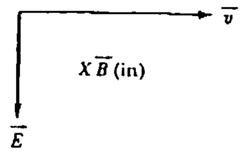
Chapter 3

1. The required field is homogeneous within the desired region and decreases in magnitude to zero as rapidly as possible outside that region. The magnitude of the field is $B = E/v_0$. The best design is an electromagnet with flat, parallel pole faces that are large compared with the distance between them. But no matter what the design, it is impossible to eliminate edge effects.

2. $eE = evB$

$$B = \frac{E}{v} = \frac{(2 \times 10^5 \text{ V/m})}{(2 \times 10^6 \text{ m/s})} = 0.10 \text{ T}$$



3. Assume the speed is exact. Non-relativistically use the energy $eV = \frac{1}{2}mv^2$

$$V = \frac{mv^2}{2e} = \frac{(9.1094 \times 10^{-31} \text{ kg})\left(2.00 \times 10^7 \text{ m/s}\right)^2}{2\left(1.6022 \times 10^{-19} \text{ C}\right)} = 1137.1 \text{ V}$$

Relativistically $eV = K = (\gamma - 1)mc^2$

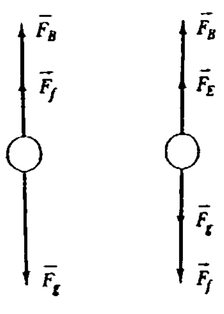
$$V = \left[\frac{1}{\sqrt{1 - \left(\frac{2.00 \times 10^7 \text{ m/s}}{2.9979 \times 10^8 \text{ m/s}}\right)^2}} - 1\right]\left[\frac{511 \text{ keV}}{e}\right] = 1141.0 \text{ V}$$

The results differ by about 4 volts, or about 0.34%. Relativity is required only if that level of precision is needed.

* 4. $eE = evB$ so $E = vB = \left(5.0 \times 10^6 \text{ m/s}\right)\left(1.3 \times 10^{-2} \text{ T}\right) = 6.50 \times 10^4 \text{ V/m}$

$$\begin{aligned} y &= \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{F}{m}\right)\left(\frac{\ell}{v_0}\right)^2 = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{\ell}{v_0}\right)^2 = \frac{eE\ell^2}{2mv_0^2} \\ &= \frac{\left(1.602 \times 10^{-19} \text{ C}\right)\left(6.50 \times 10^4 \text{ V/m}\right)(0.02 \text{ m})^2}{2\left(9.109 \times 10^{-31} \text{ kg}\right)\left(5.0 \times 10^6 \text{ m/s}\right)^2} = 9.1452 \times 10^{-2} \text{ m} = 9.15 \text{ cm} \end{aligned}$$

5. The forces are shown below.



(a) (b)

6. At terminal velocity the net force is zero, so $F_f = fv_t = mg$ and $v_t = mg/f$.

* 7.

$$v_t = \frac{mg}{f} = \frac{mg}{6\pi\eta r} \qquad m = \rho\,(\text{volume}) = \frac{4}{3}\pi\rho r^3$$

$$v_t = \left(\frac{4}{3}\pi\rho r^3\right)\left(\frac{g}{6\pi\eta r}\right) = \frac{2g\rho r^2}{9\eta}$$

Solving for r

$$r = 3\sqrt{\frac{\eta v_t}{2g\rho}}$$

8. a)

$$r = 3\sqrt{\frac{\eta v_t}{2g\rho}} = 3\sqrt{\frac{(1.82\times10^{-5}\ \text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1})(1.3\times10^{-3}\ \text{m/s})}{2\left(9.80\ \text{m/s}^2\right)\left(900\ \text{kg/m}^3\right)}} = 3.47\mu\text{m}$$

b)

$$m = \rho V = \frac{4}{3}\pi\rho r^3 = \frac{4}{3}\pi\left(900\ \text{kg/m}^3\right)\left(3.47\times10^{-6}\ \text{m}\right)^3 = 1.58\times10^{-13}\ \text{kg}$$

c)

$$f = \frac{mg}{v_t} = \frac{(1.58\times10^{-13}\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{1.3\times10^{-3}\ \text{m/s}} = 1.19\times10^{-9}\ \text{kg/s}$$

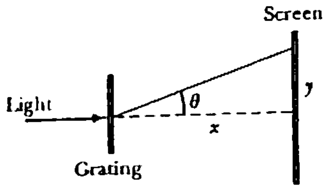
* 9. Lyman:

$$\lambda = \left[R_H\left(1-\frac{1}{\infty^2}\right)\right]^{-1} = R_H^{-1} = \left(1.096776\times10^7\ \text{m}^{-1}\right)^{-1} = 91.2\ \text{nm}$$

Balmer:

$$\lambda = \left[R_H\left(\frac{1}{2^2}-\frac{1}{\infty^2}\right)\right]^{-1} = 4R_H^{-1} = 4\left(1.096776\times10^7\ \text{m}^{-1}\right)^{-1} = 364.7\ \text{nm}$$

10. $d = \left(400\ \text{mm}^{-1}\right)^{-1} = 2.5\,\mu\text{m}$ and $\lambda = d\sin\theta$ in first order. Also $\tan\theta = y/x$ so $y = x\tan\theta = (2.5\ \text{m})\tan\left[\sin^{-1}(\lambda/d)\right]$



Red:

$$y = (2.5\ \text{m})\tan\left[\sin^{-1}\left(\frac{656.5\ \text{nm}}{2500\ \text{nm}}\right)\right] = 68.0\ \text{cm}$$

Blue-green:

$$y = (2.5\ \text{m})\tan\left[\sin^{-1}\left(\frac{486.3\ \text{nm}}{2500\ \text{nm}}\right)\right] = 49.6\ \text{cm}$$

Violet:

$$y = (2.5\ \text{m})\tan\left[\sin^{-1}\left(\frac{434.2\ \text{nm}}{2500\ \text{nm}}\right)\right] = 44.1\ \text{cm}$$

11. $d = \left(420\ \text{mm}^{-1}\right)^{-1} = 2.381\,\mu\text{m}$ $\lambda = 656.5$ nm for red

We know $n\lambda = d\sin\theta$ so $\theta = \sin^{-1}\left(\frac{n\lambda}{d}\right)$. For $n = 1$ (first order) we find

$$y = x\tan\theta = (2.5\text{ m})\tan\left[\sin^{-1}\left(\frac{656.5\text{ nm}}{2381\text{ nm}}\right)\right] = 71.7\text{ cm}$$

Similarly for $n = 2$ we find $y = 165.3$ cm and for $n = 3$ we find $y = 368.0$ cm. Therefore the separations are: between $n = 1$ and $n = 2$, $\Delta y = 165.3\text{ cm} - 71.7\text{ cm} = 93.6$ cm; between $n = 2$ and $n = 3$, $\Delta y = 368.0\text{ cm} - 165.3\text{ cm} = 202.7$ cm.

12. Use Equation (3.13) with $n = 4$ for the Brackett series and $n = 5$ for the Pfund series. The largest wavelengths occur for the smallest values of k.
Brackett:

$$\lambda = \left[R_H\left(\frac{1}{4^2} - \frac{1}{k^2}\right)\right]^{-1} = \left[\left(1.096776\times 10^7\text{ m}^{-1}\right)\left(\frac{1}{4^2} - \frac{1}{k^2}\right)\right]^{-1}$$

For $k = 5$ $\lambda = 4.052\,\mu$m; for $k = 6$ $\lambda = 2.626\,\mu$m; for $k = 7$ $\lambda = 2.166\,\mu$m; for $k = 8$ $\lambda = 1.945\,\mu$m.

Pfund:

$$\lambda = \left[R_H\left(\frac{1}{5^2} - \frac{1}{k^2}\right)\right]^{-1} = \left[\left(1.096776\times 10^7\text{ m}^{-1}\right)\left(\frac{1}{5^2} - \frac{1}{k^2}\right)\right]^{-1}$$

For $k = 6$ $\lambda = 7.460\,\mu$m; for $k = 7$ $\lambda = 4.654\,\mu$m; for $k = 8$ $\lambda = 3.741\,\mu$m; for $k = 9$ $\lambda = 3.297\,\mu$m.

* 13. Beginning with Equation (3.10) with $n = 1$, we have $\lambda = d\sin\theta$ with $d = 0.20$ mm. Therefore $\frac{d\lambda}{d\theta} = d\cos\theta$. Assuming the spectra was viewed in the forward direction, then $\theta \approx 0$ and $\cos\theta \approx 1$ so $\frac{d\lambda}{d\theta} \approx \frac{\Delta\lambda}{\Delta\theta} = d$. An angle of 0.50 minutes of arc corresponds to 1.45×10^{-4} rad so

$$\Delta\lambda = d\Delta\theta = \left(0.20\times 10^{-3}\text{ m}\right)1.45\times 10^{-4}\text{ rad} = 29\text{ nm}.$$

14. a) Use Equation (3.13) $\frac{1}{\lambda} = R_H\left(\frac{1}{n^2} - \frac{1}{k^2}\right)$ with $R_H = 1.096776\times 10^7\text{ m}^{-1}$ and $n = 2$ for the Balmer series. With $k = 3$ we find

$$\frac{1}{\lambda} = 1.096776\times 10^7\text{m}^{-1}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 656.47\text{ nm}.$$

This wavelength, found with the smallest value of k, is referred to as the hydrogen alpha or H_α line. Using the equation above with $k = 4$ we find the hydrogen beta or H_β wavelength equals 486.27 nm. Similarly with $k = 5$ the hydrogen gamma or H_γ wavelength equals 434.17 nm and finally with $k = 6$ the hydrogen delta or H_δ wavelength equals 410.29 nm.

b) As only three wavelengths are observed, the source must moving relative to the detector. The wavelengths have increased, so the source must be moving away from the detector. (Refer to equation (2.34) for example.) The H_α line at 656.47nm is redshifted out of the visible.

c) Since the wavelengths are known and $c = \lambda f$, we will use the reciprocal of Equation (2.33). We select this equation since the wavelengths are larger and thus the source is receding from the observer. $\dfrac{\lambda_{\text{observed}}}{\lambda_{\text{source}}} = \sqrt{\dfrac{1+\beta}{1-\beta}}$. Using algebra to solve the equation for β and substituting the smallest wavelength gives

$$\beta = \frac{\left(\frac{\lambda_{\text{observed}}}{\lambda_{\text{source}}}\right)^2 - 1}{\left(\frac{\lambda_{\text{observed}}}{\lambda_{\text{source}}}\right)^2 + 1} = \frac{\left(\frac{453.4\,\text{nm}}{410.29\,\text{nm}}\right)^2 - 1}{\left(\frac{453.4\,\text{nm}}{410.29\,\text{nm}}\right)^2 + 1} = 0.0995$$

Therefore the speed is $v = 0.10\,c$ or $v = 3.0 \times 10^7$ m/s. Using other pairs of wavelengths gives a similar result. If the object is rotating, then one side would be moving toward and another side away from the detector so a range of wavelengths would be observed. This effect can be used to determine the rotation speeds.

15. a) Use Equation (3.13) $\dfrac{1}{\lambda} = R_{\text{H}}\left(\dfrac{1}{n^2} - \dfrac{1}{k^2}\right)$ with $R_{\text{H}} = 1.096776 \times 10^7\,\text{m}^{-1}$ and $n = 3$ for the Paschen series. With $k = 4$ we find

$$\frac{1}{\lambda} = 1.096776 \times 10^7\text{m}^{-1}\left(\frac{1}{3^2} - \frac{1}{4^2}\right); \quad \lambda = 1875.63\,\text{nm}.$$

With $k = 5$ $\lambda = 1282.17\,\text{nm}$; with $k = 6$ $\lambda = 1094.12\,\text{nm}$; with $k = 7$ $\lambda = 1005.22\,\text{nm}$; with $k = 8$ $\lambda = 954.86\,\text{nm}$;

b) The observed spectral lines have been Doppler shifted. It might appear as if the wavelengths have been blueshifted since the largest observed wavelength is *smaller* than the largest expected wavelength using the Paschen series. However it is more likely that the wavelengths have been redshifted and the calculated wavelength just below 1000 nm in part a) corresponds to the observed wavelength of 1046.1 nm.

c) Using the formula from problem 14 and noting from part b) that the star is receding from the detector

$$\beta = \frac{\left(\frac{\lambda_{\text{observed}}}{\lambda_{\text{source}}}\right)^2 - 1}{\left(\frac{\lambda_{\text{observed}}}{\lambda_{\text{source}}}\right)^2 + 1} = \frac{\left(\frac{1334.5\,\text{nm}}{1282.17\,\text{nm}}\right)^2 - 1}{\left(\frac{1334.5\,\text{nm}}{1282.17\,\text{nm}}\right)^2 + 1} = 0.040 \text{ or } v = 1.20 \times 10^7\ \text{m/s}.$$

16. a) To get a charge of +1 with three quarks requires two charges of $+2e/3$ and one of charge $-e/3$. Three quarks with charge $+e/3$ would violate the Pauli exclusion principle for spin 1/2 particles.

b) To get a charge of zero we could have either two $+e/3$ and one $-2e/3$ or one $+2e/3$ and two $-e/3$. At this point in the text there is no reason to prefer either choice (the latter turns out to be correct).

17. a)

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{4.2\ \text{K}} = 0.69\ \text{mm}$$

b)

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{293\ \text{K}} = 9.89\,\mu\text{m}$$

c)

$$\lambda_{\text{max}} = \frac{2.898 \times 10^{-3}\ \text{m}\cdot\text{K}}{2500\ \text{K}} = 1.16\,\mu\text{m}$$

18. a)

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{10^{-14}\ \mathrm{m}} = 2.898 \times 10^{11}\ \mathrm{K}$$

b)

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{10^{-9}\ \mathrm{m}} = 2.898 \times 10^{6}\ \mathrm{K}$$

c)

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{670 \times 10^{-9}\ \mathrm{m}} = 4325\ \mathrm{K}$$

d)

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{1\ \mathrm{m}} = 2.898 \times 10^{-3}\ \mathrm{K}$$

e)

$$T = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{204\ \mathrm{m}} = 1.42 \times 10^{-5}\ \mathrm{K}$$

* 19.

$$\frac{P_1}{P_0} = \frac{\sigma T_1^4}{\sigma T_0^4} \quad \text{so} \quad P_1 = P_0 \frac{T_1^4}{T_0^4} = \left(\frac{1900\ \mathrm{K}}{900\ \mathrm{K}}\right)^4 P_0 = 19.9\ P_0$$

The power increases by a factor of 19.9.

* 20. a)

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{310\,\mathrm{K}} = 9.35\,\mu\mathrm{m}$$

b) At this temperature the power per unit area is

$$R = \sigma T^4 = \left(5.67 \times 10^{-8}\,\mathrm{W \cdot m^{-2} \cdot K^4}\right)(310\,\mathrm{K})^4 = 524\,\mathrm{W/m^2}$$

The total surface area of a cylinder is $2\pi r(r+h) = 2\pi\,(0.13\,\mathrm{m})\,(1.78\,\mathrm{m}) = 1.45\,\mathrm{m}^2$ so the total power is

$$P = \left(524\,\mathrm{W/m^2}\right)\left(1.45\,\mathrm{m^2}\right) = 760\,\mathrm{W}\,.$$

21.

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{200{,}273\ \mathrm{K}} = 1.447 \times 10^{-8}\mathrm{m}$$

22. We know from example 3.8 that in the long-wavelength limit, the two expressions are the same. By comparing the expressions, we can also note that the Rayleigh-Jeans spectral distribution will be larger than the Planck expression for a given λ and T. (Compare graphs or calculations using Equations (3.22) and (3.23) one can obtain using Excel, Mathcad, or similar program.) So want

$$(0.95)\left(\frac{2\pi ckT}{\lambda^4}\right) = \left(\frac{2\pi c^2 h}{\lambda^5}\right)\left(\frac{1}{\left[e^{hc/\lambda kT} - 1\right]}\right)$$

Simplifying we find

$$(0.95)\left(\frac{\lambda kT}{hc}\right) = \frac{1}{\left[e^{hc/\lambda kT} - 1\right]}\,.$$

Let $\dfrac{\lambda kT}{hc} = x$ and this simplifies to

$$(0.95)\, x = \frac{1}{\left[e^{1/x} - 1\right]} .$$

We can solve this transcendental equation to find $x = 9.83$ so $9.83 = \dfrac{\lambda kT}{hc}$. Substituting numerical values for the constants we find

$$\lambda = \frac{9.83hc}{kT} = \frac{9.83\left(1.986 \times 10^{-25}\,\mathrm{J \cdot m}\right)}{1.38 \times 10^{-23}\,\mathrm{J \cdot K^{-1}}\,(5800\,\mathrm{K})} = 2.44 \times 10^{-5}\,\mathrm{m} = 24.4\,\mu\mathrm{m}\,.$$

* 23.

$$\lambda_{\max} = \frac{2.898 \times 10^{-3}\ \mathrm{m \cdot K}}{3000\ \mathrm{K}} = 966\ \mathrm{nm}$$

which is in the near infrared.

24. The graph is a characteristic Planck law curve with a maximum at $\lambda = 966$ nm (see Problem 23).

a) Numerical integration of the $\mathcal{I}(\lambda, T)$ function shows that approximately 8.1% of the radiated power is between 400 nm and 700 nm. Details of the calculation are:

$$\begin{aligned} 2\pi c^2 h \int_{4\times10^{-7}}^{7\times10^{-7}} \exp\left(-\frac{hc}{\lambda kT}\right) \lambda^{-5}\, d\lambda &= \left(3.74 \times 10^{-16}\right) \int_{4\times10^{-7}}^{7\times10^{-7}} \exp\left(-\frac{4.798 \times 10^{-6}}{\lambda}\right) \lambda^{-5}\, d\lambda \\ &= 3.71 \times 10^{5}\ \mathrm{W/m^2} \end{aligned}$$

That is the power per unit area emitted over visible wavelengths. Over all wavelengths we know the power per unit area is $R = \sigma T^4 = 4.59 \times 10^6\ \mathrm{W/m^2}$. Therefore the fraction emitted in the visible is

$$\frac{3.7128 \times 10^5\ \mathrm{W/m^2}}{4.593 \times 10^6\ \mathrm{W/m^2}} = 0.081$$

b) Using computed numerical intensity values

$$\frac{\mathcal{I}(400\ \mathrm{nm}, T)}{\mathcal{I}(966\ \mathrm{nm}, T)} \approx 0.073$$

$$\frac{\mathcal{I}(700\ \mathrm{nm}, T)}{\mathcal{I}(966\ \mathrm{nm}, T)} \approx 0.754$$

25. In this limit $\exp(hc/\lambda kT) >> 1$ so

$$\mathcal{I}(\lambda, T) \approx \frac{2\pi c^2 h}{\lambda^5} \exp\left(-\frac{hc}{\lambda kT}\right)$$

The exponential goes to zero faster than λ^5, so the intensity approaches zero in this limit.

26. a) At this temperature the power per unit area is

$$R = \sigma T^4 = \left(5.67 \times 10^{-8}\ \mathrm{W \cdot m^{-2} \cdot K^{-4}}\right)(293\ \mathrm{K})^4 = 419\ \mathrm{W/m^2}$$

For the basketball, a sphere of radius $r = 12.5$ cm, we get

$$P = R\left(4\pi r^2\right) = \left(419\ \mathrm{W/m^2}\right)(4\pi)(0.125\ \mathrm{m})^2 = 82.3\ \mathrm{W}$$

b) At this temperature the power per unit area is

$$R = \sigma T^4 = \left(5.67\times10^{-8}\ \mathrm{W\cdot m^{-2}\cdot K^{-4}}\right)(310\ \mathrm{K})^4 = 524\ \mathrm{W/m^2}$$

Assume the body is roughly cylindrical with a radius of 13 cm and a height of 1.65 m. The total surface area of a cylinder is $2\pi r\,(r+h) = 2\pi\,(0.13\,\mathrm{m})\,(1.78\,\mathrm{m}) = 1.45\,\mathrm{m}^2$ so the total power is

$$P = \left(524\,\mathrm{W/m^2}\right)\left(1.45\,\mathrm{m^2}\right) = 760\,\mathrm{W}\,.$$

Numerical values will vary depending on estimates of the human body size.

27.

$$\lambda_{\max} = \frac{2.898\times10^{-3}\ \mathrm{m\cdot K}}{310\ \mathrm{K}} = 9.35\ \mu\mathrm{m}$$

* 28. Taking derivatives

$$\frac{\partial^2\psi}{\partial t^2} = \frac{\partial^2 a}{\partial t^2}\sin\left(\frac{n\pi x}{L}\right) \qquad \frac{\partial^2\psi}{\partial x^2} = -a\,\frac{n^2\pi^2}{L^2}\sin\left(\frac{n\pi x}{L}\right)$$

Substituting these values into the wave equation produces

$$\frac{1}{c^2}\frac{\partial^2 a}{\partial t^2}\sin\left(\frac{n\pi x}{L}\right) - \left(-a\,\frac{n^2\pi^2}{L^2}\sin\left(\frac{n\pi x}{L}\right)\right) = 0$$

$$\frac{\partial^2 a}{\partial t^2} = -a\frac{n^2\pi^2c^2}{L^2} = -\Omega^2 a$$

where $\Omega = \dfrac{n\pi c}{L}$. Since $\lambda = \dfrac{2L}{n}$ for this system and $c = \lambda f$, then $\Omega = 2\pi f$.

29. Let $r = \sqrt{n_x^2 + n_y^2 + n_z^2}$ be the radius of a three-dimensional number space with the n_i the three components of that space. Then let dN be the number of allowed states between r and $r + dr$. This corresponds to the number of points in a spherical shell of number space, which is

$$dN = \frac{1}{8}\left(4\pi r^2\right)\,dr$$

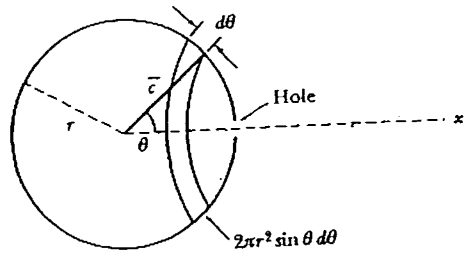
where we have used the fact that $4\pi r^2\,dr$ is the "volume" of the shell (area $4\pi r^2$ by thickness dr), and the 1/8 is due to the fact that only positive numbers n_i are allowed, so only 1/8 of the space is available. Also

$$r^2 = n_x^2 + n_y^2 + n_z^2 = \frac{\Omega^2 L^2}{\pi^2 c^2} = \frac{4L^2 f^2}{c^2}$$

or $r = \dfrac{2Lf}{c}$. Then from this $dr = \left(\dfrac{2L}{c}\right)df$. Putting everything together:

$$dN = \frac{1}{8}\left(4\pi r^2\right)\,dr = \frac{\pi}{2}r^2\,dr = \frac{\pi}{2}\left(\frac{2Lf}{c}\right)^2\frac{2L}{c}\,df = \frac{4\pi L^3}{c^3}\,f^2 df$$

30. From the diagram at right we see that the average x-component of the velocity (c) of electromagnetic radiation within the cavity is



$$\langle c_x \rangle = \frac{\int_0^{\pi/2} (c\cos\theta)\, 2\pi r^2 \sin\theta\, d\theta}{\int_0^{\pi/2} 2\pi r^2 \sin\theta\, d\theta}$$

Letting $u = \cos\theta$ we have

$$\langle c_x \rangle = \frac{c\int_0^1 u\, du}{\int_0^1 du} = \frac{c}{2}$$

On average only one-half of the photons are traveling to the right. Thus the mean velocity of photons traveling to the right is $c/4$. Therefore

$$\text{power} = (\text{intensity})\,(\text{area}) = \frac{c}{4}\, dU\ (\Delta A)$$

31. For classical oscillators the Maxwell-Boltzmann distribution gives

$$n(E) = A\exp(-E/kT) = A\exp(-\beta E)$$

where $E = nhf$ and $\beta = 1/kT$. The mean energy is

$$\overline{E} = \frac{\sum_{n=0}^{\infty} E\, n(E)}{\sum_{n=0}^{\infty} n(E)} = \frac{\sum_{n=0}^{\infty} nhf\exp(-\beta nhf)}{\sum_{n=0}^{\infty} \exp(-\beta nhf)}$$

Notice that

$$\overline{E} = \frac{\partial}{\partial\beta}\ln\sum_{n=0}^{\infty}\exp(-\beta nhf)$$

Now letting $x = \exp(-\beta hf)$ we see that by Taylor series

$$\sum_{n=0}^{\infty}\exp(-\beta nhf) = 1 + x + x^2 + \ldots = (1-x)^{-1} \text{ for } x^2 < 1.$$

$$\overline{E} = \frac{\partial}{\partial\beta}\ln(1-x)^{-1} = -\frac{\partial}{\partial\beta}\ln(1-x) = \frac{hf\exp(-\beta hf)}{1-\exp(-\beta hf)}$$

$$\overline{E} = \frac{hf}{\exp(hf/kT)-1}$$

Using the result of Problem 29 (along with a factor of 2 for two photon polarizations) we can see that

$$U(f,T) = 2\frac{4\pi}{c^3}f^2\frac{hf}{\exp(hf/kT)-1} = \frac{8\pi hf^3/c^3}{\exp(hf/kT)-1}$$

To change from U to $\mathcal{I}$ requires the factor $c/4$ (Problem 30), and changing from a frequency distribution requires a factor c/λ^2 (because with $f = c/\lambda$ we have $|df| = (c/\lambda^2)\, d\lambda$). Putting these together

$$\mathcal{I}(\lambda,T) = \frac{8\pi h/\lambda^3}{\exp(hf/kT)-1}\left(\frac{c}{\lambda^2}\right)\left(\frac{c}{4}\right) = \frac{2\pi c^2 h}{\lambda^5}\frac{1}{\exp(hc/\lambda kT)-1}$$

* 32.

$$\text{energy per photon } = hf = \left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(107.7 \times 10^{6}\ \text{s}^{-1}\right) = 7.14 \times 10^{-26}\ \text{J}$$

$$\left(5.0 \times 10^{4}\ \text{J/s}\right)\frac{1\ \text{photon}}{7.14 \times 10^{-26}\ \text{J}} = 7.00 \times 10^{29}\ \text{photons/s}$$

33. a)

$$\text{energy per photon } = hf = \left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(1100 \times 10^{3}\ \text{s}^{-1}\right) = 7.29 \times 10^{-28}\ \text{J}$$

$$\left(150\ \text{J/s}\right)\frac{1\ \text{photon}}{7.29 \times 10^{-28}\ \text{J}} = 2.06 \times 10^{29}\ \text{photons/s}$$

b)

$$\text{energy per photon } = h\frac{c}{\lambda} = \left(6.626 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(\frac{3.00 \times 10^{8}\ \text{m/s}}{8 \times 10^{-9}\ \text{m}}\right) = 2.48 \times 10^{-17}\ \text{J}$$

$$\left(150\ \text{J/s}\right)\frac{1\ \text{photon}}{2.48 \times 10^{-17}\ \text{J}} = 6.05 \times 10^{18}\ \text{photons/s}$$

c)

$$\left(150\ \text{J/s}\right)\frac{1\ \text{photon}}{4\ \text{MeV}}\frac{1\ \text{MeV}}{1.60 \times 10^{-13}\ \text{J}} = 2.34 \times 10^{14}\ \text{photons/s}$$

34.

$$f_t = \frac{\phi}{h} = \frac{2.9\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}} = 7.01 \times 10^{14}\ \text{Hz}$$

$$eV_0 = \frac{hc}{\lambda} - \phi \quad \text{so} \quad V_0 = \frac{1}{e}\left[\frac{hc}{\lambda} - \phi\right]$$

$$V_0 = \frac{1}{e}\left[\frac{1240\ \text{eV}\cdot\text{nm}}{400\ \text{nm}} - 2.9\ \text{eV}\right] = 0.20\ \text{eV}$$

* 35.

$$\lambda_t = \frac{hc}{\phi} = \frac{1240\ \text{eV}\cdot\text{nm}}{4.64\ \text{eV}} = 267.2\ \text{nm}$$

If the wavelength is halved (to $\lambda = 133.6$ nm)

$$K = \frac{hc}{\lambda} - \phi = \frac{1240\ \text{eV}\cdot\text{nm}}{133.6\ \text{nm}} - 4.64\ \text{eV} = 4.64\ \text{eV}$$

36. Notice that $\dfrac{hc}{\lambda} = 2.34\ \text{eV} > \phi$, so photoelectrons will be produced.

$$\left(2 \times 10^{-3}\ \text{J/s}\right)\left(10^{-5}\right)\left(\frac{1\ \text{photoelectron}}{2.34\ \text{eV}}\right)\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right)\left(\frac{1.60 \times 10^{-19}\ \text{C}}{\text{electron}}\right) = 8.55\ \text{nA}$$

37.

$$\phi = \frac{hc}{\lambda_t} = \frac{1240\ \text{eV}\cdot\text{nm}}{270\ \text{nm}} = 4.59\ \text{eV}$$

$$K = 2.0\ \text{eV} = hf - \phi$$

$$f = \frac{K + \phi}{h} = \frac{2.0\ \text{eV} + 4.59\ \text{eV}}{4.136 \times 10^{-15}\ \text{eV}\cdot\text{s}} = 1.59 \times 10^{15}\ \text{Hz}$$

38.

$$E = 100\left(\frac{hc}{\lambda}\right) = 100\frac{1240\ \text{eV}\cdot\text{nm}}{580\ \text{nm}} = 214\ \text{eV}$$

39. $eV_{01} = hc/\lambda_1 - \phi$ and $eV_{02} = hc/\lambda_2 - \phi$. Subtracting these equations and rearranging we find

$$h = \frac{e\,(V_{02} - V_{01})}{c\left(\frac{1}{\lambda_2} - \frac{1}{\lambda_1}\right)} = \frac{e\,(2.3\ \text{V}\ - 1.0\ \text{V})}{(3.00\times10^8\ \text{m/s})\left(\frac{1}{207\ \text{nm}} - \frac{1}{260\ \text{nm}}\right)} = 4.40\times10^{-15}\ \text{eV}\cdot\text{s}$$

This is about 6% from the accepted value. For the work function we use the first set of data (the second set should give the same result):

$$\phi = \frac{hc}{\lambda_1} - eV_{01} = \frac{(4.40\times10^{-15}\ \text{eV}\cdot\text{s})\,(3.00\times10^8\ \text{m/s})}{260\times10^{-9}\ \text{m}} - 1.0\ \text{eV}\ = 4.1\ \text{eV}$$

40. For 400 nm:

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{400\ \text{nm}} = 3.10\ \text{eV} \qquad f = \frac{c}{\lambda} = \frac{3.00\times10^8\ \text{m/s}}{400\ \text{nm}} = 7.50\times10^{14}\ \text{Hz}$$

For 700 nm:

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{700\ \text{nm}} = 1.77\ \text{eV} \qquad f = \frac{c}{\lambda} = \frac{3.00\times10^8\ \text{m/s}}{700\ \text{nm}} = 4.29\times10^{14}\ \text{Hz}$$

41.

$$\lambda_{\text{min}} = \frac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{V} = \frac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{30\ \text{kV}} = 0.0413\ \text{nm}$$

42.

$$\lambda = \frac{hc}{K} = \frac{1240\ \text{eV}\cdot\text{nm}}{5\times10^{10}\ \text{eV}} = 2.48\times10^{-17}\ \text{m}$$

A photon produced by bremsstrahlung is still an x ray, even though this falls outside the normal range for x rays.

* 43.

$$\lambda = \frac{1240\ \text{eV}\cdot\text{nm}}{2\times10^4\ \text{eV}} = 0.0620\ \text{nm}$$

44. Since the electron is accelerated through a potential difference, it has kinetic energy equal to $K = eV_0$. As described in the text, Equation (3.37) gives the minimum wavelength (assuming the work function is small). So

$$\lambda_{\text{min}} = \left(\frac{hc}{e}\right)\left(\frac{1}{V_0}\right) = \frac{1.240\times10^{-6}\ \text{V}\cdot\text{m}}{3.5\times10^4\ \text{V}} = 3.54\times10^{-11}\ \text{m} = 3.54\times10^{-2}\ \text{nm}$$

The work function for tungsten from Table 3.3 is 4.63 eV. If we include the work function, the energy available for the photon and the wavelength of the photon will change by 4.63 parts out of 35000 or about 0.013 % which is very small.

* 45. From Figure (3.19) we observe that the two characteristic spectral lines occur at wavelengths of 6.4×10^{-11} m and 7.2×10^{-11} m. Rearrange Equation (3.37) to solve for the potential V_0:

$$V_0 = \frac{hc}{e}\frac{1}{\lambda_{\min}} = \frac{1.24 \times 10^{-6}\,\mathrm{V \cdot m}}{7.2 \times 10^{-11}\,\mathrm{m}} = 17.2\,\mathrm{kV}$$

and we have used the larger of the two wavelengths in order to determine the minimum potential.

46.

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta)$$

so at maximum $\cos\theta = -1$ and

$$\frac{\Delta\lambda}{\lambda} = \frac{2h}{\lambda mc} = \frac{2hc}{mc^2\lambda} = \frac{2\,(1240\ \mathrm{eV \cdot nm})}{(511.0\ \mathrm{keV})\,(530\ \mathrm{nm})} = 9.16 \times 10^{-6}$$

This corresponds to $\Delta\lambda = 4.9 \times 10^{-12}$ m and therefore is not easily observed.

47. The maximum change in the photon's energy is obtained in backscattering ($\theta = 180°$), so $1 - \cos\theta = 2$ and $\Delta\lambda = \dfrac{2h}{mc} = 4.853 \times 10^{-12}$ m. The photon's original wavelength was

$$\lambda = \frac{hc}{E} = \frac{1240\ \mathrm{eV \cdot nm}}{40000\ \mathrm{eV}} = 0.0310\ \mathrm{nm} = 3.10 \times 10^{-11}\ \mathrm{m}$$

and the new wavelength is $\lambda' = \lambda + \Delta\lambda = 3.585 \times 10^{-11}$ m. The electron's recoil energy equals the change in the photon's energy, or

$$K = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{1240\ \mathrm{eV \cdot nm}}{3.10 \times 10^{-2}\ \mathrm{nm}} - \frac{1240\ \mathrm{eV \cdot nm}}{3.585 \times 10^{-2}\ \mathrm{nm}} = 5411\ \mathrm{eV} = 5.41\ \mathrm{keV}$$

48. Use the Compton scattering formula but with the proton's mass and $\theta = 90°$:

$$\Delta\lambda = \frac{h}{mc}(1 - \cos\theta) = \frac{h}{mc} = \frac{hc}{mc^2} = \frac{1240\ \mathrm{eV \cdot nm}}{938.27\ \mathrm{MeV}} = 1.32\ \mathrm{fm}$$

49. We find the Compton wavelength using

$$\lambda_c = \frac{h}{mc} = \frac{hc}{mc^2} = \frac{1240\ \mathrm{eV \cdot nm}}{938.27\ \mathrm{MeV}} = 1.32\ \mathrm{fm}$$

The photon energy is

$$E = \frac{hc}{\lambda_c} = 938\ \mathrm{MeV}$$

In principle this could be observed, but the energy requirements are high.

* 50.

$$\frac{\Delta\lambda}{\lambda} = 0.004 = \frac{\lambda_c}{\lambda}(1 - \cos\theta) \qquad \text{so} \qquad \lambda = 250\lambda_c(1 - \cos\theta)$$

a)

$$\lambda = 250\,(2.43 \times 10^{-12}\ \mathrm{m})\,(1 - \cos 30°) = 8.14 \times 10^{-11}\ \mathrm{m}$$

b)

$$\lambda = 250\,(2.43 \times 10^{-12}\ \mathrm{m})\,(1 - \cos 90°) = 6.08 \times 10^{-10}\ \mathrm{m}$$

c)

$$\lambda = 250\,(2.43 \times 10^{-12}\ \mathrm{m})\,(1 - \cos 170°) = 1.21 \times 10^{-9}\ \mathrm{m}$$

51. By conservation of energy we know the electron's recoil energy equals the energy lost by the photon:

$$K = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc(\lambda' - \lambda)}{\lambda'\lambda} = \frac{hc\,\Delta\lambda}{\lambda'\lambda}$$

Using $\lambda' = \lambda + \Delta\lambda$

$$K = \frac{hc\,\Delta\lambda}{\lambda(\lambda + \Delta\lambda)} = \frac{hf\,\Delta\lambda}{\lambda + \Delta\lambda} = \frac{hf\,\Delta\lambda}{\lambda(1 + \Delta\lambda/\lambda)} = \frac{(\Delta\lambda/\lambda)}{1 + \frac{\Delta\lambda}{\lambda}}\,hf$$

Conservation of p_x:

$$p_e \cos\phi + \frac{h}{\lambda'}\cos\theta = \frac{h}{\lambda}$$

$$p_e \cos\phi = \frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta \qquad (1)$$

Conservation of p_y:

$$p_e \sin\phi - \frac{h}{\lambda'}\sin\theta = 0$$

$$p_e \sin\phi = \frac{h}{\lambda'}\sin\theta \qquad (2)$$

Dividing equation (2) by equation (1)

$$\tan\phi = \frac{\frac{h}{\lambda'}\sin\theta}{\frac{h}{\lambda} - \frac{h}{\lambda'}\cos\theta}$$

Using $\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$

$$\tan\phi = \frac{\frac{h\sin\theta}{\lambda + \frac{h}{mc}(1-\cos\theta)}}{\frac{h}{\lambda} - \frac{h\cos\theta}{\lambda + \frac{h}{mc}(1-\cos\theta)}}$$

Multiplying numerator and denominator by $\lambda\left[\lambda + \frac{h}{mc}(1 - \cos\theta)\right]$.

$$\tan\phi = \frac{\lambda h\sin\theta}{\lambda h + \frac{h^2}{mc}(1 - \cos\theta) - \lambda h\cos\theta} = \frac{\lambda\sin\theta}{\left(\lambda + \frac{h}{mc}\right)(1 - \cos\theta)}$$

Trig identity: $\dfrac{\sin\theta}{(1 - \cos\theta)} = \cot\left(\dfrac{\theta}{2}\right)$

$$\tan\phi = \frac{\lambda}{\lambda + \frac{h}{mc}}\cot\left(\frac{\theta}{2}\right) = \frac{1}{1 + \frac{h}{mc\lambda}}\cot\left(\frac{\theta}{2}\right) = \frac{1}{1 + \frac{hf}{mc^2}}\cot\left(\frac{\theta}{2}\right)$$

Inverting the equation gives

$$\cot\phi = \left[1 + \frac{hf}{mc^2}\right]\tan\left(\frac{\theta}{2}\right)$$

52.

$$\lambda' = \lambda + \lambda_c(1-\cos\theta) = \frac{hc}{E} + \lambda_c(1-\cos\theta)$$

$$\lambda' = \frac{1240\ \text{eV}\cdot\text{nm}}{700\times 10^3\ \text{eV}} + \left(2.43\times 10^{-3}\ \text{nm}\right)\left(1-\cos 110^\circ\right) = 5.03\ \text{pm}$$

$$E' = \frac{hc}{\lambda'} = \frac{1240\ \text{eV}\cdot\text{nm}}{5.03\times 10^{-3}\ \text{nm}} = 2.46\times 10^5\ \text{eV} = 246\ \text{keV}$$

By conservation of energy

$K_e = E - E' = 700$ keV -246 keV $= 454$ keV (agrees with K formula in the previous problem)

From Problem 51

$$\cot\phi = \left[1+\frac{hf}{mc^2}\right]\tan\left(\frac{\theta}{2}\right) = \left[1+\frac{700\ \text{keV}}{511\ \text{keV}}\right]\tan\left(\frac{110^\circ}{2}\right) = 3.3845$$

$$\phi = 16.5^\circ$$

* 53. For $\theta = 90^\circ$ we know $\lambda' = \lambda + \lambda_c = 2.00243$ nm

$$\frac{\Delta\lambda}{\lambda} = \frac{\lambda_c}{\lambda} = \frac{2.43\times 10^{-3}\ \text{nm}}{2\ \text{nm}} = 1.22\times 10^{-3} = 0.122\%$$

* 54.

$$E = 2mc^2 = 2\,(938.3\ \text{MeV}) = 1877\ \text{MeV}$$

This energy could come from a particle accelerator.

55. a) To find the minimum energy consider the zero-momentum frame. Let E_e be the energy of the electron in that frame, and E_0 is the rest energy of the electron. From conservation of energy and momentum:

$$\text{momentum: } \frac{hf}{c} = p_e = \frac{\sqrt{E_e^2 - E_0^2}}{c} \qquad \text{or} \qquad hf = \sqrt{E_e^2 - E_0^2}$$

$$\text{energy: } hf + E_e = 3E_0 \qquad \text{or} \qquad hf = 3E_0 - E_e$$

Squaring and subtracting these two equations gives

$$0 = -10E_0^2 + 6E_eE_0 \qquad \text{or} \qquad E_e = \frac{5}{3}E_0$$

This tells us that for the transformation form the lab frame to the zero-momentum frame, $\gamma = 5/3$ and $v = 0.8c$. Then from the momentum equation we have in the zero-momentum frame

$$hf = \sqrt{\frac{25E_0^2}{9} - E_0^2} = \frac{4}{3}E_0$$

In the lab the photon's energy is obtained using a Doppler shift:

$$hf_{\text{lab}} = hf\sqrt{\frac{1+\beta}{1-\beta}} = \frac{4}{3}E_0\sqrt{\frac{1+0.8}{1-0.8}} = 4E_0 = 2.04\ \text{MeV}$$

b) The proton's rest energy is Mc^2. Now as in (a) we let the proton's energy in the lab frame be E_p and conservation of momentum and energy give

$$\text{momentum: } hf = \sqrt{E_p^2 - (Mc^2)^2}$$

$$\text{energy: } hf + E_p = 2E_0 + Mc^2$$

Squaring and subtracting, we find

$$E_p = \frac{(Mc^2)^2 + 2E_0^2 + 2E_0Mc^2}{2E_0 + Mc^2}$$

This is very close to $E_p = Mc^2$ Therefore the zero-momentum and lab frames are equivalent, and we conclude $hf_{\text{lab}} \approx 2E_0 = 1.02$ MeV.

56. The maximum energy transfer occurs when $\theta = 180°$ so that $\Delta\lambda = (h/mc)(1-\cos\theta) = 2h/mc$. By conservation of energy the kinetic energy of the electron is

$$K = E - E' = \frac{hc}{\lambda} - \frac{hc}{\lambda'} = \frac{hc}{\lambda} - \frac{hc}{\lambda + \Delta\lambda}$$

Multiplying through by $\lambda(\lambda + \Delta\lambda)$ we find

$$\lambda(\lambda + \Delta\lambda)K = hc(\lambda + \Delta\lambda) - hc\lambda = hc\,\Delta\lambda$$

$$\lambda^2 K + \lambda\Delta\lambda K - hc\Delta\lambda = 0$$

This is a quadratic equation that with numerical values can be solved for λ to find $\lambda = 1.20 \times 10^{-11}$ m. Then

$$E = \frac{hc}{\lambda} = \frac{1.24 \text{ keV}\cdot\text{nm}}{1.20 \times 10^{-2} \text{ nm}} = 104 \text{ keV}$$

57. To find the asteroid mass m note that the earth (matter) would supply an equal mass m to the process, so

$$2mc^2 = \frac{GM_E^2}{2R_E}$$

$$m = \frac{GM_E^2}{4R_Ec^2} = \frac{(6.67 \times 10^{-11} \text{ m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2})(5.98 \times 10^{24} \text{ kg})^2}{4(6378 \times 10^3 \text{ m})(3.00 \times 10^8 \text{ m/s})^2} = 1.04 \times 10^{15} \text{ kg}$$

Then

$$r = \left[\frac{3m}{4\pi\rho}\right]^{1/3} = \left(\frac{3(1.04 \times 10^{15} \text{ kg})}{4\pi\left(5000 \text{ kg/m}^3\right)}\right)^{1/3} = 3.68 \text{ km}$$

which is relatively small. Evaluating the energy:

$$E = \frac{GM_E^2}{2R_e} = \frac{(6.67 \times 10^{-11} \text{ m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2})(5.98 \times 10^{24} \text{ kg})^2}{2(6378 \times 10^3 \text{ m})} = 1.87 \times 10^{32} \text{ J}$$

$$\frac{E}{\text{nuclear arsenals}} = \frac{1.87 \times 10^{32} \text{ J}}{2000(4.2 \times 10^{15} \text{ J})} = 2 \times 10^{13}$$

There is a lot of energy in the annihilation process!

* 58. For maximum recoil energy the scattering angle is $\theta = 180°$ and $\phi = 0$. Then as usual $\Delta\lambda = \frac{2h}{mc}$. Using the result of Problem 56

$$K = \frac{\Delta\lambda/\lambda}{1+\Delta\lambda/\lambda}hf = \frac{2h/mc\lambda}{1+2h/mc\lambda}hf = \frac{2hf/mc^2}{1+2hf/mc^2}hf$$

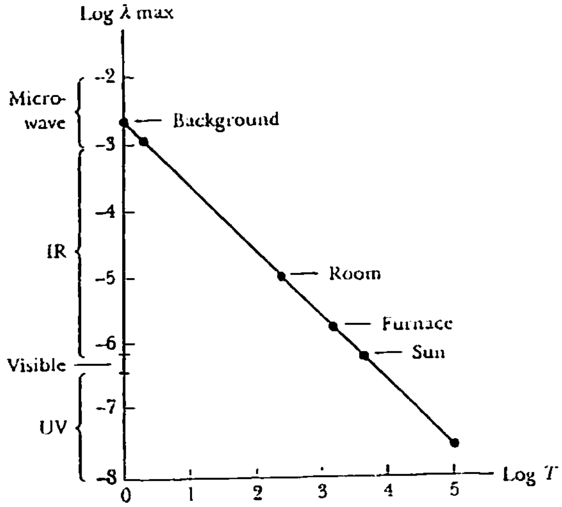
For the given value of $K = 100$ keV we can solve this equation:

$$K\left(1+\frac{2hf}{mc^2}\right) = \frac{2(hf)^2}{mc^2}$$

$$\left(\frac{2}{mc^2}\right)(hf)^2 - \left(\frac{2K}{mc^2}\right)(hf) - K = 0$$

This constitutes a quadratic equation in hf which can be solved numerically to yield $hf = 217$ keV.

59. See graph below.



60. a) For $\theta = 180°$ we have $\lambda' - \lambda = 2h/mc$. Therefore

$$E' = \frac{hc}{\lambda'} = \frac{hc}{\lambda + \frac{2h}{mc}}$$

b) With $\lambda = \frac{hc}{E}$ we find

$$E' = \frac{hc}{\frac{hc}{E}+\frac{2h}{mc}} = \frac{1}{\frac{1}{E}+\frac{2}{mc^2}} = \left(\frac{1}{1\times10^5 \text{ eV}} + \frac{2}{511000 \text{ eV}}\right)^{-1} = 71.9 \text{ keV}$$

61. a) The energy per second detection capability of the detector in the Hubble Space Telescope would be $\left(2.0\times10^{-20}\text{W/m}^2\right)0.30\,\text{m}^2 = 6.0\times10^{-21}$ J/s. Each photon at 486 nm would have energy

$$E = hf = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{J}\cdot\text{s})(3.0\times10^8\text{m/s})}{486\times10^{-9}\text{m}} = 4.09\times10^{-19}\text{ J}$$

Therefore the average number of photons per second would be

$$n = \frac{6.0 \times 10^{-21}\,\text{J/s}}{4.09 \times 10^{-19}\,\text{J/photon}} = 0.015\,\text{photons/s}$$

Long exposure times would be required.

b) If 30$^{\text{th}}$ magnitude corresponds to $2 \times 10^{-20}\text{W/m}^2$ and each change in magnitude by one represents a change in brightness of 2.5119, then we must increase by a factor of $(2.5119)^{24} = 3.98 \times 10^9$. So 6th magnitude corresponds to 7.96×10^{-11} W/m^2. If the diameter of the pupil is 6.5 mm, then the power reaching the retina is

$$P = 7.96 \times 10^{-11}\,\text{W/m}^2 \left[\pi \left(3.25 \times 10^{-3}\,\text{m}\right)^2\right] = 2.64 \times 10^{-15}\,\text{J/s}.$$

With each photon carrying energy 4.09×10^{-19} J, then

$$2.64 \times 10^{-15}\,\text{J/s} = 4.09 \times 10^{-19}\,\text{J}\left(n\,\frac{\text{photons}}{\text{s}}\right)$$

and $n = 6.45 \times 10^3$ photons each second.

62. The threshold (minimum) frequency for sodium is given as 4.39×10^{14}Hz in the figure caption. This corresponds to an energy of

$$E = hf = \left(6.6261 \times 10^{-34}\text{J} \cdot \text{s}\right)\left(4.39 \times 10^{14}\text{Hz}\right)\left(\frac{1\,\text{eV}}{1.602 \times 10^{-19}\,\text{J}}\right) = 1.82\,\text{eV}$$

The work function for sodium shown in Table 3.3 is 2.36 eV which represents an error of about 23 %. Planck's constant can be found from the slope of the graph and is determined as

$$h = \frac{\Delta E}{\Delta f} = \frac{3.2\,\text{eV}}{(12 - 4.4) \times 10^{14}\,\text{Hz}} = 4.21 \times 10^{-15}\,\text{eV} \cdot \text{s} = 6.75 \times 10^{-34}\text{J} \cdot \text{s}$$

This differs from the accepted value by about 1.8 %.