Chapter 21 Solutions

***21.1** One mole of helium contains Avogadro's number of molecules and has a mass of 4.00 g. Let us call *m* the mass of one atom, and we have

$$N_{\mathrm{A}}m = 4.00\ \mathrm{g/mol}$$

or $$m = \frac{4.00\ \mathrm{g/mol}}{6.02\times10^{23}\ \mathrm{molecules/mol}} = 6.64\times10^{-24}\ \mathrm{g/molecule}$$

$$m = \boxed{6.64\times10^{-27}\ \mathrm{kg}}$$

***21.2** We first find the pressure exerted by the gas on the wall of the container.

$$P = \frac{NkT}{V} = \frac{3N_{\mathrm{A}}k_{\mathrm{B}}T}{V} = \frac{3RT}{V} = \frac{3(8.315\ \mathrm{N\cdot m/mol\cdot K})(293\ \mathrm{K})}{8.00\times10^{-3}\ \mathrm{m}^3} = 9.13\times10^5\ \mathrm{Pa}$$

Thus, the force on one of the walls of the cubical container is

$$F = PA = (9.13\times10^5\ \mathrm{Pa})(4.00\times10^{-2}\ \mathrm{m}^2) = \boxed{3.65\times10^4\ \mathrm{N}}$$

21.3 $$\overline{F} = Nm\frac{\Delta v}{\Delta t} = 500(5.00\times10^{-3}\ \mathrm{kg})\frac{[8.00\sin45.0^\circ - (-8.00\sin45.0^\circ)]\mathrm{m/s}}{30.0\ \mathrm{s}} = \boxed{0.943\ \mathrm{N}}$$

$$P = \frac{\overline{F}}{A} = 1.57\ \mathrm{N/m^2} = \boxed{1.57\ \mathrm{Pa}}$$

21.4 Consider the *x* axis to be perpendicular to the plane of the window. Then, the average force exerted on the window by the hail stones is

$$\overline{F} = Nm\frac{\Delta v}{\Delta t} = Nm\frac{[v_{xf} - v_{xi}]}{t} = Nm\frac{[v\sin\theta - (-v\sin\theta)]}{t} = \boxed{Nm\frac{2v\sin\theta}{t}}$$

Thus, the pressure on the window pane is

$$P = \frac{\overline{F}}{A} = \boxed{Nm\frac{2v\sin\theta}{At}}$$

***21.5** $$\overline{F} = \frac{(5.00\times10^{23})(2\times4.68\times10^{-26}\ \mathrm{kg}\times300\ \mathrm{m/s})}{1.00\ \mathrm{s}} = 14.0\ \mathrm{N}$$

and $$P = \frac{\overline{F}}{A} = \frac{14.0\ \mathrm{N}}{8.00\times10^{-4}\ \mathrm{m}^2} = \boxed{17.6\ \mathrm{kPa}}$$

21.6 Use Equation 21.2, $P = \frac{2N}{3V}\left(\frac{mv^2}{2}\right)$, so that

$$K_{\text{av}} = \frac{mv^2}{2} = \frac{3PV}{2N} \quad \text{where } N = nN_{\text{A}} = 2N_{\text{A}}$$

$$K_{\text{av}} = \frac{3PV}{2(2N_{\text{A}})} = \frac{3(8.00 \text{ atm})(1.013 \times 10^5 \text{ Pa/atm})(5.00 \times 10^{-3} \text{ m}^3)}{2(2 \text{ mol})(6.02 \times 10^{23} \text{ molecules/mol})}$$

$$K_{\text{av}} = \boxed{5.05 \times 10^{-21} \text{ J/molecule}}$$

21.7 $P = \frac{2}{3}\frac{N}{V}(\overline{KE})$ Equation 21.2

$$N = \frac{3}{2}\frac{PV}{(\overline{KE})} = \frac{3}{2}\frac{(1.20 \times 10^5)(4.00 \times 10^{-3})}{(3.60 \times 10^{-22})} = 2.00 \times 10^{24} \text{ molecules}$$

$$n = \frac{N}{N_A} = \frac{2.00 \times 10^{24} \text{ molecules}}{6.02 \times 10^{23} \text{ molecules/mol}} = \boxed{3.32 \text{ mol}}$$

Goal Solution

G: The balloon has a volume of 4.00 L and a diameter of about 20 cm, which seems like a reasonable size for a typical helium balloon. The pressure of the balloon is only slightly more than 1 atm, and if the temperature is anywhere close to room temperature, we can use the estimate of 22 L/mol for an ideal gas at STP conditions. If this is valid, the balloon should contain about 0.2 moles of helium.

O: The average kinetic energy can be used to find the temperature of the gas, which can be used with $PV=nRT$ to find the number of moles.

A: The gas temperature must be that implied by $\frac{1}{2}mv^2 = \frac{3}{2}k_BT$ for a monatomic gas like He.

$$T = \frac{2}{3}\left(\frac{\frac{1}{2}mv^2}{k_a}\right) = \frac{2}{3}\left(\frac{3.6 \times 10^{-22} \text{ J}}{1.38 \times 10^{-23} \text{ J/K}}\right) = 17.4 \text{ K}$$

Now $PV = nRT$ gives

$$n = \frac{PV}{RT} = \frac{(1.20 \times 10^5 \text{ N/m}^2)(4.00 \times 10^{-3} \text{ m}^3)}{(8.315 \text{ J/mol} \cdot \text{K})(17.4 \text{ K})}$$

$$n = 3.32 \text{ mol}$$

L: This result is more than ten times the number of moles we predicted, primarily because the temperature of the helium is *much colder* than room temperature! In fact, T is only slightly above the temperature at which the helium would liquify (4.2 K at 1 atm). We should hope this balloon is not being held by a child; not only would the balloon sink in the air, it is cold enough to cause frostbite!

21.8 $v = \sqrt{\dfrac{3k_BT}{m}}$

$$\frac{v_{\text{O}}}{v_{\text{He}}} = \sqrt{\frac{M_{\text{He}}}{M_O}} = \sqrt{\frac{4.00}{32.0}} = \sqrt{\frac{1}{8.00}}$$

$$v_O = \frac{1350\text{ m/s}}{\sqrt{8.00}} = \boxed{477\text{ m/s}}$$

21.9 (a) $PV = Nk_BT$

$$N = \frac{PV}{k_BT} = \frac{(1.013\times 10^5\text{ Pa})\frac{4}{3}\pi(0.150\text{ m})^3}{(1.38\times 10^{-23}\text{ J/K})(293\text{ K})} = \boxed{3.53\times 10^{23}\text{ atoms}}$$

(b) $\overline{K} = \dfrac{3}{2}k_BT = \dfrac{3}{2}(1.38\times 10^{-23})(293)\text{ J} = \boxed{6.07\times 10^{-21}\text{ J}}$

(c) $\dfrac{1}{2}m\overline{v^2} = \dfrac{3}{2}k_BT \quad \therefore v_{\text{rms}} = \sqrt{\dfrac{3k_BT}{m}} = \boxed{1.35\text{ km/s}}$

21.10 (a) $PV = nRT = \dfrac{Nmv^2}{3} \qquad K = \dfrac{Nmv^2}{2} = E_{\text{trans}}$

$$E_{\text{trans}} = \frac{3PV}{2} = \frac{3}{2}(3.00\times 1.013\times 10^5)(5.00\times 10^{-3}) = \boxed{2.28\text{ kJ}}$$

(b) $\dfrac{mv^2}{2} = \dfrac{3k_BT}{2} = \dfrac{3RT}{2N_A} = \dfrac{3}{2}\dfrac{(8.315)(300)}{(6.02\times 10^{23})} = \boxed{6.22\times 10^{-21}\text{ J}}$

21.11 (a) $\overline{K} = \dfrac{3}{2}k_{\text{B}}T = \dfrac{3}{2}(1.38\times 10^{-23}\text{ J/K})(423\text{ K}) = \boxed{8.76\times 10^{-21}\text{ J}}$

(b) $\overline{K} = \dfrac{1}{2}mv_{\text{rms}}^2 = 8.76\times 10^{-21}\text{ J}$

so $v_{\text{rms}} = \sqrt{\dfrac{1.75\times 10^{-20}\text{ J}}{m}}$ (1)

For helium,

$$m = \frac{4.00\text{ g/mol}}{6.02\times 10^{23}\text{ molecules/mol}} = 6.64\times 10^{-24}\text{ g/molecule}$$

$$m = 6.64\times 10^{-27}\text{ kg/molecule}$$

Similarly for argon,

$$m = \frac{39.9 \text{ g/mol}}{6.02 \times 10^{23} \text{ molecules/mol}} = 6.63 \times 10^{-23} \text{ g/molecule}$$

$$m = 6.63 \times 10^{-26} \text{ kg/molecule}$$

Substituting in (1) above, we find

for helium, $\boxed{v_{\text{rms}} = 1.62 \text{ km/s}}$; and for argon, $\boxed{v_{\text{rms}} = 514 \text{ m/s}}$

***21.12** (a) $1 \text{ Pa} = (1 \text{ Pa})\left(\frac{1 \text{ N/m}^2}{1 \text{ Pa}}\right)\left(\frac{1 \text{ J}}{1 \text{ N} \cdot \text{m}}\right) = \boxed{1 \frac{\text{J}}{\text{m}^3}}$

(b) For a monatomic ideal gas, $E_{\text{int}} = \frac{3}{2} nRT$

For any ideal gas, the energy of molecular translation is the same,

$$E_{\text{trans}} = \frac{3}{2} nRT = \frac{3}{2} PV$$

Thus, the energy per volume is $\frac{E_{\text{trans}}}{V} = \boxed{\frac{3}{2} P}$

21.13 $E_{\text{int}} = \frac{3}{2} nRT$

$$\Delta E_{\text{int}} = \frac{3}{2} nR\,\Delta T = \frac{3}{2}(3.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(2.00 \text{ K}) = \boxed{75.0 \text{ J}}$$

21.14 The piston moves to keep pressure constant. Since $V = \frac{nRT}{P}$, then

$\Delta V = \frac{nR\,\Delta T}{P}$ for a constant pressure process.

$$Q = nC_P\,\Delta T = n(C_V + R)\Delta T \quad \text{so} \quad \Delta T = \frac{Q}{n(C_V + R)} = \frac{Q}{n(5R/2 + R)} = \frac{2Q}{7nR}$$

and $\Delta V = \frac{nR}{P}\left(\frac{2Q}{7nR}\right) = \frac{2Q}{7P} = \frac{2}{7}\frac{QV}{nRT}$

$$\Delta V = \frac{2}{7}\frac{(4.40 \times 10^3 \text{ J})(5.00 \text{ L})}{(1.00 \text{ mol})(8.315 \text{ J/mol} \cdot \text{K})(300 \text{ K})} = 2.52 \text{ L}$$

Thus, $V_f = V_i + \Delta V = 5.00 \text{ L} + 2.52 \text{ L} = \boxed{7.52 \text{ L}}$

21.15 Use C_P and C_V from Table 21.2.

(a) $Q = nC_P\,\Delta T = (1.00\text{ mol})(28.8\text{ J/mol}\cdot\text{K})(420 - 300)\text{ K} = \boxed{3.46\text{ kJ}}$

(b) $\Delta E_{\text{int}} = nC_V\,\Delta T = (1.00\text{ mol})(20.4\text{ J/mol}\cdot\text{K})(120\text{ K}) = \boxed{2.45\text{ kJ}}$

(c) $W = Q - \Delta E_{\text{int}} = 3.46\text{ kJ} - 2.45\text{ kJ} = \boxed{1.01\text{ kJ}}$

21.16 $n = 1.00$ mol, $T_i = 300$ K

(b) Since V = constant, $W = \boxed{0}$

(a) $\Delta E_{\text{int}} = Q - \text{W} = 209\text{ J} - 0 = \boxed{209\text{ J}}$

(c) $\Delta E_{\text{int}} = nC_V\,\Delta T = n\left(\frac{3}{2}R\right)\Delta T$

so $$\Delta T = \frac{2(\Delta E_{\text{int}})}{3nR} = \frac{2(209\text{ J})}{3(1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = 16.8\text{ K}$$

$$T = T_i + \Delta T = 300\text{ K} + 16.8\text{ K} = \boxed{317\text{ K}}$$

21.17 (a) Consider heating it at constant pressure. Oxygen and nitrogen are diatomic, so $C_P = 7R/2$

$$Q = nC_P\,\Delta T = \frac{7}{2}\,nR\,\Delta T = \frac{7}{2}\left(\frac{PV}{T}\right)\Delta T$$

$$Q = \frac{7}{2}\frac{(1.013\times10^5\text{ N/m}^2)(100\text{ m}^3)}{300\text{ K}}(1.00\text{ K}) = \boxed{118\text{ kJ}}$$

(b) $U_g = mgy$

$$m = \frac{U_g}{gy} = \frac{1.18\times10^5\text{ J}}{(9.80\text{ m/s}^2)2.00\text{ m}} = \boxed{6.03\times10^3\text{ kg}}$$

***21.18** (a) $C_V = \frac{5}{2}R = \frac{5}{2}\left(8.315\,\frac{\text{J}}{\text{mol}\cdot\text{K}}\right)\left(\frac{1.00\text{ mol}}{0.0289\text{ kg}}\right) = 719\,\frac{\text{J}}{\text{kg}\cdot\text{K}} = \boxed{0.719\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}}$

(b) $m = nM = M\left(\frac{PV}{RT}\right)$

$$m = \left(0.0289\,\frac{\text{kg}}{\text{mol}}\right)\frac{(200\times10^3\text{ Pa})(0.350\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})} = \boxed{0.811\text{ kg}}$$

(c) We consider a constant volume process where no work is done.

$$Q = mC_V\,\Delta T = (0.811\text{ kg})\left(0.719\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)(700\text{ K} - 300\text{ K}) = \boxed{233\text{ kJ}}$$

(d) We now consider a constant pressure process where the internal energy of the gas is increased and work is done.

$$Q = mC_P\,\Delta T = m(C_V + R)\Delta T = m(7R/2)\Delta T = m(7C_V/5)\Delta T$$

$$Q = (0.811\text{ kg})\left[\frac{7}{5}\left(0.719\,\frac{\text{kJ}}{\text{kg}\cdot\text{K}}\right)\right](400\text{ K}) = \boxed{327\text{ kJ}}$$

***21.19** Consider 800 cm^3 of (flavored) water at 90.0˚C mixing with 200 cm^3 of diatomic ideal gas at 20.0˚C:

$$Q_{\text{cold}} = -Q_{\text{hot}} \quad\text{or}\quad m_{\text{air}}C_{P,\text{air}}(T_f - T_{i,\text{air}}) = -m_w c_w(\Delta T)_w$$

$$(\Delta T)_w = -\frac{m_{\text{air}}C_{P,\text{air}}(T_f - T_{i,\text{air}})}{m_w c_w} = \frac{[\rho V]_{\text{air}}\,C_{P,\text{air}}(90.0°\text{C} - 20.0°\text{C})}{(\rho_w V_w)c_w}$$

where we have anticipated that the final temperature of the mixture will be close to 90.0˚C.

$$C_{P,\text{air}} = \frac{7}{2}R = \frac{7}{2}\left(8.315\,\frac{\text{J}}{\text{mol}\cdot\text{C}°}\right)\left(\frac{1.00\text{ mol}}{28.9\text{ g}}\right) = 1.01\text{ J/g}\cdot\text{C}°$$

$$(\Delta T)_w = -\frac{[(1.20\times10^{-3}\text{ g/cm}^3)(200\text{ cm}^3)](1.01\text{ J/g}\cdot\text{C}°)(70.0\text{ C}°)}{[(1.00\text{ g/cm}^3)(800\text{ cm}^3)](4.186\text{ J/g}\cdot\text{C}°)}$$

or $(\Delta T)_w \approx 5.05\times10^{-3}\text{ C}°$

The change of temperature for the water is $\boxed{\text{between }10^{-3}\text{ °C and }10^{-2}\text{ °C}}$

21.20 $Q = (nC_P\,\Delta T)_{\text{isobaric}} + (nC_V\,\Delta T)_{\text{isovolmetric}}$

In the isobaric process, V doubles so T must double, to $2T_i$.
In the isovolumetric process, P triples so T changes from $2T_i$ to $6T_i$.

$$Q = n\left(\frac{7}{2}R\right)(2T_i - T_i) + n\left(\frac{5}{2}R\right)(6T_i - 2T_i)$$

$$Q = 13.5nRT_i = \boxed{13.5PV}$$

21.21 In the isovolumetric process $A \to B$, $W = 0$ and $Q = nC_V \Delta T = 500\text{ J}$

$$500\text{ J} = n(3R/2)(T_B - T_A) \quad \text{or} \quad T_B = T_A + \frac{2(500\text{ J})}{3nR}$$

$$T_B = 300\text{ K} + \frac{2(500\text{ J})}{3(1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = 340\text{ K}$$

In the isobaric process $B \to C$,

$$Q = nC_P \Delta T = \frac{5nR}{2}(T_C - T_B) = -500\text{ J}$$

Thus,

(a) $$T_C = T_B - \frac{2(500\text{ J})}{5nR} = 340\text{ K} - \frac{1000\text{ J}}{5(1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = \boxed{316\text{ K}}$$

(b) The work done by the gas during the isobaric process is

$$W_{BC} = P_B \Delta V = nR(T_C - T_B) = (1.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(316\text{ K} - 340\text{ J})$$

or $W_{BC} = -200\text{ J}$

The total work done <u>on</u> the gas is then

$$W_{\text{on gas}} = -W_{\text{by gas}} = -(W_{AB} + W_{BC}) = -(0 - 200\text{ J})$$

or $W_{\text{on gas}} = \boxed{+200\text{ J}}$

21.22 (a) The heat required to produce a temperature change is

$$Q = n_1 C_1 \Delta T + n_2 C_2 \Delta T$$

The number of molecules is $N_1 + N_2$, so the number of "moles of the mixture" is $n_1 + n_2$ and $Q = (n_1 + n_2) C \Delta T$,

so $$C = \boxed{\frac{n_1 C_1 + n_2 C_2}{n_1 + n_2}}$$

(b) $$Q = \sum_{i=1}^{m} n_i C_i \Delta T = \left(\sum_{i=1}^{m} n_i\right) C \Delta T$$

$$C = \boxed{\sum_{i=1}^{m} n_i C_i \Big/ \sum_{i=1}^{m} n_i}$$

21.23 The rms speed of the gas molecules is $v = \sqrt{\dfrac{3RT}{M}}$. Thus, to double the rms speed, the temperature must increase by a factor of 4: $T_f = 4T_i$.

Since the pressure is proportional to volume in this process,

$$\frac{P}{V} = \frac{P_i}{V_i} = \text{constant} \quad \text{or} \quad P = \left(\frac{P_i}{V_i}\right) V$$

Then, $PV = nRT$ becomes

$$\left(\frac{P_i}{V_i}\right) V^2 = nRT \quad \text{or} \quad V^2 = \left(\frac{V_i}{P_i}\right) nRT$$

Therefore, $\dfrac{V_f^2}{V_i^2} = \dfrac{T_f}{T_i} = 4$ or $V_f = 2V_i$.

The work done by the gas is then

$$W = \int_{V_i}^{V_f} P\, dV = \frac{P_i}{V_i}\int_{V_i}^{2V_i} V\, dV = \frac{3}{2}\, P_i V_i$$

The change in the internal energy of the gas is

$$\Delta E_{\text{int}} = nC_V\, \Delta T = n\left(\frac{5}{2} R\right)(4T_i - T_i) = \frac{15}{2}\, nRT_i = \frac{15}{2}\, P_i V_i$$

and the energy transferred to the gas as heat is

$$Q = \Delta E_{\text{int}} + W = \frac{15}{2}\, P_i V_i + \frac{3}{2}\, P_i V_i = \boxed{9P_i V_i}$$

***21.24** (a) $P_i V_i^{\gamma} = P_f V_f^{\gamma}$ so $\dfrac{V_f}{V_i} = \left(\dfrac{P_i}{P_f}\right)^{1/\gamma} = \left(\dfrac{1.00}{20.0}\right)^{5/7} = \boxed{0.118}$

(b) $\dfrac{T_f}{T_i} = \dfrac{P_f V_f}{P_i V_i} = \left(\dfrac{P_f}{P_i}\right)\left(\dfrac{V_f}{V_i}\right) = (20.0)(0.118) = \boxed{2.35}$

(c) Since the process is adiabatic, $\boxed{Q = 0}$

Since $\gamma = 1.40 = \dfrac{C_P}{C_V} = \dfrac{R + C_V}{C_V}$, $C_V = \dfrac{5}{2} R$, and $\Delta T = 2.35T_i - T_i = 1.35T_i$

$$\Delta E_{\text{int}} = nC_V\, \Delta T = (0.0160\text{ mol})\left(\frac{5}{2}\right)\left(8.315\, \frac{\text{J}}{\text{mol} \cdot \text{K}}\right)[1.35(300\text{ K})] = \boxed{135\text{ J}}$$

and $W = Q - \Delta E_{\text{int}} = 0 - 135\text{ J} = \boxed{-135\text{ J}}$

***21.25** (a) $P_i V_i^\gamma = P_f V_f^\gamma$

$$P_f = \left(\frac{V_i}{V_f}\right)^\gamma P_i = (5.00\text{ atm})\left(\frac{12.0}{30.0}\right)^{1.40} = \boxed{1.39\text{ atm}}$$

(b) $$T_i = \frac{P_i V_i}{nR} = \frac{(5.00)(1.013\times10^5\text{ Pa})(12.0\times10^{-3}\text{ m}^3)}{(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = \boxed{366\text{ K}}$$

$$T_f = \frac{P_f V_f}{nR} = \frac{(1.39)(1.013\times10^5\text{ Pa})(30.0\times10^{-3}\text{ m}^3)}{(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = \boxed{253\text{ K}}$$

(c) The process is adiabatic: $\boxed{Q = 0}$

$$\gamma = 1.40 = \frac{C_P}{C_V} = \frac{R + C_V}{C_V} \qquad C_V = \frac{5}{2}R$$

$$\Delta E_{\text{int}} = nC_V\,\Delta T = (2.00\text{ mol})\left(\frac{5}{2}(8.315\text{ J/mol}\cdot\text{K})\right)(366 - 253)\text{ K} = \boxed{4.66\text{ kJ}}$$

$$W = Q - \Delta E_{\text{int}} = 0 - 4.66\text{ kJ} = \boxed{-4.66\text{ kJ}}$$

21.26 $V_i = \pi(2.50\times10^{-2}\text{ m}/2)^2\ 0.500\text{ m} = 2.45\times10^{-4}\text{ m}^3$

The quantity of air we find from $P_i V_i = nRT_i$

$$n = \frac{P_i V_i}{RT_i} = \frac{(1.013\times10^5\text{ Pa})(2.45\times10^{-4}\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})}$$

$$n = 9.97\times10^{-3}\text{ mol}$$

Adiabatic compression: $P_f = 101.3\text{ kPa} + 800\text{ kPa} = 901.3\text{ kPa}$

(a) $P_i V_f^\gamma = P_f V_f^\gamma$

$$V_f = V_i(P_i/P_f)^{1/\gamma} = 2.45\times10^{-4}\text{ m}^3(101.3/901.3)^{5/7}$$

$$V_f = \boxed{5.15\times10^{-5}\text{ m}^3}$$

(b) $P_f V_f = nRT_f$

$$T_f = T_i\frac{P_f V_f}{P_i V_i} = T_i\frac{P_f}{P_i}\left(\frac{P_i}{P_f}\right)^{1/\gamma} = T_i(P_i/P_f)^{(1/\gamma - 1)}$$

$$T_f = 300\text{ K}(101.3/901.3)^{(5/7 - 1)} = \boxed{560\text{ K}}$$

(c) The work put into the gas in compressing it is $\Delta E_{\text{int}} = nC_V\,\Delta T$

$$W = (9.97 \times 10^{-3}\text{ mol})\frac{5}{2}(8.315\text{ J/mol}\cdot\text{K})(560 - 300)\text{ K}$$

$$W = 53.9\text{ J}$$

Now imagine this energy being shared with the inner wall as the gas is held at constant volume. The pump wall has outer diameter 25.0 mm + 2.00 mm + 2.00 mm = 29.0 mm, and volume

$$[\pi(14.5 \times 10^{-3}\text{ m})^2 - \pi(12.5 \times 10^{-3}\text{ m})^2]4.00 \times 10^{-2}\text{ m} = 6.79 \times 10^{-6}\text{ m}^3$$

and mass $\rho V = (7.86 \times 10^3\text{ kg/m}^3)(6.79 \times 10^{-6}\text{ m}^3) = 53.3\text{ g}$

The overall warming process is described by

$$53.9\text{ J} = nC_V\,\Delta T + mc\,\Delta T$$

$$53.9\text{ J} = (9.97 \times 10^{-3}\text{ mol})\frac{5}{2}(8.315\text{ J/mol}\cdot\text{K})(T_{ff} - 300\text{ K})$$

$$+ (53.3 \times 10^{-3}\text{ kg})(448\text{ J/kg}\cdot\text{K})(T_{ff} - 300\text{ K})$$

$$53.9\text{ J} = (0.207\text{ J/K} + 23.9\text{ J/K})(T_{ff} - 300\text{ K})$$

$$T_{ff} - 300\text{ K} = \boxed{2.24\text{ K}}$$

21.27 $\dfrac{T_f}{T_i} = \left(\dfrac{V_i}{V_f}\right)^{\gamma - 1} = \left(\dfrac{1}{2}\right)^{0.400}$

If $T_i = 300$ K, then $T_f = \boxed{227\text{ K}}$

Goal Solution

G: The air should cool as it expands, so we should expect $T_f < 300$ K.

O: The air expands adiabatically, losing no heat but dropping in temperature as it does work on the air around it, so we assume that PV^γ = constant (where $\gamma = 1.40$ for an ideal gas).

A: Combine $P_1V_1^\gamma = P_2V_2^\gamma$ and $P_1 = \dfrac{nRT_1}{V_1}$ with $P_2 = \dfrac{nRT_2}{V_2}$

to find $T_1V_1^{\gamma - 1} = T_2V_2^{\gamma - 1}$

$$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma - 1} = 300\text{ K}\left(\frac{1}{2}\right)^{(1.40 - 1)} = 227\text{ K}$$

L: The air does cool, but the rate is not linear with the change in volume (the temperature drops only 24% while the volume doubles)

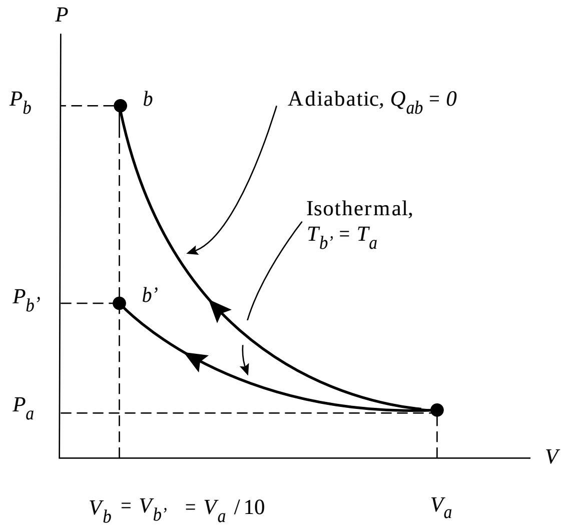
21.28 (a) The work done *on* the gas is

$$-W_{ab} = -\int_{V_a}^{V_b} P\,dV$$

For the isothermal process,

$$-W_{ab'} = -nRT_a \int_{V_a}^{V_{b'}} \left(\frac{1}{V}\right) dV$$

$$= -nRT_a \ln\left(\frac{V_{b'}}{V_a}\right) = nRT_a \ln\left(\frac{V_a}{V_{b'}}\right)$$



Thus, $-W_{ab'} = (5.00\text{ mol})\left(8.315\,\dfrac{\text{J}}{\text{mol}\cdot\text{K}}\right)(293\text{ K})\ \ln(10.0)$

$$= \boxed{28.0\text{ kJ}}$$

(b) For the adiabatic process, we must first find the final temperature, T_b. Since air consists primarily of diatomic molecules, we shall use

$$\gamma_{\text{air}} = 1.40 \quad \text{and} \quad C_{V,\text{air}} = 5R/2 = 5(8.315)/2 = 20.8\text{ J/mol}\cdot\text{K}$$

Then, from Equation 21.20,

$$T_b = T_a\left(\frac{V_a}{V_b}\right)^{\gamma-1} = (293\text{ K})(10.0)^{0.400} = 736\text{ K}$$

Thus, the work done on the gas during the adiabatic process is

$$-W_{ab} = -(Q - \Delta E_{\text{int}})_{ab} = -(0 - nC_V\,\Delta T)_{ab} = nC_V(T_b - T_a)$$

or $\quad -W_{ab} = (5.00\text{ mol})(20.8\text{ J/mol}\cdot\text{K})(736 - 293)\text{ K} = \boxed{46.1\text{ kJ}}$

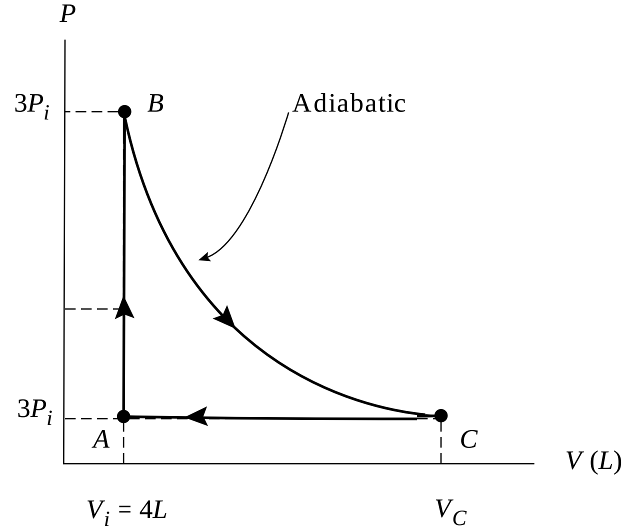
(c) For the isothermal process, we have $P_{b'}V_{b'} = P_aV_a$

Thus, $P_{b'} = P_a\left(\dfrac{V_{a'}}{V_{b'}}\right) = (1.00\text{ atm})(10.0) = \boxed{10.0\text{ atm}}$

For the adiabatic process, we have $P_{b'}V_b^\gamma = P_aV_a^\gamma$

Thus, $P_b = P_a\left(\dfrac{V_a}{V_b}\right)^\gamma = (1.00\text{ atm})(10.0)^{1.40} = \boxed{25.1\text{ atm}}$

21.29 (a) See the diagram at the right.



(b) $P_B V_B^{\gamma} = P_C V_C^{\gamma}$

$$3P_i V_i^{\gamma} = P_i V_C^{\gamma}$$

$$V_C = 3^{1/\gamma} V_i = 3^{5/7} V_i = 2.19 V_i$$

$$V_C = 2.19(4.00\text{ L}) = \boxed{8.79\text{ L}}$$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$$T_B = 3T_i = 3(300\text{ K}) = \boxed{900\text{ K}}$$

(d) After one whole cycle, $T_A = T_i = \boxed{300\text{ K}}$

(e) In AB, $Q_{AB} = nC_V\,\Delta T = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$Q_{BC} = 0$ as this process is abiabatic

$P_C V_C = nRT_C = P_i(2.19V_i) = 2.19nRT_i$ so $T_C = 2.19T_i$

$$Q_{CA} = nC_P\,\Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = -4.17nRT_i$$

For the whole cycle,

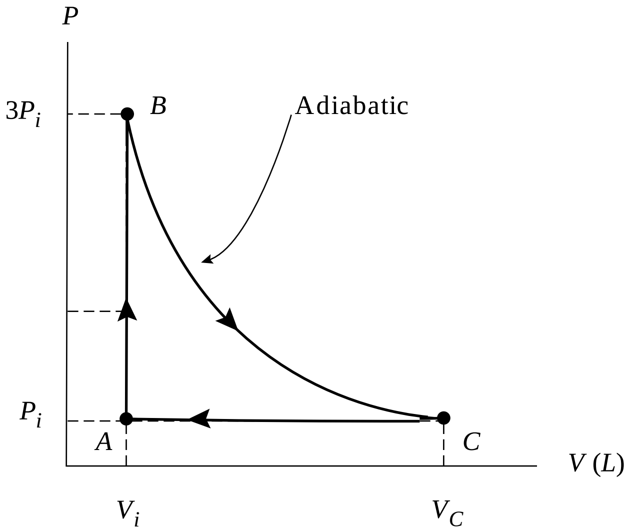
$$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = 0.830nRT_i$$

$$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} - W_{ABCA}$$

$$W_{BACA} = Q_{ABCA} = 0.830nRT_i = 0.830P_i V_i$$

$$W_{ABCA} = 0.830(1.013 \times 10^5\text{ Pa})(4.00 \times 10^{-3}\text{ m}^3) = \boxed{336\text{ J}}$$

21.30 (a) See the diagram at the right.



(b) $P_B V_B^{\gamma} = P_C V_C^{\gamma}$

$3P_i V_i^{\gamma} = P_i V_C^{\gamma}$

$V_C = 3^{1/\gamma} V_i = 3^{5/7} V_i = \boxed{2.19 V_i}$

(c) $P_B V_B = nRT_B = 3P_i V_i = 3nRT_i$

$T_B = \boxed{3T_i}$

(d) After one whole cycle, $T_A = \boxed{T_i}$

(e) In AB, $Q_{AB} = nC_V\,\Delta T = n\left(\frac{5}{2}R\right)(3T_i - T_i) = (5.00)nRT_i$

$Q_{BC} = 0$ as this process is abiabatic

$P_C V_C = nRT_C = P_i(2.19V_i) = 2.19nRT_i$ so $T_C = 2.19T_i$

$Q_{CA} = nC_P\,\Delta T = n\left(\frac{7}{2}R\right)(T_i - 2.19T_i) = -4.17nRT_i$

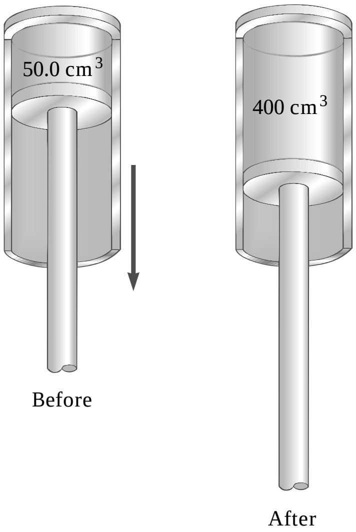
For the whole cycle,

$Q_{ABCA} = Q_{AB} + Q_{BC} + Q_{CA} = (5.00 - 4.17)nRT_i = 0.830nRT_i$

$(\Delta E_{\text{int}})_{ABCA} = 0 = Q_{ABCA} - W_{ABCA}$

$W_{ABCA} = Q_{ABCA} = 0.830nRT_i = \boxed{0.830P_iV_i}$

21.31 We suppose the air plus burnt gasoline behaves likes a diatomic ideal gas. We find its final absolute pressure:



$$21.0\ \text{atm}(50.0\ \text{cm}^3)^{7/5} = P_f(400\ \text{cm}^3)^{7/5}$$

$$P_f = 21.0\ \text{atm}\ (1/8)^{7/5} = 1.14\ \text{atm}$$

Now $Q = 0$, and $W = -\Delta E_{\text{int}} = -nC_V(T_f - T_i)$

$$\therefore\ W = -n\frac{5}{2}\ RT_f + \frac{5}{2}\ nRT_i = \frac{5}{2}(-P_fV_f + P_iV_i)$$

$$= \frac{5}{2}\ [-(1.14\ \text{atm})(400\ \text{cm}^3) + (21.0\ \text{atm})(50.0\ \text{cm}^3)]$$

$$\left(\frac{1.013 \times 10^5\ \text{N}/\text{m}^2}{1\ \text{atm}}\right)\left(\frac{10^{-6}\ \text{m}^3}{\text{cm}^3}\right)$$

$$W = 150\ \text{J}$$

The time for this stroke is $\frac{1}{4}\frac{1\ \text{min}}{2500}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = 6.00 \times 10^{-3}\ \text{s}$

So $\wp = \frac{W}{t} = \frac{150\ \text{J}}{6.00 \times 10^{-3}\ \text{s}} = \boxed{25.0\ \text{kW}}$

21.32 (1) $E_{\text{int}} = Nf\frac{k_BT}{2} = f\frac{nRT}{2}$

(2) $C_V = \frac{1}{n}\left(\frac{dE_{\text{int}}}{dT}\right) = \frac{1}{2}\ fR$

(3) $C_P = C_V + R = \frac{1}{2}(f + 2)\ R$

(4) $\gamma = \frac{C_P}{C_V} = \frac{(f + 2)}{f}$

21.33 (a) $C'_V = \frac{5}{2}\ nR = \boxed{9.95\ \text{cal}/\text{K}}$ $C'_P = \frac{7}{2}\ nR = \boxed{13.9\ \text{cal}/\text{K}}$

(b) $C'_V = \frac{7}{2}\ nR = \boxed{13.9\ \text{cal}/\text{K}}$ $C'_P = \frac{9}{2}\ nR = \boxed{17.9\ \text{cal}/\text{K}}$

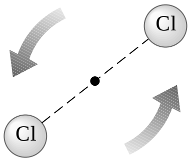
21.34 A more massive diatomic or polyatomic molecule will generally have a lower frequency of vibration. At room temperature, vibration has more chance of being excited than in a less massive molecule. Absorbing energy into vibration shows up in higher specific heats.

21.35 Rotational Kinetic Energy $= \frac{1}{2} I\omega^2$

$I = 2mr^2$, $m = 35.0 \times 1.67 \times 10^{-27}$ kg, $r = 10^{-10}$ m

$I = 1.17 \times 10^{-45}\ \text{kg} \cdot \text{m}^2 \qquad \omega = 2.00 \times 10^{12}\ \text{s}^{-1}$

$$\therefore K_{\text{rot}} = \frac{1}{2} I\omega^2 = \boxed{2.33 \times 10^{-21}\ \text{J}}$$



21.36 The ratio of the number at higher energy to the number at lower energy is $e^{-\Delta E / k_B T}$ where ΔE is the energy difference. Here,

$$\Delta E = (10.2\ \text{eV})(1.60 \times 10^{-19}\ \text{J}/1\ \text{eV}) = 1.63 \times 10^{-18}\ \text{J}$$

and at 0°C,

$$k_B T = (1.38 \times 10^{-23}\ \text{J}/\text{K})(273\ \text{K}) = 3.77 \times 10^{-21}\ \text{J}$$

Since this is much less than the excitation energy, nearly all the atoms will be in the ground state and the number excited is

$$(2.70 \times 10^{25})\exp(-1.63 \times 10^{-18}\ \text{J}/3.77 \times 10^{-21}\ \text{J}) = (2.70 \times 10^{25})e^{-433}$$

This number is much less than one, so almost all of the time $\boxed{\text{no atom is excited}}$.

At 10000°C,

$$k_B T = (1.38 \times 10^{-23}\ \text{J}/\text{K})10273\ \text{K} = 1.42 \times 10^{-19}\ \text{J}$$

The number excited is

$$(2.70 \times 10^{25})\exp(-1.63 \times 10^{-18}\ \text{J}/1.42 \times 10^{-19}\ \text{J})$$

$$= (2.70 \times 10^{25})e^{-11.5} = \boxed{2.70 \times 10^{20}}$$

21.37 Call n_{00} the sea-level number density of oxygen molecules, $n_{\text{N}0}$ the sea-level number of nitrogen per volume, and n_0 and n_{N} their respective densities at $y = 10.0$ km.

Then, $n_0 = n_{00} \exp(-m_0 gy / k_B T)$

$$n_{\text{N}} = n_{\text{NO}} \exp(-m_{\text{N}} gy / k_B T)$$

and $$\frac{n_0}{n_{\text{N}}} = \frac{n_{00}}{n_{\text{N}0}} \exp(-m_0 gy / k_B T + m_{\text{N}} gy / k_B T)$$

So $$\frac{(n_0/n_N)}{(n_{00}/n_{N0})} = \exp\,[-(m_0 - m_N)gy/k_BT]$$

$$= \exp\left(-\frac{(32.0 - 28.0)\text{ u }(1.66\times10^{-27}\text{ kg/u})(9.80\text{ m/s}^2)10^4\text{ m}}{(1.38\times10^{-23}\text{ J/K})300\text{ K}}\right)$$

$$= 0.855$$

$\boxed{\text{The ratio of oxygen to nitrogen molecules decreases to 85.5\% of its sea-level value.}}$

21.38 (a) $$\frac{V_{\text{rms},35}}{V_{\text{rms},37}} = \frac{\sqrt{3RT/M_{35}}}{\sqrt{3RT/M_{37}}} = \left(\frac{37.0\text{ g/mol}}{35.0\text{ g/mol}}\right)^{1/2} = \boxed{1.03}$$

(b) The lighter atom, $\boxed{^{35}\text{Cl}}$, moves faster.

21.39 (a) $$v_{\text{av}} = \frac{\sum n_i v_i}{N} = \frac{1}{15}\,[1(2) + 2(3) + 3(5) + 4(7) + 3(9) + 2(12)] = \boxed{6.80\text{ m/s}}$$

(b) $$(v^2)_{\text{av}} = \frac{\sum n_i v_i^2}{N} = 54.9\text{ m}^2/\text{s}^2$$

so $$v_{\text{rms}} = \sqrt{(v^2)_{\text{av}}} = \sqrt{54.9} = \boxed{7.41\text{ m/s}}$$

(c) $v_{\text{mp}} = \boxed{7.00\text{ m/s}}$

21.40 Following Equation 21.29,

$$v_{\text{mp}} = \sqrt{\frac{2k_BT}{m}} = \sqrt{\frac{2(1.38\times10^{-23}\text{ J/K})(4.20\text{ K})}{6.64\times10^{-27}\text{ kg}}} = \boxed{132\text{ m/s}}$$

21.41 Use Equation 21.26.

Take $\frac{dN_v}{dv} = 0$: $$4\pi N\left(\frac{m}{2\pi k_BT}\right)^{3/2}\exp\left(-\frac{mv^2}{2k_BT}\right)\left(2v - \frac{2mv^3}{2k_BT}\right) = 0$$

and solve for v_{mp} to get Equation 21.29. Reject the solutions $v = 0$ and $v = \infty$.
Retain only $2 - \frac{mv^2}{k_BT} = 0$.

21.42 (a) From $v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}}$, we find the temperature as

$$T = \frac{(6.64\times10^{-27}\text{ kg})(1.12\times10^4\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/mol}\cdot\text{K})} = \boxed{2.01\times10^4\text{ K}}$$

(b) $$T = \frac{(6.64\times10^{-27}\text{ kg})(2.37\times10^3\text{ m/s})^2}{3(1.38\times10^{-23}\text{ J/mol}\cdot\text{K})} = \boxed{9.01\times10^2\text{ K}}$$

21.43 At 0°C, $\frac{1}{2}\, m v_{\text{rms}0}^{2} = \frac{3}{2}\, k_B T_0$

At the higher temperature, $\frac{1}{2}\, m(2v_{\text{rms}0})^2 = \frac{3}{2}\, k_B T$

$$T = 4T_0 = 4(273\text{ K}) = 1092\text{ K} = \boxed{819°\text{C}}$$

21.44 Visualize the molecules in liquid water at 20°C jostling about randomly. One happens to get kinetic energy corresponding to 2430 J/g, and happens to be at the surface and headed upward. Then this molecule can break out of the liquid.

(a) $2430\text{ J/g} = \frac{2430\text{ J}}{\text{g}}\left(\frac{18.0\text{ g}}{1\text{ mol}}\right)\left(\frac{1\text{ mol}}{6.02\times 10^{23}\text{ molecules}}\right)$

$$= \boxed{7.27\times 10^{-20}\text{ J/molecule}}$$

(b) $7.27\times 10^{-20}\text{ J} = \frac{1}{2}\, m v^2$

$$v = \sqrt{\frac{2(7.27\times 10^{-20}\text{ J})}{18.0\text{ u}(1.66\times 10^{-27}\text{ kg/1 u})}} = \boxed{2.21\text{ km/s}}$$

(c) If these were typical molecules in an ideal gas instead of exceptional molecules in liquid water,

$$\frac{1}{2}\, m v^2 = \frac{3}{2}\, k_B T$$

$$T = \frac{2}{3}\frac{7.27\times 10^{-20}\text{ J}}{1.38\times 10^{-23}\text{ J/K}} = \boxed{3510\text{ K}}$$

These molecules got to be fast-moving in collisions that made other molecules slow-moving; the average molecular energy is unaffected.

21.45 (a) $PV = \left(\frac{N}{N_A}\right) RT$ and $N = \frac{PVN_A}{RT}$ so that

$$N = \frac{(1.00\times 10^{-10})(133)(1.00)(6.02\times 10^{23})}{(8.315)(300)} = \boxed{3.21\times 10^{12}\text{ molecules}}$$

(b) $l = \frac{1}{n_V \pi d^2 2^{1/2}} = \frac{V}{N\pi d^2 2^{1/2}} = \frac{1.00\text{ m}^3}{(3.21\times 10^{12}\text{ molecules})\pi(3.00\times 10^{-10}\text{ m})^2(2)^{1/2}}$

$$l = \boxed{778\text{ km}}$$

(c) $f = \frac{v}{l} = \boxed{6.42\times 10^{-4}\text{ s}^{-1}}$

Goal Solution

G: Since high vacuum means low pressure as a result of a low molecular density, we should expect a relatively low number of molecules, a long free path, and a low collision frequency compared with the values found in Example 21.7 for normal air. Since the ultrahigh vacuum is 13 orders of magnitude lower than atmospheric pressure, we might expect corresponding values of $N \sim 10^{12}$ molecules / m^3, $l \sim 10^6$ m, and $f \sim 0.0001$ / s.

O: The equation of state for an ideal gas can be used with the given information to find the number of molecules in a specific volume. The mean free path can be found directly from equation 21.30, and this result can be used with the average speed to find the collision frequency.

A: (a) $PV = \left(\dfrac{N}{N_A}\right) RT$ and $N = \dfrac{PVN_A}{RT}$ so that

$$N = \frac{(1.00 \times 10^{-10}\text{ torr})(133\text{ Pa/torr})(6.02 \times 10^{23}\text{ molecules/mol})}{(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})} = 3.21 \times 10^{12}\text{ molecules}$$

(b) $$l = \frac{1}{\sqrt{2}\,\pi d^2 n_v} = \frac{V}{\sqrt{2}\,N\pi d^2} = \frac{1.00\text{ m}^3}{\sqrt{2}\,(3.21 \times 10^{12}\text{ molecules})\pi(3.00 \times 10^{-10}\text{ m})^2}$$

$$l = 7.78 \times 10^5\text{ m} = 778\text{ km}$$

(c) $$f = \frac{v}{l} = \frac{500\text{ m/s}}{7.78 \times 10^5\text{ m}} = 6.42 \times 10^{-4}\text{ s}^{-1}$$

L: The pressure and the calculated results differ from the results in Example 21.7 by about 13 orders of magnitude as we expected. This ultrahigh vacuum provides conditions that are extremely different from normal atmosphere, and these conditions provide a "clean" environment for a variety of experiments and manufacturing processes that would otherwise be impossible.

21.46 The average molecular speed is

$$v = \sqrt{8k_BT/\pi m} = \sqrt{8k_BN_AT/\pi N_Am}$$

$$v = \sqrt{8RT/\pi M}$$

$$v = \sqrt{8(8.315\text{ J/mol}\cdot\text{K})3.00\text{ K}/\pi(2.016 \times 10^{-3}\text{ kg/mol})}$$

$$v = 178\text{ m/s}$$

(a) The mean free path is

$$\lambda = \frac{1}{\sqrt{2}\pi d^2 n_V} = \frac{1}{\sqrt{2}\pi(0.200 \times 10^{-9}\text{ m})^2\ 1/\text{m}^3}$$

$$\lambda = \boxed{5.63 \times 10^{18}\text{ m}}$$

The mean free time is

$$\lambda / v = 5.63 \times 10^{18}\text{ m}/178\text{ m/s} = 3.17 \times 10^{16}\text{ s} = \boxed{1.00 \times 10^{9}\text{ yr}}$$

(b) Now n_V is 10^6 times larger, to make λ smaller by 10^6 times:

$$\lambda = \boxed{5.63 \times 10^{12}\text{ m}}$$

Thus, $\lambda / v = 3.17 \times 10^{10}\text{ s} = \boxed{1.00 \times 10^{3}\text{ yr}}$

21.47 From Equation 21.30, $\lambda = \dfrac{1}{\sqrt{2}\,\pi d^2 n_V}$

For an ideal gas, $n_V = \dfrac{N}{V} = \dfrac{P}{k_B T}$

Therefore, $\lambda = \dfrac{k_B T}{\sqrt{2}\pi d^2 P}$, as required.

21.48 $\lambda = [\sqrt{2}\ \pi d^2 n_V]^{-1}$ $\qquad n_V = \dfrac{P}{k_B T}$

$d = 3.60 \times 10^{-10}\text{ m}$ $\qquad n_V = \dfrac{1.013 \times 10^5}{(1.38 \times 10^{-23})(293)} = 2.51 \times 10^{25}/\text{m}^3$

$\therefore \lambda = 6.93 \times 10^{-8}$ m, or about $\boxed{\text{193 molecular diameters}}$

21.49 Using $P = n_V k_B T$, Equation 21.30 becomes $\lambda = \dfrac{k_B T}{\sqrt{2}\ \pi\ P d^2}$ (1)

(a) $\lambda = \dfrac{(1.38 \times 10^{-23}\text{ J/K})(293\text{ K})}{\sqrt{2}\ \pi(1.0113 \times 10^{5}\text{ Pa})(3.10 \times 10^{-10}\text{ m})^2} = \boxed{9.36 \times 10^{-8}\text{ m}}$

(b) Equation (1) shows that $P_1\lambda_1 = P_2\lambda_2$. Taking $P_1\lambda_1$ from (a) and with $\lambda_2 = 1.00$ m, we find

$$P_2 = \frac{(1.00\text{ atm})(9.36 \times 10^{-8}\text{m})}{1.00\text{ m}} = \boxed{9.36 \times 10^{-8}\text{ atm}}$$

(c) For $\ell_3 = 3.10 \times 10^{-10}$ m, we have

$$P_3 = \frac{(1.00\text{ atm})(9.36 \times 10^{-8}\text{ m})}{3.10 \times 10^{-10}\text{ m}} = \boxed{302\text{ atm}}$$

***21.50** (a) $n = \frac{PV}{RT} = \frac{(1.013 \times 10^5\text{ Pa})(4.20\text{ m} \times 3.00\text{ m} \times 2.50\text{ m})}{(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})} = 1.31 \times 10^3\text{ mol}$

$N = nN_A = (1.31 \times 10^3\text{ mol})(6.02 \times 10^{23}\text{ molecules/mol})$

$N = \boxed{7.88 \times 10^{26}\text{ molecules}}$

(b) $m = nM = (1.31 \times 10^3\text{ mol})(0.0289\text{ kg/mol}) = \boxed{37.9\text{ kg}}$

(c) $\frac{1}{2}\, m_0 v^2 = \frac{3}{2}\, k_B T = \frac{3}{2}(1.38 \times 10^{-23}\text{ J/K})(293\text{ K}) = \boxed{6.07 \times 10^{-21}\text{ J/molecule}}$

(d) For one molecule,

$$m_0 = \frac{M}{N_A} = \frac{0.0289\text{ kg/mol}}{6.02 \times 10^{23}\text{ molecules/mol}} = 4.80 \times 10^{-26}\text{ kg/molecule}$$

$$v_{\text{rms}} = \sqrt{\frac{2(6.07 \times 10^{-21}\text{ J/molecule})}{4.80 \times 10^{-26}\text{ kg/molecule}}} = \boxed{503\text{ m/s}}$$

(e) and (f)

$$E_{\text{int}} = nC_V T = n\left(\frac{5}{2}R\right)T = \frac{5}{2}\, PV$$

$$E_{\text{int}} = \frac{5}{2}(1.013 \times 10^5\text{ Pa})(31.5\text{ m}^3) = \boxed{7.98\text{ MJ}}$$

***21.51** (a) $P_f = \boxed{100\text{ kPa}}$ $T_f = \boxed{400\text{ K}}$

$$V_f = \frac{nRT_f}{P_f} = \frac{(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(400\text{ K})}{100 \times 10^3\text{ Pa}} = 0.0665\text{ m}^3 = \boxed{66.5\text{ L}}$$

$$\Delta E_{\text{int}} = 3.50nR\,\Delta T = 3.50(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(100\text{ K}) = \boxed{5.82\text{ kJ}}$$

$$W = P\,\Delta V = nR\,\Delta T = (2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(100\text{ K}) = \boxed{1.66\text{ kJ}}$$

$$Q = \Delta E_{\text{int}} + W = 5.82\text{ kJ} + 1.66\text{ kJ} = \boxed{7.48\text{ kJ}}$$

(b) $T_f = \boxed{400\text{ K}}$

$$V_f = V_i = \frac{nRT_i}{P_i} = \frac{(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})}{100\times 10^3\text{ Pa}} = 0.0499\text{ m}^3 = \boxed{49.9\text{ L}}$$

$$P_f = P_i\left(\frac{T_f}{T_i}\right) = (100\text{ kPa})\left(\frac{400\text{ K}}{300\text{ K}}\right) = \boxed{133\text{ kPa}}$$

$W = \int P\,dV = \boxed{0}$ since V = constant

$\Delta E_{\text{int}} = \boxed{5.82\text{ kJ}}$ as in (a)

$Q = \Delta E_{\text{int}} + W = 5.82\text{ kJ} + 0 = \boxed{5.82\text{ kJ}}$

(c) $T_f = \boxed{300\text{ K}}$ $\quad P_f = \boxed{120\text{ kPa}}$

$$V_f = V_i\left(\frac{P_i}{P_f}\right) = (49.9\text{ L})\left(\frac{100\text{ kPa}}{120\text{ kPa}}\right) = \boxed{41.6\text{ L}}$$

$\Delta E_{\text{int}} = 3.50nR\,\Delta T = \boxed{0}$ since T = constant

$$W = \int P\,dV = nRT_i\int_{V_i}^{V_f}\frac{dV}{V} = nRT_i\ln\left(\frac{V_f}{V_i}\right) = nRT_i\ln\left(\frac{P_i}{P_f}\right)$$

$$W = (2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})\ln\left(\frac{100\text{ kPa}}{120\text{ kPa}}\right) = \boxed{-910\text{ J}}$$

$Q = \Delta E_{\text{int}} + W = 0 - 910\text{ J} = \boxed{-910\text{ J}}$

(d) $P_f = \boxed{120\text{ kPa}}$ $\quad \gamma = \dfrac{C_P}{C_V} = \dfrac{C_V + R}{C_V} = \dfrac{3.50R + R}{3.50R} = \dfrac{4.50}{3.50R} = \dfrac{4.50}{3.50} = \dfrac{9}{7}$

$$P_fV_f^\gamma = P_iV_i^\gamma \quad\text{so}\quad V_f = V_i\left(\frac{P_i}{P_f}\right)^{1/\gamma} = (49.9\text{ L})\left(\frac{100\text{ kPa}}{120\text{ kPa}}\right)^{7/9} = \boxed{43.3\text{ L}}$$

$$T_f = T_i\left(\frac{P_fV_f}{P_iV_i}\right) = (300\text{ K})\left(\frac{120\text{ kPa}}{100\text{ kPa}}\right)\left(\frac{43.3\text{ L}}{49.9\text{ L}}\right) = \boxed{312\text{ K}}$$

$$\Delta E_{\text{int}} = 3.50nR\,\Delta T = 3.50(2.00\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(12.4\text{ K}) = \boxed{722\text{ J}}$$

$Q = \boxed{0}$ (abiabatic process)

$W = Q - \Delta E_{\text{int}} = 0 - 722\text{ J} = \boxed{-722\text{ J}}$

21.52 (a) The average speed v_{av} is just the weighted average of all the speeds.

$$v_{\text{av}} = \frac{[2(v) + 3(2v) + 5(3v) + 4(4v) + 3(5v) + 2(6v) + 1(7v)]}{(2 + 3 + 5 + 4 + 3 + 2 + 1)} = \boxed{3.65v}$$

(b) First find the average of the square of the speeds,

$$v_{\text{av}}^2 = \frac{[2(v)^2 + 3(2v)^2 + 5(3v)^2 + 4(4v)^2 + 3(5v)^2 + 2(6v)^2 + 1(7v)^2]}{2 + 3 + 5 + 4 + 3 + 2 + 1} = 15.95v^2$$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{v_{\text{av}}^2} = \boxed{3.99v}$

(c) The most probable speed is the one that most of the particles have; i.e., five particles have speed $\boxed{3.00v}$

(d) $PV = \frac{1}{3} Nmv_{\text{av}}^2$

Therefore, $P = \frac{20}{3}\frac{[m(15.95)v^2]}{V} = \boxed{106\left(\frac{mv^2}{V}\right)}$

(e) The average kinetic energy for each particle is

$$\overline{K} = \frac{1}{2} mv_{\text{av}}^2 = \frac{1}{2} m(15.95v^2) = \boxed{7.98mv^2}$$

21.53 (a) $PV^\gamma = k$. So, $W = \int_{\text{i}}^{\text{f}} P\,dV = k\int_i^f \frac{dV}{V^\gamma} = \frac{P_iV_i - P_fV_f}{\gamma - 1}$

(b) $dE_{\text{int}} = dQ - dW$ and $dQ = 0$ for an adiabatic process.

Therefore, $W = -\Delta E_{\text{int}} = -\frac{3}{2} nR\,\Delta T = nC_V(T_i - T_f)$

To show consistency between these 2 equations, consider that $\gamma = C_P / C_V$ and $C_P - C_V = R$. Therefore, $1/(\gamma - 1) = C_V / R$.

Using this, the result found in part (a) becomes

$$W = (P_iV_i - P_fV_f)\frac{C_V}{R}$$

Also, for an ideal gas $\frac{PV}{R} = nT$ so that $W = nC_V(T_i - T_f)$

21.54 (a) Maxwell's speed distribution function is

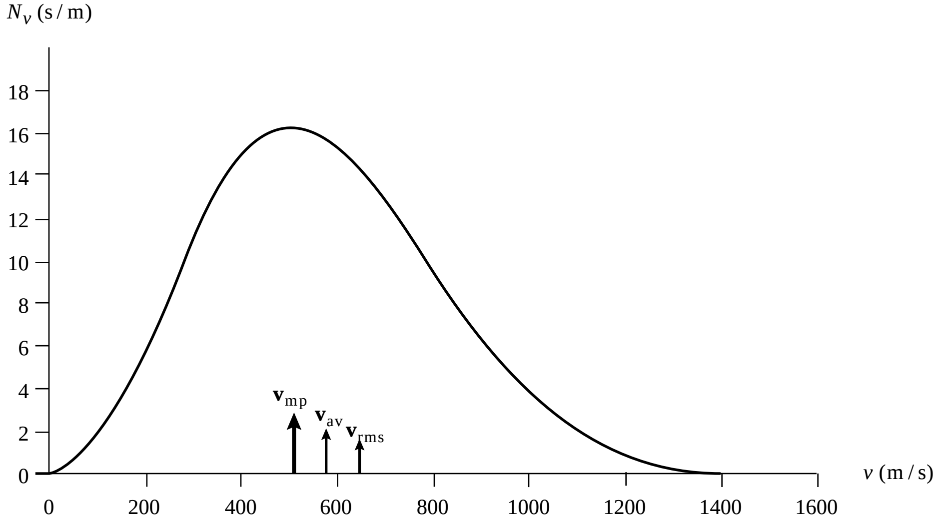
$$N_v = 4\pi N\left(\frac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{-mv^2/2k_BT}$$

With $N = 1.00 \times 10^4$, $m = \dfrac{M}{N_A} = \dfrac{0.032\text{ kg}}{6.02 \times 10^{23}} = 5.32 \times 10^{-26}\text{ kg}$

$T = 500$ K, and $k_B = 1.38 \times 10^{-23}$ J / molecule · K; this becomes

$$N_v = (1.71 \times 10^{-4})v^2 e^{-(3.85 \times 10^{-6})v^2}$$

The following is a plot of this function for the range $0 \le v \le 1400$ m / s.



(b) The most probable speed occurs where N_v is a maximum.

From the graph, $\boxed{v_{\text{mp}} \approx 510\text{ m/s}}$

(c) $v_{\text{av}} = \sqrt{\dfrac{8k_BT}{\pi m}} = \sqrt{\dfrac{8(1.38 \times 10^{-23})(500)}{\pi(5.32 \times 10^{-26})}} = \boxed{575\text{ m/s}}$

Also, $v_{\text{rms}} = \sqrt{\dfrac{3k_BT}{m}} = \sqrt{\dfrac{3(1.38 \times 10^{-23})(500)}{5.32 \times 10^{-26}}} = \boxed{624\text{ m/s}}$

(d) The fraction of particles in the range $300 \le v \le 600$ m / s is $\dfrac{\int_{300}^{600} N_v\,dv}{N}$ where $N = 10^4$ and the integral of N_v is read from the graph as the area under the curve. This is approximately 4400 and the fraction is 0.44 or $\boxed{44\%}$.

21.55 The pressure of the gas in the lungs of the diver will be the same as the absolute pressure of the water at this depth of 50.0 meters. This is:

$$P = P_0 + \rho g h = 1.00\text{ atm} + (1.00\times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(50.0\text{ m})$$

or $$P = 1.00\text{ atm} + (4.90\times 10^5\text{ Pa})\left(\frac{1.00\text{ atm}}{1.013\times 10^5\text{ Pa}}\right) = 5.84\text{ atm}$$

If the partial pressure due to the oxygen in the gas mixture is to be 1.00 atmosphere or less (or approximately one-sixth of the total pressure), oxygen molecules should make up only about one-sixth of the total number of molecules. This will be true if 1.00 mole of oxygen is used for every 5.00 moles of helium. The ratio by weight is therefore,

$$\frac{(5.00\text{ mol He})g}{(1.00\text{ mol O}_2)g} = \frac{(20.0\text{ g})g}{(32.0\text{ g})g} = \boxed{0.625}$$

***21.56** $n = \dfrac{m}{M} = \dfrac{1.20\text{ kg}}{0.0289\text{ kg/mol}} = 41.5\text{ mol}$

(a) $$V_i = \frac{nRT_i}{P_i} = \frac{(41.5\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(298\text{ K})}{200\times 10^3\text{ Pa}} = \boxed{0.514\text{ m}^3}$$

(b) $$\frac{P_f}{P_i} = \frac{\sqrt{V_f}}{\sqrt{V_i}} \quad\text{so}\quad V_f = V_i\left(\frac{P_f}{P_i}\right)^2 = (0.514\text{ m}^3)\left(\frac{400}{200}\right)^2 = \boxed{2.06\text{ m}^3}$$

(c) $$T_f = \frac{P_f V_f}{nR} = \frac{(400\times 10^3\text{ Pa})(2.06\text{ m}^3)}{(41.5\text{ mol})(8.315\text{ J/mol}\cdot\text{K})} = \boxed{2.38\times 10^3\text{ K}}$$

(d) $$W = \int_{V_i}^{V_f} P\,dV = C\int_{V_i}^{V_f} V^{1/2}dV = \left(\frac{P_i}{V_i^{1/2}}\right)\frac{2V^{3/2}}{3}\Bigg|_{V_i}^{V_f} = \frac{2}{3}\left(\frac{P_i}{V_i^{1/2}}\right)(V_f^{3/2} - V_i^{3/2})$$

$$W = \frac{2}{3}\left(\frac{200\times 10^3\text{ Pa}}{\sqrt{0.514\text{ m}}}\right)[(2.06\text{ m}^3)^{3/2} - (0.514\text{ m})^{3/2}] = \boxed{4.80\times 10^5\text{ J}}$$

(e) $$\Delta E_{\text{int}} = nC_V\,\Delta T = (41.5\text{ mol})\left[\frac{5}{2}(8.315\text{ J/mol}\cdot\text{K})\right](2.38\times 10^3 - 298)\text{ K}$$

$$\Delta E_{\text{int}} = 1.80\times 10^6\text{ J}$$

$$Q = \Delta E_{\text{int}} + W = 1.80\times 10^6\text{ J} + 4.80\times 10^5\text{ J} = 2.28\times 10^6\text{ J} = \boxed{2.28\text{ MJ}}$$

21.57 (a) Since $\boxed{\text{pressure increases as volume decreases}}$ (and vice versa) ,

$$\frac{dV}{dP} < 0 \quad \text{and} \quad -\frac{1}{V}\left[\frac{dV}{dP}\right] > 0$$

(b) For an ideal gas, $V = \frac{nRT}{P}$ and $\kappa_1 = -\frac{1}{V}\frac{d}{dP}\left(\frac{nRT}{P}\right)$

If the compression is isothermal, T is constant and

$$\kappa_1 = -\frac{nRT}{V}\left(-\frac{1}{P^2}\right) = \frac{1}{P}$$

(c) For an adiabatic compression, $PV^\gamma = C$ (where C is a constant) and

$$\kappa_2 = -\frac{1}{V}\frac{d}{dP}\left(\frac{C}{P}\right)^{1/\gamma} = \frac{1}{V}\left(\frac{1}{\gamma}\right)\frac{C^{1/\gamma}}{P^{(1/\gamma)+1}} = \frac{P^{1/\gamma}}{\gamma P^{1/\gamma+1}} = \frac{1}{\gamma P}$$

(d) $\kappa_1 = \frac{1}{P} = \frac{1}{(2.00\text{ atm})} = \boxed{0.500\text{ atm}^{-1}}$

$\gamma = \frac{C_P}{C_V}$ and for a monatomic ideal gas, $\gamma = 5/3$, so that

$$\kappa_2 = \frac{1}{\gamma P} = \frac{1}{(5/3)(2.00\text{ atm})} = \boxed{0.300\text{ atm}^{-1}}$$

***21.58** (a) The speed of sound is $v = \sqrt{\frac{B}{\rho}}$ where $B = -V\frac{dP}{dV}$.

According to Problem 57, in an adiabatic process, this is $B = \frac{1}{\kappa_2} = \gamma P$.

Also, $\rho = \frac{m_s}{V} = \frac{nM}{V} = \frac{(nRT)M}{V(RT)} = \frac{PM}{RT}$ where m_s is the sample mass. Then, the speed of sound in the ideal gas is $v = \sqrt{\frac{B}{\rho}} = \sqrt{\gamma P\left(\frac{RT}{PM}\right)} = \boxed{\sqrt{\frac{\gamma RT}{M}}}$

(b) $v = \sqrt{\frac{1.40(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})}{0.0289\text{ kg/mol}}} = \boxed{344\text{ m/s}}$

This nearly agrees with the 343 m/s listed in Table 17.1.

(c) We use $k_B = \dfrac{R}{N_A}$ and $M = mN_A$: $v = \sqrt{\dfrac{\gamma R\,T}{M}} = \sqrt{\dfrac{\gamma k_B N_A T}{mN_A}} = \sqrt{\dfrac{\gamma k_B T}{m}}$

The most probable molecular speed is $\sqrt{\dfrac{2k_B T}{m}}$,

the average speed is $\sqrt{\dfrac{8k_B T}{\pi m}}$, and the rms speed is $\sqrt{\dfrac{3k_B T}{m}}$.

$\boxed{\text{All are somewhat larger}}$ than the speed of sound.

21.59 $N_v(v) = 4\pi N \left(\dfrac{m}{2\pi k_B T}\right)^{3/2} v^2 \exp(-mv^2/2k_B T)$

Note that $v_{\text{mp}} = (2k_B T/m)^{1/2}$

Thus, $N_v(v) = 4\pi N \left(\dfrac{m}{2\pi k_B T}\right)^{3/2} v^2 e^{(-v^2/v_{\text{mp}}^2)}$

and $\dfrac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\dfrac{v}{v_{\text{mp}}}\right)^2 e^{(1 - v^2/v_{\text{mp}}^2)}$

For $v = v_{\text{mp}}/50$, $\dfrac{N_v(v)}{N_v(v_{\text{mp}})} = \left(\dfrac{1}{50}\right)^2 e^{[1-(1/50)^2]} = 1.09 \times 10^{-3}$

The other values are computed similarly, with the following results:

$\dfrac{v}{v_{\text{mp}}}$	$\dfrac{N_v(v)}{N_v(v_{\text{mp}})}$
1/50	1.09×10^{-3}
1/10	2.69×10^{-2}
1/2	0.529
1	1.00
2	0.199
10	1.01×10^{-41}
50	1.25×10^{-1082}

To find the last value, note:

$$(50)^2 e^{1-2500} = 2500e^{-2499}$$

$$= 10^{\log 2500} e^{(\ln 10)(-2499/\ln 10)} = 10^{\log 2500} 10^{-2499/\ln 10}$$

$$= 10^{\log 2500 - 2499/\ln 10} = 10^{-1081.904}$$

***21.60** The ball loses energy

$$\frac{1}{2}mv_i^2 - \frac{1}{2}mv_f^2 = \frac{1}{2}(0.142\text{ kg})\,[(47.2)^2 - (42.5)^2]\text{m}^2/\text{s}^2 = 29.9\text{ J}$$

The air volume is $V = \pi(0.0370\text{ m})^2(19.4\text{ m}) = 0.0834\text{ m}^3$,

and its quantity is $n = \dfrac{PV}{RT} = \dfrac{(1.013\times 10^5\text{ Pa})(0.0834\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(293\text{ K})} = 3.47\text{ mol}$

The air absorbs energy according to $Q = nC_P\,\Delta T$, so

$$\Delta T = \frac{Q}{nC_P} = \frac{29.9\text{ J}}{(3.47\text{ mol})(7/2)(8.315\text{ J/mol}\cdot\text{K})} = \boxed{0.296\text{ C}^\circ}$$

21.61 (a) The effect of high angular speed is like the effect of a very high gravitational field on an atmosphere. The result is:

$\boxed{\text{The larger-mass molecules settle to the outside}}$ while the region at smaller r has a higher concentration of low-mass molecules.

(b) Consider a single kind of molecules, all of mass m. To supply the centripetal force on the molecules between r and $r + dr$, the pressure must increase outward according to $\sum F_r = ma_r$. Thus,

$$PA - (P + dP)A = -(nmA\ dr)(r\omega^2)$$

where n is the number of molecules per unit volume and A is the area of any cylindrical surface. This reduces to $dP = nm\omega^2 r dr$.

But also $P = nk_BT$, so $dP = k_BT\,dn$. Therefore, the equation becomes

$$\frac{dn}{n} = \frac{m\omega^2}{k_BT}\,r\,dr \quad \text{giving} \quad \int_{n_0}^{n}\frac{dn}{n} = \frac{m\omega^2}{k_BT}\int_0^r r\,dr \quad \text{or}$$

$$\ln(n)\Big|_{n_0}^{n} = \frac{m\omega^2}{k_BT}\left(\frac{r^2}{2}\right)\Bigg|_0^r$$

$$\ln\left(\frac{n}{n_0}\right) = \frac{m\omega^2}{2k_BT}\,r^2 \text{ and solving for } n\text{: } \boxed{n = n_0 e^{mr^2\omega^2/2k_BT}}$$

21.62 First find v_{av}^2 as $v_{\text{av}}^2 = \frac{1}{N}\int_0^\infty v^2 N_v dv$. Let $a = \frac{m}{2k_BT}$.

Then, $v_{\text{av}}^2 = \frac{[4N\pi^{-1/2}a^{3/2}]}{N}\int_0^\infty v^4 e^{-av^2}dv = [4a^{3/2}\pi^{-1/2}]\frac{3}{8a^2}\sqrt{\frac{\pi}{a}} = \frac{3k_BT}{m}$

The root-mean square speed is then $v_{\text{rms}} = \sqrt{v_{\text{av}}^2} = \boxed{\sqrt{\frac{3k_BT}{m}}}$

To find the average speed, we have

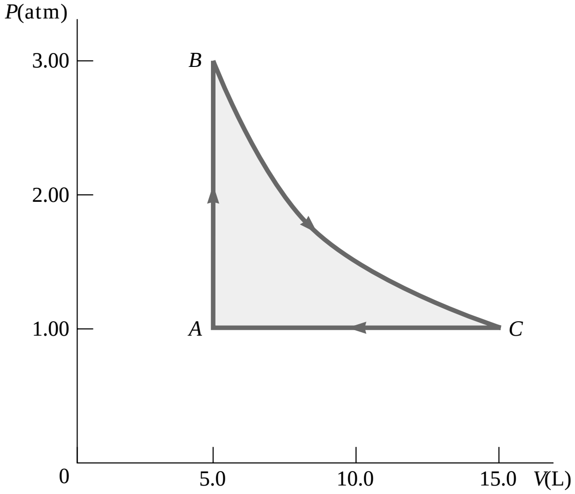
$$v_{\text{av}} = \frac{1}{N}\int_0^\infty vN_v dv =$$

$$\frac{(4Na^{3/2}\pi^{-1/2})}{N}\int_0^\infty v^3 e^{-av^2}dv = \frac{4a^{3/2}\pi^{-1/2}}{2a^2} =$$

$$\boxed{\sqrt{\frac{8k_BT}{\pi\, m}}}$$

***21.63** (a) $n = \frac{PV}{RT} = \frac{(1.013\times10^5\text{ Pa})(5.00\times10^{-3}\text{ m}^3)}{(8.315\text{ J/mol}\cdot\text{K})(300\text{ K})}$

$n = \boxed{0.203\text{ mol}}$



(b) $T_B = T_A\left(\frac{P_B}{P_A}\right) = (300\text{ K})\left(\frac{3.00}{1.00}\right) = \boxed{900\text{ K}}$

$T_C = T_B = \boxed{900\text{ K}}$

$V_C = V_A\left(\frac{T_C}{T_A}\right) = (5.00\text{ L})\left(\frac{900}{300}\right) = \boxed{15.0\text{ L}}$

(c) $E_{\text{int},A} = \frac{3}{2}nRT_A = \frac{3}{2}(0.203\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(300\text{ K}) = \boxed{760\text{ J}}$

$E_{\text{int},C} = E_{\text{int},B} = \frac{3}{2}nRT_B = \frac{3}{2}(0.203\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(900\text{ K}) = \boxed{2.28\text{ kJ}}$

(d)

	P(atm)	V(L)	T(K)	E_{int} (kJ)
A	1.00	5.00	300	0.760
B	3.00	5.00	900	2.28
C	1.00	15.00	900	2.28

(e) For the process AB, lock the piston in place and put the cylinder into an oven at 900 K. For BC, keep the sample in the oven while gradually letting the gas expand to lift a load on the piston as far as it can. For *CA*, carry the cylinder back into the room at 300 K and let the gas cool without touching the piston.

(f) For *AB*: $W = \boxed{0}$, $\Delta E_{\text{int}} = E_{\text{int},B} - E_{\text{int},A} = (2.28 - 0.769)\text{ kJ} = \boxed{1.52\text{ kJ}}$

$Q = \Delta E_{\text{int}} + W = \boxed{1.52\text{ kJ}}$

For *BC*: $\Delta E_{\text{int}} = \boxed{0}$, $W = nRT_B \ln(V_C / V_B)$

$W = (0.203\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(900\text{ K})\ln(3.00) = \boxed{1.67\text{ kJ}}$

$Q = \Delta E_{\text{int}} + W = \boxed{1.67\text{ kJ}}$

For *CA*: $\Delta E_{\text{int}} = E_{\text{int},A} - E_{\text{int},C} = (0.760 - 2.28)\text{ kJ} = \boxed{-1.52\text{ kJ}}$

$W = P\,\Delta V = nR\,\Delta T$

$W = (0.203\text{ mol})(8.315\text{ J/mol}\cdot\text{K})(-600\text{ K}) = \boxed{-1.01\text{ kJ}}$

$Q = \Delta E_{\text{int}} + W = -1.52\text{ kJ} - 1.01\text{ kJ} = \boxed{-2.53\text{ kJ}}$

(g) We add the amounts of energy for each process to find them for the whole cycle.

$Q_{ABCA} = +1.52\text{ kJ} + 1.67\text{ kJ} - 2.53\text{ kJ} = \boxed{0.656\text{ kJ}}$

$W_{ABCA} = 0 + 1.67\text{ kJ} - 1.01\text{ kJ} = \boxed{0.656\text{ kJ}}$

$(\Delta E_{\text{int}})_{ABCA} = +1.52\text{ kJ} + 0 - 1.52\text{ kJ} = \boxed{0}$

21.64 With number-per-volume $n_0 e^{(-mgy/k_BT)}$, the number of molecules above unit ground area is $\int_0^\infty n(y)dy$, and the number below altitude h is $\int_0^h n(y)dy$. So,

(a) $$f = \frac{\int_0^h n(y)dy}{\int_0^\infty n(y)dy} = \frac{n_0\int_0^h e^{(-mgy/k_BT)}dy}{n_0\int_0^\infty e^{(-mgy/k_BT)}dy} = \frac{-k_BT/mg\int_0^h e^{(-mgy/k_BT)}(-mg\,dy/k_BT)}{-k_BT/mg\int_0^\infty e^{(-mgy/k_BT)}(-mg\,dy/k_BT)}$$

$$= \frac{e^{(-mgy/k_BT)}\Big|_0^h}{e^{(-mgy/k_BT)}\Big|_0^\infty} = \frac{e^{(-mgh/k_BT)}-1}{0-1} = \boxed{1-e^{(-mgh/k_BT)}}$$

(b) $$\frac{1}{2} = 1 - e^{(-mgh'/k_BT)}$$

$$e^{(-mgh'/k_BT)} = \frac{1}{2} \quad \text{or} \quad e^{(+mgh'/k_BT)} = 2$$

$mgh'/k_BT = \ln 2$ so

$$h' = \frac{k_BT\ln 2}{mg} = \frac{(1.38\times10^{-23}\text{ J/K})(270\text{ K})(\ln 2)}{(28.9\text{ u})(1.66\times10^{-27}\text{ kg/u})(9.80\text{ m/s}^2)} = \boxed{5.47\text{ km}}$$

***21.65** (a) $$(10\,000\text{ g})\left(\frac{1.00\text{ mol}}{18.0\text{ g}}\right)\left(\frac{6.02\times10^{23}\text{ molecules}}{1.00\text{ mol}}\right) = \boxed{3.34\times10^{26}\text{ molecules}}$$

(b) After one day, 10^{-1} of the original molecules would remain. After two days, the fraction would be 10^{-2}, and so on. After 26 days, only 3 of the original molecules would likely remain, and after $\boxed{27\text{ days}}$, likely none.

(c) The soup is this fraction of the hydrosphere: $\left(\frac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)$

Therefore, today's soup likely contains this fraction of the original molecules. The number of original molecules likely in the pot again today is:

$$\left(\frac{10.0\text{ kg}}{1.32\times10^{21}\text{ kg}}\right)(3.34\times10^{26}\text{ molecules}) = \boxed{2.53\times10^{6}\text{ molecules}}$$

21.66 (a) For escape, $\frac{1}{2}mv^2 = \frac{GmM}{R}$. Since the free-fall acceleration at the surface is $g = \frac{GM}{R^2}$, this can also be written as: $\frac{1}{2}mv^2 = \frac{GmM}{R} = \boxed{mgR}$

(b) For O_2, the mass of one molecule is

$$m = \frac{0.0320\text{ kg/mol}}{6.02\times10^{23}\text{ molecules/mol}} = 5.32\times10^{-26}\text{ kg/molecule}$$

Then, if $mgR = 10(3k_BT/2)$, the temperature is

$$T = \frac{mgR}{15k_B} = \frac{(5.32 \times 10^{-26}\text{ kg})(9.80\text{ m/s}^2)(6.37 \times 10^6\text{ m})}{15(1.38 \times 10^{-23}\text{ J/mol} \cdot \text{K})} = \boxed{1.60 \times 10^4\text{ K}}$$

21.67 (a) For sodium atoms (with a molar mass $M = 32.0$ g/mol)

$$\frac{1}{2}mv^2 = \frac{3}{2}k_BT$$

$$\frac{1}{2}\left(\frac{M}{N_A}\right)v^2 = \frac{3}{2}k_BT$$

(a) $$v_{\text{rms}} = \sqrt{\frac{3RT}{M}} = \sqrt{\frac{3(8.315\text{ J/mol} \cdot \text{K})(2.40 \times 10^{-4}\text{ K})}{23.0 \times 10^{-3}\text{ kg}}} = \boxed{0.510\text{ m/s}}$$

(b) $$t = \frac{d}{v_{\text{rms}}} = \frac{0.010\text{ m}}{0.510\text{ m/s}} \approx \boxed{20\text{ ms}}$$