

Chapter 16

* 1.

$$1 \text{ pc} = \frac{1 \text{ au}}{\tan 1''} (1.496 \times 10^{11} \text{ m/au}) = 3.086 \times 10^{16} \text{ m}$$

$$1 \text{ ly} = (2.9979 \times 10^8 \text{ m/s}) (365.25 \text{ d/y}) (86400 \text{ s/d}) = 9.461 \times 10^{15} \text{ m}$$

$$1 \text{ pc} = \frac{3.086 \times 10^{16} \text{ m}}{9.461 \times 10^{15} \text{ m/ly}} = 3.26 \text{ ly}$$

2. As in Example 16.3 we have $7 = \exp(\Delta mc^2/kT)$ so $\ln 7 = \Delta mc^2/kT$. Then

$$T = \frac{\Delta mc^2}{k \ln 7} = \frac{939.56563 \text{ MeV} - 938.27231 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (\ln 7)} = 7.71 \times 10^9 \text{ K}$$

3. Using $mc^2 = 135 \text{ MeV} = kT$

$$T = \frac{135 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 1.57 \times 10^{12} \text{ K}$$

From Figure 16.8 the time associated with this temperature is about 10^{-4} s .

4.

$$\frac{\text{number of d}}{\text{number of u}} = \exp\left(\frac{10 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 1.00$$

$$\frac{\text{number of u}}{\text{number of c}} = \exp\left(\frac{1300 \text{ MeV} - 10 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 1.16$$

$$\frac{\text{number of u}}{\text{number of s}} = \exp\left(\frac{115 \text{ MeV} - 10 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 1.01$$

Similarly,

$$\frac{\text{number of d}}{\text{number of s}} = \exp\left(\frac{115 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 1.01$$

$$\frac{\text{number of u or d}}{\text{number of c}} = \exp\left(\frac{1300 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 1.16$$

$$\frac{\text{number of u or d}}{\text{number of t}} = \exp\left(\frac{174000 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 5.9 \times 10^8$$

$$\frac{\text{number of u or d}}{\text{number of b}} = \exp\left(\frac{4250 \text{ MeV} - 6 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 1.64$$

$$\frac{\text{number of s}}{\text{number of c}} = \exp\left(\frac{1300 \text{ MeV} - 115 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 1.15$$

$$\frac{\text{number of s}}{\text{number of t}} = \exp\left(\frac{174000 \text{ MeV} - 115 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 5.8 \times 10^8$$

$$\frac{\text{number of s}}{\text{number of b}} = \exp\left(\frac{4250 \text{ MeV} - 115 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 1.62$$

$$\frac{\text{number of c}}{\text{number of t}} = \exp\left(\frac{174000 \text{ MeV} - 1300 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K}) (10^{14} \text{ K})}\right) = 5.1 \times 10^8$$

$$\frac{\text{number of c}}{\text{number of b}} = \exp\left(\frac{4250 \text{ MeV} - 1300 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K})(10^{14} \text{ K})}\right) = 1.41$$

$$\frac{\text{number of b}}{\text{number of t}} = \exp\left(\frac{174000 \text{ MeV} - 4250 \text{ MeV}}{(8.617 \times 10^{-11} \text{ MeV/K})(10^{14} \text{ K})}\right) = 3.6 \times 10^8$$

5. $mc^2 = kT$ so we have

$$\text{electron: } T = \frac{mc^2}{k} = \frac{0.5110 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 5.93 \times 10^9 \text{ K}$$

$$\text{muon: } T = \frac{mc^2}{k} = \frac{105.66 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 1.23 \times 10^{12} \text{ K}$$

6. As in the previous problem we have for the 5 eV mass

$$T = \frac{mc^2}{k} = \frac{5 \text{ eV}}{8.617 \times 10^{-5} \text{ eV/K}} = 5.80 \times 10^4 \text{ K.}$$

Using Figure 16.7, this temperature corresponds to a time of approximately 10 seconds. If the mass of the neutrino is $10^{-4} \text{ eV}/c^2$

$$T = \frac{mc^2}{k} = \frac{10^{-4} \text{ eV}}{8.617 \times 10^{-5} \text{ eV/K}} = 1.16 \text{ K}$$

Note this answer is lower than the present temperature!

* 7. The π^+ ($E_0 = 140 \text{ MeV}$) is more massive than the π^0 ($E_0 = 135 \text{ MeV}$), so the π^+ was formed first. With $\Delta mc^2 = k\Delta T$ we have

$$\Delta T = \frac{\Delta mc^2}{k} = \frac{5 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 5.80 \times 10^{10} \text{ K}$$

* 8. Set the deuteron binding energy 2.22 MeV equal to kT :

$$T = \frac{2.22 \text{ MeV}}{k} = \frac{2.22 \text{ MeV}}{8.617 \times 10^{-11} \text{ MeV/K}} = 2.58 \times 10^{10} \text{ K}$$

9. Set the hydrogen binding energy 13.6 eV equal to kT :

$$T = \frac{13.6 \text{ eV}}{k} = \frac{13.6 \text{ eV}}{8.617 \times 10^{-5} \text{ eV/K}} = 1.58 \times 10^5 \text{ K}$$

10.

$$\frac{0.3GN^2m^2}{V^{4/3}} = \frac{3.9\hbar^2N^{5/3}}{2mV^{5/3}}$$

$$V^{1/3} = \frac{3.9\hbar^2}{0.6m^3N^{1/3}G} = \frac{6.5\hbar^2}{N^{1/3}m^3G}$$

11. Notice that Example 16.5 indicates a neutron star with a mass of two solar masses.

$$\rho = \frac{\text{mass}}{\text{volume}} = \frac{2M}{\frac{4}{3}\pi R^3} = \frac{3M}{2\pi R^3}$$

where $M = 1.99 \times 10^{30}$ kg (mass of sun) and $R = 11$ km.

$$\rho = \frac{3M}{2\pi R^3} = \frac{3(1.99 \times 10^{30} \text{ kg})}{2\pi (11 \times 10^3 \text{ m})^3} = 7.14 \times 10^{17} \text{ kg/m}^3$$

For the nucleon we have

$$\rho = \frac{m_p}{\frac{4}{3}\pi r_0^3} = \frac{3(1.67 \times 10^{-27} \text{ kg})}{4\pi (1.2 \times 10^{-15} \text{ m})^3} = 2.31 \times 10^{17} \text{ kg/m}^3$$

and the neutron star is about three times as dense.

12. We see from Equation (16.16) that $V^{1/3}$ is proportional to $N^{-1/3}$. However, $V^{1/3}$ is proportional to R , so R is proportional to $N^{-1/3}$. The reason this makes sense is that the gravitational pressure increases in proportion to N^2 [see Equation (16.12)] while the neutron pressure increase in proportion to $N^{5/3}$ [see Equation (16.14)]. The gravitational pressure increases more rapidly than the neutron pressure so that the radius of the neutron star decreases as the number of neutrons increases.

13. a)

$$\begin{aligned} P &= 0.3G \frac{(Nm)^2}{V^{4/3}} = 0.3G \frac{M^2}{\left(\frac{4}{3}\pi R^3\right)^{4/3}} \\ &= 0.3(6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}) \frac{(1.99 \times 10^{30} \text{ kg})^2}{\left(\frac{4}{3}\pi (6.96 \times 10^8 \text{ m})^3\right)^{4/3}} = 5.00 \times 10^{13} \text{ N/m}^2 \end{aligned}$$

- b)

$$\begin{aligned} P &= 0.3G \frac{M^2}{\left(\frac{4}{3}\pi R^3\right)^{4/3}} \\ &= 0.3(6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}) \frac{4(1.99 \times 10^{30} \text{ kg})^2}{\left(\frac{4}{3}\pi (1.1 \times 10^4 \text{ m})^3\right)^{4/3}} = 3.21 \times 10^{33} \text{ N/m}^2 \end{aligned}$$

- * 14. We begin with Equation (16.1).

$$v = HR = \frac{71 \text{ km/s}}{\text{Mpc}} (4 \times 10^9 \text{ ly}) \left(\frac{1 \text{ Mpc}}{3.26 \times 10^6 \text{ ly}} \right) = 8.71 \times 10^4 \text{ km/s} = 0.29 c$$

- * 15.

$$R = \frac{v}{H} = \frac{15000 \text{ km/s}}{71 \text{ km/s/Mpc}} = 211 \text{ Mpc or about } 689 \text{ Mly}$$

16. a) From Equation (16.18) we see that for a redshift of 3.8 we have $\Delta\lambda/\lambda_0 = 3.8$, so

$$3.8 = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

which can be solved to find $\beta = 0.917$. Then

$$R = \frac{v}{H} = \frac{0.917 (299790 \text{ km/s})}{71 \text{ km/s/Mpc}} = 3872 \text{ Mpc}$$

b) $3872 \text{ Mpc} \left(\frac{3.26 \text{ Mly}}{1 \text{ Mpc}} \right) = 12.6 \text{ Gly}$

- * 17. a) From Equation (16.18) we see that for a redshift of 10 we have $\Delta\lambda/\lambda_0 = 10$, so

$$10 = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

which can be solved to find $\beta = 0.984$ or $v = 0.984c$.

b)

$$R = \frac{v}{H} = \frac{0.984 (299790 \text{ km/s})}{71 \text{ km/s/Mpc}} = 4155 \text{ Mpc}$$

$$4155 \text{ Mpc} \left(\frac{3.26 \text{ Mly}}{1 \text{ Mpc}} \right) = 13.5 \text{ Gly}$$

18. a) Type I supernova have no hydrogen spectral lines. Type Ia has strong silicon spectral lines but no helium lines. Type Ib has very weak or no silicon lines and has neutral helium lines. Type Ic has weak or no helium lines and weak or no silicon lines.

b) All supernovas other than Type Ia are caused by gravitational contraction of stars much more massive than our Sun.

19. a)

$$\left(3 \times 10^{-28} \text{ kg/m}^3 \right) \left(\frac{1 \text{ nucleon}}{1.67 \times 10^{-27} \text{ kg}} \right) = 0.18 \text{ nucleons/m}^3$$

b)

$$R = (16.6 \text{ ly}) (9.46 \times 10^{15} \text{ m/ly}) = 1.57 \times 10^{17} \text{ m}$$

$$60 (1.99 \times 10^{30} \text{ kg}) \frac{1 \text{ nucleon}}{1.67 \times 10^{-27} \text{ kg}} = 7.15 \times 10^{58} \text{ nucleons}$$

The nucleon density is the number of nucleons divided by the volume, or

$$\frac{7.15 \times 10^{58}}{\frac{4}{3}\pi (1.57 \times 10^{17} \text{ m})^3} = 4.41 \times 10^6 \text{ nucleons/m}^3$$

The nucleon density in our neighborhood is much larger than it is for the universe as a whole.

20. a) The solid blue curve represents the current, best idea of the expansion of the universe. The expansion of the universe is accelerating. Only 30% of the mass is known in the universe. Most of the mass is represented by dark energy. The solid black curve represents the known mass of the universe. It is an open, low density universe, and the expansion of the universe is slowing down. The blue dashed curve represents a flat, critical density universe. The expansion rate slows down until the curve becomes more horizontal. The black dashed curve represents a closed, high density universe which will expand for a few more billion years, but eventually will turn around and collapse. There is not enough mass in the universe for this to happen.

b) Yes, the black dashed curve represents a closed universe. See the description in (a).

21.

$$H = (22 \text{ km/s/Mly}) \frac{1 \text{ Mly}}{10^6 (9.461 \times 10^{12} \text{ km})} = 2.32 \times 10^{-18} \text{ s}^{-1}$$

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3 (2.32 \times 10^{-18} \text{ s}^{-1})^2}{8\pi (6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})} = 9.63 \times 10^{-27} \text{ kg/m}^3 = 9.63 \times 10^{-30} \text{ g/cm}^3$$

22. From Wien's law $\lambda_{\text{max}} T = 2.898 \times 10^{-3} \text{ m} \cdot \text{K}$.

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3} \text{ m} \cdot \text{K}}{2.725 \text{ K}} = 1.06 \text{ mm}$$

- * 23. For $H = 55 \text{ km/s/Mpc}$ we have

$$H = (55000 \text{ m/s/Mpc}) \left(\frac{1 \text{ Mpc}}{3.086 \times 10^{22} \text{ m}} \right) = 1.78 \times 10^{-18} \text{ s}^{-1}$$

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3 (1.78 \times 10^{-18} \text{ s}^{-1})^2}{8\pi (6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})} = 5.67 \times 10^{-27} \text{ kg/m}^3$$

For $H = 85 \text{ km/s/Mpc}$ we have

$$H = (85000 \text{ m/s/Mpc}) \left(\frac{1 \text{ Mpc}}{3.086 \times 10^{22} \text{ m}} \right) = 2.75 \times 10^{-18} \text{ s}^{-1}$$

$$\rho_c = \frac{3H^2}{8\pi G} = \frac{3 (2.75 \times 10^{-18} \text{ s}^{-1})^2}{8\pi (6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})} = 1.35 \times 10^{-26} \text{ kg/m}^3$$

24. a) The only combination of the constants that gives time is $m = \frac{1}{2}$, $n = \frac{1}{2}$, and $l = -\frac{5}{2}$.

b)

$$t_p = \sqrt{\frac{Gh}{c^5}} = \sqrt{\frac{(6.67 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2}) (6.626 \times 10^{-34} \text{ J} \cdot \text{s})}{(2.998 \times 10^8 \text{ m/s})^5}} = 1.35 \times 10^{-43} \text{ s}$$

25. A complete derivation can be found in Section 2.10.

26. Let dm be the mass of a spherical shell of thickness dr , so $dm = \rho d(\text{vol}) = 4\pi\rho r^2 dr$.

$$dV = -\frac{GM' dm}{r}$$

where $M' = \frac{4}{3}\rho\pi r^3$ is the mass inside radius r . Thus

$$dV = -G\rho^2 \frac{16\pi^2}{3} r^4 dr$$

$$V = \int_0^R dV = -\frac{16\pi^2\rho^2 R^5 G}{15}$$

Now $\rho = 3M/4\pi R^3$, so

$$V = -\frac{16\pi^2\rho^2 R^5 G}{15} \left(\frac{9M^2}{16\pi^2 R^6} \right) = -\frac{3GM^2}{5R}$$

27. From the slopes it appears that $t_0 \approx \tau/2$.

28. In general, as in Example 16.7, we have $\Delta t = \left(\frac{d}{2c} \right) \left(\frac{mc^2}{E} \right)^2$. For a distance of 50 kpc

$$\frac{d}{2c} = \frac{(50 \text{ kpc})}{2(3.00 \times 10^8 \text{ m/s})} \frac{3.086 \times 10^{19} \text{ m}}{\text{kpc}} = 2.57 \times 10^{12} \text{ s}$$

Note that $2.57 \times 10^{12} \text{ s} = (2.57 \text{ s}) (10 \text{ MeV}/10 \text{ eV})^2$, so for any distance (in kpc)

$$\Delta t = (2.57 \text{ s}) \left(\frac{\text{distance}}{50 \text{ kpc}} \right) \left(\frac{m_\nu c^2}{10 \text{ eV}} \right)^2 \left(\frac{10 \text{ MeV}}{E} \right)^2$$

29. We know from Section 9.7 [see the comment after Equation (9.56)] that the energy flux rate $\Phi = \sigma T^4$ is related to the energy density ρ_e by a factor of $c/4$, such that $\Phi = (c/4)\rho_e$. But using $E = mc^2$ we have $\rho_e = \rho_{\text{rad}}c^2$, so

$$\sigma T^4 = \frac{c}{4}\rho_e = \frac{c}{4}\rho_{\text{rad}}c^2$$

or, rearranging

$$\rho_{\text{rad}} = \frac{4\sigma T^4}{c^3}$$

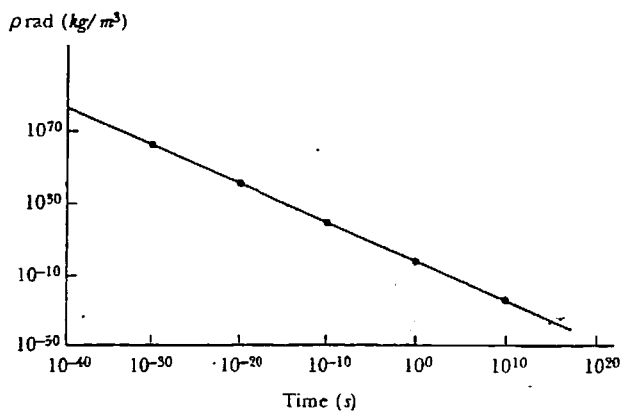
- 30.

$$\rho_{\text{rad}} = \frac{4\sigma T^4}{c^3} = \frac{4(5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4})(2.725 \text{ K})^4}{(3.00 \times 10^8 \text{ m/s})^3} = 4.63 \times 10^{-31} \text{ kg/m}^3$$

This is substantially less than the matter density of about $3 \times 10^{-28} \text{ kg/m}^3$, so the universe is matter dominated.

31. At 400,000 years the temperature is about 10^2 K. Then

$$\rho_{\text{rad}} = \frac{4\sigma T^4}{c^3} = \frac{4(5.67 \times 10^{-8} \text{ W} \cdot \text{m}^{-2} \cdot \text{K}^{-4})(10^2 \text{ K})^4}{(3.00 \times 10^8 \text{ m/s})^3} = 8.4 \times 10^{-25} \text{ kg/m}^3$$



32. The 300 day mark is almost exactly 200 days after the peak, so use $t = 200$ d in the exponential decay formula $\exp(-(\ln 2)t/t_{1/2})$.

$$^{56}\text{Ni}: \exp(-(\ln 2)t/t_{1/2}) = \exp(-(\ln 2)(200 \text{ d}) / (6.1 \text{ d})) = 1.35 \times 10^{-10}$$

$$^{56}\text{Co}: \exp(-(\ln 2)t/t_{1/2}) = \exp(-(\ln 2)(200 \text{ d}) / (77.1 \text{ d})) = 0.166$$

The cobalt must be primarily responsible, with a small contribution from the nickel.

* 33.

$$\text{Redshift} = \frac{\Delta\lambda}{\lambda_0} = \frac{582.5 \text{ nm} - 121.6 \text{ nm}}{121.6 \text{ nm}} = 3.79$$

$$3.79 = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

so $\beta = 0.92$ and $v = 0.92c$.

* 34. From the Doppler effect we know

$$\frac{\lambda}{\lambda_0} = \sqrt{\frac{1+\beta}{1-\beta}} = \frac{\lambda_0 + \Delta\lambda}{\lambda_0} = 1 + \frac{\Delta\lambda}{\lambda_0} = 1 + z$$

35. Using the binomial expansion on the result of the previous problem

$$1 + z \approx \left(1 + \frac{\beta}{2} + \dots\right) \left(1 + \frac{\beta}{2} + \dots\right) \approx 1 + \beta$$

so we see that to first order $z = \beta$.

36.

$$5.34 = \sqrt{\frac{1+\beta}{1-\beta}} - 1$$

so $\beta = 0.951$ and $v = 0.951c$.

$$R = \frac{v}{H} = \frac{0.951 (299790 \text{ km/s})}{71 \text{ km/s/Mpc}} = 4016 \text{ Mpc}$$

37.

$$Q = (2m_p - m_d - m_e) u \cdot c^2 = 0.43 \text{ MeV}$$

There are three particles in the final state, but it is possible for the deuteron and positron to have negligible energy, in which case the neutrino energy is 0.43 MeV.

* 38. We begin with Equation (16.21):

$$\begin{aligned} \rho_c &= \frac{3H^2}{8\pi G} = \left(\frac{H \text{ in (km/s)/Mpc}}{100} \right)^2 \frac{3 \times 10^4}{8\pi (6.6726 \times 10^{-11} \text{ m}^3 \cdot \text{kg}^{-1} \cdot \text{s}^{-2})} \\ &= \left(\frac{H}{100} \right)^2 (1.789 \times 10^{13} \text{ km}^2 \cdot \text{s}^{-2} \cdot \text{Mpc}^{-2} \cdot \text{m}^{-3} \cdot \text{kg}^{-1} \cdot \text{s}^2) \left(\frac{\text{Mpc}}{3.086 \times 10^{22} \text{ m}} \right)^2 \left(10^3 \frac{\text{m}}{\text{km}} \right)^2 \\ &= \left(\frac{H}{100} \right)^2 (1.88 \times 10^{-26} \text{ kg/m}^3) = \left(\frac{H}{100} \right)^2 (1.88 \times 10^{-29} \text{ g/m}^3) \end{aligned}$$

$$39. \text{ a) } a = Ct^n \quad da/dt = nCt^{n-1} \quad d^2a/dt^2 = n(n-1)Ct^{n-2}$$

$$q = -a \frac{d^2a/dt^2}{(da/dt)^2} = -\frac{an(n-1)Ct^{n-2}}{n^2C^2t^{2n-2}} = -\frac{an(n-1)}{n^2Ct^n} = -\frac{n(n-1)}{n^2}$$

where in the last step we used the definition $a = Ct^n$. From this we see that there is deceleration ($q > 0$) only if $0 < n < 1$.

b)

$$H = \frac{1}{a} \frac{da}{dt} = \frac{nCt^{n-1}}{Ct^n} = \frac{n}{t}$$

with $0 < n < 1$. There is an inverse dependence on time.

40. a)

$$\frac{X_n}{X_p} = \frac{\exp(-E_n/kT)}{\exp(-E_p/kT)} = \exp[-(E_n - E_p)/kT] = \exp[-(m_n - m_p)c^2/kT]$$

where in last step uses the fact that since $E = K + mc^2$, the kinetic energy term will cancel. The difference in rest energy equals 1.3 MeV so we have

$$\frac{X_n}{X_p} = \exp(-1.3 \text{ MeV}/kT)$$

b)

$$\frac{X_n}{X_p} = \frac{1}{6.7} = \exp(-1.3 \text{ MeV}/kT)$$

Take the natural log of both sides and solve for kT . We find $kT = 0.68 \text{ MeV}$ and thus $T = 7.9 \times 10^9 \text{ K}$.