-15} \text{ m}} = 4.456 \times 10^{-20} \text{ kg} \cdot \text{m/s}$$

and the kinetic energy is

$$K = \frac{p^2}{2m} = \frac{(4.456 \times 10^{-20} \text{ kg} \cdot \text{m/s})^2}{2(1.675 \times 10^{-27} \text{ kg})} = 5.927 \times 10^{-13} \text{ J} = 3.70 \text{ MeV}$$

This is still much less than the neutron's rest energy, which justifies the non-relativistic treatment. Then in thermal equilibrium

$$T = \frac{2K}{3k} = \frac{2(5.927 \times 10^{-13} \text{ J})}{3(1.381 \times 10^{-23} \text{ J/K})} = 2.86 \times 10^{10} \text{ K}$$

Clearly this is not a realistic temperature in a reactor, so the proposed experiment is not possible.

47. The alpha decay reaction is  $^{239}\text{Pu} \rightarrow ^{235}\text{U} + ^4\text{He}$ , and the energy released in each decay is

$$Q = [M(^{239}\text{Pu}) - M(^{235}\text{U}) - M(^4\text{He})] \text{ u} \cdot c^2 = 5.25 \text{ MeV}$$

The activity of a 10 kg sample is

$$R = \lambda N = \frac{\ln 2}{(24110 \text{ y})(3.156 \times 10^7 \text{ s/y})} (10 \text{ kg}) \left( \frac{6.022 \times 10^{23}}{0.239 \text{ kg}} \right) = 2.30 \times 10^{13} \text{ Bq}$$

Then the power is

$$P = (0.6)(5.25 \text{ MeV}) (1.602 \times 10^{-13} \text{ J/MeV}) (2.30 \times 10^{13} \text{ s}^{-1}) = 11.6 \text{ W}$$

\*48. a)

$$(1 \text{ kg}) \frac{6.022 \times 10^{23}}{\text{mol}} \frac{1 \text{ mol}}{0.001 \text{ kg}} \frac{13.6 \text{ eV}}{1 \text{ atom}} = 8.19 \times 10^{27} \text{ eV}$$

b)

$$(1 \text{ kg}) \frac{6.022 \times 10^{23}}{\text{mol}} \frac{1 \text{ mol}}{0.002 \text{ kg}} \frac{2.22 \times 10^6 \text{ eV}}{1 \text{ atom}} = 6.68 \times 10^{32} \text{ eV}$$

c)

$$(1 \text{ kg}) \frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}} \frac{931.49 \text{ MeV}}{\text{u}} = 5.61 \times 10^{35} \text{ eV}$$

\*49. a)

$$Q = K_{\text{out}} - K_{\text{in}} = 86.63 \text{ MeV} - 100 \text{ MeV} = -13.37 \text{ MeV}$$

This agrees with the  $Q$  computed using atomic masses.

b)

$$M = -\frac{Q}{c^2} + M(^{18}\text{O}) + M(^{30}\text{Si}) - M(^{14}\text{O}) = 33.979 \text{ u}$$

which matches the value for the mass of  $^{34}\text{Si}$  in Appendix 8 pretty well.

50. a) The number of atoms is

$$1000 (100 \text{ kg}) \frac{1 \text{ mol}}{0.235 \text{ kg}} \frac{6.022 \times 10^{23}}{\text{mol}} (0.04) = 1.03 \times 10^{28}$$

b)

$$\frac{1.03 \times 10^{28}}{4\pi (6 \times 10^6 \text{ m})^2} = 2.28 \times 10^{13} \text{ m}^{-2}$$

Then the activity for each square meter is

$$R = \lambda N = \frac{\ln 2}{(28.8 \text{ y}) (3.156 \times 10^7 \text{ s/y})} (2.28 \times 10^{13}) = 1.74 \times 10^4 \text{ Bq}$$

51. a) Starting with  $F(E) = (\text{const.})e^{-E/kT}E^{1/2}$  and setting  $dF/dE = 0$ , we find that the most probable energy is  $E^* = \frac{1}{2}kT$ .

$$E^* = \frac{1}{2}kT = \frac{1}{2} (8.621 \times 10^{-5} \text{ eV/K}) (2 \times 10^8 \text{ K}) = 8620 \text{ eV}$$

b)

$$\frac{n(2E)}{n(E)} = \exp(-\beta(2E - E)) = \exp(-\beta E) = \exp\left(-\frac{1}{2}\right) = 0.607$$

$$\frac{n(5E)}{n(E)} = \exp(-\beta(5E - E)) = \exp(-4\beta E) = \exp(-2) = 0.135$$

$$\frac{n(10E)}{n(E)} = \exp(-\beta(10E - E)) = \exp(-9\beta E) = \exp\left(-\frac{9}{2}\right) = 0.0111$$

52. a)

$$P = \sigma nt = (90 \times 10^{-27} \text{ cm}^2) (1.85 \text{ g/cm}^3) (3 \text{ cm}) (6.022 \times 10^{23} \text{ mol}^{-1}) \left(\frac{1 \text{ mol}}{9 \text{ g}}\right) = 0.0334$$

b) We know that  $10^5$  alpha particles must interact each second to get an activity of  $10^5 \text{ Bq}$ . With this probability the number of incident alpha particles is

$$\frac{10^5 \text{ s}^{-1}}{0.0334} = 2.99 \times 10^6 \text{ s}^{-1}$$

c)

$$N = \frac{R}{\lambda} = (2.99 \times 10^6 \text{ s}^{-1}) \frac{2.41 \times 10^4 \text{ y}}{\ln 2} \frac{3.156 \times 10^7 \text{ s}}{\text{y}} = 3.28 \times 10^{18}$$

$$m = \left(3.28 \times 10^{18}\right) \frac{0.239 \text{ kg}}{6.022 \times 10^{23}} = 1.30 \times 10^{-6} \text{ kg}$$

which is reasonable.

\*53.

$$m(^{40}\text{K}) = (70 \text{ kg}) (0.0035) (0.00012) = 2.94 \times 10^{-5} \text{ kg}$$

$$N = \left(2.94 \times 10^{-5} \text{ kg}\right) \frac{6.022 \times 10^{23}}{0.040 \text{ kg}} = 4.43 \times 10^{20}$$

$$R = \lambda N = \frac{\ln 2}{1.28 \times 10^9 \text{ y}} \frac{1 \text{ y}}{3.156 \times 10^7 \text{ s}} \left(4.43 \times 10^{20}\right) = 7600 \text{ Bq}$$

The beta activity is  $(7600 \text{ Bq}) (0.893) = 6790 \text{ Bq}$ .