Chapter 10

* 1. a) For each state the energy is given by $E_{\text{rot}} = \dfrac{\hbar^2 \ell(\ell+1)}{2I}$, so the transition energy is

$$\Delta E = \frac{\hbar^2}{2I}\left(2(3) - 1(2)\right) = \frac{2\hbar^2}{I} = \frac{2\left(1.055 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{10^{-46}\ \text{kg}\cdot\text{m}^2} = 2.2 \times 10^{-22}\ \text{J} = 1.4 \times 10^{-3}\ \text{eV}$$

b) As in part (a)

$$\Delta E = \frac{\hbar^2}{2I}\left(15(16) - 14(15)\right) = \frac{15\hbar^2}{I} = 1.67 \times 10^{-21}\ \text{J} = 1.04 \times 10^{-2}\,\text{eV}$$

This is still in the infrared part of the spectrum.

2. From Table 10.1 $\kappa = 1530$ N/m and $f = 5.63 \times 10^{13}$ Hz

a)

$$\Delta E = hf = \left(4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}\right)\left(5.63 \times 10^{13}\ \text{Hz}\right) = 0.233\ \text{eV}$$

b) Set $\Delta E = kT$ with two degrees of freedom in the vibrational mode. Then

$$T = \frac{\Delta E}{k} = \frac{0.233\ \text{eV}}{8.617 \times 10^{-5}\ \text{eV/K}} = 2702\ \text{K}$$

3. In the ground state $E = \left(n + \frac{1}{2}\right)\hbar\omega = \frac{1}{2}\hbar\omega = \frac{1}{2}kA^2 = \frac{1}{2}\mu\omega^2 A^2$. Solving for A:

$$A = \sqrt{\frac{\hbar}{\mu\omega}}$$

Using the ^{35}Cl isotope

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{35}{36}\ \text{u} = 1.614 \times 10^{-27}\ \text{kg}$$

From Table 10.1 $f = 8.66 \times 10^{13}$ Hz. With $\omega = 2\pi f$ we have

$$A = \sqrt{\frac{\hbar}{\mu\omega}} = \sqrt{\frac{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}{\left(1.614 \times 10^{-27}\ \text{kg}\right)(2\pi)\left(8.66 \times 10^{13}\ \text{s}^{-1}\right)}} = 1.10 \times 10^{-11}\ \text{m}$$

4. a) Using the result of Problem 8 for the rotational inertia we have

$$I = \mu R^2 = \left(1.614 \times 10^{-27}\ \text{kg}\right)\left(1.28 \times 10^{-10}\ \text{m}\right)^2 = 2.64 \times 10^{-47}\ \text{kg}\cdot\text{m}^2$$

For $\ell = 1$ we have

$$E_{\text{rot}} = \frac{\ell(\ell+1)\hbar^2}{2I} = \frac{\hbar^2}{I} = \frac{1}{2}I\omega^2$$

Therefore

$$\omega = \sqrt{\frac{2\hbar^2}{I^2}} = \sqrt{\frac{2\left(1.055 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2}{\left(2.64 \times 10^{-47}\ \text{kg}\cdot\text{m}^2\right)^2}} = 5.65 \times 10^{12}\ \text{rad/s}$$

For $\ell = 10$ we have

$$E_{\text{rot}} = \frac{\ell(\ell+1)\hbar^2}{2I} = \frac{55\hbar^2}{I} = \frac{1}{2}I\omega^2$$

Therefore

$$\omega = \sqrt{\frac{110\hbar^2}{I^2}} = \sqrt{\frac{110\,(1.055 \times 10^{-34}\ \text{J}\cdot\text{s})^2}{(2.64 \times 10^{-47}\ \text{kg}\cdot\text{m}^2)^2}} = 4.19 \times 10^{13}\ \text{rad/s}$$

b) The distance of the center of mass from the H atom is

$$x_{\text{cm}} = \frac{m_{\text{Cl}}x_{\text{Cl}}}{M} = \frac{35}{36}R = 1.24 \times 10^{-10}\ \text{m}$$

Then from rotational kinematics we have for $\ell = 1$

$$v = \omega x_{\text{cm}} = \left(5.65 \times 10^{12}\ \text{rad/s}\right)\left(1.24 \times 10^{-10}\ \text{m}\right) = 700\ \text{m/s}$$

and for $\ell = 10$:

$$v = \omega x_{\text{cm}} = \left(4.19 \times 10^{13}\ \text{rad/s}\right)\left(1.24 \times 10^{-10}\ \text{m}\right) = 5200\ \text{m/s}$$

c)

$$\omega = \frac{v}{x_{\text{cm}}} = \frac{0.1c}{x_{\text{cm}}} = \frac{2.998 \times 10^{7}\ \text{m/s}}{1.24 \times 10^{-10}\ \text{m}} = 2.42 \times 10^{17}\ \text{rad/s}$$

Then

$$\omega = \sqrt{\frac{\ell(\ell+1)\hbar^2}{I^2}}$$

$$\ell(\ell+1) = \frac{\omega^2 I^2}{\hbar^2} = \left(\frac{\left(2.42 \times 10^{17}\ \text{rad/s}\right)\left(2.64 \times 10^{-47}\ \text{kg}\cdot\text{m}^2\right)}{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}\right)^2 = 3.67 \times 10^{9}$$

from which it follows that $\ell \approx 6.1 \times 10^4$.

d) $E_{\text{rot}} = kT = \dfrac{\hbar^2\ell(\ell+1)}{2I}$. Thus

$$T = \frac{\hbar^2 l\,(l+1)}{2Ik} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\cdot\text{s}\right)^2\left(3.67 \times 10^{9}\right)}{2\,(2.64 \times 10^{-47}\ \text{kg}\cdot\text{m}^2)\,(1.38 \times 10^{-23}\ \text{J/K})} = 5.6 \times 10^{10}\ \text{K}$$

* 5. With Bohr's condition $L = n\hbar$ we find

$$E_{\text{rot}} = \frac{L^2}{2I} = \frac{n^2\hbar^2}{2I}$$

The Bohr version and the correct version become similar for large values of quantum number n or ℓ, but they are quite different for small ℓ.

* 6. $\Delta E = E_1 - E_0 = E_1 = \dfrac{\hbar^2}{I} = \dfrac{hc}{\lambda}$.

$$I = \frac{\hbar^2\lambda}{hc} = \frac{\hbar\lambda}{2\pi c} = \frac{\left(1.055 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(1.3 \times 10^{-3}\ \text{m}\right)}{2\pi\,(2.998 \times 10^{8}\ \text{m/s})} = 7.28 \times 10^{-47}\ \text{kg}\cdot\text{m}^2$$

b) The minimum energy in a vibrational transition is $\Delta E = hf$. From Table 10.1 $f = 6.42 \times 10^{13}$ Hz, which corresponds to a photon of wavelength $\lambda = c/f = 4.67\mu$m. A photon of this wavelength or less is required to excite the vibrational mode, so the 1.30 mm photon is too weak.

7.

$$\Delta E = hf = \left(6.626 \times 10^{-34}\ \mathrm{J \cdot s}\right)\left(6.42 \times 10^{13}\ \mathrm{Hz}\right) = 4.25 \times 10^{-20}\ \mathrm{J}$$

Using this energy for a rotational transition from the ground state to the ℓth state we have

$$\Delta E = \frac{\hbar^2 \ell(\ell+1)}{2I}$$

so

$$\ell(\ell+1) = \frac{2I\Delta E}{\hbar^2} = \frac{2\left(7.28 \times 10^{-47}\ \mathrm{kg \cdot m^2}\right)\left(4.25 \times 10^{-20}\ \mathrm{J}\right)}{\left(1.055 \times 10^{-34}\ \mathrm{J \cdot s}\right)^2} = 556$$

so $\ell \approx 24$. This is prohibited by the $\Delta\ell = \pm 1$ selection rule.

8. Combining $R = r_1 + r_2$ with $m_1 r_1 = m_2 r_2$ we find that

$$r_1 = \frac{m_2}{m_1 + m_2} R$$

and

$$r_2 = \frac{m_1}{m_1 + m_2} R$$

Therefore

$$\begin{aligned} I &= m_1 r_1^2 + m_2 r_2^2 = m_1 \left(\frac{m_2}{m_1+m_2} R\right)^2 + m_2 \left(\frac{m_1}{m_1+m_2} R\right)^2 \\ &= \frac{m_1 m_2 (m_1 + m_2)}{(m_1+m_2)^2} R^2 = \mu R^2 \end{aligned}$$

9. a)

$$\Delta E = \frac{\hbar^2}{2I}\left(3(4) - 2(3)\right) = \frac{3\hbar^2}{I}$$

$$I = \frac{3\hbar^2}{\Delta E} = \frac{3\left(1.055 \times 10^{-34}\ \mathrm{J \cdot s}\right)^2}{\left(1.43 \times 10^{-3}\ \mathrm{eV}\right)\left(1.602 \times 10^{-19}\ \mathrm{J/eV}\right)} = 1.46 \times 10^{-46}\ \mathrm{kg \cdot m^2}$$

b)

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{(12)(16)}{28}\mathrm{u} = 1.139 \times 10^{-26}\ \mathrm{kg}$$

Then $I = \mu R^2$, so

$$R = \sqrt{\frac{I}{\mu}} = \sqrt{\frac{1.46 \times 10^{-46}\ \mathrm{kg \cdot m^2}}{1.139 \times 10^{-26}\ \mathrm{kg}}} = 1.13 \times 10^{-10}\ \mathrm{m}$$

which is a reasonable answer.

* 10. a) The distance of each H atom from the line is $d = (0.0958\ \mathrm{nm})(\sin 52.5°) = 7.60 \times 10^{-2}$ nm. Then

$$I = 2m_{\mathrm{H}} d^2 = 2\left(1.67 \times 10^{-27}\ \mathrm{kg}\right)\left(7.60 \times 10^{-11}\ \mathrm{m}\right)^2 = 1.93 \times 10^{-47}\ \mathrm{kg \cdot m^2}$$

b)

$$E_1 = \frac{\hbar^2}{I} = \frac{\left(1.055 \times 10^{-34}\ \mathrm{J \cdot s}\right)^2}{1.93 \times 10^{-47}\ \mathrm{kg \cdot m^2}} = 5.77 \times 10^{-22}\ \mathrm{J} = 3.61\ \mathrm{MeV}$$

$$E_2 = \frac{3\hbar^2}{I} = \frac{3\left(1.055 \times 10^{-34}\ \mathrm{J \cdot s}\right)^2}{1.93 \times 10^{-47}\ \mathrm{kg \cdot m^2}} = 1.73 \times 10^{-21}\ \mathrm{J}\ = 10.81\ \mathrm{MeV}$$

c)

$$\lambda = \frac{hc}{E_1} = \frac{\left(6.626 \times 10^{-34}\ \mathrm{J \cdot s}\right)\left(2.998 \times 10^{8}\ \mathrm{m/s}\right)}{5.77 \times 10^{-22}\ \mathrm{J}} = 344\,\mu\mathrm{m}$$

11. First we need to compute the rotational inertia. Including both the nucleus and electrons we have

$$\begin{aligned} I &= \frac{2}{5} m_\alpha r_{\mathrm{nuc}}^2 + \frac{2}{5}\left(2m_e\right) a_0^2 \\ &= \frac{2}{5}\left(\left(6.64 \times 10^{-27}\ \mathrm{kg}\right)\left(1.9 \times 10^{-15}\ \mathrm{m}\right)^2 + 2\left(9.109 \times 10^{-31}\ \mathrm{kg}\right)\left(5.29 \times 10^{-11}\ \mathrm{m}\right)^2\right) \\ &= 2.04 \times 10^{-51}\ \mathrm{kg \cdot m^2} \end{aligned}$$

Notice that the nuclear contribution was negligible.

a)

$$E_{\mathrm{rot}} = \frac{\hbar^2}{I} = \frac{\left(1.055 \times 10^{-34}\ \mathrm{J \cdot s}\right)^2}{2.04 \times 10^{-51}\ \mathrm{kg \cdot m^2}} = 5.46 \times 10^{-18}\ \mathrm{J}\ = 34.1\ \mathrm{eV}$$

b) This is greater than the ionization energy for helium and not likely to be observed.

12.

$$f' = f \pm \frac{\hbar}{2\pi I}\left(2\ell + 3\right)$$

with $f = c/\lambda = 1.00 \times 10^{13}$ Hz. Thus

$$f' = 1.00 \times 10^{13}\ \mathrm{Hz}\ \pm \frac{1.055 \times 10^{-34}\ \mathrm{J \cdot s}}{2\pi\left(1.46 \times 10^{-45}\ \mathrm{kg \cdot m^2}\right)}\left(2\ell + 3\right)$$

or

$$f' = 1.00 \times 10^{13}\ \mathrm{Hz}\ \pm \left(\left(2.30 \times 10^{10}\ \mathrm{Hz}\right)\ell + 3.45 \times 10^{10}\ \mathrm{Hz}\right)$$

where ℓ is an integer and then as usual $\lambda = \dfrac{c}{f'}$.

13.

$$\Delta E = hf = \frac{\hbar^2}{2I}\left[\left(\ell + 1\right)\left(\ell + 2\right) - \ell\left(\ell + 1\right)\right]$$

Therefore we can say that for a particular transition $f = C/\mu$ where C is a constant. because $I = \mu R^2$. Then $\dfrac{df}{d\mu} = -\dfrac{C}{\mu^2}$, or. taking absolute values, $\dfrac{df}{d\mu} = C/\mu^2 = \dfrac{f}{\mu}$. Thus $\dfrac{\Delta f}{f} = \dfrac{\Delta\mu}{\mu}$ as required.

14. a) The Maxwell-Boltzmann factor is $F_{\mathrm{MB}} = A\exp\left(-E/kT\right)$. The energies of the three rotational levels are

$$E_0 = 0 \qquad E_1 = \frac{\hbar^2}{I} \qquad E_2 = \frac{3\hbar^2}{I}$$

Thus the Maxwell-Boltzmann factors are

$$l = 0: \qquad F_{\mathrm{MB}} = A$$

$\ell = 1$:

$$F_{\text{MB}} = A\exp\left(-\frac{\hbar^2}{IkT}\right) = A\exp\left(-\frac{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{(10^{-46}\ \text{kg}\cdot\text{m}^2)\,(1.381\times10^{-23}\ \text{J/K})\,(293\ \text{K})}\right) = 0.973A$$

$\ell = 2$:

$$F_{\text{MB}} = A\exp\left(-\frac{3\hbar^2}{IkT}\right) = A\exp\left(-\frac{3\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{(10^{-46}\ \text{kg}\cdot\text{m}^2)\,(1.381\times10^{-23}\ \text{J/K})\,(293\ \text{K})}\right) = 0.921A$$

b) The degeneracy factor $g(E)$ is 1 for $\ell = 0$, 3 for $\ell = 1$, and 5 for $\ell = 2$. Therefore the level populations $n(E)$ are

$$\ell = 0: \quad n(E) = g(E)F_{\text{MB}} = A$$

$$\ell = 1: \quad n(E) = g(E)F_{\text{MB}} = 3\,(0.973A) = 2.92A$$

$$\ell = 2: \quad n(E) = g(E)F_{\text{MB}} = 5\,(0.921A) = 4.61A$$

c) For the lower rotational states the degeneracy factor causes the state populations to increase with increasing ℓ. However, as ℓ increases and the rotational energy increases, the exponential factor begins to take over and decrease the state populations.

* 15. The gap between adjacent lines is $h\,(\Delta f) = \hbar^2/I$.

a)

$$I = \frac{\hbar^2}{h\,(\Delta f)} = \frac{\left(1.055\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})\,(7\times10^{11}\ \text{Hz})} = 2.4\times10^{-47}\ \text{kg}\cdot\text{m}^2$$

b) $\omega = 2\pi f = \sqrt{\kappa/\mu}$ with $f = 8.65\times10^{13}$ Hz. The reduced mass is (using the ^{35}Cl isotope)

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{35}{36}\ \text{u} = 1.614\times10^{-27}\ \text{kg}$$

Solving for κ we find

$$\kappa = 4\pi^2 f^2 \mu = 4\pi^2\left(8.65\times10^{13}\ \text{Hz}\right)^2\left(1.614\times10^{-27}\ \text{kg}\right) = 478\ \text{N/m}$$

in good agreement with Table 10.1.

16. a)

$$P = qx = \left(1.602\times10^{-19}\ \text{C}\right)\left(2.67\times10^{-10}\ \text{m}\right) = 4.28\times10^{-29}\ \text{C}\cdot\text{m}$$

b) The fractional ionic character is the ratio

$$\frac{4.28\times10^{-29}\ \text{C}\cdot\text{m}}{5.41\times10^{-29}\ \text{C}\cdot\text{m}} = 0.79$$

17. $\Delta E = hf = \dfrac{hc}{\lambda}$. Using the data from Table 10.1, the wavelength is

$$\lambda = \frac{c}{f} = \frac{2.998\times10^{8}\ \text{m/s}}{8.72\times10^{13}\ \text{Hz}} = 3.44\,\mu\text{m}$$

which corresponds to an energy of 5.78×10^{-20} J. This photon is in the infrared.

* 18. a) Using dimensional analysis and the fact that the energy of each photon is $hc/\lambda = 3.14 \times 10^{-19}$ J,

$$\frac{N}{t} = \frac{5 \times 10^{-3}\ \text{J/s}}{3.14 \times 10^{-19}\ \text{J/photon}} = 1.59 \times 10^{16}\ \text{photon/s}$$

b) 0.02 mole is equal to $0.02N_A = 1.20 \times 10^{22}$ atoms. Then the fraction participating is

$$\frac{1.59 \times 10^{16}}{1.20 \times 10^{22}} = 1.33 \times 10^{-6}$$

c) The transitions involved have a fairly low probability, even with stimulated emission. We are saved by the large number of atoms available.

19. a) The energy of each photon is

$$E_2 - E_1 = 1.15\,\text{eV/photon} \cdot \frac{1.602 \times 10^{-19}\text{J}}{1\,\text{eV}} = 1.84 \times 10^{-19}\,\text{J/photon}\ .$$

Since $P = \dfrac{E}{t}$ then we have $P = \left(1.84 \times 10^{-19}\,\text{J/photon}\right)\left(5.50 \times 10^{18}\,\text{photons/s}\right) = 1.00\,\text{W}$.

b)

$$\lambda = \frac{hc}{\Delta E} = \frac{1.986 \times 10^{-25}\ \text{J}\cdot\text{m}}{1.84 \times 10^{-19}\,\text{J}} = 1.08 \times 10^{-6}\,\text{m} = 1.08\,\mu\text{m}$$

20. With the given wavelength, we know that each photon has energy - $E = \dfrac{hc}{\lambda} = 2.88 \times 10^{-19}$ J. From the power and time pulse given, we require an energy of $E = Pt$ so $E = 500\,\text{MW}\,\left(1.0 \times 10^{-9}\,\text{s}\right) = 0.5\,\text{J}$. Therefore the number of transitions required is $N = \dfrac{0.5\,\text{J}}{2.88 \times 10^{-19}\,\text{J/photon}} = 1.74 \times 10^{18}\,\text{photons}$.

21. a) From Chapter 9 we have

$$\Delta f = \frac{f_0}{c}\sqrt{\frac{kT}{m}}$$

We also have in general $f = c/\lambda$ so $\Delta f = \left(c/\lambda^2\right)\Delta\lambda$. Therefore

$$\Delta\lambda = \frac{\lambda^2 \Delta f}{c} = \frac{\lambda^2 f_0}{c^2}\sqrt{\frac{kT}{m}} = \frac{\lambda}{c}\sqrt{\frac{kT}{m}}$$

Using the neon mass 3.32×10^{-26} kg we calculate

$$\Delta\lambda = \frac{6.328 \times 10^{-7}\ \text{m}}{2.998 \times 10^{8}\ \text{m/s}}\sqrt{\frac{(1.381 \times 10^{-23}\ \text{J/K})\,(293\ \text{K})}{3.32 \times 10^{-26}\ \text{kg}}} = 7.37 \times 10^{-13}\ \text{m}$$

b)

$$\Delta E = \frac{hc}{\lambda^2}\Delta\lambda = \frac{\hbar}{2\tau}$$

$$\Delta\lambda = \frac{\hbar\lambda^2}{2hc\tau} = \frac{\lambda^2}{4\pi c\tau} = \frac{\left(6.328 \times 10^{-7}\ \text{m}\right)^2}{4\pi\,(2.998 \times 10^{8}\ \text{m/s})\,(10^{-3}\ \text{s})} = 1.06 \times 10^{-19}\ \text{m}$$

The Doppler broadening is much more significant than the Heisenberg broadening.

22. a)

$$\Delta t = \frac{2\,(1\text{ m})}{2.998 \times 10^8\text{ m/s}} = 6.67 \times 10^{-9}\text{ s}$$

b) For the 16 km round trip

$$\Delta t = \frac{16 \times 10^3\text{ m}}{2.998 \times 10^8\text{ m/s}} - \frac{16 \times 10^3\text{ m}}{(1 - 3 \times 10^{-4})\,2.998 \times 10^8\text{ m/s}} = -1.6 \times 10^{-8}\text{ s}$$

Because this result is larger than the desired uncertainty in timing, it is important to take atmospheric effects into account.

23. In a three-level system the population of the upper level must exceed the population of the ground state. This is not necessary in a four-level system.

24. Because the 3s state has two possible configurations and the $n = 2$ level has eight, let us assign a density of states $g = 2$ to the excited state and $g = 8$ to the ground state.

a)

$$\begin{aligned}\frac{f_3}{f_2} &= \frac{2\exp(-\beta E_3)}{8\exp(-\beta E_2)} = 0.25\exp\left(-\frac{E_3 - E_2}{kT}\right)\\ &= 0.25\exp\left(-\frac{16.6\text{ eV}}{(8.617 \times 10^{-5}\text{ eV/K})\,(293\text{ K})}\right) = 7 \times 10^{-287}\end{aligned}$$

b)

$$\frac{f_3}{f_2} = 0.25\exp\left(-\frac{16.6\text{ eV}}{(8.617 \times 10^{-5}\text{ eV/K})\,(200\text{ K})}\right) = 1.2 \times 10^{-419}$$

c)

$$\frac{f_3}{f_2} = 0.25\exp\left(-\frac{16.6\text{ eV}}{(8.617 \times 10^{-5}\text{ eV/K})\,(500\text{ K})}\right) = 1.2 \times 10^{-168}$$

d) Thermal excitations can be neglected.

* 25. Using dimensional analysis the number density is

$$1980\text{ kg/m}^3\,\frac{1\text{ mol}}{0.07455\text{ kg}}\,\frac{2\,(6.022 \times 10^{23})}{\text{mol}} = 3.20 \times 10^{28}\text{ m}^{-3}$$

Therefore the distance is

$$d = \left(3.20 \times 10^{28}\text{ m}^{-3}\right)^{-1/3} = 3.15 \times 10^{-10}\text{ m} = 0.315\text{ nm}$$

* 26. Each charge has two unlike charges a distance r away, two like charges a distance $2r$ away, and so on:

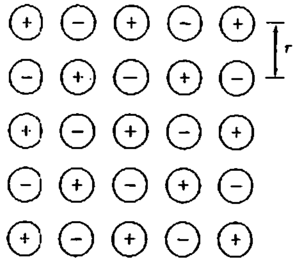
$$V = -\frac{2e^2}{4\pi\epsilon_0 r}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots\right)$$

The bracketed expression is the Taylor series expansion for $\ln 2$, so

$$V = -\frac{2e^2}{4\pi\epsilon_0 r}\ln 2 = -\frac{\alpha e^2}{4\pi\epsilon_0 r}$$

and we see that $\alpha = 2\ln 2$.

27. Using the positive central charge as a guide, we find



$$V = \frac{e^2}{4\pi\epsilon_0 r}\left(-\frac{4}{r} + \frac{4}{\sqrt{2}r} + \frac{4}{2r} - \frac{8}{\sqrt{5}r} + \frac{4}{\sqrt{8}r} - \ldots\right)$$

so by definition of α we have

$$\alpha = 4 - \frac{4}{\sqrt{2}} - 2 + \frac{8}{\sqrt{5}} - \frac{4}{\sqrt{8}} + \ldots$$

28.

$$F = -\frac{dV}{dr} = -\frac{\alpha e^2}{4\pi\epsilon_0 r^2} + \frac{\lambda}{\rho}e^{-r/\rho}$$

From Equation (10.20a) we have

$$1 = e^{r_0/\rho}\frac{\rho\alpha e^2}{4\pi\epsilon_0 r_0^2 \lambda}$$

Multiplying this factor of 1 by the last term in the force equation, we obtain

$$\begin{aligned} F &= -\frac{\alpha e^2}{4\pi\epsilon_0 r^2} + \frac{\lambda}{\rho}e^{-r/\rho}e^{r_0/\rho}\frac{\rho\alpha e^2}{4\pi\epsilon_0 r_0^2\lambda} \\ &= \frac{\alpha e^2}{4\pi\epsilon_0 r_0^2}\left(-\frac{r_0^2}{r^2} + e^{-(r-r_0)/\rho}\right) \end{aligned}$$

29. Inserting $r = r_0 + \delta r$ into the force equation from Problem 28

$$F = \frac{\alpha e^2}{4\pi\epsilon_0 r_0^2}\left(-\frac{r_0^2}{(r_0+\delta r)^2} + e^{-\delta r/\rho}\right)$$

Factoring the first term in the parentheses leaves

$$F = \frac{\alpha e^2}{4\pi\epsilon_0 r_0^2}\left(-\frac{1}{\left(1+\frac{\delta r}{r_0}\right)^2} + e^{-\delta r/\rho}\right)$$

Applying the binomial theorem

$$\left(1+\frac{\delta r}{r_0}\right)^{-2} = 1 - \frac{2}{r_0}\delta r + \frac{3}{r_0^2}(\delta r)^2 - \ldots$$

and we end the series at that point for small δr. The Taylor series for the exponential term is

$$e^{-\delta r/\rho} = 1 - \frac{\delta r}{\rho} + \frac{(\delta r)^2}{2\rho^2} - \ldots$$

Putting these two series approximations together:

$$F \approx \frac{\alpha e^2}{4\pi\epsilon_0 r_0^2}\left(-1 + \frac{2}{r_0}\delta r - \frac{3}{r_0^2}(\delta r)^2 + 1 - \frac{\delta r}{\rho} + \frac{(\delta r)^2}{2\rho^2}\right)$$

Collecting terms:

$$\begin{aligned} F &\approx \frac{\alpha e^2}{4\pi\epsilon_0 r_0^2}\left(\left(\frac{2}{r_0} - \frac{1}{\rho}\right)(\delta r) + \left(-\frac{3}{r_0^2} + \frac{1}{2\rho^2}\right)(\delta r)^2\right) \\ &\approx K_1(\delta r) + K_2(\delta r)^2 \end{aligned}$$

where

$$K_1 = \frac{\alpha e^2}{4\pi\epsilon_0 r_0^2}\left(\frac{2}{r_0} - \frac{1}{\rho}\right) \quad \text{and} \quad K_2 = \frac{\alpha e^2}{4\pi\epsilon_0 r_0^2}\left(-\frac{3}{r_0^2} + \frac{1}{2\rho^2}\right)$$

30. a) Looking at the result of the previous problem we see the spring constant is $\kappa = -K_1$, and we know that for the harmonic oscillator $\omega = \sqrt{\kappa/\mu}$. For NaCl

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = 2.32 \times 10^{-26}\ \text{kg}$$

Recall that for NaCl we know $\alpha = 1.7476$, $r_0 = 0.282$ nm, and $\rho = 0.0316$ nm. These values give

$$-K_1 = \kappa = -\frac{\alpha e^2}{4\pi\epsilon_0 r_0^2}\left(\frac{2}{r_0} - \frac{1}{\rho}\right) = 124.7\ \text{N/m}$$

Then the oscillation frequency is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{\kappa}{\mu}} = \frac{1}{2\pi}\sqrt{\frac{124.7\ \text{N/m}}{2.32 \times 10^{-26}\ \text{kg}}} = 1.17 \times 10^{13}\ \text{Hz}$$

b)

$$\lambda = \frac{c}{f} = \frac{2.998 \times 10^8\ \text{m/s}}{1.17 \times 10^{13}\ \text{Hz}} = 25.6\,\mu\text{m}$$

which is about half the observed value.

31. a)

$$\overline{F} = 0 = K_1\overline{\delta r} + K_2\overline{(\delta r)^2}$$

Therefore

$$\overline{\delta r} = -\frac{K_2}{K_1}\overline{(\delta r)^2}$$

b) From the equipartition theorem

$$K_1\overline{(\delta r)^2} = kT$$

so from (a)

$$\overline{\delta r} = \frac{K_2}{K_1^2}kT$$

The coefficient of thermal expansion α comes from $\Delta L = L\alpha\,\Delta T$, or

$$\alpha = \frac{1}{L}\frac{\Delta L}{\Delta T}$$

Therefore in our nomenclature

$$\alpha = \frac{1}{r_0}\frac{d\left(\overline{\delta r}\right)}{dT} = \frac{1}{r_0}\frac{kK_2}{K_1^2}$$

c) Evaluating with $K_1 = -124.7$ N/m and $K_2 = 2.35 \times 10^{12}$ N/m^2, we find $\alpha = 7.4 \times 10^{-6}$ K^{-1}, which is on the right order of magnitude.

* 32. We begin with Equation (10.32): $L \equiv \dfrac{K}{\sigma T} = \dfrac{4k^2}{\pi e^2}$. Solving for the thermal conductivity, K, we find

$$K = \frac{4\sigma k^2 T}{\pi e^2} = \frac{4\left(6.0 \times 10^7\,\Omega^{-1}\cdot\text{m}^{-1}\right)\left(1.3807 \times 10^{-23}\,\text{J}\cdot\text{K}^{-1}\right)^2 293\,\text{K}}{\pi\left(1.6022 \times 10^{-19}\,\text{C}\right)^2} = 166\,\text{W}\cdot\text{K}^{-1}\cdot\text{m}^{-1}$$

As mentioned in the text, the Wiedemann-Franz Law was derived using classical expressions for the mean speed and the molar heat capacity. Quantum mechanical corrections give an additional factor of $\dfrac{\pi^3}{12} = 2.58$. With this correction, our answer would be $428\,\text{W}\cdot\text{K}^{-1}\cdot\text{m}^{-1}$ which is much closer to the measured value.

33. We begin with Equation (10.26) and compare that equation to the graph given in Figure 10.23. We notice that in the figure the mean displacement is plotted on the vertical axis and the absolute temperature is on the horizontal axis. Equation (10.26) is of the form $y = mx+b$ where the the mean displacement represents the vertical or y quantity and the absolute temperature is the horizontal or x quantity. Thus the slope, m is the constant multiplying the temperature, or in this case, $m = \dfrac{3bk}{4a^2}$.

We can estimate the slope from the graph. In the region between temperatures of about 40 K and 90 K the graph is nearly linear. The slope is approximately

$$m = \frac{0.546\,\text{nm} - 0.534\,\text{nm}}{85\,\text{K} - 40\,\text{K}} = 2.67 \times 10^{-4}\,\text{nm}\cdot\text{K}^{-1}$$

From above, we also know that

$$\frac{b}{a^2} = \frac{4m}{3k} = \frac{4\left(2.67 \times 10^{-4}\,\text{nm}\cdot\text{K}^{-1}\right)}{3\left(1.3807 \times 10^{-23}\,\text{J}\cdot\text{K}^{-1}\right)} = 2.58 \times 10^{10}\,\text{N}^{-1}$$

34. a) Substituting the results from Equations (10.24) and (10.25) into Equation (10.23), we have

$$\frac{\frac{3\sqrt{\pi}}{4}ba^{-5/2}\beta^{-3/2}}{\pi^{1/2}a^{-1/2}\beta^{-1/2}} = \frac{3}{4}ba^{-2}\beta^{-1} = \frac{3bkT}{4a^2}$$

b) Let $\dfrac{3bk}{4a^2} = C_0$. Then $\langle x\rangle = C_0 T$.

$$C_0 = \frac{\Delta\langle x\rangle}{\Delta T} = \alpha x = \left(1.67 \times 10^{-5}\ \text{K}^{-1}\right)\left(8.47 \times 10^{28}\ \text{m}^{-3}\right)^{-1/3} = 3.80 \times 10^{-15}\ \text{m/K}$$

where we used the number density of copper from Chapter 9.

35. The potential energy of a harmonic oscillator would have only the quadratic term and be of the form $\frac{1}{2}\kappa x^2$. Examine Equation (10.22) and notice that a is the multiplicative constant for the x^2 term. Thus $a = (1/2)\kappa$. From Table 10.1 we see that an average value might be $\kappa \approx 1000$ N/m. Using one-half this value for a along with the definition of C_0 from the previous problem we find

$$b = \frac{4a^2C_0}{3k} = \frac{4\left(5 \times 10^2 \text{ N/m}\right)^2 \left(3.80 \times 10^{-15} \text{ m/K}\right)}{3\left(1.381 \times 10^{-23} \text{ J/K}\right)} = 9.2 \times 10^{13} \text{ N/m}^2$$

36. a) $\overline{E} = \frac{3}{2}kT$ so $T = \dfrac{2\overline{E}}{3k}$ and the ideal gas law becomes

$$PV = NkT = Nk\frac{2\overline{E}}{3k} = \frac{2N\overline{E}}{3}$$

b) From Chapter 9 $\dfrac{N}{V} = 8.47 \times 10^{28}$ m^{-3} and we know $E = \frac{3}{5}E_F = 6.76 \times 10^{-19}$ J. Thus in SI units we have

$$P = \frac{2N\overline{E}}{3V} = \frac{2}{3}\left(8.47 \times 10^{28} \text{ m}^{-3}\right)\left(6.76 \times 10^{-19} \text{ J}\right) = 3.8 \times 10^{10} \text{ N/m}^2$$

which is quite high. The ideal gas law may not be the best assumption for conduction electrons.

37. From the previous problem

$$P = \frac{2N\overline{E}}{3V} = \frac{2N}{3V}\frac{3}{5}E_F = \frac{2NE_F}{5V}$$

We must be careful, because E_F depends on the volume:

$$E_F = \frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3}$$

$$P = \frac{2N}{5V}\frac{h^2}{8m}\left(\frac{3N}{\pi V}\right)^{2/3} = \left(\frac{3}{\pi}\right)^{2/3}\frac{N^{5/3}h^2}{20m}V^{-5/3}$$

The bulk modulus is

$$\begin{aligned} B &= -V\frac{\partial P}{dV} = -V\left(\frac{3}{\pi}\right)^{2/3}\frac{N^{5/3}h^2}{20m}\left(-\frac{5}{3}\right)V^{-8/3} \\ &= \frac{5}{3}\left(\frac{3}{\pi}\right)^{2/3}\frac{N^{5/3}h^2}{20m}V^{-5/3} = \frac{5P}{3} \end{aligned}$$

Using the fact from above that $P = \dfrac{2NE_F}{5V}$ we find

$$B = \frac{5}{3}\frac{2NE_F}{5V} = \frac{2NE_F}{3V}$$

38. For silver $N/V = 5.86 \times 10^{28}\ \text{m}^{-3}$ and $E_F = 5.49$ eV.

a)

$$B = \frac{2NE_F}{3V} = \frac{2}{3}\left(5.86 \times 10^{28}\ \text{m}^{-3}\right)(5.49\ \text{eV})\left(1.602 \times 10^{-19}\ \text{J/eV}\right) = 3.44 \times 10^{10}\ \text{N/m}^2$$

b) The computed result is about one-third of the measured value.

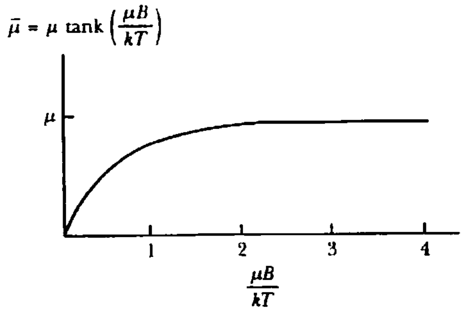
* 39. a) Following the arguments in the text, regardless of the sense of rotation of the electron, as the magnetic field increases from zero, the flux upward through the loop increases. Then, as in the text, the tangential electric field is still directed clockwise. The torque does not depend on the sense of rotation either, so the torque is directed out of the page. Thus the ΔL is out of the page which means that $\Delta\mu = -\frac{e}{2m}\Delta L = \frac{e^2 r^2 B}{4m}$ is *into* the page and thus opposite the direction of B as before. (If the reader is uncertain that the sense of rotation is irrelevant, recall that $\Phi_B = \int \vec{B} \cdot d\vec{a}$. You use the right-hand rule to determine the sense of $d\vec{a}$. While the direction of $d\vec{a}$ will depend on the sense of rotation, the other side of the equation for Faraday's law contains the expression $\oint \vec{E} \cdot d\vec{\ell}$. The sense of transit along the increment $d\vec{\ell}$ will also change.)

b) With the field directed into the paper, now the magnetic flux increases downward through the loop. Thus Faraday's law will indicate that the electric field is tangent to the orbit but now directed counterclockwise. Thus the torque will directed into the page and thus ΔL will be directed into the page. Finally the negative sign in $\Delta\mu = -\frac{e}{2m}\Delta L$ will mean that $\Delta\mu = \frac{e^2 r^2 B}{4m}$ is directed *out of* the page.

40. We use the result of the previous problem and the semi-classical model that m_ℓ represents a projection of the angular momentum vector on the z-axis and thus represents a sense of rotation. If we say that $m_\ell = +1$ represents a clockwise rotation, then $m_\ell = -1$ represents a counterclockwise rotation. Regardless of the sense of rotation, there is a diamagnetic effect for both. The electron with $m_\ell = 0$ will not contribute to the change in the magnetic moment. The angular momentum of this electron is perpendicular to the magnetic field. There is not magnetic flux through the orbit so Faraday's law indicates there will be no tangential electric field. So the total inducted magnetic moment is twice the effect of the previous problem. That is $\Delta\mu = 2\left(\frac{e^2 r^2 B}{4m}\right) = \frac{e^2 r^2 B}{2m}$.

41. This is the same as the high-field limit. With $\mu B/kT \gg 1$ we have $\tanh(\mu B/kT) \approx 1$ so $\overline{\mu} \approx \mu$.

42. a) See graph:



b)

$$\overline{\mu} = \mu \tanh(5) = 0.99991\mu$$

and the approximate result of Problem 41 is off by only 0.009%.

c)

$$\overline{\mu} = \mu \tanh(0.10) = 0.0997\mu$$

The approximate result is off by just 0.3%.

* 43. The magnetic dipole moment has units $\text{A}\cdot\text{m}^2$, so M has units $\text{A}\cdot\text{m}^2/\text{m}^3 = \text{A/m}$. μ_0 has units $\text{T}\cdot\text{m/A}$ and B has units T, so $\chi = \frac{\mu_0 M}{B}$ has units $\frac{(\text{T}\cdot\text{m/A})(\text{A/m})}{T}$ which reduces to no units.

44. a) If we assume that every atom's magnetic moment is a Bohr magneton aligned in the same direction, $M = n\mu_B$ where n is the number density.

$$n = \frac{7.87 \times 10^3 \text{ kg}}{\text{m}^3} \frac{1 \text{ mol}}{0.05585 \text{ kg}} \left(\frac{6.022 \times 10^{23}}{1 \text{ mol}}\right) = 8.49 \times 10^{28} \text{ m}^{-3}$$

Thus

$$M = n\mu_B = \left(8.49 \times 10^{28} \text{ m}^{-3}\right)\left(9.274 \times 10^{-24} \text{ J/T}\right) = 7.87 \times 10^5 \text{ A/m}$$

b) The computed value is almost exactly one-half the measured value.

c) This implies that there are two unpaired spins per atom.

* 45.

$$B_c = B_c(0)\left(1 - \left(\frac{T}{T_c}\right)^2\right) = 0.1B_c(0)$$

Thus $(T/T_c)^2 = 0.9$ and $T = \sqrt{0.9}T_c \approx 0.95T_c$.

Similarly for a ratio of 0.5 we find $T = \sqrt{0.5}T_c \approx 0.71T_c$, and for a ratio of 0.9 we find $T = \sqrt{0.1}T_c \approx 0.32T_c$.

46. The energy gap at $T = 2$ K is

$$E_g(2\text{ K}) = \frac{hc}{\lambda} = \frac{1240 \text{ eV}\cdot\text{nm}}{5.68 \times 10^5 \text{ nm}} = 2.18 \times 10^{-3} \text{ eV}$$

Inserting Equation (10.46) into Equation (10.47) gives

$$E_g(T) = 1.74\,(3.54kT_c)\left(1 - \frac{T}{T_c}\right)^{1/2}$$

Using $E_g(2\ \mathrm{K}) = 2.18 \times 10^{-3}$ eV and $k = 8.62 \times 10^{-5}$ eV/K we get

$$4.11\ \mathrm{K} = T_c\left(1 - \frac{T}{T_c}\right)^{1/2} = T_c\left(1 - \frac{2}{T_c}\right)^{1/2}$$

Solving using Mathcad we obtain $T_c = 5.23$ K which is closest to vanadium.

* 47. Using the value given in the text just below Equation (10.43), $T_c = 4.146$ K for a mass of 203.4 u, we find

$$M^{0.5}T_c = \text{ constant } = 59.1296\ \mathrm{u}^{0.5} \cdot \mathrm{K}$$

For a mass of 201 u we find $T_c = 4.171$ K and for a mass of 204 u we find $T_c = 4.140$ K.

48. With ^{16}O the molar mass in grams is

$$88.906 + 2(137.34) + 3(63.546) + 7(16.00) = 666.224$$

Replacing all the ^{16}O atoms with ^{18}O adds 14 grams per mole, changing the mass to 680.224. Using the BCS formula for the isotope effect

$$M_1^{0.5}T_{c1} = M_2^{0.5}T_{c2}$$

and assuming $T_c = 93$ K (exactly) for the first sample

$$T_{c2} = \left(\frac{M_1}{M_2}\right)^{1/2} T_{c1} = \left(\frac{666.224}{680.224}\right)^{1/2} (93\ \mathrm{K}) = 92.0\ \mathrm{K}$$

a change of 1.0 Kelvin.

49. Extrapolating on the graph in Figure 10.37, it could be at about 130 K.

* 50.

$$B = \mu_0 In = \left(4\pi \times 10^{-7}\ \mathrm{N/A^2}\right)(5.0\ \mathrm{A})\left(2500\ \mathrm{m^{-1}}\right) = 15.71\,\mathrm{mT}$$

$$\Phi = BA = \frac{B\pi d^2}{4} = \left(15.71 \times 10^{-3}\ \mathrm{T}\right)\frac{\pi\,(0.028\ \mathrm{m})^2}{4} = 9.67 \times 10^{-6}\ \mathrm{T\cdot m^2}$$

$$\frac{\Phi}{\Phi_0} = \frac{9.67 \times 10^{-6}\ \mathrm{T\cdot m^2}}{2.068 \times 10^{-15}\ \mathrm{T\cdot m^2}} = 4.7 \times 10^9 \text{ flux quanta}$$

This large number shows how small the flux quantum is.

51. We know that for niobium $B_c = 0.206$ T. Then the diameter (twice the radius) is

$$D = 2R = \frac{\mu_0 I}{\pi B} = \frac{\left(4\pi \times 10^{-7}\ \mathrm{N/A^2}\right)(4.5\ \mathrm{A})}{\pi\,(0.206\ \mathrm{T})} = 8.74 \times 10^{-6}\mathrm{m}$$

which is quite small.

52.

$$P = I^2 R = \frac{I^2 \rho L}{A}$$

where ρ is resistivity, L is length, and A is area. Now $A = \pi r^2$, so

$$P = \frac{I^2 \rho L}{\pi r^2}$$

The surface area is $2\pi r L$ so the power per unit area is

$$\frac{P}{\text{area}} = \frac{I^2 \rho L}{\pi r^2 (2\pi r L)} = \frac{I^2 \rho}{2\pi^2 r^3} = 100\ \text{W/m}^2$$

Using $r = 3.75 \times 10^{-4}$ m we find

$$I^2 = \frac{2\pi^2 \left(3.75 \times 10^{-4}\ \text{m}\right)^3 \left(100\ \text{W/m}^2\right)}{1.72 \times 10^{-8}\ \Omega \cdot \text{m}} = 6.052\ \text{A}^2$$

or $I = 2.46$ A.

53. a) From the BCS theory we have

$$B_c(4.2\ \text{K}) = B_c(0)\left(1 - \left(\frac{T}{T_c}\right)^2\right) = (0.206\ \text{T})\left(1 - \left(\frac{4.2}{9.25}\right)^2\right) = 0.1635\ \text{T}$$

From the result of Problem 51 we know that

$$I = \frac{\pi B_c D}{\mu_0} = \frac{\pi\,(0.1635\ \text{T})\left(7.5 \times 10^{-4}\ \text{m}\right)}{4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A}} = 307\ \text{A}$$

b) This is more than 100 times the current that the copper wire can carry.

54. Because f_j is directly proportional to V, the frequency is known to within one part in 10^{10}, or

$$\left(10^{-10}\right)\left(483.6 \times 10^9\ \text{Hz}\right) = 48.36\ \text{Hz}$$

which is pretty fine tuning.

55. By conversion 420 km/h = 119.44 m/s. Then from kinematics $v^2 = 2ax$, so

$$a = \frac{v^2}{2x} = \frac{(119.44\ \text{m/s})^2}{2\,(3000\ \text{m})} = 2.38\ \text{m/s}^2$$

This is about $g/4$, which would certainly be noticeable.

56. a) To compute escape speed use conservation of energy, with $\frac{1}{2}mv_{esc}^2 = \dfrac{GMm}{R_e}$:

$$v_{esc} = \sqrt{\frac{2GM}{R_e}} = \sqrt{\frac{2\left(6.673 \times 10^{-11}\ \text{m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-1}\right)\left(5.98 \times 10^{24}\ \text{kg}\right)}{6.378 \times 10^6\ \text{m}}} = 11.1\ \text{km/s}$$

b) From Chapter 9

$$\bar{v} = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{kT}{m}} = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{(1.381 \times 10^{-23}\ \text{J/K})\,(293\ \text{K})}{4\left(1.661 \times 10^{-27}\ \text{kg}\right)}} = 1245\ \text{m/s}$$

c) There are always enough helium atoms on the (high-speed) tail of the Maxwell-Boltzmann distribution that a significant number can escape, given enough time.

57. Equating the centripetal force with the Lorentz (magnetic) force we get

$$\frac{mv^2}{R} = qvB$$

or $mv = p = qBR$. The formula $p = qBR$ is also correct relativistically, and note that for these extremely high energies $E \approx pc = qBRc$. Therefore the energy is

$$\begin{aligned} E &\approx qBRc = \left(1.602 \times 10^{-19}\ \text{C}\right)(13.5\ \text{T})\left(\frac{27000\ \text{m}}{2\pi}\right)\left(2.998 \times 10^8\ \text{m/s}\right) \\ &= 2.786 \times 10^{-6}\ \text{J} = 17.4\ \text{TeV} \end{aligned}$$

* 58. a) In a RL circuit the current is

$$I = I_0 e^{-Rt/L}$$

For small values of R let us approximate the exponential with the Taylor expansion $1 - Rt/L$. Then

$$10^{-9} = 1 - \frac{I}{I_0} = 1 - e^{-Rt/L} \approx \frac{Rt}{L}$$

$$R \le 10^{-9}\frac{L}{t} = 10^{-9}\left(\frac{3.14 \times 10^{-8}\ \text{H}}{2.5\ \text{y}\ \left(3.16 \times 10^7\ \text{s/y}\right)}\right) = 4.0 \times 10^{-25}\ \Omega$$

b) For a 10% loss

$$t = \frac{0.1}{10^{-9}}(2.5\ \text{y}) = 2.5 \times 10^8\ \text{y}$$

59. From the BCS theory

$$B = B_c(0)\left(1 - \left(\frac{T}{T_c}\right)^2\right)$$

Then

$$\frac{\Delta S}{V} = -\frac{\partial}{\partial T}\left(\frac{B^2}{2\mu_0}\right) = \frac{2B_c^2(0)}{\mu_0 T_c}\left(\frac{T}{T_c} - \left(\frac{T}{T_c}\right)^3\right)$$

For numerical values use $T = 6$ K, $T_c = 9.25$ K, and $B_c(0) = 0.206$ T.

$$\frac{\Delta S}{V} = \frac{2\,(0.206\ \text{T})^2}{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(9.25\ \text{K})}\left(\frac{6}{9.25} - \left(\frac{6}{9.25}\right)^3\right) = 2743\ \text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}$$

The volume of one mole of niobium is

$$V = \frac{92.91\ \text{g}}{8.57\ \text{g/cm}^3} = 10.84\ \text{cm}^3 = 1.084 \times 10^{-5}\ \text{m}^3$$

Thus

$$\Delta S = \left(2743\ \text{J}\cdot\text{m}^{-3}\cdot\text{K}^{-1}\right)\left(1.084 \times 10^{-5}\ \text{m}^3\right) = 2.97 \times 10^{-2}\ \text{J/K}$$

for one mole of niobium. The superconducting state has a lower entropy than the normal state.

60. Provided the superconducting wire is in its superconducting state and remains in that state, the resistance is zero. Therefore, the resistance of the circuit is determined by the copper wire.

a) The current, from Ohm's law is $I = \frac{V}{R} = \frac{12.0\,\mathrm{V}}{1.50\,\Omega} = 8\,\mathrm{A}$.

b) The potential difference across the copper is 12.0 V. The superconducting wire will carry the current but have no potential difference across it.