Chapter 14

*1. By conservation of momentum the photons must have the same energy. For each one $E = h\nu = mc^2$ and

$$\nu = \frac{mc^2}{h} = \frac{938.27 \times 10^6 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 2.27 \times 10^{23} \text{ Hz}$$

*2. As in the text let $mc^2 = \hbar c/R$, so

$$R = \frac{\hbar c}{mc^2} = \frac{197.3 \text{ eV} \cdot \text{nm}}{140 \times 10^6 \text{ eV}} = 1.41 \times 10^{-6} \text{ nm} \; = 1.41 \text{ fm}$$

3. We will use a small nucleus (helium, diameter 3.8 fm) and a large nucleus (uranium, diameter 14.9 fm) to get a range of values. For helium:

$$\Delta t = \frac{3.8 \times 10^{-15} \text{ m}}{2.998 \times 10^8 \text{ m/s}} = 1.27 \times 10^{-23} \text{ s}$$

For uranium:

$$\Delta t = \frac{14.9 \times 10^{-15} \text{ m}}{2.998 \times 10^8 \text{ m/s}} = 4.97 \times 10^{-23} \text{ s}$$

4. a) $\overline{\nu}_\mu$ and ν_e b) $\overline{\nu}_e$ c) $\overline{\nu}_\mu$ d) $\overline{\nu}_\mu$ e) ν_μ

5. Assuming roughly an equal number of protons and neutrons, we can use the average of their masses per baryon, or 1.674×10^{-27} kg. The mass of the earth is 5.98×10^{24} kg, so the baryon number is

$$\frac{5.98 \times 10^{24} \text{ kg}}{1.674 \times 10^{-27} \text{ kg}} = 3.57 \times 10^{51}$$

6. In the first reaction strangeness is violated ($+1 \rightarrow -1$). The second reaction is allowed. The fact that the K^0 has strangeness $+1$ and $\overline{K}^0$ has strangeness -1 is consistent with the idea that it is not its own antiparticle. However, M. Gell-Mann and A. Pais found that $K^0 \rightarrow \pi^+ + \pi^- \rightarrow \overline{K}^0$ (and the reverse operation $\overline{K}^0 \rightarrow \pi^+ + \pi^- \rightarrow K^0$) can occur by the laws of quantum mechanics (see Feynman Lectures in Physics vol. 3 p. 11-16). So in this case the K^0 does act as its own antiparticle.

7. In both (a) and (b) the baryon number is not conserved.

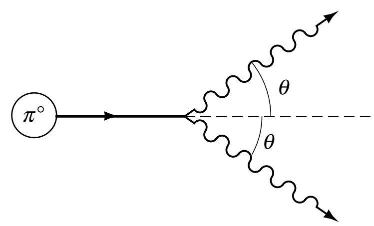
8. a) μ and e lepton numbers are not conserved
   b) charge is not conserved
   c) momentum and spin are not conserved

*9. Let subscript 1 refer to the Σ, subscript 2 to the Λ, and no subscript to the photon. From conservation of momentum $p = p_2 = E/c$. From conservation of energy

$$m_1c^2 = \sqrt{p_2^2c^2 + (m_2c^2)^2} + E = \sqrt{E^2 + (m_2c^2)^2} + E$$

$$E = \frac{(m_1c^2)^2 - (m_2c^2)^2}{2m_1c^2} = \frac{(1193\text{ MeV})^2 - (1116\text{ MeV})^2}{2\,(1193\text{ MeV})} = 74.5\text{ MeV}$$

10. The π^0 begins with energy $E = K + E_0 = 735$ MeV. Because the photon energies are equal, we know by conservation of momentum that the two photons make equal angles θ above and below the original line of motion for the π^0. Let each photon energy be E'. Then conservation of energy and momentum give:



$$E' = \frac{E}{2} = \frac{735}{2}\text{ MeV } = 367.5\text{ MeV}$$

$$p = \frac{2E'}{c}\cos\theta = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(735\text{ MeV})^2 - (135\text{ MeV})^2}}{c} = 722.5\text{ MeV}/c$$

$$\cos\theta = \frac{pc}{2E'} = \frac{722.5\text{ MeV}}{2\,(367.5\text{ MeV})} = 0.9830$$

from which we find $\theta = 10.6°$.

11. a) ν_μ is needed to conserve lepton number.
    b) a kaon is needed to conserve strangeness, and to conserve charge too it must be a K^+.

*12. n (udd): $q = \frac{2e}{3} - \frac{e}{3} - \frac{e}{3} = 0$; $B = 3\left(\frac{1}{3}\right) = 1$; $S = 3\,(0) = 0$

Σ^+ (uus): $q = \frac{2e}{3} + \frac{2e}{3} - \frac{e}{3} = e$; $B = 3\left(\frac{1}{3}\right) = 1$; $S = 0 + 0 - 1 = -1$

Λ_C^+ (udc): $q = \frac{2e}{3} - \frac{e}{3} + \frac{2e}{3} = e$; $B = 3\left(\frac{1}{3}\right) = 1$; $S = 0 + 0 + 0 = 0$

13. $\pi^+(u\overline{d})$: $q = \frac{2e}{3} + \frac{e}{3} = e$; $B = \frac{1}{3} - \frac{1}{3} = 0$; $S = 0 + 0 = 0$

$K^+(u\overline{s})$: $q = \frac{2e}{3} + \frac{e}{3} = e$; $B = \frac{1}{3} - \frac{1}{3} = 0$; $S = 0 + 1 = 1$

D^0:$(c\overline{u})$: $q = \frac{2e}{3} - \frac{2e}{3} = 0$; $B = \frac{1}{3} - \frac{1}{3} = 0$; $S = 0 + 0 = 0$

14. D^0: $c\overline{u}$ $\qquad$ D^+: $c\overline{d}$

15. The B^+ and B^- have zero strangeness and zero charm, but they have masses greater than 5000 MeV/c^2 and charge +1. This leads one to conclude that the quark configuration should be $\overline{b}u$ for the B^+ and $b\overline{u}$ for the B^-. The B^0 has zero charge, so it should be either $b\overline{d}$ or $\overline{b}d$. The only way to distinguish between these two possibilities is by looking at the conservation laws in the appropriate decay reactions. it turns out that the B^0 is $\overline{b}d$ and the antiparticle $\overline{B}^0$ is $b\overline{d}$.

16. Charge, baryon number, strangeness, topness, and bottomness are zero. Charm = −1. From the table in the text D^0 has configuration $c\overline{u}$, so $\overline{c}u$ must be $\overline{D^0}$.

17. Assuming an average nucleon mass of 1.674×10^{-27} kg and noting that 10 out of 18 nucleons in water are protons, we have

$$\left(10^5 \text{ gal}\right) \frac{3.786 \text{ liter}}{\text{gal}} \frac{10^{-3} \text{ m}^3}{\text{liter}} \frac{1000 \text{ kg}}{\text{m}^3} \frac{1}{1.674 \times 10^{-27} \text{ kg}} \frac{10}{18} = 1.26 \times 10^{32} \text{ protons}$$

Then if one half of the protons decay in 10^{33} years,

$$R = \lambda N = \frac{\ln 2}{10^{33} \text{ y}} \left(1.26 \times 10^{32}\right) = 0.087 \text{ y}^{-1}$$

18. Baryon number is not conserved in any of the three. This is a common problem in proton decay schemes, because there are no lighter baryons. In addition, (a) violates electron lepton number, (b) violates muon lepton number, and (c) violates strangeness.

19.

$$\frac{25 \text{ kg}}{1.673 \times 10^{-27} \text{ kg}} \frac{\ln 2 \text{ decays}}{10^{32} \text{ y}} \frac{1 \text{ y}}{365.25 \text{ d}} = 2.84 \times 10^{-7} \text{ decays/d}$$

20. The lifetime of the Σ^+ is 8.0×10^{-11} s. Due to relativity it travels farther then one might expect.

$$\gamma = \frac{K + E_0}{E_0} = \frac{3 \text{ GeV} + 1.109 \text{ GeV}}{1.109 \text{ GeV}} = 3.705 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/3.705^2} = 0.9629\, c$$

$$d = vt' = \gamma vt = (3.705)\,(0.9629)\left(2.998 \times 10^8 \text{ m/s}\right)\left(8.0 \times 10^{-11} \text{ s}\right) = 8.56 \text{ cm}$$

21.

$$\gamma = \frac{K + E_0}{E_0} = \frac{7000 \text{ GeV } + 0.938 \text{ GeV}}{0.938 \text{ GeV}} = 7464 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/7464^2} = c\left(1 - 9.0 \times 10^{-9}\right)$$

22. The maximum energies result from a head-on collision. Before the collision the proton has energy $E = K + E_0 = 998.27$ MeV. After the collision the proton has energy and momentum E_1, p_1 and the recoiling particle has E_2, p_2. From conservation of momentum and energy

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(998.27 \text{ MeV})^2 - (938.27 \text{ MeV})^2}}{c} = 340.87 \text{ MeV}/c = p_1 + p_2$$

$$E + E_0\,(d) = 998.27 \text{ MeV } + 1875.61 \text{ MeV } = 2873.9 \text{ MeV } = E_1 + E_2$$

Using the energy-momentum invariant $E_0^2 = E^2 - p^2c^2$ for both the proton and deuteron, these two equations can be solved to yield $p_1 = -111$ MeV/c, $p_2 = 452$ MeV/c, $E_1 = 944$ MeV, $E_2 = 1930$ MeV, so the deuteron's kinetic energy is

$$K = 1930 \text{ MeV } - 1876 \text{ MeV } = 54 \text{ MeV}$$

Similarly for the triton (t) with $E_0 = 2809$ MeV, we can follow the same procedure:

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(998.27 \text{ MeV})^2 - (938.27 \text{ MeV})^2}}{c} = 340.87 \text{ MeV}/c = p_1 + p_2$$

$$E + E_0\,(t) = 998.27 \text{ MeV } + 2809 \text{ MeV } = 3807 \text{ MeV } = E_1 + E_2$$

Using the energy-momentum invariant $E_0^2 = E^2 - p^2c^2$ for both the proton and triton, these two equations can be solved to yield $p_1 = -166$ MeV/c, $p_2 = 507$ MeV/c, $E_1 = 953$ MeV, $E_2 = 2854$ MeV, so the triton's kinetic energy is

$$K = 2854 \text{ MeV } - 2809 \text{ MeV } = 45 \text{ MeV}$$

23. To conserve baryon number we must produce both a proton and antiproton, so in the cm system we need $E_{\text{cm}} = 4E_0$. We can use Equation (14.10) and solve for K to find

$$K = \frac{E_{\text{cm}}^2 - \left(m_1c^2 + m_2c^2\right)^2}{2m_2c^2} = \frac{E_{\text{cm}}^2 - 4\left(mc^2\right)^2}{2mc^2}$$

where we have used $m_1 = m_2 = m$. With $E_{\text{cm}} = 4mc^2$ we have

$$K = \frac{\left(4mc^2\right)^2 - 4\left(mc^2\right)^2}{2mc^2} = 6mc^2 = 6\,(938.27 \text{ MeV}) = 5630 \text{ MeV}$$

*24. a)

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(948.27 \text{ MeV})^2 - (938.27 \text{ MeV})^2}}{c} = 137.35 \text{ MeV}/c$$

$$R = \frac{p}{qB} = \frac{137.35 \text{ MeV}}{(2.998 \times 10^8 \text{ m/s})\,(e)\,(1 \text{ T})} = 0.458 \text{ m}$$

b) As in Problem 25 we have

$$\nu = \frac{eB}{2\pi m}\sqrt{1 - v^2/c^2} = \frac{eB}{2\pi m\gamma}$$

$$\gamma = \frac{E}{E_0} = \frac{948.27 \text{ MeV}}{938.27 \text{ MeV}} = 1.0107$$

$$\nu = \frac{(e)\,(1 \text{ T})}{2\pi\,(938.27 \text{ MeV})\,(1.0107)}\left(2.998 \times 10^8 \text{ m/s}\right)^2 = 1.51 \times 10^7 \text{ Hz}$$

25. With $p = \gamma m v$ we have

$$\nu = \frac{v}{2\pi R} = \frac{p}{2\pi\gamma m R}$$

With $p = qBR$ we find

$$\nu = \frac{qBR}{2\pi\gamma m R} = \frac{qB}{2\pi m\gamma} = \frac{qB}{2\pi m}\sqrt{1 - v^2/c^2}$$

26.

$$E_{\text{lab}} = K + m_1c^2 + m_2c^2$$

$$p_{\text{lab}}c = \sqrt{\left(K + m_1c^2\right)^2 - \left(m_1c^2\right)^2}$$

Using the fact that $E^2 - p^2c^2$ is invariant, so

$$\begin{aligned}
E_{\text{cm}}^2 &= E_{\text{lab}}^2 - \left(p_{\text{lab}}c\right)^2 = \left(K + m_1c^2 + m_2c^2\right)^2 - \left(K + m_1c^2\right)^2 + \left(m_1c^2\right)^2 \\
&= K^2 + \left(m_1c^2 + m_2c^2\right)^2 + 2K\left(m_1c^2 + m_2c^2\right) - K^2 - 2Km_1c^2 - \left(m_1c^2\right)^2 + \left(m_1c^2\right)^2 \\
&= \left(m_1c^2 + m_2c^2\right)^2 + 2Km_2c^2
\end{aligned}$$

$$E_{\text{cm}} = \sqrt{\left(m_1c^2 + m_2c^2\right)^2 + 2Km_2c^2} = \left(m_1c^2 + m_2c^2\right)\sqrt{1 + \frac{2Km_2c^2}{\left(m_1c^2 + m_2c^2\right)^2}}$$

For the nonrelativistic limit we have $K << mc^2$. We can look at the binomial expansion of the square root:

$$E_{\text{cm}} \cong \left(m_1c^2 + m_2c^2\right)\left(1 + \frac{Km_2c^2}{\left(m_1c^2 + m_2c^2\right)^2}\right)$$

$$K_{\text{cm}} = E_{\text{cm}} - \left(m_1c^2 + m_2c^2\right) \cong \frac{Km_2c^2}{m_1c^2 + m_2c^2} = \frac{m_2}{m_1 + m_2}K$$

*27. As in the previous problem

$$E_{\mathrm{cm}} = \sqrt{(m_1c^2 + m_2c^2)^2 + 2Km_2c^2} = \sqrt{(2mc^2)^2 + 2Km_2c^2}$$

a) If $K << mc^2$ we neglect K, so $E_{\mathrm{cm}} \cong 2mc^2$.

b) If $K >> mc^2$ we neglect the first term and $E_{\mathrm{cm}} \cong \sqrt{2Km_2c^2} = \sqrt{2Kmc^2}$.

In (a) we interpret the result to mean that at very low energies there is no extra energy available (beyond the masses of the two original particles). In (b) we see that the available center of mass energy increases only in proportion to $\sqrt{K}$, thus illustrating the great advantage of colliding beam experiments over fixed target experiments.

28.

$$\gamma = \frac{K + E_0}{E_0} = \frac{50000 \text{ MeV } + 0.511 \text{ MeV}}{0.511 \text{ MeV}} = 97848 = \frac{1}{\sqrt{1 - v^2/c^2}}$$

$$v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/97848^2} = c\left(1 - 5.2 \times 10^{-11}\right)$$

29. In each case the desired ratio is simply equal to the relativistic factor γ.

a)

$$\gamma = \frac{K + E_0}{E_0} = \frac{10 \text{ MeV } + 938.27 \text{ MeV}}{938.27 \text{ MeV}} = 1.0107$$

b)

$$\gamma = \frac{K + E_0}{E_0} = \frac{100 \text{ MeV } + 938.27 \text{ MeV}}{938.27 \text{ MeV}} = 1.107$$

c)

$$\gamma = \frac{K + E_0}{E_0} = \frac{1000 \text{ MeV } + 938.27 \text{ MeV}}{938.27 \text{ MeV}} = 2.066$$

30.

$$\gamma = \frac{K + E_0}{E_0} = \frac{33000 \text{ MeV } + 938.27 \text{ MeV}}{938.27 \text{ MeV}} = 36.17$$

Therefore in the lab frame $v \cong c$ and the time for 160,000 revolutions is

$$t = N\frac{2\pi R}{v} \cong (160000)\,\frac{800 \text{ m}}{2.998 \times 10^8 \text{ m/s}} = 0.427 \text{ s}$$

The statement is accurate.

*31. a) For a stationary target the sum of the rest energies of the products equals the total center of mass energy, so

$$
\begin{aligned}
E_{\mathrm{cm}} &= \sqrt{2E_0\left(2E_0+K\right)} = \left(m_p + m_\Lambda + m_{\mathrm{K}}\right)c^2 \\
&= 938 \text{ MeV } + 1116 \text{ MeV } + 494 \text{ MeV } = 2548 \text{ MeV}
\end{aligned}
$$

Rearranging we have

$$\frac{E_{\mathrm{cm}}^2}{2E_0} = 2E_0 + K$$

$$K = \frac{E_{\mathrm{cm}}^2}{2E_0} - 2E_0 = \frac{\left(2548 \text{ MeV}\right)^2}{2\left(938 \text{ MeV}\right)} - 2\left(938 \text{ MeV}\right) = 1585 \text{ MeV}$$

b) In a colliding beam experiment the total momentum is zero, and we have by conservation of energy

$$2E_0 + 2K = E_0 + \left(m_\Lambda + m_{\mathrm{K}}\right)c^2$$

$$K = \frac{-E_0 + \left(m_\Lambda + m_{\mathrm{K}}\right)c^2}{2} = \frac{-938 \text{ MeV } + 1116 \text{ MeV } + 494 \text{ MeV}}{2} = 336 \text{ MeV}$$

32. a) strangeness is not conserved (changes by 2 units)
b) allowed if neutrinos are added to conserve lepton number
c) allowed (strangeness changes by one unit)
d) baryon number is not conserved
e) baryon number and μ lepton number are not conserved

33. a) allowed
b) ν_e should be $\overline{\nu}_e$ to conserve electron lepton number
c) strangeness is not conserved (changes by 2 units)
d) allowed (strangeness changes by one unit)
e) allowed

*34. a) baryon number and electron lepton number not conserved
b) allowed c) allowed d) allowed

*35. a) allowed (strangeness changes by one unit)
b) charge and strangeness are not conserved
c) baryon number is not conserved
d) allowed (strangeness changes by one unit)

36. a) $\lambda = h/p$ and $p = \sqrt{2mK}$, so

$$\lambda = \frac{h}{\sqrt{2mK}}$$

b) $p \cong E/c \cong K/c$, so

$$\lambda = \frac{hc}{K}$$

37. a) From Problem 26 we have

$$E_{\rm cm} = \sqrt{\left(m_1c^2 + m_2c^2\right)^2 + 2Km_2c^2}$$

and with $m_1 = m_2 = m$ and $K >> mc^2$, we have

$$E_{\rm cm} \cong \sqrt{2Kmc^2} = \sqrt{2\,(0.938\text{ GeV})\,(7000\text{ GeV})} = 114.5\text{ GeV}$$

b) For colliding beams the available energy is the sum of the two beam energies, or 14 TeV. This is an improvement over the fixed-target result by a factor of

$$\frac{14000\text{ GeV}}{114.5\text{ GeV}} = 122$$