Chapter 5

1. Starting with Equation (5.1), with $n = 1$ and $\theta = 15°$, $\sin\theta_1 = \dfrac{\lambda}{2d} = 0.259$

   Second order:

$$\sin\theta_2 = \frac{2\lambda}{2d} = 2\sin\theta_1 \qquad \theta_2 = \sin^{-1}(2\sin\theta_1) = 31.2°$$

$$\theta_3 = \sin^{-1}(3\sin\theta_1) = 50.9°$$

* 2. Use $\lambda = 0.186$ nm, and we know from the text that $d = 0.282$ nm for NaCl.

$$n = 1: \qquad \sin\theta = \frac{n\lambda}{2d} = \frac{\lambda}{2d} = 0.284 \qquad \theta = 19.3°$$

$$n = 2: \qquad \sin\theta = \frac{n\lambda}{2d} = \frac{\lambda}{d} = 0.567 \qquad \theta = 41.3°$$

$$\Delta\lambda = 41.3° - 19.3° = 22.0°$$

3. For $n = 1$ we have $\lambda = 2d\sin\theta = 2\,(0.314\text{ nm})\,(\sin 12°) = 0.131$ nm

$$E = \frac{hc}{\lambda} = \frac{1240\text{ eV}\cdot\text{nm}}{0.131\text{ nm}} = 9.47\,\text{keV}$$

   The largest order n is the largest integer for which $\dfrac{n\lambda}{2d} < 1$

$$n < \frac{2d}{\lambda} = 4.79$$

   so we can observe up through $n = 4$.

4. As in Davisson-Germer scattering $n\lambda = D\sin\phi$

$$\phi = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{hc}{ED}\right) = \sin^{-1}\left(\frac{1240\text{ eV}\cdot\text{nm}}{(10^5\text{ eV})\,(0.24\text{ nm})}\right) = 3.0°$$

5.

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{(3.0\text{ kg})\,(6.0\text{ m/s})} = 3.68\times10^{-35}\text{ m}$$

   No, the wavelength of the water waves depends on the medium; they are strictly mechanical waves.

6. Using the mean speed from kinetic theory (Chapter 9)

$$\bar{v} = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{kT}{m}} = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{(1.38\times10^{-23}\text{ J/K})\,(310.15\text{ K})}{28\,(1.66\times10^{-27}\text{ kg})}} = 484.2\,\text{m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{28\,(1.66\times10^{-27}\text{ kg})\,(484.2\text{ m/s})} = 2.94\times10^{-11}\text{ m}$$

   or roughly 3% of the size of the molecule.

7. Using the approach of Example 5.2, and with $K = e\Delta V$ we have, assuming relativistic effects are small,

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2\,(mc^2)\,K}} .$$

If we do not assume that relativistic effects are small, then

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2\,(mc^2)\,K}} .$$

See problem 11 for details. When $K = 40\,\text{keV}$, if we use the non-relativistic formula then

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{2\,(0.511\times10^6\,\text{eV})\,(40\times10^3\,\text{eV})}} = 6.13\times10^{-3}\,\text{nm} = 6.13\,\text{pm} .$$

Using the relativistic formula, we find

$$\lambda = \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{(40\times10^3\,\text{eV})^2 + 2\,(0.511\times10^6\,\text{eV})\,(40\times10^3\,\text{eV})}} = 6.02\,\text{pm} .$$

This represents a 2 % difference. Clearly when $K = 100\,\text{keV}$, we must use the relativistic approach and we find $\lambda = 3.70\times10^{-3}\,\text{nm} = 3.70\,\text{pm}$.

* 8. The resolution will be comparable to the de Broglie wavelength. The energy of the microscope requires a relativistic treatment, so

$$\begin{aligned}\lambda &= \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2\,(mc^2)\,K}}\\ &= \frac{1240\,\text{eV}\cdot\text{nm}}{\sqrt{(3\times10^6\,\text{eV})^2 + 2\,(0.511\times10^6\,\text{eV})\,(3\times10^6\,\text{eV})}}\\ &= 3.57\times10^{-4}\,\text{nm} = 0.357\,\text{pm}\end{aligned}$$

9. $(50\text{ eV})\left(1.602\times10^{-19}\text{ J/eV}\right) = 8.01\times10^{-18}\text{ J}$

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{6.626\times10^{-34}\text{ J}\cdot\text{s}}{\sqrt{2\,(9.109\times10^{-31}\text{ kg})\,(8.01\times10^{-18}\text{ J})}} = 1.73\times10^{-10}\text{ m}$$

This is the same as in the textbook's example.

10. When $E >> E_0$ then $E \approx pc$ for the particle; $E = pc$ for a photon. Therefore the electron's energy is approximately equal to the photon energy.

If $E = 2E_0$ then we cannot use $E \approx pc$. The exact expression is

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{3}E_0}{c}$$

for the electron's momentum. Then the de Broglie wavelength is

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{3}E_0}$$

If the photon has the same wavelength, its energy is

$$E = \frac{hc}{\lambda} = \sqrt{3}E_0$$

* 11. a) Relativistically

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(K + mc^2)^2 - (mc^2)^2}}{c} = \frac{\sqrt{K^2 + 2Kmc^2}}{c}$$

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}}$$

b) Non-relativistically, as in the text

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2mc^2K}}$$

* 12. The rest energy of the electron is very small compared to the total energy.

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = 50 \text{ GeV}/c$$

$$\lambda = \frac{h}{p} = \frac{1240 \text{ eV} \cdot \text{nm}}{50 \text{ GeV}} = 2.48 \times 10^{-17} \text{ m}$$

$$\text{fraction} = \frac{2.48 \times 10^{-17} \text{ m}}{2 \times 10^{-15} \text{ m}} = 0.012$$

13. a) For photons kinetic energy equals total energy and de Broglie wavelength is wavelength

$$K = E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{0.15 \text{ nm}} = 8.27 \text{ keV}$$

b) The energy is low enough that we can use the non-relativistic formula:

$$K = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{2\,(511 \times 10^3 \text{ eV})\,(0.15 \text{ nm})^2} = 66.9 \text{ eV}$$

c)

$$K = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{2\,(939 \times 10^6 \text{ eV})\,(0.15 \text{ nm})^2} = 0.036 \text{ eV}$$

d)

$$K = \frac{h^2}{2m\lambda^2} = \frac{(hc)^2}{2mc^2\lambda^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{2\,(3727 \times 10^6 \text{ eV})\,(0.15 \text{ nm})^2} = 9.17 \times 10^{-3} \text{ eV}$$

14. a) As in Problem 6 we use the mean speed formula from kinetic theory:

$$\bar{v} = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{kT}{m}} = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{(1.38 \times 10^{-23} \text{ J/K})\,(5 \text{ K})}{(1.675 \times 10^{-27} \text{ kg})}} = 324 \text{ m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.675 \times 10^{-27} \text{ kg})\,(324 \text{ m/s})} = 1.22 \text{ nm}$$

b)

$$\bar{v} = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{kT}{m}} = \frac{4}{\sqrt{2\pi}}\sqrt{\frac{(1.38 \times 10^{-23} \text{ J/K})\,(0.01 \text{ K})}{(1.675 \times 10^{-27} \text{ kg})}} = 14.5 \text{ m/s}$$

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.675 \times 10^{-27} \text{ kg})\,(14.5 \text{ m/s})} = 27.3 \text{ nm}$$

* 15. From the accelerating potential we know $K = eV = 3$ keV.

$$E = K + E_0 = 514 \text{ keV}$$

$$p = \frac{\sqrt{E^2 - E_0^2}}{c} = \frac{\sqrt{(514 \text{ keV})^2 - (511 \text{ keV})^2}}{c} = 55.4 \text{ keV}/c$$

$$\lambda = \frac{h}{p} = \frac{hc}{pc} = \frac{1240 \text{ eV} \cdot \text{nm}}{55.4 \times 10^3 \text{ eV}} = 22.4 \text{ pm}$$

16. We use the relativistic formula derived in Problem 11:

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}}$$

a) $\lambda = 0.194$ nm b) $\lambda = 6.13 \times 10^{-2}$ nm c) $\lambda = 1.94 \times 10^{-2}$ nm

d) $\lambda = 6.02 \times 10^{-3}$ nm e) $\lambda = 1.64 \times 10^{-3}$ nm f) $\lambda = 2.77 \times 10^{-4}$ nm

From Example 5.1 in the text, we know the spacing of the NaCl lattice is 0.282 nm. So even the lowest energy electrons here could be used to probe the crystal structure.

17. a)

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{32\,(1.661 \times 10^{-27} \text{ kg})\,(480 \text{ m/s})} = 2.60 \times 10^{-11} \text{ m}$$

b)

$$\lambda = \frac{h}{p} = \frac{6.626 \times 10^{-34} \text{ J} \cdot \text{s}}{(1.5 \times 10^{-15} \text{ kg})\,(10^{-6} \text{ m/s})} = 4.42 \times 10^{-13} \text{ m}$$

18. Using the relativistic formula from Problem 11

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{K^2 + 2Kmc^2}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{(10^{12} \text{ eV})^2 + 2\,(10^{12} \text{ eV})\,(938 \times 10^6 \text{ eV})}} = 1.24 \times 10^{-18} \text{ m}$$

19.

$$d = D\sin(\phi/2) = (0.23 \text{ nm})\sin 16^\circ = 0.063 \text{ nm}$$

$$\lambda = D\sin\phi = (0.23 \text{ nm})\sin 32^\circ = 0.122 \text{ nm}$$

$$p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1240 \text{ eV} \cdot \text{nm}}{(0.122 \text{ nm})\,c} = 10.2 \text{ keV}/c$$

$$E = \sqrt{p^2c^2 + E_0^2} = \sqrt{(10.2 \text{ keV})^2 + (511 \text{ keV})^2} = 511.102 \text{ keV}$$

$$K = E - E_0 = 102 \text{ eV}$$

20. Beginning with Equation (5.7) and with $V_0 = 48$ V and $D = 0.215$ nm for nickel, we have

$$\lambda = \frac{1.226 \text{ nm} \cdot V^{1/2}}{\sqrt{48 \text{ V}}} = 0.177 \text{ nm}$$

$$\phi = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{0.177 \text{ nm}}{0.215 \text{ nm}}\right) = 55.4^\circ$$

At 64 eV

$$\lambda = \frac{1.226 \text{ nm} \cdot V^{1/2}}{\sqrt{64 \text{ V}}} = 0.153 \text{ nm}$$

$$\phi = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{0.153 \text{ nm}}{0.215 \text{ nm}}\right) = 45.4^\circ$$

21. First we compute the wavelength of the electrons:

$$\lambda = \frac{h}{p} = \frac{hc}{\sqrt{E^2 - E_0^2}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{(513 \text{ keV})^2 - (511 \text{ keV})^2}} = 2.74 \times 10^{-2} \text{ nm}$$

From Figure 5.7(a) we see that $2\theta_1 = \tan^{-1}(2.1 \text{ cm}/35 \text{ cm}) = 3.434^\circ$ or $\theta_1 = 1.717^\circ$. Now since $\lambda = 2d\sin\theta$ we have

$$d_1 = \frac{\lambda}{2\sin\theta_1} = \frac{2.74 \times 10^{-2} \text{ nm}}{2\sin(1.717^\circ)} = 0.457 \text{ nm}$$

$$\theta_2 = \frac{1}{2}\tan^{-1}(2.3 \text{ cm}/35 \text{ cm}) = 1.880^\circ$$

$$d_2 = \frac{\lambda}{2\sin\theta_2} = \frac{2.74 \times 10^{-2} \text{ nm}}{2\sin(1.880^\circ)} = 0.412 \text{ nm}$$

$$\theta_3 = \frac{1}{2}\tan^{-1}(3.2 \text{ cm}/35 \text{ cm}) = 2.612^\circ$$

$$d_3 = \frac{\lambda}{2\sin\theta_3} = \frac{2.74 \times 10^{-2} \text{ nm}}{2\sin(2.612^\circ)} = 0.301 \text{ nm}$$

* 22.

$$\lambda = \frac{h}{\sqrt{2mK}} = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(939 \times 10^6 \text{ eV})(0.025 \text{ eV})}} = 0.181 \text{ nm}$$

$$\lambda = D\sin\phi \qquad \phi = \sin^{-1}\left(\frac{\lambda}{D}\right) = \sin^{-1}\left(\frac{0.181 \text{ nm}}{0.45 \text{ nm}}\right) = 23.7^\circ$$

23. We begin with Equation (5.23) $\Delta\omega\,\Delta t = 2\pi$. Since $\omega = 2\pi f$ the first equation is equivalent to $\Delta f \Delta t = 1$. Then $\Delta t = \frac{1}{\Delta f} = \frac{1}{0.3\text{ Hz}} = 3.33\,\text{s}$. If we want the time to be one-half of that found from the bandwidth relation, then we must monitor the frequency of the system every 1.67 s.

24. Refer to problem 11. The 5 eV electrons are nonrelativistic. We must treat the 500 keV electrons relativistically.

For the 5 eV electrons, $\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2(0.511 \times 10^6\,\text{eV})(5\,\text{eV})}} = 0.549\,\text{nm}$. The electron speed can be found using $p = mv = \frac{h}{\lambda}$. Therefore

$$v = \frac{6.626 \times 10^{-34}\,\text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\,\text{kg})(0.549 \times 10^{-9}\,\text{m/s})} = 1.33 \times 10^6 \text{ m/s}$$

and $\beta = \dfrac{v}{c} = 4.42 \times 10^{-3}$. Alternatively for this nonrelativistic case you could use $K = 5\,\text{eV} = \frac{1}{2}mv^2$ to find v.

The phase velocity is found using a modification of Equation (5.32). We must note that E represents the total energy of the particle, so

$$v_{ph} = \frac{\omega}{k} = \frac{E/\hbar}{p/\hbar} = \frac{E}{p} = \frac{0.511 \times 10^6\,\text{eV}\,(1.602 \times 10^{-19}\,\text{J/eV})}{(9.11 \times 10^{-31}\,\text{kg})\,(1.33 \times 10^6\,\text{m/s})} = 6.76 \times 10^{10}\,\text{m/s}\,.$$

This speed exceeds the speed of light but is not associated with the transmission of information. In problem 28 we will show that a simpler approach to find the phase velocity is to use $v_{ph} = \dfrac{c}{\beta}$ but either approach yields the same answer. We find the group velocity from Equation (5.31), namely $u_{gr} = \dfrac{pc^2}{E}$. In problem 73 from chapter 2, we showed that $\beta = \dfrac{pc}{E}$ so $u_{gr} = \beta c = 1.33 \times 10^6\,\text{m/s}$ which is the same as the particle speed.

For the 500 keV electrons,

$$\lambda = \frac{hc}{\sqrt{K^2 + 2\,(mc^2)\,K}} = \frac{1240\ \text{eV}\cdot\text{nm}}{\sqrt{(500 \times 10^3\,\text{eV})^2 + 2\,(0.511 \times 10^6\,\text{eV})\,(500 \times 10^3\,\text{eV})}}$$

so $\lambda = 1.42 \times 10^{-3}\,\text{nm}$. Using the same approach as before $p = \dfrac{h}{\lambda} = 4.67 \times 10^{-22}\,\text{kg}\cdot\text{m/s}$. However, this is a relativistic momentum so $p = \gamma m v$. We find $\gamma = \dfrac{K + E_0}{E_0} = 1.979$ and therefore $v = 2.59 \times 10^8\,\text{m/s} = 0.863\,c$. The phase velocity is $v_{ph} = \dfrac{c}{\beta} = 1.16\,c$ and again the group velocity is the same as the particle velocity, $u_{gr} = 0.863\,c$.

25. a) $f = \dfrac{v}{\lambda} = 0.571$ Hz

b) From the initial conditions given, we should use a cosine function.

$$\Psi = A\cos\left(\left(\frac{2\pi}{\lambda}\right)(x - vt)\right) = (3.0\ \text{cm})\cos\left(\left(\frac{2\pi}{7\ \text{cm}}\right)(10\ \text{cm} - (4\ \text{cm/s})\,(13\ \text{s}))\right) = 3.0\ \text{cm}$$

26. a) $f = \dfrac{v}{\lambda} = \dfrac{4.2\,\text{cm/s}}{4.0\,\text{cm}} = 1.05$ Hz

b) $T = 1/f = 0.95$ s

c) $k = 2\pi/\lambda = \pi/2\ \text{cm}^{-1}$

d) $\omega = 2\pi/T = 2.1\pi$ rad/s

27. a)

$$\Psi = \Psi_1 + \Psi_2 = 0.003\,[\sin(6.0x - 300t) + \sin(7.0x - 250t]$$

We can use a trig identity $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

$$\Psi = 0.006\sin(6.5x - 275t)\cos(-0.5x - 25t)$$

or because cosine is an even function

$$\Psi = 0.006\sin(6.5x - 275t)\cos(0.5x + 25t)$$

At 64 eV

$$\lambda = \frac{1.226\ \text{nm} \cdot V^{1/2}}{\sqrt{64\ \text{V}}} = 0.153\ \text{nm}$$

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From Figure 5.7(a) we see that $2\theta_1 = \tan^{-1}(2.1\ \text{cm}/35\ \text{cm}) = 3.434^\circ$ or $\theta_1 = 1.717^\circ$. Now since $\lambda = 2d\sin\theta$ we have

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24. Refer to problem 11. The 5 eV electrons are nonrelativistic. We must treat the 500 keV electrons relativistically.

For the 5 eV electrons, $\lambda = \frac{hc}{\sqrt{2mc^2K}} = \frac{1240\ \text{eV} \cdot \text{nm}}{\sqrt{2(0.511 \times 10^6\,\text{eV})(5\,\text{eV})}} = 0.549\,\text{nm}$. The electron speed can be found using $p = mv = \frac{h}{\lambda}$. Therefore

$$v = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})(0.549 \times 10^{-9}\ \text{m/s})} = 1.33 \times 10^6\ \text{m/s}$$

and $\beta = \dfrac{v}{c} = 4.42 \times 10^{-3}$. Alternatively for this nonrelativistic case you could use $K = 5\,\text{eV} = \frac{1}{2}mv^2$ to find v.
The phase velocity is found using a modification of Equation (5.32). We must note that E represents the total energy of the particle, so

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This speed exceeds the speed of light but is not associated with the transmission of information. In problem 28 we will show that a simpler approach to find the phase velocity is to use $v_{ph} = \dfrac{c}{\beta}$ but either approach yields the same answer. We find the group velocity from Equation (5.31), namely $u_{gr} = \dfrac{pc^2}{E}$. In problem 73 from chapter 2, we showed that $\beta = \dfrac{pc}{E}$ so $u_{gr} = \beta c = 1.33 \times 10^6$ m/s which is the same as the particle speed.
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d) $\omega = 2\pi/T = 2.1\pi$ rad/s

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or because cosine is an even function

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b)

$$v_{ph} = \frac{\omega_1 + \omega_2}{k_1 + k_2} = \frac{550 \text{ rad/s}}{13 \text{ m}^{-1}} = 42.3 \text{ m/s}$$

$$u_{gr} = \frac{\Delta\omega}{\Delta k} = \frac{50 \text{ rad/s}}{1 \text{ m}^{-1}} = 50 \text{ m/s}$$

c) As in Equation (5.22) $\Delta x = 2\pi/\Delta k = 2\pi$ m is the separation between zeros.

d) $\Delta k \Delta x = \left(1 \text{ m}^{-1}\right)(2\pi \text{ m}) = 2\pi$

28.

$$u_{gr} = \frac{d\omega}{dk} = \frac{dE}{dp} = \frac{d}{dp}\left(p^2c^2 + E_0^2\right)^{1/2} = \frac{pc^2}{\sqrt{p^2c^2 + E_0^2}} = \frac{pc^2}{E} = \beta c$$

$$v_{ph} = \lambda v = \frac{h}{p}\frac{\omega}{2\pi} = \frac{E}{p} = \frac{pc^2/v}{p} = \frac{c^2}{v} = \frac{c}{\beta}$$

The particle and its "signal" are associated with the group velocity, not the phase velocity.

* 29. As in Example 5.6 we start with $v_{ph} = c\lambda^n$ where c is a constant. We also know from Equation (5.33) that $u_g = v_{ph} + k\,\frac{dv_{ph}}{dk}$. We note further that $\frac{dv_{ph}}{dk} = \frac{dv_{ph}}{d\lambda}\left(\frac{d\lambda}{dk}\right)$. Since $\lambda = \frac{2\pi}{k}$ then $\frac{d\lambda}{dk} = \frac{-2\pi}{k^2} = \frac{-\lambda^2}{2\pi}$. Therefore

$$u_g = v_{ph} + \left(\frac{2\pi}{\lambda}\right)\frac{dv_{ph}}{d\lambda}\left(\frac{-\lambda^2}{2\pi}\right) = v_{ph} - (\lambda)\frac{dv_{ph}}{d\lambda}$$

Setting $u_g = v_{ph} = c\lambda^n$, we find $c\lambda^n = c\lambda^n - cn\lambda^n$. This can be satisfied only if $n = 0$, so v_{ph} is independent of λ. This is consistent with the idea that when a medium is non-dispersive, the phase and group velocities are equal and the speed independent of wavelength.

30. Protons:

$$\gamma = \frac{K + E_0}{E_0} = \frac{946.27 \text{ MeV}}{938.27 \text{ MeV}} = 1.00853 \qquad \beta = \sqrt{1 - \frac{1}{1.00853^2}} = 0.130$$

$$u_{gr} = \beta c = 0.130c \qquad v_{ph} = \frac{c}{\beta} = 7.7c$$

Electrons:

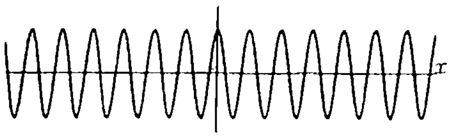
$$\gamma = \frac{K + E_0}{E_0} = \frac{8.511 \text{ MeV}}{0.511 \text{ MeV}} = 16.66 \qquad \beta = \sqrt{1 - \frac{1}{16.66^2}} = 0.9982$$

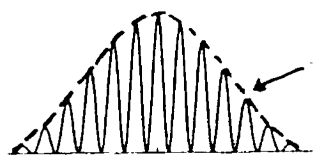
$$u_{gr} = \beta c = 0.9982c \qquad v_{ph} = \frac{c}{\beta} = 1.002c$$

31.

$$\begin{aligned}
\Psi(x,0) &= \int \tilde{A}(k)\cos(kx)\,dk = A_0\int_{k_0-\Delta k/2}^{k_0+\Delta k/2}\cos(kx)\,dk \\
&= A_0\,\frac{\sin(kx)}{x}\Bigg|_{k_0-\Delta k/2}^{k_0+\Delta k/2} = \frac{A_0}{x}\left(\sin\left(k_0 - \Delta k/2\right)x - \sin\left(k_0 - \Delta k/2\right)x\right) \\
&= \frac{2A_0}{x}\sin\left(\frac{\Delta k x}{2}\right)\cos\left(k_0 x\right)
\end{aligned}$$

See the diagrams below. At the half-width of $|\Psi(x,0)|^2$, we have $\sin^2\left(\frac{\Delta k x}{2}\right) = \frac{1}{2}$, so $\frac{\Delta k x}{2} = \frac{\pi}{4}$ and $x = \frac{\pi}{(2\Delta k)}$. Then $\Delta x = 2x = \frac{\pi}{\Delta k}$, and $\Delta k \Delta x = \pi$.





* 32. Relativistically

$$u = \frac{dE}{dp} = \frac{d}{dp}\left(p^2c^2 + E_0^2\right)^{1/2} = \frac{pc^2}{\sqrt{p^2c^2 + E_0^2}} = \frac{pc^2}{E}$$

Classically

$$u = \frac{dE}{dp} = \frac{d}{dp}\left(\frac{p^2}{2m}\right) = \frac{p}{m}$$

33. For a double slit, the amplitude of E is doubled, and hence the intensity (proportional to E^2) is higher by a factor of four for the double slit.

34.

$$d = \frac{\lambda}{\sin\theta} = \frac{h/p}{\sin\theta} = \frac{hc}{\left(\sqrt{E^2 - E_0^2}\right)\sin\theta} = \frac{1240 \text{ eV}\cdot\text{nm}}{\left(\sqrt{(512 \text{ keV})^2 - (511 \text{ keV})^2}\right)\sin 1^\circ} = 2.22 \text{ nm}$$

* 35. We make use of the small angle approximations: $\sin\theta \approx \tan\theta$ and $\sin\theta \approx \theta$.

$$\sin\theta \approx \tan\theta = \frac{0.3 \text{ mm}}{0.8 \text{ m}} = 3.75 \times 10^{-4}$$

$$\lambda = d\sin\theta \approx d\theta = (2000 \text{ nm})\left(3.75 \times 10^{-4}\right) = 0.75 \text{ nm}$$

$$p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1240 \text{ eV}\cdot\text{nm}}{(0.75 \text{ nm})\, c} = 1.653 \text{ keV}/c$$

$$K = E - E_0 = \sqrt{p^2c^2 + E_0^2} - E_0 = \sqrt{(1.653 \text{ keV})^2 + (511 \text{ keV})^2} - 511 \text{ keV} = 2.67 \text{ eV}$$

Such low energies will present problems, because low-energy electrons are more easily deflected by stray electric fields.

36. By the uncertainty principle $\Delta p \Delta x = \frac{\hbar}{2}$ at minimum. Non-relativistically (with $\Delta x = d$)

$$E_{\min} = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{8md^2} = \frac{(\hbar c)^2}{8mc^2 d^2} = \frac{(197.3 \text{ eV} \cdot \text{nm})^2}{8\,(938.27 \times 10^6 \text{ eV})\,(1.6 \times 10^{-5} \text{ nm})^2} = 20.3 \text{ keV}$$

37. For the $n = 1$ energy level

$$E = \frac{h^2}{8md^2} = \frac{h^2c^2}{8mc^2d^2} = \frac{(1240 \text{ eV} \cdot \text{nm})^2}{8\,(939.57 \times 10^6 \text{ eV})\,(2 \times 10^{-6} \text{ nm})^2} = 51.1 \text{ MeV}$$

By the uncertainty principle $\Delta p \Delta x = \frac{\hbar}{2}$ at minimum. Non-relativistically (with $\Delta x = d$)

$$E_{\min} = \frac{(\Delta p)^2}{2m} = \frac{\hbar^2}{8md^2} = 1.30 \text{ MeV}$$

The two answers differ by the factor of $(2\pi)^2$ in using $\hbar$ or h in the energy formula.

* 38. The uncertainty ratio is the same for any mass and independent of the box length.

$$\Delta p \Delta x = m \Delta v \Delta x \geq \frac{\hbar}{2} \quad \text{or} \quad \Delta v \geq \frac{\hbar}{2m\Delta x} = \frac{\hbar}{2mL}$$

$$E = \frac{1}{2}mv^2 = \frac{h^2}{8mL^2} \quad \text{or} \quad v = \sqrt{\frac{h^2}{4m^2L^2}} = \frac{h}{2mL}$$

$$\frac{\Delta v}{v} = \frac{\hbar/2mL}{h/2mL} = \frac{1}{2\pi}$$

39. For circular motion $L = rp$ and so $\Delta L = r\Delta p$. Along the circle $x = r\theta$ and $\Delta x = r\Delta\theta$. Thus

$$\Delta p \Delta x = \frac{\Delta L}{r}(r\Delta\theta) = \Delta L \Delta\theta \geq \frac{\hbar}{2}$$

For complete uncertainty $\Delta\theta = 2\pi$ and

$$\Delta L = \frac{\hbar/2}{2\pi} = \frac{\hbar}{4\pi}$$

40. $\Delta E \Delta t \geq \frac{\hbar}{2}$ so

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{1.054 \times 10^{-34} \text{ J} \cdot \text{s}}{(2 \times 10^{36} \text{ y})\,(3.16 \times 10^7 \text{ s/y})} = 1.67 \times 10^{-78} \text{ J}$$

41. If we use Equation (5.44), $\Delta\omega\Delta t = \frac{1}{2}$ we find

$$\Delta\omega = \frac{1}{2\Delta t} = \frac{1}{2\,(2.4\,\mu\text{s})} = 2.1 \times 10^5 \text{ Hz}$$

42. $\Delta p \Delta x = m \Delta v \Delta x \geq \frac{\hbar}{2}$ so at minimum uncertainty

$$\Delta v = \frac{\hbar}{2m\Delta x} = \frac{1.054 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\,(3 \times 10^{-15}\ \text{kg})\,(10^{-6}\ \text{m})} = 1.76 \times 10^{-14}\ \text{m/s}$$

* 43. a) $\Delta E \Delta t \geq \frac{\hbar}{2}$ so

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{6.582 \times 10^{-16}\ \text{eV}\cdot\text{s}}{2\,(1 \times 10^{-13}\ \text{s})} = 3.29 \times 10^{-3}\ \text{eV}$$

b) Using the photon relation $E = \frac{hc}{\lambda}$ and taking a derivative

$$dE = -\frac{hc}{\lambda^2}\,d\lambda = -\frac{E^2}{hc}\,d\lambda$$

Then letting $\Delta\lambda = d\lambda$ and $\Delta E = dE$ we have

$$|\Delta\lambda| = hc\frac{\Delta E}{E^2} = (1240\ \text{eV}\cdot\text{nm})\,\frac{3.29 \times 10^{-3}\ \text{eV}}{(4.7\ \text{eV})^2} = 0.185\,\text{nm}$$

* 44. The wavelength of the electrons should be 0.14 nm or less. For this wavelength

$$p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1240\ \text{eV}\cdot\text{nm}}{(0.14\ \text{nm})\ c} = 8.86\ \text{keV}/c$$

$$K = E - E_0 = \sqrt{p^2c^2 + E_0^2} - E_0 = \sqrt{(8.86\ \text{keV})^2 + (511\ \text{keV})^2} - 511\ \text{keV} = 77\ \text{eV}$$

45. For the angle θ_R we find $\theta_R \approx \tan\theta_R = \frac{4000\,\text{nm}}{20\,\text{cm}} = 2 \times 10^{-5}$. The wavelength is

$$\lambda = \frac{d\,\theta_R}{1.22} = \frac{(0.05\ \text{m})\,(2 \times 10^{-5})}{1.22} = 820\ \text{nm}$$

a)

$$E = \frac{hc}{\lambda} = \frac{1240\ \text{eV}\cdot\text{nm}}{820\ \text{nm}} = 1.51\ \text{eV}$$

b) For non-relativistic electrons

$$K = \frac{p^2}{2m} = \frac{h^2}{2m\lambda^2} = \frac{h^2c^2}{2mc^2\lambda^2} = \frac{(1240\ \text{eV}\cdot\text{nm})^2}{2\,(511 \times 10^3\ \text{eV})\,(820\ \text{nm})^2} = 2.24 \times 10^{-6}\ \text{eV}$$

Check with the uncertainty principle:

$$\Delta p \geq \frac{\hbar}{2\Delta x} = \frac{1.055 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\,(4000 \times 10^{-9}\,\text{m})} = 1.32 \times 10^{-29}\ \text{kg}\cdot\text{m/s}$$

The actual momentum is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J}\cdot\text{s}}{820 \times 10^{-9}\ \text{m}} = 8.08 \times 10^{-28}\ \text{kg}\cdot\text{m/s}$$

This is allowed because $p > \Delta p$.

46.

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{1240 \text{ eV} \cdot \text{nm}}{\sqrt{2\,(3727 \text{ MeV})\,(5.5 \text{ MeV})}} = 6.12 \text{ fm}$$

The minimum kinetic energy according to the uncertainty principle is

$$K = \frac{(\Delta p)^2}{2m} = \frac{(\hbar c)^2}{8mc^2\,(\Delta x)^2} = \frac{(197.33 \text{ eV} \cdot \text{nm})^2}{8\,(3727 \text{ MeV})\,(16 \times 10^{-6} \text{ nm})^2} = 5.10 \text{ keV}$$

Since the kinetic energy exceeds the minimum, it is allowed.

* 47. The proof is done in Example 6.11. With $\omega = \sqrt{k/m}$ we have a minimum energy

$$E = \frac{\hbar\omega}{2} = \frac{\hbar}{2}\sqrt{\frac{k}{m}} = \frac{1.055 \times 10^{-34} \text{ J} \cdot \text{s}}{2}\sqrt{\frac{8.2 \text{ N/m}}{0.0023 \text{ kg}}} = 3.15 \times 10^{-33} \text{ J}$$

48. We can determine the value using the normalization condition:

$$\int_0^L A^2 \sin^2\left(\frac{\pi x}{L}\right) dx = 1 = A^2 \frac{L}{2} \qquad \text{or} \qquad A = \sqrt{\frac{2}{L}}$$

No answers are provided for questions 49 through 53.

54.

$$\psi_1 = A \sin\left(\frac{\pi x}{L}\right) \qquad \psi_2 = A \sin\left(\frac{2\pi x}{L}\right) \qquad \psi_3 = A \sin\left(\frac{3\pi x}{L}\right)$$

where $A = \sqrt{2/L}$. Refer to problem 48.

55. As before $\lambda = \dfrac{2L}{n}$ so $p_n = \dfrac{nh}{2L}$. At high energies we must use relativity, so

$$E = \sqrt{p^2c^2 + E_0^2} = E_0\sqrt{\frac{p^2c^2}{E_0^2} + 1}$$

$$\frac{E_2}{E_1} = \left[\frac{1 + h^2c^2/\left(L^2E_0^2\right)}{1 + h^2c^2/\left(4L^2E_0^2\right)}\right]^{1/2}$$

$$\frac{E_3}{E_1} = \left[\frac{1 + 9h^2c^2/\left(4L^2E_0^2\right)}{1 + h^2c^2/\left(4L^2E_0^2\right)}\right]^{1/2}$$

$$\frac{E_4}{E_1} = \left[\frac{1 + 4h^2c^2/\left(L^2E_0^2\right)}{1 + h^2c^2/\left(4L^2E_0^2\right)}\right]^{1/2}$$

These are quite different from the non-relativistic results, as one might expect. They do reduce to the non-relativistic results in the low-energy limit.

56. At time $t = 0$ the velocity is uncertain by at least $\Delta v_0 = \dfrac{\Delta p}{m} = \dfrac{\hbar}{2m\Delta x}$. After a time $t = \dfrac{m\,(\Delta x)^2}{\hbar}$ we have

$$\Delta x' = (\Delta v_0)\,t = \frac{\hbar}{2m\Delta x}\,\frac{m\,(\Delta x)^2}{\hbar} = \frac{\Delta x}{2}$$

which means the uncertainty equals half the distance of travel.

57. Both the spatial distribution $\psi(x)$ and the wavenumber distribution $\phi(k)$ should have the same Gaussian form:

$$\psi(x) \propto \exp\left[-\frac{x^2}{(2\Delta x)^2}\right] \qquad \phi(k) \propto \exp\left[-\frac{k^2}{(2\Delta k)^2}\right]$$

For conjugate variables (x, k) it is possible to obtain one distribution by taking a Fourier transform of the other. Letting A be a normalization constant for $\phi(k)$ we have

$$\psi(x) = \frac{A}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \exp\left[-\frac{k^2}{(2\Delta k)^2}\right] \exp(ikx)$$

The integral is done by completing the square:

$$\begin{aligned} \psi(x) &= \frac{A}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dk \exp\left[-\frac{k^2}{4\Delta k^2} + ikx - x^2\Delta k^2 + x^2\Delta k^2\right] \\ &= \frac{A}{\sqrt{2\pi}} \exp\left(-x^2\Delta k^2\right) \int_{-\infty}^{\infty} dk \exp\left[\left(-\frac{1}{4\Delta k^2}\right)\left(k - 2ix\Delta k^2\right)^2\right] \end{aligned}$$

Letting $u = \dfrac{(k - 2ix\Delta k^2)}{2\Delta k}$ we have

$$\psi(x) = A\sqrt{\frac{2}{\pi}}\, \Delta k \exp\left(-x^2\Delta k^2\right) \int_{-\infty}^{\infty} \exp\left(-u^2\right) du$$

The integral has a value $\sqrt{\pi}$ so

$$\psi(x) = \sqrt{2} A \Delta k \exp\left(-x^2\Delta k^2\right)$$

Now comparing with the Gaussian form

$$\psi(x) \propto \exp\left[-\frac{x^2}{(2\Delta x)^2}\right]$$

we see that

$$(\Delta k)^2 = \frac{1}{(2\Delta x)^2} \qquad \text{or} \qquad \Delta k \Delta x = \frac{1}{2}$$

58. $\Delta E \Delta t \geq \hbar/2$

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{6.582 \times 10^{-16}\ \text{eV} \cdot \text{s}}{2 \times 10^{-16}\ \text{s}} = 3.29\ \text{eV}$$

* 59.

$$\Delta t = \frac{d}{c} = \frac{1.2 \times 10^{-15}\ \text{m}}{3.0 \times 10^{8}\ \text{m/s}} = 4.0 \times 10^{-24}\ \text{s}$$

$$\Delta E \geq \frac{\hbar}{2\Delta t} = \frac{6.582 \times 10^{-16}\ \text{eV} \cdot \text{s}}{2\,(4.0 \times 10^{-24}\ \text{s})} = 82\ \text{MeV}$$

This "lower bound" estimate of the rest mass is $\Delta E/c^2$ which is within a factor of two of the rest energy.

60. a) In general $n\lambda = 2d\sin\theta$ so

$$\sin\theta = \frac{n\lambda}{2d} = n\frac{0.5\text{ nm}}{2\,(0.8\text{ nm})} = 0.3125\,n$$

It is required that $\sin\theta \leq 1$ so the allowed values of n are 1, 2, 3. Substituting we find $\theta = 18.2°$ for $n = 1$, $\theta = 38.7°$ for $n = 2$, and $\theta = 69.6°$ for $n = 3$.

b) For electrons

$$p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1240\text{ eV}\cdot\text{nm}}{(0.5\text{ nm})\,c} = 2480\text{ eV}/c$$

$$K = E - E_0 = \sqrt{p^2c^2 + E_0^2} - E_0 = \sqrt{(2.480\text{ keV})^2 + (511\text{ keV})^2} - 511\text{ keV} = 6.0\text{ eV}$$

61. The uncertainty of the strike zone width is $\Delta x < 0.38$ m. There is a second uncertainty, in the x-component of the ball's velocity, given by

$$\Delta p_x = m\Delta v_x \geq \frac{\hbar}{2\,\Delta x}$$

Due to the uncertainty in velocity the ball's x position will be uncertain by the time it reaches home plate in an amount

$$\Delta v_x t = \frac{\hbar t}{2m\,\Delta x} \leq 0.38\text{ m}$$

where we have added the inequality because this is the condition for a strike. Since this inequality must be satisfied simultaneously with the first one $\Delta x < 0.38$ m, we multiply the two inequalities together to find

$$\frac{\hbar t}{2m} \leq (0.38\text{ m})^2 = 0.144\text{ m}^2$$

Rearranging we find

$$\hbar \leq \frac{2m\,(0.144\text{ m}^2)}{t} = \frac{2\,(0.145\text{ kg})\,(0.144\text{ m}^2)}{(18\text{ m})\,/\,(35\text{ m/s})} = 0.081\text{ J}\cdot\text{s}$$

62. Assume that the given angle corresponds to the first order reflection. We have:

$$\lambda = 2d\sin\theta = 2\,(0.156\,\text{nm})\sin 26° = 0.1368\,\text{nm}\,.$$

Next we find the energy:

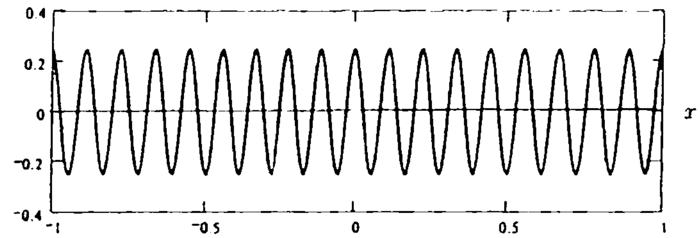
$$p = \frac{h}{\lambda} = \frac{hc}{\lambda c} = \frac{1240\text{ eV}\cdot\text{nm}}{(0.1368\,\text{nm})\,c} = 9.06\times 10^3\text{ eV}/c$$

$$\begin{aligned} K &= E - E_0 = \sqrt{p^2c^2 + E_0^2} - E_0 = \sqrt{(9.06\text{ keV})^2 + (939.57\text{ MeV})^2} - 939.57\text{ MeV} \\ &= 4.36\times 10^{-2}\text{eV} \end{aligned}$$

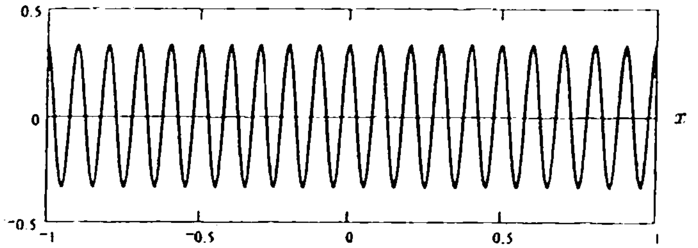
The Oak Ridge Electron Linear Accelerator Pulsed Neutron Source (ORELA) produces intense, nanosecond bursts of neutrons, each burst containing neutrons with energies from 10^{-3} to 10^8 eV.

63. a) Using the given information, the wave packet will equal: $\psi = \sum_n \psi_n = \frac{1}{4}\cos(18\pi x) + \frac{1}{3}\cos(20\pi x) + \frac{1}{2}\cos(22\pi x) + \cos(24\pi x) + \frac{1}{2}\cos(26\pi x) + \frac{1}{3}\cos(28\pi x) + \frac{1}{4}\cos(30\pi x)$. A graph of the first two terms and a graph of the entire packet are shown below. ψ_9 has a frequency of 9 and an amplitude of 0.25. ψ_{10} has a frequency of 10 and an amplitude of 0.33. Other terms would have a frequency of 11 and amplitude of 0.5, etc. The peak value of the packet is approximately 3.16.

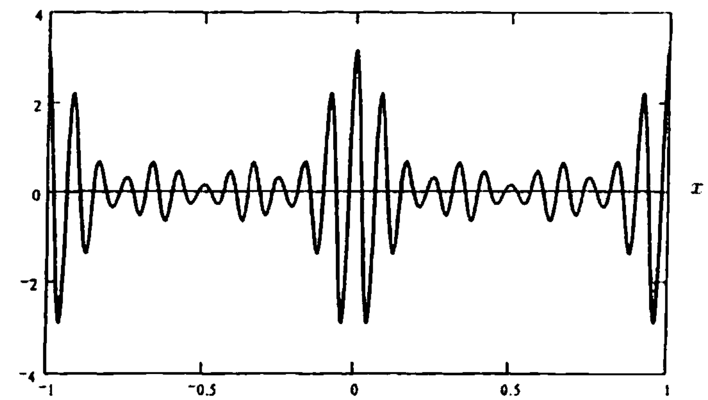
$$\psi_1 = \frac{1}{4}\cos(18\pi x)$$



$$\psi_2 = \frac{1}{3}\cos(20\pi x)$$



$$\psi = \sum_n \psi_n$$



b) The packet is centered about $x = 0$ but extends to $\pm\infty$. The packet repeats every unit along the x axis.

* 64. a) Starting from Equation (5.45), $\Delta E \Delta t \geq \frac{\hbar}{2}$ and substituting we have $\left(\frac{\Gamma}{2}\right)\tau \geq \frac{\hbar}{2}$. Therefore $\Gamma\tau \geq \hbar$.

b) We can find the minimum value for Γ from the equation above. Using the data for the neutron, for example.

$$\Gamma_{\text{neutron}} = \frac{6.582 \times 10^{-16}\,\text{eV}\cdot\text{s}}{887\,\text{s}} = 7.42 \times 10^{-19}\,\text{eV}\,.$$

The other values follow in a similar fashion.

$$\Gamma_{\text{pion}} = 2.53 \times 10^{-8}\,\text{eV} \qquad \Gamma_{\text{upsilon}} = 65.8\,\text{keV}$$