Chapter 4

1. With more than one electron we are almost forced into some kind of Bohr-like orbits. This was the dilemma faced by physicists in the early 20th Century.

2. Non-relativistically $K = \frac{1}{2}mv^2$ and

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2\,(7.7\text{ MeV})}{3727\text{ MeV}/c^2}} = 6.4281 \times 10^{-2}c$$

Relativistically $K = (\gamma - 1)\,mc^2$ so $\gamma = 1 + K/mc^2$ and

$$\gamma = 1 + \frac{7.7\text{ MeV}}{3727\text{ MeV}} = 1.002066$$

$$\beta = \sqrt{1 - \frac{1}{\gamma^2}} = 6.4181 \times 10^{-2}c$$

The difference is about $10^{-4}c$ or about 0.16% of the velocity.

3. Conserving momentum and energy:

$$M_\alpha v_\alpha = M_\alpha v_\alpha^{'} + m_e v_e^{'} \tag{1}$$

$$M_\alpha v_\alpha^2 = M_\alpha v_\alpha^{'2} + m_e v_e^{'2} \tag{2}$$

From (1) we see

$$v_\alpha^{'} = v_\alpha - \frac{m_e}{M_\alpha} v_e^{'}$$

which inserted into (2) gives

$$M_\alpha v_\alpha^2 = M_\alpha \left[v_\alpha - \frac{m_e}{M_\alpha} v_e^{'} \right]^2 + m_e v_e^{'2}$$

This can be solved to find

$$v_e^{'} \left[1 + \frac{m_e}{M_\alpha} \right] = 2v_\alpha$$

But with $m_e << M_\alpha$ we have $v_e^{'} \approx 2v_\alpha$.

4.

$$P(\theta) = \exp\left(-\frac{80^2}{1^2} \right) = 3 \times 10^{-2780}$$

Therefore multiple scattering does not provide an adequate explanation.

* 5. a) With $Z_1 = 2$, $Z_2 = 79$, and $\theta = 1°$ we have

$$b = \frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K} \cot\left(\frac{\theta}{2}\right) = \frac{(2)\,(79)\,(1.44 \times 10^{-9}\text{ eV}\cdot\text{m})}{2\,(7.7 \times 10^6\text{ eV})} \cot(0.5°) = 1.69 \times 10^{-12}\text{ m}$$

b) For $\theta = 90°$

$$b = \frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K} \cot\left(\frac{\theta}{2}\right) = \frac{(2)\,(79)\,(1.44 \times 10^{-9}\text{ eV}\cdot\text{m})}{2\,(7.7 \times 10^6\text{ eV})} \cot(45°) = 1.48 \times 10^{-14}\text{ m}$$

* 6.

$$f = \pi n t \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\right)^2 \cot^2\left(\frac{\theta}{2}\right)$$

For the two different angles everything is the same except the angles, so

$$\frac{f(1^\circ)}{f(2^\circ)} = \frac{\cot^2(0.5^\circ)}{\cot^2(1.0^\circ)} = 4.00$$

7. The fraction f is proportional to nt and to Z^2 from Equation (4.12). The question states, however, the number of scattering nuclei per unit area is equal so nt is the same for either target. Therefore

$$\frac{N(\mathrm{Au})}{N(\mathrm{Al})} = \frac{n(\mathrm{Au})t(79)^2}{n(\mathrm{Al})t(13)^2} = \frac{79^2}{13^2} = 36.93$$

8. From Example 4.2 we know $n = 5.90 \times 10^{28}\ \mathrm{m}^{-3}$. Thus

$$\begin{aligned} f &= \pi n t \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\right)^2 \cot^2\left(\frac{\theta}{2}\right) \\ &= \pi\left(5.90 \times 10^{28}\ \mathrm{m}^{-3}\right)\left(10^{-8}\ \mathrm{m}\right)\left(\frac{(2)(79)\left(1.44 \times 10^{-9}\ \mathrm{eV}\cdot\mathrm{m}\right)}{2\left(5 \times 10^{6}\ \mathrm{eV}\right)}\right)^2 \cot^2(3^\circ) \\ &= 3.49 \times 10^{-4} \end{aligned}$$

9. a) With all other parameters equal the number depends only on the scattering angles, so

$$\frac{f(90^\circ)}{f(50^\circ)} = \frac{\cot^2(45^\circ)}{\cot^2(25^\circ)} = 0.217$$

so the number scattered through angles greater than 90° is (10000) (0.217) = 2170.

b) Similarly

$$\frac{f(70^\circ)}{f(50^\circ)} = \frac{\cot^2(35^\circ)}{\cot^2(25^\circ)} = 0.4435$$

$$\frac{f(80^\circ)}{f(50^\circ)} = \frac{\cot^2(40^\circ)}{\cot^2(25^\circ)} = 0.3088$$

The numbers for the two angles are thus 4435 and 3088 and the number scattered between 70° and 80° is 4435 − 3088 = 1347.

* 10. From the Rutherford scattering result, the number detected through a small angle is inversely proportional to $\sin^4\left(\frac{\theta}{2}\right)$. Thus

$$\frac{n(50^\circ)}{n(6^\circ)} = \frac{\sin^4(3^\circ)}{\sin^4(25^\circ)} = 2.35 \times 10^{-4}$$

and if they count 2000 at 6° the number counted at 50° is $(2000)\left(2.35 \times 10^{-4}\right) = 0.47$ which is *insufficient*.

11. In each case all the kinetic energy is changed to potential energy:

$$K = -\Delta V = |V| = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 (r_1 + r_2)}$$

where r_1 and r_2 are the radii of the particular particles (as the problem indicates the particles just touch).

a) $Z_1 = 2$ and $Z_2 = 13$ for Al, $Z_2 = 79$ for Au

$$\text{Al: } K = \frac{(2)(13)\left(1.44 \times 10^{-9}\ \text{eV}\cdot\text{m}\right)}{2.6 \times 10^{-15}\ \text{m} + 3.6 \times 10^{-15}\ \text{m}} = 6.04\ \text{MeV}$$

$$\text{Au: } K = \frac{(2)(79)\left(1.44 \times 10^{-9}\ \text{eV}\cdot\text{m}\right)}{2.6 \times 10^{-15}\ \text{m} + 7.0 \times 10^{-15}\ \text{m}} = 23.7\ \text{MeV}$$

b) Now $Z_1 = 1$ and for the two different values of Z_2

$$\text{Al: } K = \frac{(1)(13)\left(1.44 \times 10^{-9}\ \text{eV}\cdot\text{m}\right)}{1.3 \times 10^{-15}\ \text{m} + 3.6 \times 10^{-15}\ \text{m}} = 3.82\ \text{MeV}$$

$$\text{Au: } K = \frac{(1)(79)\left(1.44 \times 10^{-9}\ \text{eV}\cdot\text{m}\right)}{1.3 \times 10^{-15}\ \text{m} + 7.0 \times 10^{-15}\ \text{m}} = 13.7\ \text{MeV}$$

12. a) The maximum Coulomb force is at the surface and equal to $2Z_2e^2/4\pi\epsilon_0 R^2$. Then

$$\Delta p = F\Delta t = \frac{2Z_2e^2}{4\pi\epsilon_0 R^2}\frac{2R}{v} = \frac{4Z_2e^2}{4\pi\epsilon_0 Rv}$$

For maximum deflection

$$\theta \approx \tan\theta = \frac{\Delta p}{p} = \frac{1}{mv}\frac{4Z_2e^2}{4\pi\epsilon_0 Rv} = \frac{2Z_2e^2}{4\pi\epsilon_0 KR}$$

b)

$$\theta = \frac{(2)\,(79)\,(1.44\ \text{eV}\cdot\text{nm})}{(8\ \text{MeV})\,(0.1\ \text{nm})} = 2.84 \times 10^{-4}\ \text{rad} = 0.016^\circ$$

13. a)

$$\theta = \frac{(2)\,(79)\,(1.44\ \text{eV}\cdot\text{nm})}{(10\ \text{MeV})\,(0.1\ \text{nm})} = 2.28 \times 10^{-4}\ \text{rad} = 0.013^\circ$$

b) These results are comparable in magnitude with those obtained by electron scattering (Example 4.1).

14. a)

$$v = \frac{e}{\sqrt{4\pi\epsilon_0 mr}} = \frac{ec}{\sqrt{4\pi\epsilon_0 mc^2 r}} = \frac{\sqrt{1.44\ \text{eV}\cdot\text{nm}}}{\sqrt{(511000\ \text{eV})\,(1.2 \times 10^{-6}\ \text{nm})}}\,c = 1.53c$$

which is not an allowed speed.

b)

$$E = -\frac{e^2}{8\pi\epsilon_0 r} = -\frac{1.44\ \text{eV}\cdot\text{nm}}{2\,(1.2 \times 10^{-6}\ \text{nm})} = -600\ \text{keV}$$

c) Clearly (a) is not allowed and (b) is too much energy.

* 15. a)

$$\begin{aligned} v &= \frac{e}{\sqrt{4\pi\epsilon_0 mr}} = \frac{ec}{\sqrt{4\pi\epsilon_0 mc^2 r}} = \frac{\sqrt{1.44\ \text{eV}\cdot\text{nm}}}{\sqrt{(938\times10^6\ \text{eV})\,(0.05\ \text{nm})}}c = 1.75\times10^{-4}c \\ &= 5.25\times10^4\ \text{m/s} \end{aligned}$$

b)

$$E = -\frac{e^2}{8\pi\epsilon_0 r} = -\frac{1.44\ \text{eV}\cdot\text{nm}}{2\,(0.05\ \text{nm})} = -14.4\ \text{eV}$$

c) The "nucleus" is too light to be fixed, and there is no way to reconcile this model with the results of Rutherford scattering.

16. For hydrogen:

$$\begin{aligned} v &= \frac{e}{\sqrt{4\pi\epsilon_0 mr}} = \frac{ec}{\sqrt{4\pi\epsilon_0 mc^2 r}} = \frac{\sqrt{1.44\ \text{eV}\cdot\text{nm}}}{\sqrt{(511\times10^3\ \text{eV})\,(0.0529\ \text{nm})}}c \\ &= 7.30\times10^{-3}c = 2.19\times10^6\ \text{m/s} \end{aligned}$$

$$a = \frac{v^2}{r} = \frac{\left(2.19\times10^6\ \text{m/s}\right)^2}{5.29\times10^{-11}\ \text{m}} = 9.07\times10^{22}\ \text{m/s}^2$$

For the hydrogen-like Li^{++}

$$F = \frac{Ze^2}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \quad \text{or} \quad v^2 = \frac{Ze^2}{4\pi\epsilon_0 rm}$$

But we also know

$$r = \frac{4\pi\epsilon_0\hbar^2}{Zme^2} = \frac{a_0}{Z}$$

$$v^2 = \frac{Z^2e^2}{4\pi\epsilon_0 a_0 m} = \frac{3^2\,(1.44\ \text{eV}\cdot\text{nm})}{(5.29\times10^{-11}\ \text{m})\,(511\times10^3\ \text{eV}/c^2)} = 4.79\times10^{-4}c^2$$

or $v = 2.19\times10^{-2}c = 6.57\times10^6$ m/s. This is a factor of 3 greater than the speed for hydrogen.

$$a = \frac{v^2}{r} = \frac{\left(6.57\times10^6\ \text{m/s}\right)^2}{(5.29\times10^{-11}\ \text{m})\,/3} = 2.45\times10^{24}\ \text{m/s}^2$$

17. For a hydrogen-like atom $E = \dfrac{-Z^2E_0}{n^2} = -Z^2E_0$ for $n = 1$.

$$\text{H: } E = -E_0 = -13.6\ \text{eV}$$

$$\text{He}^+\text{: } E = -4E_0 = -54.4\ \text{eV}$$

$$\text{Li}^{++}\text{: } E = -9E_0 = -122.5\ \text{eV}$$

The binding energy is larger for atoms with larger Z values, due to the greater attractive force between the nucleus and electron.

18. The total energy of the atom is $-e^2/(8\pi\epsilon_0 r)$. Differentiating with respect to time:

$$\frac{dE}{dt} = \frac{e^2}{8\pi\epsilon_0 r^2}\frac{dr}{dt}$$

Equating this result with the given equation from electromagnetic theory

$$\frac{e^2}{8\pi\epsilon_0 r^2}\frac{dr}{dt} = -\frac{1}{4\pi\epsilon_0}\frac{2e^2}{3c^3}\left(\frac{d^2r}{dt^2}\right)^2$$

$$\frac{e^2}{2r^2}\frac{dr}{dt} = -\frac{2e^2}{3c^3}\left(\frac{d^2r}{dt^2}\right)^2$$

In a circular orbit $\frac{d^2r}{dt^2}$ is just the centripetal acceleration, which is also given by

$$a = \frac{F}{m} = \frac{e^2}{4\pi\epsilon_0 m r^2}$$

Substituting:

$$\frac{e^2}{2r^2}\frac{dr}{dt} = -\frac{2e^2}{3c^3}\left(\frac{e^2}{4\pi\epsilon_0 m r^2}\right)^2$$

$$\frac{dr}{dt} = -\frac{4e^4}{(4\pi\epsilon_0)^2\, 3m^2c^3r^2}$$

Solving by separation of variables:

$$dt = -(4\pi\epsilon_0)^2\frac{3m^2c^3}{4e^4}r^2\,dr$$

$$t = -(4\pi\epsilon_0)^2\frac{3m^2c^3}{4e^4}\int_{a_0}^{0} r^2\,dr = (4\pi\epsilon_0)^2\frac{m^2c^3}{4e^4}a_0^3$$

Inserting numerical values we find $t = 1.55 \times 10^{-11}$ s.

19.

$$f = \frac{c}{\lambda} = Z^2Rc\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

$$f(K_\alpha) = Z^2Rc\left(1 - \frac{1}{4}\right) \qquad f(K_\beta) = Z^2Rc\left(1 - \frac{1}{9}\right) \qquad f(L_\alpha) = Z^2Rc\left(\frac{1}{4} - \frac{1}{9}\right)$$

With $1 - 1/4 + (1/4 - 1/9) = 1 - 1/9$ we see

$$f(K_\alpha) + f(L_\alpha) = f(K_\beta)$$

20. As in Problem 16, $v = 2.19 \times 10^6$ m/s and

$$L = mvr = \left(9.11 \times 10^{-31}\text{ kg}\right)\left(2.19 \times 10^6\text{ m/s}\right)\left(5.29 \times 10^{-11}\text{ m}\right) = 1.0554 \times 10^{-34}\text{ kg}\cdot\text{m}^2/\text{s}$$

Notice that $L = \hbar$.

21.

$$hc = \left(4.135669 \times 10^{-15}\ \text{eV} \cdot \text{s}\right)(299792458\ \text{m/s}) = 1239.8\ \text{eV} \cdot \text{nm}$$

$$\frac{e^2}{4\pi\epsilon_0} = \frac{\left(1.6021733 \times 10^{-19}\ \text{C}\right)^2}{4\pi\,(8.8541878 \times 10^{-12}\ \text{F/m})} \frac{1\ \text{eV}}{1.6021733 \times 10^{-19}\ \text{N} \cdot \text{m}} \frac{10^9\ \text{nm}}{\text{m}} = 1.4400\ \text{eV} \cdot \text{nm}$$

$$mc^2 = \left(510.99906\ \text{keV}/c^2\right) c^2 = 511.00\ \text{keV}$$

$$\begin{aligned} a_0 &= \frac{4\pi\epsilon_0\hbar^2}{me^2} = \frac{4\pi\left(8.8541878 \times 10^{-12}\ \text{F/m}\right)\left(1.05457 \times 10^{-34}\ \text{J} \cdot \text{s}\right)^2}{(9.1093897 \times 10^{-31}\ \text{kg})\,(1.6021733 \times 10^{-19}\ \text{C})^2} \\ &= 5.2918 \times 10^{-11}\ \text{m} = 5.2918 \times 10^{-2}\ \text{nm} \end{aligned}$$

$$\begin{aligned} E_0 &= \frac{e^2}{8\pi\epsilon_0 a_0} = -\frac{\left(1.6021733 \times 10^{-19}\ \text{C}\right)^2}{8\pi\,(8.8541878 \times 10^{-12}\ \text{F/m})\,(5.2917725 \times 10^{-11}\ \text{m})} \\ &= 2.179874 \times 10^{-18}\ \text{J} = 13.606\ \text{eV} \end{aligned}$$

* 22. From Equation (4.31) $v_n = (1/n)(\hbar/ma_0)$

$n = 1$: $$v_1 = \frac{1}{1} \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})\,(5.29 \times 10^{-11}\ \text{m})} = 2.19 \times 10^6\ \text{m/s} = 0.0073c$$

$n = 2$: $$v_2 = \frac{1}{2} \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})\,(5.29 \times 10^{-11}\ \text{m})} = 1.09 \times 10^6\ \text{m/s} = 0.0036c$$

$n = 3$: $$v_3 = \frac{1}{3} \frac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})\,(5.29 \times 10^{-11}\ \text{m})} = 7.30 \times 10^5\ \text{m/s} = 0.0024c$$

* 23. The photon energy is

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV} \cdot \text{nm}}{434\ \text{nm}} = 2.86\ \text{eV}$$

This is the energy difference between the two states in hydrogen. From Figure 4.16, we see $E_3 = -1.51$ eV so the initial state must be $n = 2$. We notice that this energy difference exists between $n = 2$ (with $E_2 = -3.40$ eV) and $n = 5$ (with $E_5 = -0.54$ eV).

24. The photon energy is

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV} \cdot \text{nm}}{95\ \text{nm}} = 13.05\ \text{eV}$$

This can only be a transition to $n = 1$ ($E_1 = -13.6$ eV) and because of the energy difference it comes from $n = 5$ with $E_5 = -0.54$ eV.

25. In general the ground state energy is $Z^2 E_0$.

a)

$$E = 1^2 E_0 = 13.6\ \text{eV}$$

The reduced mass does not change this result to three significant digits.

b)

$$E = 2^2 E_0 = 54.4\ \text{eV}$$

c)

$$E = 4^2 E_0 = 218\ \text{eV}$$

26. Following the strategy of example 4.8, we use Equation (4.37) to determine appropriate Rydberg constants for atomic hydrogen, deuterium, and tritium. The Balmer series has n lower equal to 2. The α refers to $n_u = 3$; β refers to $n_u = 4$, etc. Then we use Equation (4.30) to calculate the "isotope shifted" wavelengths.

$$\frac{1}{\lambda} = R_{\infty}\left(\frac{1}{n_{\ell}^2} - \frac{1}{n_u^2}\right)$$

The example with n upper equal to 3 is completed in example 4.8; the results are repeated here:
$\lambda\,(\mathrm{H}_\alpha\,,\mathrm{hydrogen}) = 656.47\,\mathrm{nm}$ $\lambda\,(\mathrm{H}_\alpha\,,\mathrm{deuterium}) = 656.29\,\mathrm{nm}$ $\lambda\,(\mathrm{H}_\alpha\,,\mathrm{tritium}) = 656.23\,\mathrm{nm}$

$\lambda\,(\mathrm{H}_\beta\,,\mathrm{hydrogen}) = 486.27\,\mathrm{nm}$ $\lambda\,(\mathrm{H}_\beta\,,\mathrm{deuterium}) = 486.14\,\mathrm{nm}$ $\lambda\,(\mathrm{H}_\beta\,,\mathrm{tritium}) = 486.10\,\mathrm{nm}$

$\lambda\,(\mathrm{H}_\gamma\,,\mathrm{hydrogen}) = 434.17\,\mathrm{nm}$ $\lambda\,(\mathrm{H}_\gamma\,,\mathrm{deuterium}) = 434.05\,\mathrm{nm}$ $\lambda\,(\mathrm{H}_\gamma\,,\mathrm{tritium}) = 434.02\,\mathrm{nm}$

$\lambda\,(\mathrm{H}_\delta\,,\mathrm{hydrogen}) = 410.29\,\mathrm{nm}$ $\lambda\,(\mathrm{H}_\delta\,,\mathrm{hydrogen}) = 410.18\,\mathrm{nm}$ $\lambda\,(\mathrm{H}_\delta\,,\mathrm{hydrogen}) = 410.15\,\mathrm{nm}$

27. We know from Equations (4.25) and (4.29) that $E_1 = -hcR_{\infty}$. We must adjust the Rydberg constant to account for the finite mass of the nucleus. Use the calculations of example 4.8 that provide the constants for deuterium $R_D = 0.99973\,R_{\infty}$and tritium $R_T = 0.99982\,R_{\infty}$. Then we see

$$E_{1,D} = hc\,(0.99973)\,R_{\infty} = 1239.8\,\mathrm{eV}\cdot\mathrm{nm}\,(0.99973)\left(1.097373\times10^{-2}\,\mathrm{nm}^{-1}\right) = 13.602\,\mathrm{eV}$$

$$E_{1,T} = hc\,(0.99982)\,R_{\infty} = 1239.8\,\mathrm{eV}\cdot\mathrm{nm}\,(0.99982)\left(1.097373\times10^{-2}\,\mathrm{nm}^{-1}\right) = 13.603\,\mathrm{eV}$$

In problem 21 we found that $E_{1,\mathrm{H}} = 13.606\,\mathrm{eV}$.

28. a) It is only the first four lines of the Balmer series, with wavelengths 656.5 nm, 486.3 nm, 434.2 nm, and 410.2 nm.

b) To determine the energy levels in helium, perform the same analysis as in the text but with e^2 replaced by $Ze^2 = 2e^2$. This results in an extra factor of $Z^2 = 4$ in the energy, so the revised Rydberg-Ritz equation is

$$\frac{1}{\lambda} = \frac{4E_0}{hc}\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right) = \left(4.377\times10^{7}\ \mathrm{m}^{-1}\right)\left(\frac{1}{n_l^2} - \frac{1}{n_u^2}\right)$$

We need the wavelength to be between 400 and 700 nm. The combinations of n_l and n_u that work are tabulated below:

n_l	n_u	λ (nm)	comments
3	4	470	
4	6	658	
4	7	543	
4	8	487	etc. with $n_l = 4$ to $n_u = 13$
4	13	404	but $n_l = 4$ and $n_u > 13$ gives $\lambda < 400$ nm
5	12	691	
5	13	670	etc. with $n_l = 5$ all the way to...
5	∞	571	a series limit

29. From Problem 22 the speed in the $n = 3$ state is $v = 7.30 \times 10^5$ m/s. The radius of the orbit is $n^2 a_0 = 9a_0$. Then from kinematics

$$\text{number of revolutions} = \frac{vt}{2\pi r} = \frac{(7.30 \times 10^5 \text{ m/s})(10^{-8} \text{ s})}{2\pi (9)(5.29 \times 10^{-11} \text{ m})} = 2.44 \times 10^6$$

* 30. The energy of each photon is $hc/\lambda = 12.4$ eV. Looking at the energy difference between levels in hydrogen we see that $E_2 - E_1 = 10.2$ eV, $E_3 - E_1 = 12.1$ eV, and $E_4 - E_1 = 12.8$ eV. There is enough energy to excite only to the second or third level. In theory it is possible for a second photon to come along and take the atom from one of these excited states to a higher one, but this is unlikely, because the $n = 2$ and $n = 3$ states are short-lived.

* 31. We must use the reduced mass for the muon:

$$\mu = \frac{mM}{m+M} = \frac{(106 \text{ MeV}/c^2)(938 \text{ MeV}/c^2)}{106 \text{ MeV}/c^2 + 938 \text{ MeV}/c^2} = 95.2 \text{ MeV}/c^2$$

a)

$$a_0 = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2} = \frac{(6.58 \times 10^{-16} \text{ eV}\cdot\text{s})^2}{(1.44 \times 10^{-9} \text{ eV}\cdot\text{m})(95.2 \times 10^6 \text{ eV}/c^2)} \frac{(3.00 \times 10^8 \text{ m/s})^2}{c^2} = 2.84 \times 10^{-13} \text{ m}$$

b)

$$E = \frac{e^2}{8\pi\varepsilon_0 a_0} = \frac{(1.44 \times 10^{-9} \text{ eV}\cdot\text{m})}{2(2.84 \times 10^{-13} \text{ m})} = 2535 \text{ eV}$$

c) First series:

$$\lambda = \frac{hc}{E} = \frac{1240 \text{ eV}\cdot\text{nm}}{2535 \text{ eV}} = 0.49 \text{ nm}$$

Second series:

$$\lambda = \frac{4hc}{E} = \frac{4(1240 \text{ eV}\cdot\text{nm})}{2535 \text{ eV}} = 1.96 \text{ nm}$$

Third series:

$$\lambda = \frac{9hc}{E} = \frac{9(1240 \text{ eV}\cdot\text{nm})}{2535 \text{ eV}} = 4.40 \text{ nm}$$

32. The reduced mass for this system is $\mu = mm/(m+m) = m/2$ where m is the mass of each particle. Then

$$r = \frac{4\pi\varepsilon_0\hbar^2}{\mu e^2} = 2a_0$$

$$E = -\frac{e^2}{8\pi\varepsilon_0 r} = -\frac{e^2}{8\pi\varepsilon_0 (2a_0)} = -\frac{E_0}{2} = -6.8 \text{ eV}$$

33. a) As shown in Problem 32, the radius of the orbit is $2a_0$.

b) With $E_0 = 6.8$ eV (see Problem 32) we have

$$\Delta E = E_2 - E_1 = -\frac{E_0}{2^2} - \left(-\frac{E_0}{1^1}\right) = \frac{3E_0}{4} = 5.1 \text{ eV}$$

Then

$$\lambda = \frac{hc}{\Delta E} = \frac{1240 \text{ eV}\cdot\text{nm}}{5.1 \text{ eV}} = 243 \text{ nm}$$

34. a)

$$r_2 - r_1 = 4a_0 - a_0 = 3a_0 = 1.59 \times 10^{-10} \text{ m}$$

b)

$$r_6 - r_5 = 36a_0 - 25a_0 = 11a_0 = 5.83 \times 10^{-10} \text{ m}$$

c)

$$r_{11} - r_{10} = 121a_0 - 100a_0 = 21a_0 = 1.11 \times 10^{-9} \text{ m}$$

Note that in each case $r_m - r_n = (m+n)a_0$ which is valid when m and n differ by 1.

35.

$$\text{hydrogen: } \frac{1}{\lambda} = R_{\text{H}}\left(\frac{1}{4} - \frac{1}{n_u^2}\right)$$

$$\text{helium: } \frac{1}{\lambda} = Z^2 R_{\text{He}}\left(\frac{1}{4^2} - \frac{1}{n_u^2}\right) = R_{\text{He}}\left(\frac{1}{4} - \frac{4}{n_u^2}\right)$$

We see that the lines match very well when n_u is even for helium but not when it is odd. For example, when $n_u = 6$, for the ionized helium, then the $n_u = 3$ state for hydrogen will be very similar. In the same fashion, the $n_u = 8$ for the ionized helium will be very similar to the $n_u = 4$ for hydrogen. Also, all the "matched" lines differ slightly because of the different Rydberg constant (which is due to the different reduced masses). The differ by a factor of $R_{\text{He}}/R_{\text{H}} = 0.99986/0.99946 = 1.0004$.

36. As mentioned in the text below Equation (4.38), if each atom can be treated as single-electron atoms (and the problem states we can make this assumption), then

$$R = \frac{1}{1+\frac{m}{M}} R_\infty$$

where $R_\infty = 1.0973731534 \times 10^7 \text{ m}^{-1}$ and $m = 0.0005485799$ u.

Using ^{4}He ($M = 4.0026$ u), $R = 0.999863 R_\infty = 1.097223 \times 10^7 \text{ m}^{-1}$ (off by 0.14%)

Using ^{39}K ($M = 38.963708$ u), $R = 0.9999859 R_\infty = 1.097358 \times 10^7 \text{ m}^{-1}$ (off by 0.0014%)

Using ^{238}U ($M = 238.05078$ u), $R = 0.9999977 R_\infty = 1.097371 \times 10^7 \text{ m}^{-1}$ (off by 0.00023%)

37. The derivation of the Rydberg equation is the same as in the text. Because He^+ is hydrogen-like it works with $R = Z^2 R_{\text{He}}$ and $R_{\text{He}} = 1.097223 \times 10^7 \text{ m}^{-1}$ as in Problem 36. Then

$$R = 4\left(1.097223 \times 10^7 \text{ m}^{-1}\right) = 4.38889 \times 10^7 \text{ m}^{-1}$$

* 38. For L_α we have

$$\lambda = \frac{c}{f} = \frac{36}{5R(Z-7.4)^2}$$

$Z = 43$: $$\lambda = \frac{36}{5R(43-7.4)^2} = 0.52 \text{ nm}$$

$Z = 61$: $$\lambda = \frac{36}{5R(61-7.4)^2} = 0.23 \text{ nm}$$

$Z = 75$: $$\lambda = \frac{36}{5R(75-7.4)^2} = 0.14 \text{ nm}$$

39. For Pt $Z = 78$ and for Au $Z = 79$. For the K_α lines

$$\lambda = \frac{4}{3R(Z-1)^2}$$

$$\text{Pt: } \lambda = \frac{4}{3R(77)^2} = 20.49 \text{ pm} \qquad \text{Au: } \lambda = \frac{4}{3R(78)^2} = 19.97 \text{ pm}$$

Therefore $\Delta\lambda = 0.52$ pm which is less than the specified resolution, so it will not work.

40.

$$f(K_\beta) = \frac{c}{\lambda(K_\beta)} = cR(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \frac{8cR}{9}(Z-1)^2$$

This is higher than the K_α frequency by a factor of $(8/9)(3/4) = 32/27$, which seems to be in agreement with Figure 4.19.

* 41. Helium:

$$\lambda(K_\alpha) = \frac{4}{3R(Z-1)^2} = 122 \text{ nm} \qquad \lambda(K_\beta) = \frac{9}{8R(Z-1)^2} = 103 \text{ nm}$$

Lithium:

$$\lambda(K_\alpha) = \frac{4}{3R(Z-1)^2} = 30.4 \text{ nm} \qquad \lambda(K_\beta) = \frac{9}{8R(Z-1)^2} = 25.6 \text{ nm}$$

42. Refer to Figure 4.19 and Equation (4.46). λ is inversely proportional to $(Z-1)^2$ for the K series and to $(Z-7.4)^2$ for the L series.

a)

$$\frac{\lambda(\text{U})}{\lambda(\text{C})} = \frac{(6-1)^2}{(92-1)^2} = 0.0030$$

b)

$$\frac{\lambda(\text{W})}{\lambda(\text{Ca})} = \frac{(20-7.4)^2}{(74-7.4)^2} = 0.036$$

43. The longest wavelengths are produced for an electron vacancy in the K-shell. We begin with Equation (4.43) and notice that the longest wavelengths will occur when the expression on the right of the equal sign is as small as possible; thus we choose n equal to 2, 3, and 4. The series limit occurs for $n = \infty$. Molybdenum (Mo) has $Z = 42$. Then

$$\frac{1}{\lambda_{K_\alpha}} = R(Z-1)^2\left(1 - \frac{1}{n^2}\right) = \left(1.09737 \times 10^7 \text{ m}^{-1}\right)(41)^2\left(1 - \frac{1}{2^2}\right) = 72.28 \text{ pm}$$

In a similar fashion, we find

$$\lambda(K_\beta) = 60.99 \text{ pm} \qquad \lambda(K_\gamma) = 57.82 \text{ pm}$$

and the K series limit to be $\lambda = 54.21$ pm.

* 44. The longest wavelengths occur for an electron vacancy in K shell. The two longest wavelengths correspond to the K_α and the K_β. Use Equation (4.41) and rearrange it to solve for $(Z-1)^2$. Then

$$(Z-1)^2 = \frac{4}{3}\frac{1}{R\,\lambda_{K_\alpha}} = \frac{4}{3\,(1.09737 \times 10^7\,\mathrm{m}^{-1})\,(0.155 \times 10^{-9}\,\mathrm{m})} = 783.89$$

This gives $(Z-1) = 28$ or $Z = 29$. Using the expression for λ_{K_β} from problem 40, we have $(Z-1)^2 = \frac{9}{8}\frac{1}{R\,\lambda_{K_\beta}}$. Using the second wavelength given, 0.131 nm, we find

$$(Z-1)^2 = \frac{9}{8}\frac{1}{R\,\lambda_{K_\beta}} = \frac{9}{8}\frac{1}{(1.09737 \times 10^7\,\mathrm{m}^{-1})\,(0.131 \times 10^{-9}\,\mathrm{m})} = 782.58$$

This yields $(Z-1) = 27.97$ or $Z = 29$. Therefore we can conclude the target must be copper.

45. Non-relativistically 40 eV $= \frac{1}{2}mv_1^2$ so

$$v_1 = \sqrt{\frac{2\,(40\text{ eV})}{511\text{ keV}/c^2}} = 0.0125c = 3.75 \times 10^6\text{ m/s}$$

From elementary physics

$$v_2' = \frac{2m_1}{m_1+m_2}v_1 = \frac{2\,(0.0005486\text{ u})}{0.0005486\text{ u} + 202.97\text{ u}}\left(3.75 \times 10^6\text{ m/s}\right) = 20.3\text{ m/s}$$

where we have used the most abundant mercury isotope. Then

$$\begin{aligned} K_2' &= \frac{1}{2}m_2 v_2'^2 = \frac{1}{2}\,(202.97\text{ u})\left(931.49\text{ MeV/u}\cdot c^2\right)\left(\frac{c^2}{9 \times 10^{16}\text{ m}^2/\text{s}^2}\right)(20.3\text{ m/s})^2 \\ &= 4.33 \times 10^{-4}\text{ eV} \end{aligned}$$

46. Without the negative potential an electron with any energy, no matter how small, could drift into the collector plate. As a result the electron could give up its kinetic energy to a Hg atom and still contribute to the plate current. The Franck-Hertz curve would not show the distinguishing periodic drops, but rather would rise monotonically.

47. Using $\Delta E = hc/\lambda$ we find

$$h = \frac{\lambda \Delta E}{c} = \frac{(254\text{ nm})\,(4.88\text{ eV})}{3.00 \times 10^8\text{ m/s}} = 4.13 \times 10^{-15}\text{ eV}\cdot\text{s}$$

48. 3.6 eV, 4.6 eV, 2(3.6 eV) = 7.2 eV, 3.6 eV + 4.6 eV = 8.2 eV, etc. with other combinations giving 10.8 eV, 11.8 eV, 12.6 eV, 14.4 eV, 15.4 eV, 16.2 eV, 16.4 eV, 17.2 eV, 18.0 eV.

49. Magnesium:

$$\lambda(K_\alpha) = \frac{4}{3R\,(Z-1)^2} = 1.00\text{ nm} \qquad \lambda(L_\alpha) = \frac{36}{5R\,(Z-7.4)^2} = 31.0\text{ nm}$$

Iron:

$$\lambda(K_\alpha) = \frac{4}{3R\,(Z-1)^2} = 0.194\text{ nm} \qquad \lambda(L_\alpha) = \frac{36}{5R\,(Z-7.4)^2} = 1.90\text{ nm}$$

* 50. K_α is a transition from $n = 2$ to $n = 1$ and K_β is from $n = 3$ to $n = 1$. We know those wavelengths in the Lyman series are 121.6 nm and 102.6 nm, respectively. The redshift factor (λ/λ_0) is (with $\beta = 1/6$)

$$\sqrt{\frac{1+\beta}{1-\beta}} = \sqrt{\frac{1+1/6}{1-1/6}} = 1.183$$

Then the redshifted wavelengths are higher by 18.3% in each case. The observed wavelengths are:

$$K_\alpha\text{: } \lambda = (1.183)\,(121.6 \text{ nm}) = 143.9 \text{ nm}$$

$$K_\beta\text{: } \lambda = (1.183)\,(102.6 \text{ nm}) = 121.4 \text{ nm}$$

* 51.

$$f = \pi n t \left(\frac{Z_1 Z_2 e^2}{8\pi\epsilon_0 K}\right)^2 \cot^2\left(\frac{\theta}{2}\right)$$

a)

$$\begin{aligned} f(1^\circ) &= \pi\left(5.90 \times 10^{28} \text{ m}^{-3}\right)\left(4 \times 10^{-7} \text{ m}\right)\left(\frac{(2)\,(79)\left(1.44 \times 10^{-9} \text{ eV}\cdot\text{m}\right)}{2\,(8 \times 10^6 \text{ eV})}\right)^2 \cot^2(0.5^\circ) \\ &= 0.197 \end{aligned}$$

$$\begin{aligned} f(2^\circ) &= \pi\left(5.90 \times 10^{28} \text{ m}^{-3}\right)\left(4 \times 10^{-7} \text{ m}\right)\left(\frac{(2)\,(79)\left(1.44 \times 10^{-9} \text{ eV}\cdot\text{m}\right)}{2\,(8 \times 10^6 \text{ eV})}\right)^2 \cot^2(1^\circ) \\ &= 0.0492 \end{aligned}$$

The fraction scattered between 1° and 2° is $0.197 - 0.0492 = 0.148$.

b)

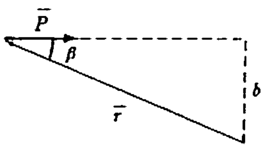
$$\frac{f(1^\circ)}{f(10^\circ)} = \frac{\cot^2(0.5^\circ)}{\cot^2(5^\circ)} = 100.5$$

$$\frac{f(1^\circ)}{f(90^\circ)} = \frac{\cot^2(0.5^\circ)}{\cot^2(45^\circ)} = 1.31 \times 10^4$$

52. From classical mechanics we know that $\vec{L}$ is conserved for central forces.

$L = \left|\vec{L}\right| = |\vec{r} \times \vec{p}| = r p_0 \sin\beta$

But $r\sin\beta = b$ so $L = p_0 b = m v_0 b$.



53. If the positions of the electron and proton are respectively (along a line) r_e and r_M, then putting the center of mass at $R = 0$ we have

$$R = 0 = \frac{m r_e + M r_M}{m + M} \qquad \text{or} \qquad r_M = -\frac{m}{M} r_e$$

$$r = r_e - r_M = r_e - \left(-\frac{m}{M} r_e\right) = r_e\left(1 + \frac{m}{M}\right) \qquad \text{or} \qquad r_e = \frac{M}{M+m} r\,.$$

Similarly we can show that $r_M = \left(\frac{m}{M+m}\right) r$. From these two expressions we can see that $\frac{r_M}{r_e} = \frac{m}{M}$. Now the centripetal force on the electron and the nucleus must be equal (in magnitude) by Newton's third law so we have

$$\frac{mv_e^2}{r_e} = \frac{Mv_M^2}{r_M} \qquad \text{or} \qquad \frac{v_M^2}{v_e^2} = \frac{mr_M}{Mr_e} = \frac{m^2}{M^2} \qquad \text{so} \qquad \frac{v_M}{v_e} = \frac{m}{M}.$$

The total angular momentum of the atom is the sum of the electron and nuclear orbital angular momenta, so

$$L = mv_e r_e + Mv_M r_M = mv_e r_e + M\left(\frac{m}{M}v_e\right)\left(\frac{m}{M}r_e\right) = mv_e r_e\left(1+\frac{m}{M}\right) = mv_e r.$$

where the last substitution comes from using the expression for r_e from above. Since the total L must equal $n\hbar$ we find $v_e = \frac{n\hbar}{mr}$. This is the equivalent of Equation (4.22b). Now the centripetal force depends on the distance of the electron from the center of mass while the Coulomb force depends on r, the distance between the electron and the nucleus. So Equation (4.18) becomes

$$\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{e^2}{r^2}\right) = \frac{mv_e^2}{r_e} \qquad \text{or} \qquad v_e^2 = \left(\frac{e^2}{4\pi\epsilon_0}\right)\left(\frac{1}{r^2}\right)\left(\frac{r_e}{m}\right) = \left(\frac{n\hbar}{mr}\right)^2.$$

This can be solved to give $mr_e = \frac{4\pi\epsilon_0 n^2\hbar^2}{e^2}$ or $m\left(\frac{M}{M+m}\right) r = \frac{4\pi\epsilon_0 n^2\hbar^2}{e^2}$. Finally this can be solved for r and the subscript added to account for allowed orbits. We find

$$r_n = \frac{4\pi\epsilon_0 n^2\hbar^2}{\frac{mM}{M+m}e^2} = \frac{4\pi\epsilon_0 n^2\hbar^2}{\mu e^2} \qquad \text{with} \qquad \mu = \frac{mM}{M+m} = \frac{m}{1+m/M}.$$

This is the modification to Equation (4.24).

54. a)

$$f = \frac{me^4}{4\varepsilon_0^2 h^3} = \frac{(9.11\times10^{-31}\ \text{kg})\left(1.60\times10^{-19}\ \text{C}\right)^4}{4\left(8.85\times10^{-12}\ \text{F/m}\right)^2\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)^3} = 6.55\times10^{15}\ \text{Hz}$$

b) As determined in previous problems $v = 2.19\times10^6$ m/s and $r = a_0 = 5.29\times10^{-11}$ m.

$$f = \frac{v}{2\pi r} = \frac{2.19\times10^6\ \text{m/s}}{2\pi\left(5.29\times10^{-11}\ \text{m}\right)} = 6.59\times10^{15}\ \text{Hz}$$

c) We know

$$E = -\frac{e^2}{8\pi\varepsilon_0 a_0} \qquad \text{and} \qquad K = \frac{e^2}{8\pi\varepsilon_0 a_0} = |E|$$

d) Since $K = nhf_{\text{orb}}/2 = hf_{\text{orb}}/2$ for $n = 1$ we have

$$f_{\text{orb}} = \frac{2K}{h} = \frac{2(13.6\ \text{eV})}{4.136\times10^{-15}\ \text{eV}\cdot\text{s}} = 6.58\times10^{15}\ \text{Hz}$$

which agrees with (a) and (b) to within rounding errors.

* 55. We start with $K = nhf_{\text{orb}}/2$. From classical mechanics we have for a circular orbit $f = v/2\pi r$, or $r = v/2\pi f$:

$$L = mvr = mv\left(\frac{v}{2\pi f}\right) = \left(\frac{mv^2}{2}\right)\left(\frac{1}{\pi f}\right)$$

Using $K = \frac{1}{2}mv^2$,

$$L = \frac{K}{\pi f} = \frac{nh\nu}{2\pi f} = \frac{nh}{2\pi} = n\hbar$$

56. From problem 32 we know that the reduced mass of the system $\mu = m/2$ where m is the mass of either particle. Then from Equation (4.37), we see that the effective Rydberg constant would be $R_{\text{eff}} = R_\infty/2$. We know that

$$E_n = -\frac{(hc)(R_{\text{eff}})}{n^2} = -\frac{(1239.8\,\text{eV}\cdot\text{nm})\left(\frac{1}{2}\,1.09737\times 10^{-2}\,\text{nm}^{-1}\right)}{n^2} = \frac{6.803\,\text{eV}}{n^2}$$

Therefore $E_1 = -6.803\,\text{eV}$, $E_2 = -1.701\,\text{eV}$, $E_3 = -0.756\,\text{eV}$, and $E_4 = 0.425\,\text{eV}$.
As in other problems, we know the K_α wavelength occurs for a $n_u = 2$ to an $n_\ell = 1$ and the L_α wavelength occurs for a $n_u = 3$ to an $n_\ell = 2$ transition, etc. Further, we know that $\lambda = \dfrac{hc}{[E_u - E_\ell]}$. For example, we have

$$\lambda(K_\alpha) = \frac{1239.8\,\text{eV}\cdot\text{nm}}{[(-1.701\,\text{eV}) - (-6.803\,\text{eV})]} = 243.0\,\text{nm}$$

In a similar fashion we can show that $\lambda(K_\beta) = 205.0\,\text{nm}$, $\lambda(L_\alpha) = 1312\,\text{nm}$, and $\lambda(L_\beta) = 971.6\,\text{nm}$.