

## Introduction Part II: What Physics Education Research Can Teach Us About the Introductory Physics Courses

Over the last twenty-five years, a growing number of physicists, science education researchers, and cognition specialists have been carefully studying how students learn physics and how they solve physics problems. These physics education researchers have succeeded in understanding many of the difficulties students have with the introductory physics course and developing effective ways to address them. This introduction to Physics Education Research (PER) and the introductory course is designed to give physics instructors a working knowledge of the research into student learning difficulties and the methods developed to help students overcome them. Section I gives a brief summary of the main findings on student difficulties with learning introductory physics and their implications for instruction. Sections II and III offer a review of the physics education research literature. Section II describes three case studies of what happens in introductory physics lecture classes. Section III is a PER overview of findings from research on conceptual understanding, problem solving, and cognitive expectations.

### I. Physics Education research summary

#### A. Student Difficulties

PER gives instructors tools to better recognize the knowledge state of their students and understand the difficulties students have in learning physics. The findings from PER on student difficulties in the introductory physics sequence can be summarized as follows:

- 1. Traditional lecture instruction is not helping many students in the introductory course develop a functional understanding of physics.** Even many students who have mastered typical end-of-chapter problems demonstrate a weak understanding of basic concepts and fundamental principles, have difficulty expressing these ideas in graphs, diagrams, and their own words, and do not use the concepts and principles in problem solving. In addition, they are rarely able to apply what they know to new situations.<sup>1, 2, 3, 4, 5, & 6</sup>
- 2. Students do not come into the introductory physics sequence as blank slates.** They have very definite attitudes and beliefs about how things work,<sup>7, 8</sup> how to solve problems,<sup>9</sup> and how to learn physics based on years of experience both in and out of the classroom.<sup>10</sup> These attitudes and beliefs are often vague, ill defined, and may contain inconsistent elements. Nevertheless, students use them in making sense out of the course material. Note that because students' common-sense beliefs about how things work are often not well thought out, they are referred to as preconceptions rather than as misconceptions.
- 3. Many student attitudes and beliefs are incompatible with what is taught in introductory physics and can distort what students get out of the course.** These common-sense attitudes and beliefs shape students' perception of everything they do and see in the course.<sup>11</sup> Students receive a firehouse of information from the homework, readings, and lecture; much more than they can easily comprehend. The students filter this information by using their attitudes and beliefs about what physics is and how to learn it to select which activities they use to build their understanding of the course material.<sup>12</sup> These attitudes and beliefs also determine the type of understanding the students build and affect how students solve physics' problems. Because these naive views sometimes conflict with the view instructors are trying to teach,

what students learn can be very different from what is desired. For example, when viewing demonstrations that should challenge their physics preconceptions, many students either see a result that supports their beliefs (a result different from what happened) or see the demonstration as a special case that is not relevant.

4. **These student attitudes and beliefs are very resistant to change.** Traditional lecture instruction (lectures, homework, exams and labs) helps only a small fraction of the students to change their attitudes and beliefs in developing a physics perspective. For example, At New Mexico State University, van Heuvelen found that 80% of the beginning engineering students believed that the net force on an object is proportional to the velocity based on the results of a special test<sup>13</sup> designed to look at students conceptual understanding.<sup>14</sup> After one semester of introductory physics, 60% of these students still believed that net force is proportional to velocity. Other studies have found similar results.<sup>15, 16</sup>
5. **Students have little or no coherent framework for their physics knowledge.** While physicists organize their physics knowledge in large-scale functional units organized around basic physical principles, student knowledge often consists of random facts and equations that have little conceptual meaning.<sup>17, 18</sup> Their poorly organized knowledge structure is largely responsible for their formula-driven approach to problem solving.<sup>19</sup> When solving a problem, students first identify "surface features" of the problem (a pulley, an inclined plane, or a spring) and then search for an equation they associate with that feature. This is one reason why students often have great difficulty working a problem that uses the same physical principles as one they have solved previously, but applied to a different physical situation.
6. **Carefully constructed activities can make use of what students think and what they believe to help them develop a physics perspective.** Since many students often share a small number of difficulties, it is possible to develop research-based teaching methods that improve student learning by helping students change their attitudes and beliefs, particularly their common-sense conceptual beliefs (preconceptions). Students taught with PER-based curricula such as *Tutorials in Introductory Physics*, *Cooperative Group Problem Solving*, *Workshop Physics*, *Overview*, *Case Study* and others have demonstrated improved conceptual understanding and problem solving ability compared with students taught with traditional lecture instruction.<sup>20, 21, 22, 23, 24 & 25</sup>

## B. Implications for Instruction

Although PER does not give us the formula for the best way to teach, it does provide general guidance as to why some teaching methods are more effective than others. PER clearly shows that lecture can be good for efficiently presenting material to students as well as exciting and motivating them. However, regardless of the skill or proficiency of the lecturer, lecture by itself or in conjunction with traditional homework problems is not sufficient to help the majority of students build a good understanding of the course material or to change the students' naive beliefs and attitudes.

The traditional lecture method reinforces the naive student view that instructors hold the knowledge students need to learn and all they need to do to learn it is to listen to the instructor tell it to them. Unfortunately, teaching by telling is not effective for helping most students learn. Although the information may be presented flawlessly to the students, passive listening is not sufficient to learn to understand the course material or be able to reason with it. PER shows that students must be intellectually active (actively engaged) to develop a functional understanding and that instruction must take into account where students are to help bring them to where instructors would like them to be. Note that PER results do not show that lecture is not important or never effective,<sup>26</sup> but for the introductory physics sequence the traditional lecture method must be augmented<sup>27</sup> or modified to better facilitate student understanding.<sup>28</sup>

The PER lessons for physics instructors can be summarized as follows:

1. **Instructors need to change their role as the authoritative source of knowledge to being more of a facilitator providing activities and feedback to help students build their understanding of the material and apply that understanding in solving problems.** Even if lecture material is presented perfectly, students cannot learn it until they have built their own understanding of the material. McDermott, who heads the University of Washington PER group, explains,<sup>29</sup>

*All individuals must construct their own concepts, and the knowledge they already have (or think they have) significantly affects what they can learn. The student is viewed not as a passive recipient of knowledge but as an active participant in its creation.*

In this model, student learning is a growth process, rather than a knowledge transfer. The focus of instruction is on what the students are learning, not the presentation. It is important for students to begin seeing themselves as authorities on the physics they are learning.

2. **Instruction must deal explicitly with students' common sense attitudes and beliefs.** Unfortunately, just telling students about their preconceptions and then telling them the correct information does not help them build the correct perspective. Some of the most effective PER-based strategies for facilitating conceptual change use "cognitive conflict." The most common form used by PER-based curricula is known as "elicit-confront-resolve."<sup>30</sup> In this approach, the activities are specially designed to trigger the most common student preconceptions revealed by PER. In small groups, students make and explain predictions to one another about what will happen. The situations at the focus of the activity are selected so that most students will recognize the conflict between their reasoning and what actually happens. The students are then guided into resolving the conflict using the correct concept. Because students' common sense beliefs are highly resistant to change, students will need to see their preconceptions challenged in several different contexts.
3. **Students must be actively engaged and receive feedback.**

*I hear and I forget, I see and I remember, I do and I understand.*  
Chinese proverb

Although students must build their own understanding, many students lack the skills to do this on their own, outside of class. Because of this, instructors need to provide activities where students are actively engaged in doing or talking about physics. The activities should be designed to give students more experience with related physical situations, help students work through their conceptual difficulties, or improve their problem-solving skills. By discussing, listening, and explaining physics in small groups, students can build a much deeper understanding and a more sophisticated approach to learning physics. However, it is also important that students receive prompt feedback for active learning to promote conceptual change. For example, during an in-class activity, it is important that students receive feedback that allows them to recognize the correct answer and the conflict between their preconceptions and physics while they are still thinking about the activity.

4. **Instructors should emphasize understanding physical situations rather than abstractions.** The goal of physics as a science is to make sense of the world around us. Since our main goal as physics instructors is to help students learn to reason about and understand physical processes, instruction should emphasize physical situations over abstract concepts and mathematics. This has several advantages. It makes physics more accessible to the students since many of them are concrete thinkers<sup>31</sup> and see basic physics concepts and the equations that concisely describe them as "the theory." The following suggestions have been shown to improve student reasoning about physical phenomena:
  - Using experiential or discovery labs to help students become familiar with physical examples of concepts.
  - In presenting material, work inductively from the concrete to the abstract. This links the physics to real world situations and helps students from losing the physics in the math.
  - Asking questions that get students to explain their reasoning such as "How do we know?," "Why do we believe?," and "Can you explain that without using equations?"
5. **Students should be taught and use explicit problem solving skills and strategies.** Studies of experts and novices solving physics problems find that most introductory physics students solve problems using a means-end analysis.<sup>32</sup> Typically, this means finding an equation that contains the given and desired unknown quantities and if there is another unknown quantity in the equation, repeating the process until the problem is solved. Because this approach emphasizes finding the right formula (one that matches the quantities listed in the problem) and the numeric calculation, it encourages students to solve problems without understanding the underlying physics concepts. PER studies have shown that one way to help students become better problem solvers is to teach students an expert-like problem solving strategy like the GOAL protocol and require them to use it.<sup>33, 34, 35, & 36</sup> Strategies like goal help students learn how to do qualitative analysis of problems, plan how to approach the problem, and to check to see if the answer makes sense; all steps that most students forgo in their pursuit of the right numeric answer.
6. **Homework and exam problems must go beyond symbol manipulation and numeric calculation.** Many of the physics' problems typically used on homework and exams encourage student behavior that prevents the students from learning physics with understanding. For example, the typical end-of-chapter problems often use ideal situations that have little connection to the students' real world. In addition, they are often over-defined so that the desired unknown quantity is specified and almost all the other quantities needed to solve the problem are given in the problem statement. This encourages students to use the formula-driven pattern-matching approach to problem solving described above. If we want students to learn to use an expert-like problem solving strategy like GOAL, to solve problems with understanding, and to build a functional understanding of the basic ideas in introductory physics, we must give them exercises that help them learn and practice these skills. In addition, while the GOAL protocol can be used with any homework problem, students are more likely to see the benefit of GOAL with problems that require more thought than plug and chug exercises. With this in mind, instructors should assign exam and homework problems that
  - Balance qualitative and quantitative reasoning
  - Emphasize more reasoning and less algebra and calculations
  - Relate directly to observations and physical situations

(Note that it is important to provide in-class activities that help students develop the skills to solve these problems. Studies have shown that just giving students conceptual problems or more complex quantitative problems like this stumps students using a plug-and-chug approach, does not encourage them to use a more sophisticated problem-solving strategy, and does not foster an understanding of the underlying concepts.)

7. **Instructors need to model what they want students to learn.** While activities are useful for students to learn by doing, many of the behaviors and skills we want students to learn are new to them and must be demonstrated in several contexts to help students learn them. For example, PER studies of curricula using problem-solving strategies like GOAL found that it is important for instructors to model the problem-solving strategy for the students.<sup>37</sup> There is a tendency for some instructors to take shortcuts with notation and problem-solving steps on the board with the idea that the students can fill in the missing steps. However at the introductory level, students already want to skip steps and notation (particularly symbol notation) they see as unnecessary, not seeing these steps as linking a chain of reasoning from the given information to the final answer. The best way for instructors to help students learn the problem solving and reasoning techniques they want them to use is to model them in class and in the posted solutions. Discussing examples of good student solutions and showing why they are good can help model how to evaluate a solution.
8. **Assessment should be used to learn how students think about the course material and to reinforce instructional goals.** In the model of instruction proposed here, assessment plays two new roles: helping the instructor learn more about how students are thinking about the physics they are learning and reinforcing the instructor's learning goals for the course. The former is necessary because, as noted above, knowing what students are thinking is necessary for facilitating effective learning. The examples of Mazur, Hammer, and Tobias in the next section show that assessment needs to go beyond traditional problem solving to understand student difficulties and student thinking. In addition, student mistakes on carefully written problems are often more revealing of student thinking than correct responses. It is also necessary to use homework and particularly exams to reinforce learning goals to the students. While students will listen to what instructors say they want students to do, students quickly figure out what they need to learn to get the best grade with the least amount of effort, which is usually the minimum they need to learn to do well on the exams. Since many introductory physics students can solve traditional physics exam problems without really thinking about the underlying physics, this is one reason why traditional problems reinforce student preconceptions on physics problem solving and learning.

## II. Warning Signs: A look at traditional lecture courses

PER has found that a major part of the problems in teaching physics is the nature of the traditional lecture method of teaching. The current lecture method of teaching introductory physics was developed a hundred years ago to train professional scientists. It is designed to efficiently transfer information to large numbers of students. However, as the examples below show, many students (perhaps, a majority of students in the calculus-based physics course) are not well served by this method of teaching.

These three studies represent a small fraction of the available literature; they were selected primarily for their value as specific illustrations of what happens in traditional introductory physics lecture courses. The first example is from Eric Mazur,<sup>38</sup> a physics professor at Harvard, the second from David Hammer,<sup>39</sup> a physics education researcher, and the third from Sheila Tobias,<sup>40</sup> a non-traditional researcher of student difficulties with math and science.

### A. Mazur's Example: Student Difficulties With Conceptual Understanding

Eric Mazur is the Gordon McKay Professor of Applied Physics and a professor of physics at Harvard University. He currently leads a research program in applied physics and maintains an active interest in innovative instruction. He has developed his own strategy for teaching physics to lecture classes which is described in his book for physics instructors, *Peer Instruction: A User's Manual*. Mazur taught the introductory course for science and engineering majors for six years in the traditional lecture format before he became aware of a series of PER articles by Halloun and Hestenes concerning the persistence of students' common sense beliefs about mechanics.<sup>41</sup> In Mazur's words,<sup>42</sup>

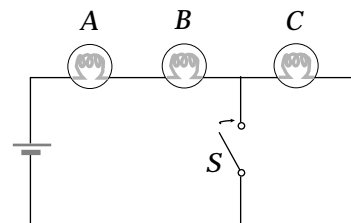
... I taught a fairly conventional course consisting of lectures enlivened by classroom demonstrations. I was generally satisfied with my teaching—my students did well on what I considered difficult problems and the evaluations I received from them were very positive. As far as I knew there were not many problems in my class.

Like most instructors when they first become aware of the poor performance of students on multiple choice concept tests like the Force Concept Inventory (FCI) following instruction, Mazur thought, "Not my students." When he gave his students the test he read about,<sup>43</sup> his first warning that there was a problem was when one of his students asked, "Professor Mazur, how should I answer these questions? According to what you taught us, or by the way I think about these things?" Despite the concept tests' simplicity, the students scored little better than they had on their last midterm examination which Mazur thought was of far greater difficulty.

To better understand these results, Mazur gave his students the two problems shown in Figure 1 as the first and last problems on a midterm examination in a traditional lecture class. Note that the first problem is purely conceptual and requires only a basic understanding of simple DC circuits. The second is a traditional circuit problem that deal with the same concepts as problem 1 but also requires setting up and solving equations using Kirchhoff's laws. Most physics instructors consider the first problem to be easier than the second. The results shown in Figure 2 show that the students disagree. Both problems were graded out of ten points. The average on the conventional problem is significantly higher than on the conceptual problem (6.9 vs. 4.9). Figures 2a and b are histograms of the student scores for the two problems. Note that two-thirds of the students scored at least 7 out of 10 on the conventional problem while only one third of the students did as well on the conceptual problem. In fact, almost one third of the students were unable to answer more than one of the five parts correctly.

Figure 1: Conceptual (top) and Traditional exam problems on the subject of DC circuits. Eric Mazur of Harvard gave these problems as the first and last problems on a midterm examination in the spring of 1991. (Figures from E. Mazur, *Peer Instruction: A Users Manual*, Prentice Hall 1997.)

1. A series circuit consists of three identical light bulbs connected to a battery as shown here. When the switch  $S$  is closed, do the following increase, decrease, or stay the same?
  - (a) The intensities of bulbs  $A$  and  $B$
  - (b) The intensity of bulb  $C$
  - (c) The current drawn from the battery
  - (d) The voltage drop across each bulb
  - (e) The power dissipated in the circuit



2. For the circuit shown, calculate (a) the current in the  $2\Omega$  resistor and (b) the potential difference between points P and Q.

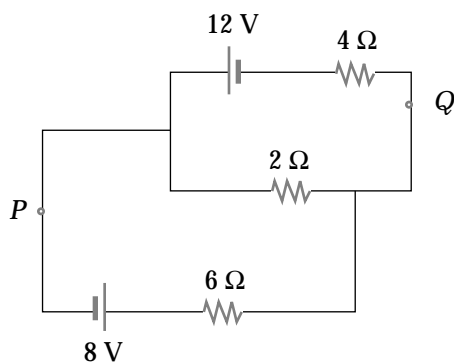


Figure 2: Test scores for the problems shown in Figure 1. For the conceptual problem, each part was worth a maximum of 2 points. (a) Test score distribution for the conceptual problem. (b) Test score distribution for the traditional problem. (c) Correlation between the scores on each problem. The radius of each data point is a measure of the number of students represented by that point. (Figures from E. Mazur, *Peer Instruction: A User's Manual*, Prentice Hall 1997.)

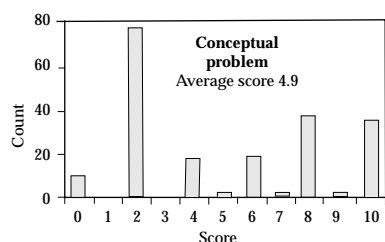


Figure 2a: Problem 1 Histogram

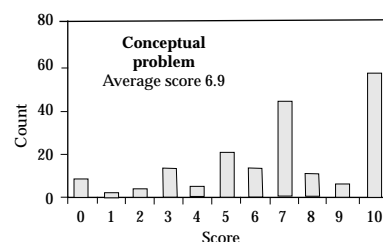


Figure 2b: Problem 2 Histogram

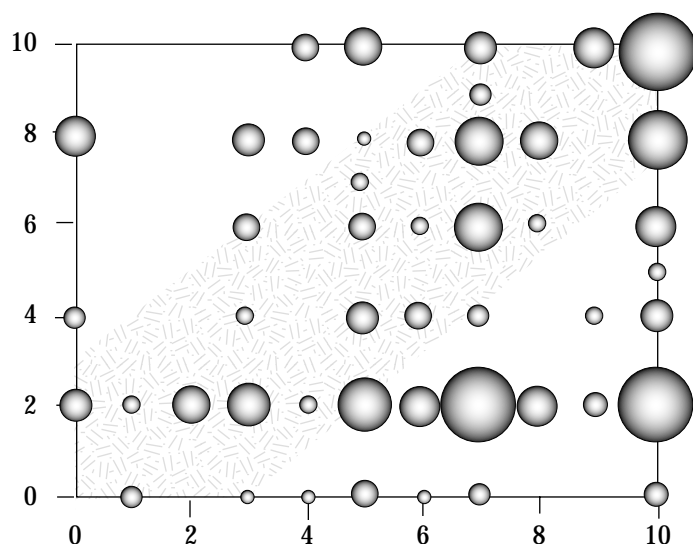


Figure 2c: Students Conceptual Problem Score vs. Traditional Problem Score

In Figure 2c the student scores for each problem are plotted against each other. Notice the apparent lack of correlation between student scores on the two problems. The broad diagonal band represents the boundary for roughly equal scores ( $\pm 3$  points) on the two problems. Although 52% of the students lie on the band, 39% of the students did substantially worse on the conceptual problem. A large number of these students managed to score 2 points or less on the conceptual problem while getting 10 points on the traditional problem.

According to Mazur, this result was repeated many times on pairs of problems during the remainder of the semester. The students performed significantly better on the traditional problems. Mazur suggests this example have three implications for physics instructors:

1. It is possible for students to do well on conventional problems by memorizing algorithms (recipes) without understanding the underlying physics.
2. It is possible even for an experienced instructor to be misled by traditional assessments into thinking that students have been taught effectively.
3. Students who use this algorithmic approach may believe they have mastered the material and then are severely frustrated when they learn that their plug-and-chug strategies don't work on all problems.

## **B. Hammer's Example, Two Approaches to Learning Physics**

David Hammer conducted case studies on students in the introductory "pre-med" physics course at the University of California at Berkeley to study student epistemological beliefs on learning and physics in introductory undergraduate physics courses.<sup>44</sup> In one study, randomly selected students from an introductory "pre-med" physics course were interviewed individually five times during the semester. The interviews consisted of a variety of tasks including open-ended discussions of the student's impressions of the course and of physics, semi-directed tasks such as going through a midterm exam, and specific discussions of physics concepts as well as quantitative and qualitative problem solving.

Hammer reported on two of these students, Liza and Ellen (pseudonyms) who on paper appeared very similar. They were both planning to go to medical school; both had math SAT scores around 700 and had A's in mathematics courses through calculus.<sup>45</sup> Liza's record was stronger, including a 5 on the BC Calculus Advanced Placement Exam, an A- in her first semester chemistry course, and an A in her high school physics class. Ellen received a C in the same chemistry course and had not taken a physics course beforehand.

### **1. Two approaches to learning physics**

Hammer chose to report on these two students because, unlike the other subjects in the study, Liza and Ellen both put a great deal of time and effort into the course and, while their grades were similar (Liza B+, Ellen B), initially their approaches to the course were not. Liza relied heavily on memorization and pattern matching without trying to understand the material. Hammer noted that throughout her interviews, Liza's indicated that her approach to the course was to learn formulas and facts based on the authority of the instructor and the text. To Liza, the formulas were the physics. If the professor stated the formula in lecture or the formula was in the book, this constituted sufficient justification for the formula to be true. Hammer notes, "She almost always said she understood the lecture or the reading; it was only when I pressed for explanations beyond citations that she 'didn't think about it' or was 'not sure'." Liza was not concerned whether or not the formulas made sense. She solved problems by 'figuring out which formula to use, but having an explanation for the formula itself did not seem relevant.



Liza: *This if  $v_f$  equals  $v_0$  plus  $at$  ( $v_f = v_0 + at$ ).*

Hammer: OK, and where did that come from?

Liza: *It came from here (laughs and indicates book).*

Hammer: Why is it true?

Liza: *Well, there is an initial velocity, and there is a final velocity, we know what the  $a$  is which is the acceleration, and I just use this to find what time it takes.*

On the other hand, Ellen tried to build her own understanding of the course material. She worked very hard to reconcile what she was learning in class with her experiences and her intuition. She was more independent, and, to her, the formalism was only one way of looking at physics. She did want to make sense of the material, to integrate it with her own intuitions. Ellen criticized lectures for failing to connect 'theory' with 'reality,' for emphasizing the formalism over simple explanations. She felt, for example, that the professor's treatment of finding the range of a projectile was needlessly complicated [excerpt from transcript follows]:

Ellen: *. . . obviously when it stops being in the air it stops going horizontally. . . . it seems like we spent a couple of lectures just trying to get that through people's heads, and I'm sure if you just say that to someone they'll (say) well obviously. . . . I guess that's what it is, we get theory with theory, and then we get application with application.*

Hammer: What do you mean by theory?

Ellen: *It means formulas, . . . let's use this formula because it has the right variable, . . . instead of saying OK, we want to know how fast the ball goes in this directions, because if we know that, all we have to do is find out how long it goes in that direction before it hits the ground and we can find out how far.* [Note that this quote seems to indicate she believes that the theory is the mathematical formalism.]

After the first few weeks she became frustrated, finding it difficult to reconcile different parts of the formalism with each other and with her intuition . . . Eventually she compromised her standards:

Ellen: *I'd rather know why for real.*

Hammer: What do you mean by 'know why for real?'

Ellen: *Well, sometimes I rationalize things, and I know that it's probably not the soundest rationalization, or something, it's just that I can kind of make sense for the time being, if I don't have time to really figure everything about it . . . even though I don't really know how it was derived.*

Like Liza, Ellen was able to apply prescribed methods to solve problems, but for her this was not sufficient:

Hammer: so part b, can you do it?

Ellen: *No. (laughs) I can pretend to. I can do it the way I know it's supposed to be done, but I pretty much already did that.*

Both students put in a great deal of time and effort to learn physics in this course. Liza worked extra problems and attended every lecture, recitation, and lab. While Liza felt she was succeeding in learning physics, Ellen did not feel she was getting much out of the course. Although initially she became comfortable with the material, even then she was aware of large gaps in her understanding.

## 2. Two approaches to problem solving

The two approaches to learning physics had a significant effect on how the two students approached physics problems. As Hammer relates,

An assigned problem asked: "One ball is thrown horizontally with a velocity  $v_0$  from a height  $h$ , and another is thrown straight down with the same initial speed.

Which ball will land first?"<sup>46</sup> Liza wrote out  $x = x_0t + \left(\frac{1}{2}\right)at^2$  for the horizontal

and vertical components of each ball, substituting appropriate initial positions, speeds, and accelerations. [Author's note: this problem is very difficult for introductory students to solve symbolically using only the equations for position and velocity as a function of time.] She then used the vertical components to show, not without some difficulty, that the second ball travels a greater vertical distance after time  $t$  than the first, so it hits first. Ellen, in contrast, said immediately that the answer was 'obvious,' and went on to explain qualitatively that the second ball hits first because it has a greater speed downward to cover the same vertical distance.

... When Ellen was able to make sense of the material, her intuition helped her understand the formalism and guided her solutions. After the first few weeks, however, Ellen's attitude interfered with her ability to answer the course questions. For a while she continued to try to reason based on her own sense of things, rather than accepting what she was told or answering by rote, but her intuitions were not generally in accord with Newtonian physics.

Liza, following the algorithms provided, often failed to apply her own knowledge from everyday life. She almost certainly had the common sense to know that the ball thrown down would hit first, but she did not think to make use of it in this context. At no time did she acknowledge that this was the answer one would expect. However, with her methodical adherence to the procedures, she was more reliable than Ellen in answering correctly.

## 3. Implications

For Liza, understanding physics meant knowing the facts and formulas and being able to apply them to problems. She did not deliberately or consciously use her intuition or her experience. According to Hammer, the effect of Liza's approach was

that she was satisfied with learning physics as a set of relatively unrelated pieces, isolated both from each other and from her own everyday experience. Her understanding remained incoherent and fragmented ... , but it served her comparatively well on problem sets and examinations. Ellen, when her approach was successful, was able to bring fragments of her understanding together, to integrate different pieces from the course with each other and with her intuitive knowledge. Unfortunately, it was not successful after the first few weeks.

Most physics instructors would prefer their students to approach an introductory physics more like Ellen than Liza, to think and reflect on the material while trying to reconcile intuition based on experience and the course material. Yet, as the course proceeded, Ellen had more and more trouble trying to build a comprehensive understanding of the course material while Liza consistently did well in the course. By the second midterm exam, Ellen was struggling and scored below the mean. Eventually she was only able to get a B in the course by abandoning her approach and doing "what everyone else does" by adopting an approach more like Liza's.

It is also worth noting Liza's response in her last interview when she was asked if she had liked the course. She answered, "Not really . . . it was kind of boring . . . all those formulas and derivations." When asked if she was describing the course or physics in general, she replied, "Both."

The course did not support Ellen's thinking, reflective approach but seemed to reward and encourage Liza's memorization approach. While Liza's approach allowed her to be 'successful in the class' as measured by grades, the course left her with a very naive and boring view of what physics is and what it means to do physics. Hammer comments,

This was a standard introductory physics course. The method of instruction was to present material, demonstrate its validity, provide examples of its application, and assign further problems for students to do on their own. Lectures paralleled the text at a pace of about a chapter a week. . . . Similarly, two of the ten labs involved determination of an empirical rule . . . All others concerned verification of known results or the use of known results to measure some quantity. In every case, the procedures were specified.

This description fits many traditional lecture format classes. Hammer indicates that while he only reported on the results for two students, the other students in the study made similar comments implying that these two students are not atypical. What is happening in the typical traditional lecture course to cause results like these?

The previous two examples illustrate some of the difficulties with the introductory physics course taught by the traditional lecture method uncovered by PER. Note that in both cases, traditional assessment offered no indications of problems with the course. Both Mazur and Hammer offer explanations of their findings. Mazur uses an analogy to explain.

If I were lecturing not on physics but, say, on Shakespeare, I would certainly not spend the lectures reading plays to the student. Instead, I would ask the students to read the plays before coming to the lecture and I would use the lecture periods to discuss the plays and deepen the students' understanding of and appreciation of Shakespeare.

Yet in physics (and many other introductory courses) lecturing is often little more than reading the book to the students with demonstrations as illustrations. But the problem with traditional lectures is more complicated than that. In explaining his findings, Hammer offers a more analytical view.

The flow of reasoning was always from the theory to the phenomena, the flow of information always from the professor and text to the students. The students applied the theoretical concepts of the course, but they were not involved in or even witness to the formation of those concepts. The laws were simply provided, and students were to become familiar with them through practice in solving problems. . . .

The emphasis on formalism and the goal of problem solving facility seem consonant with conceptions of physics as a collection of facts and formulas. The style of instruction, with knowledge passing from instructor to student, procedures and results to experimentation specified in advance, seems consistent with a reliance on authority. Furthermore, the pace of the course, with a chapter of reading and ten problems due every week, would not allow much independent exploration, except for those students who find the material easy.

Both Mazur's and Hammer's comments suggest that the nature and structure of the traditional lecture course, specifically the emphasis on problem solving and the over-reliance on authority, can have a negative effect on student learning. Similar results were found in the study conducted by Sheila Tobias.

### C. Tobias' Example: Observations on the Traditional Lecture Teaching Method

In her study, Tobias was trying to learn why many qualified undergraduate students with good math and science backgrounds decide to turn away from majoring in science after exposure to introductory courses.<sup>47</sup> Of college freshman who switch out of science and engineering majors, only about a third switch because they found the course work too difficult. For the rest, 40% find other fields more interesting and 25% believe they would have better job prospects elsewhere.<sup>48</sup> She refers to these students who have the ability to pursue science but choose not to as "the second tier." These second tier students are often the same students who are not being reached by traditional lecture instruction. In this study, she looked at second tier students' view of the process and problems of learning science in introductory physics classes.<sup>49</sup>

Because she wanted the observers to record their thoughts on the learning process as it was happening and compare with their experiences in other classes, she chose to use mature postgraduate or senior undergraduate students with good high school math and science background who chose not to take science in college. All the observers had demonstrated ability in their own fields. None of them had taken undergraduate introductory science courses. While Tobias' student observers were not typical introductory physics students, their comments correlate strongly with an independent survey of students who switched out of being science majors.<sup>50</sup>

Three humanities graduate students (Eric, Jackie, & Michel) and one fifth-year social science undergraduate (Vicki) acted as student observers in three different calculus-based introductory physics classes at the University of Arizona and the University of Nebraska, Lincoln. To make sure they had the proper background, each candidate was required to have taken at least one semester of college calculus in addition to four years each of high school mathematics and science. Two of the student observers with rusty but solid calculus skills,<sup>51</sup> Eric & Jackie, did very well in their respective courses. Another student observer, Michel, whose math skills were weaker, struggled in the course but was able to learn the concepts and solve the assigned problems. The remaining student observer, Vicki, struggled and would have dropped the course if she were not involved in the project. Tobias considers Vicki's viewpoint to be closest to that of an average "C" student.

Each student observer was paid to seriously audit one semester of different calculus-based introductory physics courses as observers. In return for the stipend, they were expected to perform as well as they could in the class including attending all classes, submitting all work, and taking all examinations up to the final. In addition, they were expected to keep a journal where they would monitor the instructor's style of presentation, the material in the book, and the assignments, as well as to record their personal experiences with the course and where possible those of their classmates.

While the student observers' comments on their own progress and difficulties are very illuminating, it is their observations on how the class was taught that can help us understand what is happening in traditional lecture classes. For example:

*Eric: To some extent science is hard because it simply is hard. That is to say, the material to be learned involves a great many concepts, some of which are counterintuitive. The process of mastering these concepts and being able to demonstrate a computational understanding of actual or theoretical situations required a great deal of time and devotion. In my experience, this fact is well understood by the students, the professor, and the general public. What is not as well understood are the various ways in which this already hard subject matter is made even harder and more frustrating by the pedagogy itself.*

All four student observers commented on three particular aspects of the class that they felt made learning physics more difficult:

1. The lack of a narrative or story line,
2. An overemphasis on quantitative problem solving, and
3. A classroom culture that discourages discussion and cooperation.

To understand the nature of these difficulties, it is instructive to review some of their comments on the standard lecture course. Despite being in different classes, the comments are very similar.

#### 1. Lack of a narrative or story line

In Hammer's study, Liza, the student who focused on developing recipes for problem solving from the start, indicated that to her physics was a collection of equations and formulas that applied to many different situations. She did not see physics as the process of learning and applying a few fundamental ideas to understand many different situations. While she could solve the assigned problems, she missed the big picture. This may have contributed significantly to why she found both the course and the subject boring.

In Tobias' study, the professors' lectures apparently failed to help the student observers see common themes and links in the material. In fact, Eric, Jackie, and Michel all found it patronizing not to be told in advance where they were headed or what they needed to understand as indicated by the comments below.

*Eric: We had marched through the chapters, doing the required work but never digging deeper . . . I was able to keep myself on track by concentrating on one chapter at a time. But I never got the idea that the professor had any understanding of how the concepts were related, as he rarely tied together information from more than one chapter. His lectures did not seem to build upon each other, and he gave no indication of a linear movement through a group of concepts . . . The final then asked the most primary basic questions about only the most important laws of physics. We were not required, at any time, to interrelate concepts or to try to understand the "bigger picture."*

*Jackie: Why, I wanted to know, did we begin by studying only the idealized motion of particles in straight lines? What about other kinds of motion? If he [the professor] could tell us what's coming next, why we moved from projectile motion to circular motion, for example, I would find it easier to concentrate; I'd know what to focus on. In college, I always wanted to know how to connect the small parts of a large subject. In humanities classes, I searched for themes in novels, connections in history, and organizing principles in poetry.*

Although all four of the student observers were accomplished students with at least four years of experience in undergraduate courses in other fields, not one of them was able to see either the connections or the context in what they were learning in the introductory physics course.

## 2. Overemphasis on quantitative problem solving

The exams, homework assignments, and lecture emphasized algorithmic problem solving, especially 'how much' problems. All four student observers were more interested in building an understanding of the physics concepts, in particular, the 'why' questions. However, they felt that their curiosity was not supported by the course. As Michelle notes,

*Michele: My curiosity simply did not extend to the quantitative solution. I just didn't care how much. I was more interested in the why and the how. I wanted verbal explanations with formulae and computations only as a secondary aid. Becoming capable at problem solving was not a major goal of mine. But it was a major goal of the course.*

Eric also initially noted that "for the most part, 'why' questions are neither asked nor answered." Later he found,

*Eric: As I am able to ask more knowledgeable questions, class becomes more interesting. I am finding that while the professor is happy to do example problems for the entire period, he will discuss the real world ramifications of a theory if asked.*

Note that this professor was only willing to discuss the ramifications in response to Eric's questions. It is also worth noting that his fellow classmates did not appreciate Eric's questions. They lost patience with his silly 'why' questions when they got in the way of finding the right solution to the assigned problems. According to Eric, this was what physics was all about for them. The other student observers made similar observations about their classmates.

*Vicki: When [John, a classmate she was studying with,] brought up an equation, I would try to relate it to another equation to help me learn them both all the better. This confused the effort. When John works a problem he uses only what is necessary and brings nothing else into the process. If I am going to learn a subject I need to know what is similar and dissimilar about an item. Why, when you are pushing down on a moving block, is there no "work" done? Isn't there a force downward on the block? I know that force and displacement have to be in the same direction; I needed to relate this to the concepts in the problem . . . John seemed disturbed by this and thought I had not yet mastered the fundamentals.*

*Jackie: [In her study group, the other students'] concerns focused on the kinds of problems they would encounter on the exams, not at all on a general understanding of the concepts . . . They ignored all the fun parts, seeing the whole picture, laying out the equations and solving these. Instead, they wanted to know what equaled what and solve for an answer. The elegance of problem solving was lost . . .*

Jackie later speculated that this might be less a difference of mind and more a matter of efficiency.

*Jackie: I think the students around me are having the same sort of thought-provoking questions about the material that I put into my journal, but under time pressure they don't pursue them, eventually they learn to disregard "extraneous" thoughts and to stick only to the details of what they'll need to know for the exam. Since the only feedback we get is on the homework assignments, the students cannot help but conclude that their ability to solve problems is the only important goal of this class.*

Part of the reason for this student attitude may have been caused by the exams. The student observers noted that the exam problems were usually more like the easier homework problems rather than (in their opinion) the more interesting harder problems.

Michele: *Too easy exams in contrast to too hard homework. On philosophy exams, instructors expect their students to do more than what they've done before, not less.*

Eric: *The problems [on exams] seldom required the use of more than one concept or physical principle. Only once were we asked to explain or comment on something rather than complete a calculation.*

Eric found the class exams biased towards computation and away from conceptual understanding. While he understood that some level of conceptual understanding was required to complete the computations, he found that the level was not particularly high.

One consequence of this emphasis on quantitative problems is that physics becomes less of a creative process and more of a craft.

Eric: *I do not feel that what this professor is doing can be considered teaching in any complex or complete sense. My understanding is that we are to learn primarily by reading the text, secondarily by doing problems on our own and comparing our solutions to those on sale in the physics office, and thirdly by mimicking the professor's problem solving examples. Simply by intuition I know physics, and more generally science to involve creativity and finesse; but this man makes it more into a craft, like cooking, where if someone follows the recipe, he or she will do well.*

Jackie: *Learning to solve physics problems is a process, not a matter of insight . . . Understanding the free body diagram means knowing how to do them.*

[Tobias comments:] The problems were of limited interest because they had all been solved before. Only occasionally did these exercises provide intellectual satisfaction; rarely were they a source of new insight. [The student observers] looked upon the effort to be training at the expense of education in science, too many scales, not enough music.

### 3. A classroom culture that discourages discussion and cooperation

The way material is presented in the traditional lecture format makes it appear that the book and the instructor are the acknowledged authorities and they are passing what they know to students who learn it and repeat it back in homework and exams. The student is trying to learn what the instructor knows. In this transmissionist view (knowledge is transmitted from the authority to the student) of learning, the emphasis is on learning facts, not understanding. It is interesting to note that the student observers perceived their courses to be 'low' on concepts & theory and mired in facts (*dry formulas and dull reality*).

As Eric's comment below indicates, this model of instruction can cause students to approach the course in ways that hinder student learning.

Eric: *I still get the feeling that unlike a humanities course, here the professor is the keeper of the information, the one who knows all the answers. This does little to propagate discussion or dissent. The professor does examples the 'right way' and we are to mimic this as accurately as possible. Our opinions are not valued, especially since there is only one right answer, and at this level, usually only one right way to get it.*

Part of this perception may be due to the course emphasis on quantitative problems discussed above, but part of it is due to a lack of student discussion in and out of class on the ideas and concepts being learned. The students do not feel they have ownership of the material; they have not made it an intrinsic part of what they know and understand. All of the observers commented and speculated on the passive nature of their classmates in lecture. Vicki and Eric noted,

Vicki: *No one ever asks questions in class except for that rare "Will you clarify?" question. I feel like it is grab and run.*

Eric: [after learning of the low class average on quizzes and exams] *What this means is that there are a good many people sitting quietly and not asking questions. This is always the case to some extent in college, but physics seems harder on these people than the humanities.*

The other observers also speculated as to why students are so passive in physics lectures.

Jackie: *When he goes through these problems, the work seems so obvious, the equations so inevitable, that I tend not to question what he's doing . . . Lectures in physics can be incredibly passive experiences for students, particularly dangerous for those who believe that if they can follow the professor, they've mastered the material.*

Compare these comments with the situations that the student observers thought were helpful to them in learning the course material:

[Tobias comments:] Vicky eventually found that her best studying came while working with at least one other student, *teaching the material to one another*. Here she was able to do what she could not do in class: question and try out what she thought she understood, and then question again. . . . The best class in Eric's view was one where the professor brought in five or six demonstrations, the results of which were counter-intuitive, and then asked the class to speculate as to why the particular results occurred. In this class, there was substantial interchange.

Some of the student observer comments suggest that one reason why students were so passive in lecture is the course format and culture.

Eric: *The lack of community, together with the lack of interchange between the professor and the students combines to produce a totally passive classroom experience . . . The best classes I had were classes in which I was constantly engaged, constantly questioning and pushing the limits of myself and the subject and myself. The way this course is organized accounts for the lack of student involvement . . . The students are given pre-masticated information simply to mimic and apply to problems. Let them rather be exposed to conceptual problems, try to find solutions to them on their own, and then help them to understand the mistakes they make along the way.*



Both Eric and Vicki believed that one problem was the classroom competition. Eric noted,

Eric: *[The sense of competition is in no way beneficial.] It automatically precludes any desire to work with or to help other people. Suddenly your classmates are your enemies. . . . My class is full of intellectual warriors who will some day hold jobs in technologically-based companies where they will be assigned to teams or groups in order to collectively work on projects. [But] these people will have had no training in working collectively. In fact, their experience will have taught them to fear cooperation, and that another person's intellectual achievement will be detrimental to their own.*

#### D. Implications

These three studies demonstrate that traditional lecture instruction by itself is not helping many students build an understanding of physics concepts and principles. Tobias' student observers, like Hammer's student who tried to build her own understanding, exhibited beliefs and attitudes toward learning and physics that most instructors would greatly desire to see in their students, in particular, their drive for a deep, useful understanding of physics beyond traditional problem solving, their need for debate and discussion, and their need for a coherent structure in what they were learning. Yet, like Hammer's student, the class did not support these attitudes. They felt that the physics classes lacked coherent themes, overemphasized quantitative problem solving, and lacked meaningful student discussion on the principles, ideas, and concepts of introductory physics in and out of class.

In addition, each of the four student observers felt that the traditional exams were narrowly focused on quantitative problem solving and were not reflective of what they were learning. This suggests that if instructors wish to improve student learning in the introductory course, both the traditional lecture format and assessment methods may need to change. Note that in all three studies, traditional assessments and student course evaluations showed no indication of any of the problems discussed.

### III. Physics Education Research overview

Physics Education Research (PER) is the study of how students learn physics, the study of the difficulties they have in learning physics, and the development of more effective strategies/curriculum for teaching physics. Although the field began with observational studies,<sup>52</sup> in the early 1980's researchers like McDermott, Viennot, Reif, Larkin, Clement, Halloun, Hestenes and others developed PER into an area of active research within physics departments.

McDermott has described the process of physics education research as an iterative cycle with three parts:<sup>53</sup>

1. conducting systematic investigations of student understanding
2. applying the results to the development of specific instructional strategies to address specific difficulties, and
3. designing, testing, modifying, and revising the materials in a continuous cycle on the basis of classroom experience and systematic investigations with the target population.

Often several iterations of this cycle are necessary to develop effective instruction. There are currently three major areas of active research in PER.

- Studying the conceptual knowledge students develop about how the world works
- Studying the problem solving techniques and strategies students use
- Studying students' cognitive attitudes about what physics is and how it should be learned

In all three areas, researchers study what students bring into the physics classroom and how it changes as a result of instruction. The most important finding in all three areas is that students do not enter the introductory physics course as "blank slates." They come to the course with their own attitudes and beliefs of how the world works, how to solve problems, and how to learn physics based on their experiences and observations. These beliefs shape students' initial state coming into the introductory physics class.

The next step is to look at how to bring the student from their initial state to a more desired state where they have begun building a more functional understanding of physics. In Part 1 of the introduction, we saw that active learning where the students debate and discuss the material is needed to change student beliefs. But why are some group activities better than others? What is happening as the student learns?

This overview is divided into three sections corresponding to the three areas of active research in PER. It discusses results of research into the initial state of students coming into the introductory physics course and some of the learning processes that help students change their beliefs to improve their conceptual understanding and their problem solving skills. Surprisingly, change in student beliefs in two areas—conceptual understanding and problem solving—is likely to occur only when the students learn to recognize an incompatibility with what they believe and what they discover to be true. This cognitive conflict is the stimulus for change. Because researchers have only recently begun studying expectations and expectations are more difficult to assess, much less is known about how to teach students to improve their expectation beliefs. However, what is known suggests expectations also change as a result of cognitive conflict.

### A. Conceptual Understanding

In the early 1980's, physics education researchers such as McDermott,<sup>54</sup> Clement,<sup>55</sup> Viennot,<sup>56</sup> Hestenes & Halloun,<sup>57</sup> and others<sup>58</sup> began studying undergraduate students' conceptual understanding of physics in the area of mechanics. Using a combination of interviews and specially designed problems, they began studying students' understanding of selected concepts in mechanics like velocity, acceleration, and force in specific contexts. They found that each student brings into a physics course their own system of common sense beliefs and intuitions about how the world works derived from extensive personal experience. Furthermore, they found that many students' common sense beliefs are incompatible with the physics taught in the introductory course and often appear to outlast instruction. Unfortunately, these common-sense beliefs have been found to be very stable and often persist even after instruction. We will refer to these common sense beliefs and intuitions as preconceptions since they are often partially formed ideas based on experience.

This research on student preconceptions led Hestenes *et al.* to claim that "instruction that does not take the students initial state of conceptual understanding into account is almost totally ineffective, at least for the majority of the students."<sup>59</sup> Traditional instruction does little to change students' preconceptions causing them to misinterpret the course material. However, many students have the same preconceptions. This makes it possible to design curriculum that can take the more prevalent preconceptions into account and help students learn concepts more effectively.

## 1. Student Preconceptions: Kinematics and Newton's Laws

This section summarizes what is generally known about the initial state of students' conceptual understanding in introductory physics in the context of the best understood and longest studied areas of students' conceptual understanding: kinematics and Newton's laws of motion. Typically, the most frequent student difficulties with common-sense preconceptions of motion and force are language, pre-Newtonian views of motion and force, and representations.

### a. The language of physics

The language difficulty occurs mainly because many words used in the basic description of mechanics are also used in common everyday language. The way students use words like force, momentum, energy, acceleration, speed, and velocity in common speech can cause difficulties in the context of the physics class. Many of these words have different meanings and connotations in common speech and many are used interchangeably without regard to the meaning they have for physicists. As a result, students often use the language of physics either without understanding the meaning of the words in the physics context<sup>60</sup> and/or without differentiating between words for related concepts. For example, many students have trouble differentiating between distance, velocity, and acceleration and equate them all with a generalized idea of "motion."<sup>61</sup>

### b. Common sense beliefs on force and motion

Halloun and Hestenes found from interviews and diagnostic tests (see chapter 4) that the majority of responses from 478 students on force and motion questions at the beginning of a University Physics class were consistent with both Aristotelian (83% on the diagnostic) and Newtonian (17%) models of motion.<sup>62</sup> Moreover, nearly every student used a mixture of models of motion and appeared to apply the models inconsistently in different contexts. Interviews were conducted with 22 students to probe their preconceptions about how and why things move the way they do and verify the diagnostic results. This study led to the development by Halloun *et al.* of the taxonomy of student preconceptions about force and motion is shown in Table 1.

Halloun and Hestenes caution that the common-sense alternatives to Newtonian views are not just misconceptions. Many of them were held by some of the greatest intellectuals in the past including Galileo and even Newton.<sup>63</sup> Many are reasonable hypotheses grounded in everyday experiences. For example, the common-sense belief that something must cause the motion of an object comes from observations that some force must be applied to most objects like cars, trains, and boxes to keep them in motion at constant speed because of frictional effects. However, the fact that the students apply their common-sense preconceptions inconsistently implies that their knowledge structure coming into the class is fragmented, incoherent, and context dependent. Studies by Minstrell<sup>64</sup> and diSessa<sup>65</sup> indicate that the student preconceptions can indeed be characterized as loosely organized, ill-defined bits and pieces of knowledge that are dependent upon the specific circumstance in question.<sup>66</sup> Studies by diSessa found that many student preconceptions are rooted in pieces of knowledge he calls "psychological primitives" or "p-prims." P-prims are simple, general isolated pieces of mental models that are cued by particular situations and are used to explain the events of the situation. They are usually strongly tied to real world experiences. The common sense example discussed in the paragraph above is an expression of the

"force as mover" p-prim and the "continuous force" p-prim. The "force as mover" p-prim is the belief that objects go in the direction they are pushed. This p-prim tends to be used when an object is given a short, instantaneous push. The "continuous force" p-prim is the belief that a force is required to keep an object moving. This p-prim tends to be used in situations where the force is continuous rather than instantaneous.

Table 1. Taxonomy of preconceptions of force and motion. Reprinted with permission from Hestenes D., Wells M., Swackhamer G., "Force Concept Inventory," *The Physics Teacher*, **30** (3), 1992, pp 141-158. Copyright 1992, American Association of Physics Teachers.

### **Kinematics Preconceptions**

- K1. Position-velocity indiscriminated
- K2. velocity-acceleration indiscriminated
- K3. nonvectorial velocity composition

### **Impetus Preconceptions**

- I1. impetus supplied by 'hit'
- I2. loss/recovery of original impetus
- I3. impetus dissipation
- I4. gradual/delayed impetus build-up
- I5. circular impetus

### **Active Force Preconceptions**

- AF1. only active agents exert forces
- AF2. motion implies active force
- AF3. no motion implies no force
- AF4. velocity proportional to applied force
- AF5. acceleration implies increasing force
- AF6. force causes acceleration to terminal velocity
- AF7. active force wears out

### **Action/Reaction Pair Preconceptions**

- AR1. greater mass implies greater force
- AR2. most active agent produces greatest force

### **Concatenation of Influence Preconceptions**

- CI1. largest force determines motion
- CI2. force compromise determines motion
- CI3. last force to act determines motion

### **Other Preconceptions on Motion**

- CF. Centrifugal force
- Ob. Obstacles exert no force
- Resistance
  - R1. mass makes things stop
  - R2. motion when force overcomes resistance
  - R3. resistance opposes force/impetus
- Gravity
  - G1. air pressure-assisted gravity
  - G2. gravity intrinsic to mass
  - G3. heavier objects fall faster
  - G4. gravity increases as objects fall
  - G5. gravity acts after impetus wears down

Studies using pre and post testing and/or interviews have shown that traditional lecture classes show little improvement in understanding of basic concepts and many students still have these common-sense preconceptions after traditional instruction.<sup>67</sup> From their interview study, Halloun and Hestenes found that as a rule,

students held firm to mistaken beliefs even when confronted with phenomena that contradicted those beliefs. When a contradiction was recognized or pointed out, they tended at first not to question their own beliefs, but to argue that the observed instance was governed by some other law or principle and the principle they were using applied to a slightly different state. . . . Careful interviews of students who have just witnessed a demonstration are enough to make one dubious about the effectiveness of typical classroom physics demonstrations in altering mistaken physics beliefs. We doubt that a demonstration can be effective unless it is performed in a context that elicits and helps to resolve conflicts between common sense and specific scientific concepts.

This last finding of the ineffectiveness of typical demonstrations to resolve conflicts between common-sense beliefs and specific physics concepts has also been reported in studies by Kraus *et al.*<sup>68</sup> as well as Redish *et al.*<sup>69</sup>

Recent studies by Francis and Larson<sup>70</sup> at Montana State University have shown that even after students appear to have acquired a Newtonian view of linear force and motion, rotational analogs to the students' common sense beliefs from linear motion appear when the students begin discussing rotational motion and dynamics. This suggests that students actually hold onto both the physics concept and the common-sense preconception and that their response will depend on what is triggered by the cues of the situation. (This has been more directly observed in the University of Maryland PER group's study of student understanding of waves.<sup>71,72, &73</sup>) The Francis and Larson study further suggests that in situations outside the context where they learn specific physics concepts, students have a tendency to revert to their common-sense beliefs.

### c. Representations

In addition to the above-mentioned difficulties with language and concepts, students also have difficulties with the abstract representations of physics such as graphs, equations, free-body diagrams, and vectors. One of the main goals of physics instruction is to help students become fluent with the multiple representations of physics, a necessity for students trying to develop a robust, functional knowledge of physics. Although there is a great deal of research on this issue by both math and physics education researchers, here we will only consider student difficulties with understanding graphs since graphs are one of the most useful and powerful representations of ideas and data, both in class and in everyday life.

Several studies have found that students coming into introductory physics classes understand the basic construction of graphs but have difficulty applying their understanding to the tasks they encounter in physics.<sup>74 & 75</sup> The two most common types of student errors are thinking of a graph as a literal picture of an object's motion and confusing the meaning of the slope of the line with the height of the line.<sup>76</sup> An example of the former is when a student asked to draw a velocity vs. time graph of a bicycle going along on a hilly road draws a velocity graph that resembles the hills and valleys traversed by the bicycle. An example of the latter

is when students asked to find the point of maximum rate of change indicate the point of largest value. In general, students tend to find interpreting slopes more difficult than individual data points. They also have difficulty separating the meanings of position, velocity, and acceleration graphs.

Beichner at North Carolina State University developed a multiple-choice diagnostic test for studying the prevalence of student difficulties with the understanding of kinematics graphs.<sup>77</sup> In a study of 900 students at both the high school and college level, there was no significant difference in the overall score between the high school and the college students. However, the calculus-based students did score significantly better than the algebra/trig students (with a mean of 9.8 vs. 7.4 out of a maximum of 21) did. In addition, he uncovered a consistent set of student difficulties with graphs of position, velocity, and acceleration. One of the main difficulties was that approximately 25% of the students believed that switching variables would not change the appearance of the graph. This is an indication that the students are interpreting the graphs as pictures of motion. It is interesting to note that the students who could correctly translate from one variable to another had the best scores on this diagnostic test. Another result was that 73% of the students correctly identified the slope of a line passing through the origin while only 25% were able to do so for a line that did not go through the origin. Approximately 25% of the students gave responses that are consistent with the slope/height mix-up described above. He also found that the students had trouble using the area under the curve to go from one graph to another such as using the acceleration vs. time graph to find the velocity vs. time graph. About one third of the students gave responses that used the slope of the line when they needed to find the area, and less than a third of the students were correctly able to determine the change in velocity from an acceleration graph.

Although Beichner did not use interviews to see why students answered the way they did, a similar study by Thornton of 10 multiple-choice velocity and acceleration graph questions did.<sup>78</sup> Thornton found student error rates after traditional instruction of about 40% on the velocity questions and 70-95% error rates on the acceleration questions. The instructors of these classes felt that these questions were simple (they expected error rates of 5-10%) and that students who were unable to answer these questions correctly had a very poor understanding of kinematics. Thornton noted that the problem was not that the students were unable to read graphs. Almost all (95%) of the students could answer similar questions about distance graphs correctly and interviews with the students showed that the students were picking graphs consistent with their verbal or written explanations of velocity and acceleration. This implies that students' difficulties with kinematics graphs are directly related to their conceptual difficulties with acceleration and velocity.

Research into students' conceptual understanding has been extended to many areas besides kinematics and forces to be an active area of PER. Some of the other areas include:

- energy and momentum<sup>79</sup>
- heat and temperature<sup>80</sup>
- waves<sup>81</sup>
- electricity and magnetism<sup>82</sup>
- light and optics<sup>83</sup>

Many of these findings are discussed in the PER topic summaries throughout the instructor's manual.

## 2. Mechanisms for Changing Students' Preconceptions

### a. Cognitive Conflict

As discussed previously, the common sense beliefs that students bring to the introductory physics class are ill-defined, vague notions based on real world experience. Because they are neither fully developed concept nor truly misconceptions (since they are based on real situations), we will use the term preconception to describe them.

In order to see how student preconceptions can be changed, we first need to discuss two mechanisms for learning introduced by Piaget, "assimilation" and "accommodation."<sup>84</sup> Piaget defines assimilation as the process by which people learn new ideas that match or extend on their existing conceptual knowledge. He defines accommodation as the process where people learn new ideas that don't fit into their existing conceptual knowledge either because the ideas are completely new or because the idea conflicts with what they already know, i.e. cognitive conflict. Students find it much easier to learn physics concepts that fit their view of how things work, i.e. to assimilate new ideas.<sup>85</sup> Accommodation tends to be much harder because the students must change or rethink their existing views. One reason for this is that often students will perceive and interpret what they learn in a way that makes sense in terms of their existing views. For example, in the case of the demonstration studies described previously, many students tried to assimilate rather than accommodate what they observed by interpreting it either as an example that demonstrates what they believe or as a special case that is unrelated. This tendency to assimilate rather than accommodate is one reason that students' conceptual knowledge may contain contradictory elements.

However, even though helping students learn to change their conceptual understanding is difficult, it is not impossible. Research has shown that it is possible to stimulate conceptual change for most of the students in a class.<sup>86</sup> In his review of cognitive science research relevant to physics instruction, Redish notes that the mechanism for conceptual change appears to critically involve prediction and observation.<sup>87</sup> "The prediction must be made by the individual and the observation must be a clear and compelling contradiction to the existing [conceptual knowledge]."

He notes Posner *et al.*'s suggestion that in order to change students' existing conceptual understanding the proposed replacement must have the following characteristics: <sup>88</sup>

- The replacement must be understandable.
- The replacement must be plausible.
- There must be a strong conflict with predictions based on the subject's existing conceptual understanding.
- The replacement concept must be seen as useful.

Redish adds, "The clearer the prediction and the stronger the conflict, the better the effect." However, since students can hold conflicting ideas in their models at the same time, it is crucial that the students be made to resolve the conflict for conceptual change to be effective. This method of conceptual change by direct confrontation is known as "cognitive dissonance."

Almost all of the active learning methods discussed in Part I of this introduction make use of cognitive dissonance. Two examples, *Tutorials in Introductory Physics* developed by the McDermott PER group at University of Washington<sup>89</sup> and *Workshop Physics* developed by Priscilla Laws at Dickinson College,<sup>90</sup> were specifically designed to improve students' conceptual understanding of physics based on the research discussed in this section. (The curricula are described in detail in Part I.) Both curricula use strategies that resemble Posner *et al.*'s four conditions of conceptual conflict and change.

In the *Tutorials in Introductory Physics* curricula, traditional recitations are replaced by groups of three to four students working through a specially designed worksheet by consensus. The tutorials use the strategy of elicit, confront, and resolve. The worksheet problems first put students into situations where they use their common sense beliefs to explain what is going on. Then it puts them in a situation where they are forced to recognize an inconsistency between their reasoning and something they believe is true, and helps them to reconcile the inconsistency.

In the *Workshop Physics* curricula, the traditional lecture/lab/recitation format is abandoned in favor of an all-laboratory approach. Each laboratory unit is designed to take the collaborative student groups through a four-part process that can be summarized as follows:

1. Students make predictions about a physical situation.
2. The students perform discovery-oriented experiments and try to reconcile the differences between their experimental observations and their predictions.
3. The students develop definitions and equations from theoretical considerations.
4. Finally, they perform experiments to verify their theoretical model and apply their understanding of the phenomenon to the solution of problems.

While these two curricula seem very different, there are some common factors between them. Both of them build on students' existing common sense beliefs to help them develop a physicists' understanding of the concepts. They are also constructed on the principle that the students must be "engaged" in the learning process by thinking about situations, committing to a prediction of what will happen, discussing that prediction with their fellow students, and resolving that prediction with what really happens.

**b. Using Interactive Technology to Improve Conceptual Understanding:  
The Role of Microcomputer-based Laboratory and Video-based Laboratory  
Instruction**

In physics research, new technology often makes possible experiments that could not be done before as well as experiments that explore familiar phenomena in new ways. In physics education, the technology of personal computers allows students to explore physical situations in ways that help them build a better understanding of physics concepts and their representations. In particular, microcomputer-based laboratory (MBL) and video-based laboratory (VBL) activities have been demonstrated to be effective tools for improving students understanding of the basic concepts and graphical representations related to kinematics and Newton's laws of motion.

Note that here the terms MBL and VBL refer not simply to the technology of using computers to collect and display data, but to laboratory activities using this technology where students are actively engaged in constructing an understanding of relevant concepts. When used with a predict-observe-explain approach to elicit and resolve cognitive dissonance (see the description of Interactive Lecture



demonstrations in Part I of the introduction), MBL and VBL activities can help students accommodate physics concepts that contradict their preconceptions. MBL technology often consists of an interface box that connects the computer to various inexpensive probes that can make physical measurements including distance, force, temperature, current, and voltage.

One of the best studies of MBL activities in university physics was conducted by Thornton and Sokoloff.<sup>91</sup> They reported that introductory physics students' understanding of velocity graphs could be significantly improved using MBL activities they developed. They evaluated the effect of their activities using five multiple-choice velocity questions in which students were required to match a description of a motion to a velocity graph. A large fraction of the students taught with traditional lecture missed all but the simplest velocity graph questions while after four hours of group-learning, guided-discovery, active-engagement MBL activities, error rates drop to below 10% on all questions. The result is extremely robust and has been reproduced at many schools with lecturers using a variety of teaching styles.<sup>92</sup> The poor results achieved by students on the velocity graph questions, despite traditional instruction in the subject, might be interpreted as difficulties reading graphs and not as a confusion on concept. However, the fact that students succeed at a much higher rate on position graphs, plus the reports of interviews from many researchers indicate that this is a confusion of concept, not just of representation.

In these MBL activities, students first use sonic rangers and microcomputer data acquisition to display position and velocity graphs of their own motions as walk towards and away from the sonic ranger. Next, they graph the motions of a cart under various conditions. There are two types of activities. Both use a strategy to promote conceptual change similar to the elicit-confront - resolve approach discussed above. In the first type of activities, guiding questions require that students make predictions as to what the graphs would look like, carry out the experiments, and reflect on their own thinking. In the second type of activities, students have to plan how to match a displayed graph and then carry out the experiment. (Note that this second type of activity with velocity graphs is particularly helpful for getting students to evaluate their thinking about velocity. Even many physics faculty have to think carefully about how to move to match a velocity graph.)

A follow-up study by Redish *et al.* looked at whether the improved response on the multiple-choice velocity questions was due to the MBL activity or the additional time spent on the topic.<sup>93</sup> After reading the Thornton and Sokoloff article, Redish was sure that after he lectured his students on velocity they would do well on the multiple-choice questions<sup>94</sup> and he felt that four extra hours of instruction gave the MBL students too much of an advantage. The following semester, Redish did his best to teach the material explicitly in lecture, devoting nearly three full hours to the topic of instantaneous velocity. In his words,<sup>95</sup>

I tried to make sure most of the students were mentally engaged. I wrote clear definitions on the board and walked a pattern and made them graph it in their notebooks. I gave examples that were realistic and related to their experience. I used our high quality demonstration equipment—including the [MBL] equipment Thornton and Sokoloff used in their labs.

Note that while the students were prompted for predictions for many of the demonstrations, they were not given time to explain their prediction and their reasoning with one another. In the recitation section, graduate-teaching assistants spent one hour going over velocity problems from the textbook. Redish continues,

And then I gave their problem on my mid-semester exam. The results were both humbling and elating. Despite my best efforts in lectures, the results my students obtained were very close to the [result] Thornton and Sokoloff reported from lectures at other universities. On the other hand, I was pleased at the robustness of the result. ... This was exhilarating—just as in my freshman lab when I measured  $g$  with a long pendulum and got the answer in the textbook.

After completing a sabbatical with McDermott's PER group at University of Washington and learning about the *Tutorials in Introductory Physics* curriculum, Redish implemented the tutorials at University of Maryland. However, Redish *et al.* developed and substituted some MBL tutorials into the curriculum including a one-hour velocity tutorial based on the Thornton and Sokoloff MBL activities.

This time Redish taught velocity for one hour of lecture and one hour of MBL tutorial (tutorials replace traditional recitation sections). The questions were again given on the mid-semester exam. The result, shown in Figure 4, is substantially improved from Redish's "best lecture efforts," but not as good as that obtained with four hour of MBL activities. In addition, Redish *et al.* also gave open ended questions on the final exam to courses taught with and without tutorials by other instructors. The tutorial students performed significantly better on these open-ended questions including one that asked them to draw a velocity graph for a complex situation.

Thornton and Sokoloff conjecture that their MBL activities are unusually successful for enhancing student learning in introductory physics for the following reasons:<sup>96</sup>

- Students learn concepts by investigating the physical world (often kinesthetically)<sup>97</sup> rather than only manipulate symbols or discuss abstractions as is common in traditional course.
- The immediate feedback helps students to see graphing as a useful scientific representation and aids them in understanding physical concepts by making the abstract more concrete.<sup>98</sup>
- Because the data are presented quickly in an understandable way, this encourages students to discuss the validity, the meaning, and the implications of their data with their peers.<sup>99</sup>
- Powerful tools reduce unnecessary drudgery<sup>100</sup> so that more student time is spent observing physical situations, analyzing graphical representations, and thinking about the underlying physics concepts.<sup>101</sup>
- The MBL activities guide students to understand the specific and familiar before moving to consideration of more general and abstract examples.

Like MBL, VBL technology can produce real-time graphs and data tables of the motion of the objects being investigated. VBL technology refers to computer tools that allow students to collect data from digitized video clips that can be shown graphically on the same screen as the video display. In more sophisticated implementations, students can digitize and analyze video clips from any video source including cameras, VCRs, and videodisk players. Like MBL, students can use the data collected to create mathematical models to explain their observations. Although VBL activities cannot provide kinesthetic or hands-on experiences, they

do have some advantages over MBL activities. For example VBL can be used to analyze complex two-dimensional motion such as projectile motion and real-world situations from outside the classroom. In addition, VBL can also be used to analyze frame by frame the motions of physical situations that occur too fast for the eye to observe well such as collisions or wave interference.

In one of the few studies that evaluates student learning from VBL activities, Beichner showed that "the greater the integration of video analysis into the kinematics curriculum, the larger the educational impact."<sup>102</sup> In this study, Beichner ranked seven classes according to how often they used VBL activities and the nature of the activities. The classes used varying degrees of VBL activities from none or one VBL demonstration to courses that had extensive VBL demonstrations and VBL labs. The educational impact was studied by looking at student performance on a diagnostic test on kinematics graphs. Classes that had extensive VBL activities including both labs and demonstrations had much higher average scores (70% vs. 48%) than classes with few or no VBL activities.<sup>103</sup> Classes that had either extensive video-based labs or demonstrations performed significantly better (60%) than classes with few or no VBL activities but not as well as classes with both video labs and demonstrations.

While the studies described above show that technology can be used effectively to improve student learning, just introducing technology into the classroom and having students work in groups does not by itself improve student learning. An example of this comes from a study of *Studio Physics* classes at RPI by Cummings *et al.*<sup>104</sup> *Studio Physics* is an introductory university physics curriculum with integrated lecture/laboratory sections, collaborative group work, and extensive use of computers for classes of 30-45 students. Cummings *et al.* found studio classes where student groups worked on activities were "predominately 'traditional' exercises adapted to fit the studio environment and incorporate the use of computers" showed much less improvement in understanding basic physics concepts than studio classes that incorporated elements from research-based physics curricula.<sup>105</sup> They used the FCI and FMCE (conceptual diagnostics that look at student understanding of basic Newtonian force and motion concepts) to measure improvement improvements in students' conceptual understanding. The studio classes that implemented research-based activities had twice the normalized gain of the classes that used modified traditional activities. This shows that technology and working in groups are not sufficient for promoting conceptual change, the activities need to be carefully designed to engage the students and help them address their preconceptions.

This is but one example. In another study by Hestenes *et al.* of high school instructors implementing a research-based curriculum into their physics classes following a summer workshop, many of the high school instructors reported little or no gain in their students' conceptual understanding after one year. On further inquiry, the instructors were found to have paid more attention in teaching their classes to the mechanics of the curriculum—computers, lab activities, discussion technique—than the crucial pedagogy that makes it effective.<sup>106</sup> Hestenes *et al.* suggests that this example demonstrates that technology by itself cannot improve instruction; the best technology can do is to enhance the effectiveness of good pedagogy. This conclusion is supported by several other studies of computers used in a variety of educational situations.<sup>107, 108, & 109</sup>

### c. Bridging

In the methods discussed above, students are confronted with situations where their preconceptions give predictions that many students recognize as incorrect. The students are then given support to construct the reasoning behind the correct answer. Another approach for working with student preconceptions to help them understand the physicist's view of fundamental concepts is the "bridging" method developed by Camp and Clement.<sup>110</sup>

"Bridging" means helping students build an understanding of a difficult situation by using a series of intermediate examples starting from a situation that most students understand. Like the elicit-confront-resolve approach discussed above, students first discuss a target problem designed to elicit students' preconceptions that are not consistent with the physics view. Then, rather than confront the students immediately with evidence that their preconceptions are wrong, the students are presented with a related situation in which most students' intuition is consistent with the physics view of the situation. Camp and Clement refer to these situations as "anchor analogies." After this, intermediate analogies are used to "bridge" the conceptual gap between the anchoring analogy and the target problem, as shown in Figure 3 below. Choosing the right anchor analogy is crucial to the process.

Consider a sample lesson on the normal force for high school students that addresses the preconception that solid, unmoving objects like walls and tables do not exert forces.<sup>111</sup> The lesson would proceed in the following way:

1. The concept of force is introduced as "a push or a pull of one object on another." (Note that this definition is only a starting point and will be refined as the course progresses.)
2. The target problem is introduced as the instructor draws a picture of a book on a table and asks the class "Does the table exert an upward force on the book?" This should lead to a lively discussion of student ideas where many students may argue that the table does not exert a force on the book because it doesn't move. (The instructor is urged to remain neutral until the end of the lesson.)

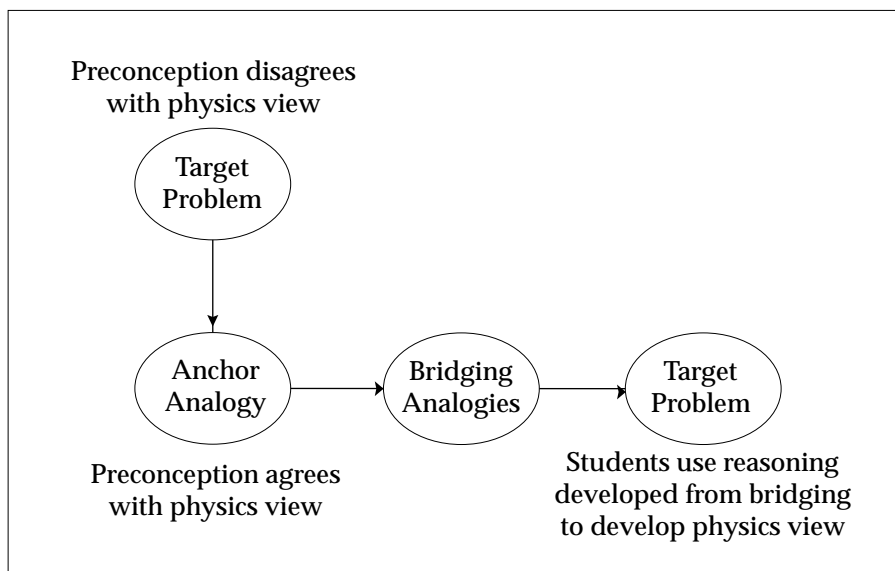


Figure 3. Using bridging to help students understand a situation that is contrary to their preconceptions, i.e. the target problem, by helping them reason from a situation where their preconception is correct by using a series of intermediate or bridging analogies.

3. The instructor then introduces the anchor analogy, the case of a hand pressing down on a spring, and asks the students about the similarities and differences between this situation and the target problem. (The choice of this anchoring analogy is useful because most students know from studying properties of springs earlier in the term that the spring exerts a force on the hand.)
4. Then two bridging analogies are introduced: a book resting on a foam pad, and a book resting on a flexible board. Students are asked to compare these cases with the anchor analogy and the target problem.
5. At this point the teacher introduces the microscopic model of rigid objects being made up of atoms connected by spring-like bonds.
6. Finally the students view a demonstration where a laser beam is reflected off a mirror on desk onto a wall. The students see that the beam is deflected downward when the teacher stands on the desk. (This demonstrates that the table compresses slightly when the teacher stands on the desk.)

The bridging analogies also involves student predictions (with reasoning), peer discussion, and sometimes experiments.

Clement investigated how bridging activities like these improved student learning of three basic mechanics concepts: normal forces, frictional forces, and Newton's third Law.<sup>112</sup> A specially designed test was used as a pre-post instruction measure to assess improvements in student understanding of these basic concepts. He found that the bridging section had much larger pre-post gains (average gain difference of 27% or 1.3 std. dev.) than the traditional lecture classes. All classroom instructors were blind to the test and clinical interviews were used to validate that the test questions were indicative of students' conceptual understanding.

## B. Problem Solving

### 1. Characterizing Expert and Novice Problem Solvers

Over the last twenty-five years, cognitive scientists and artificial intelligence researchers as well as math and physics education researchers have studied how people solve physics problems. The key finding is that there are significant differences between novice and expert problem solvers. This section will briefly summarize the nature and results of some of the more pertinent studies. For further information, refer to David Maloney's excellent review article on problem solving in physics, which is paraphrased extensively in this section.<sup>113</sup>

A number of problem-solving studies have focused on identifying differences between expert and novice problem solvers as well as good and bad problem solvers. These studies find differences in knowledge structure, approach to problem solving, and the use of examples and readings. The studies are generally conducted by interviewing individuals as they perform a problem-solving task or by studying computer models designed to simulate human problem-solving behavior. Most of the interview studies classify graduate students and professors as expert problem solvers while students from introductory physics sequences are considered novices. Novice students are further classified as either good problem solvers or poor problem solvers on the basis of how well they do on traditional textbook-style physics problems on class exams.

**a. Knowledge structure**

Although physics content knowledge is an integral part of physics problem solving,<sup>114,115, 116, & 117</sup> de Jong and Ferguson-Hessler argue "that just having the knowledge is not sufficient, it must be organized in a useful manner."<sup>118</sup> The following results tend to support this view.

- Experts problem solvers have extensive domain knowledge that is organized hierarchically, with general physics principles at the top. Novices have significantly less domain knowledge that, when organized, tends to be organized around the surface features of physical situations.<sup>119,120,121,122, & 123</sup>
- Experts tend to classify problems by the physical principles involved in the solution of the problem. Novices tend to classify problems based on physical objects, configurations, properties, and concepts explicitly described in the problem statement.<sup>124,125,126, & 127</sup>
- Good problem solvers had better knowledge organization than poor problem solvers.<sup>128 & 129</sup>

**b. Problem solving approaches**

One major emphasis of the studies on problem solving from the late 1970s through the early 1980s was to explore the differences between the problem solving approaches of novices and experts. The results can be summarized as follows:

- Expert problem solvers make use of and often require a detailed qualitative representation to plan their solution before working forward to a solution. Novice problem solvers tend to start with an equation that contains the unknown quantity asked for by the problem and work backwards to the given information (means-end analysis).<sup>130 & 131</sup>
- Expert problem solvers plan their solutions more carefully and in greater detail before carrying them out.<sup>132 & 133</sup>
- Expert problem solvers make greater use of physical reasoning.<sup>134</sup>
- Expert problem solvers also conduct an exploratory analysis, evaluate their thinking, and check their solutions to make sure they are reasonable when solving problems.<sup>135</sup>

Two of the studies found "strong evidence" that one of the main causes of poor problem solvers' difficulties is their failure and/or inability to construct an appropriate qualitative representation.<sup>136 & 137</sup> They suggest that the inability to use qualitative representations is due to poor understanding of the physics concepts.

**c. Use of example problems and reading**

In 1989, Chi *et al.* reported on a study of how students used worked examples when they are trying to learn how to solve problems.<sup>138</sup> They observed what students did when studying the examples and how they used the examples when they were solving problems. Chi *et al.* found the following differences between good and poor students:

- The good students were significantly better at working out the missing steps and identifying the points they did not understand in an example. They would continue to study an example until they understood it. The poor students tended to walk through the examples without checking to see if they understood them.
- Good students worked out the procedural aspects of the problems that are usually left implicit in textbook presentations, but poor students needed assistance in identifying and understanding these aspects.

- Good students tended to refer to a short segment of an example when they needed a specific relation or when they wanted to compare what they had done on a problem. Poor students tend to go back and read major segments of an example in search of procedures to use on the problem they were trying to solve.

Fergusson-Hessler and de Jong conducted a similar study investigating how students read segments of text.<sup>139</sup> At selected points in the text, the students would be asked to think aloud about what they were doing. The students were judged as "good" or "poor" performers on the basis of test scores from three exams. The think-aloud interviews from five students who did well on all the exams and from five students who did poorly on all three exams were analyzed. Ferguson-Hessler and de Jong found the following result:

- Good performers used more deep processing than the poor performers when reading the text. Deep processing here means imposing structure not given in the text and making procedures and assumptions explicit.
- Poor performers tended to take more for granted and focused more on declarative knowledge (principles, formulas, and concepts) when reading the text. Good performers focused more on procedural (when a particular relation is used) and situational (characteristics of problem situations) knowledge.

## 2. How to Help Novices Become Experts

Although it is useful to understand the difference between expert and novice problem-solvers, the real question from an instructional standpoint is, "how do we help students learn to become good problem solvers?" Although this is a current area of research, few studies have been done so far and not much is known. However, the results that do exist are encouraging.

### a. Instruction to improve problem solving

As part of their research identifying the factors that contribute to good problem solving, Larkin and Reif tested methods of improving problem solving ability.<sup>140</sup> They found that students who were taught with either an explicit problem solving strategy or taught in a way that emphasized a hierarchical physics knowledge structure with the main principles at the top performed significantly better at problem solving than control groups. Although these studies were successful, they were conducted on a small scale over a relatively short time period and not incorporated into a regular introductory physics course.

The problem solving strategy used by Reif and Larking was based on the following four-step problem-solving strategy advocated by Polya in his 1945 book, *How to Solve It*.<sup>141</sup>

1. Understand the problem
2. Devise a plan
3. Execute the plan
4. Look back to review the results

Subsequent efforts to teach an expert-like problem-solving strategy to introductory physics students continue to make use of strategies based on Polya's four-step approach. GOAL is essentially the same strategy in mnemonic form so that it is easier for students to remember.

Three classroom-based studies on improving physics problem solving have been reported since the characteristics of good and poor problem solvers discussed previously appeared in the PER literature. All three studies were conducted in modified undergraduate introductory physics classes with typical class-size samples.

### *WISE*

In the first study, conducted by Wright and Williams at a community college in 1986, students were taught and encouraged to use an explicit problem solving strategy as part of the Explicitly Structured Physics Instruction (ESPI) system.<sup>142</sup> The "WISE" problem solving strategy they used is similar to the GOAL approach.

Wright and Williams found that students who used the WISE strategy on homework and exams performed significantly better (as determined by course grade) than students who did not use the strategy. In addition, student comments were strongly favorable to the WISE strategy and class retention was better for the experimental class. However, the authors found that the students were reluctant, even actively opposed in some cases to adopting the WISE problem strategy because of the extra work.

### *Overview, Case Study*

In 1991, Van Heuvelen reported on the Overview, Case Study (OCS) approach to physics instruction.<sup>143</sup> Although this approach restructures the entire format of the class, it does specifically address problem solving. An integral feature of the OCS approach is the explicit development of multiple representations, especially qualitative physics representations, in problem solving. The approach also includes explicit discussion of knowledge hierarchy, an emphasis on active reasoning, and the use of cooperative group activities. Students who were taught with the OCS approach scored significantly better on problems from the College Board Advanced Placement physics test than students who were taught with traditional instruction at the same school.

### *Group Problem Solving with Context-Rich Problems*

In 1992, Heller and the Physics Education Group at University of Minnesota reported on a research-based lecture-recitation-laboratory curriculum they developed that emphasizes group problem solving.<sup>144</sup> In their curriculum, the lecture component stresses underlying themes, i.e. the main principles of physics, and an explicit problem solving strategy that is presented and modeled for the students. In recitation, the students do group problem solving using the strategy, and in lab they work in groups on laboratory problems where they must decide on what measurements to make and how to analyze the data they collect to answer the lab problem.

In lecture, the students were taught a five-step problem-solving strategy very similar to the GOAL protocol.<sup>145</sup> To get the students to use this strategy productively, they found it necessary to use specially constructed problems called "context-rich problems" for the groups to use in recitation and lab. These context-rich problems are more complex than typical textbook problems and are described in more detail below.

Heller *et al.* designed this curriculum so that three out of six hours of class time each week are spent on group problem solving. They hypothesized that,

... in well functioning groups, students share conceptual and procedural knowledge as they solve a problem together. [That is, the students discuss their ideas, their strategies, and their understanding of the physics of the problem.] During this joint construction of a solution, individual group members can request explanations and justifications from one another. This mutual critique would clarify all the members'



thinking about the concepts and principles to be used, and how those concepts and principles should be applied to the particular problem. Moreover, each member can observe others perform the varied thinking strategies that he or she must perform independently and silently on individual problem assignments.

To evaluate this curriculum, Heller *et al.* made extensive observations, studied the copied solutions, surveyed the students, and interviewed them. Although they did not report on how this type of activity affects the students' thinking and reasoning processes, they did show that groups that have more elaborate discussions produce better qualitative descriptions<sup>146</sup> and that students who were taught with this curriculum produced better solutions.

Class exams included both an individual and a group component. By comparing group and individual solutions to exam problems rated equally difficult by a rating scheme of their own design, Heller *et al.* showed that the group solutions were consistently better than the solutions of the best individuals in each group. The greatest difference between the individual and group solutions was in the qualitative analysis of the problem.

The students also did better on traditional textbook problems. One exam composed entirely of typical textbook problems was given to one class taught with the modified curriculum and one class taught with the traditional lecture, recitation, and laboratory curriculum. The solutions were judged on the expertness of the approach to the solution, not on the final answer.

#### **b. Context-rich problems**

While developing the group problem solving curriculum described above, Heller *et al.* studied what type of problems would be useful for teaching students to use an expert-like problem solving strategy. They found that this is a difficult proposition for individual problem solvers because in their words,<sup>147</sup>

if the problems are simple enough to be solved moderately well using their novice strategy, then students see no reason to abandon this strategy—even if the prescribed strategy works as well or better. If the problems are complex enough so the novice approach clearly fails, then students are initially unsuccessful at using the prescribed strategy, so they revert back to their novice strategy.

In researching what types of problems are effective for promoting the use of their prescribed "expert" problem solving strategy in group problem solving, Heller and Hollabaugh first looked at how student groups solved standard textbook.<sup>148</sup> An example problem of motion on an inclined plane is shown in Table 2, Problem A. When solving problems of this type, the group discussions tended to revolve around "what formulas should we use?" rather than "what physics concepts and principles should be applied to this problem?" One group, whose solution was typical, used the following solution process:

1. Rather than begin with a discussion and analysis of the forces acting on the block, this group began by attempting to recall the force diagram and formulas from their text. The example from the text was for a book sliding down the ramp. As a result, their frictional force was in the wrong direction and there was a sign error in their force equation.
2. At no point did the students plan a solution. They plugged numbers into formulas and manipulated equations until they had a numeric solution.

3. The group discussion was concerned with finding additional formulas that contained the same symbols as the unknown variables. They did not discuss the meaning of either the symbols or the formulas. They incorrectly tried to substitute the formula for instantaneous velocity into the formula for average velocity.

Table 2: Comparison of (A) a typical textbook problem with (B) a context rich problem for an object on an inclined plane. Reprinted with permission from Heller, P. and Hollabaugh M., "Teaching problem solving through cooperative grouping. Part 2: Designing problems and structuring groups," *American Journal of Physics* **60** (7), 1992, pp 637-344. Copyright 1992, American Association of Physics Teachers.

**Problem A. A Typical textbook style problem**

A 5.0 kg block slides 0.5 m up an inclined plane to a stop. The plane is inclined at an angle of  $20^\circ$  to the horizontal, and the coefficient of kinetic friction between the block and the plane is 0.60. What is the initial velocity of the block?

**Problem B. Context-rich problem**

While visiting a friend in San Francisco, you decide to drive around the city. You turn a corner and find yourself going up a steep hill. Suddenly a small boy runs out on the street chasing a ball. You slam on the brakes and skid to a stop, leaving a skid mark 50 ft long on the street. The boy calmly walks away, but a policeman watching from the sidewalk comes over and gives you a ticket for speeding. You are still shaking from the experience when he points out that the speed limit on this street is 25 MPH.

After you recover your wits, you examine the situation more closely. You determine that the street makes an angle of  $20^\circ$  and that the coefficient of static friction between your tires and the street is 0.80. Your car's information book tells you that the mass of your car is 1570 kg. You weigh 130 lb and a witness tells you that the boy had a weight of about 60 lbs and took 3.0 s to cross the 15-ft wide street. Will you fight the ticket in court?

Heller *et al.* estimate that two-thirds of the groups used this type of formula-based approach. They concluded that standard textbook problems were not effective in promoting group discussions that would help the student become better problem solvers.

Next, Heller and Hollabaugh compared textbook problems to real world problems to determine which characteristics of textbook problems encourage the use of novice strategies and which characteristics of real world problems require the use of an expert strategy. They found several characteristics of textbook problems that encourage students to use the formulaic approach described above "despite the instructor's effort to teach a more effective strategy." Textbook problems typically use idealized objects and events that have little or no connection to the student's real world. They suggest that this reinforces the student's tendency to memorize algorithms to deal with specific objects or situations. The unknown quantity is specified in the problem (usually in the last sentence) and all the other quantities needed to solve the problem are given (usually with the correct units). This encourages students to solve the problem by searching for formulas that have the right quantities and then plugging in numbers until a numeric answer is obtained. This numeric answer can then be checked in the back of the book.

On the other hand, solutions to real world problems are motivated by the solver wanting to know something about actual objects or events with which the solver is familiar. Before any calculations can be done, the solver must decide which quantities are useful for solving the question, which physics concepts and principles are relevant, what additional information is needed, and determine

which information can be determined and which must be estimated. In other words, students solving a real world problem have to think about the problem, try to understand what is going on, and make a number of decisions before reducing the problem to plug and chug mathematics. Since most textbook problems have removed the need for this type of analysis, they make algorithmic problem solving appear to be the correct way to solve problems.

To encourage students to use the prescribed problem solving strategy, Heller and her group created what they call "context-rich problems." The context rich problems are designed to utilize many of the characteristics of real world problems. They are short stories that include a reason (although sometimes humorous or farfetched) for calculating specific quantities about real objects or events. They are more complex than typical textbook problems. In addition, they typically have one or more of the following characteristics:

1. The problem statement may not specify the unknown variable.
2. More information may be provided than is necessary to solve the problem.
3. Some information may need to be estimated or recalled.
4. Reasonable assumptions may need to be made to simplify the problem.

An example of a context rich problem is shown in Table 3B. This is the inclined-plane textbook problem rewritten in context-rich form. According to Heller and Hollabaugh,<sup>149</sup>

Because context-rich problems are complex and involve making decisions about physics concepts and principles new to beginning students, they are difficult and frustrating even for the best students. In cooperative groups, however, students share the thinking load and can solve these problems. Because decisions must be made, the context-rich problems forced the groups to discuss physics issues while practicing effective problem-solving techniques. The group practice enhanced the students' ability individually [as well] . . .

The students have to pool what they know of the actual behavior of objects and the physics principles and concepts that describe this behavior to solve context-rich problems. Heller and Hollabaugh describe how a group came up with this solution to the traffic ticket problem shown in Table 3B,

The students first sketched the situation and discussed what variable was needed to answer the question: "Will you fight the traffic ticket in court?" They decided they should calculate the initial velocity of the car just before the brakes were applied to see if this velocity was above the speed limit of 25 mph. After drawing the kinematics diagram, they then discussed what information they needed to find the initial velocity. They decided they could ignore the information about the child, since "*the car stopped before it hit the child.*" They then spent several minutes drawing free body diagrams of the car and discussing whether they needed to use static friction, kinetic friction, or both. During this discussion, they referred several times to the friction experiments they were doing in the laboratory. Once this issue was resolved and the force diagram agreed upon, they systematically planned a solution following the planning procedure modeled during lectures.

Notice that this group focused on "what physics concepts and principles should be applied to this problem" rather than "what formulas should we use." While student attitudes towards using the prescribed strategy for context-rich problems improved, they still found using the strategy "annoying" or "frustrating" to use on

simple textbook problems because the strategy required them to write down more than they thought was necessary. This reaction is particularly interesting since Heller and Hollabaugh note that "these students were not usually successful at solving these problems using the formulaic strategy they preferred." However these same students did agree that the prescribed strategy was useful for solving the more difficult textbook problems in addition to individual and group context-rich problems

**c. Implications**

The results from these problem-solving studies indicate that research-based instruction methods can help introductory physics students develop expert problem solving skills. Students can be taught an expert-like problem solving strategy and they can learn to use qualitative representations. In each case learning these skills had a positive effect on the students' problem solving abilities. But is there evidence that students can use their improved knowledge and problem solving skills to solve problems in new contexts?

A related study by Volet on modified instruction in computer science found that students taught a programming strategy that involved reflection and evaluation of a computer program did not know significantly more content than the control group, but they were able to apply their computing knowledge to an unfamiliar complex problem on the final.<sup>150</sup> The modified instruction strongly resembled Heller *et al.*'s Cooperative Group Problem Solving approach, again using a 5-step programming strategy very similar to the GOAL strategy. Volet compared students in two experimental sections to an equal number of students in the same class but in different recitation sections with 4 different instructors. The experimental and control students were paired based on the students' background in computing, overall program of study, gender, interest in computing, and initial study goals for the computing course. Parts one and two of the final exam asked students questions on their factual and procedural knowledge, for example, asking them to describe programming concepts, functions, and procedures and asking them why certain techniques are used. Part three required them to solve an unfamiliar, fairly complex programming problem. While the experimental group performed about the same as the control group on parts one and two, they did do significantly better on part three. Volet claims this result indicates "that experimental and control students did not differ in the amount of computer programming knowledge they had acquired (as assessed in parts one and two [on the final])," but the result on part three "indicates that the experimental students' computing knowledge was more accessible and more usable than control students' knowledge." In addition, the experimental group did significantly better in more advanced computer science classes.

We see from the various studies described above that it is possible for introductory physics instruction to help students acquire at least some expert problem-solving skills. But there are additional issues to consider.

Heller and Hollabaugh's study strongly suggests one reason why students who do well on traditional textbook problems may not do as well on qualitative problems. Because students don't use physical reasoning to solve problems and their grades depend on how well they do on the typical textbook problems, they may not see conceptual understanding and physical reasoning as important. This result is consistent with the findings of Maloney and Siegler as well as the previously described studies characterizing novice problem solvers.<sup>151</sup>

Maloney and Siegler gave novice undergraduate physics students a set of problems in which the students were asked to compare five objects and determine which objects had the largest and smallest values of either momentum or kinetic energy. Some of the questions were phrased in everyday language and some were phrased in explicit physics language asking about momentum and kinetic energy. The object of the study was to observe what strategies the students used to solve the problems and if the students had multiple strategies. Although Maloney and Siegler did observe students using multiple strategies, in both types of problems the strategies emphasized mathematical formulas, not physics concepts. They also found that while students tended to use momentum on most of the problems in everyday language, students tended to use a different strategy if the question asked about kinetic energy explicitly. This study is one of many that indicates students approach problems differently depending on the cues in the problem.<sup>152</sup>

### C. Expectations and the Hidden Curriculum

Students bring more than just naive or novice views of problem solving and physics concepts to the introductory physics classroom. Each student, based on his or her own experiences, also brings to the class a set of attitudes, beliefs, and assumptions about what science is, what sorts of things they will learn, what skills will be required, and what they will be expected to do. In particular, their view of the nature of scientific information affects how they interpret what they hear. The phrase "expectations" is used to cover this rich set of understandings—beliefs and attitudes about their understanding of the process of learning physics and the structure of physics knowledge rather than about the content of physics itself. This is in contrast to more affective expectations such as students' motivation, preferences, and feelings about science and/or scientists, etc.<sup>153</sup>

As we saw in the studies of Hammer and Tobias, students' expectations about what they were learning and what they needed to do to succeed in the class seemed to affect their views on building conceptual understanding and problem solving as well as what they took away from the class. In particular, some of the students' expectations prevented the students from building a robust understanding of physics. Furthermore, it is reasonable to assume that one reason why many of Mazur's students learned to do the traditional quantitative without learning the underlying concepts is related to these students' expectations of what they were supposed to learn.

The role of these types of cognitive beliefs on adult learners in introductory undergraduate physics courses is still not well understood. The few studies like Hammer's that exist indicate the effects may be profound and very much related to students' conceptual knowledge and problem solving skills. Furthermore, these studies indicate that like problem solving and conceptual understanding, introductory students' epistemological beliefs often differ from those of experts and may hinder their learning of physics. Several of the most pertinent studies are discussed below.

#### 1. Research on Cognitive Expectations in the Pre-College Classroom

There are a number of studies of student expectations in science in the pre-college classroom that show that student attitudes towards their classroom activities and their beliefs about the nature of science and knowledge affect their learning. Studies by Carey,<sup>154</sup> Linn,<sup>155</sup> and others have demonstrated that many pre-college students have misconceptions both about science and about what they should be doing in a science class. Other studies at the pre-college level indicate some of the critical items that make up the relevant elements of a student's system of expectations and beliefs. For example, Songer and Linn studied students in middle schools and found that students could be categorized as having beliefs that science was either dynamic (science is understandable, interpretive, and integrated) or static (science knowledge is

memorization-intensive, fixed, and not relevant to their everyday lives).<sup>156</sup> In a review of student expectations studies in high school physics, Gunstone concludes: "The ideas and beliefs of learners about learning, teaching, and the nature of appropriate roles for learners and teachers are major influences is the likelihood of learners choosing to undertake the demanding and risky processes of personal and conceptual change."<sup>157</sup>

Expectation studies of high school mathematics classes by Schoenfeld<sup>158</sup> and the third National Assessment of Educational Progress<sup>159</sup> have found that students' beliefs about mathematics and mathematical problem solving are shaped by their experiences in mathematics classrooms. Using national math assessments, surveys, interviews, and classroom observations, these two studies found that the majority of junior high school and high school students have the following beliefs about the nature of mathematics:

1. Mathematics is mostly memorization
2. Mathematics problems have one and only one correct solution and answer; the correct solution usually uses the rule the teacher has most recently demonstrated to the class.
3. Students who have understood the mathematics they have studied will be able to solve any assigned math problem in five minutes or less. (Note: students with this belief will give up on a problem after a few minutes of unsuccessful attempts, even though they might have solved it had they persevered.)

Schoenfeld's studies had two additional findings of typical student beliefs about mathematics that echo the expectations of Liza in Hammer's study discussed earlier in this chapter:

- Ordinary students cannot expect to understand mathematics; they expect simply to memorize it and apply what they have learned mechanically without understanding.
- The mathematics learned in school has little or nothing to do with the real world.<sup>160</sup>

In his classroom observations and interviews, Schoenfeld studied how student expectations affected their behavior in class and in problem solving. He concluded that "Student's beliefs shape their behavior in ways that have extraordinarily powerful (and often negative) consequences."

These results led Edward Silver to use the phrase "hidden curriculum"<sup>161</sup> to describe this unintentional by-product of formal mathematics education.<sup>162</sup> In his words,

Since the students' viewpoint represented by these statements is clearly inadequate, and potentially harmful to their future progress in mathematics, we need to focus our attention more clearly on those hidden products of the mathematics curriculum. . . . Our students may realize greater educational benefits from our attention to the hidden curriculum of beliefs and attitudes about mathematics than from any improvement we could make in the "transparent" curriculum of mathematics facts, procedures, and concepts.

## 2. Studies of Young Adults' Attitudes Towards Knowledge and Learning

Two important large scale studies that concern the general cognitive expectations of adult learners are those of Perry<sup>163</sup> and Belenky *et al.*<sup>164</sup> Perry tracked the attitudes of about 100 Harvard and Radcliffe students on epistemology, morals, and general world outlook throughout their college career. The students filled out an attitudinal survey

at the beginning of their college careers and were interviewed at least once or twice a year during their four years at college. Extending on Perry's study, Belenky *et al.* conducted in-depth interviews on similar issues, but with women from various walks of life. They tracked the views of 135 women in a variety of social and economic circumstances including 90 who were enrolled in one of six academic institutions varying from an inter-city community college to a prestigious four-year women's college. Twenty-five of the women in an academic setting were interviewed a second time one to five years later.

Both studies found evolution in the expectations of their subjects, especially in their attitudes about knowledge.<sup>165</sup> Both studies frequently found their young adult subjects starting in a "binary" or "received knowledge" stage in which they expected everything to be true or false, good or evil, etc., and in which they expected to learn "the truth" from authorities. Both studies observed their subjects moving through a "relativist" or "subjective" stage (nothing is true or good, every view has equal value) to a "consciously constructivist" stage. In this last, most sophisticated stage, the subjects accepted that nothing can be perfectly known, and accepted their own personal role in deciding what views were most likely to be productive and useful for them.

The two studies also had similar findings regarding the progression of their subjects through these stages. One common finding was that the progression through the stages was not always linear and some of the subjects stayed in the binary or relativist stages. The other common finding was the subjects did not usually progress from one stage to the next without some type of cognitive conflict where their previous epistemology was inadequate for their current situation. That is, until the students were placed in situations that required them to think in new ways.

Although the Perry and Belenky *et al.* studies both focused on areas other than science,<sup>166</sup> most professional scientists who teach at the undergraduate and/or graduate levels will recognize a binary stage, in which students just want to be told the "right" answers, and a constructivist stage in which the student takes charge of building his or her own understanding. Consciously constructivist students carry out their own evaluation of an approach, equation, or result, and understand both the conditions of validity and the relation to fundamental physical principles. Students who want to become creative scientists will have to move from the binary to the constructivist stage.

Another finding of the studies by Perry and Belenky *et al.* was that these expectation stages were not mutually exclusive and were often domain specific. An excellent introduction to the cognitive issues involved is given by Reif and Larkin<sup>167</sup> who compare the spontaneous cognitive activities that occur naturally in everyday life with those required for learning science. They pinpoint differences and show how application of everyday cognitive expectations in a science class causes difficulties. Another excellent introduction to the cognitive literature on the difference between everyday and in-school cognitive expectations is the paper by Brown, Collins, and Duguid, who stress the artificiality of much typical school activity and discuss the value of cognitive apprenticeships.<sup>168</sup>

### 3. Students expectations in Undergraduate Physics

The most relevant and valuable study on expectation issues for introductory physics instructors was a case study of six individual students conducted by Hammer at Berkeley.<sup>169</sup> In Hammer's more extensive study (which followed the preliminary study discussed earlier), he interviewed six students several times each from the first semester of the calculus-based sequence for engineering students at the University of California at Berkeley. Four of the students volunteered and two more agreed to participate after specifically being selected in order to add "good" students to the group. All six students had taken physics in high school and scored at least 700 on the

math SAT. Each student was interviewed for approximately ten hours during the same semester. In these interviews, Hammer used three types of activities to probe student expectations: open and semi-directed discussions, problem solving, and direct questioning. For the problem solving activities, he asked the students to solve the problems out loud. The problems included three specifically chosen to address student common-sense beliefs. The interviews were taped and transcribed, and students were classified according to their statements and how they approached the problems.

Hammer proposed three dimensions along which to classify student expectations of physics knowledge: beliefs about learning, beliefs about content, and beliefs about structure. The three dimensions are described in Table 3 below.

For their beliefs about how physics is learned, the type A students feel they need to make sense of the course material for themselves, while the type B students feel they have to trust what they learned from the authorities, i.e. the teacher and the text, and simply believe what they were given. This dimension is based largely on the binary or received knowledge state of Perry and Belenky *et al.* However, Hammer notes that while a student in the relativist or subjective state might feel that all views are equally valid, they may recognize that using the views of the instructor and the textbook (the classroom authorities) on tests and exams is expedient for getting a good grade in the course. The quote from Mazur's student about whether she should respond to the ungraded FCI according to what she was taught or by the way she really thinks is an example of this.

For their beliefs about the content of physics knowledge, the type A students feel that physics is coherent and fits together in a way they expect to understand and use. In contrast, type B students feel that knowing physics means remembering facts, and/or they cannot solve a problem without knowing the "right" equation, i.e. they treat physics knowledge as a set of independent and unrelated pieces of information. Some other indicators of this dimension are that type A students believe that understanding formula derivations is important and check for consistency in their work while type B students do not think derivations are important and are not bothered by apparent inconsistencies.

For their beliefs about the structure of physics knowledge, the type A students feel that the underlying concepts are what is essential and that the formulas are expressions of those concepts, while the type B students feel that the formulas are essential (at least for them), and the underlying concepts are not something that they need to bother with. Note that these three dimensions need not be independent.

Table 3. Hammer's expectation dimensions of students learning		
1: Beliefs about learning	<i>independent</i> self-motivated, questions material until it makes sense to them	<i>by authority</i> takes what is given by instructor or text without evaluation
2: Beliefs about content	<i>coherent</i> believes physics can be considered as a connected, consistent framework	<i>pieces</i> believes physics can be treated as separate facts of "pieces"
3: Beliefs about structure	<i>concepts</i> focuses on conceptual understanding and connects formulas to intuition and underlying concepts	<i>formulas</i> focuses on memorizing formulas and using them in problem solving.



Hammer classified students by the number of times something they said or did in the interviews indicated they held a particular type of expectation. He found that the students' expectations were generally consistent across all the activities in the interview. Of the two students from the earlier study, Liza was classified as Type B while Ellen was type A. In this study, the two "good" students were both type A and other four fell into category B for all the variables. The type A students earned an A and an A+ in the course, while the four type B students grades ranged from A- to C+.

In addition, all four of the type B students displayed misconceptions on at least one of three selected problems in interviews. In general, they displayed more misconceptions than the type A students and were usually unable to resolve them when challenged by Hammer. The two type A students displayed no misconceptions on the selected problems and in general were able to resolve their own misconceptions when challenged. It is not surprising that the two good students received a higher grade and showed fewer misconceptions, since they were selected based on their high grades on the first midterm. It is surprising that they are characterized by expectations different from the subjects that did less well in the class and who were unable to resolve their misconceptions.

It is interesting to note that one of the type A students, code named Ken, is an interesting counter example to many instructors' suppositions about successful physics students. Ken had a weaker math and science background than two of the type B students. Also, he was more concerned with getting a good grade in the class than in understanding physics but he felt conceptual understanding was essential to getting a good grade. Last, he did not value derivations except as support for his conceptual understanding. His main advantage over the two students with stronger backgrounds seems to have been his expectations, particularly his need to build a good conceptual understanding of physics.

Hammer also made the following three general observations about the involvement of expectations in the subjects learning.

1. [Type B subjects] were quite casual about making and breaking associations between different aspects of their knowledge; [type A subjects] were much more careful about building and modifying their understanding.
2. [Type B subjects] were quick to decide that they understood new information, while [the type A subjects] were more reflective and questioning.
3. [Type B subjects] were reluctant to spend time working on problems they did not know how to solve, while [the type A subjects] seemed to consider these the most interesting [problems]. In part, this appeared to reflect different goals: [the type A subjects] appeared to have a goal of understanding the material, while [the type B subjects] did not always consider understanding important.

Note that the first observation would lead to significant differences in the two types of students' knowledge structures. The second observation is remarkably similar to the result of the studies by Chi *et al.*<sup>170</sup> and Fergusson-Hessler and de Jong<sup>171</sup> (discussed earlier in the problem solving section) that found that good students process readings and example problems more deeply than poorer students who just tend to read and accept. The third observation implies that the students' personal course goals may be hindering them from achieving a robust functional understanding.

In summary, Hammer found that his six students had expectations about physics knowledge and learning that could be classified based on indications from several interviews. He found that these indications were consistent across a wide range of activities. His results indicate that expectations seem to be involved in whether the subjects were able to solve his interview problems and in whether they developed a coherent, conceptual understanding of the course material.

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- <sup>9</sup> See Ref. 5 and the references therein.
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- <sup>11</sup> See Refs. 1, 2, 3, 4, 5, 6, & 10.
- <sup>12</sup> See Ref. 10.
- <sup>13</sup> Conceptual diagnostic test of this type typically use very simple questions that most instructors believe will be easy for their students. The questions are specially designed to elicit known student misconceptions.
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- 25 Also Refs. 1, 2, & 3.
- 26 Lecture can be effective when the instructor and the students share the same learning objectives and the students are motivated enough to take the course material from their notes and from the text and put it back together for themselves. However even physics majors who are motivated to learn and understand the course material take three passes (lower division course, upper division courses, and graduate core courses) to learn physics fundamentals.
- 27 Proponents of augmented lecture suggest breaking presentation of material into 20 minutes segments separated by at least 5 minutes of student activities. In addition to helping students learn the material better, this also helps them concentrate better during the lecture presentations.
- 28 A description of various PER-based methods for augmenting or modifying traditional lecture can be found in Part I of the introduction.
- 29 L.C. McDermott, "Millikan Lecture 1990: What we teach and what is learned—Closing the gap," *American Journal of Physics* **59** (4), 301-315 (1991).
- 30 This method of conceptual change is mainly espoused by McDermott and the University of Washington Physics Education Group (see Ref. 1 ), but it forms the basis for several PER-based curricula. A commonly used variation is the Predict-Observe-Explain model.
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- 41 See Refs. 7 & 8.
- 42 See Ref. 38.
- 43 Mazur used the mechanics diagnostic test from Ref. 8.

- <sup>44</sup> In this section, there are quotes from papers and researchers as well as quotes from interview transcripts. The former are written in plain text while the latter are written in italics. Brackets [ ] are used to denote words added by this author to clarify a quote. Brackets in italics [ ] are used to denote words added by the original author to clarify a quote.
- <sup>45</sup> Liza and Ellen also went to high schools in neighborhoods with similar socioeconomic backgrounds.
- <sup>46</sup> M.M. Sternheim and J.W. Kane, *General Physics* (Wiley, New York, 1986), p38.
- <sup>47</sup> See Ref. 40.
- <sup>48</sup> NSF Division of Policy Research and Analysis, "The State of Academic Science and Engineering," 1990.
- <sup>49</sup> Tobias used student observers to look at both introductory chemistry and introductory physics. Only the student observations from introductory physics are examined here.
- <sup>50</sup> Since her study uses a very small sample set, Tobias engaged Abigail Lipson, a psychologist and senior member of the Harvard University Bureau of Study Counsel, to compare the student observers' results with an analysis of interviews of eighty science and non-science Harvard-Radcliffe students at the beginning of each of the students' four years of study. These interviews were part of a much larger study of Harvard-Radcliffe students who were traced, tested, and interviewed throughout their undergraduate careers to look at predictors of success in science and non-science degrees as well as gender differences. Although the Tobias' student observers were not typical of the students who participated in Lipson's study, the student observers' reactions to the courses and the subject matter were similar to those of Lipson's interviewed Harvard-Radcliffe students who started as science majors and then switched out.
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- <sup>54</sup> D.E. Trowbridge and L.C. McDermott, "Investigation of student understanding of the concept of velocity in one dimension," *American Journal of Physics* **48**, 1020-1028 (1980); and D.E. Trowbridge and L.C. McDermott, "Investigation of student understanding of the concept of acceleration in one dimension," *American Journal of Physics* **49**, 242-253 (1981).
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- <sup>60</sup> A. Arons, "Student patterns of thinking and reasoning," *The Physics Teacher* in three parts: **21**, 576-581 (1983); **22**, 21-26 (1984); **22**, 88-93 (1984).
- <sup>61</sup> See Ref. 7.
- <sup>62</sup> See Ref. 7.
- <sup>63</sup> M. Steinberg, D.E. Brown, and J. Clement, "Genius is not immune to persistent misconception: Conceptual difficulties impeding Issac Newton," *International Journal of Science Education* **12**, 265 (1990).

- <sup>64</sup> See Ref. 17.
- <sup>65</sup> See Ref. 18.
- <sup>66</sup> This discussion paraphrases a summary of Minstrell's and diSessa findings in D. Huffman and P. Heller, "What does the Force Concept Inventory measure?," *The Physics Teacher* **33** (3), 138-143 (1995).
- <sup>67</sup> See Refs. 1, 2, 3, 6, & 15.
- <sup>68</sup> P.A. Kraus, R.R. Harrington, P.S. Shaffer, and L.C. McDermott, "Do lectures demonstrations deepen students understanding?," *American Association of Physics Teachers Announcer* **24** (2), 56 (July 1994, Abstract only).
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- <sup>70</sup> G. Francis and M. Larson, "Student misconceptions in rotational kinematics and dynamics," *American Association of Physics Teachers Announcer* **24** (2), 64 (July 1994, abstract only); and G. Francis, "Spiraling back with rotational mechanics: Transferring conceptual change from linear to rotational motion," *American Association of Physics Teachers Announcer* **25** (4), 44 (December 1995, abstract only).
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 using MBL students can repeat experiments with a variety of changed conditions, while  
 VBL is often limited to particular clips. Also, MBL activities often have a kinesthetic  
 component. For example, one of the most effective kinematics MBL activities is where  
 students first plan how to move to match a displayed graph.
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