Chapter 23 Solutions

23.1 (a) $N = \left(\frac{10.0 \text{ grams}}{107.87 \text{ grams/mol}}\right)\left(6.02\times10^{23} \frac{\text{atoms}}{\text{mol}}\right)\left(47.0 \frac{\text{electrons}}{\text{atom}}\right) = \boxed{2.62\times10^{24}}$

(b) $\# \text{ electrons added} = \frac{Q}{e} = \frac{1.00\times10^{-3} \text{ C}}{1.60\times10^{-19} \text{ C/electron}} = 6.25\times10^{15}$

or $\boxed{2.38 \text{ electrons for every } 10^9 \text{ already present}}$

23.2 (a) $F_e = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19} \text{ C}\right)^2}{(3.80\times10^{-10} \text{ m})^2} = \boxed{1.59\times10^{-9} \text{ N}}$ (repulsion)

(b) $F_g = \frac{G m_1 m_2}{r^2} = \frac{\left(6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2\right)(1.67\times10^{-27} \text{ kg})^2}{(3.80\times10^{-10} \text{ m})^2} = \boxed{1.29\times10^{-45} \text{ N}}$

The electric force is $\boxed{\text{larger by } 1.24\times10^{36} \text{ times}}$

(c) If $k_e \frac{q_1 q_2}{r^2} = G\frac{m_1 m_2}{r^2}$ with $q_1 = q_2 = q$ and $m_1 = m_2 = m$, then

$$\frac{q}{m} = \sqrt{\frac{G}{k_e}} = \sqrt{\frac{6.67\times10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2}{8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2}} = \boxed{8.61\times10^{-11} \text{ C/kg}}$$

23.3 If each person has a mass of ≈ 70 kg and is (almost) composed of water, then each person contains

$$N \approx \left(\frac{70{,}000 \text{ grams}}{18 \text{ grams/mol}}\right)\left(6.02\times10^{23} \frac{\text{molecules}}{\text{mol}}\right)\left(10 \frac{\text{protons}}{\text{molecule}}\right) \approx 2.3\times10^{28} \text{ protons}$$

With an excess of 1% electrons over protons, each person has a charge

$$q = (0.01)(1.6\times10^{-19} \text{ C})(2.3\times10^{28}) = 3.7\times10^7 \text{ C}$$

So $F = k_e\frac{q_1 q_2}{r^2} = (9\times10^9)\frac{(3.7\times10^7)^2}{0.6^2} \text{ N} = 4\times10^{25} \text{ N} \ \boxed{\sim 10^{26} \text{ N}}$

This force is almost enough to lift a "weight" equal to that of the Earth:

$$Mg = (6\times10^{24} \text{ kg})(9.8 \text{ m/s}^2) = 6\times10^{25} \text{ N} \sim 10^{26} \text{ N}$$

23.4 We find the equal-magnitude charges on both spheres:

$$F = k_e \frac{q_1 q_2}{r^2} = k_e \frac{q^2}{r^2} \qquad \text{so} \qquad q = r\sqrt{\frac{F}{k_e}} = (1.00\text{ m})\sqrt{\frac{1.00\times 10^4\text{ N}}{8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = 1.05\times 10^{-3}\text{ C}$$

The number of electron transferred is then

$$N_{xfer} = \left(1.05\times 10^{-3}\text{ C}\right)\Big/\left(1.60\times 10^{-19}\text{ C / e}^-\right) = 6.59\times 10^{15}\text{ electrons}$$

The whole number of electrons in each sphere is

$$N_{tot} = \left(\frac{10.0\text{ g}}{107.87\text{ g / mol}}\right)\left(6.02\times 10^{23}\text{ atoms / mol}\right)\left(47\text{ e}^-\text{ / atom}\right) = 2.62\times 10^{24}\text{ e}^-$$

The fraction transferred is then

$$f = \frac{N_{xfer}}{N_{tot}} = \left(\frac{6.59\times 10^{15}}{2.62\times 10^{24}}\right) = \boxed{2.51\times 10^{-9}} = 2.51\text{ charges in every billion}$$

23.5

$$F = k_e \frac{q_1 q_2}{r^2} = \frac{\left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times 10^{-19}\text{ C}\right)^2\left(6.02\times 10^{23}\right)^2}{\left[2(6.37\times 10^6\text{ m})\right]^2} = \boxed{514\text{ kN}}$$

***23.6** (a) The force is one of attraction. The distance r in Coulomb's law is the distance between centers. The magnitude of the force is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(12.0\times 10^{-9}\text{ C}\right)\left(18.0\times 10^{-9}\text{ C}\right)}{(0.300\text{ m})^2} = \boxed{2.16\times 10^{-5}\text{ N}}$$

(b) The net charge of -6.00×10^{-9} C will be equally split between the two spheres, or -3.00×10^{-9} C on each. The force is one of repulsion, and its magnitude is

$$F = \frac{k_e q_1 q_2}{r^2} = \left(8.99\times 10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(3.00\times 10^{-9}\text{ C}\right)\left(3.00\times 10^{-9}\text{ C}\right)}{(0.300\text{ m})^2} = \boxed{8.99\times 10^{-7}\text{ N}}$$

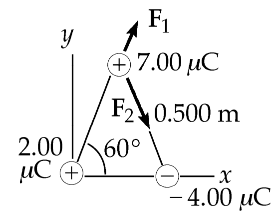
23.7 $$F_1 = k_e \frac{q_1 q_2}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(2.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} = 0.503\ \mathrm{N}$$

$$F_2 = k_e \frac{q_1 q_2}{r^2} = \frac{(8.99\times10^9\ \mathrm{N\cdot m^2/C^2})(7.00\times10^{-6}\ \mathrm{C})(4.00\times10^{-6}\ \mathrm{C})}{(0.500\ \mathrm{m})^2} = 1.01\ \mathrm{N}$$

$$F_x = (0.503 + 1.01)\cos 60.0° = 0.755\ \mathrm{N}$$

$$F_y = (0.503 - 1.01)\sin 60.0° = -0.436\ \mathrm{N}$$

$$\mathbf{F} = (0.755\ \mathrm{N})\mathbf{i} - (0.436\ \mathrm{N})\mathbf{j} = \boxed{0.872\ \mathrm{N\ at\ an\ angle\ of\ 330°}}$$



<u>Goal Solution</u>

Three point charges are located at the corners of an equilateral triangle as shown in Figure P23.7. Calculate the net electric force on the 7.00–μC charge.

G: **G**ather Information: The 7.00–μC charge experiences a repulsive force $\mathbf{F}_1$ due to the 2.00–μC charge, and an attractive force $\mathbf{F}_2$ due to the –4.00–μC charge, where $F_2 = 2F_1$. If we sketch these force vectors, we find that the resultant appears to be about the same magnitude as F_2 and is directed to the right about 30.0° below the horizontal.

O: **O**rganize: We can find the net electric force by adding the two separate forces acting on the 7.00–μC charge. These individual forces can be found by applying Coulomb's law to each pair of charges.

A: **A**nalyze: The force on the 7.00–μC charge by the 2.00–μC charge is $\mathbf{F}_1 = k_e \frac{q_1 q_2}{r^2}\hat{\mathbf{r}}$

$$\mathbf{F}_1 = \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(7.00\times10^{-6}\ \mathrm{C}\right)\left(2.00\times10^{-6}\ \mathrm{C}\right)}{(0.500\ \mathrm{m})^2}\left(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}\right) = \mathbf{F}_1 = \left(0.252\mathbf{i} + 0.436\mathbf{j}\right)\ \mathrm{N}$$

Similarly, the force on the 7.00–μC by the –4.00–μC charge is $\mathbf{F}_2 = k_e \frac{q_1 q_3}{r^2}\hat{\mathbf{r}}$

$$\mathbf{F}_2 = -\left(8.99\times10^9\ \frac{\mathrm{N\cdot m^2}}{\mathrm{C^2}}\right)\frac{\left(7.00\times10^{-6}\ \mathrm{C}\right)\left(-4.00\times10^{-6}\ \mathrm{C}\right)}{(0.500\ \mathrm{m})^2}\left(\cos 60°\,\mathbf{i} - \sin 60°\,\mathbf{j}\right) = \left(0.503\mathbf{i} - 0.872\mathbf{j}\right)\ \mathrm{N}$$

Thus, the total force on the 7.00–μC, expressed as a set of components, is

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \left(0.755\ \mathbf{i} - 0.436\ \mathbf{j}\right)\ \mathrm{N} = 0.872\ \mathrm{N\ at\ 30.0°\ below\ the\ } +x \mathrm{\ axis}$$

L: **L**earn: Our calculated answer agrees with our initial estimate. An equivalent approach to this problem would be to find the net electric field due to the two lower charges and apply $\mathbf{F}=q\mathbf{E}$ to find the force on the upper charge in this electric field.

***23.8** Let the third bead have charge Q and be located distance x from the left end of the rod. This bead will experience a net force given by

$$\mathbf{F} = \frac{k_e(3q)Q}{x^2}\mathbf{i} + \frac{k_e(q)Q}{(d-x)^2}(-\mathbf{i})$$

The net force will be zero if $\dfrac{3}{x^2} = \dfrac{1}{(d-x)^2}$, or $d - x = \dfrac{x}{\sqrt{3}}$

This gives an equilibrium position of the third bead of $x = \boxed{0.634d}$

The equilibrium is $\boxed{\text{stable if the third bead has positive charge}}$.

***23.9** (a) $F = \dfrac{k_e e^2}{r^2} = (8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)\dfrac{(1.60 \times 10^{-19}\ \text{C})^2}{(0.529 \times 10^{-10}\ \text{m})^2} = \boxed{8.22 \times 10^{-8}\ \text{N}}$

(b) We have $F = \dfrac{mv^2}{r}$ from which $v = \sqrt{\dfrac{Fr}{m}} = \sqrt{\dfrac{(8.22 \times 10^{-8}\ \text{N})(0.529 \times 10^{-10}\ \text{m})}{9.11 \times 10^{-31}\ \text{kg}}} = \boxed{2.19 \times 10^6\ \text{m/s}}$

23.10 The top charge exerts a force on the negative charge $\dfrac{k_e qQ}{(d/2)^2 + x^2}$ which is directed upward and to the left, at an angle of $\tan^{-1}(d/2x)$ to the x-axis. The two positive charges together exert force

$$\left(\frac{2k_e qQ}{\left(d^2/4 + x^2\right)}\right)\left(\frac{(-x)\mathbf{i}}{\left(d^2/4 + x^2\right)^{1/2}}\right) = m\mathbf{a} \qquad \text{or for } x \ll d/2, \qquad \mathbf{a} \approx \frac{-2k_e qQ}{md^3/8}\mathbf{x}$$

(a) The acceleration is equal to a negative constant times the excursion from equilibrium, as in $\mathbf{a} = -\omega^2\mathbf{x}$, so we have Simple Harmonic Motion with $\omega^2 = \dfrac{16k_e qQ}{md^3}$.

$T = \dfrac{2\pi}{\omega} = \boxed{\dfrac{\pi}{2}\sqrt{\dfrac{md^3}{k_e qQ}}}$, where m is the mass of the object with charge $-Q$.

(b) $v_{\max} = \omega A = \boxed{4a\sqrt{\dfrac{k_e qQ}{md^3}}}$

23.11 For equilibrium, $\mathbf{F}_e = -\mathbf{F}_g$, or $q\mathbf{E} = -mg(-\mathbf{j})$. Thus, $\mathbf{E} = \dfrac{mg}{q}\mathbf{j}$.

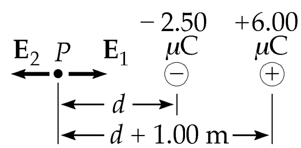
(a) $$\mathbf{E} = \frac{mg}{q}\mathbf{j} = \frac{(9.11\times10^{-31}\text{ kg})(9.80\text{ m/s}^2)}{\left(-1.60\times10^{-19}\text{ C}\right)}\mathbf{j} = \boxed{-\left(5.58\times10^{-11}\text{ N/C}\right)\mathbf{j}}$$

(b) $$\mathbf{E} = \frac{mg}{q}\mathbf{j} = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)}\mathbf{j} = \boxed{\left(1.02\times10^{-7}\text{ N/C}\right)\mathbf{j}}$$

23.12 $\sum F_y = 0$: $\quad QE\mathbf{j} + mg(-\mathbf{j}) = 0$

$$\therefore \qquad m = \frac{QE}{g} = \frac{(24.0\times10^{-6}\text{ C})(610\text{ N / C})}{9.80\text{ m / s}^2} = \boxed{1.49\text{ grams}}$$

***23.13** The point is designated in the sketch. The magnitudes of the electric fields, E_1, (due to the -2.50×10^{-6} C charge) and E_2 (due to the 6.00×10^{-6} C charge) are



$$E_1 = \frac{k_e q}{r^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(2.50\times10^{-6}\text{ C})}{d^2} \qquad (1)$$

$$E_2 = \frac{k_e q}{r^2} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(6.00\times10^{-6}\text{ C})}{(d+1.00\text{ m})^2} \qquad (2)$$

Equate the right sides of (1) and (2) to get $\quad (d + 1.00\text{ m})^2 = 2.40d^2$

or $\quad d + 1.00\text{ m} = \pm1.55d$

which yields $\quad d = 1.82\text{ m} \quad$ or $\quad d = -0.392\text{ m}$

The negative value for d is unsatisfactory because that locates a point between the charges where both fields are in the same direction. Thus, $\boxed{d = 1.82\text{ m to the left of the -2.50}\ \mu\text{C charge.}}$

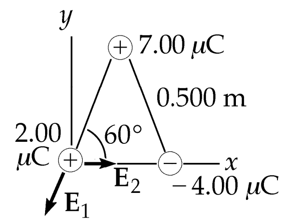
23.14 If we treat the concentrations as point charges,

$$\mathbf{E}_+ = k_e\frac{q}{r^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(40.0\text{ C})}{(1000\text{ m})^2}(-\mathbf{j}) = 3.60\times10^5\text{ N / C}\,(-\mathbf{j})\ \text{(downward)}$$

$$\mathbf{E}_- = k_e\frac{q}{r^2} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(40.0\text{ C})}{(1000\text{ m})^2}(-\mathbf{j}) = 3.60\times10^5\text{ N / C}\,(-\mathbf{j})\ \text{(downward)}$$

$$\mathbf{E} = \mathbf{E}_+ + \mathbf{E}_- = \boxed{7.20\times10^5\text{ N / C downward}}$$

***23.15** (a) $E_1 = \dfrac{k_e q}{r^2} = \dfrac{\left(8.99\times10^9\right)\left(7.00\times10^{-6}\right)}{(0.500)^2} = 2.52\times10^5 \text{ N/C}$



$E_2 = \dfrac{k_e q}{r^2} = \dfrac{\left(8.99\times10^9\right)\left(4.00\times10^{-6}\right)}{(0.500)^2} = 1.44\times10^5 \text{ N/C}$

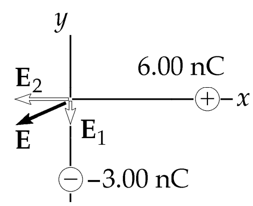
$E_x = E_2 - E_1 \cos 60° = 1.44\times10^5 - 2.52\times10^5 \cos 60.0° = 18.0\times10^3 \text{ N/C}$

$E_y = -E_1 \sin 60.0° = -2.52\times10^5 \sin 60.0° = -218\times10^3 \text{ N/C}$

$\mathbf{E} = [18.0\mathbf{i} - 218\mathbf{j}]\times10^3 \text{ N/C} = \boxed{[18.0\mathbf{i} - 218\mathbf{j}] \text{ kN/C}}$

(b) $\mathbf{F} = q\mathbf{E} = \left(2.00\times10^{-6}\text{ C}\right)\left(18.0\mathbf{i} - 218\mathbf{j}\right)\times10^3 \text{ N/C} = \left(36.0\mathbf{i} - 436\mathbf{j}\right)\times10^{-3}\text{ N} = \boxed{\left(36.0\mathbf{i} - 436\mathbf{j}\right)\text{mN}}$

***23.16** (a) $\mathbf{E}_1 = \dfrac{k_e|q_1|}{r_1^2}(-\mathbf{j}) = \dfrac{\left(8.99\times10^9\right)\left(3.00\times10^{-9}\right)}{(0.100)^2}(-\mathbf{j}) = -\left(2.70\times10^3 \text{ N/C}\right)\mathbf{j}$



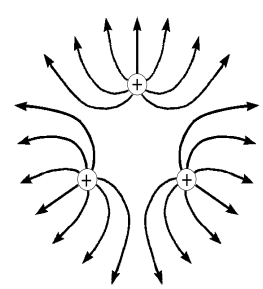
$\mathbf{E}_2 = \dfrac{k_e|q_2|}{r_2^2}(-\mathbf{i}) = \dfrac{\left(8.99\times10^9\right)\left(6.00\times10^{-9}\right)}{(0.300)^2}(-\mathbf{i}) = -\left(5.99\times10^2 \text{ N/C}\right)\mathbf{i}$

$\mathbf{E} = \mathbf{E}_2 + \mathbf{E}_1 = \boxed{-\left(5.99\times10^2 \text{ N/C}\right)\mathbf{i} - \left(2.70\times10^3 \text{ N/C}\right)\mathbf{j}}$

(b) $\mathbf{F} = q\mathbf{E} = \left(5.00\times10^{-9}\text{ C}\right)\left(-599\mathbf{i} - 2700\mathbf{j}\right)\text{N/C}$

$\mathbf{F} = \left(-3.00\times10^{-6}\,\mathbf{i} - 13.5\times10^{-6}\,\mathbf{j}\right)\text{N} = \boxed{\left(-3.00\mathbf{i} - 13.5\mathbf{j}\right)\,\mu\text{N}}$

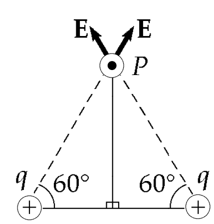
23.17 (a) The electric field has the general appearance shown. It is zero $\boxed{\text{at the center}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out.



(b) You may need to review vector addition in Chapter Three.

$\mathbf{E} = k_e \sum_i \dfrac{q_i}{r_i^2}\,\hat{\mathbf{r}}_i$

The magnitude of the field at point P due to each of the charges along the base of the triangle is $E = k_e\, q/a^2$. The direction of the field in each case is along the line joining the charge in question to point P as shown in the diagram at the right. The x components add to zero, leaving



$\mathbf{E} = \dfrac{k_e q}{a^2}(\sin 60.0°)\mathbf{j} + \dfrac{k_e q}{a^2}(\sin 60.0°)\mathbf{j} = \boxed{\sqrt{3}\,\dfrac{k_e q}{a^2}\,\mathbf{j}}$

Goal Solution

Three equal positive charges q are at the corners of an equilateral triangle of side a, as shown in Figure P23.17. (a) Assume that the three charges together create an electric field. Find the location of a point (other than ∞) where the electric field is zero. (**Hint:** Sketch the field lines in the plane of the charges.) (b) What are the magnitude and direction of the electric field at P due to the two charges at the base?

G: The electric field has the general appearance shown by the black arrows in the figure to the right. This drawing indicates that $\mathbf{E} = 0$ at the center of the triangle, since a small positive charge placed at the center of this triangle will be pushed away from each corner equally strongly. This fact could be verified by vector addition as in part (b) below.

The electric field at point P should be directed upwards and about twice the magnitude of the electric field due to just one of the lower charges as shown in Figure P23.17. For part (b), we must ignore the effect of the charge at point P, because a charge cannot exert a force on itself.

O: The electric field at point P can be found by adding the electric field vectors due to each of the two lower point charges: $\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$

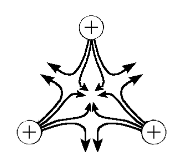
A: (b) The electric field from a point charge is $\mathbf{E} = k_e \dfrac{q}{r^2}\hat{\mathbf{r}}$

As shown in the solution figure above, $\mathbf{E}_1 = k_e \dfrac{q}{a^2}$ to the right and upward at 60°

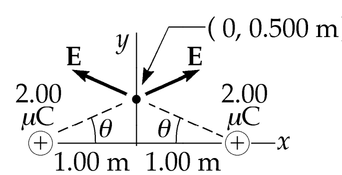
$\mathbf{E}_2 = k_e \dfrac{q}{a^2}$ to the left and upward at 60°

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 = k_e \frac{q}{a^2}\left[\left(\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}\right) + \left(-\cos 60°\,\mathbf{i} + \sin 60°\,\mathbf{j}\right)\right] = k_e \frac{q}{a^2}\left[2\left(\sin 60°\,\mathbf{j}\right)\right] = 1.73 k_e \frac{q}{a^2}\,\mathbf{j}$$

L: The net electric field at point P is indeed nearly twice the magnitude due to a single charge and is entirely vertical as expected from the symmetry of the configuration. In addition to the center of the triangle, the electric field lines in the figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.



23.18 (a) $E = \dfrac{k_e q}{r^2} = \dfrac{(8.99\times10^9)(2.00\times10^{-6})}{(1.12)^2} = 14{,}400 \text{ N/C}$



$E_x = 0$ and $E_y = 2(14{,}400) \sin 26.6° = 1.29\times10^4$ N/C

so $\boxed{\mathbf{E} = 1.29\times10^4\,\mathbf{j}\text{ N/C}}$

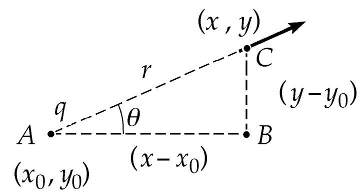
(b) $\mathbf{F} = \mathbf{E}q = (1.29\times10^4\,\mathbf{j})(-3.00\times10^{-6}) = \boxed{-3.86\times10^{-2}\,\mathbf{j}\text{ N}}$

23.19 (a) $\mathbf{E} = \dfrac{k_e q_1}{r_1^2}\hat{\mathbf{r}}_1 + \dfrac{k_e q_2}{r_2^2}\hat{\mathbf{r}}_2 + \dfrac{k_e q_3}{r_3^2}\hat{\mathbf{r}}_3 = \dfrac{k_e(2q)}{a^2}\mathbf{i} + \dfrac{k_e(3q)}{2a^2}(\mathbf{i}\cos 45.0° + \mathbf{j}\sin 45.0°) + \dfrac{k_e(4q)}{a^2}\mathbf{j}$

$$\mathbf{E} = 3.06\frac{k_e q}{a^2}\mathbf{i} + 5.06\frac{k_e q}{a^2}\mathbf{j} = \boxed{5.91\frac{k_e q}{a^2} \text{ at } 58.8°}$$

(b) $\mathbf{F} = q\mathbf{E} = \boxed{5.91\dfrac{k_e q^2}{a^2} \text{ at } 58.8°}$

23.20 The magnitude of the field at (x, y) due to charge q at (x_0, y_0) is given by $E = k_e q/r^2$ where r is the distance from (x_0, y_0) to (x, y). Observe the geometry in the diagram at the right. From triangle ABC, $r^2 = (x - x_0)^2 + (y - y_0)^2$, or



$$r = \sqrt{(x - x_0)^2 + (y - y_0)^2}, \quad \sin\theta = \frac{(y - y_0)}{r}, \quad \text{and} \quad \cos\theta = \frac{(x - x_0)}{r}$$

Thus, $E_x = E\cos\theta = \dfrac{k_e q}{r^2}\dfrac{(x - x_0)}{r} = \boxed{\dfrac{k_e q(x - x_0)}{[(x - x_0)^2 + (y - y_0)^2]^{3/2}}}$

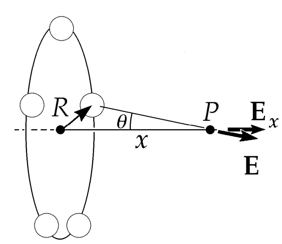
and $E_y = E\sin\theta = \dfrac{k_e q}{r^2}\dfrac{(y - y_0)}{r} = \boxed{\dfrac{k_e q(y - y_0)}{[(x - x_0)^2 + (y - y_0)^2]^{3/2}}}$

23.21 The electric field at any point x is $E = \dfrac{k_e q}{(x - a)^2} - \dfrac{k_e q}{(x - (-a))^2} = \dfrac{k_e q(4ax)}{(x^2 - a^2)^2}$

When x is much, much greater than a, we find $E \approx \boxed{\dfrac{(4a)(k_e q)}{x^3}}$

23.22 (a) One of the charges creates at P a field $\mathbf{E} = \dfrac{k_e Q/n}{R^2 + x^2}$

at an angle θ to the x-axis as shown.



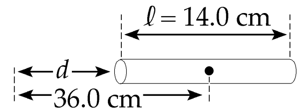
When all the charges produce field, for $n > 1$, the components perpendicular to the x-axis add to zero.

The total field is $\dfrac{nk_e(Q/n)\mathbf{i}}{R^2 + x^2}\cos\theta = \boxed{\dfrac{k_e Q x\mathbf{i}}{(R^2 + x^2)^{3/2}}}$

(b) A circle of charge corresponds to letting n grow beyond all bounds, but the result does not depend on n. Smearing the charge around the circle does not change its amount or its distance from the field point, so it $\boxed{\text{does not change the field.}}$.

23.23 $$\mathbf{E} = \sum \frac{k_e q}{r^2} \sim= \frac{k_e q}{a^2}(-\,\mathbf{i}) + \frac{k_e q}{(2a)^2}(-\,\mathbf{i}) + \frac{k_e q}{(3a)^2}(-\,\mathbf{i}) + \ \ldots \ = \frac{-\,k_e q\,\mathbf{i}}{a^2}\left(1 + \frac{1}{2^2} + \frac{1}{3^3} + \ \ldots\right) \ = \boxed{-\,\frac{\pi^2 k_e q}{6a^2}\,\mathbf{i}}$$

23.24 $$E = \frac{k_e \lambda \ell}{d(\ell + d)} = \frac{k_e (Q/\ell)\ell}{d(\ell + d)} = \frac{k_e Q}{d(\ell + d)} \ = \frac{(8.99 \times 10^9)(22.0 \times 10^{-6})}{(0.290)(0.140 + 0.290)}$$



$$\mathbf{E} \ = \boxed{1.59 \times 10^6 \text{ N/C}} \ , \quad \boxed{\text{directed toward the rod}}\ .$$

23.25 $$E = \int \frac{k_e\,dq}{x^2} \qquad \text{where } dq = \lambda_0\,dx$$

$$E = k_e\,\lambda_0 \int_{x_0}^{\infty} \frac{dx}{x^2} \ = k_e\left(-\frac{1}{x}\right)\Bigg|_{x_0}^{\infty} \ = \boxed{\frac{k_e \lambda_0}{x_0}}$$ $\boxed{\text{The direction is } -\mathbf{i} \text{ or left for } \lambda_0 > 0}$

23.26 $$\mathbf{E} = \int d\mathbf{E} = \int_{x_0}^{\infty}\left[\frac{k_e \lambda_0 x_0\;dx(-\mathbf{i})}{x^3}\right] = -\,k_e \lambda_0 x_0\,\mathbf{i}\int_{x_0}^{\infty} x^{-3}\;dx = -\,k_e \lambda_0 x_0\,\mathbf{i}\left(-\frac{1}{2x^2}\Bigg|_{x_0}^{\infty}\right) = \boxed{\frac{k_e \lambda_0}{2x_0}(-\mathbf{i})}$$

23.27 $$E = \frac{k_e x\,Q}{(x^2 + a^2)^{3/2}} \ = \frac{(8.99 \times 10^9)(75.0 \times 10^{-6})x}{(x^2 + 0.100^2)^{3/2}} \ = \frac{6.74 \times 10^5\;x}{(x^2 + 0.0100)^{3/2}}$$

(a) At $x = 0.0100$ m, $\mathbf{E} = 6.64 \times 10^6\ \mathbf{i}$ N/C = $\boxed{6.64\,\mathbf{i}\ \text{MN/C}}$

(b) At $x = 0.0500$ m, $\mathbf{E} = 2.41 \times 10^7\ \mathbf{i}$ N/C = $\boxed{24.1\,\mathbf{i}\ \text{MN/C}}$

(c) At $x = 0.300$ m, $\mathbf{E} = 6.40 \times 10^6\ \mathbf{i}$ N/C = $\boxed{6.40\,\mathbf{i}\ \text{MN/C}}$

(d) At $x = 1.00$ m, $\mathbf{E} = 6.64 \times 10^5\ \mathbf{i}$ N/C = $\boxed{0.664\,\mathbf{i}\ \text{MN/C}}$

23.28 $E = \dfrac{k_e Q x}{(x^2 + a^2)^{3/2}}$

For a maximum, $\dfrac{dE}{dx} = Qk_e\left[\dfrac{1}{(x^2+a^2)^{3/2}} - \dfrac{3x^2}{(x^2+a^2)^{5/2}}\right] = 0$

$x^2 + a^2 - 3x^2 = 0$ or $x = \dfrac{a}{\sqrt{2}}$

Substituting into the expression for E gives

$$E = \frac{k_e Qa}{\sqrt{2}\left(\frac{3}{2}a^2\right)^{3/2}} = \frac{k_e Q}{3\frac{\sqrt{3}}{2}a^2} = \boxed{\frac{2k_e Q}{3\sqrt{3}\,a^2}} = \boxed{\frac{Q}{6\sqrt{3}\pi \epsilon_0 a^2}}$$

23.29 $E = 2\pi k_e \sigma\left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)$

$$E = 2\pi\left(8.99\times10^9\right)\left(7.90\times10^{-3}\right)\left(1 - \frac{x}{\sqrt{x^2+(0.350)^2}}\right) = 4.46\times10^8\left(1 - \frac{x}{\sqrt{x^2+0.123}}\right)$$

(a) At x = 0.0500 m, $E = 3.83\times10^8$ N/C = $\boxed{383\ \text{MN/C}}$

(b) At x = 0.100 m, $E = 3.24\times10^8$ N/C = $\boxed{324\ \text{MN/C}}$

(c) At x = 0.500 m, $E = 8.07\times10^7$ N/C = $\boxed{80.7\ \text{MN/C}}$

(d) At x = 2.00 m, $E = 6.68\times10^8$ N/C = $\boxed{6.68\ \text{MN/C}}$

23.30 (a) From Example 23.9: $E = 2\pi k_e \sigma\left(1 - \dfrac{x}{\sqrt{x^2+R^2}}\right)$

$\sigma = \dfrac{Q}{\pi R^2} = 1.84\times10^{-3}\ \text{C/m}^2$

$E = (1.04\times10^8\ \text{N/C})(0.900) = 9.36\times10^7\ \text{N/C} = \boxed{93.6\ \text{MN/C}}$

appx: $E = 2\pi k_e \sigma = \boxed{104\ \text{MN/C (about 11\% high)}}$

(b) $E = (1.04\times10^8\ \text{N / C})\left(1 - \dfrac{30.0\ \text{cm}}{\sqrt{30.0^2+3.00^2}\ \text{cm}}\right) = (1.04\times10^8\ \text{N/C})(0.00496) = \boxed{0.516\ \text{MN/C}}$

appx: $E = k_e \dfrac{Q}{r^2} = (8.99\times10^9)\dfrac{5.20\times10^{-6}}{(0.30)^2} = \boxed{0.519\ \text{MN/C (about 0.6\% high)}}$

23.31 The electric field at a distance x is $E_x = 2\pi k_e \sigma\left[1 - \dfrac{x}{\sqrt{x^2+R^2}}\right]$

This is equivalent to $E_x = 2\pi k_e \sigma\left[1 - \dfrac{1}{\sqrt{1+R^2/x^2}}\right]$

For large x, $R^2/x^2 << 1$ and $\sqrt{1+\dfrac{R^2}{x^2}} \approx 1 + \dfrac{R^2}{2x^2}$

so $E_x = 2\pi k_e \sigma\left(1 - \dfrac{1}{\left[1+R^2/(2x^2)\right]}\right) = 2\pi k_e \sigma \dfrac{\left(1+R^2/(2x^2)-1\right)}{\left[1+R^2/(2x^2)\right]}$

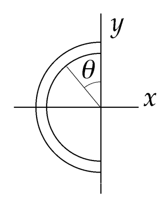
Substitute $\sigma = Q/\pi R^2$, $E_x = \dfrac{k_e Q\left(1/x^2\right)}{\left[1+R^2/(2x^2)\right]} = k_e Q\left(x^2 + \dfrac{R^2}{2}\right)$

But for $x >> R$, $\dfrac{1}{x^2+R^2/2} \approx \dfrac{1}{x^2}$, so $\boxed{E_x \approx \dfrac{k_e Q}{x^2} \text{ for a disk at large distances}}$

23.32 The sheet must have negative charge to repel the negative charge on the Styrofoam. The magnitude of the upward electric force must equal the magnitude of the downward gravitational force for the Styrofoam to "float" (i.e., $F_e = F_g$).

Thus, $-qE = mg$, or $-q\left(\dfrac{\sigma}{2e_0}\right) = mg$ which gives $\sigma = \boxed{-\dfrac{2e_0 mg}{q}}$

23.33 Due to symmetry $E_y = \int dE_y = 0$, and $E_x = \int dE \sin\theta = k_e \int \dfrac{dq \sin\theta}{r^2}$



where $dq = \lambda ds = \lambda r\, d\theta$, so that, $E_x = \dfrac{k_e\lambda}{r}\int_0^\pi \sin\theta\, d\theta = \dfrac{k_e\lambda}{r}(-\cos\theta)\Big|_0^\pi = \dfrac{2k_e\lambda}{r}$

where $\lambda = \dfrac{q}{L}$ and $r = \dfrac{L}{\pi}$. Thus, $E_x = \dfrac{2k_e q\pi}{L^2} = \dfrac{2(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(7.50\times10^{-6}\text{ C})\pi}{(0.140\text{ m})^2}$

Solving, $E = E_x = 2.16\times10^7$ N/C

Since the rod has a negative charge, $\mathbf{E} = (-2.16\times10^7\,\mathbf{i})$ N/C = $\boxed{-21.6\,\mathbf{i}\text{ MN/C}}$

23.34 (a) We define $x = 0$ at the point where we are to find the field. One ring, with thickness dx, has charge Qdx/h and produces, at the chosen point, a field

$$d\mathbf{E} = \frac{k_e x}{(x^2 + R^2)^{3/2}} \frac{Q\,dx}{h}\,\mathbf{i}$$

The total field is

$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_d^{d+h} \frac{k_e Q x\,dx}{h(x^2 + R^2)^{3/2}}\,\mathbf{i} = \frac{k_e Q\mathbf{i}}{2h}\int_{x=d}^{d+h} (x^2 + R^2)^{-3/2}\, 2x\,dx$$

$$\mathbf{E} = \frac{k_e Q\mathbf{i}}{2h}\,\frac{(x^2 + R^2)^{-1/2}}{(-1/2)}\Bigg|_{x=d}^{d+h} = \boxed{\frac{k_e Q\mathbf{i}}{h}\left[\frac{1}{(d^2 + R^2)^{1/2}} - \frac{1}{\left((d+h)^2 + R^2\right)^{1/2}}\right]}$$

(b) Think of the cylinder as a stack of disks, each with thickness dx, charge $Q\,dx/h$, and charge-per-area $\sigma = Q\,dx/\pi R^2 h$. One disk produces a field

$$d\mathbf{E} = \frac{2\pi k_e Q\,dx}{\pi R^2 h}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)\mathbf{i}$$

So, $$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = \int_{x=d}^{d+h} \frac{2k_e Q\,dx}{R^2 h}\left(1 - \frac{x}{(x^2 + R^2)^{1/2}}\right)\mathbf{i}$$

$$\mathbf{E} = \frac{2k_e Q\mathbf{i}}{R^2 h}\left[\int_d^{d+h} dx - \frac{1}{2}\int_{x=d}^{d+h} (x^2 + R^2)^{-1/2}\, 2x\,dx\right] = \frac{2k_e Q\mathbf{i}}{R^2 h}\left[x\Bigg|_d^{d+h} - \frac{1}{2}\,\frac{(x^2 + R^2)^{1/2}}{1/2}\Bigg|_d^{d+h}\right]$$

$$\mathbf{E} = \frac{2k_e Q\mathbf{i}}{R^2 h}\left[d + h - d - \left((d+h)^2 + R^2\right)^{1/2} + (d^2 + R^2)^{1/2}\right]$$

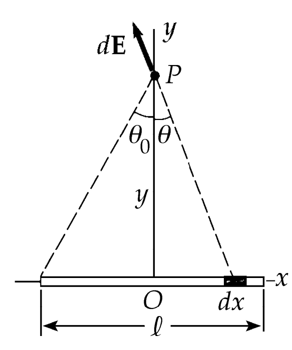
$$\mathbf{E} = \boxed{\frac{2k_e Q\mathbf{i}}{R^2 h}\left[h + (d^2 + R^2)^{1/2} - \left((d+h)^2 + R^2\right)^{1/2}\right]}$$

23.35 (a) The electric field at point P due to each element of length dx, is $dE = \dfrac{k_e dq}{(x^2 + y^2)}$ and is directed along the line joining the element of length to point P. By symmetry,

$E_x = \int dE_x = 0$ and since $dq = \lambda\,dx$,

$$E = E_y = \int dE_y = \int dE \cos\theta \quad \text{where} \quad \cos\theta = \frac{y}{(x^2 + y^2)^{1/2}}$$

Therefore, $$E = 2k_e \lambda y \int_0^{\ell/2} \frac{dx}{(x^2 + y^2)^{3/2}} = \boxed{\frac{2k_e \lambda \sin\theta_0}{y}}$$



(b) For a bar of infinite length, $\theta \to 90°$ and $E_y = \boxed{\dfrac{2k_e \lambda}{y}}$

***23.36** (a) The whole surface area of the cylinder is $A = 2\pi r^2 + 2\pi rL = 2\pi r(r+L)$.

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)2\pi(0.0250\ \mathrm{m})[0.0250\ \mathrm{m} + 0.0600\ \mathrm{m}] = \boxed{2.00\times10^{-10}\ \mathrm{C}}$$

(b) For the curved lateral surface only, $A = 2\pi rL$.

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)2\pi(0.0250\ \mathrm{m})(0.0600\ \mathrm{m}) = \boxed{1.41\times10^{-10}\ \mathrm{C}}$$

(c) $Q = \rho V = \rho\pi r^2 L = \left(500\times10^{-9}\ \mathrm{C/m^3}\right)\pi(0.0250\ \mathrm{m})^2(0.0600\ \mathrm{m}) = \boxed{5.89\times10^{-11}\ \mathrm{C}}$

***23.37** (a) Every object has the same volume, $V = 8(0.0300\ \mathrm{m})^3 = 2.16\times10^{-4}\ \mathrm{m^3}$.

For each, $Q = \rho V = \left(400\times10^{-9}\ \mathrm{C/m^3}\right)\left(2.16\times10^{-4}\ \mathrm{m^3}\right) = \boxed{8.64\times10^{-11}\ \mathrm{C}}$

(b) We must count the $9.00\ \mathrm{cm}^2$ squares painted with charge:

(i) $6\times4 = 24$ squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)24.0\left(9.00\times10^{-4}\ \mathrm{m^2}\right) = \boxed{3.24\times10^{-10}\ \mathrm{C}}$$

(ii) 34 squares exposed

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)34.0\left(9.00\times10^{-4}\ \mathrm{m^2}\right) = \boxed{4.59\times10^{-10}\ \mathrm{C}}$$

(iii) 34 squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)34.0\left(9.00\times10^{-4}\ \mathrm{m^2}\right) = \boxed{4.59\times10^{-10}\ \mathrm{C}}$$

(iv) 32 squares

$$Q = \sigma A = \left(15.0\times10^{-9}\ \mathrm{C/m^2}\right)32.0\left(9.00\times10^{-4}\ \mathrm{m^2}\right) = \boxed{4.32\times10^{-10}\ \mathrm{C}}$$

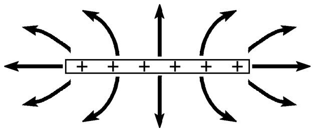
(c) (i) total edge length: $\ell = 24\times(0.0300\ \mathrm{m})$

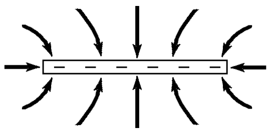
$$Q = \lambda\ell = \left(80.0\times10^{-12}\ \mathrm{C/m}\right)24\times(0.0300\ \mathrm{m}) = \boxed{5.76\times10^{-11}\ \mathrm{C}}$$

(ii) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \mathrm{C/m}\right)44\times(0.0300\ \mathrm{m}) = \boxed{1.06\times10^{-10}\ \mathrm{C}}$

(iii) $Q = \lambda\ell = \left(80.0\times10^{-12}\ \mathrm{C/m}\right)64\times(0.0300\ \mathrm{m}) = \boxed{1.54\times10^{-10}\ \mathrm{C}}$

(iv) $Q = \lambda \ell = \left(80.0 \times 10^{-12} \text{ C/m}\right) 40 \times (0.0300 \text{ m}) = \boxed{0.960 \times 10^{-10} \text{ C}}$

22.38 

22.39 

23.40 (a) $\dfrac{q_1}{q_2} = \dfrac{-6}{18} = \boxed{-\dfrac{1}{3}}$

(b) $\boxed{q_1 \text{ is negative, } \; q_2 \text{ is positive}}$

23.41 $F = qE = ma \quad a = \dfrac{qE}{m}$

$v = v_i + at \qquad v = \dfrac{qEt}{m}$

electron: $\quad v_e = \dfrac{(1.602\times10^{-19})(520)(48.0\times10^{-9})}{9.11\times10^{-31}} = \boxed{4.39\times10^{6}\text{ m/s}}$

in a direction opposite to the field

proton: $\quad v_p = \dfrac{(1.602\times10^{-19})(520)(48.0\times10^{-9})}{1.67\times10^{-27}} = \boxed{2.39\times10^{3}\text{ m/s}}$

in the same direction as the field

23.42 (a) $|a| = \dfrac{qE}{m} = \dfrac{(1.602\times10^{-19})(6.00\times10^{5})}{(1.67\times10^{-27})} = 5.76\times10^{13}\text{ m/s}$ so $\qquad \mathbf{a} = \boxed{-5.76\times10^{13}\,\mathbf{i}\text{ m/s}^2}$

(b) $v = v_i + 2a(x - x_i)$

$0 = v_i^{\,2} + 2(-5.76\times10^{13})(0.0700) \qquad \boxed{\mathbf{v}_i = 2.84\times10^{6}\,\mathbf{i}\text{ m/s}}$

(c) $v = v_i + at$

$0 = 2.84\times10^{6} + (-5.76\times10^{13})t \qquad t = \boxed{4.93\times10^{-8}\text{ s}}$

23.43 (a) $a = \dfrac{qE}{m} = \dfrac{\left(1.602 \times 10^{-19}\right)(640)}{\left(1.67 \times 10^{-27}\right)} = \boxed{6.14 \times 10^{10}\ \text{m}/\text{s}^2}$

(b) $v = v_i + at$

$1.20 \times 10^6 = (6.14 \times 10^{10})t$

$\boxed{t = 1.95 \times 10^{-5}\ \text{s}}$

(c) $x - x_i = \frac{1}{2}\left(v_i + v\right)t$

$x = \frac{1}{2}\left(1.20 \times 10^6\right)\left(1.95 \times 10^{-5}\right) = \boxed{11.7\ \text{m}}$

(d) $K = \frac{1}{2}mv^2 = \frac{1}{2}(1.67 \times 10^{-27}\ \text{kg})(1.20 \times 10^6\ \text{m}/\text{s})^2 = \boxed{1.20 \times 10^{-15}\ \text{J}}$

23.44 The required electric field will be $\boxed{\text{in the direction of motion}}$. We know that Work $= \Delta K$

So, $-Fd = -\frac{1}{2}\, m v_i^2$ (since the final velocity = 0)

This becomes $Eed = \dfrac{1}{2}mv_i^2$ or $E = \dfrac{\frac{1}{2}mv_i^2}{ed}$

$$E = \frac{1.60 \times 10^{-17}\ \text{J}}{(1.60 \times 10^{-19}\ \text{C})(0.100\ \text{m})} = \boxed{1.00 \times 10^3\ \text{N}/\text{C}}$$ (in direction of electron's motion)

23.45 The required electric field will be $\boxed{\text{in the direction of motion}}$.

Work done $= \Delta K$ so, $-Fd = -\frac{1}{2}\, m v_i^2$ (since the final velocity = 0)

which becomes $eEd = K$ and $\boxed{E = \dfrac{K}{ed}}$

Goal Solution

The electrons in a particle beam each have a kinetic energy K. What are the magnitude and direction of the electric field that stops these electrons in a distance of d?

G: We should expect that a larger electric field would be required to stop electrons with greater kinetic energy. Likewise, **E** must be greater for a shorter stopping distance, d. The electric field should be in the same direction as the motion of the negatively charged electrons in order to exert an opposing force that will slow them down.

O: The electrons will experience an electrostatic force $\mathbf{F} = q\mathbf{E}$. Therefore, the work done by the electric field can be equated with the initial kinetic energy since energy should be conserved.

A: The work done on the charge is $W = \mathbf{F}\cdot\mathbf{d} = q\mathbf{E}\cdot\mathbf{d}$

and $K_i + W = K_f = 0$

Assuming **v** is in the $+x$ direction, $K + (-e)\mathbf{E}\cdot d\mathbf{i} = 0$

$$e\mathbf{E}\cdot(d\mathbf{i}) = K$$

E is therefore in the direction of the electron beam: $\mathbf{E} = \frac{K}{ed}\mathbf{i}$

L: As expected, the electric field is proportional to K, and inversely proportional to d. The direction of the electric field is important; if it were otherwise the electron would speed up instead of slowing down! If the particles were protons instead of electrons, the electric field would need to be directed opposite to **v** in order for the particles to slow down.

23.46 The acceleration is given by $v^2 = v_i^2 + 2a(x - x_i)$ or $v^2 = 0 + 2a(-h)$

Solving, $a = -\frac{v^2}{2h}$

Now $\sum \mathbf{F} = m\mathbf{a}$: $-mg\mathbf{j} + q\mathbf{E} = -\frac{mv^2\,\mathbf{j}}{2h}$

Therefore $q\mathbf{E} = \left(-\frac{mv^2}{2h} + mg\right)\mathbf{j}$

(a) Gravity alone would give the bead downward impact velocity

$$\sqrt{2\left(9.80\text{ m / s}^2\right)(5.00\text{ m})} = 9.90\text{ m / s}$$

To change this to 21.0 m/s down, a $\boxed{\text{downward}}$ electric field must exert a downward electric force.

(b) $$q = \frac{m}{E}\left(\frac{v^2}{2h} - g\right) = \frac{1.00\times10^{-3}\text{ kg}}{1.00\times10^{4}\text{ N/C}}\left(\frac{\text{N}\cdot\text{s}^2}{\text{kg}\cdot\text{m}}\right)\left(\frac{(21.0\text{ m/s})^2}{2(5.00\text{ m})} - 9.80\text{ m/s}^2\right) = \boxed{3.43\ \mu\text{C}}$$

23.47 (a) $t = \frac{x}{v} = \frac{0.0500}{4.50 \times 10^5} = 1.11 \times 10^{-7}\text{ s} = \boxed{111\text{ ns}}$

(b) $a_y = \frac{qE}{m} = \frac{(1.602 \times 10^{-19})(9.60 \times 10^3)}{(1.67 \times 10^{-27})} = 9.21 \times 10^{11}\text{ m / s}^2$

$y - y_i = v_{yi}t + \frac{1}{2}a_y t^2$

$y = \frac{1}{2}(9.21 \times 10^{11})(1.11 \times 10^{-7})^2 = 5.67 \times 10^{-3}\text{ m} = \boxed{5.67\text{ mm}}$

(c) $v_x = \boxed{4.50 \times 10^5\text{ m/s}}$

$v_y = v_{y\,i} + a_y = (9.21 \times 10^{11})(1.11 \times 10^{-7}) = \boxed{1.02 \times 10^5\text{ m/s}}$

23.48 $a_y = \frac{qE}{m} = \frac{(1.602 \times 10^{-19})(390)}{(9.11 \times 10^{-31})} = 6.86 \times 10^{13}\text{ m / s}^2$

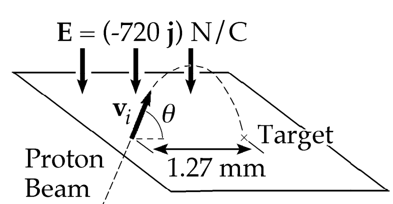
(a) $t = \frac{2v_i \sin\theta}{a_y}$ from projectile motion equations

$t = \frac{2(8.20 \times 10^5)\sin 30.0°}{6.86 \times 10^{13}} = 1.20 \times 10^{-8}\text{ s)} = \boxed{12.0\text{ ns}}$

(b) $h = \frac{v_i^2 \sin^2\theta}{2a_y} = \frac{(8.20 \times 10^5)^2 \sin^2 30.0°}{2(6.86 \times 10^{13})} = \boxed{1.23\text{ mm}}$

(c) $R = \frac{v_i^2 \sin 2\theta}{2a_y} = \frac{(8.20 \times 10^5)^2 \sin 60.0°}{2(6.86 \times 10^{13})} = \boxed{4.24\text{ mm}}$

23.49 $v_i = 9.55 \times 10^3\text{ m/s}$



(a) $a_y = \frac{eE}{m} = \frac{(1.60 \times 10^{-19})(720)}{(1.67 \times 10^{-27})} = 6.90 \times 10^{10}\text{ m/s}^2$

$R = \frac{v_i^2 \sin 2\theta}{a_y} = 1.27 \times 10^{-3}\text{ m}$ so that $\frac{(9.55 \times 10^3)^2 \sin 2\theta}{6.90 \times 10^{10}} = 1.27 \times 10^{-3}$

$\sin 2\theta = 0.961$ $\theta = \boxed{36.9°}$ $90.0° - \theta = \boxed{53.1°}$

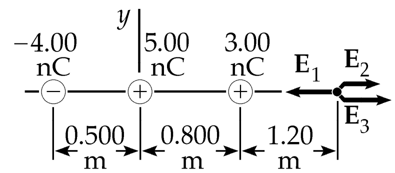
(b) $t = \frac{R}{v_{ix}} = \frac{R}{v_i \cos\theta}$ If $\theta = 36.9°$, $t = \boxed{167\text{ ns}}$ If $\theta = 53.1°$, $t = \boxed{221\text{ ns}}$

***23.50** (a) The field, E_1, due to the 4.00×10^{-9} C charge is in the $-x$ direction.

$$\mathbf{E}_1 = \frac{k_e q}{r^2}\,\hat{\mathbf{r}} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4.00 \times 10^{-9}\ \text{C})}{(2.50\ \text{m})^2}\,\mathbf{i} = -5.75\mathbf{i}\ \text{N/C}$$

Likewise, E_2 and E_3, due to the 5.00×10^{-9} C charge and the 3.00×10^{-9} C charge are

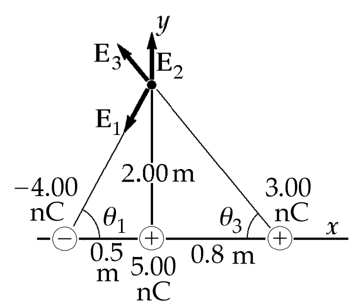
$$\mathbf{E}_2 = \frac{k_e q}{r^2}\,\hat{\mathbf{r}} = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00 \times 10^{-9}\ \text{C})}{(2.00\ \text{m})^2}\,\mathbf{i} = 11.2\ \text{N/C}$$



$$\mathbf{E}_3 = \frac{(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.00 \times 10^{-9}\ \text{C})}{(1.20\ \text{m})^2}\,\mathbf{i} = 18.7\ \text{N/C}$$

$$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \boxed{24.2\ \text{N/C}}\ \text{ in } +x \text{ direction.}$$

(b) $$\mathbf{E}_1 = \frac{k_e q}{r^2}\,\hat{\mathbf{r}} = (-8.46\ \text{N/C})(0.243\mathbf{i} + 0.970\mathbf{j})$$



$$\mathbf{E}_2 = \frac{k_e q}{r^2}\,\hat{\mathbf{r}} = (11.2\ \text{N/C})(+\mathbf{j})$$

$$\mathbf{E}_3 = \frac{k_e q}{r^2}\,\hat{\mathbf{r}} = (5.81\ \text{N/C})(-0.371\mathbf{i} + 0.928\mathbf{j})$$

$E_x = E_{1x} + E_{3x} = -4.21\mathbf{i}$ N/C $\qquad E_y = E_{1y} + E_{2y} + E_{3y} = 8.43\mathbf{j}$ N/C

$E_R = \boxed{9.42\ \text{N/C}} \qquad \theta = \boxed{63.4° \text{ above } -x \text{ axis}}$

23.51 The proton moves with acceleration $$|a_p| = \frac{qE}{m} = \frac{(1.60 \times 10^{-19}\ \text{C})(640\ \text{N/C})}{1.67 \times 10^{-27}\ \text{kg}} = 6.13 \times 10^{10}\ \text{m/s}^2$$

while the e⁻ has acceleration $$|a_e| = \frac{(1.60 \times 10^{-19}\ \text{C})(640\ \text{N/C})}{9.11 \times 10^{-31}\ \text{kg}} = 1.12 \times 10^{14}\ \text{m/s}^2 = 1836\,a_p$$

(a) We want to find the distance traveled by the proton (i.e., $d = \frac{1}{2}a_p t^2$), knowing:

$$4.00\ \text{cm} = \tfrac{1}{2}a_p t^2 + \tfrac{1}{2}a_e t^2 = 1837\left(\tfrac{1}{2}a_p t^2\right)$$

Thus, $d = \frac{1}{2}a_p t^2 = \dfrac{4.00\ \text{cm}}{1837} = \boxed{21.8\ \mu\text{m}}$

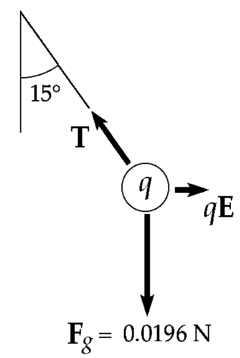
(b) The distance from the positive plate to where the meeting occurs equals the distance the sodium ion travels (i.e., $d_{\text{Na}} = \frac{1}{2}a_{\text{Na}}t^2$). This is found from:

$$4.00\text{ cm} = \tfrac{1}{2}a_{\text{Na}}t^2 + \tfrac{1}{2}a_{\text{Cl}}t^2:\ 4.00\text{ cm} = \frac{1}{2}\left(\frac{eE}{22.99\text{ u}}\right)t^2 + \frac{1}{2}\left(\frac{eE}{35.45\text{ u}}\right)t^2$$

This may be written as $$4.00\text{ cm} = \tfrac{1}{2}a_{\text{Na}}t^2 + \tfrac{1}{2}\left(0.649a_{\text{Na}}\right)t^2 = 1.65\left(\tfrac{1}{2}a_{\text{Na}}t^2\right)$$

so $$d_{\text{Na}} = \tfrac{1}{2}a_{\text{Na}}t^2 = \frac{4.00\text{ cm}}{1.65} = \boxed{2.43\text{ cm}}$$

23.52 From the free-body diagram shown, $\sum F_y = 0$



and $T\cos 15.0° = 1.96\times 10^{-2}\text{ N}$

So $T = 2.03\times 10^{-2}\text{ N}$

From $\sum F_x = 0$, we have $qE = T\sin 15.0°$

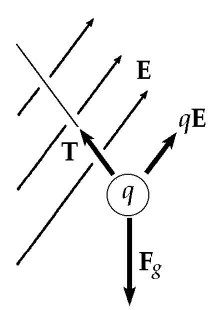
or $$q = \frac{T\sin 15.0°}{E} = \frac{(2.03\times 10^{-2}\text{ N})\sin 15.0°}{1.00\times 10^{3}\text{ N/C}} = 5.25\times 10^{-6}\text{ C} = \boxed{5.25\ \mu\text{C}}$$

23.53 (a) Let us sum force components to find

$\sum F_x = qE_x - T\sin\theta = 0,$ and $\sum F_y = qE_y + T\cos\theta - mg = 0$

Combining these two equations, we get

$$q = \frac{mg}{(E_x\cot\theta + E_y)} = \frac{(1.00\times 10^{-3})(9.80)}{(3.00\cot 37.0° + 5.00)\times 10^{5}} = 1.09\times 10^{-8}\text{ C} = \boxed{10.9\text{ nC}}$$



Free Body Diagram for Goal Solution

(b) From the two equations for $\sum F_x$ and $\sum F_y$ we also find

$$T = \frac{qE_x}{\sin 37.0°} = 5.44\times 10^{-3}\text{ N} = \boxed{5.44\text{ mN}}$$

Goal Solution

A charged cork ball of mass 1.00 g is suspended on a light string in the presence of a uniform electric field, as shown in Fig. P23.53. When $\mathbf{E} = (3.00\mathbf{i} + 5.00\mathbf{j}) \times 10^5$ N / C, the ball is in equilibrium at $\theta = 37.0°$. Find (a) the charge on the ball and (b) the tension in the string.

G: (a) Since the electric force must be in the same direction as **E**, the ball must be positively charged. If we examine the free body diagram that shows the three forces acting on the ball, the sum of which must be zero, we can see that the tension is about half the magnitude of the weight.

O: The tension can be found from applying Newton's second law to this statics problem (electrostatics, in this case!). Since the force vectors are in two dimensions, we must apply $\Sigma\mathbf{F} = m\mathbf{a}$ to both the x and y directions.

A: Applying Newton's Second Law in the x and y directions, and noting that $\Sigma\mathbf{F} = \mathbf{T} + q\mathbf{E} + \mathbf{F}_g = 0$,

$$\Sigma F_x = qE_x - T\sin 37.0° = 0 \qquad (1)$$

$$\Sigma F_y = qE_y + T\cos 37.0° - mg = 0 \qquad (2)$$

We are given $E_x = 3.00 \times 10^5$ N / C and $E_y = 5.00 \times 10^5$ N / C; substituting T from (1) into (2):

$$q = \frac{mg}{\left(E_y + \dfrac{E_x}{\tan 37.0°}\right)} = \frac{(1.00 \times 10^{-3}\text{ kg})(9.80\text{ m / s}^2)}{\left(5.00 + \dfrac{3.00}{\tan 37.0°}\right) \times 10^5\text{ N / C}} = 1.09 \times 10^{-8}\text{ C}$$

(b) Using this result for q in Equation (1), we find that the tension is $T = \dfrac{qE_x}{\sin 37.0°} = 5.44 \times 10^{-3}$ N

L: The tension is slightly more than half the weight of the ball ($F_g = 9.80 \times 10^{-3}$ N) so our result seems reasonable based on our initial prediction.

23.54 (a) Applying the first condition of equilibrium to the ball gives:

$$\Sigma F_x = qE_x - T\sin\theta = 0 \quad \text{or} \quad T = \frac{qE_x}{\sin\theta} = \frac{qA}{\sin\theta}$$

and

$$\Sigma F_y = qE_y + T\cos\theta - mg = 0 \quad \text{or} \quad qB + T\cos\theta = mg$$

Substituting from the first equation into the second gives:

$$q(A\cot\theta + B) = mg\,, \quad \text{or} \quad q = \boxed{\frac{mg}{(A\cot\theta + B)}}$$

(b) Substituting the charge into the equation obtained from ΣF_x yields

$$T = \frac{mg}{(A\cot\theta + B)}\left(\frac{A}{\sin\theta}\right) = \boxed{\frac{mgA}{A\cos\theta + B\sin\theta}}$$

Goal Solution

A charged cork ball of mass m is suspended on a light string in the presence of a uniform electric field, as shown in Figure P23.53. When $\mathbf{E} = (A\mathbf{i} + B\mathbf{j})$ N / C, where A and B are positive numbers, the ball is in equilibrium at the angle θ. Find (a) the charge on the ball and (b) the tension in the string.

G: This is the general version of the preceding problem. The known quantities are A, B, m, g, and θ. The unknowns are q and T.

O: The approach to this problem should be the same as for the last problem, but without numbers to substitute for the variables. Likewise, we can use the free body diagram given in the solution to problem 53.

A: Again, Newton's second law: $-T\sin\theta + qA = 0$ (1)

and $+T\cos\theta + qB - mg = 0$ (2)

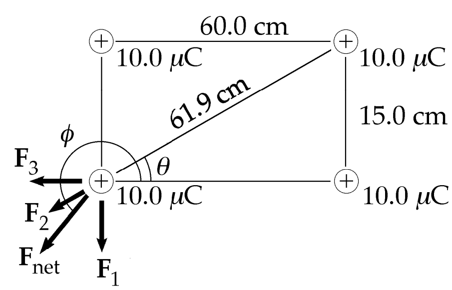
(a) Substituting $T = \dfrac{qA}{\sin\theta}$, into Eq. (2), $\dfrac{qA\cos\theta}{\sin\theta} + qB = mg$

Isolating q on the left, $q = \dfrac{mg}{(A\cot\theta + B)}$

(b) Substituting this value into Eq. (1), $T = \dfrac{mgA}{(A\cos\theta + B\sin\theta)}$

L: If we had solved this general problem first, we would only need to substitute the appropriate values in the equations for q and T to find the numerical results needed for problem 53. If you find this problem more difficult than problem 53, the little list at the Gather step is useful. It shows what symbols to think of as known data, and what to consider unknown. The list is a guide for deciding what to solve for in the Analysis step, and for recognizing when we have an answer.

23.55 $F = \dfrac{k_e q_1 q_2}{r^2}$ $\tan\theta = \dfrac{15.0}{60.0}$ $\theta = 14.0°$



$$F_1 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.150)^2} = 40.0\text{ N}$$

$$F_3 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.600)^2} = 2.50\text{ N}$$

$$F_2 = \frac{(8.99\times10^9)(10.0\times10^{-6})^2}{(0.619)^2} = 2.35\text{ N}$$

$$F_x = -F_3 - F_2\cos 14.0° = -2.50 - 2.35\cos 14.0° = -4.78\text{ N}$$

$$F_y = -F_1 - F_2\sin 14.0° = -40.0 - 2.35\sin 14.0° = -40.6\text{ N}$$

$$F_{net} = \sqrt{F_x^2 + F_y^2} = \sqrt{(-4.78)^2 + (-40.6)^2} = \boxed{40.9\text{ N}}$$

$$\tan\phi = \frac{F_y}{F_x} = \frac{-40.6}{-4.78} \qquad \phi = \boxed{263°}$$

23.56 From Fig. A: $d\cos 30.0° = 15.0\text{ cm}$, or $d = \dfrac{15.0\text{ cm}}{\cos 30.0°}$

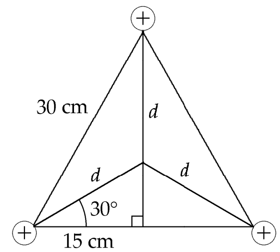


Figure A

From Fig. B: $\theta = \sin^{-1}\left(\dfrac{d}{50.0\text{ cm}}\right) = \sin^{-1}\left(\dfrac{15.0\text{ cm}}{50.0\text{ cm}(\cos 30.0°)}\right) = 20.3°$

$$\frac{F_q}{mg} = \tan\theta$$

or $F_q = mg\tan 20.3°$ (1)

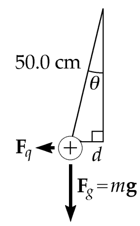


Figure B

From Fig. C: $F_q = 2F\cos 30.0° = 2\left[\dfrac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0°$ (2)

Equating equations (1) and (2), $2\left[\dfrac{k_e q^2}{(0.300\text{ m})^2}\right]\cos 30.0° = mg\tan 20.3°$

$$q^2 = \frac{mg(0.300\text{ m})^2\tan 20.3°}{2k_e\cos 30.0°}$$

$$q^2 = \frac{\left(2.00\times 10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)(0.300\text{ m})^2\tan 20.3°}{2\left(8.99\times 10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\cos 30.0°}$$

$$q = \sqrt{4.20\times 10^{-14}\text{ C}^2} = 2.05\times 10^{-7}\text{ C} = \boxed{0.205\ \mu\text{C}}$$

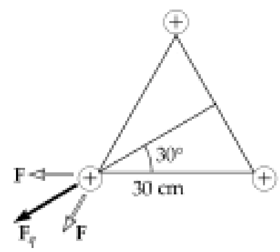


Figure C

23.57 Charge $Q/2$ resides on each block, which repel as point charges:

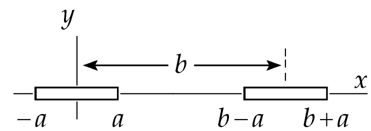
$$F = \frac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$$

$$Q = 2L\sqrt{\frac{k(L-L_i)}{k_e}} = 2(0.400\text{ m})\sqrt{\frac{(100\text{ N / m})(0.100\text{ m})}{\left(8.99\times 10^{9}\text{ N}\cdot\text{m}^2\text{ / C}^2\right)}} = \boxed{26.7\ \mu\text{C}}$$

23.58 Charge $Q/2$ resides on each block, which repel as point charges: $F = \dfrac{k_e(Q/2)(Q/2)}{L^2} = k(L - L_i)$

Solving for Q, $Q = \boxed{2L\sqrt{\dfrac{k(L-L_i)}{k_e}}}$

***23.59** According to the result of Example 23.7, the lefthand rod creates this field at a distance d from its righthand end:



$$E = \frac{k_e Q}{d(2a+d)}$$

$$dF = \frac{k_e Q Q}{2a}\frac{dx}{d(d+2a)}$$

$$F = \frac{k_e Q^2}{2a}\int_{x=b-2a}^{b}\frac{dx}{x(x+2a)} = \frac{k_e Q^2}{2a}\left(-\frac{1}{2a}\ln\frac{2a+x}{x}\right)_{b-2a}^{b}$$

$$F = \frac{+k_e Q^2}{4a^2}\left(-\ln\frac{2a+b}{b} + \ln\frac{b}{b-2a}\right) = \frac{k_e Q^2}{4a^2}\ \ln\frac{b^2}{(b-2a)(b+2a)} = \boxed{\left(\frac{k_e Q^2}{4a^2}\right)\ln\left(\frac{b^2}{b^2-4a^2}\right)}$$

***23.60** The charge moves with acceleration of magnitude a given by $\Sigma F = ma = |q|E$

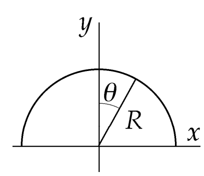
(a) $a = \dfrac{|q|E}{m} = \dfrac{1.60\times10^{-19}\text{ C}(1.00\text{ N/C})}{9.11\times10^{-31}\text{ kg}} = 1.76\times10^{11}\text{ m/s}^2$

Then $v = v_i + at = 0 + at$ gives $t = \dfrac{v}{a} = \dfrac{3.00\times10^{7}\text{ m/s}}{1.76\times10^{11}\text{ m/s}^2} = \boxed{171\ \mu\text{s}}$

(b) $t = \dfrac{v}{a} = \dfrac{vm}{qE} = \dfrac{(3.00\times10^{7}\text{ m/s})(1.67\times10^{-27}\text{ kg})}{(1.60\times10^{-19}\text{ C})(1.00\text{ N/C})} = \boxed{0.313\text{ s}}$

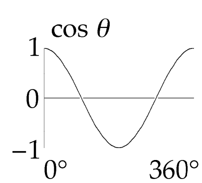
(c) From $t = \dfrac{vm}{qE}$, as E increases, t gets $\boxed{\text{shorter}}$ in inverse proportion.

23.61 $Q = \int \lambda d\ell = \int_{-90.0°}^{90.0°} \lambda_0 \cos\theta\, R d\theta = \lambda_0 R \sin\theta\Big|_{-90.0°}^{90.0°} = \lambda_0 R\,[1-(-1)] = 2\lambda_0 R$



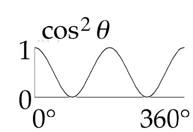
$Q = 12.0\ \mu\text{C} = (2\lambda_0)(0.600)\text{ m} = 12.0\ \mu\text{C}$ so $\lambda_0 = 10.0\ \mu\text{C/m}$

$$dF_y = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})(\lambda d\ell)}{R^2}\right)\cos\theta = \frac{1}{4\pi\epsilon_0}\left(\frac{(3.00\ \mu\text{C})\left(\lambda_0\cos^2\theta R d\theta\right)}{R^2}\right)$$



$$F_y = \int_{-90.0°}^{90.0°}\left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(3.00\times10^{-6}\text{ C})(10.0\times10^{-6}\text{ C/m})}{(0.600\text{ m})}\ \cos^2\theta d\theta$$

$$F_y = \frac{8.99(30.0)}{0.600}\left(10^{-3}\text{ N}\right)\int_{-\pi/2}^{\pi/2}\left(\tfrac{1}{2}+\tfrac{1}{2}\cos2\theta\right)d\theta$$

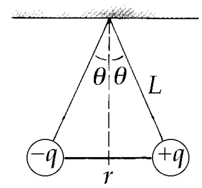


$$F_y = (0.450\text{ N})\left(\tfrac{1}{2}\pi + \tfrac{1}{4}\sin2\theta\Big|_{-\pi/2}^{\pi/2}\right) = \boxed{0.707\text{ N}}\quad \text{Downward.}$$

Since the leftward and rightward forces due to the two halves of the semicircle cancel out, $F_x = 0$.

23.62 At equilibrium, the distance between the charges is $r = 2(0.100\text{ m})\sin 10.0° = 3.47\times 10^{-2}$ m

Now consider the forces on the sphere with charge $+q$, and use $\Sigma F_y = 0$:

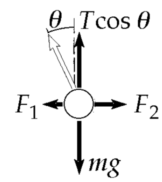


$\Sigma F_y = 0$: $\quad T\cos 10.0° = mg$, or $T = \dfrac{mg}{\cos 10.0°}$ (1)

$\Sigma F_x = 0$: $\quad F_{\text{net}} = F_2 - F_1 = T\sin 10.0°$ (2)

F_{net} is the net electrical force on the charged sphere. Eliminate T from (2) by use of (1).

$$F_{\text{net}} = \frac{mg\sin 10.0°}{\cos 10.0°} = mg\tan 10.0° = \left(2.00\times 10^{-3}\text{ kg}\right)\left(9.80\text{ m / s}^2\right)\tan 10.0° = 3.46\times 10^{-3}\text{ N}$$



F_{net} is the resultant of two forces, F_1 and F_2. F_1 is the attractive force on $+q$ exerted by $-q$, and F_2 is the force exerted on $+q$ by the external electric field.

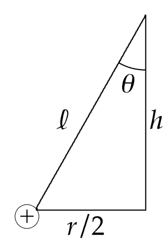
$F_{\text{net}} = F_2 - F_1$ or $F_2 = F_{\text{net}} + F_1$

$$F_1 = \left(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(5.00\times 10^{-8}\text{ C}\right)\left(5.00\times 10^{-8}\text{ C}\right)}{\left(3.47\times 10^{-3}\text{ m}\right)^2} = 1.87\times 10^{-2}\text{ N}$$

Thus, $F_2 = F_{\text{net}} + F_1$ yields $F_2 = 3.46\times 10^{-3}\text{ N} + 1.87\times 10^{-2}\text{ N} = 2.21\times 10^{-2}\text{ N}$

and $F_2 = qE$, or $\quad E = \dfrac{F_2}{q} = \dfrac{2.21\times 10^{-2}\text{ N}}{5.00\times 10^{-8}\text{ C}} = 4.43\times 10^5\text{ N/C} = \boxed{443\text{ kN/C}}$

23.63 (a) From the $2Q$ charge we have $\quad F_e - T_2\sin\theta_2 = 0$ and $mg - T_2\cos\theta_2 = 0$



Combining these we find $\dfrac{F_e}{mg} = \dfrac{T_2\sin\theta_2}{T_2\cos\theta_2} = \tan\theta_2$

From the Q charge we have $\quad F_e - T_1\sin\theta_1 = 0$ and $mg - T_1\cos\theta_1 = 0$

Combining these we find $\quad \dfrac{F_e}{mg} = \dfrac{T_1\sin\theta_1}{T_1\cos\theta_1} = \tan\theta_1$ or $\boxed{\theta_2 = \theta_1}$

(b) $F_e = \dfrac{k_e 2QQ}{r^2} = \dfrac{2k_eQ^2}{r^2}$

If we assume θ is small then $\tan\theta \approx \dfrac{(r/2)}{\ell}$. Substitute expressions for F_e and $\tan\theta$ into either equation found in part (a) and solve for r.

$\dfrac{F_e}{mg} = \tan\theta$ then $\dfrac{2k_eQ^2}{r^2}\left(\dfrac{1}{mg}\right) \approx \dfrac{r}{2\ell}$ and solving for r we find $r = \left[\dfrac{4k_eQ^2\ell}{mg}\right]^{1/3}$

23.64 At an equilibrium position, the net force on the charge Q is zero. The equilibrium position can be located by determining the angle θ corresponding to equilibrium. In terms of lengths s, $\frac{1}{2}a\sqrt{3}$, and r, shown in Figure P23.64, the charge at the origin exerts an attractive force $k_eQq/(s+\frac{1}{2}a\sqrt{3})^2$. The other two charges exert equal repulsive forces of magnitude k_eQq/r^2. The horizontal components of the two repulsive forces add, balancing the attractive force,

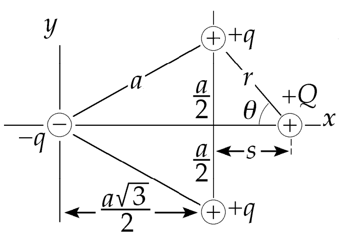
$$F_{\text{net}} = k_eQq\left\{\frac{2\cos\theta}{r^2} - \frac{1}{(s+\frac{1}{2}a\sqrt{3})^2}\right\} = 0$$

From Figure P23.64, $\quad r = \dfrac{\frac{1}{2}a}{\sin\theta} \qquad s = \frac{1}{2}a\cot\theta$

The equilibrium condition, in terms of θ, is $\quad F_{\text{net}} = \left(\dfrac{4}{a^2}\right)k_eQq\left(2\cos\theta\sin^2\theta - \dfrac{1}{(\sqrt{3}+\cot\theta)^2}\right) = 0$

Thus the equilibrium value of θ is $\qquad 2\cos\theta\sin^2\theta(\sqrt{3}+\cot\theta)^2 = 1.$

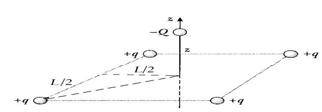
One method for solving for θ is to tabulate the left side. To three significant figures the value of θ corresponding to equilibrium is 81.7°. The distance from the origin to the equilibrium position is $x = \frac{1}{2}a(\sqrt{3}+\cot 81.7°) = \boxed{0.939a}$



θ	$2\cos\theta\sin^2\theta(\sqrt{3}+\cot\theta)^2$
60°	4
70°	2.654
80°	1.226
90°	0
81°	1.091
81.5°	1.024
81.7°	0.997

23.65 (a) The distance from each corner to the center of the square is

$$\sqrt{(L/2)^2 + (L/2)^2} = L/\sqrt{2}$$



The distance from each positive charge to $-Q$ is then $\sqrt{z^2 + L^2/2}$. Each positive charge exerts a force directed along the line joining q and $-Q$, of magnitude

$$\frac{k_eQq}{z^2 + L^2/2}$$

The line of force makes an angle with the z-axis whose cosine is $\quad \dfrac{z}{\sqrt{z^2+L^2/2}}$

The four charges together exert forces whose x and y components add to zero, while the z-components add to

$$\mathbf{F} = \boxed{-\frac{4k_eQqz}{\left(z^2+L^2/2\right)^{3/2}}\mathbf{k}}$$

(b) For $z << L$, the magnitude of this force is $F_z \approx -\dfrac{4k_eQqz}{\left(L^2/2\right)^{3/2}} = -\left(\dfrac{4(2)^{3/2}k_eQq}{L^3}\right)z = ma_z$

Therefore, the object's vertical acceleration is of the form $\boxed{a_z = -\omega^2 z}$

with $\omega^2 = \dfrac{4(2)^{3/2}k_eQq}{mL^3} = \dfrac{k_eQq\sqrt{128}}{mL^3}$

Since the acceleration of the object is always oppositely directed to its excursion from equilibrium and in magnitude proportional to it, the object will execute simple harmonic motion with a period given by

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{(128)^{1/4}}\sqrt{\frac{mL^3}{k_eQq}} = \boxed{\frac{\pi}{(8)^{1/4}}\sqrt{\frac{mL^3}{k_eQq}}}$$

23.66 (a) The total non-contact force on the cork ball is: $F = qE + mg = m\left(g + \dfrac{qE}{m}\right)$,

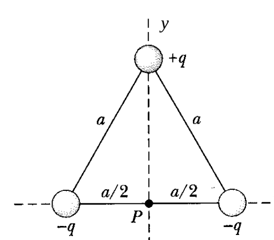
which is constant and directed downward. Therefore, it behaves like a simple pendulum in the presence of a modified uniform gravitational field with a period given by:

$$T = 2\pi\sqrt{\frac{L}{g + \dfrac{qE}{m}}} = 2\pi\sqrt{\frac{0.500\text{ m}}{9.80\text{ m / s}^2 + \dfrac{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N / C}\right)}{1.00\times10^{-3}\text{ kg}}}} = \boxed{0.307\text{ s}}$$

(b) $\boxed{\text{Yes}}$. Without gravity in part (a), we get $T = 2\pi\sqrt{\dfrac{L}{qE/m}}$

$$T = 2\pi\sqrt{\frac{0.500\text{ m}}{\left(2.00\times10^{-6}\text{ C}\right)\left(1.00\times10^{5}\text{ N / C}\right)/1.00\times10^{-3}\text{ kg}}} = 0.314\text{ s} \quad \text{(a 2.28\% difference).}$$

23.67 (a) Due to symmetry the field contribution from each negative charge is equal and opposite to each other. Therefore, their contribution to the net field is zero. The field contribution of the $+q$ charge is



$$E = \frac{k_eq}{r^2} = \frac{k_eq}{\left(3a^2/4\right)} = \frac{4k_eq}{3a^2}$$

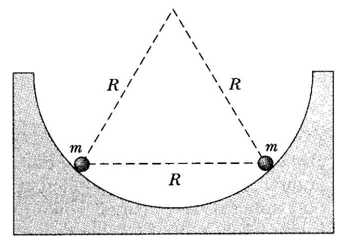
in the negative y direction, i.e., $\mathbf{E} = \boxed{-\dfrac{4k_eq}{3a^2}\mathbf{j}}$

(b) If $F_e = 0$, then E at P must equal zero. In order for the field to cancel at P, the $-4q$ must be above $+q$ on the y-axis.

Then, $E = 0 = -\frac{k_e q}{(1.00\text{ m})^2} + \frac{k_e(4q)}{y^2}$, which reduces to $y^2 = 4.00\text{ m}^2$.

Thus, $y = \pm 2.00\text{ m}$. Only the positive answer is acceptable since the $-4q$ must be located above $+q$. Therefore, the $-4q$ must be placed $\boxed{\text{2.00 meters above point } P \text{ along the } +y-\text{axis}}$.

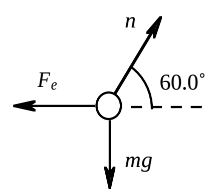
23.68 The bowl exerts a normal force on each bead, directed along the radius line or at 60.0° above the horizontal. Consider the free-body diagram of the bead on the left:



$\Sigma F_y = n\sin 60.0° - mg = 0$,

or $n = \frac{mg}{\sin 60.0°}$

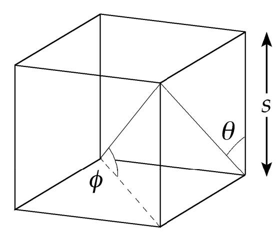
Also, $\Sigma F_x = -F_e + n\cos 60.0° = 0$,



or $\frac{k_e q^2}{R^2} = n\cos 60.0° = \frac{mg}{\tan 60.0°} = \frac{mg}{\sqrt{3}}$

Thus, $q = \boxed{R\left(\frac{mg}{k_e\sqrt{3}}\right)^{1/2}}$

23.69 (a) There are 7 terms which contribute:



3 are s away (along sides)

3 are $\sqrt{2}\ s$ away (face diagonals) and $\sin\theta = \frac{1}{\sqrt{2}} = \cos\theta$

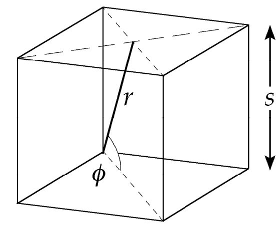
1 is $\sqrt{3}\ s$ away (body diagonal) and $\sin\phi = \frac{1}{\sqrt{3}}$

The component in each direction is the same by symmetry.

$$\mathbf{F} = \frac{k_e q^2}{s^2}\left[1 + \frac{2}{2\sqrt{2}} + \frac{1}{3\sqrt{3}}\right](\mathbf{i}+\mathbf{j}+\mathbf{k}) = \boxed{\frac{k_e q^2}{s^2}(1.90)(\mathbf{i}+\mathbf{j}+\mathbf{k})}$$

(b) $F = \sqrt{F_x^2 + F_y^2 + F_z^2} = \boxed{3.29\frac{k_e q^2}{s^2} \text{ away from the origin}}$

23.70 (a) Zero contribution from the same face due to symmetry, opposite face contributes



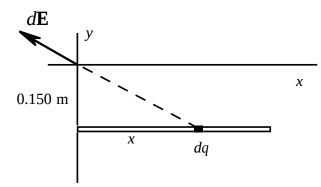
$$4\left(\frac{k_e q}{r^2}\sin\phi\right) \qquad \text{where} \qquad r = \sqrt{\left(\frac{s}{2}\right)^2 + \left(\frac{s}{2}\right)^2 + s^2} = \sqrt{1.5}\ s = 1.22\ s$$

$$E = 4\frac{k_e q s}{r^3} = \frac{4}{(1.22)^3}\frac{k_e q}{s^2} = \boxed{2.18\ \frac{k_e q}{s^2}}$$

$\sin\phi = s/r$

(b) $\boxed{\text{The direction is the } \mathbf{k} \text{ direction.}}$

***23.71** $$d\mathbf{E} = \frac{k_e\,dq}{x^2 + (0.150\text{ m})^2}\left(\frac{-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right) = \frac{k_e\lambda\left(-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j}\right)dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$



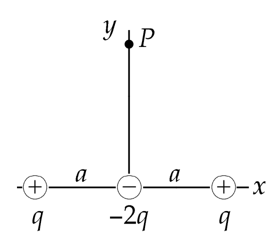
$$\mathbf{E} = \int_{\text{all charge}} d\mathbf{E} = k_e\lambda\int_{x=0}^{0.400\text{ m}} \frac{\left(-x\,\mathbf{i} + 0.150\text{ m}\,\mathbf{j}\right)dx}{\left[x^2 + (0.150\text{ m})^2\right]^{3/2}}$$

$$\mathbf{E} = k_e\lambda\left[\left.\frac{+\mathbf{i}}{\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}} + \left.\frac{(0.150\text{ m})\,\mathbf{j}\,x}{(0.150\text{ m})^2\sqrt{x^2 + (0.150\text{ m})^2}}\right|_0^{0.400\text{ m}}\right]$$

$$\mathbf{E} = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(35.0\times10^{-9}\ \frac{\text{C}}{\text{m}}\right)\left[\mathbf{i}(2.34 - 6.67)/\text{m} + \mathbf{j}(6.24 - 0)/\text{m}\right]$$

$$\mathbf{E} = \left(-1.36\,\mathbf{i} + 1.96\,\mathbf{j}\right)\times10^3\ \text{N/C} = \boxed{\left(-1.36\,\mathbf{i} + 1.96\,\mathbf{j}\right)\ \text{kN/C}}$$

23.72 By symmetry $\sum E_x = 0$. Using the distances as labeled,



$$\sum E_y = k_e\left[\frac{q}{(a^2 + y^2)}\sin\theta + \frac{q}{(a^2 + y^2)}\sin\theta - \frac{2q}{y^2}\right]$$

But $\sin\theta = \dfrac{y}{\sqrt{(a^2 + y^2)}}$, so $E = \sum E_y = 2k_e q\left[\dfrac{y}{(a^2 + y^2)^{3/2}} - \dfrac{1}{y^2}\right]$

Expand $(a^2 + y^2)^{-3/2}$ as $(a^2 + y^2)^{-3/2} = y^{-3} - (3/2)a^2 y^{-5} + \ldots$

Therefore, for $a << y$, we can ignore terms in powers higher than 2,

and we have $E = 2k_e q\left[\dfrac{1}{y^2} - \left(\dfrac{3}{2}\right)\dfrac{a^2}{y^4} - \dfrac{1}{y^2}\right]$ or $\boxed{\mathbf{E} = \left[-\dfrac{k_e 3qa^2}{y^4}\right]\mathbf{j}}$

23.73 The field on the axis of the ring is calculated in Example 23.8, $E = E_x = \dfrac{k_e x Q}{(x^2 + a^2)^{3/2}}$

The force experienced by a charge $-q$ placed along the axis of the ring is

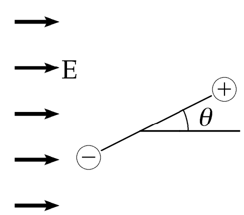
$F = -k_e Qq\left[\dfrac{x}{(x^2 + a^2)^{3/2}}\right]$ and when $x << a$, this becomes $F = \left(\dfrac{k_e Qq}{a^3}\right)x$

This expression for the force is in the form of Hooke's law,

with an effective spring constant of $k = k_e Qq / a^3$

Since $\omega = 2\pi f = \sqrt{k/m}$, we have $f = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{k_e Qq}{ma^3}}}$

23.74 The electrostatic forces exerted on the two charges result in a net torque $\tau = -2Fa \sin\theta = -2Eqa \sin\theta$.



For small θ, $\sin\theta \approx \theta$ and using $p = 2qa$, we have $\tau = -Ep\theta$.

The torque produces an angular acceleration given by $\tau = I\alpha = I\dfrac{d^2\theta}{dt^2}$

Combining these two expressions for torque, we have $\dfrac{d^2\theta}{dt^2} + \left(\dfrac{Ep}{I}\right)\theta = 0$

This equation can be written in the form $\dfrac{d^2\theta}{dt^2} = -\omega^2\theta$ where $\omega^2 = \dfrac{Ep}{I}$

This is the same form as Equation 13.17 and the frequency of oscillation is found by comparison with Equation 13.19, or $f = \dfrac{1}{2\pi}\sqrt{\dfrac{pE}{I}} = \boxed{\dfrac{1}{2\pi}\sqrt{\dfrac{2qaE}{I}}}$