Chapter 11 Solutions

11.1 (a) $K_{\text{trans}} = \frac{1}{2}mv^2 = \frac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $K_{\text{rot}} = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right)\left(\frac{v^2}{r^2}\right) = \frac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$

(c) $K_{\text{total}} = K_{\text{trans}} + K_{\text{rot}} = \boxed{750\text{ J}}$

11.2 $K = \frac{1}{2}I\omega^2 + \frac{1}{2}mv^2$

$$K = \frac{1}{2}(1.60\times 10^{-2}\text{ kg}\cdot\text{m}^2)\left(\frac{4.00\text{ m/s}}{0.100\text{ m}}\right)^2 + \frac{1}{2}(4.00\text{ kg})(4.00\text{ m/s})^2$$

$= 12.8 + 32.0 = \boxed{44.8\text{ J}}$

11.3 $W = K_f - K_i = (K_{\text{trans}} + K_{\text{rot}})_f - (K_{\text{trans}} + K_{\text{rot}})_i$

$$W = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 - 0 - 0 = \frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{2}{5}MR^2\right)\left(\frac{v}{R}\right)^2$$

or $W = \boxed{(7/10)Mv^2}$

11.4 $K = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2$ where $\omega = \frac{v}{R}$ since no slipping.

Also, $U_i = mgh$, $U_f = 0$, and $v_i = 0$

Therefore, $\frac{1}{2}\left[m + \frac{I}{R^2}\right]v^2 = mgh$

Thus, $v^2 = \frac{2gh}{[1 + (I/mR^2)]}$

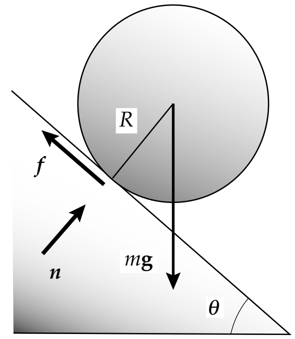
For a disk, $I = \frac{1}{2}mR^2$, so

$v^2 = \frac{2gh}{[1 + (1/2)]}$ or $\boxed{v_{\text{disk}} = \sqrt{\frac{4gh}{3}}}$

For a ring, $I = mR^2$ so $v^2 = \frac{2gh}{2}$ or $\boxed{v_{\text{ring}} = \sqrt{gh}}$

Since $v_{\text{disk}} > v_{\text{ring}}$, $\boxed{\text{the disk}}$ reaches the bottom first.

11.5 (a) $\tau = I\alpha$



$$mg\,R\sin\theta = (I_{CM} + mR^2)\alpha$$

$$a = \frac{mg\,R^2\sin\theta}{I_{CM} + mR^2}$$

$$a_{hoop} = \frac{mg\,R^2\sin\theta}{2mR^2} = \boxed{\frac{1}{2}g\sin\theta}$$

$$a_{disk} = \frac{mg\,R^2\sin\theta}{\frac{3}{2}mR^2} = \boxed{\frac{2}{3}g\sin\theta}$$

The disk moves with $\frac{4}{3}$ the acceleration of the hoop.

(b) $Rf = I\alpha$

$$f = \mu n = \mu mg\cos\theta$$

$$\mu = \frac{f}{mg\cos\theta} = \frac{I\alpha/R}{mg\cos\theta} = \frac{\left(\frac{2}{3}g\sin\theta\right)\left(\frac{1}{2}mR^2\right)}{R^2 mg\cos\theta} = \boxed{\frac{1}{3}\tan\theta}$$

<u>**Goal Solution**</u>

G: The acceleration of the disk will depend on the angle of the incline. In fact, it should be proportional to $g\sin\theta$ since the disk should not accelerate when the incline angle is zero, and since $a = g$ when the angle is 90° and the disk can fall freely. The acceleration of the disk should also be greater than a hoop since the mass of the disk is closer to its center, giving it less rotational inertia so that it can roll faster than the hoop. The required coefficient of friction is difficult to predict, but is probably between 0 and 1 since this is a typical range for μ.

O: We can find the acceleration by applying Newton's second law and considering both the linear and rotational motion. A free-body diagram will greatly assist in defining our variables and seeing how the forces are related.

A: $\Sigma F_x = mg\sin\theta - f = ma_{CM}$ (1)

$\Sigma F_y = n - mg\cos\theta = 0$ (2)

$\tau = fr = I_{CM}\alpha = \frac{I_{CM}a_{CM}}{r}$ (3)

(a) For a disk, $(I_{CM})_{disk} = \frac{1}{2} mr^2$. From (3) we find $f = \frac{\left[\frac{1}{2}mr^2\right]a_{CM}}{r^2} = \frac{1}{2} ma_{CM}$.

Substituting this into (1) gives

$$mg \sin\theta - \frac{1}{2} ma_{CM} \quad \text{so that} \quad (a_{CM})_{disk} = \frac{2}{3} g \sin\theta$$

For a hoop, $(I_{CM})_{hoop} = mr^2$

From (3), $f = \frac{mr^2 a_{CM}}{r^2} = ma_{CM}$

Substituting this into (1) gives

$$mg \sin\theta - ma_{CM} \quad \text{so} \quad (a_{CM})_{hoop} = \frac{1}{2} g \sin\theta$$

Therefore, $\frac{a_{CM\ disk}}{a_{CM\ hoop}} = \frac{\frac{2}{3}g \sin\theta}{\frac{1}{2}g \sin\theta} = \frac{4}{3}$

(b) From (2) we find $n = mg\cos\theta$, and $f = \mu n = \mu mg \cos\theta$

Likewise, from equation (1), $f = mg \sin\theta - ma_{CM}$

Setting these two equations equal,

$$\mu mg\cos\theta = mg\sin\theta - \frac{2}{3} mg\sin\theta$$

so $\mu = \frac{1}{3}\left(\frac{\sin\theta}{\cos\theta}\right) = \frac{1}{3}\tan\theta$

L: As expected, the acceleration of the disk is proportional to $g \sin\theta$ and is slightly greater than the acceleration of the hoop. The coefficient of friction result is similar to the result found for a block on an incline plane, where $\mu = \tan\theta$ (see Example 5.12). However, μ is not always between 0 and 1 as predicted. For angles greater than 72° the coefficient of friction must be larger than 1. For angles greater than 80° it must be extremely large to make the disk roll without slipping.

11.6 $I = \frac{1}{2}\ M(R_1^2\ + R_2^2) = \frac{1}{2}\ M\left(\left(\frac{3R_2}{4}\right)^2 + R_2^2\right) = \frac{25MR_2^2}{32}$

Energy is conserved between $x = 2.00$ m and $x + \Delta x$.

$$\frac{1}{2}\ Mv_i^2\ + \frac{1}{2}\ I\omega_i^2\ = Mgy_f$$

$$\frac{1}{2}\ Mv_i^2\ + \frac{1}{2}\frac{25}{32}\ MR_2^2\,(v_i/R_2)^2 = Mg\ \Delta x \sin\theta$$

$$\frac{57}{64}\ v_i^2\ = g\ \Delta x \sin\theta$$

$$\Delta x = \frac{57(2.80\ \text{m/s})^2}{64(9.80\ \text{m/s}^2)(\sin 36.9°)} = 1.19\ \text{m}$$

So the final position is 2.00 m + 1.19 m = $\boxed{3.19\ \text{m}}$

11.7 $\bar{v}\ = \frac{\Delta x}{\Delta t} = \frac{3.00\ \text{m}}{1.50\ \text{s}} = 2.00\ \text{m/s} = \frac{1}{2}(0 + v_f)$

$v_f = 4.00$ m/s and $\omega_f = \frac{v_f}{r} = \frac{4.00\ \text{m/s}}{(6.38\times 10^{-2}\ \text{m})/2} = \frac{8.00}{6.38\times 10^{-2}}$ rad/s

We ignore internal friction and suppose the can rolls without slipping.

$$(K_{\text{trans}} + K_{\text{rot}} + U_g)_i + \Delta E = (K_{\text{trans}} + K_{\text{rot}} + U_g)_f$$

$$(0 + 0 + mgy_i) + 0 = \left(\frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2 + 0\right)$$

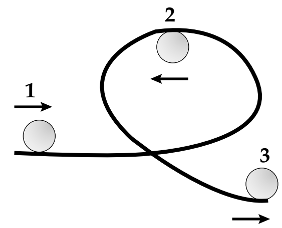
$$(0.215\ \text{kg})(9.80\ \text{m/s}^2)[(3.00\ \text{m})\sin 25.0°] = \frac{1}{2}(0.215\ \text{kg})(4.00\ \text{m/s})^2 + \frac{1}{2}\ I\left(\frac{8.00}{6.38\times 10^{-2}}\text{rad/s}\right)^2$$

$$2.67\ \text{J} = 1.72\ \text{J} + (7860/\text{s}^2)I$$

$$I = \frac{(0.951\ \text{kg}\cdot\text{m}^2/\text{s}^2)}{7860/\text{s}^2} = \boxed{1.21\times 10^{-4}\ \text{kg}\cdot\text{m}^2}$$

The $\boxed{\text{height of the can}}$ is unnecessary data.

***11.8** (a) Energy conservation between the horizontal section and top of loop:



$$\frac{1}{2}mv_2^2 + \frac{1}{2}I\omega_2^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}I\omega_1^2$$

$$\frac{1}{2}mv_2^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_2}{r}\right)^2 + mgy_2 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$\frac{5}{6}v_2^2 + gy_2 = \frac{5}{6}v_1^2$$

$$v_2 = \sqrt{v_1^2 - \frac{6}{5}gy_2} = \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(0.900\text{ m})} = \boxed{2.38\text{ m/s}}$$

The centripetal acceleration is $\frac{v_2^2}{r} = \frac{(2.38\text{ m/s})^2}{0.450\text{ m}} = 12.6\text{ m/s}^2 > g$

Thus, the ball must be in contact with the track, with the track pushing downward on it.

(b) $$\frac{1}{2}mv_3^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_3}{r}\right)^2 + mgy_3 = \frac{1}{2}mv_1^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_1}{r}\right)^2$$

$$v_3 = \sqrt{v_1^2 - \frac{6}{5}gy_3} = \sqrt{(4.03\text{ m/s})^2 - \frac{6}{5}(9.80\text{ m/s}^2)(-0.200\text{ m})} = \boxed{4.31\text{ m/s}}$$

(c) $$\frac{1}{2}mv_2^2 + mgy_2 = \frac{1}{2}mv_1^2$$

$$v_2 = \sqrt{v_1^2 - 2gy_2} = \sqrt{(4.03\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(0.900\text{ m})} = \sqrt{-1.40\text{ m}^2/\text{s}^2}$$

This result is imaginary. In the case where the ball does not roll, the ball starts with less energy than in part (a) and $\boxed{\text{never makes it to the top}}$ of the loop.

***11.9** $$\mathbf{M}\times\mathbf{N} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 2 & -1 \\ 2 & -1 & -3 \end{vmatrix} = \boxed{-7.00\mathbf{i} + 16.0\mathbf{j} - 10.0\mathbf{k}}$$

***11.10** (a) area $= |\mathbf{A}\times\mathbf{B}| = AB\sin\theta = (42.0\text{ cm})(23.0\text{ cm})\sin(65.0° - 15.0°) = \boxed{740\text{ cm}^2}$

(b) $\mathbf{A} + \mathbf{B} = [(42.0\text{ cm})\cos 15.0° + (23.0\text{ cm})\cos 65.0°]\mathbf{i} + [(42.0\text{ cm})\sin 15.0°$

$+ (23.0\text{ cm})\sin 65.0°]\mathbf{j}$

$\mathbf{A} + \mathbf{B} = (50.3\text{ cm})\mathbf{i} + (31.7\text{ cm})\mathbf{j}$

length $= |\mathbf{A} + \mathbf{B}| = \sqrt{(50.3\text{ cm})^2 + (31.7\text{ cm})^2} = \boxed{59.5\text{ cm}}$

11.11 (a) $\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ 2 & 3 & 0 \end{vmatrix} = \boxed{-17.0\mathbf{k}}$

(b) $|\mathbf{A}\times\mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin\theta$

$17 = 5\sqrt{13}\ \sin\theta$

$\theta = \arc\sin\left(\frac{17}{5\sqrt{13}}\right) = \boxed{70.6°}$

***11.12** $\mathbf{A}\cdot\mathbf{B} = (-3.00)(6.00) + (7.00)(-10.0) + (-4.00)(9.00) = -124$

$AB = \sqrt{(-3.00)^2 + (7.00)^2 + (-4.00)^2}\sqrt{(6.00)^2 + (-10.0)^2 + (9.00)^2} = 127$

(a) $\cos^{-1}\left(\frac{\mathbf{A}\cdot\mathbf{B}}{AB}\right) = \cos^{-1}(-0.979) = \boxed{168°}$

(b) $\mathbf{A}\times\mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3.00 & 7.00 & -4.00 \\ 6.00 & -10.0 & 9.00 \end{vmatrix} = 23.0\mathbf{i} + 3.00\mathbf{j} - 12.0\mathbf{k}$

$|\mathbf{A}\times\mathbf{B}| = \sqrt{(23.0)^2 + (3.00)^2 + (-12.0)^2} = 26.1$

$\sin^{-1}\left(\frac{|\mathbf{A}\times\mathbf{B}|}{AB}\right) = \sin^{-1}(0.206) = \boxed{11.9°}$ or 168°

(c) Only $\boxed{\text{the first method}}$ gives the angle between the vectors unambiguously.

11.13 (a) $\tau = \mathbf{r}\times\mathbf{F} = (4.00\mathbf{i} + 5.00\mathbf{j})\times(2.00\mathbf{i} + 3.00\mathbf{j})$

$\tau = |12.00\mathbf{k} - 10.0\mathbf{k}| = |2.00\mathbf{k}| = \boxed{2.00\ \text{N}\cdot\text{m}}$

(b) The torque vector is in the direction of the unit vector **k**, or in the $\boxed{+z\ \text{direction}}$.

11.14 The cross-product vector must be perpendicular to both of the factors, so its dot product with either factor must be zero:

Does $(2\mathbf{i} - 3\mathbf{j} + 4\mathbf{k})\cdot(4\mathbf{i} + 3\mathbf{j} - \mathbf{k}) = 0$?

$8 - 9 - 4 = -5 \neq 0$

$\boxed{\text{No}}$ The cross product could not work out that way.

11.15 (a) in the $\boxed{\text{negative } z \text{ direction}}$ given by the right-hand rule

(b) in the $\boxed{\text{positive } z \text{ direction}}$ given by the right-hand rule

11.16 (a) $\tau = \mathbf{r} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \mathbf{i}(0-0) - \mathbf{j}(0-0) + \mathbf{k}(2-9) = \boxed{(-7.00 \text{ N} \cdot \text{m})\mathbf{k}}$

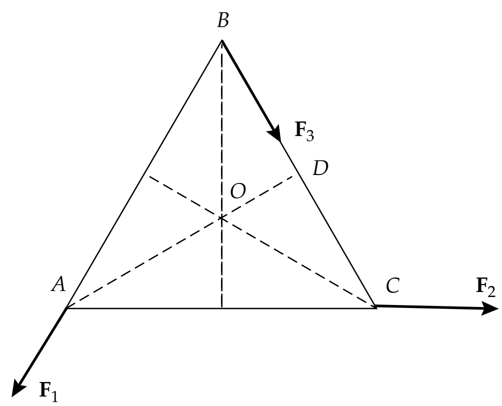
(b) $\tau = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -3 & 0 \\ 3 & 2 & 0 \end{vmatrix} = \boxed{(11.0 \text{ N} \cdot \text{m})\mathbf{k}}$

11.17 $|\mathbf{A} \times \mathbf{B}| = \mathbf{A} \cdot \mathbf{B} \Rightarrow AB \sin\theta = AB\cos\theta \Rightarrow \tan\theta = 1$ or

$\theta = \boxed{45.0°}$

11.18 $\boxed{|\mathbf{F}_3| = |\mathbf{F}_1| + |\mathbf{F}_2|}$

The torque produced by $\mathbf{F}_3$ depends on the perpendicular distance **OD**, therefore translating the point of application of $\mathbf{F}_3$ to any other point along **BC** $\boxed{\text{will not change the net torque}}$.



11.19 $L = \sum m_i v_i r_i$

$= (4.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m}) + (3.00 \text{ kg})(5.00 \text{ m/s})(0.500 \text{ m})$

$L = 17.5 \text{ kg} \cdot \text{m}^2/\text{s}$, and

$\boxed{\mathbf{L} = (17.5 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}}$

***11.20** $\mathbf{L} = \mathbf{r} \times \mathbf{p}$

$\mathbf{L} = (1.50\mathbf{i} + 2.20\mathbf{j})\text{m} \times (1.50 \text{ kg})(4.20\mathbf{i} - 3.60\mathbf{j}) \text{ m/s}$

$\mathbf{L} = (-8.10\mathbf{k} - 13.9\mathbf{k}) \text{ kg} \cdot \text{m}^2/\text{s} = \boxed{(-22.0 \text{ kg} \cdot \text{m}^2/\text{s})\mathbf{k}}$