Chapter 7 Solutions

***7.1** $W = Fd = (5000 \text{ N})(3.00 \text{ km}) = \boxed{15.0 \text{ MJ}}$

***7.2** The component of force along the direction of motion is

$$F \cos \theta = (35.0 \text{ N}) \cos 25.0° = 31.7 \text{ N}$$

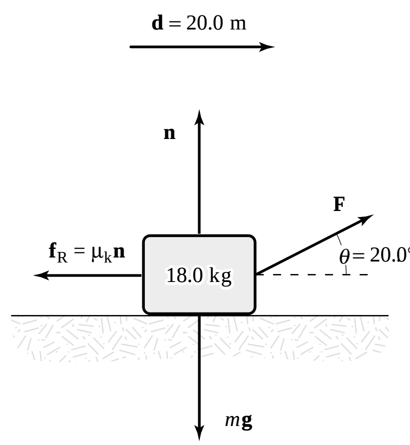
The work done by this force is

$$W = (F \cos \theta)d = (31.7 \text{ N})(50.0 \text{ m}) = \boxed{1.59 \times 10^3 \text{ J}}$$

7.3 (a) $W = mgh = (3.35 \times 10^{-5})(9.80)(100) \text{ J} = \boxed{3.28 \times 10^{-2} \text{ J}}$

(b) Since $R = mg$, $W_{\text{air resistance}} = \boxed{-3.28 \times 10^{-2} \text{ J}}$

7.4 (a) $\Sigma F_y = F \sin \theta + n - mg = 0$



$$n = mg - F \sin \theta$$

$$\Sigma F_x = F \cos \theta - \mu_k n = 0$$

$$n = \frac{F \cos \theta}{\mu_k}$$

$$\therefore \quad mg - F \sin \theta = \frac{F \cos \theta}{\mu_k}$$

$$F = \frac{\mu_k mg}{\mu_k \sin \theta + \cos \theta}$$

$$F = \frac{(0.500)(18.0)(9.80)}{0.500 \sin 20.0° + \cos 20.0°} = \boxed{79.4 \text{ N}}$$

(b) $W_F = Fd \cos \theta = (79.4 \text{ N})(20.0 \text{ m}) \cos 20.0° = \boxed{1.49 \text{ kJ}}$

(c) $f_k = F \cos \theta = 74.6 \text{ N}$

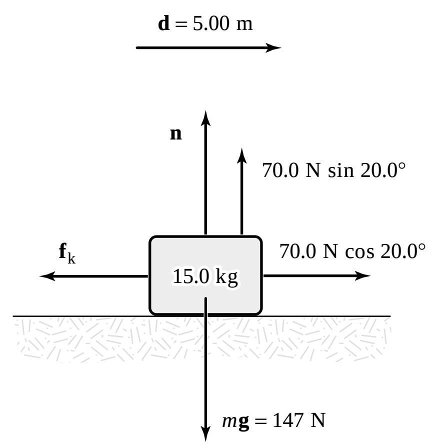
$$W_f = f_k d \cos \theta = (74.6 \text{ N})(20.0 \text{ m}) \cos 180° = \boxed{-1.49 \text{ kJ}}$$

7.5 (a) $W = Fd \cos \theta = (16.0 \text{ N})(2.20 \text{ m}) \cos 25.0° = \boxed{31.9 \text{ J}}$

(b) and (c) The normal force and the weight are both at 90° to the motion. Both do $\boxed{0}$ work.

(d) $\Sigma W = 31.9 \text{ J} + 0 + 0 = \boxed{31.9 \text{ J}}$

7.6 $\sum F_y = ma_y$



$n + (70.0\text{ N})\sin 20.0° - 147\text{ N} = 0$

$n = 123\text{ N}$

$f_k = \mu_k n = 0.300\,(123\text{ N}) = 36.9\text{ N}$

(a) $W = Fd\cos\theta$

$= (70.0\text{ N})(5.00\text{ m})\cos 20.0° = \boxed{329\text{ J}}$

(b) $W = Fd\cos\theta = (123\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0\text{ J}}$

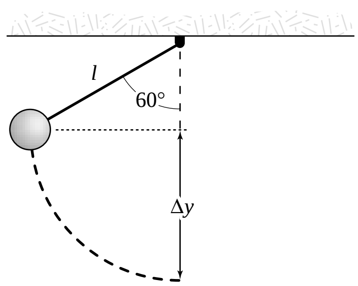
(c) $W = Fd\cos\theta = (147\text{ N})(5.00\text{ m})\cos 90.0° = \boxed{0}$

(d) $W = Fd\cos\theta = (36.9\text{ N})(5.00\text{ m})\cos 180° = \boxed{-185\text{ J}}$

(e) $\Delta K = K_f - K_i = \sum W = 329\text{ J} - 185\text{ J} = \boxed{+144\text{ J}}$

7.7 $W = mg(\Delta y) = mg(l - l\cos\theta)$

$= (80.0\text{ kg})(9.80\text{ m/s}^2)(12.0\text{ m})(1 - \cos 60.0°) = \boxed{4.70\text{ kJ}}$



7.8 $A = 5.00;\ B = 9.00;\ \theta = 50.0°$

$\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = (5.00)(9.00)\cos 50.0° = \boxed{28.9}$

7.9 $\mathbf{A}\cdot\mathbf{B} = AB\cos\theta = 7.00(4.00)\cos(130° - 70.0°) = \boxed{14.0}$

7.10 $\mathbf{A}\cdot\mathbf{B} = (A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k})\cdot(B_x\mathbf{i} + B_y\mathbf{j} + B_z\mathbf{k})$

$\mathbf{A}\cdot\mathbf{B} = A_xB_x(\mathbf{i}\cdot\mathbf{i}) + A_xB_y(\mathbf{i}\cdot\mathbf{j}) + A_xB_z(\mathbf{i}\cdot\mathbf{k}) +$

$A_yB_x(\mathbf{j}\cdot\mathbf{i}) + A_yB_y(\mathbf{j}\cdot\mathbf{j}) + A_yB_z(\mathbf{j}\cdot\mathbf{k}) +$

$A_zB_x(\mathbf{k}\cdot\mathbf{i}) + A_zB_y(\mathbf{k}\cdot\mathbf{j}) + A_zB_z(\mathbf{k}\cdot\mathbf{k})$

$\mathbf{A} \cdot \mathbf{B} = \boxed{A_x B_x + A_y B_y + A_z B_z}$

7.11 (a) $W = \mathbf{F}\cdot\mathbf{d} = F_x x + F_y y = (6.00)(3.00)\ \text{N}\cdot\text{m} + (-2.00)(1.00)\ \text{N}\cdot\text{m} = \boxed{16.0\ \text{J}}$

(b) $\theta = \cos^{-1}\dfrac{\mathbf{F}\cdot\mathbf{d}}{Fd} = \cos^{-1}\dfrac{16}{\sqrt{[(6.00)^2 + (-2.00)^2][(3.00)^2 + (1.00)^2]}} = \boxed{36.9°}$

7.12 $\mathbf{A} - \mathbf{B} = (3.00\mathbf{i} + \mathbf{j} - \mathbf{k}) - (-\mathbf{i} + 2.00\mathbf{j} + 5.00\mathbf{k})$

$\mathbf{A} - \mathbf{B} = 4.00\mathbf{i} - \mathbf{j} - 6.00\mathbf{k}$

$\mathbf{C}\cdot(\mathbf{A} - \mathbf{B}) = (2.00\mathbf{j} - 3.00\mathbf{k})\cdot(4.00\mathbf{i} - \mathbf{j} - 6.00\mathbf{k})$

$= 0 + (-2.00) + (+18.0) = \boxed{16.0}$

7.13 (a) $\mathbf{A} = 3.00\mathbf{i} - 2.00\mathbf{j}$ $\quad \mathbf{B} = 4.00\mathbf{i} - 4.00\mathbf{j}$

$\theta = \cos^{-1}\dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \cos^{-1}\dfrac{12.0 + 8.00}{\sqrt{(13.0)(32.0)}} = \boxed{11.3°}$

(b) $\mathbf{B} = 3.00\mathbf{i} - 4.00\mathbf{j} + 2.00\mathbf{k}$ $\quad \mathbf{A} = -2.00\mathbf{i} + 4.00\mathbf{j}$

$\cos\theta = \dfrac{\mathbf{A}\cdot\mathbf{B}}{AB} = \dfrac{-6.00 - 16.0}{\sqrt{(20.0)(29.0)}}$

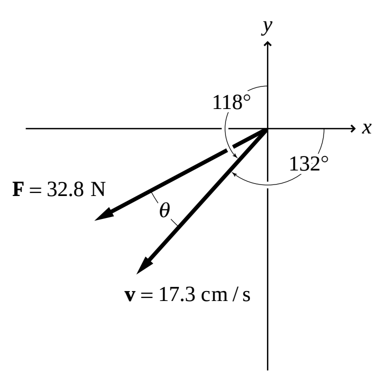
$\theta = \boxed{156°}$

(c) $\mathbf{A} = \mathbf{i} - 2.00\mathbf{j} + 2.00\mathbf{k}$ $\quad \mathbf{B} = 3.00\mathbf{j} + 4.00\mathbf{k}$

$\theta = \cos^{-1}\left(\dfrac{A\cdot B}{AB}\right) = \cos^{-1}\left(\dfrac{-6.00 + 8.00}{\sqrt{9.00}\cdot\sqrt{25.0}}\right) = \boxed{82.3°}$

***7.14** We must first find the angle between the two vectors. It is:

$\theta = 360° - 118° - 90.0° - 132° = 20.0°$



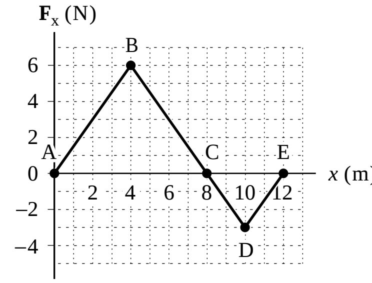
Then

$\mathbf{F}\cdot\mathbf{v} = Fv\cos\theta = (32.8\ \text{N})(0.173\ \text{m/s})\cos 20.0°$

or

$\mathbf{F}\cdot\mathbf{v} = 5.33\dfrac{\text{N}\cdot\text{m}}{\text{s}} = 5.33\dfrac{\text{J}}{\text{s}} = \boxed{5.33\ \text{W}}$

7.15 $W = \int_i^f F dx$ = area under curve from x_i to x_f



(a) $x_i = 0$ $x_f = 8.00$ m

W = area of triangle $ABC = \left(\frac{1}{2}\right) AC \times$ altitude,

$$W_{0\to8} = \left(\frac{1}{2}\right) \times 8.00 \text{ m} \times 6.00 \text{ N} = \boxed{24.0 \text{ J}}$$

(b) $x_i = 8.00$ m $x_f = 10.0$ m

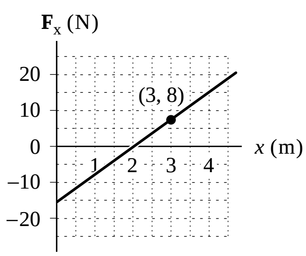
W = area of $\Delta CDE = \left(\frac{1}{2}\right) CE \times$ altitude,

$$W_{8\to10} = \left(\frac{1}{2}\right) \times (2.00 \text{ m}) \times (-3.00 \text{ N}) = \boxed{-3.00 \text{ J}}$$

(c) $W_{0\to10} = W_{0\to8} + W_{8\to10} = 24.0 + (-3.00) = \boxed{21.0 \text{ J}}$

***7.16** $F_x = (8x - 16)$ N

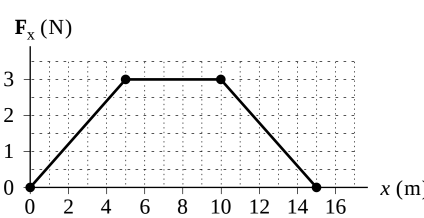
(a)



(b) $$W_{\text{net}} = \frac{-(2.00 \text{ m})(16.0 \text{ N})}{2} + \frac{(1.00 \text{ m})(8.00 \text{ N})}{2} = \boxed{-12.0 \text{ J}}$$

7.17 $W = \int F_x dx$ and

W equals the area under the Force-Displacement Curve



(a) For the region $0 \le x \le 5.00$ m,

$$W = \frac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$$

(b) For the region $5.00 \le x \le 10.0$,

$$W = (3.00 \text{ N})(5.00 \text{ m}) = \boxed{15.0 \text{ J}}$$

(c) For the region $10.0 \le x \le 15.0$,

$$W = \frac{(3.00\ \text{N})(5.00\ \text{m})}{2} = \boxed{7.50\ \text{J}}$$

(d) For the region $0 \le x \le 15.0$

$$W = (7.50 + 7.50 + 15.0)\ \text{J} = \boxed{30.0\ \text{J}}$$

7.18 $W = \int_i^f \mathbf{F} \cdot d\mathbf{s} = \int_0^{5\ \text{m}} (4x\mathbf{i} + 3y\mathbf{j})\ \text{N} \cdot dx\mathbf{i}$

$$\int_0^{5\ \text{m}} (4\ \text{N/m})\, x\, dx + 0 = (4\ \text{N/m})x^2/2 \Big|_0^{5\ \text{m}} = \boxed{50.0\ \text{J}}$$

***7.19** $k = \frac{F}{y} = \frac{Mg}{y} = \frac{(4.00)(9.80)\ \text{N}}{2.50 \times 10^{-2}\ \text{m}} = 1.57 \times 10^3\ \text{N/m}$

(a) For 1.50 kg mass $y = \frac{mg}{k} = \frac{(1.50)(9.80)}{1.57 \times 10^3} = \boxed{0.938\ \text{cm}}$

(b) Work $= \frac{1}{2}\ ky^2$

$$\text{Work} = \frac{1}{2}(1.57 \times 10^3\ \text{N} \cdot \text{m})(4.00 \times 10^{-2}\ \text{m})^2 = \boxed{1.25\ \text{J}}$$

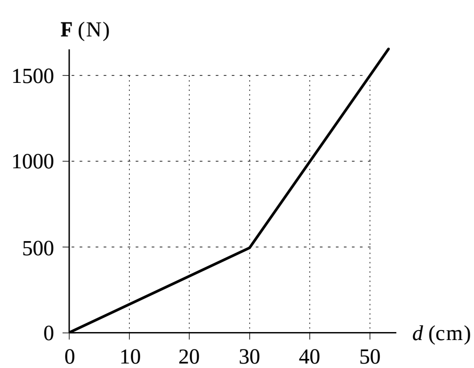
7.20 (a) Spring constant is given by $F = kx$

$$k = \frac{F}{x} = \frac{(230\ \text{N})}{(0.400\ \text{m})} = \boxed{575\ \text{N/m}}$$

(b) Work $= F_{\text{avg}}\, x = \frac{1}{2}(230\ \text{N})(0.400\ \text{m}) = \boxed{46.0\ \text{J}}$

7.21 Compare an initial picture of the rolling car with a final picture with both springs compressed

$$K_i + \sum W = K_f$$



Use equation 7.11.

$$K_i + \frac{1}{2}k_1(x_{1i}^2 - x_{1f}^2) + \frac{1}{2}k_2(x_{2i}^2 - x_{2f}^2) = K_f$$

$$\frac{1}{2}mv_i^2 + 0 - \frac{1}{2}(1600\ \text{N/m})(0.500\ \text{m})^2 + 0 - \frac{1}{2}(3400\ \text{N/m})(0.200\ \text{m})^2 = 0$$

$$\frac{1}{2}(6000\ \text{kg})v_i^2 - 200\ \text{J} - 68.0\ \text{J} = 0$$

$$v_i = \sqrt{2 \times 268\ \text{J}/6000\ \text{kg}} = \boxed{0.299\ \text{m/s}}$$

7.22 (a) $W = \int_i^f \mathbf{F} \cdot d\mathbf{s}$

$$W = \int_0^{0.600\ \text{m}} (15000\ \text{N} + 10000\,x\ \text{N/m} - 25000\,x^2\ \text{N/m}^2)\,dx \cos 0°$$

$$W = 15{,}000x + \frac{10{,}000x^2}{2} - \frac{25{,}000x^3}{3}\Bigg|_0^{0.600}$$

$$W = 9.00\ \text{kJ} + 1.80\ \text{kJ} - 1.80\ \text{kJ} = \boxed{9.00\ \text{kJ}}$$

(b) Similarly,

$$W = (15.0\ \text{kN})(1.00\ \text{m}) + \frac{(10.0\ \text{kN/m})(1.00\ \text{m})^2}{2} - \frac{(25.0\ \text{kN/m}^2)(1.00\ \text{m})^3}{3}$$

$W = \boxed{11.7\ \text{kJ}}$, larger by 29.6%

7.23 $4.00\ \text{J} = \frac{1}{2}k(0.100\ \text{m})^2$

$\therefore k = 800\ \text{N/m}$

and to stretch the spring to 0.200 m requires

$$\Delta W = \frac{1}{2}(800)(0.200)^2 - 4.00\ \text{J} = \boxed{12.0\ \text{J}}$$

Goal Solution

G: We know that the force required to stretch a spring is proportional to the distance the spring is stretched, and since the work required is proportional to the force *and* to the distance, then $W \propto x^2$. This means if the extension of the spring is doubled, the work will increase by a factor of 4, so that for $x = 20$ cm, $W = 16$ J, requiring 12 J of additional work.

O: Let's confirm our answer using Hooke's law and the definition of work.

A: The linear spring force relation is given by Hooke's law: $\mathbf{F}_s = -k\mathbf{x}$

Integrating with respect to x, we find the work done by the spring is:

$$W_s = \int_{x_x}^{x_y} F_s\,dx = \int_{x_x}^{x_y} (-kx)dx = -\frac{1}{2}\,k\,(x_f^2 - x_i^2)$$

However, we want the work done *on* the spring, which is $W = -W_s = \frac{1}{2}\,k(x_f^2 - x_i^2)$

We know the work for the first 10 cm, so we can find the force constant:

$$k = \frac{2W_{0-10}}{x_{0-10}^2} = \frac{2(4.00\text{ J})}{(0.100\text{ m})^2} = 800\text{ N/m}$$

Substituting for k, x_i and x_f, the extra work for the next step of extension is

$$W = \left(\frac{1}{2}\right)(800\text{ N/m})\ [(0.200\text{ m})^2 - (0.100\text{ m})^2] = 12.0\text{ J}$$

L: Our calculated answer agrees with our prediction. It is helpful to remember that the force required to stretch a spring is proportional to the distance the spring is extended, but the work is proportional to the square of the extension.

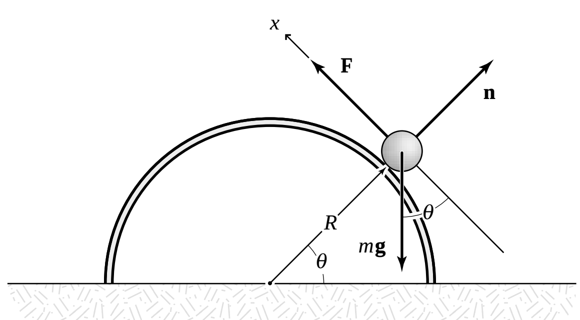
7.24 $W = \frac{1}{2}\,kd^2$

$$\therefore\ k = \frac{2W}{d^2}$$

$$\Delta W = \frac{1}{2}\,k(2d)^2 - \frac{1}{2}\,kd^2$$

$$\Delta W = \frac{3}{2}\,kd^2 = \boxed{3W}$$

7.25 (a) The radius to the mass makes angle θ with the horizontal, so its weight makes angle θ with the negative side of the x-axis, when we take the x-axis in the direction of motion tangent to the cylinder.



$$\Sigma F_x = ma_x$$

$$F - mg\cos\theta = 0$$

$$F = \boxed{mg\cos\theta}$$

(b) $W = \int_i^f \mathbf{F} \cdot d\mathbf{s}$

We use radian measure to express the next bit of displacement as $ds = r\,d\theta$ in terms of the next bit of angle moved through:

$$W = \int_0^{\pi/2} mg \cos \theta\, R d\theta = mgR \sin \theta \Big|_0^{\pi/2}$$

$$W = mgR\,(1 - 0) = \boxed{mgR}$$

***7.26** $[k] = \left[\frac{F}{x}\right] = \frac{\text{N}}{\text{m}} = \frac{\text{kg} \cdot \text{m/s}^2}{\text{m}} = \boxed{\frac{\text{kg}}{\text{s}^2}}$

7.27 (a) $K_A = \frac{1}{2}(0.600 \text{ kg})(2.00 \text{ m/s})^2 = \boxed{1.20 \text{ J}}$

(b) $\frac{1}{2}\, m v_B^2 = K_B$

$$v_B = \sqrt{\frac{2K_B}{m}} = \sqrt{\frac{(2)(7.50)}{0.600}} = \boxed{5.00 \text{ m/s}}$$

(c) $\sum W = \Delta K = K_B - K_A = \frac{1}{2}\, m(v_B^2 - v_A^2)$

$$= 7.50 \text{ J} - 1.20 \text{ J} = \boxed{6.30 \text{ J}}$$

***7.28** (a) $K = \frac{1}{2}\, mv^2 = \frac{1}{2}(0.300 \text{ kg})(15.0 \text{ m/s})^2 = \boxed{33.8 \text{ J}}$

(b) $K = \frac{1}{2}(0.300)(30.0)^2 = \frac{1}{2}(0.300)(15.0)^2(4) = 4(33.8) = \boxed{135 \text{ J}}$

7.29 $\mathbf{v}_i = (6.00\mathbf{i} - 2.00\mathbf{j})$ m/s

(a) $v_i = \sqrt{v_{ix}^2 + v_{iy}^2} = \sqrt{40.0}$ m/s

$$K_i = \frac{1}{2}\, m v_i^2 = \frac{1}{2}(3.00 \text{ kg})(40.0 \text{ m}^2/\text{s}^2) = \boxed{60.0 \text{ J}}$$

(b) $v = 8.00\mathbf{i} + 4.00\mathbf{j}$

$$v^2 = \mathbf{v} \cdot \mathbf{v} = 64.0 + 16.0 = 80.0 \text{ m}^2/\text{s}^2$$

$$\Delta K = K - K_i = \frac{1}{2}\, m(v^2 - v_i^2) = \frac{3.00}{2}(80.0) - 60.0 = \boxed{60.0 \text{ J}}$$

7.30 (a) $\Delta K = \sum W$

$\frac{1}{2}(2500 \text{ kg})\, v^2 = 5000 \text{ J}$

$v = \boxed{2.00 \text{ m/s}}$

(b) $W = \mathbf{F} \cdot \mathbf{d}$

$5000 \text{ J} = F(25.0 \text{ m})$

$F = \boxed{200 \text{ N}}$

7.31 (a) $\Delta K = \frac{1}{2} mv^2 - 0 = \sum W$, so

$v^2 = 2W/m$ and $v = \boxed{\sqrt{2W/m}}$

(b) $W = \mathbf{F} \cdot \mathbf{d} = F_x d \Rightarrow F_x = \boxed{W/d}$

7.32 (a) $\Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 5.00$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(7.50 \text{ J})}{4.00 \text{ kg}}} = \boxed{1.94 \text{ m/s}}$

(b) $\Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 10.0$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(22.5 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.35 \text{ m/s}}$

(c) $\Delta K = K_f - K_i = \frac{1}{2} mv_f^2 - 0 = \sum W =$ (area under curve from $x = 0$ to $x = 15.0$ m)

$v_f = \sqrt{\frac{2(\text{area})}{m}} = \sqrt{\frac{2(30.0 \text{ J})}{4.00 \text{ kg}}} = \boxed{3.87 \text{ m/s}}$

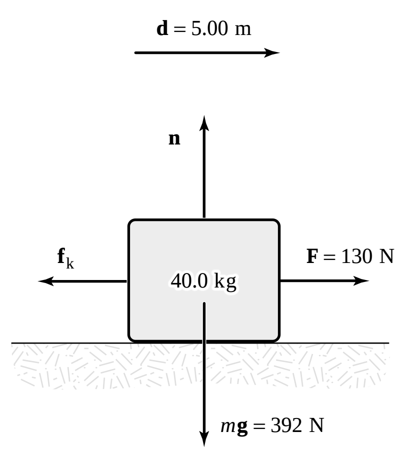
***7.33** $\sum F_y = ma_y$

$n - 392 \text{ N} = 0$ $n = 392 \text{ N}$

$f_k = \mu_k n = 0.300(392 \text{ N}) = 118 \text{ N}$

(a) $W_F = Fd \cos \theta = (130)(5.00) \cos 0° = \boxed{650 \text{ J}}$

(b) $W_{f_k} = f_k d \cos \theta = (118)(5.00) \cos 180° = \boxed{-588 \text{ J}}$



(c) $W_n = nd\cos\theta = (392)(5.00)\cos 90° = \boxed{0}$

(d) $W_g = mg\cos\theta = (392)(5.00)\cos(-90°) = \boxed{0}$

(e) $\Delta K = K_f - K_i = \sum W$

$$\frac{1}{2}mv_f^2 - 0 = 650\text{ J} - 588\text{ J} + 0 + 0 = \boxed{62.0\text{ J}}$$

(f) $v_f = \sqrt{\dfrac{2K_f}{m}} = \sqrt{\dfrac{2(62.0\text{ J})}{40.0\text{ kg}}} = \boxed{1.76\text{ m/s}}$

7.34 (a) $K_i + \sum W = K_f = \frac{1}{2}mv_f^2$

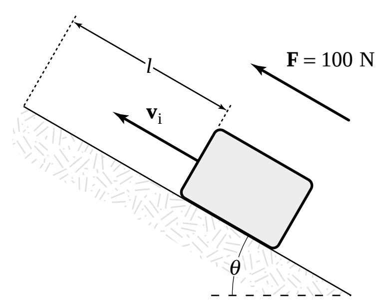
$$0 + \sum W = \frac{1}{2}(15.0\times10^{-3}\text{ kg})(780\text{ m/s})^2 = \boxed{4.56\text{ kJ}}$$

(b) $F = \dfrac{W}{d\cos\theta} = \dfrac{4.56\times10^3\text{ J}}{(0.720\text{ m})\cos 0°} = \boxed{6.34\text{ kN}}$

(c) $a = \dfrac{v_f^2 - v_i^2}{2x} = \dfrac{(780\text{ m/s})^2 - 0}{2(0.720\text{ m})} = \boxed{422\text{ km/s}^2}$

(d) $\sum F = ma = (15\times10^{-3}\text{ kg})(422\times10^3\text{ m/s}^2) = \boxed{6.34\text{ kN}}$

7.35 (a) $W_g = mgl\cos(90.0° + \theta) = (10.0\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m})\cos 110° = \boxed{-168\text{ J}}$



(b) $f_k = \mu_k n = \mu_k mg\cos\theta$

$W_f = -lf_k = l\mu_k mg\cos\theta\cos 180°$

$W_f = -(5.00\text{ m})(0.400)(10.0)(9.80)\cos 20.0° = \boxed{-184\text{ J}}$

(c) $W_F = Fl = (100)(5.00) = \boxed{500\text{ J}}$

(d) $\Delta K = \sum W = W_F + W_f + W_g = \boxed{148\text{ J}}$

(e) $\Delta K = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$

$$v_f = \sqrt{\frac{2(\Delta K)}{m} + v_i^2} = \sqrt{\frac{2(148)}{10.0} + (1.50)^2} = \boxed{5.65\text{ m/s}}$$

7.36 $\sum W = \Delta K = 0$

$$\int_0^L mg \sin 35.0° \, dl - \int_0^d kx \, dx = 0$$

$$mg \sin 35.0° \, (L) = \frac{1}{2} kd^2$$

$$d = \sqrt{\frac{2 \, mg \sin 35.0°(L)}{k}}$$

$$= \sqrt{\frac{2(12.0 \text{ kg})(9.80 \text{ m/s}^2) \sin 35.0° \, (3.00 \text{ m})}{3.00 \times 10^4 \text{ N/m}}} = \boxed{0.116 \text{ m}}$$

7.37 $v_i = 2.00 \text{ m/s}$ $\mu_k = 0.100$

$$\sum W = \Delta K$$

$$-f_k x = 0 - \frac{1}{2} mv_i^2$$

$$-\mu_k mgx = -\frac{1}{2} mv_i^2$$

$$x = \frac{v_i^2}{2\mu_k g} = \frac{(2.00 \text{ m/s})^2}{2(0.100)(9.80)} = \boxed{2.04 \text{ m}}$$

<u>Goal Solution</u>

G: Since the sled's initial speed of 2 m/s (~ 4 mph) is reasonable for a moderate kick, we might expect the sled to travel several meters before coming to rest.

O: We could solve this problem using Newton's second law, but we are asked to use the work-kinetic energy theorem: $W = K_f - K_i$, where the only work done on the sled after the kick results from the friction between the sled and ice. (The weight and normal force both act at 90° to the motion, and therefore do no work on the sled.)

A: The work due to friction is $W = -f_k d$ where $f_k = \mu_k mg$.

Since the final kinetic energy is zero, $W = \Delta K = 0 - K_i = -\frac{1}{2} mv_i^2$

Solving for the distance $d = \dfrac{mv_i^2}{2\mu_k mg} = \dfrac{v_i^2}{2\mu_k g} = \dfrac{(2.00 \text{ m})^2}{2(0.100)(9.80 \text{ m/s}^2)} = 2.04 \text{ m}$

L: The distance agrees with the prediction. It is interesting that the distance does not depend on the mass and is proportional to the square of the initial velocity. This means that a small car and a massive truck should be able to stop within the same distance if they both skid to a stop from the same initial speed. Also, doubling the speed requires 4 times as much stopping distance, which is consistent with advice given by transportation safety officers who suggest at least a 2 second gap between vehicles (as opposed to a fixed distance of 100 feet).

7.38 (a) $v_f = 0.01c = 10^{-2}(3.00 \times 10^8 \text{ m/s}) = 3.00 \times 10^6 \text{ m/s}$

$$K_f = \frac{1}{2} m v_f^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(3.00 \times 10^6 \text{ m/s})^2 = \boxed{4.10 \times 10^{-18} \text{ J}}$$

(b) $K_i + Fd \cos \theta = K_f$

$$0 + F(0.360 \text{ m}) \cos 0° = 4.10 \times 10^{-18} \text{ N} \cdot \text{m}$$

$$F = \boxed{1.14 \times 10^{-17} \text{ N}}$$

(c) $$a = \frac{\sum \mathbf{F}}{m} = \frac{1.14 \times 10^{-17} \text{ N}}{9.11 \times 10^{-31} \text{ kg}} = \boxed{1.25 \times 10^{13} \text{ m/s}^2}$$

(d) $x_f - x_i = \frac{1}{2}(v_i + v_f)\, t$

$$t = \frac{2(x_f - x_i)}{(v_i + v_f)} = \frac{2(0.360 \text{ m})}{(3.00 \times 10^6 \text{ m/s})} = \boxed{2.40 \times 10^{-7} \text{ s}}$$

7.39 (a) $\sum W = \Delta K \Rightarrow fd \cos \theta = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$

$$f(4.00 \times 10^{-2} \text{ m}) \cos 180° = 0 - \frac{1}{2}(5.00 \times 10^{-3} \text{ kg})(600 \text{ m/s})^2$$

$$f = \boxed{2.25 \times 10^4 \text{ N}}$$

(b) $$t = \frac{d}{\bar{v}} = \frac{4.00 \times 10^{-2} \text{ m}}{[0 + 600 \text{ m/s}]/2} = \boxed{1.33 \times 10^{-4} \text{ s}}$$

7.40 $\sum W = \Delta K$

$$m_1 gh - m_2 gh = \frac{1}{2}(m_1 + m_2)\, v_f^2 - 0$$

$$v_f^2 = \frac{2(m_1 - m_2)gh}{m_1 + m_2} = \frac{2(0.300 - 0.200)(9.80)(0.400)}{0.300 + 0.200} \frac{\text{m}^2}{\text{s}^2}$$

$$v_f = \sqrt{1.57} \text{ m/s} = \boxed{1.25 \text{ m/s}}$$

7.41 (a) $W_s = \frac{1}{2} k x_i^2 - \frac{1}{2} k x_f^2 = \frac{1}{2}(500)(5.00 \times 10^{-2})^2 - 0 = 0.625 \text{ J}$

$$\sum W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 = \frac{1}{2} m v_f^2 - 0$$

so $$v_f = \sqrt{\frac{2(\sum W)}{m}} = \sqrt{\frac{2(0.625)}{2.00}} \text{ m/s} = \boxed{0.791 \text{ m/s}}$$

(b) $\sum W = W_s + W_f = 0.625 \text{ J} + (-\mu_k mgd)$

$$= 0.625 \text{ J} - (0.350)(2.00)(9.80)(5.00 \times 10^{-2}) \text{ J} = 0.282 \text{ J}$$

$$v_f = \sqrt{\frac{2(\sum W)}{m}} = \sqrt{\frac{2(0.282)}{2.00}} \text{ m/s} = \boxed{0.531 \text{ m/s}}$$

***7.42** A 1300-kg car speeds up from rest to 55.0 mi/h = 24.6 m/s in 15.0 s. The output work of the engine becomes its final kinetic energy,

$$\frac{1}{2}(1300 \text{ kg})(24.6 \text{ m/s})^2 = 390 \text{ kJ}$$

with power $\frac{390000 \text{ J}}{15.0 \text{ s}} \boxed{\sim 10^4 \text{ W}}$, around 30 horsepower.

7.43 Power $= \frac{W}{t} = \frac{mgh}{t} = \frac{(700 \text{ N})(10.0 \text{ m})}{8.00 \text{ s}} = \boxed{875 \text{ W}}$

7.44 Efficiency = e = useful energy output/total energy input. The force required to lift n bundles of shingles is their weight, nmg.

$$e = \frac{n\, mgh \cos 0°}{Pt}$$

$$n = \frac{ePt}{mgh} = \frac{(0.700)(746 \text{ W})(7200 \text{ s})}{(70.0 \text{ kg})(9.80 \text{ m/s}^2)(8.00 \text{ m})} \times \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3 \cdot \text{W}} = \boxed{685 \text{ bundles}}$$

7.45 $P_a = f_a v \Rightarrow f_a = \frac{P_a}{v} = \frac{2.24 \times 10^4}{27.0} = \boxed{830 \text{ N}}$

***7.46** (a) $\sum W = \Delta K$, but $\Delta K = 0$ because he moves at constant speed. The skier rises a vertical distance of (60.0 m) sin 30.0° = 30.0 m. Thus,

$$W_{\text{in}} = -W_g = (70.0 \text{ kg})g(30.0 \text{ m}) = \boxed{2.06 \times 10^4 \text{ J}} = \boxed{20.6 \text{ kJ}}$$

(b) The time to travel 60.0 m at a constant speed of 2.00 m/s is 30.0 s. Thus,

$$P_{\text{input}} = \frac{W}{\Delta t} = \frac{2.06 \times 10^4 \text{ J}}{30.0 \text{ s}} = \boxed{686 \text{ W}} = 0.919 \text{ hp}$$

7.47 (a) The distance moved upward in the first 3.00 s is

$$\Delta y = \bar{v}\, t = \left[\frac{0 + 1.75 \text{ m/s}}{2}\right](3.00 \text{ s}) = 2.63 \text{ m}$$

$$W = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + mgy_f - mgy_i = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2 + mg(\Delta y)$$

$$W = \frac{1}{2}(650 \text{ kg})(1.75 \text{ m/s})^2 - 0 + (650 \text{ kg})g(2.63 \text{ m}) = 1.77 \times 10^4 \text{ J}$$

Also, $W = \bar{P}\, t$

so $\bar{P} = \frac{W}{t} = \frac{1.77 \times 10^4 \text{ J}}{3.00 \text{ s}} = \boxed{5.91 \times 10^3 \text{ W}} = 7.92 \text{ hp}$

(b) When moving upward at constant speed ($v = 1.75$ m/s), the applied force equals the weight $= (650 \text{ kg})(9.80 \text{ m/s}^2) = 6.37 \times 10^3$ N.

Therefore, $P = Fv = (6.37 \times 10^3 \text{ N})(1.75 \text{ m/s}) = \boxed{1.11 \times 10^4 \text{ W}} = 14.9 \text{ hp}$

***7.48** *energy = power × time*

For the 28.0 W bulb:

Energy used $= (28.0 \text{ W})(1.00 \times 10^4 \text{ h}) = 280$ kilowatt · hrs

total cost = \$17.00 + (280 kWh)(\$0.080/kWh) = \$39.40

For the 100 W bulb:

Energy used $= (100 \text{ W})(1.00 \times 10^4 \text{ h}) = 1.00 \times 10^3$ kilowatt · hrs

bulb used $= \frac{1.00 \times 10^4 \text{ h}}{750 \text{ h/bulb}} = 13.3$

total cost $= 13.3(\$0.420) + (1.00 \times 10^3 \text{ kWh})(\$0.080/\text{kWh}) = \$85.60$

Savings with energy-efficient bulb = \$85.60 – \$39.40 = $\boxed{\$46.2}$

7.49 (a) $$\text{fuel needed} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}{\text{useful energy per gallon}} = \frac{\frac{1}{2}mv_f^2 - 0}{\text{eff.} \times (\text{energy content of fuel})}$$

$$= \frac{\frac{1}{2}(900\text{ kg})(24.6\text{ m/s})^2}{(0.150)(1.34 \times 10^8\text{ J/gal})} = \boxed{1.35 \times 10^{-2}\text{ gal}}$$

(b) $\boxed{73.8}$

(c) $$\text{power} = \left(\frac{1\text{ gal}}{38.0\text{ mi}}\right)\left(\frac{55.0\text{ mi}}{1.00\text{ h}}\right)\left(\frac{1.00\text{ h}}{3600\text{ s}}\right)\left(\frac{1.34 \times 10^8\text{ J}}{1\text{ gal}}\right)(0.150) = \boxed{8.08\text{ kW}}$$

7.50 At a speed of 26.8 m/s (60.0 mph), the car described in Table 7.2 delivers a power of $P_1 = 18.3$ kW to the wheels. If an additional load of 350 kg is added to the car, a larger output power of

$$P_2 = P_1 + (\text{power input to move 350 kg at speed } v)$$

will be required. The additional power output needed to move 350 kg at speed v is:

$$\Delta P_{\text{out}} = (\Delta f)v = (\mu_r mg)v$$

Assuming a coefficient of rolling friction of $\mu_r = 0.0160$, the power output now needed from the engine is

$$P_2 = P_1 + (0.0160)(350\text{ kg})(9.80\text{ m/s}^2)(26.8\text{ m/s}) = 18.3\text{ kW} + 1.47\text{ kW}$$

With the assumption of constant efficiency of the engine, the input power must increase by the same factor as the output power. Thus, the fuel economy must decrease by this factor:

$$(\text{fuel economy})_2 = \left(\frac{P_1}{P_2}\right)(\text{fuel economy})_1 = \left(\frac{18.3}{18.3 + 1.47}\right)\left(6.40\ \frac{\text{km}}{\text{L}}\right)$$

or $$(\text{fuel economy})_2 = \boxed{5.92\ \frac{\text{km}}{\text{L}}}$$

7.51 When the car of Table 7.2 is traveling at 26.8 m/s (60.0 mph), the engine delivers a power of $P_1 = 18.3$ kW to the wheels. When the air conditioner is turned on, an additional output power of $\Delta P = 1.54$ kW is needed. The total power output now required is

$$P_2 = P_1 + \Delta P = 18.3 \text{ kW} + 1.54 \text{ kW}$$

Assuming a constant efficiency of the engine, the fuel economy must decrease by the same factor as the power output increases. The expected fuel economy with the air conditioner on is therefore

$$(\text{fuel economy})_2 = \left(\frac{P_1}{P_2}\right)(\text{fuel economy})_1 = \left(\frac{18.3}{18.3 + 1.54}\right)\left(6.40\ \frac{\text{km}}{\text{L}}\right)$$

or $$(\text{fuel economy})_2 = \boxed{5.90\ \frac{\text{km}}{\text{L}}}$$

7.52 (a) $$K = \left(\frac{1}{\sqrt{1-(v/c)^2}} - 1\right) mc^2 = \left(\frac{1}{\sqrt{1-(0.995)^2}} - 1\right)(9.11 \times 10^{-31})(2.998 \times 10^8)^2$$

$$K = \boxed{7.38 \times 10^{-13} \text{ J}}$$

(b) Classically,

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})\ [(0.995)(2.998 \times 10^8 \text{ m/s})]^2 = 4.05 \times 10^{-14} \text{ J}$$

This differs from the relativistic result by

$$\%\ \text{error} = \left(\frac{7.38 \times 10^{-13} \text{ J} - 4.05 \times 10^{-14} \text{ J}}{7.38 \times 10^{-13} \text{ J}}\right) 100\% = \boxed{94.5\%}$$

7.53 $$\sum W = K_f - K_i = \left(\frac{1}{\sqrt{1-(v_f/c)^2}} - 1\right) mc^2 - \left(\frac{1}{\sqrt{1-(v_i/c)^2}} - 1\right) mc^2$$

or $$\sum W = \left(\frac{1}{\sqrt{1-(v_f/c)^2}} - \frac{1}{\sqrt{1-(v_i/c)^2}}\right) mc^2$$

(a) $$\sum W = \left(\frac{1}{\sqrt{1-(0.750)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)(1.673 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2$$

$$\sum W = \boxed{5.37 \times 10^{-11} \text{ J}}$$

(b) $$\sum W = \left(\frac{1}{\sqrt{1-(0.995)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)(1.673 \times 10^{-27} \text{ kg})(2.998 \times 10^8 \text{ m/s})^2$$

$$\sum W = \boxed{1.33 \times 10^{-9} \text{ J}}$$

Goal Solution

G: Since particle accelerators have typical maximum energies on the order of GeV (1eV = 1.60×10^{-19} J), we could expect the work required to be $\sim 10^{-10}$ J.

O: The work-energy theorem is $W = K_f - K_i$ which for relativistic speeds ($v \sim c$) is:

$$W = \left(\frac{1}{\sqrt{1 - v_f^2/c^2}}\right) mc^2 - \left(\frac{1}{\sqrt{1 - v_i^2/c^2}}\right) mc^2$$

A: (a) $W = \left(\frac{1}{\sqrt{1 - (0.750)^2}} - 1\right)(1.67 \times 10^{-27}\text{ kg})(3.00 \times 10^8\text{ m/s})^2$

$$-\left(\frac{1}{\sqrt{1 - (0.500)^2}} - 1\right)(1.50 \times 10^{-10}\text{ J})$$

$$W = (0.512 - 0.155)(1.50\ \ 10^{-10}\text{ J}) = 5.37\ \ 10^{-11}\text{ J} \quad \Diamond$$

(b) $E = \left(\frac{1}{\sqrt{1 - (0.995)^2}} - 1\right)(1.50 \times 10^{-10}\text{ J}) - (1.155 - 1)(1.50 \times 10^{-10}\text{ J})$

$$W = (9.01 - 0.155)(1.50\ \ 10^{-10}\text{ J}) = 1.33\ \ 10^{-9}\text{ J} \quad \Diamond$$

L: Even though these energies may seem like small numbers, we must remember that the proton has very small mass, so these input energies are comparable to the rest mass energy of the proton (938 MeV = 1.50×10^{-10} J). To produce a speed higher by 33%, the answer to part (b) is 25 times larger than the answer to part (a). Even with arbitrarily large accelerating energies, the particle will never reach or exceed the speed of light. This is a consequence of special relativity, which will be examined more closely in a later chapter.

***7.54** (a) Using the classical equation,

$$K = \frac{1}{2} mv^2 = \frac{1}{2}(78.0\text{ kg})(1.06 \times 10^5\text{ m/s})^2 = \boxed{4.38 \times 10^{11}\text{ J}}$$

(b) Using the relativistic equation,

$$K = \left(\frac{1}{\sqrt{1 - (v/c)^2}} - 1\right) mc^2$$

$$= \left(\frac{1}{\sqrt{1 - (1.06 \times 10^5 / 2.998 \times 10^8)^2}} - 1\right)(78.0\text{ kg})(2.998 \times 10^8\text{ m/s})^2$$

$$K = \boxed{4.38 \times 10^{11}\text{ J}}$$

When $(v/c) << 1$, the binomial series expansion gives

$$[1-(v/c)^2]^{-1/2} \approx 1 + \frac{1}{2}(v/c)^2$$

Thus, $[1-(v/c)^2]^{-1/2} - 1 \approx (v/c)^2$

and the relativistic expression for kinetic energy becomes $K \approx \frac{1}{2}(v/c)^2 mc^2 = \frac{1}{2} mv^2$. That is, in the limit of speeds much smaller than the speed of light, the relativistic and classical expressions yield the same results.

***7.55** At start, $\mathbf{v} = (40.0 \text{ m/s}) \cos 30.0° \mathbf{i} + (40.0 \text{ m/s}) \sin 30.0° \mathbf{j}$

At apex, $\mathbf{v} = (40.0 \text{ m/s}) \cos 30.0° \mathbf{i} + 0\mathbf{j} = 34.6\mathbf{i} \text{ m/s}$

and $K = \frac{1}{2} mv^2 = \frac{1}{2}(0.150 \text{ kg})(34.6 \text{ m/s})^2 = \boxed{90.0 \text{ J}}$

***7.56** Concentration of Energy output $= (0.600 \text{ J/kg} \cdot \text{step})(60.0 \text{ kg})\left(\frac{1 \text{ step}}{1.50 \text{ m}}\right) = 24.0 \frac{\text{J}}{\text{m}}$

$$F = \left(24.0 \frac{\text{J}}{\text{m}}\right)\left(1 \frac{\text{N} \cdot \text{m}}{\text{J}}\right) = 24.0 \text{ N}$$

$$P = Fv$$

$$70.0 \text{ W} = (24.0 \text{ N})v$$

$$v = \boxed{2.92 \text{ m/s}}$$

7.57 The work-kinetic energy theorem is

$$K_i + \sum W = K_f$$

The total work is equal to the work by the constant total force:

$$\frac{1}{2} mv_i^2 + (\Sigma \mathbf{F}) \cdot (\mathbf{r} - \mathbf{r}_i) = \frac{1}{2} mv_f^2$$

$$\frac{1}{2} mv_i^2 + m\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_i) = \frac{1}{2} mv_f^2$$

$$\boxed{v_i^2 + 2\mathbf{a} \cdot (\mathbf{r} - \mathbf{r}_i) = v_f^2}$$

7.58 (a) $\mathbf{A}\cdot\mathbf{i} = (A)(1)\cos\alpha$. But also, $\mathbf{A}\cdot\mathbf{i} = A_x$.

Thus, $(A)(1)\cos\alpha = A_x$ or $\boxed{\cos\alpha = \dfrac{A_x}{A}}$

Similarly, $\boxed{\cos\beta = \dfrac{A_y}{A}}$

and $\boxed{\cos\gamma = \dfrac{A_z}{A}}$

where $A = \sqrt{A_x^2 + A_y^2 + A_z^2}$

(b) $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = \left(\dfrac{A_x}{A}\right)^2 + \left(\dfrac{A_y}{A}\right)^2 + \left(\dfrac{A_z}{A}\right)^2 = \dfrac{A^2}{A^2} = 1$

7.59 (a) $x = t + 2.00t^3$

therefore,

$$v = \frac{dx}{dt} = 1 + 6.00t^2$$

$$K = \frac{1}{2}mv^2 = \frac{1}{2}(4.00)(1 + 6.00t^2)^2 = \boxed{(2.00 + 24.0t^2 + 72.0t^4)\text{ J}}$$

(b) $a = \dfrac{dv}{dt} = \boxed{(12.0t)\text{ m/s}^2}$

$F = ma = 4.00(12.0t) = \boxed{(48.0t)\text{ N}}$

(c) $P = Fv = (48.0t)(1 + 6.00t^2) = \boxed{(48.0t + 288t^3)\text{ W}}$

(d) $W = \int_0^{2.00} P\,dt = \int_0^{2.00} (48.0t + 288t^3)\,dt = \boxed{1250\text{ J}}$

***7.60** (a) The work done by the traveler is mgh_sN where N is the number of steps he climbs during the ride.

$$N = (\text{time on escalator})(n)$$

where $(\text{time on escalator}) = \dfrac{h}{\text{vertical velocity of person}}$, and

$$\text{vertical velocity of person} = v + nh_s$$

Then, $N = \dfrac{nh}{v + nh_s}$ and the work done by the person becomes

$$W_{\text{person}} = \boxed{\dfrac{mgnhh_s}{v + nh_s}}$$

(b) The work done by the escalator is

$$W_e = (\text{power})(\text{time}) = [(\text{force exerted})(\text{speed})](\text{time}) = mgvt$$

where $t = \dfrac{h}{v + nh_s}$ as above. Thus,

$$W_e = \boxed{\dfrac{mgvh}{v + nh_s}}$$

As a check, the total work done on the person's body must add up to *mgh*, the work an elevator would do in lifting him. It does add up as follows:

$$\sum W = W_{\text{person}} + W_e = \frac{mgnhh_s}{v + nh_s} + \frac{mgvh}{v + nh_s} = \frac{mgh(nh_s + v)}{v + nh_s} = mgh$$

7.61 $W = \int_{x_i}^{x_f} F\,dx = \int_0^{x_f} (-kx + \beta x^3)\,dx$

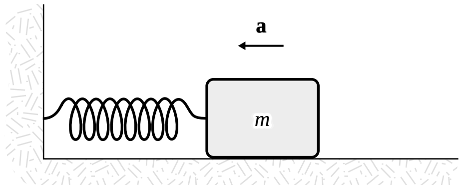
$$W = \frac{-kx^2}{2} + \frac{\beta x^4}{4}\Bigg|_0^{x_f} = \frac{-kx_f^2}{2} + \frac{\beta x_f^4}{4}$$

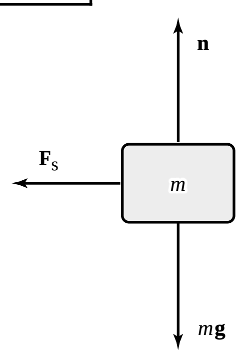
$$W = \frac{(-10.0\text{ N/m})(0.100\text{ m})^2}{2} + \frac{(100\text{ N/m}^3)(0.100\text{ m})^4}{4}$$

$$W = -5.00 \times 10^{-2}\text{ J} + 2.50 \times 10^{-3}\text{ J} = \boxed{-4.75 \times 10^{-2}\text{ J}}$$

***7.62** $\sum F_x = ma_x \Rightarrow kx = ma$

$$k = \frac{ma}{x} = \frac{(4.70 \times 10^{-3}\text{ kg})0.800(9.80\text{ m/s}^2)}{0.500 \times 10^{-2}\text{ m}} = \boxed{7.37\text{ N/m}}$$





7.63 Consider the work done on the pile driver from the time it starts from rest until it comes to rest at the end of the fall.

$$\sum W = \Delta K \Rightarrow W_{\text{gravity}} + W_{\text{beam}} = \frac{1}{2}\,mv_f^2 - \frac{1}{2}\,mv_i^2$$

so $(mg)(h + d)\cos 0° + (\bar{F})(d)\cos 180° = 0 - 0$

Thus, $\bar{F} = \dfrac{(mg)(h + d)}{d} = \dfrac{(2100\text{ kg})(9.80\text{ m/s}^2)(5.12\text{ m})}{0.120\text{ m}} = \boxed{8.78 \times 10^5\text{ N}}$

Goal Solution

G: Anyone who has hit their thumb with a hammer knows that the resulting force is greater than just the weight of the hammer, so we should also expect the force of the pile driver to be greater than its weight: $F > mg \sim 20$ kN. The force *on* the pile driver will be directed upwards.

O: The average force stopping the driver can be found from the work that results from the gravitational force starting its motion. The initial and final kinetic energies are zero.

A: Choose the initial point when the mass is elevated and the final point when it comes to rest again 5.12 m below. Two forces do work on the pile driver: gravity and the normal force exerted by the beam on the pile driver.

$$W_{\text{net}} = K_f - K_i \quad \text{so that} \quad mgs_w \cos 0 + ns_n \cos 180 = 0$$

where $m = 2\,100$ kg, $s_w = 5.12$ m, and $s_n = 0.120$ m.

In this situation, the weight vector is in the direction of motion and the beam exerts a force on the pile driver opposite the direction of motion.

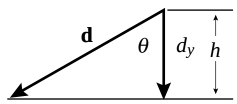
$$(2100\text{ kg})\,(9.80\text{ m/s}^2)\,(5.12\text{ m}) - n\,(0.120\text{ m}) = 0$$

Solve for *n*. $n = \dfrac{1.05 \times 10^5\text{ J}}{0.120\text{ m}} = 878$ kN (upwards) ◊

L: The normal force is larger than 20 kN as we expected, and is actually about 43 times greater than the weight of the pile driver, which is why this machine is so effective.

Additional Calculation:
Show that the work done by gravity on an object can be represented by *mgh*, where *h* is the vertical height that the object falls. Apply your results to the problem above.

By the figure, where **d** is the path of the object, and *h* is the height that the object falls,



$h = |d_y| = d\cos\theta$

Since $F = mg$, $mgh = Fd\cos\theta = \mathbf{F}\cdot\mathbf{d}$

In this problem, $mgh = n(d_n)$, or $(2100\text{ kg})(9.80\text{ m/s}^2)(5.12\text{ m}) = n(0.120\text{ m})$ and $n = 878\text{ kN}$

7.64 Let *b* represent the proportionality constant of air drag f_a to speed: $|f_a| = bv$

Let f_r represent the other frictional forces.

Take *x*-axis along each roadway.

For the gentle hill $\sum F_x = ma_x$

$$-bv - f_r + mg\sin 2.00° = 0$$

$$-b(4.00\text{ m/s}) - f_r + 25.7\text{ N} = 0$$

For the steeper hill

$$-b(8.00\text{ m/s}) - f_r + 51.3\text{ N} = 0$$

Subtracting,

$$b(4.00\text{ m/s}) = 25.6\text{ N}$$

$$b = 6.40\text{ N}\cdot\text{s/m}$$

and then $f_r = 0.0313$ N.

Now at 3.00 m/s the vehicle must pull her with force

$$bv + f_r = (6.40\text{ N}\cdot\text{s/m})(3.00\text{ m/s}) + 0.0313\text{ N} = 19.2\text{ N}$$

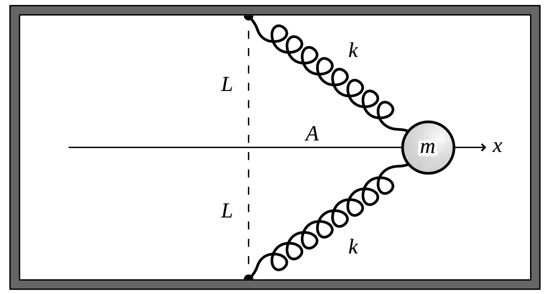
and with power

$$P = \mathbf{F}\cdot\mathbf{v} = 19.2\text{ N}(3.00\text{ m/s})\cos 0° = \boxed{57.7\text{ W}}$$

7.65 (a) $P = Fv = F(v_i + at) = F\left(0 + \frac{F}{m}t\right) = \boxed{\left(\frac{F^2}{m}\right)t}$

(b) $P = \left[\frac{(20.0\text{ N})^2}{5.00\text{ kg}}\right](3.00\text{ s}) = \boxed{240\text{ W}}$

7.66 (a) The new length of each spring is $\sqrt{x^2 + L^2}$, so its extension is $\sqrt{x^2 + L^2} - L$ and the force it exerts is $k\left(\sqrt{x^2 + L^2} - L\right)$ toward its fixed end. The y components of the two spring forces add to zero. Their x components add to



(top view)

$\mathbf{F} = -2\mathbf{i}k\left(\sqrt{x^2 + L^2} - L\right)x/\sqrt{x^2 + L^2}$

$\mathbf{F} = \boxed{-2kx\mathbf{i}\left(1 - L/\sqrt{x^2 + L^2}\right)}$

(b) $W = \int_i^f F_x\, dx$

$W = \int_A^0 -2kx\left(1 - L/\sqrt{x^2 + L^2}\right)dx$

$W = -2k\int_A^0 x\, dx + kL\int_A^0 (x^2 + L^2)^{-1/2}\, 2x\, dx$

$W = -2k\left.\frac{x^2}{2}\right|_A^0 + kL\left.\frac{(x^2 + L^2)^{1/2}}{(1/2)}\right|_A^0$

$W = -0 + kA^2 + 2kL^2 - 2kL\sqrt{A^2 + L^2}$

$W = \boxed{2kL^2 + kA^2 - 2kL\sqrt{A^2 + L^2}}$

7.67 (a) $\mathbf{F}_1 = (25.0\text{ N})(\cos 35.0°\,\mathbf{i} + \sin 35.0°\,\mathbf{j}) = \boxed{(20.5\mathbf{i} + 14.3\mathbf{j})\text{ N}}$

$\mathbf{F}_2 = (42.0\text{ N})(\cos 150°\,\mathbf{i} + \sin 150°\,\mathbf{j}) = \boxed{(-36.4\,\mathbf{i} + 21.0\,\mathbf{j})\text{ N}}$

(b) $\sum\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \boxed{(-15.9\mathbf{i} + 35.3\mathbf{j})\text{ N}}$

(c) $\mathbf{a} = \frac{\sum\mathbf{F}}{m} = \boxed{(-3.18\mathbf{i} + 7.07\mathbf{j})\text{ m/s}^2}$

(d) $\mathbf{v} = \mathbf{v}_i + \mathbf{a}t = (4.00\mathbf{i} + 2.50\mathbf{j})\text{ m/s} + (-3.18\mathbf{i} + 7.07\mathbf{j})(\text{m/s}^2)(3.00\text{ s})$

$\mathbf{v} = \boxed{(-5.54\mathbf{i} + 23.7\mathbf{j})\text{ m/s}}$

(e) $\mathbf{r} = \mathbf{r}_i + \mathbf{v}_i t + \frac{1}{2}\mathbf{a}t^2$

$$\mathbf{r} = 0 + (4.00\mathbf{i} + 2.50\mathbf{j})(\text{m/s})(3.00\text{ s}) + \frac{1}{2}(-3.18\mathbf{i} + 7.07\mathbf{j})(\text{m/s}^2)(3.00\text{ s})^2$$

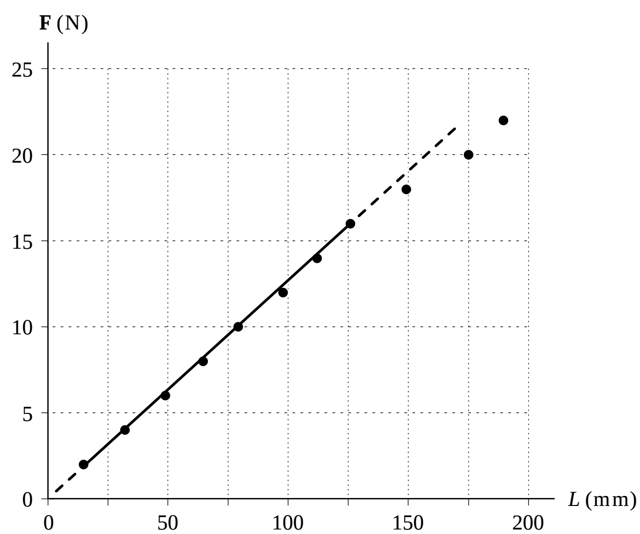
$$\mathbf{d} = \mathbf{r} = \boxed{(-2.30\mathbf{i} + 39.3\mathbf{j})\text{ m}}$$

(f) $K_f = \frac{1}{2}mv_f^2 = \frac{1}{2}(5.00\text{ kg})\ [(5.54)^2 + (23.7)^2](\text{m/s})^2 = \boxed{1.48\text{ kJ}}$

(g) $K_f = \frac{1}{2}mv_i^2 + \sum\mathbf{F}\cdot\mathbf{d} = \frac{1}{2}(5.00\text{ kg})\ [(4.00)^2 + (2.50)^2](\text{m/s})^2$

$$+ [(-15.9\text{ N})(-2.30\text{ m}) + (35.3\text{ N})(39.3\text{ m})] = 55.6\text{ J} + 1426\text{ J} = \boxed{1.48\text{ kJ}}$$

7.68 (a)



F (N)	L (mm)	F (N)	L (mm)
2.00	15.0	14.0	112
4.00	32.0	16.0	126
6.00	49.0	18.0	149
8.00	64.0	20.0	175
10.0	79.0	22.0	190
12.0	98.0		

(b) A straight line fits the first eight points, and the origin. By least-square fitting, its slope is 0.125 N / mm ± 2% = $\boxed{125\text{ N/m}}$ ± 2%. In $F = kx$, the spring constant is $k = F/x$, the same as the slope of the F-versus-x graph.

(c) $F = kx = (125\text{ N/m})(0.105\text{ m}) = \boxed{13.1\text{ N}}$

7.69 (a) $\sum W = \Delta K$

$W_s + W_g = 0$

$\frac{1}{2}(1.40 \times 10^3 \text{ N/m}) \times (0.100 \text{ m})^2 - (0.200 \text{ kg})(9.80)(\sin 60.0°)x = 0$

$x = \boxed{4.12 \text{ m}}$

(b) $\sum W = \Delta K$

$W_s + W_g + W_f = 0$

$\frac{1}{2}(1.40 \times 10^3 \text{ N/m}) \times (0.100)^2 - [(0.200)(9.80)(\sin 60.0°)$

$+ (0.200)(9.80)(0.400)(\cos 60.0°)]x = 0$

$x = \boxed{3.35 \text{ m}}$

***7.70** (a) $W = \Delta K = \frac{1}{2} m(v_f^2 - v_i^2) = \frac{1}{2}(0.400 \text{ kg}) [(6.00)^2 - (8.00)^2] (\text{m/s})^2 = \boxed{-5.60 \text{ J}}$

(b) $W = fd \cos 180° = -\mu_k mg(2\pi r)$

$-5.60 \text{ J} = -\mu_k(0.400 \text{ kg})(9.80 \text{ m/s}^2)2\pi(1.50 \text{ m})$

Thus, $\mu_k = \boxed{0.152}$

(c) After N revolutions, the mass comes to rest and $K_f = 0$. Thus,

$$W = \Delta K = 0 - K_i = -\frac{1}{2} mv_i^2 \quad \text{or} \quad -\mu_k mg [N(2\pi r)] = -\frac{1}{2} mv_i^2$$

This gives

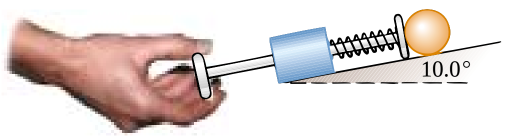
$$N = \frac{\frac{1}{2} mv_i^2}{\mu_k mg(2\pi r)} = \frac{\frac{1}{2}(8.00 \text{ m/s})^2}{(0.152)(9.80 \text{ m/s}^2)2\pi(1.50 \text{ m})} = \boxed{2.28 \text{ rev}}$$

7.71 $\frac{1}{2}\left(1.20\ \frac{\text{N}}{\text{cm}}\right)(5.00\ \text{cm})(0.0500\ \text{m})$

$= (0.100\ \text{kg})(9.80\ \text{m/s}^2)(0.0500\ \text{m})\sin 10.0° + \frac{1}{2}(0.100\ \text{kg})\ v^2$

$0.150\ \text{J} = 8.51 \times 10^{-3}\ \text{J} + (0.0500\ \text{kg})v^2$

$v = \sqrt{\frac{0.141}{0.0500}} = \boxed{1.68\ \text{m/s}}$



7.72 If positive F represents an outward force, (same direction as r), then

$$W = \int_i^f \mathbf{F} \cdot ds = \int_{r_i}^{r_f} (2F_0\sigma^{13}r^{-13} - F_0\sigma^7 r^{-7})\, dr$$

$$W = \frac{+2F_0\sigma^{13}r^{-12}}{(-12)} - \frac{F_0\sigma^7 r^{-6}}{(-6)}\Bigg|_{r_i}^{r_f}$$

$$W = \frac{-F_0\sigma^{13}(r_f^{-12} - r_i^{-12})}{6} + \frac{F_0\sigma^7(r_f^{-6} - r_i^{-6})}{6} = \frac{F_0\sigma^7}{6}[r_f^{-6} - r_i^{-6}] - \frac{F_0\sigma^{13}}{6}[r_f^{-12} - r_i^{-12}]$$

$$W = 1.03 \times 10^{-77}[r_f^{-6} - r_i^{-6}] - 1.89 \times 10^{-134}[r_f^{-12} - r_i^{-12}]$$

$$W = 1.03 \times 10^{-77}[1.88 \times 10^{-6} - 2.44 \times 10^{-6}]\ 10^{+60}$$

$$- 1.89 \times 10^{-134}[3.54 \times 10^{-12} - 5.96 \times 10^{-8}]\ 10^{120}$$

$$W = -2.49 \times 10^{-21}\ \text{J} + 1.12 \times 10^{-21}\ \text{J} = \boxed{-1.37 \times 10^{-21}\ \text{J}}$$

7.73 (a) $\sum W = \Delta K$

$$m_2 gh - \mu m_1 gh = \frac{1}{2}(m_1 + m_2)(v^2 - v_i^2)$$

$$v = \sqrt{\frac{2gh(m_2 - \mu m_1)}{(m_1 + m_2)}}$$

$$= \sqrt{\frac{2(9.80)(20.0)[0.400 - (0.200)(0.250)]}{(0.400 + 0.250)}} = \boxed{14.5\ \text{m/s}}$$

(b) $W_f + W_g = \Delta K = 0$

$$-\mu(\Delta m_1 + m_1)gh + m_2gh = 0$$

$$\mu(\Delta m_1 + m_1) = m_2$$

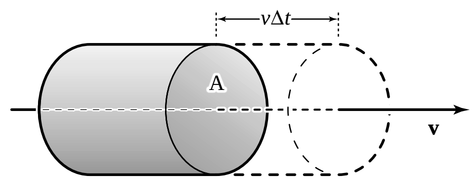
$$\Delta m_1 = \frac{m_2}{\mu} - m_1 = \frac{0.400 \text{ kg}}{0.200} - 0.250 \text{ kg} = \boxed{1.75 \text{ kg}}$$

(c) $W_f + W_g = \Delta K = 0$

$$-\mu m_1 gh + (m_2 - \Delta m_2)gh = 0$$

$$\Delta m_2 = m_2 - \mu m_1 = 0.400 \text{ kg} - (0.200)(0.250 \text{ kg}) = \boxed{0.350 \text{ kg}}$$

7.74 $P\,\Delta t = W = \Delta K = \dfrac{(\Delta m)v^2}{2}$



The density is

$$\rho = \frac{\Delta m}{\text{vol}} = \frac{\Delta m}{A\,\Delta x}$$

Substituting this into the first equation and solving for P, since

$$\frac{\Delta x}{\Delta t} = v$$

for a constant speed, we get

$$\boxed{P = \frac{\rho A v^3}{2}}$$

Also, since $P = Fv$,

$$\boxed{F = \frac{\rho A v^2}{2}}$$

7.75 We evaluate $\displaystyle\int_{12.8}^{23.7} \frac{375dx}{x^3 + 3.75x}$ by calculating

$$\frac{375(0.100)}{(12.8)^3 + 3.75(12.8)} + \frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \ldots \frac{375(0.100)}{(23.6)^3 + 3.75(23.6)} = 0.806$$

and $$\frac{375(0.100)}{(12.9)^3 + 3.75(12.9)} + \frac{375(0.100)}{(13.0)^3 + 3.75(13.0)} + \ldots \frac{375(0.100)}{(23.7)^3 + 3.75(23.7)} = 0.791$$

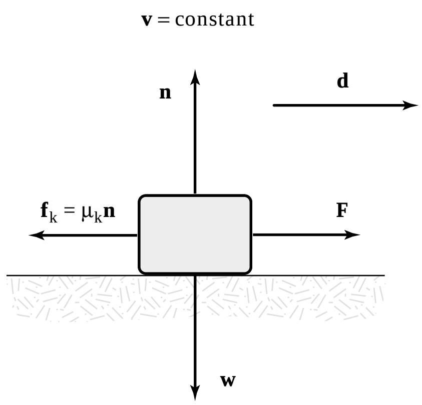
The answer must be between these two values. We may find it more precisely by using a value for Δx smaller than 0.100. Thus, we find the integral to be $\boxed{0.799 \text{ N} \cdot \text{m}}$.

***7.76** (a) The suggested equation $P\,t = bwd$ implies all of the following cases:

(1) $P\,t = b\left(\frac{w}{2}\right)(2d)$ (2) $P\left(\frac{t}{2}\right) = b\left(\frac{w}{2}\right)d$

(3) $P\left(\frac{t}{2}\right) = bw\left(\frac{d}{2}\right)$ and (4) $\left(\frac{P}{2}\right)t = b\left(\frac{w}{2}\right)d$

These are all of the proportionalities Aristotle lists.



(b) For one example, consider a horizontal force F pushing an object of weight w at constant velocity across a horizontal floor with which the object has coefficient of friction μ_k.

$\sum \mathbf{F} = m\mathbf{a}$ implies that:

$+n - w = 0$ and $F - \mu_k n = 0$

so that $F = \mu_k w$

As the object moves a distance d, the agent exerting the force does work

$W = Fd\cos\theta = Fd\cos 0° = \mu_k wd$ and puts out power $P = W/t$

This yields the equation $P\,t = \mu_k wd$ which represents Aristotle's theory with $b = \mu_k$.

Our theory is more general than Aristotle's. Ours can also describe accelerated motion.