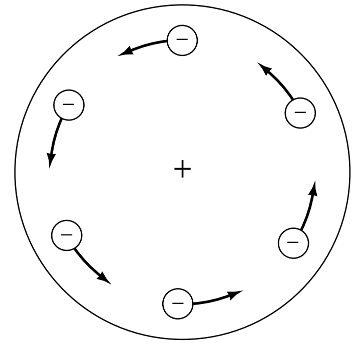
Chapter 4



1. With more than one electron we are almost forced into some kind of Bohr-like orbits. This was the dilemma faced by physicists in the early 20th Century.

2. Non-relativistically $K = \frac{1}{2}mv^2$ and

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\,(7.7\text{ MeV})}{3727\text{ MeV}/c^2}} = 6.4281 \times 10^{-2}c$$

Relativistically $K = (\gamma - 1)\,mc^2$ so $\gamma = 1 + K/mc^2$ and

$$\gamma = 1 + \frac{7.7\text{ MeV}}{3727\text{ MeV}} = 1.002066$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 6.4181 \times 10^{-2}c$$

The difference is about $10^{-4}c$ or about 0.16%.

3. Conserving momentum and energy:

$$M_\alpha v_\alpha = M_\alpha v_\alpha^{'} + m_e v_e^{'} \tag{1}$$

and

$$M_\alpha v_\alpha^2 = M_\alpha v_\alpha^{'2} + m_e v_e^{'2} \tag{2}$$

From (1) we see

$$v_\alpha^{'} = v_\alpha - \frac{m_e}{M_\alpha} v_e^{'}$$

which inserted into (2) gives

$$M_\alpha v_\alpha^2 = M_\alpha \left[v_\alpha - \frac{m_e}{M_\alpha} v_e^{'} \right]^2 + m_e v_e^{'2}$$

$$v_e^{'} \left[1 - \frac{m_e}{M_\alpha} \right] = 2v_\alpha$$

But with $m_e << M_\alpha$ we have $v_e^{'} \cong 2v_\alpha$.

4.

$$P(\theta) = \exp\left(-\frac{80^2}{1^2} \right) = 3 \times 10^{-2780}$$

Therefore multiple scattering does not provide an adequate explanation.

*5. a) With $Z_1 = 2$ and $Z_2 = 79$ we have

$$b = \frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\cot\left(\frac{\theta}{2}\right) = \frac{(2)\,(79)\,(1.44\times10^{-9}\ \mathrm{eV\cdot m})}{2\,(7.7\times10^{6}\ \mathrm{eV})}\cot\left(0.5^\circ\right) = 1.69\times10^{-12}\ \mathrm{m}$$

b) For $\theta = 90^\circ$

$$b = \frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\cot\left(\frac{\theta}{2}\right) = \frac{(2)\,(79)\,(1.44\times10^{-9}\ \mathrm{eV\cdot m})}{2\,(7.7\times10^{6}\ \mathrm{eV})}\cot\left(45^\circ\right) = 1.48\times10^{-14}\ \mathrm{m}$$

*6.

$$f = \pi n t\left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\right)^2\cot^2\left(\frac{\theta}{2}\right)$$

For the two different angles everything is the same except the angles, so

$$\frac{f(1^\circ)}{f(2^\circ)} = \frac{\cot^2(0.5^\circ)}{\cot^2(1.0^\circ)} = 4.00$$

7. The fraction f is proportional to n and to Z^2. Therefore

$$\frac{N(\mathrm{Au})}{N(\mathrm{Al})} = \frac{n(\mathrm{Au})(79)^2}{n(\mathrm{Al})(13)^2}$$

In each case $n = \rho/\left(N_A M\right)$ where ρ is density and M is atomic mass. Thus

$$\frac{N(\mathrm{Au})}{N(\mathrm{Al})} = \frac{\left(\frac{19.3\ \mathrm{g/cm^3}}{197\ \mathrm{g}}\right)(79)^2}{\left(\frac{2.70\ \mathrm{g/cm^3}}{27.0\ \mathrm{g}}\right)(13)^2} = 36.2$$

8. From Example 4.2 we know $n = 5.90\times10^{28}\ \mathrm{m^{-3}}$. Thus

$$\begin{aligned} f &= \pi n t\left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\right)^2\cot^2\left(\frac{\theta}{2}\right) \\ &= \pi\left(5.90\times10^{28}\ \mathrm{m^{-3}}\right)\left(10^{-8}\ \mathrm{m}\right)\left(\frac{(2)\,(79)\,(1.44\times10^{-9}\ \mathrm{eV\cdot m})}{2\,(5\times10^{6}\ \mathrm{eV})}\right)^2\cot(3^\circ) = 1.83\times10^{-5}\end{aligned}$$

9. a) With all other parameters equal the number depends only on the scattering angles, so

$$\frac{f(90^\circ)}{f(50^\circ)} = \frac{\cot^2(45^\circ)}{\cot^2(25^\circ)} = 0.217$$

so the number scattered through angles greater than 90° is $(10000)\,(0.217) = 2170$.

b) Similarly

$$\frac{f(70^\circ)}{f(50^\circ)} = \frac{\cot^2(35^\circ)}{\cot^2(25^\circ)} = 0.4435$$

$$\frac{f(80^\circ)}{f(50^\circ)} = \frac{\cot^2(40^\circ)}{\cot^2(25^\circ)} = 0.3088$$

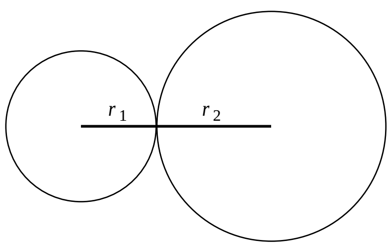
The numbers for the two angles are thus 4435 and 3088 and the number scattered between 70° and 80° is $4435 - 3088 = 1347$.

*10. From the Rutherford scattering result, the number detected through a small angle is inversely proportional to $\sin^4(\theta/2)$. Thus

$$\frac{n(50^\circ)}{n(6^\circ)} = \frac{\sin^4(3^\circ)}{\sin^4(25^\circ)} = 2.35 \times 10^{-4}$$

and if they count 2000 at 6° the number counted at 50° is $(2000)\left(2.35 \times 10^{-4}\right) = 0.47$ which is insufficient.

11. In each case all the kinetic energy is changed to potential energy:



$$K = -V = |V| = \frac{Z_1 Z_2 e^2}{4\pi\varepsilon_0 (r_1 + r_2)}$$

a) $Z_1 = 2$ and $Z_2 = 13$ for Al, $Z_2 = 79$ for Au

$$\text{Al: } K = \frac{(2)(13)\left(1.44 \times 10^{-9}\ \text{eV} \cdot \text{m}\right)}{2.6 \times 10^{-15}\ \text{m} + 3.6 \times 10^{-15}\ \text{m}} = 6.04\ \text{MeV}$$

$$\text{Au: } K = \frac{(2)(79)\left(1.44 \times 10^{-9}\ \text{eV} \cdot \text{m}\right)}{2.6 \times 10^{-15}\ \text{m} + 7.0 \times 10^{-15}\ \text{m}} = 23.7\ \text{MeV}$$

b) Now $Z_1 = 1$ and for the two different values of Z_2

$$\text{Al: } K = \frac{(1)(13)\left(1.44 \times 10^{-9}\ \text{eV} \cdot \text{m}\right)}{1.3 \times 10^{-15}\ \text{m} + 3.6 \times 10^{-15}\ \text{m}} = 3.82\ \text{MeV}$$

$$\text{Au: } K = \frac{(1)(79)\left(1.44 \times 10^{-9}\ \text{eV} \cdot \text{m}\right)}{1.3 \times 10^{-15}\ \text{m} + 7.0 \times 10^{-15}\ \text{m}} = 13.7\ \text{MeV}$$

12. a) The maximum Coulomb force is at the surface and equal to $2Z_2e^2/4\pi\epsilon_0 R^2$. Then

$$\Delta p = F\Delta t = \frac{2Z_2e^2}{4\pi\epsilon_0 R^2}\frac{2R}{v} = \frac{4Z_2e^2}{4\pi\epsilon_0 Rv}$$

For maximum deflection

$$\theta \cong \tan\theta = \frac{\Delta p}{p} = \frac{1}{mv}\frac{4Z_2e^2}{4\pi\epsilon_0 Rv} = \frac{2Z_2e^2}{4\pi\epsilon_0 KR}$$

b)

$$\theta = \frac{(2)(79)(1.44 \text{ eV} \cdot \text{nm})}{(8 \text{ MeV})(0.1 \text{ nm})} = 2.84 \times 10^{-4} \text{ rad } = 0.016^\circ$$

13. a)

$$\theta = \frac{(2)(79)(1.44 \text{ eV} \cdot \text{nm})}{(10 \text{ MeV})(0.1 \text{ nm})} = 2.28 \times 10^{-4} \text{ rad } = 0.013^\circ$$

b) These results are comparable in magnitude with those obtained by electron scattering (Example 4.1).

14. a)

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}} = \frac{ec}{\sqrt{4\pi\epsilon_0 mc^2 r}} = \frac{\sqrt{1.44 \text{ eV} \cdot \text{nm}}}{\sqrt{(511000 \text{ eV})(1.2 \times 10^{-6} \text{ nm})}} c = 1.53c$$

which is not an allowed speed.

b)

$$E = -\frac{e^2}{8\pi\epsilon_0 r} = -\frac{1.44 \text{ eV} \cdot \text{nm}}{2(1.2 \times 10^{-6} \text{ nm})} = -600 \text{ keV}$$

c) Clearly (a) is not allowed and (b) is too much energy.

*15. a)

$$\begin{aligned} v &= \frac{e}{\sqrt{4\pi\epsilon_0 mr}} = \frac{ec}{\sqrt{4\pi\epsilon_0 mc^2 r}} = \frac{\sqrt{1.44 \text{ eV} \cdot \text{nm}}}{\sqrt{(938 \times 10^6 \text{ eV})(0.05 \text{ nm})}} c = 1.75 \times 10^{-4} c \\ &= 5.25 \times 10^3 \text{ m/s} \end{aligned}$$

b)

$$E = -\frac{e^2}{8\pi\epsilon_0 r} = -\frac{1.44 \text{ eV} \cdot \text{nm}}{2(0.05 \text{ nm})} = -14.4 \text{ eV}$$

c) The "nucleus" is too light to be fixed, and there is no way to reconcile this model with the results of Rutherford scattering.

16. For hydrogen:

$$\begin{aligned} v &= \frac{e}{\sqrt{4\pi\varepsilon_0 m r}} = \frac{ec}{\sqrt{4\pi\varepsilon_0 m c^2 r}} = \frac{\sqrt{1.44 \text{ eV}\cdot\text{nm}}}{\sqrt{(511\times10^3 \text{ eV})(0.0529 \text{ nm})}}\,c \\ &= 7.30\times10^{-3}\,c = 2.19\times10^6 \text{ m/s} \end{aligned}$$

$$a = \frac{v^2}{r} = \frac{(2.19\times10^6 \text{ m/s})^2}{5.29\times10^{-11} \text{ m}} = 9.07\times10^{22} \text{ m/s}^2$$

For the hydrogen-like Li^{++}

$$F = \frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \qquad \text{or} \qquad v^2 = \frac{Ze^2}{4\pi\epsilon_0 r m}$$

But we also know

$$r = \frac{4\pi\epsilon_0\hbar^2}{Zme^2} = \frac{a_0}{Z}$$

$$v^2 = \frac{Z^2e^2}{4\pi\epsilon_0 a_0 m} = \frac{3^2\,(1.44 \text{ eV}\cdot\text{nm})}{(5.29\times10^{-11} \text{ m})\,(511\times10^3 \text{ eV}/c^2)} = 4.79\times10^{-4}c^2$$

or $v = 2.19\times10^{-2}c = 6.57\times10^6$ m/s. This is a factor of 3 greater than the speed for hydrogen.

$$a = \frac{v^2}{r} = \frac{(6.57\times10^6 \text{ m/s})^2}{(5.29\times10^{-11} \text{ m})/3} = 2.45\times10^{24} \text{ m/s}^2$$

17. For a hydrogen-like atom $E = -Z^2E_0/n = -Z^2E_0$ for $n = 1$.

$$\text{H}: E = -E_0 = -13.6 \text{ eV}$$

$$\text{He}^+\!: E = -4E_0 = -54.4 \text{ eV}$$

$$\text{Li}^{++}\!: E = -9E_0 = -122.5 \text{ eV}$$

The binding energy is larger for atoms with larger Z values, due to the greater attractive force between the nucleus and electron.

18. The total energy of the atom is $-e^2/(8\pi\epsilon_0 r)$. Differentiating with respect to time:

$$\frac{dE}{dt} = \frac{e^2}{8\pi\epsilon_0 r^2}\frac{dr}{dt}$$

Equating this result with the given equation from electromagnetic theory

$$\frac{e^2}{8\pi\epsilon_0 r^2}\frac{dr}{dt} = -\frac{1}{4\pi\epsilon_0}\frac{2e^2}{3c^3}\left(\frac{d^2r}{dt^2}\right)^2$$

$$\frac{e^2}{2r^2}\frac{dr}{dt} = -\frac{2e^2}{3c^3}\left(\frac{d^2r}{dt^2}\right)^2$$

In a circular orbit d^2r/dt^2 is just the centripetal acceleration, which is also given by

$$a = \frac{F}{m} = \frac{e^2}{4\pi\epsilon_0 r^2}$$

Substituting:

$$\frac{e^2}{2r^2}\frac{dr}{dt} = -\frac{2e^2}{3c^3}\left(\frac{e^2}{4\pi\epsilon_0 r^2}\right)^2$$

$$\frac{dr}{dt} = -\frac{4e^4}{(4\pi\epsilon_0)^2\, 3m^2c^3r^2}$$

Solving by separation of variables:

$$dt = -\left(4\pi\epsilon_0\right)^2 \frac{3m^2c^3}{4e^4} r^2\, dr$$

$$t = -\left(4\pi\epsilon_0\right)^2 \frac{3m^2c^3}{4e^4}\int_{a_0}^{0} r^2\, dr = \left(4\pi\epsilon_0\right)^2 \frac{m^2c^3}{4e^4} a_0^3$$

Inserting numerical values we find $t = 1.6 \times 10^{-11}$ s.

19.

$$\nu = \frac{c}{\lambda} = Z^2Rc\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

$$\nu(K_\alpha) = Z^2Rc\left(1 - \frac{1}{4}\right) \qquad \nu(K_\beta) = Z^2Rc\left(1 - \frac{1}{9}\right) \qquad \nu(L_\alpha) = Z^2Rc\left(\frac{1}{4} - \frac{1}{9}\right)$$

With $1 - 1/4 + (1/4 - 1/9) = 1 - 1/9$ we see

$$\nu(K_\alpha) + \nu(L_\alpha) = \nu(K_\beta)$$

20. As in Problem 16 $v = 2.19 \times 10^6$ m/s and

$$L = mvr = \left(9.11 \times 10^{-31}\text{ kg}\right)\left(2.19 \times 10^6\text{ m/s}\right)\left(5.29 \times 10^{-11}\text{ m}\right) = 1.0554 \times 10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}$$

Notice that $L = \hbar$.

21.

$$hc = \left(4.135669 \times 10^{-15}\text{ eV}\cdot\text{s}\right)(299792458\text{ m/s}) = 1239.8\text{ eV}\cdot\text{nm}$$

$$\frac{e^2}{4\pi\epsilon_0} = \frac{(1.6021733 \times 10^{-19}\text{ C})^2}{4\pi\,(8.8541878 \times 10^{-12}\text{ F/m})}\frac{1\text{ eV}}{1.6021733 \times 10^{-19}\text{ N}\cdot\text{m}}\frac{10^9\text{ nm}}{\text{m}} = 1.4400\text{ eV}\cdot\text{nm}$$

$$mc^2 = \left(510.99906\text{ keV}/c^2\right)c^2 = 511.00\text{ keV}$$

$$a_0 = \frac{4\pi\epsilon_0\hbar^2}{me^2} = \frac{(8.8541878\times10^{-12}\ \text{F/m})\left(4.135669\times10^{-15}\ \text{eV}\cdot\text{s}\right)^2/\pi}{(9.1093897\times10^{-31}\ \text{kg})\,e^2}$$
$$= 5.2918\times10^{-11}\ \text{m} = 5.2918\times10^{-2}\ \text{nm}$$

$$E_0 = \frac{e^2}{8\pi\epsilon_0 a_0} = -\frac{\left(1.6021733\times10^{-19}\ \text{C}\right)^2}{8\pi\,(8.8541878\times10^{-12}\ \text{F/m})\,(5.2917725\times10^{-11}\ \text{m})}$$
$$= 2.179874\times10^{-18}\ \text{J} = 13.606\ \text{eV}$$

*22. From Equation (4.31) $v_n = (1/n)(\hbar/ma_0)$

$n = 1$: $$v_1 = \frac{1}{1}\frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})\,(5.29\times10^{-11}\ \text{m})} = 2.19\times10^{6}\ \text{m/s} = 0.0073c$$

$n = 2$: $$v_2 = \frac{1}{2}\frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})\,(5.29\times10^{-11}\ \text{m})} = 1.09\times10^{6}\ \text{m/s} = 0.0036c$$

$n = 3$: $$v_3 = \frac{1}{3}\frac{1.055\times10^{-34}\ \text{J}\cdot\text{s}}{(9.11\times10^{-31}\ \text{kg})\,(5.29\times10^{-11}\ \text{m})} = 7.30\times10^{5}\ \text{m/s} = 0.0024c$$

*23. The photon energy is
$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{434\ \text{nm}} = 2.86\ \text{eV}$$
This is the energy difference between the two states in hydrogen. Because $E_3 = -1.51$ eV the initial state must be $n = 2$. We notice that this energy difference exists between $n = 2$ (with $E = -3.40$ eV) and $n = 5$ (with $E = -0.54$ eV).

24. The photon energy is
$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{95\ \text{nm}} = 13.05\ \text{eV}$$
This can only be a transition to $n = 1$ ($E_1 = -13.6$ eV) and because of the energy difference it comes from $n = 5$ with $E_5 = -0.54$ eV.

25. In general the ground state energy is Z^2E_0.

a)
$$E = 1^2E_0 = 13.6\ \text{eV}$$
The reduced mass does not change this result to three significant digits.

b)
$$E = 2^2E_0 = 54.4\ \text{eV}$$

c)
$$E = 4^2E_0 = 218\ \text{eV}$$

26. a) It is only the first four lines of the Balmer series, with wavelengths 656.5 nm, 486.3 nm, 434.2 nm, and 410.2 nm.

b) To get the energy levels in helium, perform the same analysis as in the text but with e^2 replaced by $Ze^2 = 2e^2$. This results in an extra factor of $Z^2 = 4$ in the energy, so the revised Rydberg-Ritz equation is

$$\frac{1}{\lambda} = \frac{4E_0}{hc}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) = \left(4.377 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

We need the wavelength to be between 400 and 700 nm. The combinations of n_l and n_u that work are tabulated below:

n_l	n_u	λ (nm)	comments
3	4	470	
4	6	658	
4	7	543	
4	8	487	etc. with $n_l = 4$ to $n_u = 13$
4	13	404	but $n_l = 4$ and $n_u > 13$ gives $\lambda < 400$ nm
5	12	691	
5	13	670	etc. with $n_l = 5$ all the way to...
5	∞	571	a series limit

27. From Problem 22 the speed in the $n = 3$ state is $v = 7.30 \times 10^5$ m/s. The radius of the orbit is $n^2 a_0 = 9a_0$. Then from kinematics

$$\text{number of revolutions } = \frac{vt}{2\pi r} = \frac{(7.30 \times 10^5 \text{ m/s})\,(10^{-8} \text{ s})}{2\pi\,(9)\,(5.29 \times 10^{-11} \text{ m})} = 2.44 \times 10^6$$

*28. The energy of each photon is $hc/\lambda = 12.4$ eV. Looking at the energy difference between levels in hydrogen we see that $E_2 - E_1 = 10.2$ eV, $E_3 - E_1 = 12.1$ eV, and $E_4 - E_1 = 12.8$ eV. There is enough energy to excite only to the second or third level. In theory it is possible for a second photon to come along and take the atom from one of these excited states to a higher one, but this is unlikely, because the $n = 2$ and $n = 3$ states are short-lived.

*29. We must use the reduced mass for the muon:

$$\mu = \frac{mM}{m+M} = \frac{(106 \text{ MeV}/c^2)\,(938 \text{ MeV}/c^2)}{106 \text{ MeV}/c^2 + 938 \text{ MeV}/c^2} = 95.2 \text{ MeV}/c^2$$

a)

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2} = \frac{(6.58 \times 10^{-16} \text{ eV} \cdot \text{s})^2}{(1.44 \times 10^{-9} \text{ eV} \cdot \text{m})\,(95.2 \times 10^6 \text{ eV}/c^2)}\,\frac{(3.00 \times 10^8 \text{ m/s})^2}{c^2} = 2.84 \times 10^{-13} \text{ m}$$

b)

$$E = \frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{(1.44 \times 10^{-9} \text{ eV} \cdot \text{m})}{2\,(2.84 \times 10^{-13} \text{ m})} = 2535 \text{ eV}$$

c) First series:

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV} \cdot \text{nm}}{2535 \text{ eV}} = 0.49 \text{ nm}$$

Second series:

$$\lambda = \frac{4hc}{E} = \frac{4\,(1240 \text{ eV} \cdot \text{nm})}{2535 \text{ eV}} = 1.96 \text{ nm}$$

Third series:

$$\lambda = \frac{9hc}{E} = \frac{9\,(1240 \text{ eV} \cdot \text{nm})}{2535 \text{ eV}} = 4.40 \text{ nm}$$

30. $\mu = mm/(m+m) = m/2$ where m is the mass of each particle. Then

$$r = \frac{4\pi\varepsilon_0 \hbar^2}{\mu e^2} = 2a_0$$

$$E = -\frac{e^2}{8\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0\,(2a_0)} = -\frac{E_0}{2} = -6.8 \text{ eV}$$

31. a) As shown in Problem 30, the radius of the orbit is $2a_0$.
b) With $E_0 = 6.8$ eV (see Problem 30) we have

$$\Delta E = E_2 - E_1 = -\frac{E_0}{2^2} - \left(-\frac{E_0}{1^1}\right) = \frac{3E_0}{4} = 5.1 \text{ eV}$$

Then

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV} \cdot \text{nm}}{5.1 \text{ eV}} = 243 \text{ nm}$$

32. a)

$$r_2 - r_1 = 4a_0 - a_0 = 3a_0 = 1.59 \times 10^{-10} \text{ m}$$

b)

$$r_6 - r_5 = 36a_0 - 25a_0 = 11a_0 = 5.83 \times 10^{-10} \text{ m}$$

c)

$$r_{11} - r_{10} = 121a_0 - 100a_0 = 21a_0 = 1.11 \times 10^{-9} \text{ m}$$

Note that in each case $r_m - r_n = (m+n)a_0$.

33.

$$\text{hydrogen: } \frac{1}{\lambda} = R_H \left(\frac{1}{4} - \frac{1}{n_u^2}\right)$$

$$\text{helium: } \frac{1}{\lambda} = Z^2 R_{He} \left(\frac{1}{4^2} - \frac{1}{n_u^2}\right) = R_{He} \left(\frac{1}{4} - \frac{4}{n_u^2}\right)$$

We see that the lines match very well when n_u is even for helium but not when it is odd. Also, all the “matched” lines differ slightly because of the different Rydberg constant (which is due to the different reduced masses). The differ by a factor of $R_{He}/R_H = 0.99986/0.99946 \cong 1.0004$.

34. In general

$$R = \frac{1}{1 + m/M} R_\infty$$

where $R_\infty = 1.0973731534 \times 10^7\ \text{m}^{-1}$ and $m = 0.0005485799$ u.
Using ^{4}He ($M = 4.0026$ u), $R = 0.999863 R_\infty = 1.097223 \times 10^7\ \text{m}^{-1}$ (off by 0.14%)
Using ^{39}K ($M = 38.963708$ u), $R = 0.9999859 R_\infty = 1.097358 \times 10^7\ \text{m}^{-1}$ (off by 0.0014%)
Using ^{238}U ($M = 238.05078$ u), $R = 0.9999977 R_\infty = 1.097371 \times 10^7\ \text{m}^{-1}$ (off by 0.00023%)

35. The derivation of the Rydberg equation is the same as in the text. Because He^+ is hydrogen-like it works with $R = Z^2 R_{He}$ and $R_{He} = 1.097223 \times 10^7\ \text{m}^{-1}$ as in Problem 34. Then

$$R = 4\left(1.097223 \times 10^7\ \text{m}^{-1}\right) = 4.38889 \times 10^7\ \text{m}^{-1}$$

*36. For L_α we have

$$\lambda = \frac{c}{\nu} = \frac{36}{5R\,(Z - 7.4)^2}$$

$$Z = 43: \qquad \lambda = \frac{36}{5R\,(43 - 7.4)^2} = 0.52\ \text{nm}$$

$$Z = 61: \qquad \lambda = \frac{36}{5R\,(61 - 7.4)^2} = 0.23\ \text{nm}$$

$$Z = 75: \qquad \lambda = \frac{36}{5R\,(75 - 7.4)^2} = 0.14\ \text{nm}$$

37. For Pt $Z = 78$ and for Au $Z = 79$. For the K_α lines

$$\lambda = \frac{4}{3R\,(Z - 1)^2}$$

$$\text{Pt: } \lambda = \frac{4}{3R(77)^2} = 20.49\ \text{pm} \qquad \text{Au: } \lambda = \frac{4}{3R(78)^2} = 19.97\ \text{pm}$$

Therefore $\Delta\lambda = 0.52$ pm which is less than the specified resolution, so it will not work.

38.

$$\nu(K_\beta) = \frac{c}{\lambda(K_\beta)} = cR(Z - 1)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8cR}{9}(Z - 1)^2$$

This is higher than the K_a frequency by a factor of $(8/9)/(3/4) = 32/27$, which seems to be in agreement with Figure 4.19.

*39. Helium:

$$\lambda(K_\alpha) = \frac{4}{3R\,(Z-1)^2} = 122 \text{ nm} \qquad \lambda(K_\beta) = \frac{9}{8R(Z-1)^2} = 103 \text{ nm}$$

Lithium:

$$\lambda(K_\alpha) = \frac{4}{3R\,(Z-1)^2} = 30.4 \text{ nm} \qquad \lambda(K_\beta) = \frac{9}{8R(Z-1)^2} = 25.6 \text{ nm}$$

40. λ is inversely proportional to $(Z-1)^2$ for the K series and to $(Z-7.4)^2$ for the L series.

a)

$$\frac{\lambda(\text{U})}{\lambda(\text{C})} = \frac{(6-1)^2}{(92-1)^2} = 0.0030$$

b)

$$\frac{\lambda(\text{W})}{\lambda(\text{Ca})} = \frac{(20-7.4)^2}{(74-7.4)^2} = 0.036$$

41. Non-relativistically 40 eV $= \frac{1}{2}mv_1^2$ so

$$v_1 = \sqrt{\frac{2\,(40 \text{ eV})}{511 \text{ keV}/c^2}} = 0.0125c = 3.75 \times 10^6 \text{ m/s}$$

From elementary physics

$$v_2' = \frac{2m_1}{m_1+m_2}v_1 = \frac{2\,(0.0005486 \text{ u})}{0.0005486 \text{ u} \ + 202.97 \text{ u}}\left(3.75 \times 10^6 \text{ m/s}\right) = 20.3 \text{ m/s}$$

where we have used the most abundant mercury isotope. Then

$$K_2' = \frac{1}{2}m_2 v_2' = \frac{1}{2}\,(202.97 \text{ u})\left(931.49 \text{ MeV/u}\cdot c^2\right)\left(\frac{c^2}{9\times 10^{16} \text{ m}^2/\text{s}^2}\right)(20.3 \text{ m/s})^2 = 4.33\times 10^{-4} \text{ eV}$$

42. Without the negative potential an electron with any energy, no matter how small, could drift into the collector plate. As a result the electron could give up its kinetic energy to a Hg atom and still contribute to the plate current. The Franck-Hertz curve would not show the distinguishing periodic drops, but rather would rise monotonically.

43. Using $\Delta E = hc/\lambda$ we find

$$h = \frac{\lambda \Delta E}{c} = \frac{(254 \text{ nm})\,(4.88 \text{ eV})}{3.00\times 10^8 \text{ m/s}} = 4.13 \times 10^{-15} \text{ eV}\cdot\text{s}$$

44. 3.6 eV, 4.6 eV, 2(3.6 eV) = 7.2 eV, 3.6 eV + 4.6 eV = 8.2 eV, etc. with other combinations giving 10.8 eV, 11.8 eV, 12.6 eV, 14.4 eV, 15.4 eV, 16.2 eV, 16.4 eV, 17.2 eV, 18.0 eV.

45. Magnesium:

$$\lambda(K_\alpha) = \frac{4}{3R\,(Z-1)^2} = 1.00 \text{ nm} \qquad \lambda(L_\alpha) = \frac{36}{5R\,(Z-7.4)^2} = 31.0 \text{ nm}$$

Iron:

$$\lambda(K_\alpha) = \frac{4}{3R\,(Z-1)^2} = 0.194 \text{ nm} \qquad \lambda(L_\alpha) = \frac{36}{5R\,(Z-7.4)^2} = 1.90 \text{ nm}$$

*46. K_α is a transition from $n = 2$ to $n = 1$ and K_β is from $n = 3$ to $n = 1$. We know those wavelengths in the Lyman series are 121.6 nm and 102.6 nm, respectively. The redshift factor (λ/λ_0) is (with $\beta = 1/6$)

$$\sqrt{\frac{1+\beta}{1-\beta}} = \sqrt{\frac{1+1/6}{1-1/6}} = 1.183$$

Then the redshifted wavelengths are higher by 18.3% in each case. The observed wavelengths are:

$$K_\alpha\text{: } \lambda = (1.183)\,(121.6 \text{ nm}) = 143.9 \text{ nm}$$

$$K_\beta\text{: } \lambda = (1.183)\,(102.6 \text{ nm}) = 121.4 \text{ nm}$$

*47.

$$f = \pi n t \left(\frac{Z_1 Z_2 e^2}{8\pi\varepsilon_0 K}\right)^2 \cot^2\left(\frac{\theta}{2}\right)$$

a)

$$f(1^\circ) = \pi\left(5.90\times10^{28}\text{ m}^{-3}\right)\left(4\times10^{-7}\text{ m}\right)\left(\frac{(2)\,(79)\,(1.44\times10^{-9}\text{ eV}\cdot\text{m})}{2\,(8\times10^6\text{ eV})}\right)^2 \cot^2(0.5^\circ) = 0.197$$

$$f(2^\circ) = \pi\left(5.90\times10^{28}\text{ m}^{-3}\right)\left(4\times10^{-7}\text{ m}\right)\left(\frac{(2)\,(79)\,(1.44\times10^{-9}\text{ eV}\cdot\text{m})}{2\,(8\times10^6\text{ eV})}\right)^2 \cot^2(1^\circ) = 0.0492$$

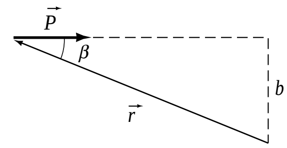
The fraction scattered between 1° and 2° is $0.197 - 0.0492 = 0.148$.

b)

$$\frac{f(1^\circ)}{f(10^\circ)} = \frac{\cot^2(0.5^\circ)}{\cot^2(5^\circ)} = 100.5$$

$$\frac{f(1^\circ)}{f(90^\circ)} = \frac{\cot^2(0.5^\circ)}{\cot^2(45^\circ)} = 1.31\times10^4$$

48. From classical mechanics we know that $\overrightarrow{L}$ is conserved for central forces.
$L = \left|\overrightarrow{L}\right| = |\overrightarrow{r} \times \overrightarrow{p}| = rp_0 \sin\beta$



But $r \sin\beta = b$ so $L = p_0 b = mv_0 b$.

49. If the positions of the electron and proton are respectively (along a line) r_e and r_p, then putting the center of mass at $R = 0$ we have

$$R = 0 = \frac{m_e r_e + m_p r_p}{m_e + m_p} \qquad \text{or} \qquad r_p = -\frac{m_e}{m} r_e$$

$$r = r_e - r_p = r_e - \left(-\frac{m_e}{m} r_e\right) = r_e \left(1 + \frac{m_e}{m_p}\right)$$

$$r_e = \frac{m_p}{m_p + m_e} r$$

Substituting this into the expression

$$v^2 = \frac{n^2\hbar^2}{m_e^2 r_e^2}$$

we find

$$v^2 = \frac{n^2\hbar^2}{m_e^2 \left(\frac{m_p}{m_p+m_e} r\right)^2} = \frac{n^2\hbar^2}{\mu^2 r^2}$$

with

$$\mu = \frac{m_e m_p}{m_e + m_p}$$

50. a)

$$\nu = \frac{me^4}{4\varepsilon_0^2 h^3} = \frac{(9.11 \times 10^{-31}\ \text{kg})\,(1.60 \times 10^{-19}\ \text{C})^4}{4\,(8.85 \times 10^{-12}\ \text{F/m})^2\,(6.626 \times 10^{-34}\ \text{J}\cdot\text{s})^3} = 6.55 \times 10^{15}\ \text{Hz}$$

b) As determined in previous problems $v = 2.19 \times 10^6$ m/s and $r = a_0 = 5.29 \times 10^{-11}$ m.

$$\nu = \frac{v}{2\pi r} = \frac{2.19 \times 10^6\ \text{m/s}}{2\pi\,(5.29 \times 10^{-11}\ \text{m})} = 6.59 \times 10^{15}\ \text{Hz}$$

c) We know

$$E = -\frac{e^2}{8\pi\varepsilon_0 a_0} \qquad \text{and} \qquad K = \frac{e^2}{8\pi\varepsilon_0 a_0} = |E|$$

d) Since $K = nh\nu_{\text{orb}}/2 = h\nu_{\text{orb}}/2$ for $n = 1$ we have

$$\nu_{\text{orb}} = \frac{2K}{h} = \frac{2(13.6\ \text{eV})}{4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}} = 6.58 \times 10^{15}\ \text{Hz}$$

which agrees with (a) and (b) to within rounding errors.

*51. We start with $K = nh\nu_{\text{orb}}/2$. From classical mechanics we have for a circular orbit $\nu = v/2\pi r$, or $r = v/2\pi\nu$:

$$L = mvr = mv\left(\frac{v}{2\pi\nu}\right) = \left(\frac{mv^2}{2}\right)\left(\frac{1}{\pi\nu}\right)$$

Using $K = \frac{1}{2}mv^2$,

$$L = \frac{K}{\pi\nu} = \frac{nh\nu}{2\pi\nu} = \frac{nh}{2\pi} = n\hbar$$