

# Chapter 16

\*1.

$$1 \text{ pc} = \frac{1 \text{ au}}{\tan 1''} (1.496 \times 10^{11} \text{ m/au}) = 3.086 \times 10^{16} \text{ m}$$

$$1 \text{ ly} = (2.9979 \times 10^8 \text{ m/s}) (365.25 \text{ d/y}) (86400 \text{ s/d}) = 9.461 \times 10^{15} \text{ m}$$

$$1 \text{ pc} = \frac{3.086 \times 10^{16} \text{ m}}{9.461 \times 10^{15} \text{ m/ly}} = 3.26 \text{ ly}$$

2. As in Example 16.3 we have  $7 = \exp(\Delta mc^2/kT)$  so  $\ln 7 = \Delta mc^2/kT$ . Then

$$T = \frac{\Delta mc^2}{k \ln 7} = \frac{939.56563 \text{ MeV} - 938.27231 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (\ln 7)} = 7.71 \times 10^9 \text{ K}$$

3. Using  $mc^2 = 135 \text{ MeV} = kT$

$$T = \frac{135 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 1.57 \times 10^{12} \text{ K}$$

From Figure 16.8 the time associated with this temperature is about  $10^{-1} \text{ s}$ .

4.

$$\frac{\text{number of d}}{\text{number of u}} = \exp \left( \frac{10 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.00$$

$$\frac{\text{number of u}}{\text{number of c}} = \exp \left( \frac{1300 \text{ MeV} - 10 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.16$$

$$\frac{\text{number of u}}{\text{number of s}} = \exp \left( \frac{115 \text{ MeV} - 10 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.01$$

Similarly,

$$\frac{\text{number of d}}{\text{number of s}} = \exp \left( \frac{115 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.01$$

$$\frac{\text{number of u or d}}{\text{number of c}} = \exp \left( \frac{1300 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.16$$

$$\frac{\text{number of u or d}}{\text{number of t}} = \exp \left( \frac{174000 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 5.9 \times 10^8$$

$$\frac{\text{number of u or d}}{\text{number of b}} = \exp \left( \frac{4250 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.64$$

$$\begin{aligned}\frac{\text{number of s}}{\text{number of c}} &= \exp \left( \frac{1300 \text{ MeV} - 115 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.15 \\ \frac{\text{number of s}}{\text{number of t}} &= \exp \left( \frac{174000 \text{ MeV} - 115 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 5.8 \times 10^8 \\ \frac{\text{number of s}}{\text{number of b}} &= \exp \left( \frac{4250 \text{ MeV} - 115 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.62 \\ \frac{\text{number of c}}{\text{number of t}} &= \exp \left( \frac{174000 \text{ MeV} - 1300 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 5.1 \times 10^8 \\ \frac{\text{number of c}}{\text{number of b}} &= \exp \left( \frac{4250 \text{ MeV} - 1300 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 1.41 \\ \frac{\text{number of b}}{\text{number of t}} &= \exp \left( \frac{174000 \text{ MeV} - 4250 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})} \right) = 3.6 \times 10^8\end{aligned}$$

5.  $mc^2 = kT$  so we have

$$\text{electron: } T = \frac{mc^2}{k} = \frac{0.5110 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 5.93 \times 10^9 \text{ K}$$

$$\text{muon: } T = \frac{mc^2}{k} = \frac{105.66 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 1.23 \times 10^{12} \text{ K}$$

6. As in the previous problem we have

$$\text{for the } 5 \text{ eV}/c^2 \text{ mass: } T = \frac{mc^2}{k} = \frac{5 \text{ eV}}{8.617 \times 10^{-5} \text{ eV/K}} = 5.80 \times 10^4 \text{ K}$$

$$\text{for the } 10^{-4} \text{ eV}/c^2 \text{ mass: } T = \frac{mc^2}{k} = \frac{10^{-4} \text{ eV}}{8.617 \times 10^{-5} \text{ eV/K}} = 1.16 \text{ K}$$

Note that the answer for  $mc^2 = 10^{-4} \text{ eV}$  is lower than the present temperature!

\*7. The  $\pi^+$  ( $E_0 = 140 \text{ MeV}$ ) is more massive than the  $\pi^0$  ( $E_0 = 135 \text{ MeV}$ ), so the  $\pi^+$  was formed first. With  $\Delta mc^2 = k\Delta T$  we have

$$\Delta T = \frac{\Delta mc^2}{k} = \frac{5 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 5.80 \times 10^{10} \text{ K}$$

\*8. Set the deuteron binding energy  $2.22 \text{ MeV}$  equal to  $kT$ :

$$T = \frac{2.22 \text{ MeV}}{k} = \frac{2.22 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 2.58 \times 10^{10} \text{ K}$$

9. Set the hydrogen binding energy 13.6 eV equal to  $kT$ :

$$T = \frac{13.6 \text{ eV}}{k} = \frac{13.6 \text{ eV}}{8.617 \times 10^{-5} \text{ eV/K}} = 1.58 \times 10^5 \text{ K}$$

10.

$$\frac{0.3GN^2m^2}{V^{4/3}} = \frac{3.9\hbar^2N^{5/3}}{2mV^{5/3}}$$

$$V^{1/3} = \frac{3.9\hbar^2}{0.6m^3N^{1/3}G} = \frac{6.5\hbar^2}{N^{1/3}m^3G}$$

11.

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{2M}{\frac{4}{3}\pi R^3} = \frac{3M}{2\pi R^3}$$

where  $M = 1.99 \times 10^{30} \text{ kg}$  (mass of sun) and  $R = 11 \text{ km}$ .

$$\rho = \frac{3M}{2\pi R^3} = \frac{3(1.99 \times 10^{30} \text{ kg})}{2\pi (11 \times 10^3 \text{ m})^3} = 7.14 \times 10^{17} \text{ kg/m}^3$$

For the nucleon (or nucleus, assuming uniform nuclear density) we have

$$\rho = \frac{m_p}{\frac{4}{3}\pi r_0^3} = \frac{3(1.67 \times 10^{-27} \text{ kg})}{4\pi (1.2 \times 10^{-15} \text{ m})^3} = 2.31 \times 10^{17} \text{ kg/m}^3$$

and the neutron star is about three times as dense.

12. We see from Equation (16.8) that  $V^{1/3}$  is proportional to  $N^{-1/3}$ . However,  $V^{1/3}$  is proportional to  $R$ , so  $R$  is proportional to  $N^{-1/3}$ . The reason this makes sense is that the gravitational pressure increases not in proportion to  $N$  but rather in proportion to  $N^{5/3}$  [see Equation (16.7)].

13. a)

$$P = 0.3G \frac{(Nm)^2}{V^{4/3}} = 0.3G \frac{M^2}{\left(\frac{4}{3}\pi R^3\right)^{4/3}}$$

$$= 0.3 \left(6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}\right) \frac{(1.99 \times 10^{30} \text{ kg})^2}{\left(\frac{4}{3}\pi (6.96 \times 10^8 \text{ m})^3\right)^{4/3}} = 5.00 \times 10^{13} \text{ N/m}^2$$

b)

$$P = 0.3G \frac{M^2}{\left(\frac{4}{3}\pi R^3\right)^{4/3}} = 0.3 \left(6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}\right) \frac{4(1.99 \times 10^{30} \text{ kg})^2}{\left(\frac{4}{3}\pi (1.1 \times 10^4 \text{ m})^3\right)^{4/3}}$$

$$= 3.21 \times 10^{33} \text{ N/m}^2$$

\*14.

$$v = HR = \frac{65 \text{ km/s}}{\text{Mpc}} (4 \times 10^9 \text{ ly}) \left( \frac{1 \text{ Mpc}}{3.26 \times 10^6 \text{ ly}} \right) = 7.98 \times 10^4 \text{ km/s} \cong 0.27c$$

\*15.

$$R = \frac{v}{H} = \frac{15000 \text{ km/s}}{65 \text{ km/s/Mpc}} = 231 \text{ Mpc or about 752 Mly}$$

16. For a redshift of 3.8 we have  $\lambda/\lambda_0 = 4.8$ , so

$$4.8 = \sqrt{\frac{1 + \beta}{1 - \beta}}$$

which can be solved to find  $\beta = 0.917$ . Then

$$R = \frac{v}{H} = \frac{0.917 (299790 \text{ km/s})}{65 \text{ km/s/Mpc}} = 4230 \text{ Mpc or about 13.8 Gly}$$

17. a)

$$(3 \times 10^{-28} \text{ kg/m}^3) \left( \frac{1 \text{ nucleon}}{1.67 \times 10^{-27} \text{ kg}} \right) = 0.18 \text{ nucleons/m}^3$$

b)

$$R = (16.6 \text{ ly}) (9.46 \times 10^{15} \text{ m/ly}) = 1.57 \times 10^{17} \text{ m}$$

$$60 (1.99 \times 10^{30} \text{ kg}) \frac{1 \text{ nucleon}}{1.67 \times 10^{-27} \text{ kg}} = 7.15 \times 10^{58} \text{ nucleons}$$

The nucleon density is the number of nucleons divided by the volume, or

$$\frac{7.15 \times 10^{58}}{\frac{4}{3}\pi (1.57 \times 10^{17} \text{ m})^3} = 4.41 \times 10^6 \text{ nucleons/m}^3$$

The nucleon density in our neighborhood is much larger than it is for the universe as a whole.

18. Using (16.2) we see that

$$H^2 = \frac{1}{a^2} \left( \frac{da}{dt} \right)^2$$

Substituting  $(da/dt)^2$  from Equation (16.14)

$$H^2 = \frac{1}{a^2} \left( \frac{8\pi}{3} G \rho a^2 - kc^2 \right) = \frac{8\pi G \rho}{3} - \frac{kc^2}{a^2}$$

19.

$$H = (21 \text{ km/s/Mly}) \frac{1 \text{ Mly}}{10^6 (9.46 \times 10^{15} \text{ m})} = 2.22 \times 10^{-18} \text{ s}^{-1}$$

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3(2.22 \times 10^{-18} \text{ s}^{-1})^2}{8\pi (6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})} = 8.82 \times 10^{-27} \text{ kg/m}^3 = 8.82 \times 10^{-30} \text{ g/cm}^3$$

20. We use  $H = 65 \text{ km/s/Mpc}$  or

$$H = (65000 \text{ m/s/Mpc}) \left( \frac{1 \text{ Mpc}}{3.086 \times 10^{22} \text{ m}} \right) = 2.1063 \times 10^{-18} \text{ s}^{-1}$$

a) For critical density  $\rho = 9 \times 10^{-27} \text{ kg/m}^3$ :

$$q = \frac{4\pi\rho G}{3H^2} = \frac{4\pi (9 \times 10^{-27} \text{ kg/m}^3) (6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})}{3(2.1063 \times 10^{-18} \text{ s}^{-1})^2} = 0.57$$

b) With the observed mass density  $3 \times 10^{-28} \text{ kg/m}^3$  we have

$$q = \frac{4\pi\rho G}{3H^2} = \frac{4\pi (3 \times 10^{-28} \text{ kg/m}^3) (6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})}{3(2.1063 \times 10^{-18} \text{ s}^{-1})^2} = 0.019$$

Note that at critical density we should get  $q = 0.5$ , so the value of  $H$  used in part (a) brings us close to critical density.

21. a)  $a = Ct^n$

$$da/dt = nCt^{n-1}$$

$$d^2a/dt^2 = n(n-1)Ct^{n-2}$$

$$q = -a \frac{d^2a/dt^2}{\left(\frac{da}{dt}\right)^2} = -\frac{an(n-1)Ct^{n-2}}{n^2C^2t^{2n-2}} = -\frac{an(n-1)}{n^2Ct^n} = -\frac{n(n-1)}{n^2}$$

where in the last step we used the definition  $a = Ct^n$ . From this we see that there is deceleration ( $q > 0$ ) only if  $0 < n < 1$ .

b)

$$H = \frac{1}{a} \frac{da}{dt} = \frac{nCt^{n-1}}{Ct^n} = \frac{n}{t}$$

with  $0 < n < 1$ . There is an inverse dependence on time.

22. From Wien's law  $\lambda_{\max}T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$ .

$$\lambda_{\max} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{2.7 \text{ K}} = 1.07 \text{ mm}$$

\*23. For  $H = 55 \text{ km/s/Mpc}$  we have

$$H = (55000 \text{ m/s/Mpc}) \left( \frac{1 \text{ Mpc}}{3.086 \times 10^{22} \text{ m}} \right) = 1.78 \times 10^{-18} \text{ s}^{-1}$$

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3 (1.78 \times 10^{-18} \text{ s}^{-1})^2}{8\pi (6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})} = 5.67 \times 10^{-27} \text{ kg/m}^3$$

For  $H = 85 \text{ km/s/Mpc}$  we have

$$H = (85000 \text{ m/s/Mpc}) \left( \frac{1 \text{ Mpc}}{3.086 \times 10^{22} \text{ m}} \right) = 2.75 \times 10^{-18} \text{ s}^{-1}$$

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3 (2.75 \times 10^{-18} \text{ s}^{-1})^2}{8\pi (6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})} = 1.35 \times 10^{-26} \text{ kg/m}^3$$

24. a) the only combination of the constants that gives time is  $m = \frac{1}{2}$ ,  $n = \frac{1}{2}$ , and  $l = \frac{5}{2}$ .

b)

$$t_p = \sqrt{\frac{Gh}{c^5}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}) (6.626 \times 10^{-34} \text{ J} \cdot \text{s})}{(2.998 \times 10^8 \text{ m/s})^5}} = 1.35 \times 10^{-43} \text{ s}$$

25. A complete derivation can be found in Section 2.10.

26. Let  $dm$  be the mass of a spherical shell of thickness  $dr$ , so  $dm = \rho d(\text{vol}) = 4\pi\rho r^2 dr$ .

$$dV = -\frac{GM' dm}{r}$$

where  $M' = \frac{4}{3}\rho\pi r^3$  is the mass inside radius  $r$ . Thus

$$dV = -G\rho^2 \frac{16\pi^2}{3} r^4 dr$$

$$V = \int_0^R dV = -\frac{16\pi^2\rho^2 R^5 G}{15}$$

Now  $\rho = 3M/4\pi R^3$ , so

$$V = -\frac{16\pi^2\rho^2 R^5 G}{15} \left( \frac{9M^2}{16\pi^2 R^6} \right) = -\frac{3GM^2}{5R}$$

27. From both figures it appears that  $t_0 \cong \tau/2$ .

28. In general as in Example 16.7 we have  $\Delta t = (d/2c) (mc^2/E)^2$ .

$$\frac{d}{2c} = \frac{(\text{distance}) (50 \text{ kpc})}{2 (3.00 \times 10^8 \text{ m/s})} \frac{3.086 \times 10^{19} \text{ m}}{\text{kpc}} = 2.57 \times 10^{12} \text{ s}$$

Note that  $2.57 \times 10^{12} \text{ s} = (2.57 \text{ s}) (10 \text{ MeV}/10 \text{ eV})^2$ , so

$$\Delta t = (2.57 \text{ s}) \left( \frac{\text{distance}}{50 \text{ kpc}} \right) \left( \frac{mc^2}{10 \text{ eV}} \right)^2 \left( \frac{10 \text{ MeV}}{E} \right)^2$$

29. We know from Section 9.7 that the energy flux rate  $\Phi = \sigma T^2$  is related to the energy density  $\rho_e$  by a factor of  $c/4$ , such that  $\Phi = (c/4)\rho_e$ . But using  $E = mc^2$  we have  $\rho_e = \rho_{\text{rad}}c^2$ , so

$$\sigma T^4 = \frac{c}{4}\rho_e = \frac{c}{4}\rho_{\text{rad}}c^2$$

or, rearranging

$$\rho_{\text{rad}} = \frac{4\sigma T^4}{c^3}$$

30.

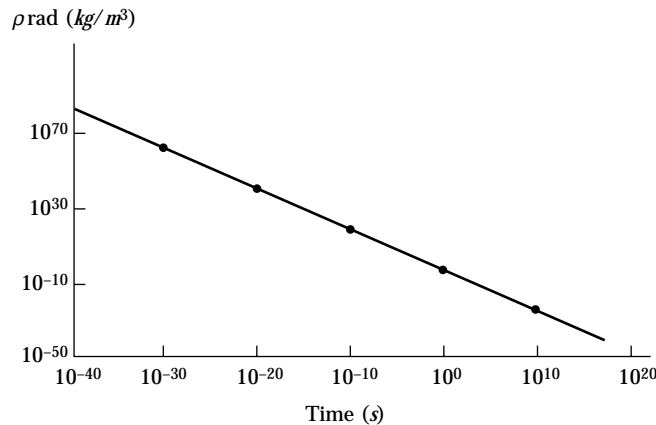
$$\rho_{\text{rad}} = \frac{4\sigma T^4}{c^3} = \frac{4 (5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}) (3 \text{ K})^4}{(3.00 \times 10^8 \text{ m/s})^3} = 6.80 \times 10^{-31} \text{ kg/m}^3$$

This is substantially less than the matter density of about  $3 \times 10^{-28} \text{ kg/m}^3$ , so the universe is matter dominated.

31. At 700,000 years the temperature is about  $10^8 \text{ K}$ . Then

$$\rho_{\text{matter}} = \rho_{\text{rad}} = \frac{4\sigma T^4}{c^3} = \frac{4 (5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4}) (10^8 \text{ K})^4}{(3.00 \times 10^8 \text{ m/s})^3} = 0.84 \text{ kg/m}^3$$

This is slightly less than the density of air at STP, so the universe was quite dense!



32. The 300 day mark is almost exactly 200 days after the peak, so use  $t = 200$  d in the exponential decay formula  $\exp(-(\ln 2)t/t_{1/2})$ .

$$^{56}\text{Ni}: \exp(-(\ln 2)t/t_{1/2}) = \exp(-(\ln 2)(200 \text{ d}) / (6.1 \text{ d})) = 1.35 \times 10^{-10}$$

$$^{56}\text{Co}: \exp(-(\ln 2)t/t_{1/2}) = \exp(-(\ln 2)(200 \text{ d}) / (77.1 \text{ d})) = 0.166$$

The cobalt must be primarily responsible, with some contribution from the nickel.

\*33.

$$\text{Redshift} = \frac{\Delta\lambda}{\lambda_0} = \frac{582.5 \text{ nm} - 121.6 \text{ nm}}{121.6 \text{ nm}} = 3.79$$

$$3.79 = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

so  $\beta = 0.92$  and  $v = 0.92c$ .

\*34. From the Doppler effect we know

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}} = \frac{\lambda_0 + \Delta\lambda}{\lambda_0} = 1 + \frac{\Delta\lambda}{\lambda_0} = 1 + z$$

35. Using the binomial expansion on the result of the previous problem

$$1 + z \cong \left(1 + \frac{\beta}{2} + \dots\right) \left(1 + \frac{\beta}{2} + \dots\right) \cong 1 + \beta$$

so we see that to first order  $z \cong \beta$ .

36.

$$5.34 = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

so  $\beta = 0.951$  and  $v = 0.951c$ .

$$R = \frac{v}{H} = \frac{0.951(299790 \text{ km/s})}{65 \text{ km/s/Mpc}} = 4390 \text{ Mpc}$$

37.

$$Q = (2m_p - m_d - m_e) c^2 = 0.43 \text{ MeV}$$

There are three particles in the final state, but it is possible for the deuteron and positron to have negligible energy, in which case the neutrino energy is 0.43 MeV.