Chapter 20 Solutions

20.1 Taking $m = 1.00$ kg, we have

$$\Delta U_g = mgh = (1.00\text{ kg})(9.80\text{ m/s}^2)(50.0\text{ m}) = 490\text{ J}$$

But $\Delta U_g = Q = mc\Delta T = (1.00\text{ kg})(4186\text{ J/kg}\cdot{}^\circ\text{C})\Delta T = 490\text{ J}$ so $\Delta T = 0.117\text{ C}^\circ$

$$T_f = T_i + \Delta T = \boxed{(10.0 + 0.117)^\circ\text{C}}$$

Goal Solution

G: Water has a high specific heat, so the difference in water temperature between the top and bottom of the falls is probably less than 1°C. (Besides, if the difference was significantly large, we might have heard about this phenomenon at some point.)

O: The temperature change can be found from the potential energy that is converted to thermal energy. The final temperature is this change added to the initial temperature of the water.

A: The change in potential energy is $\Delta U = mgy$ and the change in internal energy is $\Delta E_{\text{int}} = mc\Delta T$ so $mgy = mc\Delta T$

Therefore, $\Delta T = \dfrac{gy}{c} = \dfrac{(9.80\text{ m/s}^2)(50.0\text{ m})}{4.186\times 10^3\text{ J/kg}\cdot{}^\circ\text{C}} = 0.117^\circ\text{C}$

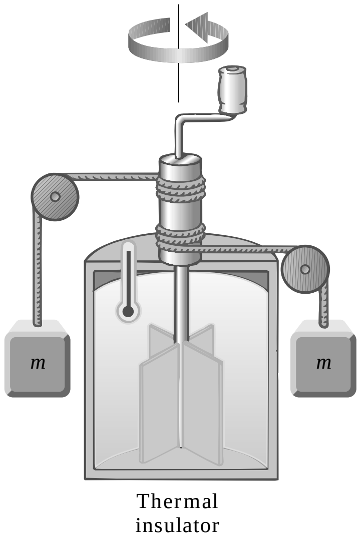
$$T_f = T_1 + \Delta T = 10.0^\circ\text{C} + 0.117^\circ\text{C} = \boxed{10.1^\circ\text{C}}$$

L: The water temperature rose less than 1°C as expected; however, the final temperature might be less than we calculated since this solution does not account for cooling of the water due to evaporation as it falls. It is interesting to note that the change in temperature is independent of the amount of water.

20.2 The container is thermally insulated, so no heat flows: $Q = 0$ and $\Delta E_{\text{int}} = Q - W_{\text{output}} = 0 - W_{\text{output}} = +W_{\text{input}} = 2mgh$. For convenience of calculation, we imagine setting the water on a stove and putting in this same amount of heat. Then we would have $2mgh = \Delta E_{\text{int}} = Q = m_{\text{water}}\, c\, \Delta T$.

$$\Delta T = \frac{2mg\,h}{m_{\text{water}}\, c} = \frac{2\times 1.50\text{ kg}(9.80\text{ m/s}^2)3.00\text{ m}}{(0.200\text{ kg})(4186\text{ J/kg}\cdot\text{C}^\circ)} = \frac{88.2\text{ J}}{837\text{ J/C}^\circ}$$

$$= \boxed{0.105\text{ C}^\circ}$$



20.3 $\Delta Q = mc_{\text{silver}}\,\Delta T$

$1.23 \text{ kJ} = (0.525 \text{ kg})c_{\text{silver}}\,(10.0°\text{C})$

$c_{\text{silver}} = \boxed{0.234 \text{ kJ/kg}\cdot°\text{C}}$

***20.4** From $Q = mc\,\Delta T$, we find

$$\Delta T = \frac{Q}{mc} = \frac{1200 \text{ J}}{(0.0500 \text{ kg})(387 \text{ J/kg}\cdot°\text{C})} = 62.0°\text{C}$$

Thus, the final temperature is $\boxed{87.0°\text{C}}$

20.5 $Q_{\text{cold}} = -Q_{\text{hot}}$

$(mc\Delta T)_{\text{water}} = -(mc\Delta T)_{\text{iron}}$

$(20.0 \text{ kg})(4186 \text{ J/kg}\cdot°\text{C})(T_f - 25.0°\text{C}) = -(1.50 \text{ kg})(448 \text{ J/kg}\cdot°\text{C})(T_f - 600°\text{C})$

$T_f = \boxed{29.6°\text{C}}$

Goal Solution

G: Even though the horseshoe is much hotter than the water, the mass of the water is significantly greater, so we might expect the water temperature to rise less than 10°C.

O: The heat lost by the iron will be gained by the water, and from this heat transfer, the change in water temperature can be found.

A: $\Delta Q_{\text{iron}} = -\Delta Q_{\text{water}}$ or $(mc\Delta T)_{\text{iron}} = -(mc\Delta T)_{\text{water}}$

$(1.50 \text{ kg})(448 \text{ J/kg}\cdot°\text{C})(T - 600°\text{C}) = -(20.0 \text{ kg})(4186 \text{ J/kg}\cdot°\text{C})(T - 25.0°\text{C})$

$T = \boxed{29.6\ °\text{C}}$

L: The temperature only rose about 5°C, so our answer seems reasonable. The specific heat of the water is about 10 times greater than the iron, so this effect also reduces the change in water temperature. In this problem, we assumed that a negligible amount of water boiled away, but in reality, the final temperature of the water would be less than what we calculated since some of the heat energy would be used to vaporize the water.

***20.6** Let us find the energy transferred in one minute.

$$Q = [m_{\text{cup}}c_{\text{cup}} + m_{\text{water}}c_{\text{water}}]\Delta T$$

$$Q = \left[(0.200 \text{ kg})\left(900 \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right) + (0.800 \text{ kg})\left(4186 \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)\right](-1.50\ \text{C}°) = -5290 \text{ J}$$

If this much energy is removed from the system each minute, the rate of removal is

$$\wp = \frac{|Q|}{\Delta t} = \frac{5290\text{ J}}{60.0\text{ s}} = 88.2\ \frac{\text{J}}{\text{s}} = \boxed{88.2\text{ W}}$$

***20.7** (a) $Q_{\text{cold}} = -Q_{\text{hot}}$

$(m_w c_w + m_c c_c)(T_f - T_c) = -m_{\text{Cu}}c_{\text{Cu}}(T_f - T_{\text{Cu}}) - m_{\text{unk}}c_{\text{unk}}(T_f - T_{\text{unk}})$

where *w* is for water, *c* the calorimeter, Cu the copper sample, and *unk* the unknown.

$$\left[(250\text{ g})\left(100\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right) + (100\text{ g})\left(0.215\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)\right](20.0 - 10.0)\ {}^\circ\text{C}$$

$$= -(50.0\text{ g})\left(0.0924\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)(20.0 - 80.0)\ {}^\circ\text{C} - (70.0\text{ g})c_{\text{unk}}(20.0 - 100){}^\circ\text{C}$$

$$2.44\times10^3\text{ cal} = (5.60\times10^3\text{ g}\cdot{}^\circ\text{C})c_{\text{unk}} \quad \text{or} \quad c_{\text{unk}} = \boxed{0.435\text{ cal/g}\cdot{}^\circ\text{C}}$$

(b) The material of the sample is $\boxed{\text{beryllium}}$.

20.8 $m = (4.00\times10^{11}\text{ m}^3)(1000\text{ kg/m}^3)$

(a) $\Delta Q = mc\,\Delta T = Pt = (4.00\times10^{14}\text{ kg})(4186\text{ J/kg}\cdot{}^\circ\text{C})(1.00{}^\circ\text{C})$

$\Delta Q = \boxed{1.68\times10^{18}\text{ J}} = Pt$

(b) $t = \dfrac{1.68\times10^{18}\text{ J}}{1.00\times10^9\text{ J/s}} = 1.68\times10^9\text{ s} = \boxed{53.1\text{ yr}}$

20.9 (a) $(f)(mgh) = mc\,\Delta T$

$$\frac{(0.600)(3.00\times10^{-3}\text{ kg})(9.80\text{ m/s})(50.0\text{ m})}{4.186\text{ J/cal}} = (3.00\text{ g})(0.0924\text{ cal/g}\cdot\text{C}^\circ)(\Delta T)$$

$\Delta T = 0.760{}^\circ\text{C};\ \boxed{T = 25.8{}^\circ\text{C}}$

(b) $\boxed{\text{No}}$ Both the change in potential energy and the heat absorbed are proportional to the mass; hence, the mass cancels in the energy relation.

20.10 $Q_{\text{cold}} = -Q_{\text{hot}}$

$$m_{\text{Al}}c_{\text{Al}}(T_f - T_c) + m_c c_w(T_f - T_c) = -m_h c_w(T_f - T_h)$$

$$(m_{\text{Al}}c_{\text{Al}} + m_c w_w)T_f - (m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c = -m_h c_w T_f + m_h c_w T_h$$

$$(m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w)T_f = (m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c + m_h c_w T_h$$

$$T_f = \boxed{\frac{(m_{\text{Al}}c_{\text{Al}} + m_c c_w)T_c + m_h c_w T_h}{(m_{\text{Al}}c_{\text{Al}} + m_c c_w + m_h c_w)}}$$

20.11 The rate of collection of heat = $\wp = (550 \text{ W/m}^2)(6.00 \text{ m}^2) = 3300 \text{ W}$. The amount of heat required to raise the temperature of 1000 kg of water by 40.0°C is:

$$Q = mc\Delta T = (1000 \text{ kg})(4186 \text{ J/kg}\cdot\text{C}°)(40.0°\text{C}) = 1.67 \times 10^8 \text{ J}$$

Thus, $\wp\Delta t = 1.67 \times 10^8 \text{ J}$

or $$\Delta t = \frac{1.67 \times 10^8 \text{ J}}{3300 \text{ W}} = \boxed{50.7 \text{ ks}} = 14.1 \text{ h}$$

***20.12** The heat needed is the sum of the following terms:

$$Q_{\text{needed}} = (\text{heat to reach melting point}) + (\text{heat to melt})$$
$$+ (\text{heat to reach boiling point})$$
$$+ (\text{heat to vaporize}) + (\text{heat to reach } 110°\text{C})$$

Thus, we have

$$Q_{\text{needed}} = 0.0400 \text{ kg}[(2090 \text{ J/kg}\cdot°\text{C})(10.0°\text{C}) + (3.33 \times 10^5 \text{ J/kg})$$
$$+ (4186 \text{ J/kg}\cdot°\text{C})(100°\text{C}) + (2.26 \times 10^6 \text{ J/kg})$$
$$+ (2010 \text{ J/kg}\cdot°\text{C})(10.0°\text{C})]$$

$$Q_{\text{needed}} = \boxed{1.22 \times 10^5 \text{ J}}$$

20.13 The bullet will not melt all the ice, so its final temperature is 0°C. Then

$$\left(\frac{1}{2}mv^2 + mc\,\Delta T\right)_{\text{bullet}} = m_w L_f$$

where m_w is the meltwater mass

$$m_w = \frac{0.500(3.00 \times 10^{-3} \text{ kg})(240 \text{ m/s})^2 + (3.00 \times 10^{-3} \text{ kg})(128 \text{ J/kg}\cdot\text{C}°)(30.0 \text{ C}°)}{3.33 \times 10^5 \text{ J/kg}}$$

$$m_w = \frac{86.4\text{ J} + 11.5\text{ J}}{333\,000\text{ J/kg}} = \boxed{0.294\text{ g}}$$

Goal Solution

G: The amount of ice that melts is probably small, maybe only a few grams based on the size, speed, and initial temperature of the bullet.

O: We will assume that all of the initial kinetic and excess internal energy of the bullet goes into internal energy to melt the ice, the mass of which can be found from the latent heat of fusion.

A: At thermal equilibrium, the energy lost by the bullet equals the energy gained by the ice:

$$\Delta K_{\text{bullet}} + \Delta Q_{\text{bullet}} = \Delta Q_{\text{ice}}$$

$$\frac{1}{2}\, m_b v^2 + m_b c_{\text{lead}} \Delta T = m_{\text{ice}} L_f$$

$$\frac{1}{2}(3 \times 10^{-3}\ \text{kg})(240\ \text{m/s})^2 + (3 \times 10^{-3}\ \text{kg})(128\ \text{J/kg}\cdot{}^\circ\text{C})(30.0^\circ\text{C}) = m_{\text{ice}}(3.33 \times 10^5\ \text{J/kg})$$

$$m_{\text{ice}} = \frac{86.4\ \text{J} + 11.5\ \text{J}}{3.33 \times 10^5\ \text{J/kg}} = 2.94 \times 10^{-4}\ \text{kg} = 0.294\ \text{g}$$

L: The amount of ice that melted is less than a gram, which agrees with our prediction. It appears that most of the energy used to melt the ice comes from the kinetic energy of the bullet (88%), while the excess internal energy of the bullet only contributes 12% to melt the ice. Small chips of ice probably fly off when the bullet makes impact. So some of the energy is transferred to their kinetic energy, so in reality, the amount of ice that would melt should be less than what we calculated. If the block of ice were colder than 0°C (as is often the case), then the melted ice would refreeze.

***20.14** (a) Q_1 = heat to melt all the ice $= (50.0 \times 10^{-3}\ \text{kg})(3.33 \times 10^5\ \text{J/kg}) = 1.67 \times 10^4\ \text{J}$

Q_2 = (heat to raise temp of ice to 100°C)

$= (50.0 \times 10^{-3}\ \text{kg})(4186\ \text{J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C}) = 2.09 \times 10^4\ \text{J}$

Thus, the total heat to melt ice and raise temp to 100°C $= 3.76 \times 10^4\ \text{J}$

$$Q_3 = \begin{matrix}\text{heat available}\\ \text{as steam condenses}\end{matrix} = (10.0 \times 10^{-3}\ \text{kg})(2.26 \times 10^6\ \text{J/kg}) = 2.26 \times 10^4\ \text{J}$$

Thus, we see that $Q_3 > Q_1$, but $Q_3 < Q_1 + Q_2$.

Therefore, all the ice melts but $T_f < 100°\text{C}$. Let us now find T_f

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$(50.0 \times 10^{-3}\text{ kg})(3.33 \times 10^5\text{ J/kg}) + (50.0 \times 10^{-3}\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(T_f - 0°\text{C})$$

$$= -(10.0 \times 10^{-3}\text{ kg})(-2.26 \times 10^6\text{ J/kg})$$

$$-(10.0 \times 10^{-3}\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(T_f - 100°\text{C})$$

From which, $\boxed{T_f = 40.4°\text{C}}$

(b) Q_1 = heat to melt all ice = 1.67×10^4 J [See part (a)]

$$Q_2 = \begin{matrix}\text{heat given up}\\ \text{as steam condenses}\end{matrix} = (10^{-3}\text{ kg})(2.26 \times 10^6\text{ J/kg}) = 2.26 \times 10^3\text{ J}$$

$$Q_3 = \begin{matrix}\text{heat given up as condensed}\\ \text{steam cools to }0°\text{C}\end{matrix} = (10^{-3}\text{ kg})(4186\text{ J/kg}\cdot°\text{C})(100°\text{C}) = 419\text{ J}$$

Note that $Q_2 + Q_3 < Q_1$. Therefore, the final temperature will be 0°C with some ice remaining. Let us find the mass of ice which must melt to condense the steam and cool the condensate to 0°C.

$$mL_f = Q_2 + Q_3 = 2.68 \times 10^3\text{ J}$$

Thus, $m = \dfrac{2.68 \times 10^3\text{ J}}{3.33 \times 10^5\text{ J/kg}} = 8.04 \times 10^{-3}\text{ kg} = 8.04\text{ g}$

Therefore, there is $\boxed{\text{42.0 g of ice left over}}$

20.15 $\Delta Q = m_{\text{Cu}}\, c_{\text{Cu}}\, \Delta T = m_{N2}\,(L_{\text{vap}})_{N2}$

$$(1.00\text{ kg})\left(0.0920\,\frac{\text{cal}}{\text{g}\cdot\text{C}°}\right)(293 - 77.3)\text{ C}° = m\left(48.0\,\frac{\text{cal}}{\text{g}}\right)$$

$m = \boxed{0.414\text{ kg}}$

***20.16** $Q_{\text{cold}} = -Q_{\text{hot}}$

$$[m_w c_w + m_c c_c](T_f - T_i) = -m_s[-L_v + c_w(T_f - 100)]$$

$$\left[(0.250\text{ kg})\left(4186\,\frac{\text{J}}{\text{kg}\cdot\text{C}°}\right) + (0.0500\text{ kg})\left(387\,\frac{\text{J}}{\text{kg}\cdot\text{C}°}\right)\right](50.0°\text{C} - 20.0°\text{C})$$

$$= -m_s\left[-2.26 \times 10^6\,\frac{\text{J}}{\text{kg}} + \left(4186\,\frac{\text{J}}{\text{kg}\cdot\text{C}°}\right)(50.0°\text{C} - 100°\text{C})\right]$$

$$m_s = \frac{3.20 \times 10^4 \text{ J}}{2.47 \times 10^6 \text{ J/kg}} = 0.0129 \text{ kg} = \boxed{12.9 \text{ g steam}}$$

20.17 (a) Since the heat required to melt 250 g of ice at 0°C *exceeds* the heat required to cool 600 g of water from 18°C to 0°C, the final temperature of the system (water + ice) must be $\boxed{0°\text{C}}$.

(b) Let m represent the mass of ice that melts before the system reaches equilibrium at 0°C.

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$mL_f = -m_w c_w(0°\text{C} - T_i)$$

$$m(3.33 \times 10^5 \text{ J/kg}) = -(0.600 \text{ kg})(4186 \text{ J/kg}\cdot°\text{C})(0°\text{C} - 18.0°\text{C})$$

$m = 136$ g, so the ice remaining = 250 g – 136 g = $\boxed{114 \text{ g}}$

20.18 The original kinetic energy all becomes thermal energy:

$$\frac{1}{2}mv^2 + \frac{1}{2}mv^2 = 2\left(\frac{1}{2}\right)(5.00 \times 10^{-3} \text{ kg})(500 \text{ m/s})^2 = 1.25 \text{ kJ}$$

Raising the temperature to the melting point requires

$$Q = mc\,\Delta T = 10.0 \times 10^{-3} \text{ kg}(128 \text{ J/kg}\cdot\text{C°})(327°\text{C} - 20.0°\text{C}) = 393 \text{ J}$$

Since 1250 J > 393 J, the lead starts to melt. Melting it all requires

$$Q = mL = (10.0 \times 10^{-3} \text{ kg})(2.45 \times 10^4 \text{ J/kg}) = 245 \text{ J}$$

Since 1250 J > 393 + 245 J, it all melts. If we assume $\boxed{\text{liquid lead}}$ has the same specific heat as solid lead, the final temperature is given by

$$1.25 \times 10^3 \text{ J} = 393 \text{ J} + 245 \text{ J} + 10.0 \times 10^{-3} \text{ kg}(128 \text{ J/kg}\cdot\text{C°})(T_f - 327°\text{C})$$

$$\boxed{T_f = 805°\text{C}}$$

20.19 $Q_{\text{cold}} = -Q_{\text{hot}}$

$$m_{\text{Fe}}c_{\text{Fe}}(\Delta T)_{\text{Fe}} = -m_{\text{Pb}}[-L_f + c\Delta T)]_{\text{Pb}}$$

$$(0.300 \text{ kg})\left(448 \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)(T_f - 20.0°\text{C})$$

$$= -0.0900 \text{ kg}\left[-2.45 \times 10^4 \frac{\text{J}}{\text{kg}} + \left(128 \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)(T_f - 327.3°\text{C})\right]$$

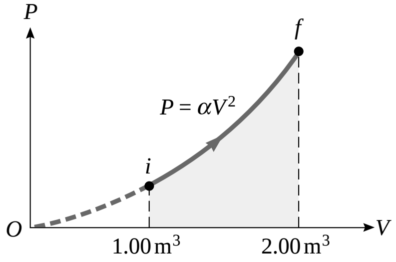
and $T = \boxed{59.4°\text{C}}$

20.20 (a) $W = \int P\,dV = P\,\Delta V = (1.50\text{ atm})(4.00\text{ m}^3) = \boxed{6.08 \times 10^5\text{ J}}$

(b) $W = \int P\,dV = P\,\Delta V = (1.50\text{ atm})(1.00 - 4.00)\text{m}^3 = \boxed{-4.56 \times 10^5\text{ J}}$

20.21 $W_{if} = \int_i^f P\,dV$

The work done by the gas is just the area under the curve $P = \alpha V^2$ between V_i and V_f.



$$W_{if} = \int_i^f \alpha V^2 dV = \frac{1}{3}\alpha\left(V_f^3 - V_i^3\right)$$

$$V_f = 2V_i = 2(1.00\text{ m}^3) = 2.00\text{ m}^3$$

$$W_{if} = \frac{1}{3}\left(5.00\,\frac{\text{atm}}{\text{m}^6} \times 1.013 \times 10^5\,\frac{\text{Pa}}{\text{atm}}\right)\left[(2.00\text{ m}^3)^3 - (1.00\text{ m}^3)^3\right] = \boxed{1.18\text{ MJ}}$$

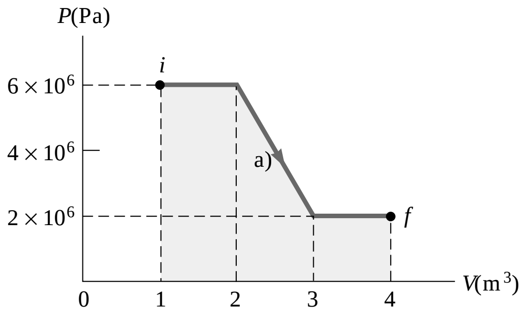
***20.22** (a) $W = \int P\,dV$

$$= (6.00 \times 10^6\text{ Pa})(2.00 - 1.00)\text{m}^3$$

$$+ (4.00 \times 10^6\text{ Pa})(3.00 - 2.00)\text{m}^3$$

$$+ (2.00 \times 10^6\text{ Pa})(4.00 - 3.00)\text{m}^3$$

$$W_{i \to f} = \boxed{+12.0\text{ MJ}}$$



(b) $W_{f \to i} = \boxed{-12.0\text{ MJ}}$

20.23 During the heating process $P = (P_i / V_i)V$.

(a) $W = \int_i^f P\,dV = \int_{V_i}^{3V_i} (P_i / V_i)\,V\,dV$

$$W = (P_i / V_i)\left.\frac{V^2}{2}\right|_{V_i}^{3V_i} = \frac{P_i}{2V_i}\left(9V_i^2 - V_i^2\right) = \boxed{4P_iV_i}$$

(b) $PV = nRT$

$[(P_i / V_i)V]V = nRT$

$\boxed{T = (P_i / nRV_i)V^2}$

Temperature must be proportional to the square of volume, rising to nine times its original value.

20.24 $W = \int_i^f P\,dV = P\int_i^f dV = PV_f - PV_i$

$W = nRT_f - nRT_i$

$$n = \frac{W}{R(\Delta T)} = \frac{20.0\text{ J}}{(8.315\text{ J/mol}\cdot\text{K})(100\text{ K})} = 0.0241\text{ mol}$$

$$m = nM = (0.0241\text{ mol})\left(\frac{4.00\text{ g}}{\text{mol}}\right) = \boxed{0.0962\text{ g}}$$

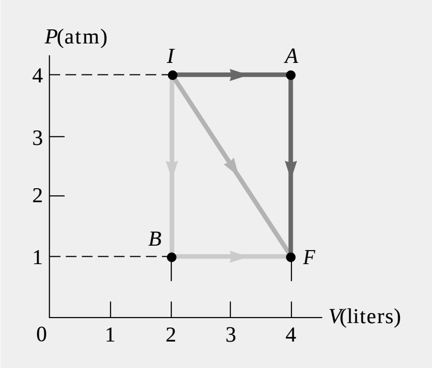
20.25 $W = P\,\Delta V = P\left(\frac{nR}{P}\right)(T_f - T_i) = nR\,\Delta T = 0.200(8.315)(280) = \boxed{466\text{ J}}$

20.26 $W = \int_i^f P\,dV = P\int_i^f dV = P(\Delta V) = nR(\Delta T) = \boxed{nR(T_2 - T_1)}$

20.27 (a) Along IAF, $W = (4.00\text{ atm})(2.00\text{ liter}) = 8.00\text{ L}\cdot\text{atm} = \boxed{810\text{ J}}$

(b) Along IF, $W = 5.00\text{ L}\cdot\text{atm} = \boxed{506\text{ J}}$

(c) Along IBF, $W = 2.00\text{ L}\cdot\text{atm} = \boxed{203\text{ J}}$



20.28 (a) $W = P\,\Delta V = (0.800\text{ atm})(-7.00\text{ L}) = \boxed{-567\text{ J}}$

(b) $\Delta E_{\text{int}} = Q - W = -400\text{ J} + 567\text{ J} = \boxed{167\text{ J}}$

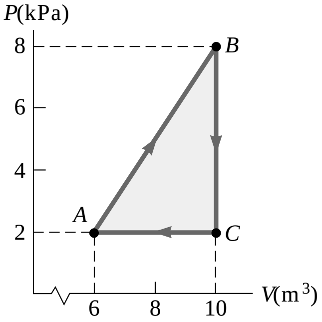
20.29 $\Delta E_{\text{int}} = Q - W$

$\underline{Q} = \Delta E_{\text{int}} + W = -500\text{ J} - 220\text{ J} = \boxed{-720\text{ J}}$

Positive heat is transferred *from* the system.

20.30 (a) $Q = W = \text{Area of triangle} = \dfrac{1}{2}(4.00\text{ m}^3)(6.00\text{ kPa}) = \boxed{12.0\text{ kJ}}$

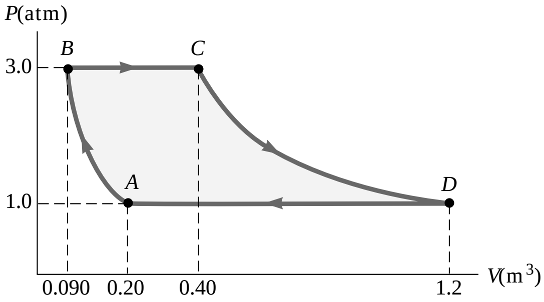
(b) $Q = W = \boxed{-12.0\text{ kJ}}$



***20.31**

	Q	W	ΔE_{int}	
BC	–	0	–	($Q = \Delta E_{\text{int}}$ since $W_{BC} = 0$)
CA	–	–	–	($\Delta E_{\text{int}} < 0$ and $W < 0$, so $Q < 0$)
AB	+	+	+	($W > 0$, $\Delta E_{\text{int}} > 0$ since $\Delta E_{\text{int}} < 0$ for $B \to C \to A$; so $Q > 0$)

20.32 $W_{\text{BC}} = P_{\text{B}}(V_{\text{C}} - V_{\text{B}}) = 3.00\text{ atm}(0.400 - 0.0900)\text{ m}^3$
$= 94.2\text{ kJ}$



$\Delta E_{\text{int}} = Q - W$

$E_{\text{int,C}} - E_{\text{int,B}} = (100 - 94.2)\text{ kJ}$

$E_{\text{int,C}} - E_{\text{int,B}} = 5.79\text{ kJ}$

Since T is constant,

$E_{\text{int,D}} - E_{\text{int,C}} = 0$

$W_{\text{DA}} = P_{\text{D}}(V_A - V_D) = 1.00\text{ atm}(0.200 - 1.20)\text{ m}^3 = -101\text{ kJ}$

$E_{\text{int,A}} - E_{\text{int,D}} = -150\text{ kJ} - (-101\text{ kJ}) = -48.7\text{ kJ}$

Now, $E_{\text{int,B}} - E_{\text{int,A}} = -[(E_{\text{int,C}} - E_{\text{int,B}}) + (E_{\text{int,D}} - E_{\text{int,C}}) + (E_{\text{int,A}} - E_{\text{int,D}})]$

$E_{\text{int,B}} - E_{\text{int,A}} = -[5.79\text{ kJ} + 0 - 48.7\text{ kJ}] = \boxed{+42.9\text{ kJ}}$

20.33 (a) $\Delta E_{\text{int}} = Q - P\,\Delta V = 12.5\text{ kJ} - (2.50\text{ kPa})(3.00 - 1.00)\text{m}^3 = \boxed{7.50\text{ kJ}}$

(b) $\dfrac{V_1}{T_1} = \dfrac{V_2}{T_2}$; $T_2 = \dfrac{V_2}{V_1}\,T_1 = \dfrac{3.00}{1.00}(300\text{ K}) = \boxed{900\text{ K}}$

20.34 (a) $W = nRT\ln\left(\dfrac{V_f}{V_i}\right) = P_f V_f \ln\dfrac{V_f}{V_i}$ so $V_i = V_f \exp\left(-\dfrac{W}{P_f V_f}\right)$

$$V_i = (0.0250)\exp\left[-\frac{3000}{(0.0250)(1.013\times 10^5)}\right] = \boxed{0.00765\text{ m}^3}$$

(b) $$T_f = \frac{P_f V_f}{nR} = \frac{(1.013 \times 10^5\ \mathrm{N/m^2})(0.0250\ \mathrm{m^3})}{(1.00\ \mathrm{mol})(8.315\ \mathrm{J/K\cdot mol})} = \boxed{305\ \mathrm{K}}$$

20.35 $$W = P\,\Delta V = P(V_s - V_w) = \frac{P(nRT)}{P} - P\left[\frac{(18.0\text{ g})}{(1.00\text{ g/cm}^3)(10^6\text{ cm}^3/\text{m}^3)}\right]$$

$$W = (1.00\text{ mol})(8.315\text{ J/K}\cdot\text{mol})(373\text{ K}) - (1.013\times10^5\text{ N/m}^2)\left(\frac{18.0\text{ g}}{10^6\text{ g/m}^3}\right)$$

$$W = \boxed{3.10\text{ kJ}}$$

$$Q = mL_v = (0.0180\text{ kg})(2.26\times10^6\text{ J/kg}) = 40.7\text{ kJ}$$

$$\Delta E_{\text{int}} = Q - W = \boxed{37.6\text{ kJ}}$$

20.36 (a) $W = P\Delta V = P\,[3\alpha V\,\Delta T]$

$$= (1.013\times10^5\text{ N}\cdot\text{m}^2)(3)(24.0\times10^{-6})(\text{C}°)^{-1}\left(\frac{1.00\text{ kg}}{2.70\times10^3\text{ kg/m}^3}\right)(18.0°\text{C})$$

$$W = \boxed{48.6\text{ mJ}}$$

(b) $Q = cm\,\Delta T = (900\text{ J/kg}\cdot°\text{C})(1.00\text{ kg})(18.0°\text{C}) = \boxed{16.2\text{ kJ}}$

(c) $\Delta E_{\text{int}} = Q - W = 16.2\text{ kJ} - 48.6\text{ mJ} = \boxed{16.2\text{ kJ}}$

20.37 (a) $P_i V_i = P_f V_f = nRT = (2.00\text{ mol})(8.315\text{ J/K}\cdot\text{mol})(300\text{ K}) = 4.99\times10^3\text{ J}$

$$V_i = \frac{nRT}{P_i} = \frac{4.99\times10^3\text{ J}}{0.400\text{ atm}}$$

$$V_f = \frac{nRT}{P_f} = \frac{4.99\times10^3\text{ J}}{1.20\text{ atm}} = \frac{1}{3}V_i = \boxed{0.0410\text{ m}^3}$$

(b) $W = \int P\,dV = nRT\ln\left(\frac{V_f}{V_i}\right) = (4.99\times10^3)\ln\left(\frac{1}{3}\right) = \boxed{-5.48\text{ kJ}}$

(c) $\Delta E_{\text{int}} = 0 = Q - W$

$$Q = \boxed{-5.48\text{ kJ}}$$

20.38 The condensing and cooling water loses heat

$$mL_v + mc\,\Delta T = 0.0180\text{ kg}\left[2.26\times10^6\text{ J/kg} + (4186\text{ J/kg}\cdot\text{C°})90.0\text{ C°}\right]$$

$$Q = 4.75\times10^4\text{ J}$$

From the First Law,

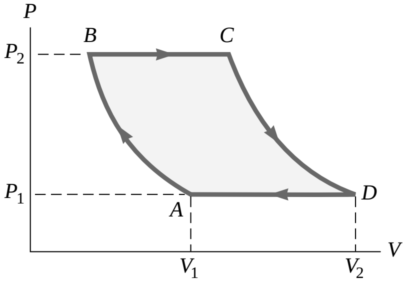
$$Q = \Delta E_{\text{int}} + W = 0 + nRT\ln(V_f/V_i)$$

$$4.75\times10^4\text{ J} = 10.0\text{ mol }(8.315\text{ J/mol}\cdot\text{K})(273\text{ K})\ln(20.0\text{ L}/V_i)$$

$$2.09 = \ln(20.0\text{ L}/V_i)$$

$$V_i = \boxed{2.47\text{ L}}$$

20.39 $W = W_{AB} + W_{BC} + W_{CD} + W_{DA}$



$$W = \int_A^B P\,dV + \int_B^C P\,dV + \int_C^D P\,dV + \int_D^A P\,dV$$

$$W = nRT_1\int_A^B\frac{dV}{V} + P_2\int_B^C dV$$

$$+\,nRT_2\int_C^D\frac{dV}{V} + P_1\int_D^A dV$$

$$W = nRT_1\ln\left(\frac{V_B}{V_1}\right) + P_2(V_C - V_B) + nRT_2\ln\left(\frac{V_2}{V_C}\right) + P_1(V_A - V_D)$$

Now $P_1V_A = P_2V_B$ and $P_2V_C = P_1V_D$, so only the logarithmic terms do not cancel out:

Also, $\dfrac{V_B}{V_1} = \dfrac{P_1}{P_2}$ and $\dfrac{V_2}{V_C} = \dfrac{P_2}{P_1}$

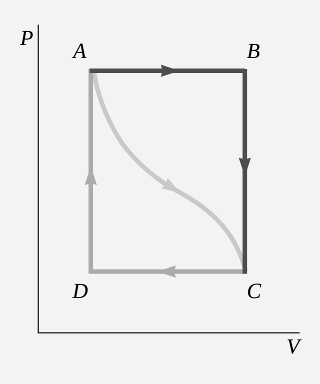
$$\sum W = nRT_1\ln\left(\frac{P_1}{P_2}\right) + nRT_2\ln\left(\frac{P_2}{P_1}\right)$$

$$= -nRT_1\ln\left(\frac{P_2}{P_1}\right) + nRT_2\ln\left(\frac{P_2}{P_1}\right)$$

$$= nR(T_2 - T_1)\ln\left(\frac{P_2}{P_1}\right)$$

Moreover $P_1V_2 = nRT_2$ and $P_1V_1 = nRT_1$

$$\sum W = \boxed{P_1(V_2 - V_1)\ln\left(\frac{P_2}{P_1}\right)}$$



20.40 $\Delta E_{\text{int},ABC} = \Delta E_{\text{int},AC}$ (conservation of energy)

(a) $\Delta E_{\text{int},ABC} = Q_{ABC} - W_{ABC}$ (First Law)

$$Q_{ABC} = 800\text{ J} + 500\text{ J} = \boxed{1300\text{ J}}$$

(b) $W_{CD} = P_C \Delta V_{CD}$, $\Delta V_{AB} = \Delta V_{CD}$, and $P_A = 5P_C$

Then, $W_{CD} = \dfrac{1}{5} P_A \, \Delta V_{AB} = \dfrac{1}{5} W_{AB} = \boxed{-100\text{ J}}$

(– means that work is done on the system)

(c) $W_{CDA} = W_{CD}$ so that $Q_{CA} = \Delta E_{\text{int},CA} + W_{CDA} = -800\text{ J} - 100\text{ J} = \boxed{-900\text{ J}}$

(– means that heat must be removed from the system)

(d) $\Delta E_{\text{int},CD} = \Delta E_{\text{int},CDA} - \Delta E_{\text{int},DA} = -800\text{ J} - 500\text{ J} = -1300\text{ J}$

and $Q_{CD} = \Delta E_{\text{int},CD} + W_{CD} = -1300\text{ J} - 100\text{ J} = \boxed{-1400\text{ J}}$

20.41 $\wp = \dfrac{kA(T_2 - T_1)}{L}$

$$\wp = \frac{(0.200\text{ cal/s}\cdot\text{cm}\cdot\text{C}°)(20.0\text{ cm})(5000\text{ cm})(180\text{ C}°)}{1.50\text{ cm}} = \boxed{2.40 \times 10^{6}\text{ cal/s}}$$

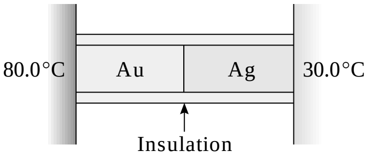
20.42 $\wp = kA\dfrac{\Delta T}{L}$

$$k = \frac{\wp L}{A\,\Delta T} = \frac{(10.0\text{ W})(0.0400\text{ m})}{(1.20\text{ m}^2)(15.0°\text{C})} = \boxed{2.22 \times 10^{-2}\text{ W/m}\cdot\text{C}°}$$

20.43 $\wp = \dfrac{kA\,\Delta T}{L} = \dfrac{(0.800\text{ W/m}\cdot°\text{C})(3.00\text{ m}^2)(25.0°\text{C})}{6.00 \times 10^{-3}\text{ m}} = 1.00 \times 10^{4}\text{ W} = \boxed{10.0\text{ kW}}$

20.44 $\wp = \dfrac{A\,\Delta T}{\sum_i \dfrac{L_i}{k_i}} = \dfrac{(6.00\text{ m}^2)(50.0°\text{C})}{\dfrac{2(4.00 \times 10^{-3}\text{ m})}{0.800\text{ W/m}\cdot°\text{C}} + \dfrac{5.00 \times 10^{-3}\text{ m}}{0.0234\text{ W/m}\cdot°\text{C}}} = \boxed{1.34\text{ kW}}$

20.45 In the steady state condition, $\wp_{\text{Au}} = \wp_{\text{Ag}}$ so that



$$k_{\text{Au}} A_{\text{Au}} \left(\frac{\Delta T}{\Delta x}\right)_{\text{Au}} = k_{\text{Ag}} A_{\text{Ag}} \left(\frac{\Delta T}{\Delta x}\right)_{\text{Ag}}$$

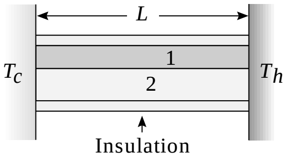
In this case $A_{\text{Au}} = A_{\text{Ag}}$, $\Delta x_{\text{Au}} = \Delta x_{\text{Ag}}$, $\Delta T_{\text{Au}} = (80.0 - T)$

and $\Delta T_{\text{Ag}} = (T - 30.0)$

where T is the temperature of the junction. Therefore,

$$k_{\text{Au}} (80.0 - T) = k_{\text{Ag}} (T - 30.0) \quad \text{and} \quad \boxed{T = 51.2°\text{C}}$$

20.46 Two rods: $\wp = (k_1 A_1 + k_2 A_2) \dfrac{\Delta T}{L}$



$$\wp = \boxed{(k_1 A_1 + k_2 A_2) \frac{(T_h - T_c)}{L}}$$

In general:

$$\wp = (\Sigma k_i A_i) \frac{\Delta T}{L} = \boxed{(\Sigma k_i A_i) \frac{(T_h - T_c)}{L}}$$

20.47 From Table 20.4,

(a) $R = \boxed{0.890 \text{ ft}^2 \cdot °\text{F} \cdot \text{h}/\text{Btu}}$

(b) The insulating glass in the table must have sheets of glass less than 1/8 inch thick. So we estimate the R-value of a 0.250-inch air space as (0.250/3.50) times that of the thicker air space. Then for the double glazing

$$R_b = \left[0.890 + \left(\frac{0.250}{3.50}\right) 1.01 + 0.890\right] \frac{\text{ft}^2 \cdot \text{F}° \cdot \text{h}}{\text{Btu}} = \boxed{1.85 \frac{\text{ft}^2 \cdot °\text{F} \cdot \text{h}}{\text{Btu}}}$$

(c) Since A and $(T_2 - T_1)$ are constants, heat flow is reduced by a factor of

$$\frac{1.85}{0.890} = \boxed{2.08}$$

20.48 $\wp = \sigma A e T^4 = (5.6696 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4) [4\pi (6.96 \times 10^8 \text{ m})^2](0.965)(5800 \text{ K})^4$

$$\wp = \boxed{3.77 \times 10^{26} \text{ W}}$$

***20.49** Suppose the pizza is 70 cm in diameter and $\ell = 2.0$ cm thick, sizzling at 100˚C. It cannot lose heat by conduction or convection. It radiates according to $\wp = \sigma AeT^4$. Here, A is its surface area,

$$A = 2\pi r^2 + 2\pi r\ell = 2\pi(0.35\text{ m})^2 + 2\pi(0.35\text{ m})(0.02\text{ m}) = 0.81\text{ m}^2$$

Suppose it is dark in the infrared, with emissivity about 0.8. Then

$$\wp = (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(0.81\text{ m}^2)(0.80)(373\text{ K})^4 = 710\text{ W}\ \boxed{\sim 10^3\text{ W}}$$

If the density of the pizza is half that of water, its mass is

$$m = \rho V = \rho\pi r^2\ell = (500\text{ kg/m}^3)\pi(0.35\text{ m})^2(0.02\text{ m}) \approx 4\text{ kg}$$

Suppose its specific heat is $c = 0.6$ cal/g · C°. The drop in temperature of the pizza is described by:

$$Q = mc(T_f - T_i)$$

$$\wp = \frac{dQ}{dt} = mc\frac{dT_f}{dt} - 0$$

$$\frac{dT_f}{dt} = \frac{\wp}{mc} = \frac{710\text{ J/s}}{(4\text{ kg})(0.6\cdot 4186\text{ J/kg}\cdot\text{C}°)} = 0.07\text{ C°/s}\ \boxed{\sim 10^{-1}\text{ K/s}}$$

***20.50** $\wp = \sigma AeT^4$

$$2.00\text{ W} = (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(0.250\times10^{-6}\text{ m}^2)(0.950)T^4$$

$$T = [1.49\times10^{14}\text{ K}^4]^{1/4} = \boxed{3.49\times10^3\text{ K}}$$

***20.51** We suppose the earth below is an insulator. The square meter must radiate in the infrared as much energy as it absorbs, $\wp = \sigma AeT^4$. Assuming that $e = 1.00$ for blackbody blacktop:

$$1000\text{ W} = (5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)(1.00\text{ m}^2)(1.00)T^4$$

$$T = (1.76\times10^{10}\text{ K}^4)^{1/4} = \boxed{364\text{ K}}$$ (You can cook an egg on it.)

***20.52** The sphere of radius R absorbs sunlight over the area of its day hemisphere, projected as a flat circle perpendicular to the light: πR^2. It radiates in all directions, over area $4\pi R^2$. Then, in steady state,

$$\wp_{\text{in}} = \wp_{\text{out}}$$

$$e(1340\text{ W/m}^2)\pi R^2 = e\sigma(4\pi R^2)T^4$$

The emissivity e, the radius R, and π all cancel. Therefore,

$$T = \left[\frac{1340\text{ W/m}^2}{4(5.67\times10^{-8}\text{ W/m}^2\cdot\text{K}^4)}\right]^{1/4} = \boxed{277\text{ K}} = 4\text{°C}$$

***20.53** 77.3 K = –195.8°C is the boiling point of nitrogen. It gains no heat to warm as a liquid, but gains heat to vaporize:

$$Q = mL_v = (0.100\text{ kg})(2.01 \times 10^5\text{ J/kg}) = 2.01 \times 10^4\text{ J}$$

The water first loses heat by cooling. Before it starts to freeze, it can lose

$$Q = mc\,\Delta T = (0.200\text{ kg})(4186\text{ J/kg}\cdot\text{C°})(5.00\text{ C°}) = 4.19 \times 10^3\text{ J}$$

The remaining $(2.01 \times 10^4 - 4.19 \times 10^3)\text{J} = 1.59 \times 10^4\text{ J}$ that is removed from the water can freeze a mass x of water:

$$Q = mL_f$$

$$1.59 \times 10^4\text{ J} = x(3.33 \times 10^5\text{ J/kg})$$

$$x = 0.0477\text{ kg} = \boxed{47.7\text{ g}}\text{ of water can be frozen}$$

***20.54** The energy required to melt 1.00 kg of snow is

$$Q = (1.00\text{ kg})(3.33 \times 10^5\text{ J/kg}) = 3.33 \times 10^5\text{ J}$$

The force of friction is

$$f = \mu n = \mu mg = (0.200)(75.0\text{ kg})(9.80\text{ m/s}^2) = 147\text{ N}$$

Therefore, the work done is

$$W = fs = (147\text{ N})s = 3.33 \times 10^5\text{ J}$$

and $s = \boxed{2.27 \times 10^3\text{ m}}$

20.55 (a) Before conduction has time to become important, the heat energy lost by the rod equals the heat energy gained by the helium. Therefore,

$$(mL_v)_{\text{He}} = (mc\Delta T)_{\text{Al}} \quad\text{or}\quad (\rho VL_v)_{\text{He}} = (\rho Vc\,\Delta T)_{\text{Al}} \quad\text{so}\quad V_{\text{He}} = \frac{(\rho Vc\,\Delta T)_{\text{Al}}}{(\rho L_v)_{\text{He}}}$$

$$V_{\text{He}} = \frac{(2.70\text{ g/cm}^3)(62.5\text{ cm}^3)(0.210\text{ cal/g}\cdot\text{°C})(295.8\text{°C})}{(0.125\text{ g/cm}^3)(2.09 \times 10^4\text{ J/kg})(1.00\text{ cal}/4.186\text{ J})(1.00\text{ kg}/1000\text{ g})}$$

$$= 1.68 \times 10^4\text{ cm}^3 = \boxed{16.8\text{ liters}}$$

(b) The rate at which energy is supplied to the rod in order to maintain constant temperatures is given by

$$\wp = kA\frac{dT}{dx} = (31.0\text{ J/s}\cdot\text{cm}\cdot\text{K})(2.50\text{ cm}^2)\left[\frac{295.8\text{ K}}{25.0\text{ cm}}\right] = 917\text{ W}$$

This power supplied to the helium will produce a "boil-off" rate of

$$\frac{\wp}{\rho L_v} = \frac{917\text{ W}}{(0.125\text{ g/cm}^3)(2.09\times10^4\text{ J/kg})(10^{-3}\text{ kg/g})} = 351\,\frac{\text{cm}^3}{\text{s}} = \boxed{0.351\,\frac{\text{L}}{\text{s}}}$$

Goal Solution

G: Demonstrations with liquid nitrogen give us some indication of the phenomenon described. Since the rod is much hotter than the liquid helium and of significant size (almost 2 cm in diameter), a substantial volume (maybe as much as a liter) of helium will boil off before thermal equilibrium is reached. Likewise, since aluminum conducts rather well, a significant amount of helium will continue to boil off as long as the upper end of the rod is maintained at 300 K.

O: Until thermal equilibrium is reached, the excess heat energy of the rod will be used to vaporize the liquid helium, which is already at its boiling point (so there is no change in the temperature of the helium).

A: As you solve this problem, be careful not to confuse L (the *conduction length* of the rod) with L_v (the *heat of vaporization* of the helium).

(a) Before heat conduction has time to become important, we suppose the heat energy lost by half the rod equals the heat energy gained by the helium. Therefore,

$$(mL_v)_{\text{He}} = (mc\Delta T)_{\text{Al}} \quad \text{or} \quad (\rho VL_v)_{\text{He}} = (\rho Vc\Delta T)_{\text{Al}}$$

so that $$v_{\text{He}} = \frac{(\rho Vc\Delta T)_{\text{Al}}}{(\rho L_v)_{\text{He}}} = \frac{(2.7\text{ g/cm}^3)(62.5\text{ cm}^3)(0.21\text{ cal/g °C})(295.8\text{°C})}{(0.125\text{ g/cm}^3)(4.99\text{ cal/g})}$$

and $v_{\text{He}} = 1.68 \times 10^4\text{ cm}^3 = 16.8$ liters

(b) Heat energy will be conducted along the rod at a rate of $\dfrac{dQ}{dt} = P = \dfrac{kA\Delta T}{L}$.

During any time interval, this will boil a mass of helium according to

$$Q = mL_v \quad \text{or} \quad \frac{dQ}{dt} = \left(\frac{dm}{dt}\right) L_v$$

Combining these two equations gives us the "boil-off" rate: $\dfrac{dm}{dt} = \dfrac{kA\Delta T}{L \cdot L_v}$

Set the conduction length $L = 25$ cm, and use $k = 31\text{ J/s cm} \cdot \text{K} = 7.41\text{ cal/s cm} \cdot \text{K}$:

$$\frac{dm}{dt} = \frac{(7.41\text{ cal/s} \cdot \text{cm} \cdot \text{K})(2.5\text{ cm}^2)(295.8\text{ K})}{(25\text{ cm})(4.99\text{ cal/g})} = 43.9\text{ g/s}$$

or $$\frac{dm}{dt} = \frac{43.9\text{ g/s}}{0.125\text{ g/cm}^3} = 351\text{ cm}^3\text{/s} = 0.351\text{ liter/s}$$

L: The volume of helium boiled off initially is much more than expected. If our calculations are correct, that sure is a lot of liquid helium that is wasted! Since liquid helium is much more expensive than liquid nitrogen, most low-temperature equipment is designed to avoid unnecessary loss of liquid helium by surrounding the liquid with a container of liquid nitrogen.

***20.56** (a) The heat thus far gained by the copper equals the heat loss by the silver. Your down parka is an excellent insulator.

$$Q_{\text{cold}} = -Q_{\text{hot}} \quad \text{or} \quad m_{\text{Cu}}c_{\text{Cu}}(T_f - T_i)_{\text{Cu}} = -m_{\text{Ag}}c_{\text{Ag}}(T_f - T_i)_{\text{Ag}}$$

$$(9.00\text{ g})(387\text{ J/kg}\cdot\text{C°})(16.0\text{°C}) = -(14.0\text{ g})(234\text{ J/kg}\cdot\text{C°})(T_f - 30.0\text{°C})_{\text{Ag}}$$

$$(T_f - 30.0\text{°C})_{\text{Ag}} = -17.0\text{°C} \quad \text{so} \quad T_{f,\text{Ag}} = \boxed{13.0\text{°C}}$$

(b) For heat flow: $m_{\text{Ag}}c_{\text{Ag}}\left(\frac{dT}{dt}\right)_{\text{Ag}} = -m_{\text{Cu}}c_{\text{Cu}}\left(\frac{dT}{dt}\right)_{\text{Cu}}$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = -\frac{m_{\text{Cu}}c_{\text{Cu}}}{m_{\text{Ag}}c_{\text{Ag}}}\left(\frac{dT}{dt}\right)_{\text{Cu}} = -\frac{(9.00\text{ g})(387\text{ J/kg}\cdot\text{C°})}{(14.0\text{ g})(234\text{ J/kg}\cdot\text{C°})}(+0.500\text{ C°/s})$$

$$\left(\frac{dT}{dt}\right)_{\text{Ag}} = \boxed{-0.532\text{ C°/s}} \quad \text{(negative sign} \Rightarrow \text{decreasing temperature)}$$

20.57 $Q = cm\,\Delta T$; $m = \rho V$; $\frac{dQ}{dt} = \rho c\,\Delta T\left(\frac{dV}{dt}\right)$

$$c = \frac{dQ/dt}{\rho\,\Delta T(dV/dt)} = \frac{(30.0\text{ J/s})}{(0.780\text{ g/cm}^2)(4.80\text{°C})(4.00\text{ cm}^3\text{/s})} = 2.00\text{ J/g}\cdot\text{C°}$$

$$c = \boxed{2.00\text{ kJ/kg}\cdot\text{C°}}$$

20.58 $Q = mc\,\Delta T = (\rho V)c\,\Delta T$

Thus, when a constant temperature difference ΔT is maintained, the rate of adding heat to the liquid is

$$\wp = \frac{dQ}{dt} = \rho\left(\frac{dV}{dt}\right)c\,\Delta T = \rho Rc\,\Delta T$$

and the specific heat of the liquid is $c = \boxed{\dfrac{\wp}{\rho R\,\Delta T}}$

20.59 Call the initial pressure P_1. In the constant volume process $1 \rightarrow 2$ the work is zero.

$$P_1V_1 = nRT_1$$

$$P_2V_2 = nRT_2$$

so $\frac{P_2V_2}{P_1V_1} = \frac{T_2}{T_1}$ $\qquad T_2 = 300\text{ K}\left(\frac{1}{4}\right)(1) = 75.0\text{ K}$

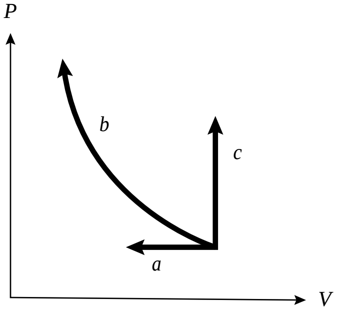
Now in $2 \rightarrow 3$

$$W = \int_2^3 P\,dV = P_2(V_3 - V_2) = P_3V_3 - P_2V_2$$

$$W = nRT_3 - nRT_2 = (1.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(300 \text{ K} - 75.0 \text{ K})$$

$$W = \boxed{1.87 \text{ kJ}}$$

***20.60** (a) Work done by the gas is the area under the PV curve



$$W = P_i\left(\frac{V_i}{2} - V_i\right) = \boxed{-\frac{P_iV_i}{2}}$$

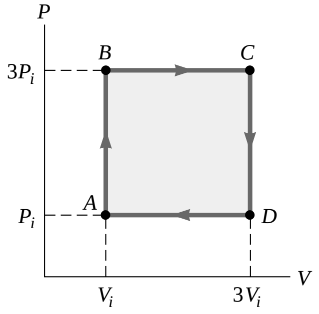
(b) In this case the area under the curve is $W = \int P\,dV$. Since the process is isothermal,

$$PV = P_iV_i = 4P_i\left(\frac{V_i}{4}\right) = nRT_i \quad \text{and}$$

$$W = \int_{V_i}^{V_i/4}\left(\frac{dV}{V}\right)(P_iV_i) = P_iV_i \ln\left(\frac{V_i/4}{V_i}\right) = -P_iV_i \ln 4 = \boxed{-1.39P_iV_i}$$

(c) The area under the curve is 0 and $\boxed{W = 0}$

20.61 (a) The work done during each step of the cycle equals the area under that segment of the PV curve



$$W = W_{DA} + W_{AB} + W_{BC} + W_{CD}$$

$$W = P_i(V_i - 3V_i) + 0 + 3P_i(3V_i - V_i) + 0 = \boxed{4P_iV_i}$$

(b) The initial and final values of T for the system are equal.

Therefore, $\Delta E_{\text{int}} = 0$, and $Q = W = \boxed{4P_iV_i}$

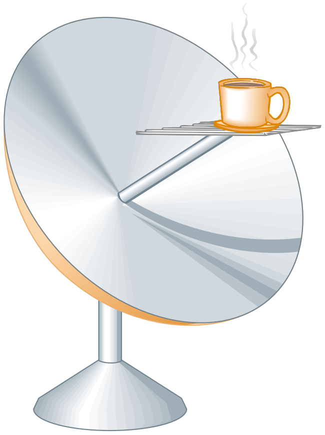
(c) $W = 4P_iV_i = 4nRT_i = 4(1.00)(8.315)(273) = \boxed{9.08 \text{ kJ}}$

20.62 (a) $Fv = (50.0 \text{ N})(40.0 \text{ m/s}) = \boxed{2000 \text{ W}}$

(b) Energy received by each object is $(1000 \text{ W})(10 \text{ s}) = 10^4 \text{ J} = 2389 \text{ cal}$. The specific heat of iron is $0.107 \text{ cal/g} \cdot °\text{C}$, so the heat capacity of each object is $5.00 \times 10^3 \times 0.107 = 535.0 \text{ cal/°C}$.

$$\Delta T = \frac{2389 \text{ cal}}{535.0 \text{ cal/°C}} = \boxed{4.47°\text{C}}$$

20.63 The power incident on the solar collector is



$$\wp_i = IA = (600\ \mathrm{W/m^2})\pi(0.300\ \mathrm{m})^2 = 170\ \mathrm{W}$$

For a 40.0% reflector, the collected power is $\wp_c = 67.9\ \mathrm{W}$

The total energy required to increase the temperature of the water to the boiling point and to evaporate it is

$$Q = cm\,\Delta T + mL_V$$

$$= (0.500\ \mathrm{kg})[(4186\ \mathrm{J/kg\cdot C°})(80.0\ \mathrm{C°}) + 2.26\times10^6\ \mathrm{J/kg}]$$

$$Q = 1.30\times10^6\ \mathrm{J}$$

The time required is $t = \dfrac{Q}{\wp_c} = \dfrac{1.30\times10^6\ \mathrm{J}}{67.9\ \mathrm{W}} = \boxed{5.31\ \mathrm{h}}$

20.64 From $Q = mL_v$ the rate of boiling is described by

$$\wp = \frac{Q}{t} = \frac{L_v m}{t} \qquad \therefore \frac{m}{t} = \frac{\wp}{L_v}$$

Model the water vapor as an ideal gas

$$P_0V = nRT = \left(\frac{m}{M}\right)RT$$

$$\frac{P_0V}{t} = \frac{m}{t}\left(\frac{RT}{M}\right)$$

$$P_0Av = \frac{\wp}{L_v}\left(\frac{RT}{M}\right)$$

$$v = \frac{\wp RT}{ML_vP_0A}$$

$$= \frac{(1000\ \mathrm{W})(8.315\ \mathrm{J/mol\cdot K})(373\ \mathrm{K})}{(0.0180\ \mathrm{kg/mol})(2.26\times10^6\ \mathrm{J/kg})(1.013\times10^5\ \mathrm{N/m^2})(2.00\times10^{-4}\ \mathrm{m^2})}$$

$$v = \boxed{3.76\ \mathrm{m/s}}$$

***20.65** To vaporize water requires an addition of 2.26×10^6 J/kg of energy, while water gives up 3.33×10^5 J/kg as it freezes. The heat to vaporize part of the water must come from the heat of fusion as some water freezes. Thus, if x kilograms of water freezes while a mass of $(1.00 \text{ kg} - x)$ is vaporized,

$$(3.33 \times 10^5 \text{ J/kg})x = (2.26 \times 10^6 \text{ J/kg})(1.00 \text{ kg} - x)$$

or $x = 6.79 \text{ kg} - 6.79x$

This yields, $7.79x = 6.79$ kg, and

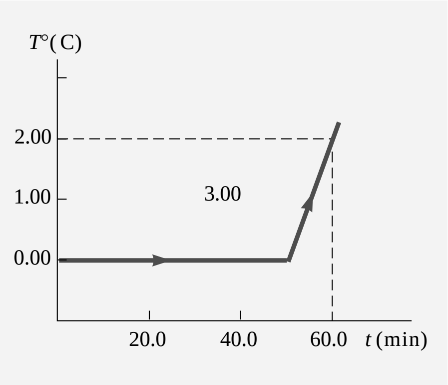
$$x = 0.872 \text{ kg} = \boxed{872 \text{ g}} \text{ of water freezes.}$$

20.66 Energy goes in at a constant rate $\wp$. For the period from

50.0 min to 60.0 min, $Q = mc\,\Delta T$

$\wp(10.0 \text{ min}) = (10 \text{ kg} + m_i)(4186 \text{ J/kg} \cdot \text{C°})(2.00 \text{ °C} - 0\text{°C})$

$\wp(10.0 \text{ min}) = 83.7 \text{ kJ} + (8.37 \text{ kJ/kg})m_i$ (1)



For the period from 0 to 50.0 min, $Q = m_i L_f$

$\wp(50.0 \text{ min}) = m_i(3.33 \times 10^5 \text{ J/kg})$

Substitute $\wp = m_i(3.33 \times 10^5 \text{ J/kg})/50.0$ min into Equation (1) to find

$$m_i(3.33 \times 10^5 \text{ J/kg})/5.00 = 83.7 \text{ kJ} + (8.37 \text{ kJ/kg})m_i$$

$$m_i = \frac{83.7 \text{ kJ}}{(66.6 - 8.37) \text{ kJ/kg}} = \boxed{1.44 \text{ kg}}$$

***20.67** (a) The block starts with $K_i = \frac{1}{2} mv_i^2 = \frac{1}{2}(1.60 \text{ kg})(2.50 \text{ m/s})^2 = 5.00$ J. All this becomes extra internal energy in the ice, melting some according to "Q" $= m_{\text{ice}} L_f$. Thus, the mass of ice that melts is

$$m_{\text{ice}} = \frac{\text{"}Q\text{"}}{L_f} = \frac{K_i}{L_f} = \frac{5.00 \text{ J}}{3.33 \times 10^5 \text{ J/kg}} = 1.50 \times 10^{-5} \text{ kg} = \boxed{15.0 \text{ mg}}$$

For the block: $Q = 0$ (no heat can flow since there is no temperature difference), $W = +5.00$ J, $\Delta E_{\text{int}} = 0$ (no temperature change), and $\Delta K = -5.00$ J. For the ice, $Q = 0$, $W = -5.00$ J, $\Delta E_{\text{int}} = +5.00$ J, and $\Delta K = 0$.

(b) Again, $K_i = 5.00$ J and $m_{\text{ice}} = \boxed{15.0 \text{ mg}}$.
For the block of ice: $Q = 0$, $\Delta E_{\text{int}} = +5.00$ J, $\Delta K = -5.00$ J, so $W = 0$.
For the copper, nothing happens: $Q = \Delta E_{\text{int}} = \Delta K = W = 0$.

(c) Again, $K_i = 5.00$ J. Both blocks must rise equally in temperature:

$$"Q" = mc\,\Delta T = \frac{"Q"}{mc} = \frac{5.00\text{ J}}{2(1.60\text{ kg})(387\text{ J/kg}\cdot\text{C°})} = \boxed{4.04\times10^{-3}\text{ C°}}$$

At any instant, the two blocks are at the same temperature, so for both $Q = 0$. For the moving block: $\Delta K = -5.00$ J and $\Delta E_{\text{int}} = +2.50$ J, so $W = +2.50$ J. For the stationary block: $\Delta K = 0$, $\Delta E_{\text{int}} = +2.50$ J, so $W = -2.50$ J.

20.68 $A = A_{\text{end walls}} + A_{\text{ends of attic}} + A_{\text{side walls}} + A_{\text{roof}}$

$$A = 2(8.00\text{ m}\times5.00\text{ m}) + 2\left[2\times\frac{1}{2}\times(4.00\text{ m})\times(4.00\text{ m}\tan 37.0°)\right]$$

$$+\,2(10.0\text{ m}\times5.00\text{ m}) + 2\left(10.0\text{ m}\times\frac{4.00\text{ m}}{\cos 37.0°}\right)$$

$A = 304\text{ m}^2$

$$\wp = \frac{kA\,\Delta T}{L} = \frac{(4.80\times10^{-4}\text{ kW/m}\cdot\text{°C})(304\text{ m}^2)(25.0\text{°C})}{0.210\text{ m}} = 17.4\text{ kW} = 4.15\frac{\text{kcal}}{\text{s}}$$

Thus, the heat lost per day = (4.15 kcal/s)(86 400 s) = 3.59×10^5 kcal/day.

The gas needed to replace this loss $= \dfrac{3.59\times10^5\text{ kcal/day}}{9300\text{ kcal/m}^3} = \boxed{38.6\text{ m}^3/\text{day}}$

20.69 $\dfrac{L\rho A\,dx}{dt} = kA\dfrac{\Delta T}{x}$ $\qquad L\rho\displaystyle\int_{4.00}^{8.00} x\,dx = k\,\Delta T\int_0^t dt$

$$L\rho\left.\frac{x^2}{2}\right|_{4.00}^{8.00} = k\,\Delta T\,t$$

$$(3.33\times10^5\text{ J/kg})(917\text{ kg/m}^3)\left(\frac{(0.0800\text{ m})^2-(0.0400\text{ m})^2}{2}\right) = \left(2.00\frac{\text{W}}{\text{m}\cdot\text{°C}}\right)(10.0\text{ C°})\,t$$

$t = 3.66\times10^4\text{ s} = \boxed{10.2\text{ h}}$

20.70 For a cylindrical shell of radius r, height L, and

thickness dr, Equation 20.14, $\dfrac{dQ}{dt} = -kA\dfrac{dT}{dx}$,

becomes $\dfrac{dQ}{dt} = -k(2\pi rL)\dfrac{dT}{dr}$

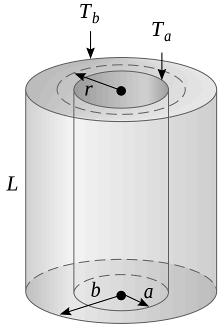
Under equilibrium conditions, $\frac{dQ}{dt}$

is constant; therefore, $dT = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\frac{dr}{r}$

and $T_b - T_a = -\frac{dQ}{dt}\left(\frac{1}{2\pi kL}\right)\ln\left(\frac{b}{a}\right)$ but $T_a > T_b$.

Therefore, $\boxed{\frac{dQ}{dt} = \frac{2\pi kL(T_a - T_b)}{\ln(b/a)}}$

20.71 From problem 70, the rate of heat flow through the wall is



$$\frac{dQ}{dt} = \frac{2\pi kL(T_a - T_b)}{\ln(b/a)}$$

$$\frac{dQ}{dt} = \frac{2\pi(4.00\times10^{-5}\ \text{cal/s}\cdot\text{cm}\cdot\text{C}^\circ)(3500\ \text{cm})(60.0\ \text{C}^\circ)}{\ln(256\ \text{cm}/250\ \text{cm})}$$

$$\frac{dQ}{dt} = 2.23\times10^3\ \text{cal/s} = \boxed{9.32\ \text{kW}}$$

This is the rate of heat loss from the plane, and consequently the rate at which energy must be supplied in order to maintain a constant temperature.

20.72 $Q_{\text{cold}} = -Q_{\text{hot}}$ or $Q_{\text{Al}} = -(Q_{\text{water}} + Q_{\text{calo}})$

$m_{\text{Al}}c_{\text{Al}}(T_f - T_i)_{\text{Al}} = -(m_w c_w + m_c c_c)(T_f - T_i)_w$

$(0.200\ \text{kg})c_{\text{Al}}(+39.3\ \text{C}^\circ)$

$$= -\left[(0.400\ \text{kg})\left(4186\ \frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}\right) + (0.0400\ \text{kg})\left(630\ \frac{\text{J}}{\text{kg}\cdot\text{C}^\circ}\right)\right](-3.70\ \text{C}^\circ)$$

$$c_{\text{Al}} = \frac{6.29\times10^3\ \text{J}}{7.86\ \text{kg}\cdot\text{C}^\circ} = \boxed{800\ \text{J/kg}\cdot\text{C}^\circ}$$