Chapter 13

1. a) ${}^{4}_{2}\text{He}$ b) ${}^{1}_{1}\text{H}$ c) ${}^{16}_{8}\text{O}$ d) ${}^{73}_{32}\text{Ge}$ e) ${}^{108}_{48}\text{Cd}$ f) ${}^{3}_{2}\text{He}$

2. a) ${}^{16}\text{O}\ (d,n)\ {}^{17}\text{F}$ b) ${}^{7}\text{Li}\ (p,\alpha)\ {}^{4}\text{He}$ c) ${}^{13}\text{C}\ (\alpha,p)\ {}^{16}\text{N}$
   d) ${}^{73}\text{Ge}\ (d,n)\ {}^{74}\text{As}$ e) ${}^{107}\text{Ag}\ ({}^{3}\text{He},{}^{4}\text{He})\ {}^{106}\text{Ag}$ f) ${}^{162}\text{Dy}\ ({}^{3}\text{He},p)\ {}^{164}\text{Ho}$

*3. Probability is equal to $nt\sigma$, or

$$nt\sigma = \frac{6.022\times10^{23}}{238\text{ g}}\frac{19\text{ g}}{\text{cm}^3}(3\text{ cm})\left(0.6\times10^{-24}\text{ cm}^2\right) = 0.087$$

4. $p+{}^{21}\text{Ne}$, ${}^{3}\text{He} + {}^{19}\text{F}$, and ${}^{6}\text{Li} + {}^{16}\text{O}$

5. The probability is $nt\sigma$.

$$nt\sigma = \frac{6.022\times10^{23}}{108\text{ g}}\frac{10.5\text{ g}}{\text{cm}^3}(0.2\text{ cm})\left(017\times10^{-24}\text{ cm}^2\right) = 0.199$$

6. We can write the probability of landing in a solid angle $\Delta\Omega$ as

$$\frac{d\sigma}{d\Omega}(\Delta\Omega)\,nt = \left(0.2\times10^{-27}\text{ cm}^2/\text{sr}\right)\left(3\times10^{-3}\text{ sr}\right)\left(\frac{6.022\times10^{23}}{12\text{ g}}\right)\left(\frac{10^{-4}\text{ g}}{\text{cm}^2}\right) = 3.01\times10^{-12}$$

The rate of incident particles is

$$\left(0.2\times10^{-6}\text{ C/s}\right)\left(\frac{1}{2\left(1.602\times10^{-19}\text{ C}\right)}\right) = 6.24\times10^{11}\text{ s}^{-1}$$

so the rate of detected particles is

$$\left(6.24\times10^{11}\text{ s}^{-1}\right)\left(3.01\times10^{-12}\right) = 1.88\text{ s}^{-1}$$

and the number detected in one hour is

$$1.88\text{ s}^{-1}(3600\text{ s}) = 6770$$

7. a) ${}^{16}\text{O}\ (n,\alpha)\ {}^{13}\text{C}$ b) ${}^{16}\text{O}\ (d,n)\ {}^{17}\text{F}$ c) ${}^{16}\text{O}\ (\gamma,p)\ {}^{15}\text{N}$
   d) ${}^{16}\text{O}\ (\alpha,p)\ {}^{19}\text{F}$ e) ${}^{16}\text{O}\ (d,{}^{3}\text{He})\ {}^{15}\text{N}$ f) ${}^{16}\text{O}\ ({}^{7}\text{Li},p)\ {}^{22}\text{Ne}$
   All the products listed above are stable except the one in part (b).

*8. a)

$$Q = \left[M(^{16}\mathrm{O}) + M(^{2}\mathrm{H}) - M(^{4}\mathrm{He}) - M(^{14}\mathrm{N})\right] \mathrm{u} \cdot c^2 = 3.11 \text{ MeV (exothermic)}$$

b)

$$Q = \left[M(^{12}\mathrm{C}) + M(^{12}\mathrm{C}) - M(^{2}\mathrm{H}) - M(^{22}\mathrm{Na})\right] \mathrm{u} \cdot c^2 = -7.95 \text{ MeV (endothermic)}$$

c)

$$Q = \left[M(^{23}\mathrm{Na}) + M(^{1}\mathrm{H}) - M(^{12}\mathrm{C}) - M(^{12}\mathrm{C})\right] \mathrm{u} \cdot c^2 = -2.24 \text{ MeV (endothermic)}$$

9. b)

$$K_{\mathrm{th}} = (7.95 \text{ MeV}) \left(\frac{24}{12}\right) = 15.9 \text{ MeV}$$

c)

$$K_{\mathrm{th}} = (2.24 \text{ MeV}) \left(\frac{24}{23}\right) = 2.34 \text{ MeV}$$

10. In the center of mass system we require

$$9.63 \text{ MeV } + M\left(^{16}\mathrm{O}\right) c^2 = 9.63 \text{ MeV } + 14899.17 \text{ MeV } = 14908.80 \text{ MeV}$$

We also have

$$M\left(^{4}\mathrm{He}\right) c^2 + M\left(^{12}\mathrm{C}\right) c^2 = 3728.40 \text{ MeV } + 11177.93 \text{ MeV } = 14906.33 \text{ MeV}$$

Therefore we need

$$14908.80 \text{ MeV } - 14906.33 \text{ MeV } = 2.47 \text{ MeV}$$

in bombarding energy. In the lab frame

$$\frac{16}{12}(2.47 \text{ MeV}) = 3.29 \text{ MeV}$$

The lifetime of the state can be computed using the uncertainty principle: $\Gamma\tau = \hbar/2$ so

$$\tau = \frac{\hbar}{2\Gamma} = \frac{6.582 \times 10^{-16} \text{ eV} \cdot \text{s}}{2\left(510 \times 10^3 \text{ eV}\right)} = 6.45 \times 10^{-22} \text{ s}$$

*11. a)

$$Q = K_y + K_Y - K_x = 1.1 \text{ MeV } + 6.4 \text{ MeV } - 5.5 \text{ MeV } = 2.0 \text{ MeV}$$

b) The Q value does not change for a particular reaction.

12.

$$Q = \left[M(^{20}\text{Ne}) + M(^{4}\text{He}) - M(^{12}\text{C}) - M(^{12}\text{C})\right] \text{u} \cdot c^2 = -4.62 \text{ MeV}$$

$$K_{\text{th}} = \frac{24}{20} (4.62 \text{ MeV}) = 5.54 \text{ MeV}$$

The sum of the carbon kinetic energies will be

$$K_y + K_Y = Q + K_x = -4.62 \text{ MeV } + 45 \text{ MeV } = 40.4 \text{ MeV}$$

13. Letting $M = m_b + m_B$, $\gamma = (1 - v^2/c^2)^{-1/2}$, and $\gamma' = (1 - v_{\text{cm}}^2/c^2)^{-1/2}$, we have from conservation of energy and conservation of momentum:

$$\gamma m_A c^2 + m_a c^2 = \gamma' M c^2$$

$$\gamma m_a v = \gamma' M v_{\text{cm}}$$

or, squaring:

$$\frac{\gamma^2 m_a^2 v^2}{c^2} = \frac{\gamma'^2 M^2 v_{\text{cm}}^2}{c^2}$$

$$\gamma^2 m_a^2 \left(1 - \gamma^{-2}\right) = \gamma'^2 M^2 \left(1 - \gamma'^{-2}\right)$$

$$\gamma^2 m_a^2 - m_a^2 = \gamma'^2 M^2 - M^2$$

From the energy equation $\gamma' M = \gamma m_A + m_a$, so

$$\gamma^2 m_a^2 - m_a^2 = (\gamma m_a + m_A)^2 - M^2$$

After multiplying the binomial and rearranging we have

$$2\gamma m_a m_A = M^2 - m_a^2 - m_A^2$$

$$2(\gamma - 1) m_a m_A = M^2 - \left(m_a^2 + 2m_A m_a + m_A^2\right) = M^2 - (m_a + m_A)^2$$

$$\begin{aligned} K_{\text{th}} &= (\gamma - 1) m_a c^2 = \frac{M^2 - (m_a + m_A)^2}{2m_A} = \frac{(M - (m_a + m_A))(M + (m_a + m_A))}{2m_A} \\ &= -Q \frac{m_A + m_a + m_B + m_b}{2m_A} \end{aligned}$$

14. The energy available for the gamma ray is

$$Q = \left[M(^{10}\text{B}) + M(n) - M(^{11}\text{B})\right] \text{u} \cdot c^2 = 11.5 \text{ MeV}$$

15. The kinetic energy is $Q + 5.1$ MeV, or

$$K = \left[M(^{9}\text{Be}) + M(^{4}\text{He}) - M(n) - M(^{12}\text{C})\right] \text{u} \cdot c^2 + 5.1 \text{ MeV } = 5.70 \text{ MeV } + 5.1 \text{ MeV } = 10.8 \text{ MeV}$$

*16. a)

$$Q = \left[M(^{16}\text{O}) + M(^{4}\text{He}) - M(^{1}\text{H}) - M(^{19}\text{F})\right] \text{u} \cdot c^2 = -8.12 \text{ MeV}$$

$$K_{\text{th}} = \frac{20}{16}(8.12 \text{ MeV}) = 10.15 \text{ MeV}$$

b)

$$Q = \left[M(^{12}\text{C}) + M(^{2}\text{H}) - M(^{3}\text{He}) - M(^{11}\text{B})\right] \text{u} \cdot c^2 = -10.46 \text{ MeV}$$

$$K_{\text{th}} = \frac{14}{12}(10.46 \text{ MeV}) = 12.20 \text{ MeV}$$

17. a)

$$N = \frac{6.022 \times 10^{23}}{59 \text{ g}}\left(40 \times 10^{-3} \text{ g}\right) = 4.083 \times 10^{20}$$

activation rate $= n\sigma\phi = \left(4.083 \times 10^{20}\right)\left(20 \times 10^{-28} \text{ m}^2\right)\left(10^{18} \text{ m}^{-2} \cdot \text{s}^{-1}\right) = 8.166 \times 10^{11} \text{ s}^{-1}$

Then in one week the number of ^{60}Co produced is

$$8.166 \times 10^{11} \text{ s}^{-1}(7(86400 \text{ s})) = 4.94 \times 10^{17}$$

b) Because the half-life of ^{60}Co is much longer than one week, we can assume almost all the nuclei produced are still present. The activity is

$$R = \lambda N = \frac{\ln 2}{t_{1/2}}N = \frac{\ln 2}{1.66 \times 10^{8} \text{ s}}\left(4.94 \times 10^{17}\right) = 2.06 \times 10^{9} \text{ Bq}$$

c)

$$10^{14} \text{ Bq}\,\frac{40 \times 10^{-3} \text{ g}}{2.06 \times 10^{9} \text{ Bq}} = 1942 \text{ g}$$

We would put 1.94 kg of ^{59}Co into the reactor for one week.

*18. The equation for K_{cm} is correct because we know from classical mechanics that the system is equivalent to a mass $M_x + M_X$ moving with a speed v_{cm}^2, so the center of mass kinetic energy is

$$K_{\text{cm}} = \frac{1}{2}\left(M_x + M_X\right)v_{\text{cm}}^2$$

Letting $M = M_x + M_X$ we have by conservation of momentum $Mv_{\text{cm}} = M_x v_x$, or $v_{\text{cm}} = M_x v_x / M$. Thus

$$K_{\text{cm}} = \frac{1}{2}Mv_{\text{cm}}^2 = \frac{1}{2}M\frac{M_x^2}{M^2}v_x^2 = \frac{1}{2}\frac{M_x^2}{M}v_x^2$$

$$K_{\text{cm}}^{\prime} = K_{\text{lab}} - K_{\text{cm}} = \frac{M_x v_x^2}{2} - \frac{1}{2}\frac{M_x^2}{M}v_x^2 = \frac{M_x v_x^2}{2}\left(1 - \frac{M_x}{M}\right) = \frac{M_x v_x^2}{2}\left(\frac{M - M_x}{M}\right)$$

$$K_{\text{cm}}^{\prime} = K_{\text{lab}}\left(\frac{M_X}{M_x + M_X}\right)$$

*19.

$$K'_{\rm cm} = \frac{M_X}{M_x + M_X} K_{\rm lab} = \frac{14}{18}\,(7.7\ {\rm MeV}) = 5.99\ {\rm MeV}$$

$$E^* = \left[M(^{14}{\rm N}) + M(^{4}{\rm He}) - M(^{18}{\rm F})\right] {\rm u}\cdot c^2 = 4.41\ {\rm MeV}$$

$$E_x = E^* + K'_{\rm cm} = 10.40\ {\rm MeV}$$

20.

$$E^* = \left[M(^{208}{\rm Pb}) + M(n) - M(^{209}{\rm Pb})\right] {\rm u}\cdot c^2 = 3.94\ {\rm MeV}$$

Because of the excited nucleus, we expect gamma decay. ^{209}Pb also undergoes β^- decay.

21. From Chapter 12 the radius of the nucleus is

$$r\left(^{14}{\rm N}\right) = (1.2\ {\rm fm})\,14^{1/3} = 2.89\ {\rm fm}$$

At that radius the potential energy is

$$V = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = \frac{(2)\,(7)\,(1.44\times10^{-9}\ {\rm eV\cdot m})}{2.89\times10^{-15}\ {\rm m}} = 6.98\ {\rm MeV}$$

$$Q = \left[M(^{14}{\rm N}) + M(^{4}{\rm He}) - M(^{1}{\rm H}) - M(^{17}{\rm O})\right] {\rm u}\cdot c^2 = -1.19\ {\rm MeV}$$

$$K_{\rm exit} = Q + K_\alpha = -1.19\ {\rm MeV}\ + 7.7\ {\rm MeV}\ = 6.51\ {\rm MeV}\ = K(p) + K(^{17}{\rm O})$$

Most of the energy at the forward angle will be in the proton, so there should be sufficient energy to overcome the 6.98 MeV barrier in a tunneling process (though it appears to be forbidden classically).

*22. From Chapter 12 Problem 23 we see that the mean lifetime is

$$\tau = \frac{t_{1/2}}{\ln 2} = \frac{109\ {\rm ms}}{\ln 2} = 157\ {\rm ms}$$

The from Equation (13.13)

$$\Gamma = \frac{\hbar}{2\tau} = \frac{6.582\times10^{-16}\ {\rm eV\cdot s}}{2\,(0.157\ {\rm s})} = 2.10\times10^{-15}\ {\rm eV}$$

^{17}Ne can decay by positron decay or electron capture.

23.

$$E^* = \left[M(^1\text{H}) + M(^{16}\text{O}) - M(^{17}\text{F})\right] \text{u} \cdot c^2 = 0.60 \text{ MeV}$$

No, this energy is greater than 0.495 MeV.

24. a)

$$E^* = \left[M(^{239}\text{Pu}) + M(n) - M(^{240}\text{Pu})\right] \text{u} \cdot c^2 = 6.53 \text{ MeV}$$

b)

$$K_{\text{cm}} = \frac{239}{240}(1.00 \text{ MeV}) = 1.00 \text{ MeV}$$
$$E = 6.53 \text{ MeV } + 1.00 \text{ MeV } = 7.53 \text{ MeV}$$

25.

$$d + {}^{21}\text{Ne} \rightarrow {}^{22}\text{Ne} + p \qquad d + {}^{22}\text{Ne} \rightarrow {}^{22}\text{Ne} + d \qquad d + {}^{23}\text{Na} \rightarrow {}^{22}\text{Ne} + {}^{3}\text{He}$$

$$\alpha + {}^{18}\text{O} \rightarrow {}^{22}\text{Ne} + \gamma \qquad \alpha + {}^{19}\text{F} \rightarrow {}^{22}\text{Ne} + p$$
$$\alpha + {}^{21}\text{Ne} \rightarrow {}^{22}\text{Ne} + {}^{3}\text{He} \qquad \alpha + {}^{22}\text{Ne} \rightarrow {}^{22}\text{Ne} + {}^{4}\text{He}$$

$$d + {}^{25}\text{Mg} \rightarrow {}^{22}\text{Ne} + {}^{5}\text{Li} \qquad d + {}^{26}\text{Mg} \rightarrow {}^{22}\text{Ne} + {}^{6}\text{Li} \qquad d + {}^{27}\text{Al} \rightarrow {}^{22}\text{Ne} + {}^{7}\text{Be} \text{ (and so on)}$$

$$\alpha + {}^{23}\text{Na} \rightarrow {}^{22}\text{Ne} + {}^{5}\text{Li} \qquad \alpha + {}^{24}\text{Mg} \rightarrow {}^{22}\text{Ne} + {}^{6}\text{Be}$$
$$\alpha + {}^{27}\text{Al} \rightarrow {}^{22}\text{Ne} + {}^{9}\text{B} \text{ (and so on)}$$

26.

$$\Delta E = \left[M(^{239}\text{Pu}) + M(n) - M(^{95}\text{Zr}) - M(^{142}\text{Xe}) - 3M(n)\right] \text{u} \cdot c^2 = 183.6 \text{ MeV}$$

*27. a)

$$m = \left(4 \times 10^{-4}\right)\left(10^6 \text{ kg}\right) = 400 \text{ kg}$$

b)

$$(400 \text{ kg}) \frac{6.022 \times 10^{23}}{0.238 \text{ kg}} = 1.01 \times 10^{27} \text{ atoms}$$

c)

$$R = \left(0.69 \text{ kg}^{-1} \cdot \text{s}^{-1}\right)(400 \text{ kg}) = 276 \text{ Bq}$$

d)

$$\left(276 \text{ s}^{-1}\right) \frac{86400 \text{ s}}{d} = 2.38 \times 10^7 \text{ d}^{-1}$$

*28.

$$\frac{N\left(^{235}\text{U}\right)}{N\left(^{238}\text{U}\right)} = \frac{7}{993} = \frac{N_0\left(^{235}\text{U}\right)e^{-\lambda_1 t}}{N_0\left(^{238}\text{U}\right)e^{-\lambda_2 t}}$$

where the subscripts 1 and 2 refer to the 235 and 238 isotopes, respectively.

$$\frac{N_0\left(^{235}\text{U}\right)}{N_0\left(^{238}\text{U}\right)} = \frac{7}{993}\exp\left(\left(\lambda_1 - \lambda_2\right)t\right)$$

$$\begin{aligned}(\lambda_1 - \lambda_2)\,t &= \ln 2\left(\frac{1}{t_{1/2}(235)} - \frac{1}{t_{1/2}(238)}\right)t \\ &= \ln 2\left(\frac{1}{7.04\times 10^8\ \text{y}} - \frac{1}{4.47\times 10^9\ \text{y}}\right)\left(2\times 10^9\ \text{y}\right) = 1.566\end{aligned}$$

Then

$$\frac{N_0\left(^{235}\text{U}\right)}{N_0\left(^{238}\text{U}\right)} = \frac{7}{993}\exp\left(\left(\lambda_1 - \lambda_2\right)t\right) = \frac{7}{993}\exp\left(1.566\right) = 0.0337$$

which is about five times higher than today. Natural fission reactors cannot operate because the relatively low abundance of ^{235}U today prohibits a critical mass being reached.

29. Here are three possibilities, out of many:

$^{236}\text{U} \rightarrow {}^{95}\text{Y} + {}^{138}\text{I} + 3n$ $\qquad$ $^{236}\text{U} \rightarrow {}^{94}\text{Y} + {}^{140}\text{I} + 2n$ $\qquad$ $^{236}\text{U} \rightarrow {}^{97}\text{Y} + {}^{136}\text{I} + 3n$

30.

$$10^3\ \text{MWe} = \left(10^9\ \text{J/s}\right)\frac{86400\ \text{s/d}}{1.602\times 10^{-13}\ \text{J/MeV}} = 5.39\times 10^{26}\ \text{MeV/d}$$

$$\left(5.39\times 10^{26}\ \text{MeV/d}\right)\left(\frac{1\ \text{(fission)}}{200\ \text{MeV}}\right)\frac{0.235\ \text{kg}}{6.022\times 10^{23}} = 1.05\ \text{kg/d}$$

*31. For uranium we assume as in the previous problem that each fission produces 200 MeV of energy.

$$(1\ \text{kg})\left(\frac{6.022\times 10^{23}\ \text{atoms}}{0.235\ \text{kg}}\right)\frac{200\ \text{MeV}}{\text{atom}} = 5.13\times 10^{26}\ \text{MeV}$$

Converting to kWh we find 2.30×10^7 kWh. On the other hand, for coal the conversion of 29000 Btu is 8.50 kWh. Therefore we see that fission produces over one million times more energy per kilogram of fuel.

*32. a)

$$\frac{3}{2}kT = \frac{3}{2}\left(8.617\times10^{-5}\ \text{eV/K}\right)(300\ \text{K}) = 3.88\times10^{-2}\ \text{eV}$$

b)

$$\frac{3}{2}kT = \frac{3}{2}\left(8.617\times10^{-5}\ \text{eV/K}\right)\left(15\times10^{6}\ \text{K}\right) = 1.94\ \text{keV}$$

33. $^{1}\text{H} + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma \rightarrow {}^{13}\text{C} + \beta^{+} + \nu$

$$Q = \left[M(^{1}\text{H}) + M(^{12}\text{C}) - M(^{13}\text{C})\right]\text{u}\cdot c^2 = 4.16\ \text{MeV}$$

$^{1}\text{H} + {}^{13}\text{C} \rightarrow {}^{14}\text{N} + \gamma$

$$Q = \left[M(^{1}\text{H}) + M(^{13}\text{C}) - M(^{14}\text{N})\right]\text{u}\cdot c^2 = 7.55\ \text{MeV}$$

$^{1}\text{H} + {}^{14}\text{N} \rightarrow {}^{15}\text{O} + \gamma \rightarrow {}^{15}\text{N} + \beta^{+} + \nu$

$$Q = \left[M(^{1}\text{H}) + M(^{14}\text{N}) - M(^{15}\text{N})\right]\text{u}\cdot c^2 = 10.05\ \text{MeV}$$

$^{1}\text{H} + {}^{15}\text{N} \rightarrow {}^{12}\text{C} + {}^{4}\text{He}$

$$Q = \left[M(^{1}\text{H}) + M(^{15}\text{N}) - M(^{12}\text{C}) - M(^{4}\text{He})\right]\text{u}\cdot c^2 = 4.97\ \text{MeV}$$

The total Q is

$$Q = 4.16\ \text{MeV} + 7.55\ \text{MeV} + 10.05\ \text{MeV} + 4.97\ \text{MeV} = 26.73\ \text{MeV}$$

34. The if the depth is d and the radius of the earth is r, volume of the oceans is

$$V = \frac{2}{3}\left(4\pi r^2\right)d = \frac{8}{3}\pi\left(6.37\times10^{6}\ \text{m}\right)^2(3000\ \text{m}) = 1.02\times10^{18}\ \text{m}^3$$

With a density of 1000 kg/m^3, the mass is 1.02×10^{21} kg, so the number of water molecules is

$$N = \left(1.02\times10^{21}\ \text{kg}\right)\frac{6.022\times10^{23}}{0.018\ \text{kg}} = 3.41\times10^{46}$$

With a 0.015% abundance, the number of deuterium atoms (in D_2O molecules) is

$$(2)\,3.41\times10^{46}\,(0.00015) = 1.02\times10^{43}$$

There will be at most half this number of fusions, or 5.1×10^{42}. The energy released is

$$\left(5.1\times10^{42}\right)(4.0\ \text{MeV})\left(1.602\times10^{-13}\ \text{J/MeV}\right) = 3.27\times10^{30}\ \text{J}$$

35. a) We use temperatures because that is what we strive for experimentally. Kinetic energy is useful for comparison with the Coulomb barrier or Q values.

b) $K = \frac{3}{2}kT$

c)

$$T = \frac{2K}{3k} = \frac{2\,(6000\text{ eV})}{3\,(8.617\times10^{-5}\text{ eV/K})} = 4.64\times10^{7}\text{ K}$$

36. $^4\text{He} + {}^3\text{He} \to {}^7\text{Be} + \gamma$

$$Q = \left[M(^4\text{H}) + M(^3\text{He}) - M(^7\text{Be})\right]\text{u}\cdot c^2 = 1.59\text{ MeV}$$

$^2\text{H} + {}^2\text{H} \to {}^3\text{H} + {}^1\text{H}$

$$Q = \left[M(^2\text{H}) + M(^2\text{H}) - M(^3\text{H}) - M(^1\text{H})\right]\text{u}\cdot c^2 = 4.03\text{ MeV}$$

$^1\text{H} + {}^2\text{H} \to {}^3\text{He} + \gamma$

$$Q = \left[M(^1\text{H}) + M(^2\text{H}) - M(^3\text{He})\right]\text{u}\cdot c^2 = 5.48\text{ MeV}$$

$^1\text{H} + {}^{12}\text{C} \to {}^{13}\text{N} + \gamma$

$$Q = \left[M(^1\text{H}) + M(^{12}\text{C}) - M(^{13}\text{N})\right]\text{u}\cdot c^2 = 1.94\text{ MeV}$$

$^3\text{He} + {}^3\text{He} \to {}^4\text{He} + 2p$

$$Q = \left[M(^3\text{He}) + M(^3\text{He}) - M(^4\text{He}) - 2M(^1\text{H})\right]\text{u}\cdot c^2 = 12.9\text{ MeV}$$

$^7\text{Li} + {}^1\text{H} \to {}^4\text{He} + {}^4\text{He}$

$$Q = \left[M(^1\text{H}) + M(^7\text{Li}) - 2M(^4\text{He})\right]\text{u}\cdot c^2 = 17.3\text{ MeV}$$

$^3\text{H} + {}^2\text{H} \to {}^4\text{He} + n$

$$Q = \left[M(^3\text{H}) + M(^2\text{H}) - M(^4\text{He}) - M(n)\right]\text{u}\cdot c^2 = 17.6\text{ MeV}$$

$^3\text{He} + {}^2\text{H} \to {}^4\text{He} + {}^1\text{H}$

$$Q = \left[M(^3\text{He}) + M(^2\text{H}) - M(^4\text{He}) - M(^1\text{H})\right]\text{u}\cdot c^2 = 18.4\text{ MeV}$$

37. The reaction is $^1\text{H} + {}^{12}\text{C} \to {}^{13}\text{N} + \gamma$.

$$Q = \left[M(^{12}\text{C}) + M(^1\text{H}) - M(^{13}\text{N})\right]\text{u}\cdot c^2 = 1.94\text{ MeV}$$

Then

$$K_{\text{th}} = -Q\left(\frac{M(^1\text{H}) + M(^{12}\text{C})}{M(^{12}\text{C})}\right)$$

which will be negative, so the threshold energy is not a concern. However, the Coulomb barrier is

$$V = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = \frac{6e^2}{4\pi\varepsilon_0 \left(r_p + r_0 A^{1/3}\right)} = \frac{6\,(1.44 \times 10^{-9}\ \text{eV} \cdot \text{m})}{\left(1.2 \times 10^{-15}\ \text{m}\ + (1.2 \times 10^{-15}\ \text{m})\,(12)^{1/3}\right)} = 2.19\ \text{MeV}$$

Setting this coulomb barrier energy equal to the thermal energy $\frac{3}{2}kT$, we have

$$T = \frac{2K}{3k} = \frac{2\,(2.19 \times 10^{6}\ \text{eV})}{3\,(8.617 \times 10^{-5}\ \text{eV/K})} = 1.69 \times 10^{10}\ \text{K}$$

38.

$$Q = \left[3M(^{4}\text{He}) - M(^{12}\text{C})\right]\text{u} \cdot c^2 = 7.27\ \text{MeV}$$

39. a)

$$K = \frac{3}{2}kT = \frac{3}{2}\left(8.617 \times 10^{-5}\ \text{eV/K}\right)(300\ \text{K}) = 3.88 \times 10^{-2}\ \text{eV}$$

b)

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\,(3.88 \times 10^{-2}\ \text{eV})\,(1.602 \times 10^{-19}\ \text{J/eV})}{1.675 \times 10^{-27}\ \text{kg}}} = 2720\ \text{m/s}$$

c)

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{(1.675 \times 10^{-27}\ \text{kg})\,(2720\ \text{m/s})} = 1.45 \times 10^{-10}\ \text{m}$$

40. $R = 512/60$ Bq $= 8.533$ Bq. The initial rate was $R_0 = 4 \times 10^5$ Bq. Over 60 days the activity has decreased to

$$R_0' = R_0 e^{-\lambda t} = \left(4 \times 10^5\ \text{Bq}\right)\exp\left(-\frac{\ln 2}{44.5\ \text{d}}\,(60\ \text{d})\right) = 1.571 \times 10^5\ \text{Bq}$$

Therefore the fraction f of the ring that has worn off into the oil is

$$f = \frac{R}{R_0'} = \frac{8.533\ \text{Bq}}{1.571 \times 10^5\ \text{Bq}} = 5.43 \times 10^{-5}$$

*41. a) With a half life of 66 h, not much is left after one week.

b)

$$R = R_0 e^{-\lambda t} = \left(10^{11}\ \text{Bq}\right)\exp\left(-\frac{\ln 2}{66\ \text{h}}\,(216\ \text{h})\right) = 1.035 \times 10^{10}\ \text{Bq}$$

c)

$$R = R_0 e^{-\lambda t} = \left(10^{11}\ \text{Bq}\right)\exp\left(-\frac{\ln 2}{66\ \text{h}}\,(96\ \text{h})\right) = 3.649 \times 10^{10}\ \text{Bq}$$

42. a) We need 1000 Bq at $t = 30$ minutes with $t_{1/2} = 83.1$ minutes.

$$R = R_0' e^{-\lambda t} = (R_0') \exp\left(-\frac{\ln 2}{83.1 \text{ min}} (30 \text{ min})\right) = 1000 \text{ Bq}$$

$$R_0' = (1000 \text{ Bq}) \exp\left(\frac{\ln 2}{83.1 \text{ min}} (30 \text{ min})\right) = 1284 \text{ Bq}$$

Because only 27% go to the first excited state, we need an activity $R_0 = R_0'/0.27 = 4756$ Bq. Then $R_0 = \lambda N = N \ln 2/t_{1/2}$, so solving for N:

$$N = \frac{R_0 t_{1/2}}{\ln 2} = \frac{(4756 \text{ s}^{-1})(83.1 \text{ min})(60 \text{ s/min})}{\ln 2} = 3.42 \times 10^7$$

b)

$$m = \left(3.42 \times 10^7\right) \frac{139 \text{ g}}{6.022 \times 10^{23}} = 7.89 \times 10^{-15} \text{ g}$$

The fraction f activated is

$$f = \frac{7.89 \times 10^{-15} \text{ g}}{(0.717)(10^{-8} \text{ g})} = 1.10 \times 10^{-6}$$

43. a)

$$I = (0.15)\left(4 \times 10^5 \text{ s}^{-1}\right)\left(2 \times 1.602 \times 10^{-19} \text{ C}\right) = 1.92 \times 10^{-14} \text{ A}$$

b) One-tenth of the above value is 1.92×10^{-15} A.

44. a)

$$R = (5000 \text{ J/s}) \frac{1 \text{ MeV}}{1.602 \times 10^{-13} \text{ J}} \frac{1 \text{ particle}}{5.3 \text{ MeV}} = 5.89 \times 10^{15} \text{ Bq}$$

$$N = \frac{R}{\lambda} = \frac{(5.89 \times 10^{15} \text{ s}^{-1})(138 \text{ d})(86400 \text{ s/d})}{\ln 2} = 1.013 \times 10^{23}$$

$$m = \left(1.013 \times 10^{23}\right)\left(\frac{0.210 \text{ kg}}{6.022 \times 10^{23}}\right) = 3.53 \times 10^{-2} \text{ kg}$$

b) For 7 kW we need a mass of $(7/5)(3.53 \times 10^{-2} \text{ kg}) = 0.0494$ kg. Letting the initial mass be m_0, we have the usual exponential decay and so

$$m_0 = m \exp(\lambda t) = (0.0494 \text{ kg}) \exp\left(\frac{(\ln 2)(730 \text{ d})}{138 \text{ d}}\right) = 1.93 \text{ kg}$$

45. The fraction incident is the patient's cross-sectional area divided by $4\pi\,(\text{distance})^2$. Note that there are two gammas per decay.

$$R = 2\left(3 \times 10^{14} \text{ Bq}\right) \frac{0.3 \text{ m}^2}{4\pi\,(4 \text{ m})^2} = 8.95 \times 10^{11} \text{ Bq}$$

46. From Chapter 12 the diameter of the uranium nucleus is

$$2r\left(^{238}\text{U}\right) = 2\,(1.2\text{ fm})\,238^{1/3} = 14.87\text{ fm}$$

The momentum of the neutron is

$$p = \frac{h}{\lambda} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{14.87\times10^{-15}\text{ m}} = 4.456\times10^{-20}\text{ kg}\cdot\text{m/s}$$

and the kinetic energy is

$$K = \frac{p^2}{2m} = \frac{\left(4.456\times10^{-20}\text{ kg}\cdot\text{m/s}\right)^2}{2\,(1.675\times10^{-27}\text{ kg})} = 5.927\times10^{-13}\text{ J} = 3.70\text{ MeV}$$

This is still much less than the neutron's rest energy, which justifies the non-relativistic treatment. Then in thermal equilibrium

$$T = \frac{2K}{3k} = \frac{2\,(5.927\times10^{-13}\text{ J})}{3\,(1.381\times10^{-23}\text{ J/K})} = 2.86\times10^{10}\text{ K}$$

Clearly this is not a realistic temperature in a reactor, so the proposed experiment is not possible.

47. The alpha decay reaction is $^{239}\text{Pu} \rightarrow {}^{235}\text{U} + {}^{4}\text{He}$, and the energy released in each decay is

$$Q = \left[M(^{239}\text{Pu}) - M(^{235}\text{U}) - M(^{4}\text{He})\right]\text{u}\cdot c^2 = 5.25\text{ MeV}$$

The activity of a 10 kg sample is

$$R = \lambda N = \frac{\ln 2}{(24110\text{ y})\,(3.156\times10^{7}\text{ s/y})}\,(10\text{ kg})\left(\frac{6.022\times10^{23}}{0.239\text{ kg}}\right) = 2.30\times10^{13}\text{ Bq}$$

Then the power is

$$P = (0.6)\,(5.25\text{ MeV})\left(1.602\times10^{-13}\text{ J/MeV}\right)\left(2.30\times10^{13}\text{ s}^{-1}\right) = 11.6\text{ W}$$

*48. a)

$$(1\text{ kg})\,\frac{6.022\times10^{23}}{\text{mol}}\,\frac{1\text{ mol}}{0.001\text{ kg}}\,\frac{13.6\text{ eV}}{1\text{ atom}} = 8.19\times10^{27}\text{ eV}$$

b)

$$(1\text{ kg})\,\frac{6.022\times10^{23}}{\text{mol}}\,\frac{1\text{ mol}}{0.002\text{ kg}}\,\frac{2.22\times10^{6}\text{ eV}}{1\text{ atom}} = 6.68\times10^{32}\text{ eV}$$

c)

$$(1\text{ kg})\,\frac{1\text{ u}}{1.66\times10^{-27}\text{ kg}}\,\frac{931.49\text{ MeV}}{\text{u}} = 5.61\times10^{35}\text{ eV}$$

*49. a)

$$Q = K_{\text{out}} - K_{\text{in}} = 86.63 \text{ MeV } - 100 \text{ MeV } = -13.37 \text{ MeV}$$

This agrees with the Q computed using atomic masses.

b)

$$M = -\frac{Q}{c^2} + M(^{18}\text{O}) + M(^{30}\text{Si}) - M(^{14}\text{O}) = 33.979 \text{ u}$$

which matches the value for the mass of ^{34}Si in Appendix 8 pretty well.

50. a) The number of atoms is

$$1000\,(100 \text{ kg}) \frac{1 \text{ mol}}{0.235 \text{ kg}} \frac{6.022 \times 10^{23}}{\text{mol}} (0.04) = 1.03 \times 10^{28}$$

b)

$$\frac{1.03 \times 10^{28}}{4\pi\,(6 \times 10^6 \text{ m})^2} = 2.28 \times 10^{13} \text{ m}^{-2}$$

Then the activity for each square meter is

$$R = \lambda N = \frac{\ln 2}{(28.8 \text{ y}) \ (3.156 \times 10^7 \text{ s/y})} \left(2.28 \times 10^{13}\right) = 1.74 \times 10^4 \text{ Bq}$$

51. a) Starting with $F(E) = (\text{const.})e^{-E/kT}E^{1/2}$ and setting $dF/dE = 0$, we find that the most probable energy is $E^* = \frac{1}{2}kT$.

$$E^* = \frac{1}{2}kT = \frac{1}{2}\left(8.621 \times 10^{-5} \text{ eV/K}\right)\left(2 \times 10^8 \text{ K}\right) = 8620 \text{ eV}$$

b)

$$\frac{n(2E)}{n(E)} = \exp\left(-\beta\,(2E - E)\right) = \exp\left(-\beta E\right) = \exp\left(-\frac{1}{2}\right) = 0.607$$

$$\frac{n(5E)}{n(E)} = \exp\left(-\beta\,(5E - E)\right) = \exp\left(-4\beta E\right) = \exp\left(-2\right) = 0.135$$

$$\frac{n(10E)}{n(E)} = \exp\left(-\beta\,(10E - E)\right) = \exp\left(-9\beta E\right) = \exp\left(-\frac{9}{2}\right) = 0.0111$$

52. a)

$$P = \sigma n t = \left(90 \times 10^{-27} \text{ cm}^2\right)\left(1.85 \text{ g/cm}^3\right)(3 \text{ cm})\left(6.022 \times 10^{23} \text{ mol}^{-1}\right)\left(\frac{1 \text{ mol}}{9 \text{ g}}\right) = 0.0334$$

b) We know that 10^5 alpha particles must interact each second to get an activity of 10^5 Bq. With this probability the number of incident alpha particles is

$$\frac{10^5 \text{ s}^{-1}}{0.0334} = 2.99 \times 10^6 \text{ s}^{-1}$$

c)

$$N = \frac{R}{\lambda} = \left(2.99 \times 10^6 \text{ s}^{-1}\right)\frac{2.41 \times 10^4 \text{ y}}{\ln 2}\frac{3.156 \times 10^7 \text{ s}}{\text{y}} = 3.28 \times 10^{18}$$

$$m = \left(3.28 \times 10^{18}\right) \frac{0.239\ \text{kg}}{6.022 \times 10^{23}} = 1.30 \times 10^{-6}\ \text{kg}$$

which is reasonable.

*53.

$$m(^{40}\text{K}) = (70\ \text{kg})\,(0.0035)\,(0.00012) = 2.94 \times 10^{-5}\ \text{kg}$$

$$N = \left(2.94 \times 10^{-5}\ \text{kg}\right) \frac{6.022 \times 10^{23}}{0.040\ \text{kg}} = 4.43 \times 10^{20}$$

$$R = \lambda N = \frac{\ln 2}{1.28 \times 10^{9}\ \text{y}} \frac{1\ \text{y}}{3.156 \times 10^{7}\ \text{s}} \left(4.43 \times 10^{20}\right) = 7600\ \text{Bq}$$

The beta activity is $(7600\ \text{Bq})\,(0.893) = 6790\ \text{Bq}$.