Chapter 13

1. a) ${}^4_2$He b) ${}^1_1$H c) ${}^{16}_8$O d) ${}^{73}_{32}$Ge e) ${}^{108}_{48}$Cd f) ${}^3_2$He

2. a) ^{16}O (d,n) ^{17}F b) ^{7}Li (p,α) ^{4}He c) ^{13}C (α,p) ^{16}N d) ^{73}Ge (d,n) ^{74}As e) ^{107}Ag (^{3}He,^{4}He) ^{106}Ag f) ^{162}Dy (^{3}He,p) ^{164}Ho

* 3. Probability is equal to $nt\sigma$, or

$$nt\sigma = \frac{6.022 \times 10^{23}}{238\text{ g}} \frac{19\text{ g}}{\text{cm}^3} (3\text{ cm}) \left(0.6 \times 10^{-24}\text{ cm}^2\right) = 0.087$$

4. $p + {}^{21}$Ne, ^{3}He + ^{19}F, and ^{6}Li + ^{16}O

5. The probability is $nt\sigma$.

$$nt\sigma = \frac{6.022 \times 10^{23}}{108\text{ g}} \frac{10.5\text{ g}}{\text{cm}^3} (0.2\text{ cm}) \left(17 \times 10^{-24}\text{ cm}^2\right) = 0.199$$

6. We can write the probability of landing in a solid angle $\Delta\Omega$ as

$$\begin{aligned} \frac{d\sigma}{d\Omega}(\Delta\Omega)\, nt &= \left(0.2 \times 10^{-27}\frac{\text{cm}^2}{\text{sr}}\right)\left(3 \times 10^{-3}\text{ sr}\right)\left(\frac{6.022 \times 10^{23}}{12\text{ g}}\right)\left(\frac{100 \times 10^{-6}\text{ g}}{\text{cm}^2}\right) \\ &= 3.01 \times 10^{-12} \end{aligned}$$

The rate of incident particles is

$$\left(0.2 \times 10^{-6}\text{ C/s}\right)\left(\frac{1}{2\left(1.602 \times 10^{-19}\text{ C}\right)}\right) = 6.24 \times 10^{11}\text{ s}^{-1}$$

so the rate of detected particles is

$$\left(6.24 \times 10^{11}\text{ s}^{-1}\right)\left(3.01 \times 10^{-12}\right) = 1.88\text{ s}^{-1}$$

and the number detected in one hour is

$$1.88\text{ s}^{-1}\,(3600\text{ s}) = 6770$$

7. a) ^{16}O (n,α) ^{13}C b) ^{16}O (d,n) ^{17}F c) ^{16}O (γ,p) ^{15}N d) ^{16}O (α,p) ^{19}F e) ^{16}O $(d,{}^3\text{He})$ ^{15}N f) ^{16}O $({}^7\text{Li},p)$ ^{22}Ne

All the products listed above are stable except the one in part (b).

* 8. a)

$$Q = \left[M({}^{16}\text{O}) + M({}^2\text{H}) - M({}^4\text{He}) - M({}^{14}\text{N})\right]\text{u} \cdot c^2 = 3.11\text{ MeV (exothermic)}$$

b)

$$Q = \left[M({}^{12}\text{C}) + M({}^{12}\text{C}) - M({}^2\text{H}) - M({}^{22}\text{Na})\right]\text{u} \cdot c^2 = -7.95\text{ MeV (endothermic)}$$

c)

$$Q = \left[M({}^{23}\text{Na}) + M({}^1\text{H}) - M({}^{12}\text{C}) - M({}^{12}\text{C})\right]\text{u} \cdot c^2 = -2.24\text{ MeV (endothermic)}$$

9. b)

$$K_{\rm th} = (7.95 \text{ MeV})\left(\frac{24}{12}\right) = 15.9 \text{ MeV}$$

c)

$$K_{\rm th} = (2.24 \text{ MeV})\left(\frac{24}{23}\right) = 2.34 \text{ MeV}$$

10. In the center of mass system we require

$$9.63 \text{ MeV } + M\left({}^{16}\text{O}\right)c^2 = 9.63 \text{ MeV } + 14899.10 \text{ MeV } = 14908.73 \text{ MeV}$$

We also have

$$M\left({}^{4}\text{He}\right)c^2 + M\left({}^{12}\text{C}\right)c^2 = 3728.39 \text{ MeV } + 11177.88 \text{ MeV } = 14906.27 \text{ MeV}$$

Therefore we need

$$14908.73 \text{ MeV } - 14906.27 \text{ MeV } = 2.46 \text{ MeV}$$

in bombarding energy. In the lab frame

$$\frac{16}{12}(2.46 \text{ MeV}) = 3.28 \text{ MeV}$$

The lifetime of the state can be computed using the uncertainty principle: $\Gamma\tau = \hbar/2$ so

$$\tau = \frac{\hbar}{2\Gamma} = \frac{6.582 \times 10^{-16} \text{ eV}\cdot\text{s}}{2\left(510 \times 10^3 \text{ eV}\right)} = 6.45 \times 10^{-22} \text{ s}$$

* 11. a)

$$Q = K_y + K_Y - K_x = 1.1 \text{ MeV } + 6.4 \text{ MeV } - 5.5 \text{ MeV } = 2.0 \text{ MeV}$$

b) The Q value does not change for a particular reaction.

12.

$$Q = \left[M({}^{20}\text{Ne}) + M({}^{4}\text{He}) - M({}^{12}\text{C}) - M({}^{12}\text{C})\right]\text{u}\cdot c^2 = -4.62 \text{ MeV}$$

$$K_{\rm th} = \frac{24}{20}(4.62 \text{ MeV}) = 5.54 \text{ MeV}$$

The sum of the carbon kinetic energies will be

$$K_y + K_Y = Q + K_x = -4.62 \text{ MeV } + 45 \text{ MeV } = 40.4 \text{ MeV}$$

13. Letting $M = m_b + m_B$, $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$, and $\gamma' = \left(1 - v_{\rm cm}^2/c^2\right)^{-1/2}$, we have from conservation of energy and conservation of momentum:

$$m_A c^2 + \gamma m_a c^2 = \gamma' M c^2$$

$$\gamma m_a v = \gamma' M v_{\rm cm}$$

Squaring the second expression and dividing by c^2, we have:

$$\frac{\gamma^2 m_a^2 v^2}{c^2} = \frac{\gamma'^2 M^2 v_{\rm cm}^2}{c^2}$$

$$\gamma^2 m_a^2 \left(1-\gamma^{-2}\right) = \gamma'^2 M^2 \left(1-\gamma'^{-2}\right)$$
$$\gamma^2 m_a^2 - m_a^2 = \gamma'^2 M^2 - M^2$$

From the energy equation $\gamma' M = m_A + \gamma m_a$, so

$$\gamma^2 m_a^2 - m_a^2 = (m_A + \gamma m_a)^2 - M^2$$

After multiplying the binomial and rearranging we have

$$2\gamma m_a m_A = M^2 - m_a^2 - m_A^2$$
$$2(\gamma-1) m_a m_A = M^2 - \left(m_a^2 + 2m_A m_a + m_A^2\right) = M^2 - (m_a + m_A)^2$$

$$\begin{aligned} K_{\text{th}} &= (\gamma-1) m_a c^2 = \frac{\left(M^2-(m_a+m_A)^2\right)c^2}{2m_A} = \frac{\left[(M-(m_a+m_A))(M+(m_a+m_A))\right]c^2}{2m_A} \\ &= -Q\frac{m_A+m_a+m_B+m_b}{2m_A} \end{aligned}$$

14. The energy available for the gamma ray is

$$Q = \left[M(^{10}\text{B}) + M(n) - M(^{11}\text{B})\right] \text{u}\cdot c^2 = 11.5 \text{ MeV}$$

15. The kinetic energy is $Q + 5.0$ MeV, or

$$\begin{aligned} K &= \left[M(^{9}\text{Be}) + M(^{4}\text{He}) - M(n) - M(^{12}\text{C})\right] \text{u}\cdot c^2 + 5.0 \text{ MeV} = 5.70 \text{ MeV} + 5.0 \text{ MeV} \\ &= 10.7 \text{ MeV} \end{aligned}$$

* 16. a)

$$Q = \left[M(^{16}\text{O}) + M(^{4}\text{He}) - M(^{1}\text{H}) - M(^{19}\text{F})\right] \text{u}\cdot c^2 = -8.11 \text{ MeV}$$
$$K_{\text{th}} = \frac{20}{16}(8.11 \text{ MeV}) = 10.14 \text{ MeV}$$

b)

$$Q = \left[M(^{12}\text{C}) + M(^{2}\text{H}) - M(^{3}\text{He}) - M(^{11}\text{B})\right] \text{u}\cdot c^2 = -10.46 \text{ MeV}$$
$$K_{\text{th}} = \frac{14}{12}(10.46 \text{ MeV}) = 12.21 \text{ MeV}$$

17. a)

$$N = \frac{6.022\times10^{23}}{59 \text{ g}}\left(40\times10^{-3} \text{ g}\right) = 4.083\times10^{20}$$

activation rate $= n\sigma\phi = \left(4.083\times10^{20}\right)\left(20\times10^{-28} \text{ m}^2\right)\left(10^{18} \text{ m}^{-2}\cdot\text{s}^{-1}\right) = 8.166\times10^{11} \text{ s}^{-1}$

Then in one week the number of ^{60}Co produced is

$$8.166\times10^{11} \text{ s}^{-1}\,(7\,(86400 \text{ s})) = 4.94\times10^{17}$$

b) Because the half-life of ^{60}Co is much longer than one week, we can assume almost all the nuclei produced are still present. The activity is

$$R = \lambda N = \frac{\ln 2}{t_{1/2}}N = \frac{\ln 2}{1.66\times10^{8} \text{ s}}\left(4.94\times10^{17}\right) = 2.06\times10^{9} \text{ Bq}$$

c)

$$10^{14}\ \text{Bq}\ \frac{40 \times 10^{-3}\ \text{g}}{2.06 \times 10^{9}\ \text{Bq}} = 1942\ \text{g}$$

We would put 1.94 kg of ^{59}Co into the reactor for one week.

* 18. The equation for K_{cm} is correct because we know from classical mechanics that the system is equivalent to a mass $M_x + M_X$ moving with a speed v_{cm}^2, so the center of mass kinetic energy is

$$K_{\text{cm}} = \frac{1}{2}(M_x + M_X)\, v_{\text{cm}}^2$$

Letting $M = M_x + M_X$ we have by conservation of momentum $Mv_{\text{cm}} = M_x v_x$, or $v_{\text{cm}} = M_x v_x / M$. Thus

$$K_{\text{cm}} = \frac{1}{2} M v_{\text{cm}}^2 = \frac{1}{2} M \frac{M_x^2}{M^2} v_x^2 = \frac{1}{2}\frac{M_x^2}{M} v_x^2$$

$$K_{\text{cm}}' = K_{\text{lab}} - K_{\text{cm}} = \frac{M_x v_x^2}{2} - \frac{1}{2}\frac{M_x^2}{M} v_x^2 = \frac{M_x v_x^2}{2}\left(1 - \frac{M_x}{M}\right) = \frac{M_x v_x^2}{2}\left(\frac{M - M_x}{M}\right)$$

$$K_{\text{cm}}' = K_{\text{lab}}\left(\frac{M_X}{M_x + M_X}\right)$$

* 19.

$$K_{\text{cm}}' = \frac{M_X}{M_x + M_X} K_{\text{lab}} = \frac{14}{18}(7.7\ \text{MeV}) = 5.99\ \text{MeV}$$

$$E^* = \left[M(^{14}\text{N}) + M(^{4}\text{He}) - M(^{18}\text{F})\right] \text{u} \cdot c^2 = 4.41\ \text{MeV}$$

$$E_x = E^* + K_{\text{cm}}' = 10.40\ \text{MeV}$$

20.

$$E^* = \left[M(^{208}\text{Pb}) + M(n) - M(^{209}\text{Pb})\right] \text{u} \cdot c^2 = 3.94\ \text{MeV}$$

Because of the excited nucleus, we expect gamma decay.

21. From Chapter 12 the radius of the nucleus is

$$r\left(^{14}\text{N}\right) = (1.2\ \text{fm})\, 14^{1/3} = 2.89\,\text{fm}$$

At that radius the potential energy is

$$V = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = \frac{(2)(7)\left(1.44 \times 10^{-9}\ \text{eV} \cdot \text{m}\right)}{2.89 \times 10^{-15}\ \text{m}} = 6.98\,\text{MeV}$$

$$Q = \left[M(^{14}\text{N}) + M(^{4}\text{He}) - M(^{1}\text{H}) - M(^{17}\text{O})\right] \text{u} \cdot c^2 = -1.19\,\text{MeV}$$

$$K_{\text{exit}} = Q + K_\alpha = -1.19\,\text{MeV} + 7.7\,\text{MeV} = 6.51\,\text{MeV} = K(p) + K(^{17}\text{O})$$

Most of the energy at the forward angle will be in the proton, so there could be sufficient energy to overcome the 6.98 MeV barrier in a tunneling process (though it appears to be forbidden classically).

* 22. From Chapter 12 Problem 27 we see that the mean lifetime is

$$\tau = \frac{t_{1/2}}{\ln 2} = \frac{109 \text{ ms}}{\ln 2} = 157 \text{ ms}$$

The from Equation (13.13)

$$\Gamma = \frac{\hbar}{2\tau} = \frac{6.582 \times 10^{-16} \text{ eV} \cdot \text{s}}{2\,(0.157 \text{ s})} = 2.10 \times 10^{-15} \text{ eV}$$

^{17}Ne can decay by positron decay or electron capture.

23.

$$E^* = \left[M(^1\text{H}) + M(^{16}\text{O}) - M(^{17}\text{F})\right] \text{u} \cdot c^2 = 0.60 \text{ MeV}$$

No, this energy is greater than 0.495 MeV.

24. a)

$$E^* = \left[M(^{239}\text{Pu}) + M(n) - M(^{240}\text{Pu})\right] \text{u} \cdot c^2 = 6.53 \text{ MeV}$$

b)

$$K_{\text{cm}} = \frac{239}{240}\,(1.00 \text{ MeV}) = 1.00 \text{ MeV}$$
$$E = 6.53 \text{ MeV } + 1.00 \text{ MeV } = 7.53 \text{ MeV}$$

25.

$$d + {}^{21}\text{Ne} \rightarrow {}^{22}\text{Ne} + p \qquad d + {}^{22}\text{Ne} \rightarrow {}^{22}\text{Ne} + d \qquad d + {}^{23}\text{Na} \rightarrow {}^{22}\text{Ne} + {}^{3}\text{He}$$

$$\alpha + {}^{18}\text{O} \rightarrow {}^{22}\text{Ne} + \gamma \qquad \alpha + {}^{19}\text{F} \rightarrow {}^{22}\text{Ne} + p$$
$$\alpha + {}^{21}\text{Ne} \rightarrow {}^{22}\text{Ne} + {}^{3}\text{He} \qquad \alpha + {}^{22}\text{Ne} \rightarrow {}^{22}\text{Ne} + {}^{4}\text{He}$$

$$d + {}^{25}\text{Mg} \rightarrow {}^{22}\text{Ne} + {}^{5}\text{Li} \qquad d + {}^{26}\text{Mg} \rightarrow {}^{22}\text{Ne} + {}^{6}\text{Li}$$
$$d + {}^{27}\text{Al} \rightarrow {}^{22}\text{Ne} + {}^{7}\text{Be} \text{ (and so on)}$$

$$\alpha + {}^{23}\text{Na} \rightarrow {}^{22}\text{Ne} + {}^{5}\text{Li} \qquad \alpha + {}^{24}\text{Mg} \rightarrow {}^{22}\text{Ne} + {}^{6}\text{Be}$$
$$\alpha + {}^{27}\text{Al} \rightarrow {}^{22}\text{Ne} + {}^{9}\text{B} \text{ (and so on)}$$

26.

$$\Delta E = \left[M(^{239}\text{Pu}) + M(n) - M(^{95}\text{Zr}) - M(^{142}\text{Xe}) - 3M(n)\right] \text{u} \cdot c^2 = 183.6 \text{ MeV}$$

* 27. a)

$$m = \left(4 \times 10^{-4}\right)\left(10^6 \text{ kg}\right) = 400 \text{ kg}$$

b)

$$(400 \text{ kg})\,\frac{6.022 \times 10^{23}}{0.238 \text{ kg}} = 1.01 \times 10^{27} \text{ atoms}$$

c)

$$R = \left(0.69 \text{ kg}^{-1} \cdot \text{s}^{-1}\right)(400 \text{ kg}) = 276 \text{ Bq}$$

d)

$$\left(276 \text{ s}^{-1}\right)\frac{86400 \text{ s}}{d} = 2.38 \times 10^7 \text{ d}^{-1}$$

* 28.

$$\frac{N\left({}^{235}\mathrm{U}\right)}{N\left({}^{238}\mathrm{U}\right)} = \frac{7}{993} = \frac{N_0\left({}^{235}\mathrm{U}\right)e^{-\lambda_1 t}}{N_0\left({}^{238}\mathrm{U}\right)e^{-\lambda_2 t}}$$

where the subscripts 1 and 2 refer to the 235 and 238 isotopes, respectively.

$$\frac{N_0\left({}^{235}\mathrm{U}\right)}{N_0\left({}^{238}\mathrm{U}\right)} = \frac{7}{993}\exp\left(\left(\lambda_1 - \lambda_2\right)t\right)$$

$$\begin{aligned}(\lambda_1 - \lambda_2)\,t &= \ln 2\left(\frac{1}{t_{1/2}(235)} - \frac{1}{t_{1/2}(238)}\right)t \\ &= \ln 2\left(\frac{1}{7.04\times 10^8\ \mathrm{y}} - \frac{1}{4.47\times 10^9\ \mathrm{y}}\right)\left(2\times 10^9\ \mathrm{y}\right) = 1.659\end{aligned}$$

Then

$$\frac{N_0\left({}^{235}\mathrm{U}\right)}{N_0\left({}^{238}\mathrm{U}\right)} = \frac{7}{993}\exp\left(\left(\lambda_1 - \lambda_2\right)t\right) = \frac{7}{993}\exp\left(1.659\right) = 0.0370$$

which is more than five times higher than today. Natural fission reactors cannot operate because of the relatively low abundance of ^{235}U today.

29. Here are three possibilities, out of many:

$${}^{236}\mathrm{U} \rightarrow {}^{95}\mathrm{Y} + {}^{138}\mathrm{I} + 3n \qquad {}^{236}\mathrm{U} \rightarrow {}^{94}\mathrm{Y} + {}^{140}\mathrm{I} + 2n \qquad {}^{236}\mathrm{U} \rightarrow {}^{97}\mathrm{Y} + {}^{136}\mathrm{I} + 3n$$

Note that the fragments (two nuclei plus some free neutrons) have to add up to mass number $A = 236$ and $Z = 92$.

30.

$$10^3\ \mathrm{MWe} = \left(10^9\ \mathrm{J/s}\right)\frac{86400\ \mathrm{s/d}}{1.602\times 10^{-13}\ \mathrm{J/MeV}} = 5.39\times 10^{26}\ \mathrm{MeV/d}$$

$$\left(5.39\times 10^{26}\ \mathrm{MeV/d}\right)\left(\frac{1\ (\mathrm{fission})}{200\ \mathrm{MeV}}\right)\frac{0.235\ \mathrm{kg}}{6.022\times 10^{23}} = 1.05\,\mathrm{kg/d}$$

* 31. For uranium we assume as in the previous problem that each fission produces 200 MeV of energy.

$$(1\ \mathrm{kg})\left(\frac{6.022\times 10^{23}\ \mathrm{atoms}}{0.235\ \mathrm{kg}}\right)\frac{200\ \mathrm{MeV}}{\mathrm{atom}} = 5.13\times 10^{26}\ \mathrm{MeV}$$

Converting to kWh we find 2.30×10^7 kWh. On the other hand, for coal the conversion of 29000 Btu is 8.50 kWh. Therefore we see that fission produces over one million times more energy per kilogram of fuel.

* 32. a)

$$\frac{3}{2}kT = \frac{3}{2}\left(8.617\times 10^{-5}\ \mathrm{eV/K}\right)(300\ \mathrm{K}) = 3.88\times 10^{-2}\ \mathrm{eV}$$

b)

$$\frac{3}{2}kT = \frac{3}{2}\left(8.617\times 10^{-5}\ \mathrm{eV/K}\right)\left(15\times 10^6\ \mathrm{K}\right) = 1.94\ \mathrm{keV}$$

33. $^1H + ^{12}C \rightarrow ^{13}N + \gamma \rightarrow ^{13}C + \beta^+ + \nu$

$$Q = \left[M(^1H) + M(^{12}C) - M(^{13}C)\right] \text{u} \cdot c^2 = 4.16 \text{ MeV}$$

$^1H + ^{13}C \rightarrow ^{14}N + \gamma$

$$Q = \left[M(^1H) + M(^{13}C) - M(^{14}N)\right] \text{u} \cdot c^2 = 7.55 \text{ MeV}$$

$^1H + ^{14}N \rightarrow ^{15}O + \gamma \rightarrow ^{15}N + \beta^+ + \nu$

$$Q = \left[M(^1H) + M(^{14}N) - M(^{15}N)\right] \text{u} \cdot c^2 = 10.05 \text{ MeV}$$

$^1H + ^{15}N \rightarrow ^{12}C + ^4He$

$$Q = \left[M(^1H) + M(^{15}N) - M(^{12}C) - M(^4He)\right] \text{u} \cdot c^2 = 4.97 \text{ MeV}$$

The total Q is

$$Q = 4.16 \text{ MeV } + 7.55 \text{ MeV } + 10.05 \text{ MeV } + 4.97 \text{ MeV } = 26.73 \text{ MeV}$$

34. a) The temperature must be very high to overcome the Coulomb barrier so the two alpha particles can interact through the nuclear force. A high density is necessary so that the probability is high enough for a third alpha particle to interact with ^{8}Be before it decays.

b) We use Equation (13.7) to find the Q value for the reaction:

$$Q = \left[3M(^4He) - M(^{12}C)\right] \text{u} \cdot c^2 = [3\,(4.002603) - 12.0]\,\text{u} \cdot c^2 = 7.27\,\text{MeV}$$

35. a) The temperature must be very high to overcome the Coulomb barrier so that the two oxygen nuclei can interact through the nuclear force. As the value of Z increases, higher and higher temperatures are required.

b) We use Equation (13.7) to find the Q value for the reaction:

$$Q = \left[2M(^{16}O) - M(^{32}S)\right] \text{u} \cdot c^2 = [2\,(15.994915) - 31.972071]\,\text{u} \cdot c^2 = 16.5\,\text{MeV}$$

This includes the energy of the γ ray in the released energy.

36. a) This problem is similar to Example 13.9. First we will determine the Coulomb potential energy that must be overcome, using Equation (12.2) to determine the radius.

$$r = r_0 A^{1/3} = 1.2 \times 10^{-15}\,\text{m}\,(28)^{1/3} = 3.6 \times 10^{-15}\,\text{m}$$

Therefore the Coulomb barrier is:

$$V = \frac{q_1 q_2}{4\pi\epsilon_0\, r} = \frac{(9 \times 10^9\,\text{N} \cdot \text{m}^2/\text{C}^2)\,(14)^2\,(1.6 \times 10^{-19}\,\text{C})^2}{3.6 \times 10^{-15}\,\text{m}} = 1.3 \times 10^{-11}\,\text{J}$$

We need at least this much kinetic energy to overcome the Coulomb barrier. We set this value equal to the thermal energy, namely $\frac{3}{2}kT$.

$$T = \frac{2V}{3k} = \frac{2\,(1.3 \times 10^{-11}\,\text{J})}{3\,(1.38 \times 10^{-23}\,\text{J/K})} = 6.3 \times 10^{11}\,\text{K}$$

b) We use Equation (13.7) to find the Q value for the reaction:

$$Q = \left[2M(^{28}Si) - M(^{56}Ni)\right] \text{u} \cdot c^2 = [2\,(27.976927) - 55.942136]\,\text{u} \cdot c^2 = 10.9\,\text{MeV}$$

This includes the energy of the γ ray in the released energy.

* 37. a) This problem is similar to Example 13.9. First we will determine the Coulomb potential energy that must be overcome, using Equation (12.2) to determine the radius.

$$r = r_0 A^{1/3} = 1.2 \times 10^{-15}\,\mathrm{m}\,(12)^{1/3} = 2.7 \times 10^{-15}\,\mathrm{m}$$

Therefore the Coulomb barrier is:

$$V = \frac{q_1 q_2}{4\pi\epsilon_0\, r} = \frac{\left(9 \times 10^9\,\mathrm{N\cdot m^2/C^2}\right)(6)^2\left(1.6 \times 10^{-19}\,\mathrm{C}\right)^2}{2.7 \times 10^{-15}\,\mathrm{m}} = 3.1 \times 10^{-12}\,\mathrm{J}$$

We need at least this much kinetic energy to overcome the Coulomb barrier. We set this value equal to the thermal energy, namely $\frac{3}{2}kT$.

$$T = \frac{2V}{3k} = \frac{2\left(3.1 \times 10^{-12}\,\mathrm{J}\right)}{3\left(1.38 \times 10^{-23}\,\mathrm{J/K}\right)} = 1.5 \times 10^{11}\,\mathrm{K}$$

b) We use Equation (13.7) to find the Q value for the reaction:

$$Q = \left[2M(^{12}\mathrm{C}) - M(^{24}\mathrm{Mg})\right]\mathrm{u}\cdot c^2 = [2\,(12.0) - 23.985042]\,\mathrm{u}\cdot c^2 = 13.9\,\mathrm{MeV}$$

This includes the energy of the γ ray in the released energy.

38. If the depth is d and the radius of the earth is r and 2/3 of the surface is covered with water, then the volume of the oceans is

$$V = \frac{2}{3}\left(4\pi r^2\right)d = \frac{8}{3}\pi\left(6.37 \times 10^6\ \mathrm{m}\right)^2(3000\ \mathrm{m}) = 1.02 \times 10^{18}\ \mathrm{m}^3$$

With a density of 1000 $\mathrm{kg/m^3}$, the mass is 1.02×10^{21} kg, so the number of water molecules is

$$N = \left(1.02 \times 10^{21}\ \mathrm{kg}\right)\frac{6.022 \times 10^{23}}{0.018\ \mathrm{kg}} = 3.41 \times 10^{46}$$

With a 0.015% abundance, the number of deuterium atoms (in D_2O molecules) is

$$(2)\,3.41 \times 10^{46}\,(0.00015) = 1.02 \times 10^{43}$$

There will be at most half this number of fusions, or 5.1×10^{42}. The energy released is

$$\left(5.1 \times 10^{42}\right)(4.0\ \mathrm{MeV})\left(1.602 \times 10^{-13}\ \mathrm{J/MeV}\right) = 3.27 \times 10^{30}\ \mathrm{J}$$

39. a) We use temperatures because that is what we strive for experimentally. Kinetic energy is useful for comparison with the Coulomb barrier or Q values.

b) $K = \frac{3}{2}kT$

c)

$$T = \frac{2K}{3k} = \frac{2\,(6000\ \mathrm{eV})}{3\left(8.617 \times 10^{-5}\ \mathrm{eV/K}\right)} = 4.64 \times 10^7\ \mathrm{K}$$

40. $^4\mathrm{He} + {}^3\mathrm{He} \rightarrow {}^7\mathrm{Be} + \gamma$

$$Q = \left[M(^4\mathrm{He}) + M(^3\mathrm{He}) - M(^7\mathrm{Be})\right]\mathrm{u}\cdot c^2 = 1.59\ \mathrm{MeV}$$

$^{2}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{H} + {}^{1}\text{H}$

$$Q = \left[M(^2\text{H}) + M(^2\text{H}) - M(^3\text{H}) - M(^1\text{H})\right] \text{u} \cdot c^2 = 4.03 \text{ MeV}$$

$^{1}\text{H} + {}^{2}\text{H} \rightarrow {}^{3}\text{He} + \gamma$

$$Q = \left[M(^1\text{H}) + M(^2\text{H}) - M(^3\text{He})\right] \text{u} \cdot c^2 = 5.48 \text{ MeV}$$

$^{1}\text{H} + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma$

$$Q = \left[M(^1\text{H}) + M(^{12}\text{C}) - M(^{13}\text{N})\right] \text{u} \cdot c^2 = 1.94 \text{ MeV}$$

$^{3}\text{He} + {}^{3}\text{He} \rightarrow {}^{4}\text{He} + 2p$

$$Q = \left[M(^3\text{He}) + M(^3\text{He}) - M(^4\text{He}) - 2M(^1\text{H})\right] \text{u} \cdot c^2 = 12.9 \text{ MeV}$$

$^{7}\text{Li} + {}^{1}\text{H} \rightarrow {}^{4}\text{He} + {}^{4}\text{He}$

$$Q = \left[M(^1\text{H}) + M(^7\text{Li}) - 2M(^4\text{He})\right] \text{u} \cdot c^2 = 17.3 \text{ MeV}$$

$^{3}\text{H} + {}^{2}\text{H} \rightarrow {}^{4}\text{He} + n$

$$Q = \left[M(^3\text{H}) + M(^2\text{H}) - M(^4\text{He}) - M(n)\right] \text{u} \cdot c^2 = 17.6 \text{ MeV}$$

$^{3}\text{He} + {}^{2}\text{H} \rightarrow {}^{4}\text{He} + {}^{1}\text{H}$

$$Q = \left[M(^3\text{He}) + M(^2\text{H}) - M(^4\text{He}) - M(^1\text{H})\right] \text{u} \cdot c^2 = 18.4 \text{ MeV}$$

41. The reaction is $^{1}\text{H} + {}^{12}\text{C} \rightarrow {}^{13}\text{N} + \gamma$.

$$Q = \left[M(^{12}\text{C}) + M(^1\text{H}) - M(^{13}\text{N})\right] \text{u} \cdot c^2 = 1.94 \text{ MeV}$$

Then

$$K_{\text{th}} = -Q\left(\frac{M(^1\text{H}) + M(^{12}\text{C})}{M(^{12}\text{C})}\right)$$

which will be negative, so the threshold energy is not a concern. However, the Coulomb barrier is

$$V = \frac{q_1 q_2}{4\pi\varepsilon_0 r} = \frac{6e^2}{4\pi\varepsilon_0\left(r_p + r_0 A^{1/3}\right)} = \frac{6\left(1.44 \times 10^{-9} \text{ eV} \cdot \text{m}\right)}{\left(1.2 \times 10^{-15} \text{ m} + (1.2 \times 10^{-15} \text{ m})(12)^{1/3}\right)} = 2.19 \text{ MeV}$$

Setting this coulomb barrier energy equal to the thermal energy $\frac{3}{2}kT$, we have

$$T = \frac{2K}{3k} = \frac{2\left(2.19 \times 10^6 \text{ eV}\right)}{3\left(8.617 \times 10^{-5} \text{ eV/K}\right)} = 1.69 \times 10^{10} \text{ K}$$

42.

$$Q = \left[3M(^4\text{He}) - M(^{12}\text{C})\right] \text{u} \cdot c^2 = 7.27 \text{ MeV}$$

43. a)

$$K = \frac{3}{2}kT = \frac{3}{2}\left(8.617 \times 10^{-5}\ \text{eV/K}\right)(300\ \text{K}) = 3.88 \times 10^{-2}\ \text{eV}$$

b)

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\left(3.88 \times 10^{-2}\ \text{eV}\right)\left(1.602 \times 10^{-19}\ \text{J/eV}\right)}{1.675 \times 10^{-27}\ \text{kg}}} = 2720\ \text{m/s}$$

c)

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{\left(1.675 \times 10^{-27}\ \text{kg}\right)(2720\ \text{m/s})} = 1.45 \times 10^{-10}\ \text{m}$$

44. $R = \frac{512}{60}$ Bq $= 8.533$ Bq. The initial rate was $R_0 = 4 \times 10^5$ Bq. Over 60 days the activity has decreased to

$$R_0' = R_0 e^{-\lambda t} = \left(4 \times 10^5\ \text{Bq}\right)\exp\left(-\frac{\ln 2}{44.5\ \text{d}}(60\ \text{d})\right) = 1.571 \times 10^5\ \text{Bq}$$

Therefore the fraction f of the ring that has worn off into the oil is

$$f = \frac{R}{R_0'} = \frac{8.533\ \text{Bq}}{1.571 \times 10^5\ \text{Bq}} = 5.43 \times 10^{-5}$$

* 45. a) With a half-life of 6.01 h. not much is left after one week.

b)

$$R = R_0 e^{-\lambda t} = \left(10^{11}\ \text{Bq}\right)\exp\left(-\frac{\ln 2}{6\,\text{h}}(216\ \text{h})\right) = 1.46\,\text{Bq}$$

c)

$$R = R_0 e^{-\lambda t} = \left(0.9 \times 10^{11}\ \text{Bq}\right)\exp\left(-\frac{\ln 2}{6\,\text{h}}(96\ \text{h})\right) = 1.37 \times 10^6\ \text{Bq}$$

46. a) We need 1000 Bq at $t = 30$ minutes with $t_{1/2} = 83.1$ minutes.

$$R = R_0' e^{-\lambda t} = \left(R_0'\right)\exp\left(-\frac{\ln 2}{83.1\ \text{min}}(30\ \text{min})\right) = 1000\ \text{Bq}$$

$$R_0' = (1000\ \text{Bq})\exp\left(\frac{\ln 2}{83.1\ \text{min}}(30\ \text{min})\right) = 1284\ \text{Bq}$$

Because only 27% go to the first excited state, we need an activity $R_0 = \frac{R_0'}{0.27} = 4756$ Bq. Then $R_0 = \lambda N = N\frac{\ln 2}{t_{1/2}}$, so solving for N:

$$N = \frac{R_0 t_{1/2}}{\ln 2} = \frac{\left(4756\ \text{s}^{-1}\right)(83.1\ \text{min})(60\ \text{s/min})}{\ln 2} = 3.42 \times 10^7$$

b)

$$m = \left(3.42 \times 10^7\right)\frac{139\ \text{g}}{6.022 \times 10^{23}} = 7.89 \times 10^{-15}\ \text{g}$$

The fraction f activated is

$$f = \frac{7.89 \times 10^{-15}\ \text{g}}{(0.717)\left(10^{-8}\ \text{g}\right)} = 1.10 \times 10^{-6}$$

47. a)

$$I = (0.15)\left(4 \times 10^5\ \text{s}^{-1}\right)\left(2 \times 1.602 \times 10^{-19}\ \text{C}\right) = 1.92 \times 10^{-14}\ \text{A}$$

b) One-tenth of the above value is 1.92×10^{-15} A.

48. a)

$$R = (5000\ \text{J/s})\,\frac{1\ \text{MeV}}{1.602 \times 10^{-13}\ \text{J}}\,\frac{1\ \text{particle}}{5.3\ \text{MeV}} = 5.89 \times 10^{15}\ \text{Bq}$$

$$N = \frac{R}{\lambda} = \frac{\left(5.89 \times 10^{15}\ \text{s}^{-1}\right)(138\ \text{d})(86400\ \text{s/d})}{\ln 2} = 1.013 \times 10^{23}$$

$$m = \left(1.013 \times 10^{23}\right)\left(\frac{0.210\ \text{kg}}{6.022 \times 10^{23}}\right) = 3.53 \times 10^{-2}\ \text{kg}$$

b) For 7 kW we need a mass of $(7/5)\left(3.53 \times 10^{-2}\ \text{kg}\right) = 0.0494$ kg. Letting the initial mass be m_0, we have the usual exponential decay and so $m = m_0 \exp(-\lambda t)$, or

$$m_0 = m \exp(\lambda t) = (0.0494\ \text{kg}) \exp\left(\frac{(\ln 2)(730\ \text{d})}{138\ \text{d}}\right) = 1.93\ \text{kg}$$

49. The fraction absorbed is the patient's cross-sectional area divided by $4\pi\,(\text{distance})^2$. Note that there are two gammas per decay:

$$R = 2\left(3 \times 10^{14}\ \text{Bq}\right)\frac{0.3\ \text{m}^2}{4\pi\,(4\ \text{m})^2} = 8.95 \times 10^{11}\ \text{Bq}$$

50. From Chapter 12 the diameter of the uranium nucleus is

$$d = 2r\left({}^{238}\text{U}\right) = 2\,(1.2\ \text{fm})\,238^{1/3} = 14.87\ \text{fm}$$

The momentum of the neutron is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{14.87 \times 10^{-15}\ \text{m}} = 4.456 \times 10^{-20}\ \text{kg}\cdot\text{m/s}$$

and the kinetic energy is

$$K = \frac{p^2}{2m} = \frac{\left(4.456 \times 10^{-20}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(1.675 \times 10^{-27}\ \text{kg}\right)} = 5.927 \times 10^{-13}\ \text{J} = 3.70\ \text{MeV}$$

This is still much less than the neutron's rest energy, which justifies the non-relativistic treatment. Then in thermal equilibrium

$$T = \frac{2K}{3k} = \frac{2\left(5.927 \times 10^{-13}\ \text{J}\right)}{3\left(1.381 \times 10^{-23}\ \text{J/K}\right)} = 2.86 \times 10^{10}\ \text{K}$$

Clearly this is not a realistic temperature in a reactor, so the proposed experiment is not possible.

51. The alpha decay reaction is $^{239}\text{Pu} \rightarrow {}^{235}\text{U} + {}^{4}\text{He}$, and the energy released in each decay is

$$Q = \left[M(^{239}\text{Pu}) - M(^{235}\text{U}) - M(^{4}\text{He})\right] \text{u} \cdot c^2 = 5.25 \text{ MeV}$$

The activity of a 10 kg sample is

$$R = \lambda N = \frac{\ln 2}{(24110 \text{ y})(3.156 \times 10^7 \text{ s/y})}(10 \text{ kg})\left(\frac{6.022 \times 10^{23}}{0.239 \text{ kg}}\right) = 2.30 \times 10^{13} \text{ Bq}$$

Then the power is

$$P = (0.6)(5.25 \text{ MeV})\left(1.602 \times 10^{-13} \text{ J/MeV}\right)\left(2.30 \times 10^{13} \text{ s}^{-1}\right) = 11.6 \text{ W}$$

52. Perhaps the best source of information for this question is the Internet site for the International Union of Pure and Applied Chemistry (IUPAC) at www.iupac.org. As of early 2005, elements with $Z = 112$, 113, 114, 115, and 116 had been observed experimentally but not officially named by IUPAC.

53. A Internet search will yield current information. An article at http:www.medreviews.compdfsarticlesRICM_31_1.pdf describes recent studies in this field. In addition the April 2000 issue of Physics Today magazine describes the relevance of radioisotopes for this procedure. There is still considerable debate about the efficacy of beta as compared to gamma emitters. Although the first study by Bottcher (1994) *et al* used a gamma emitter, ^{192}Ir, recent studies have included beta emitters such as ^{32}P, ^{90}Sr, ^{90}Y, ^{188}Re, ^{99}Tc, and ^{137}Xe. Almost all of the trials have shown low-dose radiation to be effective in preventing keloids and benign vascular malformations.

* 54. a)

$$(1 \text{ kg})\frac{6.022 \times 10^{23}}{\text{mol}}\frac{1 \text{ mol}}{0.001 \text{ kg}}\frac{13.6 \text{ eV}}{1 \text{ atom}} = 8.19 \times 10^{27} \text{ eV}$$

b)

$$(1 \text{ kg})\frac{6.022 \times 10^{23}}{\text{mol}}\frac{1 \text{ mol}}{0.002 \text{ kg}}\frac{2.22 \times 10^{6} \text{ eV}}{1 \text{ atom}} = 6.68 \times 10^{32} \text{ eV}$$

c)

$$(1 \text{ kg})\frac{1 \text{ u}}{1.66 \times 10^{-27} \text{ kg}}\frac{931.49 \times 10^{6} \text{ MeV}}{\text{u}} = 5.61 \times 10^{35} \text{ eV}$$

* 55. a)

$$Q = K_{\text{out}} - K_{\text{in}} = 86.63 \text{ MeV} - 100 \text{ MeV} = -13.37 \text{ MeV}$$

This agrees very nearly with the Q computed using atomic masses (assuming all the masses are known).

$$Q = \left[M(^{18}\text{O}) + M(^{30}\text{Si}) - M(^{14}\text{O}) - M(^{34})\text{Si}\right] \text{u} \cdot c^2 = -13.27 \text{ MeV}$$

b)

$$M = -\frac{Q}{c^2} + M(^{18}\text{O}) + M(^{30}\text{Si}) - M(^{14}\text{O}) = 33.979 \text{ u}$$

which agrees very nearly with the value for the mass of ^{34}Si in Appendix 8.

56. a) The number of atoms is

$$1000\,(100\text{ kg})\,\frac{1\text{ mol}}{0.235\text{ kg}}\,\frac{6.022\times10^{23}}{\text{mol}}\,(0.04)=1.03\times10^{28}$$

b)

$$\frac{1.03\times10^{28}}{4\pi\,(6\times10^{6}\text{ m})^{2}}=2.28\times10^{13}\text{ m}^{-2}$$

Then the activity for each square meter is

$$R=\lambda N=\frac{\ln 2}{28.8\text{ y }\,(3.156\times10^{7}\text{ s/y})}\left(2.28\times10^{13}\right)=1.74\times10^{4}\text{ Bq}$$

57. a) Starting with $F(E) = (\text{const.})e^{-E/kT}E^{1/2}$ and setting $dF/dE = 0$, we find that the most probable energy is $E^{*} = \frac{1}{2}kT$.

$$E^{*}=\frac{1}{2}kT=\frac{1}{2}\left(8.621\times10^{-5}\text{ eV/K}\right)\left(2\times10^{8}\text{ K}\right)=8620\text{ eV}$$

b)

$$\frac{n(2E)}{n(E)}=\exp\left(-\beta\left(2E-E\right)\right)=\exp\left(-\beta E\right)=\exp\left(-\frac{1}{2}\right)=0.607$$

$$\frac{n(5E)}{n(E)}=\exp\left(-\beta\left(5E-E\right)\right)=\exp\left(-4\beta E\right)=\exp\left(-2\right)=0.135$$

$$\frac{n(10E)}{n(E)}=\exp\left(-\beta\left(10E-E\right)\right)=\exp\left(-9\beta E\right)=\exp\left(-\frac{9}{2}\right)=0.0111$$

58. a)

$$P=\sigma nt=\left(90\times10^{-27}\text{ cm}^{2}\right)\left(1.85\text{ g/cm}^{3}\right)(3\text{ cm})\left(6.022\times10^{23}\text{ mol}^{-1}\right)\left(\frac{1\text{ mol}}{9\text{ g}}\right)=0.0334$$

b) We know that 10^5 alpha particles must interact each second to get an activity of 10^5 Bq. With this probability the number of incident alpha particles is

$$\frac{10^{5}\text{ s}^{-1}}{0.0334}=2.99\times10^{6}\text{ s}^{-1}$$

c)

$$N=\frac{R}{\lambda}=\left(2.99\times10^{6}\text{ s}^{-1}\right)\frac{2.41\times10^{4}\text{ y}}{\ln 2}\frac{3.156\times10^{7}\text{ s}}{\text{y}}=3.28\times10^{18}$$

$$m=\left(3.28\times10^{18}\right)\frac{0.239\text{ kg}}{6.022\times10^{23}}=1.30\times10^{-6}\text{ kg}$$

which is reasonable.

* 59.

$$m(^{40}\text{K})=(70\text{ kg})\,(0.0035)\,(0.00012)=2.94\times10^{-5}\text{ kg}$$

$$N=\left(2.94\times10^{-5}\text{ kg}\right)\frac{6.022\times10^{23}}{0.040\text{ kg}}=4.43\times10^{20}$$

$$R=\lambda N=\frac{\ln 2}{1.28\times10^{9}\text{ y}}\frac{1\text{ y}}{3.156\times10^{7}\text{ s}}\left(4.43\times10^{20}\right)=7600\text{ Bq}$$

The beta activity is (7600 Bq) (0.893) = 6790 Bq.

60. We can determine the minimum mass as follows:

$$E = (15\,\text{kilotons})\left(4.2 \times 10^{12}\,\text{J/kilton}\right) = 6.3 \times 10^{13}\,\text{J}\left(\frac{1\,\text{MeV}}{1.6 \times 10^{-13}\,\text{J}}\right) = 3.94 \times 10^{26}\,\text{MeV}$$

If each fission yields 200 MeV, then we have:

$$\text{Number of fissions} = \frac{3.94 \times 10^{26}\,\text{MeV}}{200\,\text{MeV/fission}} = 1.97 \times 10^{24}\,\text{fissions}$$

which is the minimum number of fissions. This would require a mass of:

$$M = \left(1.97 \times 10^{24}\right)\left(\frac{235\,\text{u}}{\text{atom}}\right)\left(\frac{1.66 \times 10^{-27}\,\text{kg}}{\text{u}}\right) = 0.77\,\text{kg}$$

The actual mass was larger since not every nuclei fissioned.