Chapter 8 Solutions

***8.1** (a) With our choice for the zero level for potential energy at point B, $\boxed{U_B = 0}$.

At point A, the potential energy is given by

$$U_A = mgy$$

where y is the vertical height above zero level. With

$$135 \text{ ft} = 41.1 \text{ m}$$

this height is found as:

$$y = (41.1 \text{ m}) \sin 40.0° = 26.4 \text{ m}$$

Thus,

$$U_A = (1000 \text{ kg})(9.80 \text{ m/s}^2)(26.4 \text{ m}) =$$

The change in potential energy as it moves from A to B is

$$U_B - U_A = 0 - 2.59 \times 10^5 \text{ J} =$$

(b) With our choice of the zero level at point A, we have $\boxed{U_A = 0}$.

The potential energy at B is given by $U_B = mgy$ where y is the vertical distance of point B below point A. In part (a), we found the magnitude of this distance to be 26.5 m. Because this distance is now below the zero reference level, it is a negative number. Thus,

$$U_B = (1000 \text{ kg})(9.80 \text{ m/s}^2)(-26.5 \text{ m}) =$$

The change in potential energy in going from A to B is

$$U_B - U_A = -2.59 \times 10^5 \text{ J} - 0 =$$

***8.2** (a) We take the zero level of potential energy at the lowest point of the arc. When the string is held horizontal initially, the initial position is 2.00 m above the zero level. Thus,

$$U_g = mgy = (40.0\text{ N})(2.00\text{ m}) = \boxed{80.0\text{ J}}$$

(b) From the sketch, we see that at an angle of 30.0° the ball is at a vertical height of (2.00 m)(1 – cos 30.0°) above the lowest point of the arc. Thus,

$$U_g = mgy = (40.0\text{ N})(2.00\text{ m})(1 - \cos 30.0°) = \boxed{10.7\text{ J}}$$

(c) The zero level has been selected at the lowest point of the arc. Therefore, $\boxed{U_g = 0}$ at this location.

8.3 $F_g = mg = (4.00\text{ kg})(9.80\text{ m/s}^2) = 39.2\text{ N}$

(a) Work along OAC = work along OA + work along AC

$= F_g(\text{OA}) \cos 90.0° + F_g(\text{AC}) \cos 180°$

$= (39.2\text{ N})(5.00\text{ m})(0) + (39.2\text{ N})(5.00\text{ m})(-1)$

$= \boxed{-196\text{ J}}$

(b) W along OBC = W along OB + W along BC

$= (39.2\text{ N})(5.00\text{ m}) \cos 180° + (39.2\text{ N})(5.00\text{ m}) \cos 90.0°$

$= \boxed{-196\text{ J}}$

(c) Work along OC = $F_g(\text{OC}) \cos 135°$

$$= (39.2\text{ N})(5.00 \times \sqrt{2}\text{ m})\left(-\frac{1}{\sqrt{2}}\right) = \boxed{-196\text{ J}}$$

The results should all be the same, since gravitational forces are conservative.

8.4 (a) $W =$ and if the force is constant, this can be written as

$$W = \mathbf{F} \cdot \int d\mathbf{s} =$$

(b) $W = \quad = \int (3\mathbf{i} + 4\mathbf{j}) \cdot (dx\,\mathbf{i} + dy\,\mathbf{j}) = (3.00\text{ N}) \int_0^{5.00\text{ m}} dx + (4.00\text{ N}) \int_0^{5.00\text{ m}} dy$

$W = (3.00\text{ N})\, x \Big|_0^{5.00\text{ m}} + (4.00\text{ N}) y \Big|_0^{5.00\text{ m}} = 15.0\text{ J} + 20.0\text{ J} = \boxed{35.0\text{ J}}$

The same calculation applies for all paths.

8.5 (a) $W_{OA} = \int_0^{5.00\text{ m}} dx\mathbf{i} \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00\text{ m}} 2y\, dx$ and since along this path, $y = 0$

$W_{OA} = 0$

$W_{AC} = \int_0^{5.00\text{ m}} dy\,\mathbf{j} \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00\text{ m}} x^2\, dy$. For $x = 5.00$ m, $W_{AC} = 125$ J

and $W_{OAC} = 0 + 125 = \boxed{125\text{ J}}$

(b) $W_{OB} = \quad \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00\text{ m}} x^2\, dy$ since along this path, $x = 0$

$W_{OB} = 0$

$W_{BC} = \int_0^{5.00\text{ m}} dx\mathbf{i} \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int_0^{5.00\text{ m}} 2y\, dx$ since $y = 5.00$ m, $W_{BC} = \boxed{50.0\text{ J}}$

and $W_{OBC} = 0 + 50.0 = \boxed{50.0\text{ J}}$

(c) $W_{OC} = \int (dx\mathbf{i} + dy\mathbf{j}) \cdot (2y\mathbf{i} + x^2\mathbf{j}) = \int (2y\, dx + x^2\, dy)$

Since $x = y$ along OC,

$$W_{OC} = \int_0^{5.00\text{ m}} (2x + x^2)dx = \boxed{66.7\text{ J}}$$

(d) F is $\boxed{\text{non-conservative}}$ since the work done is path dependent.

8.6 (a) $U_f = K_i - K_f + U_i$

$U_f = 30.0 - 18.0 + 10.0 = \boxed{22.0\text{ J}}$

$\boxed{E = 40.0\text{ J}}$

(b) Yes, $\Delta E = \Delta K + \Delta U$; for conservative forces $\Delta K + \Delta U = 0$.

8.7 (a) $W = \vee F_x\, dx = \int_1^{5.00\text{ m}} (2x + 4)dx = \left(\frac{2x^2}{2} + 4x\right)_1^{5.00\text{ m}}$

$$= 25.0 + 20.0 - 1.00 - 4.00 = \boxed{40.0\text{ J}}$$

(b) $\Delta K + \Delta U = 0$

$\Delta U = -\Delta K = -W = \boxed{-40.0\text{ J}}$

(c) $\Delta K = K_f - \dfrac{mv_1^2}{2}$

$K_f = \Delta K + \dfrac{mv_1^2}{2} = \boxed{62.5\text{ J}}$

8.8 (a) $\mathbf{F} = (3.00\mathbf{i} + 5.00\mathbf{j})$ N $\qquad m = 4.00$ kg

$\mathbf{r} = (2.00\mathbf{i} - 3.00\mathbf{j})$ m

$W = 3.00(2.00) + 5.00(-3.00) = \boxed{-9.00\text{ J}}$

The result does not depend on the path since the force is conservative.

(b) $W = \Delta K$

$-9.00 = \dfrac{4.00v^2}{2} - 4.00\left(\dfrac{(4.00)^2}{2}\right)$

so $\quad v = \sqrt{\dfrac{32.0 - 9.00}{2.00}} = \boxed{3.39\text{ m/s}}$

(c) $\Delta U = -W = \boxed{9.00\text{ J}}$

8.9 (a) $U = -\int_0^x (-Ax + Bx^2)dx = \boxed{\dfrac{Ax^2}{2} - \dfrac{Bx^3}{3}}$

(b) $\Delta U = -\int_{2.00\text{ m}}^{3.00\text{ m}} F\,dx = \dfrac{A[(3.00)^2 - (2.00)^2]}{2} - \dfrac{B[(3.00)^3 - (2.00)^3]}{3} = \boxed{\dfrac{5.00}{2}A - \dfrac{19.0}{3}B}$

$\Delta K = \boxed{\left(-\dfrac{5.00}{2}A + \dfrac{19.0}{3}B\right)}$

8.10 (a) Energy is conserved between point P and the apex of the trajectory.

Since the horizontal component of velocity is constant,

$$\frac{1}{2}mv_i^2 = \frac{1}{2}mv_{ix}^2 + \frac{1}{2}mv_{iy}^2 = \frac{1}{2}mv_{ix}^2 + mgh$$

$$v_{iy} = \qquad = \boxed{19.8 \text{ m/s}}$$

(b) $\Delta K \&_{P \varnothing B} = W_g = mg(60.0\text{ m}) = (0.500\text{ kg})(9.80\text{ m/s}^2)(60.0\text{ m}) = \boxed{294\text{ J}}$

(c) Now let the final point be point B.

$$v_{ix} = v_{fx} = 30.0\text{ m/s}$$

$$\Delta K \&_{P \varnothing B} = \frac{1}{2}\ mv_{fy}^2\ \ - \frac{1}{2}\ mv_{iy}^2\ \ = 294\text{ J}$$

$$v_{fy}^2\ \ = \frac{2}{m}(294)\ \ + v_{iy}^2\ \ = 1176 + 392$$

$$v_{fy} = -39.6\text{ m/s}$$

$$\mathbf{v}_B = \boxed{(30.0\text{ m/s})\mathbf{i} - (39.6\text{ m/s})\mathbf{j}}$$

8.11 $mgh = \frac{1}{2}\ kx^2$

$$(3.00\text{ kg})(9.80\text{ m/s}^2)(d + 0.200\text{ m})\sin 30.0° = \frac{1}{2}\ 400(0.200\text{ m})^2$$

$$14.7d + 2.94 = 8.00$$

$$d = \boxed{0.344\text{ m}}$$

8.12 Choose the zero point of gravitational potential energy at the level where the mass comes to rest. Then because the incline is frictionless, we have

$$E_B = E_A \Rightarrow K_B + U_{gB} + U_{sB} = K_A + U_{gA} + U_{sA}$$

or $\quad 0 + mg(d + x)\sin\theta + 0 = 0 + 0 + \frac{1}{2}\ kx^2$

Solving for d gives $\qquad d = \boxed{\dfrac{kx^2}{2mg\sin\theta} - x}$

8.13 (a) $(\Delta K)_{A\varnothing B} = \Sigma W = W_g = mg\Delta h = mg(5.00 - 3.20)$

$$\frac{1}{2}\ mv_B^2 - \frac{1}{2}\ mv_A^2 = m(9.80)(1.80)$$

$$v_B = \boxed{5.94\text{ m/s}}$$

Similarly, $v_C = \sqrt{v_A^2 + 2g(5.00 - 2.00)} = \boxed{7.67\text{ m/s}}$

(b) $W_g \&_{A\,\varnothing\,C} = mg(3.00\text{ m}) = \boxed{147\text{ J}}$

***8.14** $K_i + U_i + \Delta E = K_f + U_f$

$$0 + m(9.80\text{ m/s}^2)(2.00\text{ m} - 2.00\text{ m}\cos 25.0°) = \frac{1}{2}\ mv_f^2 + 0$$

$$v_f = \sqrt{(2)(9.80\text{ m/s}^2)(0.187\text{ m})} = \boxed{1.92\text{ m/s}}$$

8.15 $U_i + K_i = U_f + K_f$

$$mgh + 0 = mg(2R) + \frac{1}{2}\ mv^2$$

$$g(3.50\ R) = 2\ g(R) + \frac{1}{2}\ v^2$$

$$\boxed{v = \sqrt{3.00\ g\ R}}$$

$$\cdot F = m\frac{v^2}{R}$$

$$n + mg = m\frac{v^2}{R}$$

$$n = m\left[\frac{v^2}{R} - g\right] = m\left[\frac{3.00\ g\ R}{R} - g\right]$$

$$n = 2.00\ mg$$

$$n = 2.00\ (5.00 \infty 10^{-3}\text{ kg})(9.80\text{ m/s}^2)$$

$$n = \boxed{0.0980\text{ N downward}}$$

Goal Solution

G: Since the bead is released above the top of the loop, it will have enough potential energy to reach point A and still have excess kinetic energy. The energy of the bead at the top will be proportional to h and g. If it is moving relatively slowly, the track will exert an upward force on the bead, but if it is whipping around fast, the normal force will push it toward the center of the loop.

O: The speed at the top can be found from the conservation of energy, and the normal force can be found from Newton's second law.

A: We define the bottom of the loop as the zero level for the gravitational potential energy.

Since $v_i = 0$,

$$E_i = K_i + U_i = 0 + mgh = mg(3.50R)$$

The total energy of the bead at point A can be written as

$$E_A = K_A + U_A = \frac{1}{2}mv_A^2 + mg(2R)$$

Since mechanical energy is conserved, $E_i = E_A$, and we get

$$\frac{1}{2}mv_A^2 + mg(2R) = mg(3.50R)$$

$$v_A^2 = 3.00gR \quad \text{or} \quad v_A = \sqrt{3.00gR}$$

To find the normal force at the top, we may construct a free-body diagram as shown, where we assume that **n** is downward, like $m\mathbf{g}$. Newton's second law gives $F = ma_c$, where a_c is the centripetal acceleration.

$$n + mg = \frac{mv_A^2}{R} = \frac{m(3.00gR)}{R} = 3.00mg$$

$$n = 3.00mg - mg = 2.00mg$$

$$n = 2.00(5.00 \infty 10^{-3}\text{ kg})(9.80\text{ m/s}^2) = 0.0980\text{ N downward}$$

L: Our answer represents the speed at point A as proportional to the square root of the product of g and R, but we must not think that simply increasing the diameter of the loop will increase the speed of the bead at the top. In general, the speed will increase with increasing release height, which for this problem was defined in terms of the radius. The normal force may seem small, but it is twice the weight of the bead.

8.16 (a) At the equilibrium position for the mass, the tension in the spring equals the weight of the mass. Thus, elongation of the spring when the mass is at equilibrium is:

$$kx_o = mg \Rightarrow x_o = \frac{mg}{k} = \frac{(0.120)(9.80)}{40.0} = 0.0294\text{ m}$$

The mass moves with maximum speed as it passes through the equilibrium position. Use energy conservation, taking $U_g = 0$ at the initial position of the mass, to find this speed:

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si}$$

$$\frac{1}{2}mv_{\text{max}}^2 + mg(-x_o) + \frac{1}{2}kx_0^2 = 0 + 0 + 0$$

$$v_{\text{max}} = \sqrt{2gx_0 - \frac{kx_0^2}{m}} = \sqrt{2(9.80)(0.0294) - \frac{(40.0)(0.0294)^2}{0.120}} = \boxed{0.537\text{ m/s}}$$

(b) When the mass comes to rest, $K_f = 0$. Therefore,

$$K_f + U_{gf} + U_{sf} = K_i + U_{gi} + U_{si} \quad \text{becomes}$$

$$0 + mg(-x) + \frac{1}{2}kx^2 = 0 + 0 + 0 \quad \text{which becomes}$$

$$x = \frac{2mg}{k} = 2x_0 = \boxed{0.0588 \text{ m}}$$

8.17 From conservation of energy, $U_{gf} = U_{si}$, or

$(0.250 \text{ kg})(9.80 \text{ m/s}^2)h = (1/2)(5000 \text{ N/m})(0.100 \text{ m})^2$

This gives a maximum height $h = \boxed{10.2 \text{ m}}$

***8.18** From leaving ground to highest point

$$K_i + U_i = K_f + U_f$$

$$\frac{1}{2}m(6.00 \text{ m/s})^2 + 0 = 0 + m(9.80 \text{ m/s}^2)y$$

The mass makes no difference:

$$\therefore y = \frac{(6.00 \text{ m/s})^2}{(2)(9.80 \text{ m/s}^2)} = \boxed{1.84 \text{ m}}$$

***8.19** (a) $\frac{1}{2}mv^2 = mgh$

$$v = \sqrt{2gh} = \boxed{19.8 \text{ m/s}}$$

(b) $E = mgh = \boxed{78.4 \text{ J}}$

(c) $K_{10} + U_{10} = 78.4 \text{ J}$

$$K_{10} = 39.2 \text{ J} \quad U_{10} = 39.2 \text{ J} \quad \frac{K_{10}}{U_{10}} = \boxed{1.00}$$

8.20 Choose $y = 0$ at the river. Then $y_i = 36.0$ m, $y_f = 4.00$ m, the jumper falls 32.0 m, and the cord stretches 7.00 m. Between balloon and bottom,

$$K_i + U_{gi} + U_{si} = K_f + U_{gf} + U_{sf}$$

$$0 + mgy_i + 0 = 0 + mgy_f + \frac{1}{2}\ kx_f^2$$

$$(700\text{ N})(36.0\text{ m}) = (700\text{ N})(4.00\text{ m}) + \frac{1}{2}\ k(7.00\text{ m})^2$$

$$k = \frac{22400\text{ J}}{24.5\text{ m}^2} = \boxed{914\text{ N/m}}$$

8.21 Using conservation of energy

(a) $(5.00\text{ kg})g(4.00\text{ m}) = (3.00\text{ kg})g(4.00\text{ m}) + \frac{1}{2}(5.00 + 3.00)\ v^2$

$$v = \sqrt{19.6} = \boxed{4.43\text{ m/s}}$$

(b) $\frac{1}{2}(3.00)\ v^2 = mg\ \Delta y = 3.00g\ \Delta y$

$$\Delta y = 1.00\text{ m}$$

$$y_{\text{max}} = 4.00\text{ m} + \Delta y = \boxed{5.00\text{ m}}$$

8.22 $m_1 > m_2$

(a) $m_1gh = \frac{1}{2}(m_1 + m_2)\ v^2 + m_2gh$

$$v = \boxed{\sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2)}}}$$

(b) Since m_2 has kinetic energy $\frac{1}{2}\ m_2v^2$, it will rise an additional height Δh determined from

$$m_2g\ \Delta h = \frac{1}{2}\ m_2v^2$$

or from (a),

$$\Delta h = \frac{v^2}{2g} = \frac{(m_1 - m_2)h}{(m_1 + m_2)}$$

The total height m_2 reaches is $h + \Delta h = \boxed{\frac{2m_1 h}{m_1 + m_2}}$

8.23 (a) $K_i + U_{gi} = K_f + U_{gf}$

$$\frac{1}{2} mv_i^2 + 0 = \frac{1}{2} mv_f^2 + mgy_f$$

$$\frac{1}{2} mv_{xi}^2 + \frac{1}{2} mv_{yi}^2 = \frac{1}{2} mv_{xf}^2 + mgy_f$$

But $v_{xi} = v_{xf}$, so for the first ball

$$y_f = \frac{v_{yi}^2}{2g} = \frac{(1000 \sin 37.0°)^2}{2(9.80)} =$$

and for the second

$$y_f = \frac{(1000)^2}{2(9.80)} =$$

(b) The total energy of each is constant with value

$$\frac{1}{2}(20.0 \text{ kg})(1000 \text{ m/s})^2 =$$

8.24 In the swing down to the breaking point, energy is conserved:

$$mgr \cos \theta = \frac{1}{2} mv^2$$

at the breaking point consider radial forces

$$\cdot F_r = ma_r$$

$$+T_{\max} - mg \cos \theta = m\frac{v^2}{r}$$

Eliminate $\frac{v^2}{r} = 2g \cos \theta$

$$T_{\max} - mg \cos \theta = 2\, mg \cos \theta$$

$$T_{\max} = 3\, mg \cos \theta$$

$$\theta = \text{Arc cos}\left(\frac{T_{\max}}{3\, mg}\right) = \text{Arc cos}\left(\frac{44.5 \text{ N}}{3(2.00 \text{ kg})(9.80 \text{ m/s}^2)}\right)$$

$$\theta = \boxed{40.8°}$$

***8.25** (a) The force needed to hang on is equal to the force F the trapeze bar exerts on the performer.

From the free-body diagram for the performer's body, as shown,

$$F - mg\cos\theta = m\frac{v^2}{\ell}$$

or $$F = mg\cos\theta + m\frac{v^2}{\ell}$$

Apply conservation of mechanical energy between the starting point and any later point:

$$mg(\ell - \ell\cos\theta_i) = mg(\ell - \ell\cos\theta) + \frac{1}{2}mv^2$$

Solve for mv^2/ℓ and substitute into the force equation to obtain

$$F = \boxed{mg(3\cos\theta - 2\cos\theta_i)}$$

(b) At the bottom of the swing, $\theta = 0°$ so $F = mg(3 - 2\cos\theta_i)$.

$F = 2mg = mg(3 - 2\cos\theta_i)$, which gives

$\theta_i = 60.0°$

***8.26** (a) At point 3, $\Sigma F_y = ma_y$ gives $n + mg = m\dfrac{v_3^2}{R}$.

For apparent weightlessness, $n = 0$. This gives

$$v_3 = \sqrt{Rg} = \sqrt{(20.0)(9.80)} = \boxed{14.0\text{ m/s}}$$

(b) Now, from conservation of energy applied between points 1 and 3,

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_3^2 + mgy_3$$

so $$v_1 = \sqrt{v_3^2 + 2g(y_3 - y_1)} = \sqrt{(14.0)^2 + 2(9.80)(40.0)} = \boxed{31.3\text{ m/s}}$$

(c) The total energy is the same at points 1 and 2:

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2, \text{ which yields}$$

$$v_2 = \sqrt{v_1^2 + 2g(y_1 - y_2)} = \sqrt{(31.3)^2 + 2(9.80)(-20.0)} = \boxed{24.2 \text{ m/s}}$$

(d) Between points 1 and 4:

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_4^2 + mgy_4, \text{ giving}$$

$$H = y_4 - y_1 = \frac{v_1^2 - v_4^2}{2g} = \frac{(31.3)^2 - (10.0)^2}{2(9.80)}$$

$$= \boxed{44.9 \text{ m}}$$

***8.27** The force of tension and subsequent force of compression in the rod do no work on the ball, since they are perpendicular to each step of displacement. Consider energy conservation between the instant just after you strike the ball and the instant when it reaches the top. The speed at the top is zero if you hit it just hard enough to get it there.

$$K_i + U_{gi} = K_f + U_{gf}$$

$$\frac{1}{2}mv_i^2 + 0 = 0 + mg(2L)$$

$$v_i = \sqrt{4gL} = \sqrt{4(9.80)(0.770)} = \boxed{5.49 \text{ m/s}}$$

***8.28** We shall take the zero level of gravitational potential energy to be at the lowest level reached by the diver under the water, and consider the energy change from when the diver started to fall until he came to rest.

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i = f_k\, d \cos 180–$$

$$0 - 0 - mg(y_i - y_f) = -f_k\, d$$

$$f_k = \frac{mg(y_i - y_f)}{d} = \frac{(70.0 \text{ kg})(9.80 \text{ m/s}^2)(10.0 \text{ m} + 5.00 \text{ m})}{5.00 \text{ m}} = \boxed{2.06 \text{ kN}}$$

***8.29** $\frac{1}{2}mv^2 = \int_0^x F_x\,dx$ = area under the F_x vs x curve.

for $x = 2.00$ m $\qquad \int_0^{2.00} F_x\,dx = 10.0\ \text{N}\cdot\text{m}$

$$\therefore v\&_{x = 2.00\ \text{m}} = \sqrt{\frac{2(10.0)}{5.00}} = \boxed{2.00\ \text{m/s}}$$

Similarly,

$$v\&_{x = 4.00\ \text{m}} = \sqrt{\frac{2(19.5)}{5.00}} = \boxed{2.79\ \text{m/s}}$$

$$v\&_{x = 6.00\ \text{m}} = \sqrt{\frac{2(25.5)}{5.00}} = \boxed{3.19\ \text{m/s}}$$

***8.30** The distance traveled by the ball from the top of the arc to the bottom is $s = \pi r$. The work done by the non-conservative force, the force exerted by the pitcher, is $\Delta E = Fs\cos 0° = F(\pi R)$.

We shall choose the gravitational potential energy to be zero at the bottom of the arc. Then

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i \text{ becomes}$$

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2 + mgy_i + F(\pi R)$$

or $$v_f = \sqrt{v_i^2 + 2gy_i + \frac{2F(\pi R)}{m}} = \sqrt{(15.0)^2 + 2(9.80)(1.20) + \frac{2(30.)\pi(0.600)}{0.250}}$$

$$v_f = \boxed{26.5\ \text{m/s}}$$

8.31 $U_i + K_i + \Delta E = U_f + K_f$

$$m_2gh - fh = \frac{1}{2}m_1v^2 + \frac{1}{2}m_2v^2$$

$$f = \mu n = \mu m_1 g$$

$$m_2gh - \mu m_1gh = \frac{1}{2}(m_1 + m_2)\,v^2$$

$$v^2 = \frac{2(m_2 - \mu m_1)(hg)}{m_1 + m_2}$$

$$v = \sqrt{\frac{2(9.80\ \text{m/s}^2)(1.50\ \text{m})[5.00\ \text{kg} - 0.400(3.00\ \text{kg})]}{8.00\ \text{kg}}} = \boxed{3.74\ \text{m/s}}$$

Goal Solution

G: Assuming that the block does not reach the pulley within the 1.50 m distance, a reasonable speed for the ball might be somewhere between 1 and 10 m/s based on common experience.

O: We could solve this problem by using $\Sigma F = ma$ to give a pair of simultaneous equations in the unknown acceleration and tension; then we would have to solve a motion problem to find the final speed. We may find it easier to solve using the work-energy theorem.

A: For objects A (block) and B (ball), the work-energy theorem is

$$(K_A + K_B + U_A + U_B)_i + W_{\text{app}} - f_k d = (K_A + K_B + U_A + U_B)_f$$

Choose the initial point before release and the final point after each block has moved 1.50 m. For the 3.00-kg block, choose $U_g = 0$ at the tabletop. For the 5.00-kg ball, take the zero level of gravitational energy at the final position. So $K_{Ai} = K_{Bi} = U_{Ai} = U_{Af} = U_{Bf} = 0$. Also, since the only external forces are gravity and friction, $W_{app} = 0$.

We now have $0 + 0 + 0 + m_B g y_{Bi} - f_1 d = \frac{1}{2} m_A v_f^2 + \frac{1}{2} m_B v_f^2 + 0 + 0$

where the frictional force is $f_1 = \mu_1 n = \mu_1 m_A g$ and does negative work since the force opposes the motion. Since all of the variables are known except for v_f, we can substitute and solve for the final speed.

$$(5.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m}) - (0.400)(3.00\text{ kg})(9.80\text{ m/s}^2)(1.50\text{ m})$$

$$= \frac{1}{2}(3.00\text{ kg})\, v_f^2 + \frac{1}{2}(5.00\text{ kg})\, v_f^2$$

$$73.5\text{ J} - 17.6\text{ J} = \frac{1}{2}(8.00\text{ kg})\, v_f^2 \quad \text{or} \quad v_f = \sqrt{\frac{2(55.9\text{ J})}{8.00\text{ kg}}} = 3.74\text{ m/s}$$

L: The final speed seems reasonable based on our expectation. This speed must also be less than if the rope were cut and the ball simply fell, in which case its final speed would be

$$v_f' = \sqrt{2gy} = \sqrt{2(9.80\text{ m/s}^2)(1.50\text{ m})} = 5.42\text{ m/s}$$

***8.32** The initial vertical height of the car above the zero reference level at the base of the hill is

$$h = (5.00\text{ m})\sin 20.0° = 1.71\text{ m}$$

The energy lost through friction is

$$\Delta E = -fs = -(4000\text{ N})(5.00\text{ m}) = -2.00 ∞ 10^4\text{ J}$$

We now use,

$$\Delta E = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 + mgy_f - mgy_i$$

$$-2.00\times 10^4\text{ J} = \frac{1}{2}(2000\text{ kg})\ v^2 - 0 + 0 - (2000\text{ kg})g(1.71\text{ m})$$

and $v = \boxed{3.68\text{ m/s}}$

8.33 (a) $\Delta K = \frac{1}{2}m(v^2 - v_i^2) = -\frac{1}{2}mv_i^2 = \boxed{-160\text{ J}}$

(b) $\Delta U = mg(3.00\text{ m})\sin 30.0° = \boxed{73.5\text{ J}}$

(c) The energy lost to friction is 86.5 J

$$f = \frac{86.5\text{ J}}{3.00\text{ m}} = \boxed{28.8\text{ N}}$$

(d) $f = \mu_k n = \mu_K mg\cos 30.0° = 28.8\text{ N}$

$$\mu = \frac{28.8\text{ N}}{(5.00\text{ kg})(9.80\text{ m/s}^2)\cos 30.0°} = \boxed{0.679}$$

***8.34** $\Delta E = (K_f - K_i) + (U_{gf} - U_{gi})$

But $\Delta E = W_{app} + fs\cos 180°$ where W_{app} is the work the boy did pushing forward on the wheels.

Thus, $W_{app} = (K_f - K_i) + (U_{gf} - U_{gi}) + fs$, or

$$W_{app} = \frac{1}{2}m(v_f^2 - v_i^2) + mg(-h) + fs$$

$$W_{app} = \frac{1}{2}(47.0)\ [(6.20)^2 - (1.40)^2] - (47.0)(9.80)(2.60) + (41.0)(12.4)$$

$$W_{app} = \boxed{168\text{ J}}$$

8.35 $\Delta E = mgh_i - \frac{1}{2}\ mv_f^2$

$= (50.0)(9.80)(1000) - \frac{1}{2}(50.0)(5.00)\ ^2$

$\Delta E = \boxed{489 \text{ kJ}}$

8.36 Consider the whole motion: $K_i + U_i + \Delta E = K_f + U_f$

(a) $0 + mgy_i + f_1 d_1 \cos 180° + f_2 d_2 \cos 180° = \frac{1}{2}\ mv_f^2\ + 0$

$(80.0 \text{ kg})(9.80 \text{ m/s}^2)1000 \text{ m} - (50.0 \text{ N})(800 \text{ m}) - (3600 \text{ N})(200 \text{ m}) = \frac{1}{2}(80.0 \text{ kg})\ v_f^2$

$784{,}000 \text{ J} - 40{,}000 \text{ J} - 720{,}000 \text{ J} = \frac{1}{2}(80.0 \text{ kg})\ v_f^2$

$v_f = \sqrt{\frac{2(24{,}000 \text{ J})}{80.0 \text{ kg}}} = \boxed{24.5 \text{ m/s}}$

(b) $\boxed{\text{Yes}}$, this is too fast for safety.

(c) Now in the same work-energy equation d_2 is unknown, and $d_1 = 1000 \text{ m} - d_2$:

$784{,}000 \text{ J} - (50.0 \text{ N})(1000 \text{ m} - d_2) - (3600 \text{ N})d_2 = \frac{1}{2}(80.0 \text{ kg})(5.00 \text{ m/s})\ ^2$

$784{,}000 \text{ J} - 50{,}000 \text{ J} - (3550 \text{ N})d_2 = 1000 \text{ J}$

$d_2 = \frac{733{,}000 \text{ J}}{3550 \text{ N}} = \boxed{206 \text{ m}}$

(d) Really the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

***8.37** (a) $(K + U)_i + \Delta E = (K + U)_f$

$0 + \frac{1}{2}\ kx^2 - fd = \frac{1}{2}\ mv^2 + 0$

$^2 - (3.20 \times 10^{-2} \text{ N})(0.150 \text{ m}) =$ v^2

$v =$ $= \boxed{1.40 \text{ m/s}}$

(b) When the spring force just equals the friction force, the ball will stop speeding up. Here $\left|F_s\right| = kx$, the spring is compressed by

$$= 0.400 \text{ cm}$$

and the ball has moved 5.00 cm – 0.400 cm = $\boxed{\text{4.60 cm from the start.}}$

(c) Between start and maximum speed points,

$$\frac{1}{2} kx_i^2 - fx = \frac{1}{2} mv^2 + \frac{1}{2} kx_f^2$$

$$\frac{1}{2} \; 8.00(5.00 \times 10^{-2})^2 - (3.20 \times 10^{-2})(4.60 \times 10^{-2})$$

$$= \qquad v^2 + \frac{1}{2} \; 8.00(4.00 \times 10^{-3})^2$$

$$v = \boxed{1.79 \text{ m/s}}$$