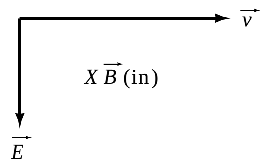
Chapter 3

1. The required field is homogeneous within the desired region and decreases in magnitude to zero as rapidly as possible outside that region. The magnitude of the field is $B = E/v_0$. The best design is an electromagnet with flat, parallel pole faces that are large compared with the distance between them. But no matter what the design, it is impossible to eliminate edge effects.

2. $eE = evB$



$B = E/v = (2 \times 10^5 \text{ V/m})/(2 \times 10^6 \text{ m/s}) = 0.10 \text{ T}$

3. Assume the speed is exact. Non-relativistically use the energy $eV = \frac{1}{2}mv^2$

$$V = \frac{mv^2}{2e} = \frac{(9.1094 \times 10^{-31} \text{ kg})\,(2.00 \times 10^7 \text{ m/s})^2}{2\,(1.6022 \times 10^{-19} \text{ C})} = 1137.1 \text{ V}$$

Relativistically $eV = K = (\gamma - 1)mc^2$

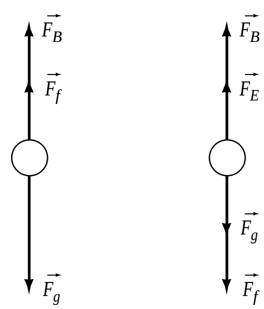
$$V = \left[\frac{1}{\sqrt{1 - \left(\frac{2.00\times10^7 \text{ m/s}}{2.9979\times10^8 \text{ m/s}}\right)^2}} - 1\right]\left[\frac{511 \text{ keV}}{e}\right] = 1141.0 \text{ V}$$

The results differ by about 4 volts, or about 0.34%. Relativity is required only if that level of precision is needed.

*4. $eE = evB$ so $E = vB = (5.0 \times 10^6 \text{ m/s})\,(1.3 \times 10^{-2} \text{ T}) = 6.50 \times 10^4 \text{ V/m}$

$$\begin{aligned} y &= \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{l}{v_0}\right)^2 = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{l}{v_0}\right)^2 = \frac{eEl^2}{2mv_0^2} \\ &= \frac{(1.602 \times 10^{-19} \text{ C})\,(6.50 \times 10^4 \text{ V/m})\,(0.02 \text{ m})^2}{2\,(9.109 \times 10^{-31} \text{ kg})\,(5.0 \times 10^6 \text{ m/s})^2} = 9.1452 \times 10^{-2} \text{ m} \; = 9.15 \text{ cm} \end{aligned}$$

5.



(a) **(b)**

6. At terminal velocity the net force is zero, so $F_f = fv_t = mg$ and $v_t = mg/f$.

*7.

$$v_t = \frac{mg}{f} = \frac{mg}{6\pi\eta r} \qquad m = \rho\,(\text{volume}) = \frac{4}{3}\pi\rho r^3$$

$$v_t = \left(\frac{4}{3}\pi\rho r^3\right)\left(\frac{g}{6\pi\eta r}\right) = \frac{2g\rho r^2}{9\eta}$$

Solving for r

$$r = 3\sqrt{\frac{\eta v_t}{2g\rho}}$$

8. a)

$$r = 3\sqrt{\frac{\eta v_t}{2g\rho}} = 3\sqrt{\frac{(1.82\times10^{-5}\ \text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1})\,(1.3\times10^{-3}\ \text{m/s})}{2\,(9.80\ \text{m/s}^2)\,(900\ \text{kg/m}^3)}} = 3.47\mu\text{m}$$

b)

$$m = \rho V = \frac{4}{3}\pi\rho r^3 = \frac{4}{3}\pi\left(900\ \text{kg/m}^3\right)\left(3.47\times10^{-6}\ \text{m}\right)^3 = 1.58\times10^{-13}\ \text{kg}$$

c)

$$f = \frac{mg}{v_t} = \frac{(1.58\times10^{-13}\ \text{kg})\,(9.80\ \text{m/s}^2)}{1.3\times10^{-3}\ \text{m/s}} = 1.19\times10^{-9}\ \text{kg/s}$$

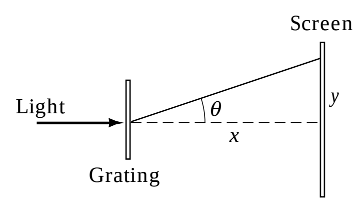
*9. Lyman:

$$\lambda = \left[R_H\left(1-\frac{1}{\infty^2}\right)\right]^{-1} = R_H^{-1} = \left(1.096776\times10^7\ \text{m}^{-1}\right)^{-1} = 91.2\ \text{nm}$$

Balmer:

$$\lambda = \left[R_H\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right)\right]^{-1} = 4R_H^{-1} = 4\left(1.096776\times10^7\ \text{m}^{-1}\right)^{-1} = 364.7\ \text{nm}$$

10. $d = (400\ \text{mm}^{-1})^{-1} = 2.5\,\mu\text{m}$ and $\lambda = d\sin\theta$ in first order. Also $\tan\theta = y/x$ so $y = x\tan\theta = (2.5\ \text{m})\tan\left[\sin^{-1}(\lambda/d)\right]$



Red:

$$y = (2.5\ \text{m})\tan\left[\sin^{-1}\left(\frac{656.5\ \text{nm}}{2500\ \text{nm}}\right)\right] = 68.0\ \text{cm}$$

Blue-green:

$$y = (2.5\text{ m})\tan\left[\sin^{-1}\left(\frac{486.3\text{ nm}}{2500\text{ nm}}\right)\right] = 49.6\text{ cm}$$

Violet:

$$y = (2.5\text{ m})\tan\left[\sin^{-1}\left(\frac{434.2\text{ nm}}{2500\text{ nm}}\right)\right] = 44.1\text{ cm}$$

11. $d = \left(420\text{ mm}^{-1}\right)^{-1} = 2.381\,\mu\text{m}$ $\lambda = 656.5$ nm for red
For $n = 1$ (first order) we have $n\lambda = d\sin\theta$ so $\theta = \sin^{-1}(\lambda/d)$

$$y = x\tan\theta = (2.5\text{ m})\tan\left[\sin^{-1}\left(\frac{656.5\text{ nm}}{2381\text{ nm}}\right)\right] = 71.7\text{ cm}$$

Similarly for $n = 2$ we find $y = 165.3$ cm and for $n = 3$ we find $y = 368.0$ cm. Therefore the separations are: between $n = 1$ and $n = 2$, $\Delta y = 165.3\text{ cm} - 71.7\text{ cm} = 93.6$ cm; between $n = 2$ and $n = 3$, $\Delta y = 368.0\text{ cm} - 165.3\text{ cm} = 202.7$ cm.

12. a) To get a charge of +1 with three quarks requires two charges of $+2e/3$ and one of charge $-e/3$.

b) To get a charge of zero we could have either two $+e/3$ and one $-2e/3$ or one $+2e/3$ and two $-e/3$. At this point in the text there is no reason to prefer either choice (the latter turns out to be correct).

13. a)

$$\lambda_{\text{max}} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{4.2\text{ K}} = 0.69\text{ mm}$$

b)

$$\lambda_{\text{max}} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{293\text{ K}} = 9.89\ \mu\text{m}$$

c)

$$\lambda_{\text{max}} = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{2500\text{ K}} = 1.16\,\mu\text{m}$$

14. a)

$$T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{10^{-14}\text{ m}} = 2.898\times10^{11}\text{ K}$$

b)

$$T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{10^{-9}\text{ m}} = 2.898\times10^{6}\text{ K}$$

c)

$$T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{670\times10^{-9}\text{ m}} = 4325\text{ K}$$

d)

$$T = \frac{2.898\times10^{-3}\text{ m}\cdot\text{K}}{1\text{ m}} = 2.898\times10^{-3}\text{ K}$$

e)

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{204\ \mathrm{m}} = 1.42 \times 10^{-5}\ \mathrm{K}$$

*15.

$$\frac{P_1}{P_0} = \frac{\sigma T_1^4}{\sigma T_0^4} \qquad \text{so} \qquad P_1 = P_0 \frac{T_1^4}{T_0^4} = \left(\frac{1900\ \mathrm{K}}{900\ \mathrm{K}}\right)^4 P_0 = 19.9\ P_0$$

The power increases by a factor of 19.9.

16. Let $hc/\lambda kT = x$. For $\lambda >> hc/kT$ we have $x << 1$. Then using a Taylor series

$$e^x = 1 + x + \frac{x^2}{2} + ... \cong 1 + x$$

because $x << 1$. Then $e^x - 1 \cong x$ and

$$\mathcal{I} = \frac{2\pi c^2 h/\lambda^5}{e^x - 1} \cong \frac{2\pi c^2 h}{\lambda^5 x} = \frac{2\pi c^2 h}{\lambda^5} \frac{\lambda kT}{hc} = \frac{2\pi ckT}{\lambda^4}$$

*17.

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{3000\ \mathrm{K}} = 966\ \mathrm{nm}$$

which is in the near infrared.

18. The graph is a characteristic Planck law curve with a maximum at $\lambda = 966$ nm (see Problem 17).

a) Numerical integration of the $\mathcal{I}(\lambda, T)$ function shows that approximately 8.1% of the radiated power is between 400 nm and 700 nm. Details of the calculation are:

$$\begin{aligned} 2\pi c^2 h \int_{4\times10^{-7}}^{7\times10^{-7}} \exp\left(-\frac{hc}{\lambda kT}\right) \lambda^{-5}\, d\lambda &= \left(3.74 \times 10^{-16}\right) \int_{4\times10^{-7}}^{7\times10^{-7}} \exp\left(-\frac{4.796 \times 10^{-6}}{\lambda}\right) \lambda^{-5}\, d\lambda \\ &= 3.71 \times 10^5\ \mathrm{W/m^2} \end{aligned}$$

That is the power per unit area emitted over visible wavelengths. Over all wavelengths we know the power per unit area is $R = \sigma T^4 = 4.59 \times 10^6\ \mathrm{W/m^2}$. Therefore the fraction emitted in the visible is

$$\frac{3.7128 \times 10^5\ \mathrm{W/m^2}}{4.59 \times 10^6\ \mathrm{W/m^2}} = 0.081$$

b) Using computed numerical intensity values

$$\frac{\mathcal{I}(400\ \mathrm{nm}, T)}{\mathcal{I}(966\ \mathrm{nm}, T)} \cong 0.073$$

$$\frac{\mathcal{I}(700 \text{ nm}, T)}{\mathcal{I}(966 \text{ nm}, T)} \cong 0.754$$

19. In this limit $\exp(hc/\lambda kT) >> 1$ so

$$\mathcal{I}(\lambda, T) \cong \frac{2\pi c^2 h}{\lambda^5} \exp\left(-\frac{hc}{\lambda kT}\right)$$

The exponential goes to zero faster than λ^5, so the intensity approaches zero in this limit.

20. a) At this temperature the power per unit area is

$$R = \sigma T^4 = \left(5.68 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}\right)(293 \text{ K})^4 = 419 \text{ W/m}^2$$

For the basketball, a sphere of radius $r = 12.5$ cm, we get

$$P = R\left(4\pi r^2\right) = \left(419 \text{ W/m}^2\right)(4\pi)(0.125 \text{ m})^2 = 82.3 \text{ W}$$

b) At this temperature the power per unit area is

$$R = \sigma T^4 = \left(5.68 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}\right)(310 \text{ K})^4 = 525 \text{ W/m}^2$$

For the human body assume a rectangular solid roughly 1.7 m by 0.3 m by 0.2 m, so the net area of the six surfaces is $2\left[(1.7 \text{ m})(0.3\text{m}) + (1.7 \text{ m})(0.2 \text{ m}) + (0.3 \text{ m})(0.2 \text{ m})\right] = 1.82 \text{ m}^2$. Then

$$P = 525 \text{ W/m}^2\left(1.82 \text{ m}^2\right)$$

or about 1000 W. Numerical values will vary depending on estimates of the human body size.

21.

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{310 \text{ K}} = 9.35 \ \mu\text{m}$$

*22. Taking derivatives

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\partial^2 a}{\partial t^2} \sin\left(\frac{n\pi x}{L}\right) \qquad \frac{\partial^2 \psi}{\partial x^2} = -a\frac{n^2\pi^2}{L^2} \sin\left(\frac{n\pi x}{L}\right)$$

Substituting these values into the wave equation produces

$$\frac{1}{c^2}\frac{\partial^2 a}{\partial t^2} \sin\left(\frac{n\pi x}{L}\right) - \left(-a\frac{n^2\pi^2}{L^2} \sin\left(\frac{n\pi x}{L}\right)\right) = 0$$

$$\frac{\partial^2 a}{\partial t^2} = -a\frac{n^2\pi^2 c^2}{L^2} = -\Omega^2 a$$

where $\Omega = nhc/L$.

23. Let $r = \sqrt{n_x^2 + n_y^2 + n_z^2}$ be the radius of a three-dimensional number space with the n_i the three components of that space. Then let dN be the number of allowed states between r and $r + dr$. This corresponds to the number of points in a spherical shell of number space, which is

$$dN = \frac{1}{8}\left(4\pi r^2\right)\, dr$$

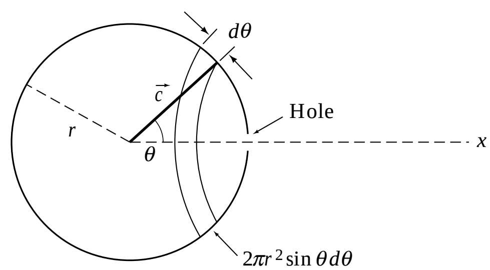
where we have used the fact that $4\pi r^2\, dr$ is the "volume" of the shell (area $4\pi r^2$ by thickness dr), and the 1/8 is due to the fact that only positive numbers n_i are allowed, so only 1/8 of the space is available. Therefore

$$r^2 = n_x^2 + n_y^2 + n_z^2 = \frac{\Omega^2 L^2}{\pi^2 c^2} = \frac{4L^2\nu^2}{c^2}$$

or $r = 2L\nu/c$. Then from this $dr = (2L/c)\, d\nu$. Putting everything together:

$$dN = \frac{1}{8}\left(4\pi r^2\right)\, dr = \frac{\pi}{2} r^2\, dr = \frac{\pi}{2}\left(\frac{2L\nu}{c}\right)^2 \frac{2L}{c}\, d\nu = \frac{4\pi L^3 \nu^2}{c^3}\, d\nu$$

24. From the diagram at right we see that the average x-component of the velocity (c) of electromagnetic radiation within the cavity is



$$\langle c_x \rangle = \frac{\int_0^{\pi/2} (c\, \cos\theta)\; 2\pi r^2\, \sin\theta\, d\theta}{\int_0^{\pi/2} 2\pi r^2\, \sin\theta\, d\theta}$$

Letting $u = \cos\theta$ we have

$$\langle c_x \rangle = \frac{c\int_0^1 u\, du}{\int_0^1 du} = \frac{c}{2}$$

On average only one-half of the photons are traveling to the right. Thus the mean velocity of photons traveling to the right is $c/4$. Therefore

$$\text{power} = \text{(intensity)}\,\text{(area)} = \frac{c}{4}\, dU\; (\Delta A)$$

25. For classical oscillators the Maxwell-Boltzmann distribution gives

$$n(E) = A \exp\left(-E/kT\right) = A\exp\left(-\beta E\right)$$

where $E = nh\nu$ and $\beta = 1/kT$. The mean energy is

$$\overline{E} = \frac{\sum_{n=0}^{\infty} E\, n(E)}{\sum_{n=0}^{\infty} n(E)} = \frac{\sum_{n=0}^{\infty} nh\nu \exp\left(-\beta n h\nu\right)}{\sum_{n=0}^{\infty} \exp\left(-\beta n h\nu\right)}$$

Notice that

$$\overline{E} = \frac{\partial}{\partial \beta} \ln \sum_{n=0}^{\infty} \exp\left(-\beta n h\nu\right)$$

Now letting $x = \exp\left(-\beta h\nu\right)$ we see that by Taylor series

$$\sum_{n=0}^{\infty} \exp\left(-\beta n h\nu\right) = 1 + x + x^2 + ... = \left(1 - x\right)^{-1}$$

$$\overline{E} = \frac{\partial}{\partial \beta} \ln\left(1 - x\right)^{-1} = -\frac{\partial}{\partial \beta} \ln\left(1 - x\right) = \frac{h\nu \exp\left(-\beta h\nu\right)}{1 - \exp\left(-\beta h\nu\right)}$$

$$\overline{E} = \frac{h\nu}{\exp\left(h\nu/kT\right) - 1}$$

Using the result of Problem 23 (along with a factor of 2 for two photon polarizations) we can see that

$$U(\nu, T) = 2\frac{4\pi}{c^3}\,\nu^2\,\frac{h\nu}{\exp\left(h\nu/kT\right) - 1} = \frac{8\pi h\nu^3/c^3}{\exp\left(h\nu/kT\right) - 1}$$

To change from U to $\mathcal{I}$ requires the factor $c/4$ (Problem 24), and changing from a frequency distribution requires a factor c/λ^2 (because with $\nu = c/\lambda$ we have $|d\nu| = \left(c/\lambda^2\right)\, d\lambda$). Putting these together

$$\mathcal{I}(\lambda, T) = \frac{8\pi h/\lambda^3}{\exp\left(h\nu/kT\right) - 1}\frac{c}{\lambda^2}\frac{c}{4} = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{\exp\left(hc/\lambda kT\right) - 1}$$

*26.

$$\text{energy per photon } = h\nu = \left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(107.7 \times 10^6\ \text{s}^{-1}\right) = 7.14 \times 10^{-26}\ \text{J}$$

$$\left(5.0 \times 10^4\ \text{J/s}\right)\frac{1\ \text{photon}}{7.14 \times 10^{-26}\ \text{J}} = 7.00 \times 10^{29}\ \text{photons/s}$$

27. a)

$$\text{energy per photon} = h\nu = \left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(1100 \times 10^3\ \text{s}^{-1}\right) = 7.29 \times 10^{-28}\ \text{J}$$

$$(150\ \text{J/s})\,\frac{1\ \text{photon}}{7.29 \times 10^{-28}\ \text{J}} = 2.06 \times 10^{29}\ \text{photons/s}$$

b)

$$\text{energy per photon } = h\frac{c}{\lambda} = \left(6.626 \times 10^{-34}\ \text{J} \cdot \text{s}\right)\left(\frac{3.00 \times 10^8\ \text{m/s}}{8 \times 10^{-9}\ \text{m}}\right) = 2.48 \times 10^{-17}\ \text{J}$$

$$(150\ \text{J/s})\,\frac{1\ \text{photon}}{2.48 \times 10^{-17}\ \text{J}} = 6.05 \times 10^{18}\ \text{photons/s}$$

c)

$$(150\ \text{J/s})\,\frac{1\ \text{photon}}{4\ \text{MeV}}\,\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}} = 2.34 \times 10^{14}\ \text{photons/s}$$

28.

$$\nu_t = \frac{\phi}{h} = \frac{2.9 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 7.01 \times 10^{14} \text{ Hz}$$

$$eV_0 = \frac{hc}{\lambda} - \phi \qquad \text{so} \qquad V_0 = \frac{1}{e}\left[\frac{hc}{\lambda} - \phi\right]$$

$$V_0 = \frac{1}{e}\left[\frac{1240 \text{ eV} \cdot \text{nm}}{400 \text{ nm}} - 2.9 \text{ eV}\right] = 0.20 \text{ eV}$$

*29.

$$\lambda_t = \frac{hc}{\phi} = \frac{1240 \text{ eV} \cdot \text{nm}}{4.7 \text{ eV}} = 264 \text{ nm}$$

If the wavelength is halved (to $\lambda = 132$ nm)

$$K = \frac{hc}{\lambda} - \phi = \frac{1240 \text{ eV} \cdot \text{nm}}{132 \text{ nm}} - 4.7 \text{ eV} = 4.7 \text{ eV}$$

30. Notice that $hc/\lambda = 2.34 \text{ eV} > \phi$, so photoelectrons will be produced.

$$\left(2 \times 10^{-3} \text{ J/s}\right)\left(10^{-5}\right)\left(\frac{1 \text{ photoelectron}}{2.34 \text{ eV}}\right)\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right)\left(\frac{1.60 \times 10^{-19} \text{ C}}{\text{electron}}\right) = 8.55 \text{ nA}$$

31.

$$\phi = \frac{hc}{\lambda_t} = \frac{1240 \text{ eV} \cdot \text{nm}}{230 \text{ nm}} = 5.39 \text{ eV}$$

$$K = 2.0 \text{ eV} = h\nu - \phi$$

$$\nu = \frac{K + \phi}{h} = \frac{2.0 \text{ eV} + 5.39 \text{ eV}}{4.136 \times 10^{-15} \text{ eV} \cdot \text{s}} = 1.79 \times 10^{15} \text{ Hz}$$

32.

$$E = 100\left(\frac{hc}{\lambda}\right) = 100\frac{1240 \text{ eV} \cdot \text{nm}}{580 \text{ nm}} = 214 \text{ eV}$$

33. $eV_{01} = hc/\lambda_1 - \phi$ and $eV_{02} = hc/\lambda_2 - \phi$. Subtracting these equations and rearranging we find

$$h = \frac{e\left(V_{02} - V_{01}\right)}{c\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)} = \frac{e\left(2.3 \text{ V} - 1.0 \text{ V}\right)}{\left(3.00 \times 10^8 \text{ m/s}\right)\left(\frac{1}{207 \text{ nm}} - \frac{1}{260 \text{ nm}}\right)} = 4.40 \times 10^{-15} \text{ eV} \cdot \text{s}$$

This is about 6% from the accepted value. For the work function we use the first set of data (the second set should give the same result):

$$\phi = \frac{hc}{\lambda_1} - eV_{01} = \frac{\left(4.40 \times 10^{-15} \text{ eV} \cdot \text{s}\right)\left(3.00 \times 10^8 \text{ m/s}\right)}{260 \times 10^{-9} \text{ m}} - 1.0 \text{ eV} = 4.1 \text{ eV}$$

34. For 400 nm:

$$E = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{400\text{ nm}} = 3.10\text{ eV} \qquad \nu = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{400\text{ nm}} = 7.50\times10^{14}\text{ Hz}$$

For 700 nm:

$$E = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{700\text{ nm}} = 1.77\text{ eV} \qquad \nu = \frac{c}{\lambda} = \frac{3.00\times10^{8}\text{ m/s}}{700\text{ nm}} = 4.29\times10^{14}\text{ Hz}$$

35.

$$\lambda_{\text{min}} = \frac{1.24\times10^{-6}\text{ V}\cdot\text{m}}{V} = \frac{1.24\times10^{-6}\text{ V}\cdot\text{m}}{30\text{ kV}} = 0.0413\text{ nm}$$

36.

$$\lambda = \frac{hc}{K} = \frac{1240\text{ eV}\cdot\text{nm}}{5\times10^{10}\text{ eV}} = 2.48\times10^{-17}\text{ m}$$

A photon produced by bremsstrahlung is still an x ray, even though this falls outside the normal range for x rays.

*37.

$$\lambda = \frac{1240\text{ eV}\cdot\text{nm}}{2\times10^{4}\text{ eV}} = 0.0620\text{ nm}$$

38.

$$\Delta\lambda = \frac{h}{mc}(1-\cos\theta)$$

so at maximum $\cos\theta = -1$ and

$$\frac{\Delta\lambda}{\lambda} = \frac{2h}{\lambda mc} = \frac{2hc}{mc^2\lambda} = \frac{2\,(1240\text{ eV}\cdot\text{nm})}{(511.0\text{ keV})\,(530\text{ nm})} = 9.16\times10^{-6}$$

This corresponds to $\Delta\lambda \cong 5\times10^{-12}$ m and therefore is not easily observed.

39. The maximum change in the photon's energy is obtained in backscattering ($\theta = 180°$), so $1-\cos\theta = 2$ and $\Delta\lambda = 2h/mc = 4.853\times10^{-12}$ m. The photon's original wavelength was

$$\lambda = \frac{hc}{E} = \frac{1240\text{ eV}\cdot\text{nm}}{40000\text{ eV}} = 0.0310\text{ nm} = 3.10\times10^{-11}\text{ m}$$

and the new wavelength is $\lambda' = \lambda + \Delta\lambda = 3.586\times10^{-11}$ m. The electron's recoil energy equals the change in the photon's energy, or

$$K = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{1240 \text{ eV} \cdot \text{nm}}{3.10 \times 10^{-2} \text{ nm}} - \frac{1240 \text{ eV} \cdot \text{nm}}{3.586 \times 10^{-2} \text{ nm}} = 5420 \text{ eV} \;\; = 5.42 \text{ keV}$$

40. Use the Compton scattering formula but with the proton's mass:

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta) = \frac{h}{mc} = \frac{hc}{mc^2} = \frac{1240 \text{ eV} \cdot \text{nm}}{938.27 \text{ MeV}} = 1.32 \text{ fm}$$

41.

$$\lambda_c = \frac{h}{mc} = \frac{hc}{mc^2} = \frac{1240 \text{ eV} \cdot \text{nm}}{938.27 \text{ MeV}} = 1.32 \text{ fm}$$

The photon energy is

$$E = \frac{hc}{\lambda_c} = 938 \text{ MeV}$$

In principle this could be observed, but the energy requirements are high.

*42.

$$\frac{\Delta\lambda}{\lambda} = 0.004 = \frac{\lambda_c}{\lambda}(1 - \cos\theta) \qquad \text{so} \qquad \lambda = 250\lambda_c(1 - \cos\theta)$$

a)

$$\lambda = 250\left(2.43 \times 10^{-12} \text{ m}\right)(1 - \cos 30°) = 8.14 \times 10^{-11} \text{ m}$$

b)

$$\lambda = 250\left(2.43 \times 10^{-12} \text{ m}\right)(1 - \cos 90°) = 6.08 \times 10^{-10} \text{ m}$$

c)

$$\lambda = 250\left(2.43 \times 10^{-12} \text{ m}\right)(1 - \cos 170°) = 1.21 \times 10^{-9} \text{ m}$$

43. By conservation of energy we know the electron's recoil energy equals the energy lost by the photon:

$$K = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc(\lambda' - \lambda)}{\lambda'\lambda} = \frac{hc\,\Delta\lambda}{\lambda'\lambda}$$

Using $\lambda' = \lambda + \Delta\lambda$

$$K = \frac{hc\,\Delta\lambda}{\lambda(\lambda + \Delta\lambda)} = \frac{h\nu\,\Delta\lambda}{\lambda + \Delta\lambda} = \frac{h\nu\,\Delta\lambda}{\lambda(1 + \Delta\lambda/\lambda)}$$

Conservation of p_x:

$$p_e\cos\phi + \frac{h}{\lambda'}\cos\theta = \frac{h}{\lambda}$$

$$p_e\cos\phi = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta \qquad (1)$$

Conservation of p_y:

$$p_e \sin\phi - \frac{h}{\lambda'}\sin\theta = 0$$

$$p_e \sin\phi = \frac{h}{\lambda'}\sin\theta \qquad (2)$$

Dividing equation (2) by equation (1)

$$\tan\phi = \frac{\frac{h}{\lambda'}\sin\theta}{\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta}$$

Using $\lambda' = \lambda + \frac{h}{mc}(1-\cos\theta)$

$$\tan\phi = \frac{\frac{h\sin\theta}{\lambda + \frac{h}{mc}(1-\cos\theta)}}{\frac{h}{\lambda} - \frac{h\cos\theta}{\lambda + \frac{h}{mc}(1-\cos\theta)}}$$

Multiplying above and below by $\lambda\left[\lambda + \frac{h}{mc}(1-\cos\theta)\right]$,

$$\tan\phi = \frac{\lambda h \sin\theta}{\lambda h + \frac{h^2}{mc}(1-\cos\theta) - \lambda h\cos\theta} = \frac{\lambda\sin\theta}{\left(\lambda + \frac{h}{mc}\right)(1-\cos\theta)}$$

Trig identity: $(\sin\theta)\,/\,(1-\cos\theta) = \cot(\theta/2)$

$$\tan\phi = \frac{\lambda}{\lambda + \frac{h}{mc}}\cot\left(\frac{\theta}{2}\right) = \frac{1}{1+\frac{h}{mc\lambda}}\cot\left(\frac{\theta}{2}\right) = \frac{1}{1+\frac{h\nu}{mc^2}}\cot\left(\frac{\theta}{2}\right)$$

Inverting

$$\cot\phi = \left[1 + \frac{h\nu}{mc^2}\right]\tan\left(\frac{\theta}{2}\right)$$

44.

$$\lambda' = \lambda + \lambda_c(1-\cos\theta) = \frac{hc}{E} + \lambda_c(1-\cos\theta)$$

$$\lambda' = \frac{1240\text{ eV}\cdot\text{nm}}{700\times10^3\text{ eV}} + \left(2.43\times10^{-3}\text{ nm}\right)(1-\cos 110^\circ) = 5.03\text{ pm}$$

$$E' = \frac{hc}{\lambda'} = \frac{1240\text{ eV}\cdot\text{nm}}{5.03\times10^{-3}\text{ nm}} = 2.47\times10^5\text{ eV} = 247\text{ keV}$$

By conservation of energy

$K_e = E - E' = 700\text{ keV} - 247\text{ keV} = 453\text{ keV}$ (agrees with K formula in the previous problem)

From Problem 43

$$\cot\phi = \left[1 + \frac{h\nu}{mc^2}\right]\tan\left(\frac{\theta}{2}\right) = \left[1 + \frac{700\text{ keV}}{511\text{ keV}}\right]\tan\left(\frac{110^\circ}{2}\right) = 3.3845$$

$$\phi = 16.5^\circ$$

*45. For $\theta = 90^\circ$ we know $\lambda' = \lambda + \lambda_c = 2.00243$ nm

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_c}{\lambda} = \frac{2.43 \times 10^{-3}\ \text{nm}}{2\ \text{nm}} = 1.22 \times 10^{-3} = 0.122\%$$

*46.

$$E = 2mc^2 = 2\,(938.3\ \text{MeV}) = 1877\ \text{MeV}$$

This energy could come from a particle accelerator.

47. a) To find the minimum energy consider the zero-momentum frame. Let E_e be the energy of the electron in that frame, and E_0 is the rest energy of the electron. From conservation of energy and momentum:

$$\text{momentum: } \frac{h\nu}{c} = p_e = \frac{\sqrt{E_e^2 - E_0^2}}{c} \quad \text{or} \quad h\nu = \sqrt{E_e^2 - E_0^2}$$

$$\text{energy: } h\nu + E_e = 3E_0 \quad \text{or} \quad h\nu = 3E_0 - E_e$$

Squaring and subtracting these two equations gives

$$0 = -10E_0^2 + 6E_eE_0 \quad \text{or} \quad E_e = \frac{5}{3}E_0$$

This tells us that for the transformation form the lab frame to the zero-momentum frame, $\gamma = 5/3$ and $v = 0.8c$. Then from the momentum equation we have in the zero-momentum frame

$$h\nu = \sqrt{\frac{25E_0^2}{9} - E_0^2} = \frac{4}{3}E_0$$

In the lab the photon's energy is obtained using a Doppler shift:

$$h\nu_{\text{lab}} = h\nu\sqrt{\frac{1+\beta}{1-\beta}} = \frac{4}{3}E_0\sqrt{\frac{1+0.8}{1-0.8}} = 4E_0 = 2.04\ \text{MeV}$$

b) The proton's rest energy is Mc^2. Now as in (a) we let the proton's energy in the lab frame be E_p and conservation of momentum and energy give

$$\text{momentum: } h\nu = \sqrt{E_p^2 - (Mc^2)^2}$$

$$\text{energy: } h\nu + E_p = 2E_0 + Mc^2$$

Squaring and subtracting, we find

$$E_p = \frac{(Mc^2)^2 + 2E_0^2 + 2E_0Mc^2}{2E_0 + Mc^2}$$

This is very close to $E_p = Mc^2$ Therefore the zero-momentum and lab frames are equivalent, and we conclude $h\nu_{\text{lab}} \cong 2E_0 = 1.02$ MeV.

48. The maximum energy transfer occurs when $\theta = 180°$ so that $\Delta\lambda = (h/mc)(1-\cos\theta) = 2h/mc$. By conservation of energy the kinetic energy of the electron is

$$K = E - E' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc}{\lambda} - \frac{hc}{\lambda + \Delta\lambda}$$

Multiplying through by $\lambda(\lambda + \Delta\lambda)$ we find

$$\lambda(\lambda + \Delta\lambda)K = hc(\lambda + \Delta\lambda) - hc\lambda = hc\,\Delta\lambda$$

$$\lambda^2 K + \lambda\,\Delta\lambda K - hc\Delta\lambda = 0$$

This is a quadratic equation that with numerical values can be solved for λ to find $\lambda = 1.20 \times 10^{-11}$ m. Then

$$E = \frac{hc}{\lambda} = \frac{1.24 \text{ keV} \cdot \text{nm}}{1.20 \times 10^{-2} \text{ nm}} = 104 \text{ keV}$$

49. To find the asteroid mass m note that the earth (matter) would supply an equal mass m to the process, so

$$2mc^2 = \frac{GM_E^2}{2R_E}$$

$$m = \frac{GM_E^2}{4R_E c^2} = \frac{(6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})\,(5.98 \times 10^{24} \text{ kg})^2}{4\,(6378 \times 10^3 \text{ m})\,(3.00 \times 10^8 \text{ m/s})^2} = 1.04 \times 10^{15} \text{ kg}$$

Then

$$r = \left[\frac{3m}{4\pi\rho}\right]^{1/3} = \left(\frac{3\,(1.04 \times 10^{15} \text{ kg})}{4\pi\,(5000 \text{ kg/m}^3)}\right)^{1/3} = 3.68 \text{ km}$$

which is pretty small. Evaluating the energy:

$$E = \frac{GM_E^2}{2R_e} = \frac{(6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})\,(5.98 \times 10^{24} \text{ kg})^2}{2\,(6378 \times 10^3 \text{ m})} = 1.87 \times 10^{32} \text{ J}$$

$$\frac{E}{\text{nuclear arsenals}} = \frac{1.87 \times 10^{32} \text{ J}}{2000\,(4.2 \times 10^{15} \text{ J})} = 2 \times 10^{13}$$

There is a lot of energy in the annihilation process!

*50. For maximum recoil energy the scattering angle is $\theta = 180°$ and $\phi = 0$. Then as usual $\Delta\lambda = 2h/mc$. Using the result of Problem 43

$$K = \frac{\Delta\lambda/\lambda}{1 + \Delta\lambda/\lambda}\,h\nu = \frac{2h/mc\lambda}{1 + 2h/mc\lambda}\,h\nu = \frac{2h\nu/mc^2}{1 + 2h\nu/mc^2}\,h\nu$$

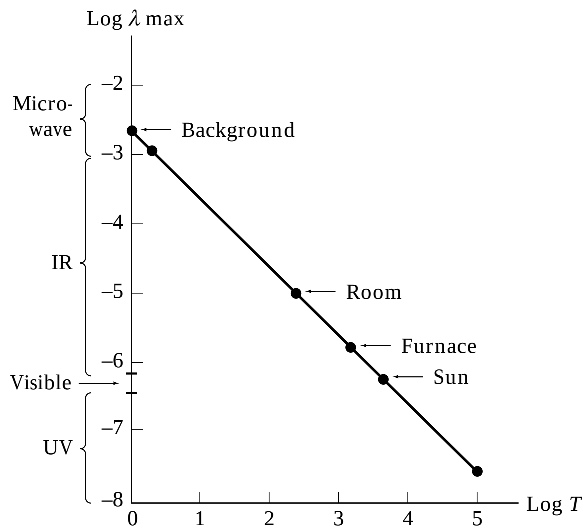
For the given value of $K = 100$ keV we can solve this equation:

$$K\left(1 + \frac{2h\nu}{mc^2}\right) = \frac{2\,(h\nu)^2}{mc^2}$$

$$\left(\frac{2}{mc^2}\right)(h\nu)^2 - \left(\frac{2K}{mc^2}\right)(h\nu) - K = 0$$

This constitutes a quadratic equation in $h\nu$ which can be solved numerically to yield $h\nu = 217$ keV.

51. See graph below.



52. a) For $\theta = 180°$ we have $\lambda' - \lambda = 2h/mc$. Therefore

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda + \frac{2h}{mc}}$$

b) With $\lambda = hc/E$ we find

$$E' = \frac{hc}{\frac{hc}{E} + \frac{2h}{mc}} = \frac{1}{\frac{1}{E} + \frac{2}{mc^2}} = \left(\frac{1}{1 \times 10^5 \text{ eV}} + \frac{2}{511000 \text{ eV}}\right)^{-1} = 71.9 \text{ keV}$$