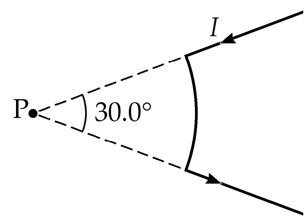
Chapter 30 Solutions

30.1 $B = \dfrac{\mu_0 I}{2R} = \dfrac{\mu_0 q(v/2\pi R)}{2R} = \boxed{12.5\text{ T}}$

***30.2** We use the Biot-Savart law. For bits of wire along the straight-line sections, $d\mathbf{s}$ is at 0° or 180° to ~, so $d\mathbf{s} \times \sim = 0$. Thus, only the curved section of wire contributes to **B** at *P*. Hence, $d\mathbf{s}$ is tangent to the arc and ~ is radially inward; so $d\mathbf{s} \times \sim = \left|ds\right|$ l $\sin 90° = \left|ds\right|$ ⊗. All points along the curve are the same distance $r = 0.600$ m from the field point, so

$$B = \int_{\text{all current}} \left|d\mathbf{B}\right| = \int \frac{\mu_0}{4\pi} \frac{I\left|d\mathbf{s} \times \sim\right|}{r^2} = \frac{\mu_0}{4\pi} \frac{I}{r^2} \int \left|ds\right| = \frac{\mu_0}{4\pi} \frac{I}{r^2}\, s$$



where *s* is the arclength of the curved wire,

$$s = r\theta = (0.600\text{ m})30.0°\left(\frac{2\pi}{360°}\right) = 0.314\text{ m}$$

Then, $B = \left(10^{-7}\,\dfrac{\text{T}\cdot\text{m}}{\text{A}}\right)\dfrac{(3.00\text{ A})}{(0.600\text{ m})^2}(0.314\text{ m})$

$B = \boxed{261\text{ nT into the page}}$

30.3 (a) $B = \dfrac{4\mu_0 I}{4\pi a}\left(\cos\dfrac{\pi}{4} - \cos\dfrac{3\pi}{4}\right)$ where $a = \dfrac{1}{2}$

is the distance from any side to the center.

$$B = \frac{4.00 \times 10^{-6}}{0.200}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right) = 2\sqrt{2} \times 10^{-5}\text{ T} = \boxed{28.3\ \mu\text{T into the paper}}$$

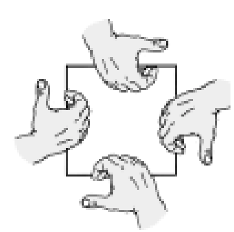


Figure for Goal Solution

(b) For a single circular turn with $4\ell = 2\pi R$,

$$B = \frac{\mu_0 I}{2R} = \frac{\mu_0 \pi I}{4\ell} = \frac{(4\pi^2 \times 10^{-7})(10.0)}{4(0.400)} = \boxed{24.7\ \mu\text{T into the paper}}$$

Goal Solution

(a) A conductor in the shape of a square of edge length $\ell = 0.400$ m carries a current I = 10.0 A (Fig. P30.3). Calculate the magnitude and direction of the magnetic field at the center of the square. (b) If this conductor is formed into a single circular turn and carries the same current, what is the value of the magnetic field at the center?

G: As shown in the diagram above, the magnetic field at the center is directed into the page from the clockwise current. If we consider the sides of the square to be sections of four infinite wires, then we could expect the magnetic field at the center of the square to be a little less than four times the strength of the field at a point $\ell/2$ away from an infinite wire with current I.

$$B < 4\frac{\mu_0 I}{2\pi a} = 4\left(\frac{\left(4\pi\times 10^{-7}\text{ T}\cdot\text{m / A}\right)(10.0\text{ A})}{2\pi(0.200\text{ m})}\right) = 40.0\ \mu\text{T}$$

Forming the wire into a circle should not significantly change the magnetic field at the center since the average distance of the wire from the center will not be much different.

O: Each side of the square is simply a section of a thin, straight conductor, so the solution derived from the Biot-Savart law in Example 30.1 can be applied to part (a) of this problem. For part (b), the Biot-Savart law can also be used to derive the equation for the magnetic field at the center of a circular current loop as shown in Example 30.3.

A: (a) We use Equation 30.4 for the field created by each side of the square. Each side contributes a field away from you at the center, so together they produce a magnetic field:

$$B = \frac{4\mu_0 I}{4\pi a}\left(\cos\frac{\pi}{4} - \cos\frac{3\pi}{4}\right) = \frac{4\left(4\pi\times 10^{-6}\text{ T}\cdot\text{m / A}\right)(10.0\text{ A})}{4\pi(0.200\text{ m})}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}\right)$$

so at the center of the square, $\mathbf{B} = 2.00\sqrt{2}\times 10^{-5}\text{ T} = 28.3\ \mu\text{T}$ perpendicularly into the page

(b) As in the first part of the problem, the direction of the magnetic field will be into the page. The new radius is found from the length of wire: $4\ell = 2\pi R$, so $R = 2\ell/\pi = 0.255$ m. Equation 30.8 gives the magnetic field at the center of a circular current loop:

$$B = \frac{\mu_0 I}{2R} = \frac{(4\pi\times 10^{-7}\text{ T}\cdot\text{m / A})(10.0\text{ A})}{2(0.255\text{ m})} = 2.47\times 10^{-5}\text{ T} = 24.7\ \mu\text{T}$$

Caution! If you use your calculator, it may not understand the keystrokes: To get the right answer, you may need to use .

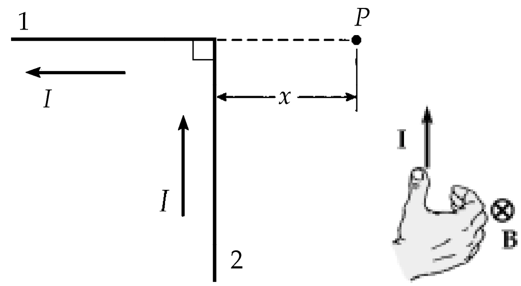
L: The magnetic field in part (a) is less than 40μT as we predicted. Also, the magnetic fields from the square and circular loops are similar in magnitude, with the field from the circular loop being about 15% less than from the square loop.

Quick tip: A simple way to use your right hand to find the magnetic field due to a current loop is to curl the fingers of your right hand in the direction of the current. Your extended thumb will then point in the direction of the magnetic field within the loop or solenoid.

30.4 $B = \dfrac{\mu_0 I}{2\pi r} = \dfrac{4\pi \times 10^{-7}\,(1.00\text{ A})}{2\pi(1.00\text{ m})} = \boxed{2.00 \times 10^{-7}\text{ T}}$

30.5 For leg 1, $d\mathbf{s} \times \sim = 0$, so there is no contribution to the field from this segment. For leg 2, the wire is only semi-infinite; thus,

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right) = \boxed{\frac{\mu_0 I}{4\pi x}\ \text{into the paper}}$$



30.6 $B = \dfrac{\mu_0 I}{2R}$ $\quad R = \dfrac{\mu_0 I}{2B} = \dfrac{20.0\pi \times 10^{-7}}{2.00 \times 10^{-5}} = \boxed{31.4\text{ cm}}$

30.7 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\mu_0 I/2\pi R$ and directed into the page) and the field due to the circular loop (having magnitude $\mu_0 I/2R$ and directed into the page). The resultant magnetic field is:

$$B = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R} = \left(1 + \frac{1}{\pi}\right)\frac{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m / A}\right)(7.00\text{ A})}{2(0.100\text{ m})} = 5.80 \times 10^{-5}\text{ T}$$

or $\boxed{\mathbf{B} = 58.0\ \mu\text{T} \quad (\text{directed into the page})}$

30.8 We can think of the total magnetic field as the superposition of the field due to the long straight wire (having magnitude $\mu_0 I/2\pi R$ and directed into the page) and the field due to the circular loop (having magnitude $\mu_0 I/2R$ and directed into the page). The resultant magnetic field is:

$$\boxed{\mathbf{B} = \left(1 + \frac{1}{\pi}\right)\frac{\mu_0 I}{2R} \quad (\text{directed into the page})}$$

30.9 For the straight sections $d\mathbf{s} \times \sim = 0$. The quarter circle makes one-fourth the field of a full loop:

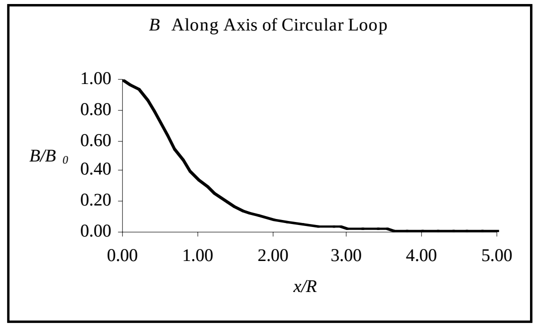
$\mathbf{B} = \dfrac{1}{4}\dfrac{\mu_0 I}{2R} = \dfrac{\mu_0 I}{8R}$ into the paper $\qquad \mathbf{B} = \dfrac{(4\pi \times 10^{-7}\text{ T}\cdot\text{m / A})(5.00\text{ A})}{8(0.0300\text{ m})} = \boxed{26.2\ \mu\text{T into the paper}}$

30.10 Along the axis of a circular loop of radius R,

$$B = \frac{\mu_0 I R^2}{2\left(x^2 + R^2\right)^{3/2}}$$

or $$\frac{B}{B_0} = \left[\frac{1}{(x/R)^2 + 1}\right]^{3/2}$$

where $B_0 \equiv \mu_0 I/2R$.



x/R	B/B_0
0.00	1.00
1.00	0.354
2.00	0.0894
3.00	0.0316
4.00	0.0143
5.00	0.00754

30.11 $$dB = \frac{\mu_0 I}{4\pi} \frac{\left|d\mathbf{l} \times \hat{\mathbf{r}}\right|}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\left(\frac{\frac{1}{6} 2\pi a}{a^2} - \frac{\frac{1}{6} 2\pi b}{b^2}\right)$$

$$\boxed{\mathbf{B} = \frac{\mu_0 I}{12}\left(\frac{1}{a} - \frac{1}{b}\right) \text{ directed out of the paper}}$$

30.12 Apply Equation 30.4 three times:

$$\mathbf{B} = \frac{\mu_0 I}{4\pi a}\left(\cos 0 - \frac{d}{\sqrt{d^2 + a^2}}\right) \text{ toward you}$$

$$+ \frac{\mu_0 I}{4\pi d}\left(\frac{a}{\sqrt{d^2 + a^2}} + \frac{a}{\sqrt{d^2 + a^2}}\right) \text{ away from you}$$

$$+ \frac{\mu_0 I}{4\pi a}\left(\frac{-d}{\sqrt{d^2 + a^2}} - \cos 180°\right) \text{ toward you}$$

$$\mathbf{B} = \boxed{\frac{\mu_0 I\left(a^2 + d^2 - d\sqrt{a^2 + d^2}\right)}{2\pi a d\sqrt{a^2 + d^2}} \text{ away from you}}$$

30.13 The picture requires $L = 2R$

$$\mathbf{B} = \frac{1}{2}\left(\frac{\mu_0 I}{2R}\right) + \frac{\mu_0 I}{4\pi R}(\cos 90.0° - \cos 135°) + \frac{\mu_0 I}{4\pi R}(\cos 45.0° - \cos 135°)$$

$$+ \frac{\mu_0 I}{4\pi R}(\cos 45.0° - \cos 90.0°) \text{ into the page}$$

$$\mathbf{B} = \frac{\mu_0 I}{R}\left(\frac{1}{4} + \frac{1}{\pi\sqrt{2}}\right) = \boxed{0.475\left(\frac{\mu_0 I}{R}\right) \text{ (into the page)}}$$

30.14 Label the wires 1, 2, and 3 as shown in Figure (a) and let the magnetic field created by the currents in these wires be B_1, B_2, and B_3 respectively.

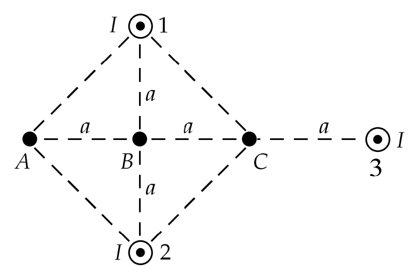


Figure (a)

(a) At Point A: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi(3a)}$.

The directions of these fields are shown in Figure (b). Observe that the horizontal components of B_1 and B_2 cancel while their vertical components both add to B_3.

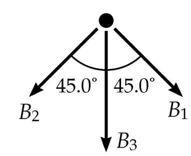


Figure (b)

Therefore, the net field at point A is:

$$B_A = B_1 \cos 45.0° + B_2 \cos 45.0° + B_3 = \frac{\mu_0 I}{2\pi a}\left[\frac{2}{\sqrt{2}}\cos 45.0° + \frac{1}{3}\right]$$

$$B_A = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi\left(1.00 \times 10^{-2}\ \text{m}\right)}\left[\frac{2}{\sqrt{2}}\cos 45° + \frac{1}{3}\right] = \boxed{53.3\ \mu\text{T}}$$

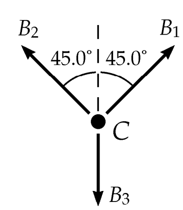


Figure (c)

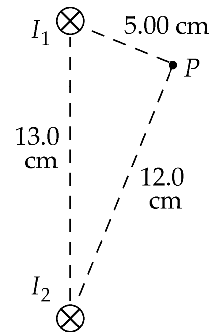
(b) At point B: B_1 and B_2 cancel, leaving $B_B = B_3 = \dfrac{\mu_0 I}{2\pi(2a)}$.

$$B_B = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(2.00\ \text{A})}{2\pi(2)\left(1.00 \times 10^{-2}\ \text{m}\right)} = \boxed{20.0\ \mu\text{T}}$$

(c) At point C: $B_1 = B_2 = \dfrac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}$ and $B_3 = \dfrac{\mu_0 I}{2\pi a}$ with the directions shown in Figure (c). Again, the horizontal components of B_1 and B_2 cancel. The vertical components both oppose B_3 giving

$$B_C = 2\left[\frac{\mu_0 I}{2\pi\left(a\sqrt{2}\right)}\cos 45.0°\right] - \frac{\mu_0 I}{2\pi a} = \frac{\mu_0 I}{2\pi a}\left[\frac{2\cos 45.0°}{\sqrt{2}} - 1\right] = \boxed{0}$$

30.15 Take the x-direction to the right and the y-direction up in the plane of the paper. Current 1 creates at P a field



$$B_1 = \frac{\mu_0 I}{2\pi a} = \frac{(2.00\times10^{-7}\ \text{T}\cdot\text{m})(3.00\ \text{A})}{\text{A}(0.0500\ \text{m})}$$

$\mathbf{B}_1 = 12.0\ \mu\text{T}$ downward and leftward, at angle 67.4° below the $-x$ axis.

Current 2 contributes

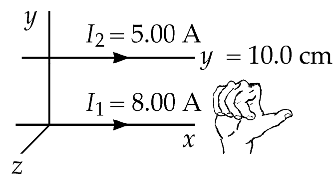
$$B_2 = \frac{(2.00\times10^{-7}\ \text{T}\cdot\text{m})(3.00\ \text{A})}{\text{A}(0.120\ \text{m})}$$ clockwise perpendicular to 12.0 cm

$\mathbf{B}_2 = 5.00\ \mu\text{T}$ to the right and down, at angle –22.6°

Then, $\mathbf{B} = \mathbf{B}_1 + \mathbf{B}_2 = (12.0\ \mu\text{T})(-\mathbf{i}\cos 67.4° - \mathbf{j}\sin 67.4°) + (5.00\ \mu\text{T})(\mathbf{i}\cos 22.6° - \mathbf{j}\sin 22.6°)$

$$\mathbf{B} = (-11.1\ \mu\text{T})\mathbf{j} - (1.92\ \mu\text{T})\mathbf{j} = \boxed{(-13.0\ \mu\text{T})\mathbf{j}}$$

***30.16** Let both wires carry current in the x direction, the first at $y = 0$ and the second at $y = 10.0$ cm.

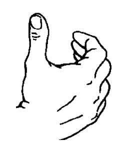


(a) $\mathbf{B} = \frac{\mu_0 I}{2\pi r}\mathbf{k} = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})}{2\pi(0.100\ \text{m})}\mathbf{k}$

$\mathbf{B} = \boxed{1.00\times10^{-5}\ \text{T out of the page}}$

(b) $\mathbf{F}_B = I_2\mathbf{L}\times\mathbf{B} = (8.00\ \text{A})\left[(1.00\ \text{m})\mathbf{i}\times(1.00\times10^{-5}\ \text{T})\mathbf{k}\right] = (8.00\times10^{-5}\ \text{N})(-\mathbf{j})$

$\boxed{\mathbf{F}_B = 8.00\times10^{-5}\ \text{N toward the first wire}}$



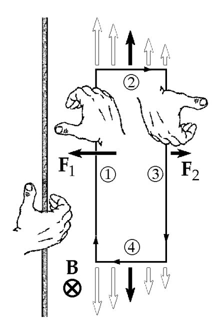
(c) $\mathbf{B} = \frac{\mu_0 I}{2\pi r}(-\mathbf{k}) = \frac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(8.00\ \text{A})}{2\pi(0.100\ \text{m})}(-\mathbf{k}) = (1.60\times10^{-5}\ \text{T})(-\mathbf{k})$

$\mathbf{B} = \boxed{1.60\times10^{-5}\ \text{T into the page}}$

(d) $\mathbf{F}_B = I_1\mathbf{L}\times\mathbf{B} = (5.00\ \text{A})\left[(1.00\ \text{m})\mathbf{i}\times(1.60\times10^{-5}\ \text{T})(-\mathbf{k})\right] = (8.00\times10^{-5}\ \text{N})(+\mathbf{j})$

$\mathbf{F}_B = \boxed{8.00\times10^{-5}\ \text{N toward the second wire}}$

30.17 By symmetry, we note that the magnetic forces on the top and bottom segments of the rectangle cancel. The net force on the vertical segments of the rectangle is (using Equation 30.12)



$$\mathbf{F}_B = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\mathbf{i}$$

Substituting given values $\mathbf{F}_B = -2.70\times10^{-5}\ \mathbf{i}$ N = $\boxed{-27.0\ \mu\text{N}\ \mathbf{i}}$

Goal Solution

In Figure P30.17, the current in the long, straight wire is $I_1 = 5.00$ A and the wire lies in the plane of the rectangular loop, which carries 10.0 A. The dimensions are $c = 0.100$ m, $a = 0.150$ m, and $\ell = 0.450$ m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

G: Even though there are forces in opposite directions on the loop, we must remember that the magnetic field is stronger near the wire than it is farther away. By symmetry the forces exerted on sides 2 and 4 (the horizontal segments of length a) are equal and opposite, and therefore cancel. The magnetic field in the plane of the loop is directed into the page to the right of I_1. By the right-hand rule, $\mathbf{F} = I\boldsymbol{\ell}\times\mathbf{B}$ is directed toward the **left** for side 1 of the loop and a smaller force is directed toward the **right** for side 3. Therefore, we should expect the net force to be to the left, possibly in the μN range for the currents and distances given.

O: The magnetic force between two parallel wires can be found from Equation 30.11, which can be applied to sides 1 and 3 of the loop to find the net force resulting from these opposing force vectors.

A: $$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\mathbf{i} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{-a}{c(c+a)}\right)\mathbf{i}$$

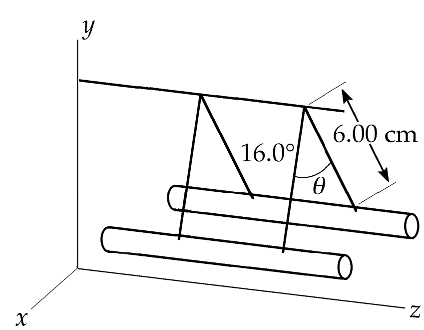
$$\mathbf{F} = \frac{\left(4\pi\times10^{-7}\ \text{N}/\text{A}^2\right)(5.00\ \text{A})(10.0\ \text{A})(0.450\ \text{m})}{2\pi}\left(\frac{-0.150\ \text{m}}{(0.100\ \text{m})(0.250\ \text{m})}\right)\mathbf{i}$$

$$\mathbf{F} = (-2.70\times10^{-5}\,\mathbf{i})\ \text{N} \qquad \text{or} \qquad \mathbf{F} = 2.70\times10^{-5}\ \text{N} \quad \text{toward the left}$$

L: The net force is to the left and in the μN range as we expected. The symbolic representation of the net force on the loop shows that the net force would be zero if either current disappeared, if either dimension of the loop became very small ($a \to 0$ or $\ell \to 0$), or if the magnetic field were uniform ($c \to \infty$).

30.18 The separation between the wires is

$a = 2(6.00\ \text{cm})\sin 8.00° = 1.67$ cm.



(a) Because the wires repel, the currents are in

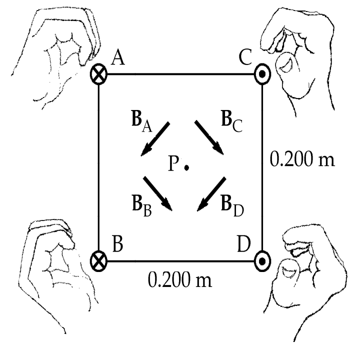
opposite directions.

(b) Because the magnetic force acts horizontally,

$$\frac{F_B}{F_g} = \frac{\mu_0 I^2 \ell}{2\pi a\, mg} = \tan 8.00°$$

$$I^2 = \frac{mg\, 2\pi a}{\ell\, \mu_0}\tan 8.00° \qquad \text{so} \qquad I = \boxed{67.8\ \text{A}}$$

30.19 Each wire is distant from P by (0.200 m) cos 45.0° = 0.141 m



Each wire produces a field at P of equal magnitude:

$$B_A = \frac{\mu_0 I}{2\pi a} = \frac{(2.00 \times 10^{-7}\ \text{T} \cdot \text{m})(5.00\ \text{A})}{\text{A}(0.141\ \text{m})} = 7.07\ \mu\text{T}$$

Carrying currents into the page, A produces at P a field of 7.07 μT to the left and down at –135°, while B creates a field to the right and down at – 45°. Carrying currents toward you, C produces a field downward and to the right at – 45°, while D's contribution is downward and to the left. The total field is then

4 (7.07 μT) sin 45.0° = **20.0 μT** toward the page's bottom

30.20 Let the current I flow to the right. It creates a field $B = \mu_0 I/2\pi d$ at the proton's location. And we have a balance between the weight of the proton and the magnetic force

$$mg(-\,\mathbf{j}) + qv(-\,\mathbf{i}) \times \frac{\mu_0 I}{2\pi d}(\mathbf{k}) = 0 \quad \text{at a distance } d \text{ from the wire}$$

$$d = \frac{qv\mu_0 I}{2\pi mg} = \frac{(1.60 \times 10^{-19}\ \text{C})(2.30 \times 10^{4}\ \text{m / s})(4\pi \times 10^{-7}\ \text{T} \cdot \text{m / A})(1.20 \times 10^{-6}\ \text{A})}{2\pi(1.67 \times 10^{-27}\ \text{kg})\,(9.80\ \text{m / s}^2)} = \boxed{5.40\ \text{cm}}$$

30.21 From Ampère's law, the magnetic field at point a is given by $B_a = \mu_0 I_a / 2\pi r_a$, where I_a is the net current flowing through the area of the circle of radius r_a. In this case, $I_a = 1.00$ A out of the page (the current in the inner conductor), so

$$B_a = \frac{\left(4\pi \times 10^{-7}\ \text{T} \cdot \text{m / A}\right)(1.00\ \text{A})}{2\pi(1.00 \times 10^{-3}\ \text{m})} = \boxed{200\ \mu\text{T toward top of page}}$$

Similarly at point b: $B_b = \dfrac{\mu_0 I_b}{2\pi r_b}$, where I_b is the net current flowing through the area of the circle having radius r_b.

Taking out of the page as positive, $I_b = 1.00\ \text{A} - 3.00\ \text{A} = -2.00\ \text{A}$, or $I_b = 2.00$ A into the page. Therefore,

$$B_b = \frac{(4\pi \times 10^{-7}\ \text{T} \cdot \text{m / A})(2.00\ \text{A})}{2\pi(3.00 \times 10^{-3}\ \text{m})} = \boxed{133\ \mu\text{T toward bottom of page}}$$

***30.22** (a) In $B = \dfrac{\mu_0 I}{2\pi r}$, the field will be one-tenth as large at a ten-times larger distance: $\boxed{400\ \text{cm}}$

(b) $\mathbf{B} = \dfrac{\mu_0 I}{2\pi r_1}\,\mathbf{k} + \dfrac{\mu_0 I}{2\pi r_2}(-\mathbf{k})$ so $B = \dfrac{4\pi\times 10^{-7}\ \text{T}\cdot\text{m}\ (2.00\ \text{A})}{2\pi\ \text{A}}\left(\dfrac{1}{0.3985\ \text{m}} - \dfrac{1}{0.4015\ \text{m}}\right) = \boxed{7.50\ \text{nT}}$

(c) Call r the distance from cord center to field point and $2d = 3.00$ mm the distance between conductors.

$$B = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\frac{2d}{r^2 - d^2}$$

$$7.50\times 10^{-10}\ \text{T} = \left(2.00\times 10^{-7}\ \frac{\text{T}\cdot\text{m}}{\text{A}}\right)(2.00\ \text{A})\frac{(3.00\times 10^{-3}\ \text{m})}{r^2 - 2.25\times 10^{-6}\ \text{m}^2} \quad \text{so} \quad r = \boxed{1.26\ \text{m}}$$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère's law encloses zero total current. Shall we sell coaxial-cable power cords to people who worry about biological damage from weak magnetic fields?

30.23 (a) $B_{\text{inner}} = \dfrac{\mu_0 NI}{2\pi r} = \boxed{3.60\ \text{T}}$

(b) $B_{\text{outer}} = \dfrac{\mu_0 NI}{2\pi r} = \boxed{1.94\ \text{T}}$

***30.24** (a) $B = \dfrac{\mu_0 I}{2\pi a^2}\,r$ for $r \le a$ so $B = \dfrac{\mu_0(2.50\ \text{A})}{2\pi(0.0250\ \text{m})^2}(0.0125\ \text{m}) = \boxed{10.0\ \mu\text{T}}$

(b) $r = \dfrac{\mu_0 I}{2\pi B} = \dfrac{\mu_0(2.50\ \text{A})}{2\pi(10.0\times 10^{-6}\ \text{T})} = 0.0500\ \text{m} = \boxed{\text{2.50 cm beyond the conductor's surface}}$

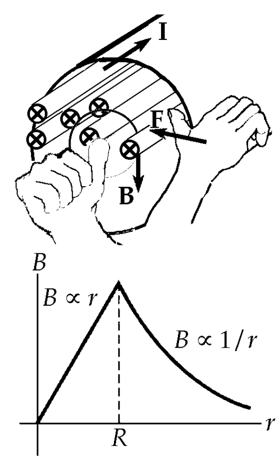
30.25 (a) One wire feels force due to the field of the other ninety-nine.

Within the bundle, $B = \left(\dfrac{\mu_0 I}{2\pi R^2}\right) r = 3.17\times 10^{-3}\ \text{T}.$

The force, *acting inward*, is $F_B = I\ell B$, and the force per unit length is

$$\frac{F_B}{\ell} = \boxed{6.34\times 10^{-3}\ \text{N/m inward}}$$

(b) $B \propto r$, so B is greatest at the outside of the bundle. Since each wire carries the same current, F is $\boxed{\text{greatest at the outer surface}}$.



Figures for Goal Solution

Goal Solution

A packed bundle of 100 long, straight, insulated wires forms a cylinder of radius R = 0.500 cm. (a) If each wire carries 2.00 A, what are the magnitude and direction of the magnetic force per unit length acting on a wire located 0.200 cm from the center of the bundle? (b) Would a wire on the outer edge of the bundle experience a force greater or less than the value calculated in part (a)?

G: The force **on** one wire comes from its interaction with the magnetic field created **by** the other ninety-nine wires. According to Ampere's law, at a distance r from the center, only the wires enclosed within a radius r contribute to this net magnetic field; the other wires outside the radius produce magnetic field vectors in opposite directions that cancel out at r. Therefore, the magnetic field (and also the force on a given wire at radius r) will be greater for larger radii within the bundle, and will decrease for distances beyond the radius of the bundle, as shown in the graph to the right. Applying $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$, the magnetic force on a single wire will be directed toward the center of the bundle, so that all the wires tend to attract each other.

O: Using Ampere's law, we can find the magnetic field at any radius, so that the magnetic force $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$ on a single wire can then be calculated.

A: (a) Ampere's law is used to derive Equation 30.15, which we can use to find the magnetic field at r = 0.200 cm from the center of the cable:

$$B = \frac{\mu_o I_o r}{2\pi R^2} = \frac{\left(4\pi\times10^{-7}\ \mathrm{T\cdot m/A}\right)(99)(2.00\ \mathrm{A})\left(0.200\times10^{-2}\ \mathrm{m}\right)}{2\pi\,(0.500\times10^{-2}\ \mathrm{m})^2} = 3.17\times10^{-3}\ \mathrm{T}$$

This field points tangent to a circle of radius 0.200 cm and exerts a force $\mathbf{F} = I\mathbf{l}\times\mathbf{B}$ toward the center of the bundle, on the single hundredth wire:

$$F/l = IB\sin\theta = (2.00\ \mathrm{A})\left(3.17\times10^{-3}\ \mathrm{T}\right)(\sin 90°) = 6.34\ \mathrm{mN/m}$$

(b) As is shown above in Figure 30.12 from the text, the magnetic field increases linearly as a function of r until it reaches a maximum at the outer surface of the cable. Therefore, the force on a single wire at the outer radius $r = 5.00$ cm would be greater than at $r = 2.00$ cm by a factor of $5/2$.

L: We did not estimate the expected magnitude of the force, but 200 amperes is a lot of current. It would be interesting to see if the magnetic force that pulls together the individual wires in the bundle is enough to hold them against their own weight: If we assume that the insulation accounts for about half the volume of the bundle, then a single copper wire in this bundle would have a cross sectional area of about

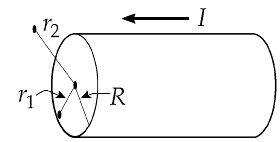
$$(1/2)(0.01)\pi(0.500\ \mathrm{cm})^2 = 4\times10^{-7}\ \mathrm{m}^2$$

with a weight per unit length of $\rho gA = \left(8920\ \mathrm{kg/m^3}\right)(9.8\ \mathrm{N/kg})\left(4\times10^{-7}\ \mathrm{m^2}\right) = 0.03\ \mathrm{N/m}$

Therefore, the outer wires experience an inward magnetic force that is about half the magnitude of their own weight. If placed on a table, this bundle of wires would form a loosely held mound without the outer sheathing to hold them together.

30.26 From $\oint \mathbf{B}\cdot d\mathbf{l} = \mu_0 I$, $I = \dfrac{2\pi rB}{\mu_0} = \dfrac{(2\pi)(1.00\times10^{-3})(0.100)}{4\pi\times10^{-7}} = \boxed{500\ \mathrm{A}}$

30.27 Use Ampère's law, $\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 I$. For current density $\mathbf{J}$, this becomes



$$\oint \mathbf{B}\cdot d\mathbf{s} = \mu_0 \int \mathbf{J}\cdot d\mathbf{A}$$

(a) For $r_1 < R$, this gives

$$2\pi r_1 = \mu_0 \int_0^{r_1} (br)(2\pi r\, dr) \quad \text{and}$$

$$B = \boxed{\frac{\mu_0 b r_1^2}{3} \quad \left(\text{for } r_1 < R \text{ or inside the cylinder}\right)}$$

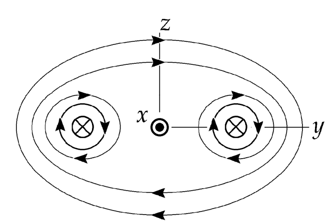
(b) When $r_2 > R$, Ampère's law yields

$$(2\pi r_2)B = \mu_0 \int_0^{R} (br)(2\pi r\, dr) = 2\pi \mu_0 b R^3/3,$$

or $$B = \boxed{\frac{\mu_0 b R^3}{3 r_2} \quad \left(\text{for } r_2 > R \text{ or outside the cylinder}\right)}$$

30.28 (a) See Figure (a) to the right.

(b) At a point on the z axis, the contribution from each wire has magnitude $B = \dfrac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}$ and is perpendicular to the line from this point to the wire as shown in Figure (b). Combining fields, the vertical components cancel while the horizontal components add, yielding

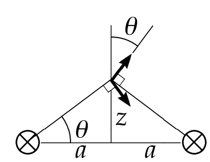


(Currents are into the paper)

Figure (a)

$$B_y = 2\left(\frac{\mu_0 I}{2\pi\sqrt{a^2+z^2}}\sin\theta\right) = \frac{\mu_0 I}{\pi\sqrt{a^2+z^2}}\left(\frac{z}{\sqrt{a^2+z^2}}\right) = \frac{\mu_0 I\, z}{\pi\left(a^2+z^2\right)}$$

The condition for a maximum is:



At a distance z above the plane of the conductors

Figure (b)

$$\frac{dB_y}{dz} = \frac{-\mu_0 I\, z(2z)}{\pi\left(a^2+z^2\right)^2} + \frac{\mu_0 I}{\pi\left(a^2+z^2\right)} = 0, \quad \text{or} \quad \frac{\mu_0 I}{\pi}\frac{\left(a^2-z^2\right)}{\left(a^2+z^2\right)^2} = 0$$

Thus, along the z axis, the field is a maximum at $\boxed{d = a}$.

30.29 $B = \mu_0 \frac{N}{\ell} I$ so $I = \frac{B}{\mu_0 n} = \boxed{31.8 \text{ mA}}$

30.30 (a) $I = \frac{10.0}{(4\pi \times 10^{-7})(2000)} = \boxed{3.98 \text{ kA}}$

(b) $\frac{F_B}{\ell} = IB = \boxed{39.8 \text{ kN/m radially outward}}$

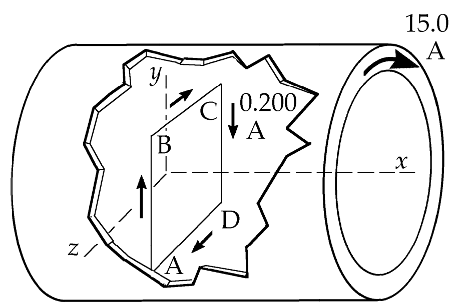
This is the force the windings will have to resist when the magnetic field in the solenoid is 10.0 T.

30.31 The resistance of the wire is $R_e = \frac{\rho \ell}{\pi r^2}$, so it carries current $I = \frac{\mathcal{E}}{R_e} = \frac{\mathcal{E}\pi r^2}{\rho \ell}$.

If there is a single layer of windings, the number of turns per length is the reciprocal of the wire diameter: $n = 1/2r$.

So, $$B = n\mu_0 I = \frac{\mu_0 \mathcal{E}\pi r^2}{\rho \ell 2r} = \frac{\mu_0 \mathcal{E}\pi r}{2\rho \ell} = \frac{(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(20.0 \text{ V})\pi(2.00 \times 10^{-3} \text{ m})}{2(1.70 \times 10^{-8} \, \Omega\cdot\text{m})(10.0 \text{ m})} = \boxed{464 \text{ mT}}$$

***30.32** The field produced by the solenoid in its interior is given by



$$\mathbf{B} = \mu_0 n I(-\mathbf{i}) = \left(4\pi \times 10^{-7} \frac{\text{T}\cdot\text{m}}{\text{A}}\right)\left(\frac{30.0}{10^{-2} \text{ m}}\right)(15.0 \text{ A})(-\mathbf{i})$$

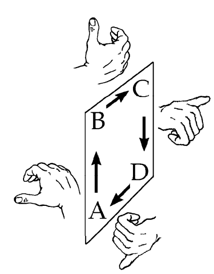
$$\mathbf{B} = -\left(5.65 \times 10^{-2} \text{ T}\right)\mathbf{i}$$

The force exerted on side AB of the square current loop is

$$(\mathbf{F}_B)_{AB} = I\mathbf{L} \times \mathbf{B} = (0.200 \text{ A})\left[\left(2.00 \times 10^{-2} \text{ m}\right)\mathbf{j} \times \left(5.65 \times 10^{-2} \text{ T}\right)(-\mathbf{i})\right]$$

$$(\mathbf{F}_B)_{AB} = \left(2.26 \times 10^{-4} \text{ N}\right)\mathbf{k}$$

Similarly, each side of the square loop experiences a force, lying in the plane of the loop, of $\boxed{226 \, \mu\text{N directed away from the center}}$. From the above result, it is seen that the net torque exerted on the square loop by the field of the solenoid should be zero. More formally, the magnetic dipole moment of the square loop is given by



$$\mu = I\mathbf{A} = (0.200 \text{ A})\left(2.00 \times 10^{-2} \text{ m}\right)^2(-\mathbf{i}) = -80.0 \, \mu\text{A}\cdot\text{m}^2 \, \mathbf{i}$$

The torque exerted on the loop is then $\tau = \mu \times \mathbf{B} = \left(-80.0 \, \mu\text{A}\cdot\text{m}^2 \, \mathbf{i}\right) \times \left(-5.65 \times 10^{-2} \text{ T}\mathbf{i}\right) = \boxed{0}$

30.33 (a) $\Phi_B = \int \mathbf{B}\cdot d\mathbf{A} = \mathbf{B}\cdot\mathbf{A} = (5\mathbf{i}+4\mathbf{j}+3\mathbf{k})\text{T}\cdot\left(2.50\times10^{-2}\text{ m}\right)^2\mathbf{i}$

$\Phi_B = 3.13\times10^{-3}\text{ T}\cdot\text{m}^2 = 3.13\times10^{-3}\text{ Wb} = \boxed{3.13\text{ mWb}}$

(b) $(\Phi_B)_{\text{total}} = \oint \mathbf{B}\cdot d\mathbf{A} = \boxed{0}$ for *any* closed surface (Gauss's law for magnetism)

30.34 (a) $\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA$ where A is the cross-sectional area of the solenoid.

$\Phi_B = \left(\frac{\mu_0 NI}{l}\right)(\pi r^2) = \boxed{7.40\ \mu\text{Wb}}$

(b) $\Phi_B = \mathbf{B}\cdot\mathbf{A} = BA = \left(\frac{\mu_0 NI}{l}\right)\left[\pi\left(r_2^2 - r_1^2\right)\right]$

$\Phi_B = \left[\frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(300)(12.0\text{ A})}{(0.300\text{ m})}\right]\pi\left[(8.00)^2-(4.00)^2\right]\left(10^{-3}\text{ m}\right)^2 = \boxed{2.27\ \mu\text{Wb}}$

30.35 (a) $(\Phi_B)_{\text{flat}} = \mathbf{B}\cdot\mathbf{A} = B\pi R^2\cos(180-\theta) = \boxed{-B\pi R^2\cos\theta}$

(b) The net flux out of the closed surface is zero: $(\Phi_B)_{\text{flat}} + (\Phi_B)_{\text{curved}} = 0$

$(\Phi_B)_{\text{curved}} = \boxed{B\pi R^2\cos\theta}$

30.36 $\frac{d\Phi_E}{dt} = \frac{d}{dt}(EA) = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0}$

(a) $\frac{dE}{dt} = \frac{I}{\epsilon_0 A} = \boxed{7.19\times10^{11}\text{ V/m}\cdot\text{s}}$

(b) $\oint B\cdot ds = \epsilon_0\mu_0\frac{\Phi_E}{dt}$ so $2\pi r B = \epsilon_0\mu_0\frac{d}{dt}\left[\frac{Q}{\epsilon_0 A}\cdot\pi r^2\right]$

$B = \frac{\mu_0 I r}{2A} = \frac{\mu_0(0.200)(5.00\times10^{-2})}{2\pi(0.100)^2} = \boxed{2.00\times10^{-7}\text{ T}}$

30.37 (a) $\frac{d\Phi_E}{dt} = \frac{dQ/dt}{\epsilon_0} = \frac{I}{\epsilon_0} = \frac{(0.100\text{ A})}{8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2} = \boxed{11.3\times10^9\text{ V}\cdot\text{m/s}}$

(b) $I_d = \epsilon_0\frac{d\Phi_E}{dt} = I = \boxed{0.100\text{ A}}$

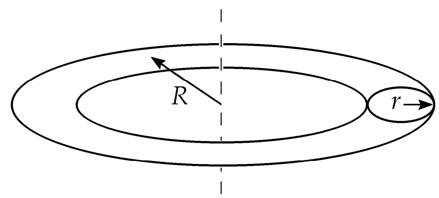
30.38 (a) $I = \dfrac{ev}{2\pi r}$

$$\mu = IA = \left(\frac{ev}{2\pi r}\right)\pi r^2 = \boxed{9.27\times10^{-24}\ \text{A}\cdot\text{m}^2}$$

The Bohr model predicts the correct magnetic moment. However, the "planetary model" is seriously deficient in other regards.

(b) Because the electron is (–), its [conventional] current is clockwise, as seen from above, and μ points $\boxed{\text{downward}}$.

30.39 Assuming a uniform B inside the toroid is equivalent to assuming $r << R$, then $B_0 \cong \mu_0 \dfrac{NI}{2\pi R}$ and a *tightly* wound solenoid.



$$B_0 = \mu_0 \frac{(630)(3.00)}{2\pi(0.200)} = 0.00189\ \text{T}$$

With the steel, $B = \kappa_m B_0 = (1+\chi)B_0 = (101)(0.00189\ \text{T})$ $\boxed{B = 0.191\ \text{T}}$

30.40 $B = \mu n I = \mu\left(\dfrac{N}{2\pi r}\right)I$ so $I = \dfrac{(2\pi r)B}{\mu N} = \dfrac{2\pi(0.100\ \text{m})(1.30\ \text{T})}{5000\left(4\pi\times10^{-7}\ \text{Wb/A}\cdot\text{m}\right)(470)} = \boxed{277\ \text{mA}}$

30.41 $\Phi_B = \mu n I A$

$$B = \mu n I = (750\times4\pi\times10^{-7})\left(\frac{500}{2\pi(0.200)}\right)(0.500) = 0.188\ \text{T}$$

$A = 8.00\times10^{-4}\ \text{m}^2$ and $\Phi_B = (0.188\ \text{T})(8.00\times10^{-4}\ \text{m}^2) = 1.50\times10^{-4}\ \text{T}\cdot\text{m}^2) = \boxed{150\ \mu\text{T}\cdot\text{m}^2}$

30.42 The period is $T = 2\pi/\omega$. The spinning constitutes a current $I = \dfrac{Q}{T} = \dfrac{Q\omega}{2\pi}$.

$$\mu = IA = \frac{Q\omega}{2\pi}\pi R^2 = \frac{Q\omega R^2}{2} \quad \text{in the direction of } \omega$$

$$\mu = \frac{(6.00\times10^{-6}\ \text{C})(4.00\ /\ \text{s})(0.0200\ \text{m})^2}{2} = \boxed{4.80\times10^{-9}\ \text{A}\cdot\text{m}^2}$$

30.43 $B = \mu_0(H + M)$ so $H = \dfrac{B}{\mu_0} - M = \boxed{2.62 \times 10^6 \text{ A/m}}$

30.44 $B = \mu_0(H + M)$

If $\mu_0 M = 2.00$ T, then the magnetization of the iron is $M = \dfrac{2.00 \text{ T}}{\mu_0}$.

But $M = xn\mu_B$ where μ_B is the Bohr magneton, n is the number of atoms per unit volume, and x is the number of electrons that contribute per atom. Thus,

$$x = \frac{M}{n\mu_B} = \frac{2.00 \text{ T}}{n\mu_B\mu_0} = \frac{2.00 \text{ T}}{\left(8.50 \times 10^{28} \text{ m}^{-3}\right)\left(9.27 \times 10^{-24} \text{ N} \cdot \text{m/T}\right)\left(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}\right)} = \boxed{2.02}$$

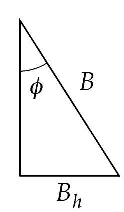
***30.45** (a) Comparing Equations 30.29 and 30.30, we see that the applied field is described by $\mathbf{B}_0 = \mu_0\mathbf{H}$. Then Eq. 30.35 becomes $M = C\dfrac{B_0}{T} = \dfrac{C}{T}\mu_0 H$, and the definition of susceptibility (Eq. 30.32) is

$$\boxed{\chi = \frac{M}{H} = \frac{C}{T}\mu_0}$$

(b) $C = \dfrac{\chi T}{\mu_0} = \dfrac{\left(2.70 \times 10^{-4}\right)(300 \text{ K})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} = \boxed{6.45 \times 10^4 \ \dfrac{\text{K} \cdot \text{A}}{\text{T} \cdot \text{m}}}$

30.46 (a) $B_h = B_{\text{coil}} = \dfrac{\mu_0 NI}{2R} = \dfrac{(4\pi \times 10^{-7})(5.00)(0.600)}{0.300} = \boxed{12.6 \ \mu\text{T}}$

(b) $B_h = B\sin\phi \rightarrow B = \dfrac{B_h}{\sin\phi} = \dfrac{12.6 \ \mu T}{\sin 13.0°} = \boxed{56.0 \ \mu\text{T}}$



30.47 (a) Number of unpaired electrons = $\dfrac{8.00 \times 10^{22} \text{ A} \cdot \text{m}^2}{9.27 \times 10^{-24} \text{ A} \cdot \text{m}^2} = \boxed{8.63 \times 10^{45}}$

Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}\left(8.63 \times 10^{45}\right)$.

(b) Mass = $\dfrac{(4.31 \times 10^{45} \text{ atoms})(7900 \text{ kg/m}^3)}{8.50 \times 10^{28} \text{ atoms/m}^3} = \boxed{4.01 \times 10^{20} \text{ kg}}$

Goal Solution

The magnetic moment of the Earth is approximately 8.00×10^{22} A·m². (a) If this were caused by the complete magnetization of a huge iron deposit, how many unpaired electrons would this correspond to? (b) At two unpaired electrons per iron atom, how many kilograms of iron would this correspond to? (Iron has a density of 7900 kg/m³, and approximately 8.50×10^{28} atoms/m³.)

G: We know that most of the Earth is not iron, so if the situation described provides an accurate model, then the iron deposit must certainly be less than the mass of the Earth ($M_{\text{Earth}} = 5.98 \times 10^{24}$ kg). One mole of iron has a mass of 55.8 g and contributes $2(6.02 \times 10^{23})$ unpaired electrons, so we should expect the total unpaired electrons to be less than 10^{50}.

O: The Bohr magneton μ_B is the measured value for the magnetic moment of a single unpaired electron. Therefore, we can find the number of unpaired electrons by dividing the magnetic moment of the Earth by μ_B. We can then use the density of iron to find the mass of the iron atoms that each contribute two electrons.

A: (a) $\mu_B = \left(9.27 \times 10^{-24} \frac{\text{J}}{\text{T}}\right)\left(1 \frac{\text{N}\cdot\text{m}}{\text{J}}\right)\left(\frac{1\text{ T}}{\text{N}\cdot\text{s/C}\cdot\text{m}}\right)\left(\frac{1\text{ A}}{\text{C/s}}\right) = 9.27 \times 10^{-24}\ \text{A}\cdot\text{m}^2$

The number of unpaired electrons is $N = \dfrac{8.00 \times 10^{22}\ \text{A}\cdot\text{m}^2}{9.27 \times 10^{-24}\ \text{A}\cdot\text{m}^2} = 8.63 \times 10^{45}\ \text{e}^-$

(b) Each iron atom has two unpaired electrons, so the number of iron atoms required is $\frac{1}{2}N = \frac{1}{2}(8.63 \times 10^{45}) = 4.31 \times 10^{45}$ iron atoms.

Thus, $M_{Fe} = \dfrac{(4.31 \times 10^{45}\ \text{atoms})(7900\ \text{kg/m}^3)}{8.50 \times 10^{28}\ \text{atoms/m}^3} = 4.01 \times 10^{20}\ \text{kg}$

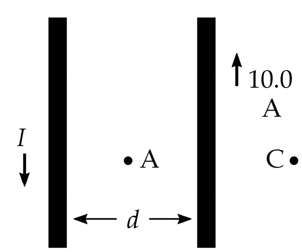
L: The calculated answers seem reasonable based on the limits we expected. From the data in this problem, the iron deposit required to produce the magnetic moment would only be about 1/15 000 the mass of the Earth and would form a sphere 500 km in diameter. Although this is certainly a large amount of iron, it is much smaller than the inner core of the Earth, which is estimated to have a diameter of about 3000 km.

30.48 $B = \dfrac{\mu_0 I}{2\pi R} = 2.00 \times 10^{-5}\ \text{T} = \boxed{20.0\ \mu\text{T}}$

30.49 $B = \dfrac{\mu_0 I R^2}{2(R^2 + R^2)^{3/2}}$ so $\boxed{I = 2.00 \times 10^9\ \text{A}}$ flowing west

30.50 (a) $B_C = \dfrac{\mu_0 I}{2\pi(0.270)} - \dfrac{\mu_0(10.0)}{2\pi(0.0900)} = 0$ so $\boxed{I = 30.0\ \text{A}}$

(b) $B_A = \dfrac{4\mu_0(10.0)}{2\pi(0.0900)} = \boxed{88.9\ \mu\text{T}}$ out of paper



***30.51** Suppose you have two 100-W headlights running from a 12-V battery, with the whole $\frac{200\text{ W}}{12\text{ V}} = 17$ A current going through the switch 60 cm from the compass. Suppose the dashboard contains little iron, so $\mu \cong \mu_0$. Model the current as straight. Then,

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi \times 10^{-7})17}{2\pi(0.6)} \boxed{\sim 10^{-5}\text{ T}}$$

If the local geomagnetic field is 5×10^{-5} T, this is $\boxed{\sim 10^{-1}\text{ times as large,}}$ enough to affect the compass noticeably.

30.52 A ring of radius r and width dr has area $dA = 2\pi r\,dr$. The current inside radius r is

$$I = \int_0^r 2\pi J r\,dr = 2\pi J_0 \int_0^r r\,dr - 2\pi\left(J_0/R^2\right)\int_0^r r^3\,dr = 2\pi J_0 r^2/2 - 2\pi\left(J_0/R^2\right)\left(r^4/4\right)$$

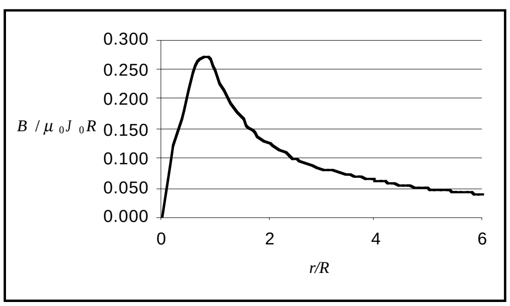
(a) Ampère's law says $B(2\pi r) = \mu_0 I = \mu_0 \pi J_0\left(r^2 - r^4/2R^2\right)$,

or $$\boxed{B = \mu_0 J_0 R\left[\frac{1}{2}\left(\frac{r}{R}\right) - \frac{1}{4}\left(\frac{r}{R}\right)^3\right] \text{ for } r \le R}$$

and $$B(2\pi r) = \mu_0 I_{\text{total}} = \mu_0\left[\pi J_0 R^2 - \pi J_0 R^2/2\right] = \mu_0 \pi J_0 R^2/2$$

or $$\boxed{B = \frac{\mu_0 J_0 R^2}{4r} = \frac{\mu_0 J_0 R}{4(r/R)} \text{ for } r \ge R}$$

(b)

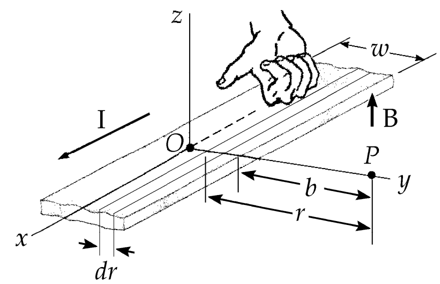


(c) To locate the maximum in the region $r \le R$, require that $\frac{dB}{dr} = \frac{\mu_0 J_0}{2} - 3\frac{\mu_0 J_0 r^2}{4R^2} = 0$

This gives the position of the maximum as $\boxed{r = \sqrt{2/3}\,R}$.

Here $B = \mu_0 J_0 R\left[\frac{1}{2}\left(\frac{2}{3}\right)^{1/2} - \frac{1}{4}\left(\frac{2}{3}\right)^{3/2}\right] = \boxed{0.272\,\mu_0 J_0 R}$

30.53 Consider a longitudinal filament of the strip of width dr as shown in the sketch. The contribution to the field at point P due to the current dI in the element dr is



$$dB = \frac{\mu_0 dI}{2\pi r} \quad \text{where} \quad dI = I(dr/w)$$

$$\mathbf{B} = \int d\mathbf{B} = \int_b^{b+w} \frac{\mu_0 I\, dr}{2\pi w r}\,\mathbf{k} = \boxed{\frac{\mu_0 I}{2\pi w} \ln\left(1 + \frac{w}{b}\right)\mathbf{k}}$$

30.54 We find the total number of turns: $B = \dfrac{\mu_0 N I}{l}$

$$N = \frac{Bl}{\mu_0 I} = \frac{(0.0300\text{ T})(0.100\text{ m})\text{A}}{(4\pi \times 10^{-7}\text{ T}\cdot\text{m})(1.00\text{ A})} = 2.39 \times 10^3$$

Each layer contains $(10.0\text{ cm}/0.0500\text{ cm}) = 200$ closely wound turns

so she needs $(2.39 \times 10^3/200) = \boxed{12\text{ layers}}$.

The inner diameter of the innermost layer is 10.0 mm. The outer diameter of the outermost layer is $10.0\text{ mm} + 2 \times 12 \times 0.500\text{ mm} = 22.0\text{ mm}$. The average diameter is 16.0 mm, so the total length of wire is

$$(2.39 \times 10^3)\pi(16.0 \times 10^{-3}\text{ m}) = \boxed{120\text{ m}}$$

30.55 On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. The magnetic field is directed away from the center, with a strength of

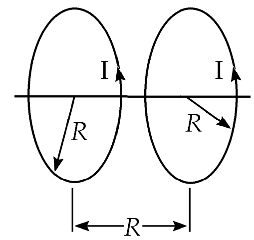
$$B = \frac{\mu_0 \omega R^2 q}{4\pi(x^2 + R^2)^{3/2}} = \frac{\mu_0(20.0)(0.100)^2(10.0 \times 10^{-6})}{4\pi\left[(0.0500)^2 + (0.100)^2\right]^{3/2}} = \boxed{1.43 \times 10^{-10}\text{ T}}$$

30.56 On the axis of a current loop, the magnetic field is given by $B = \dfrac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$

where in this case $I = \dfrac{q}{(2\pi/\omega)}$. Therefore, $B = \dfrac{\mu_0 \omega R^2 q}{4\pi(x^2 + R^2)^{3/2}}$

when $x = \dfrac{R}{2}$, then $B = \dfrac{\mu_0 \omega q R^2}{4\pi\left(\frac{5}{4}R^2\right)^{3/2}} = \boxed{\dfrac{\mu_0 q \omega}{2.5\sqrt{5}\,\pi R}}$

30.57 (a) Use Equation 30.7 twice: $B_x = \dfrac{\mu_0 I R^2}{2(x^2 + R^2)^{3/2}}$



$$B = B_{x1} + B_{x2} = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(x^2 + R^2)^{3/2}} + \frac{1}{\left((R - x)^2 + R^2\right)^{3/2}}\right]$$

$$\boxed{B = \frac{\mu_0 I R^2}{2}\left[\frac{1}{(x^2 + R^2)^{3/2}} + \frac{1}{(2R^2 + x^2 - 2xR)^{3/2}}\right]}$$

(b) $\dfrac{dB}{dx} = \dfrac{\mu_0 I R^2}{2}\left[-\dfrac{3}{2}(2x)\left(x^2 + R^2\right)^{-5/2} - \dfrac{3}{2}\left(2R^2 + x^2 - 2xR\right)^{-5/2}(2x - 2R)\right]$

Substituting $x = \dfrac{R}{2}$ and cancelling terms, $\boxed{\dfrac{dB}{dx} = 0}$

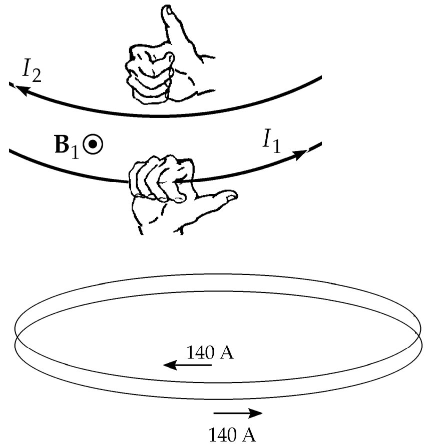
$$\frac{d^2B}{dx^2} = -\frac{3\mu_0 I R^2}{2}\left[(x^2 + R^2)^{-5/2} - 5x^2(x^2 + R^2)^{-7/2} + (2R^2 + x^2 - 2xR)^{-5/2} - 5(x - R)^2(2R^2 + x^2 - 2xR)^{-7/2}\right]$$

Again substituting $x = \dfrac{R}{2}$ and cancelling terms, $\boxed{\dfrac{d^2B}{dx^2} = 0}$

30.58 "Helmholtz pair" $\rightarrow$ separation distance = radius

$$B = \frac{2\mu_0 I R^2}{2\left[(R/2)^2 + R^2\right]^{3/2}} = \frac{\mu_0 I R^2}{\left[\frac{1}{4} + 1\right]^{3/2} R^3} = \frac{\mu_0 I}{1.40R} \quad \text{for 1 turn}$$

For N turns in each coil, $B = \dfrac{\mu_0 N I}{1.40R} = \dfrac{\left(4\pi \times 10^{-7}\right)100(10.0)}{1.40(0.500)} = \boxed{1.80 \times 10^{-3}\ \text{T}}$



30.59 Model the two wires as straight parallel wires (!)

(a) $F_B = \dfrac{\mu_0 I^2 L}{2\pi a}$ (Equation 30.12)

$$F_B = \frac{(4\pi \times 10^{-7})(140)^2 2\pi(0.100)}{2\pi(1.00 \times 10^{-3})} = \boxed{2.46\ \text{N}} \quad \text{upward}$$

(b) $$a_{\text{loop}} = \frac{2.46\ \text{N} - m_{\text{loop}}\, g}{m_{\text{loop}}} = \boxed{107\ \text{m/s}^2} \quad \text{upward}$$

***30.60** (a) In $d\mathbf{B} = \dfrac{\mu_0}{4\pi r^2} I d\mathbf{s} \times \sim$, the moving charge constitutes a bit of current as in $I = nqvA$. For a positive charge the direction of $d\mathbf{s}$ is the direction of $\mathbf{v}$, so $d\mathbf{B} = \dfrac{\mu_0}{4\pi r^2} nqA(ds)\mathbf{v} \times \sim$. Next, $A(ds)$ is the volume occupied by the moving charge, and $nA(ds) = 1$ for just one charge. Then,

$$\boxed{\mathbf{B} = \frac{\mu_0}{4\pi r^2} q\mathbf{v} \times \sim}$$

(b) $$B = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.00 \times 10^{7}\ \text{m/s}\right)}{4\pi\left(1.00 \times 10^{-3}\right)^2} \sin 90.0° = \boxed{3.20 \times 10^{-13}\ \text{T}}$$

(c) $$F_B = q\left|\mathbf{v} \times \mathbf{B}\right| = \left(1.60 \times 10^{-19}\ \text{C}\right)\left(2.00 \times 10^{7}\ \text{m/s}\right)\left(3.20 \times 10^{-13}\ \text{T}\right) \sin 90.0°$$

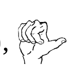
$$F_B = \boxed{1.02 \times 10^{-24}\ \text{N directed away from the first proton}}$$

(d) $$F_e = qE = \frac{k_e q_1 q_2}{r^2} = \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(1.00 \times 10^{-3}\right)^2}$$

$$F_e = \boxed{2.30 \times 10^{-22}\ \text{N directed away from the first proton}}$$

Both forces act together. The electrical force is stronger by two orders of magnitude. It is productive to think about how it would look to an observer in a reference frame moving along with one proton or the other.

***30.61** (a) $$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(24.0\ \text{A})}{2\pi(0.0175\ \text{m})} = \boxed{2.74 \times 10^{-4}\ \text{T}}$$

(b) At point C, conductor AB produces a field $\frac{1}{2}\left(2.74 \times 10^{-4}\ \text{T}\right)(-\mathbf{j})$,  conductor DE produces a field of $\frac{1}{2}\left(2.74 \times 10^{-4}\ \text{T}\right)(-\mathbf{j})$, BD produces no field, and AE produces negligible field. The total field at C is $\boxed{2.74 \times 10^{-4}\ \text{T}(-\mathbf{j})}$.

(c) $\mathbf{F}_B = I\mathbf{L}\times\mathbf{B} = (24.0\text{ A})(0.0350\text{ m}\,\mathbf{k})\times\left[5\left(2.74\times10^{-4}\text{ T}\right)(-\mathbf{j})\right] = \boxed{\left(1.15\times10^{-3}\text{ N}\right)\mathbf{i}}$

(d) $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m} = \frac{\left(1.15\times10^{-3}\text{ N}\right)\mathbf{i}}{3.00\times10^{-3}\text{ kg}} = \boxed{\left(0.384\ \frac{\text{m}}{\text{s}^2}\right)\mathbf{i}}$

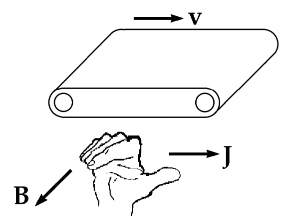
(e) The bar is already so far from *AE* that it moves through nearly constant magnetic field. The force acting on the bar is constant, and therefore the bar's $\boxed{\text{acceleration is constant}}$.

(f) $v_f^2 = v_i^2 + 2ax = 0 + 2\left(0.384\text{ m/s}^2\right)(1.30\text{ m})$, so $\mathbf{v}_f = \boxed{(0.999\text{ m/s})\mathbf{i}}$

30.62 At equilibrium, $\frac{F_B}{\ell} = \frac{\mu_0 I_A I_B}{2\pi a} = \frac{mg}{\ell}$ or $I_B = \frac{2\pi a(m/\ell)g}{\mu_0 I_A}$

$$I_B = \frac{2\pi(0.0250 \text{ m})(0.0100 \text{ kg/m})(9.80 \text{ m/s}^2)}{(4\pi\times10^{-7} \text{ T}\cdot\text{m/A})(150 \text{ A})} = \boxed{81.7 \text{ A}}$$

30.63 (a) The magnetic field due to an infinite sheet of charge (or the magnetic field at points near a large sheet of charge) is given by $B = \mu_0 J_s/2$. The current density $J_s = I/\ell$ and in this case the equivalent current of the moving charged belt is



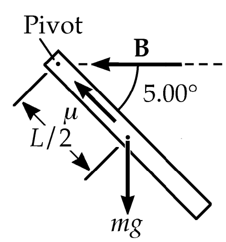
$$I = \frac{dq}{dt} = \frac{d}{dt}(\sigma \ell x) = \sigma \ell v; \qquad v = \frac{dx}{dt}$$

Therefore, $J_s = \sigma v$ and $\boxed{B = \frac{\mu_0 \sigma v}{2}}$

(b) If the sheet is positively charged and moving in the direction shown, the magnetic field is $\boxed{\text{out of the page, parallel to the roller axes.}}$

30.64 $$C = \frac{TM}{B} = \frac{(4.00 \text{ K})(10.0\%)(8.00\times10^{27} \text{ atoms / m}^3)(5.00)(9.27\times10^{-24} \text{ J / T}^2)}{5.00 \text{ T}} = \boxed{2.97\times10^4 \ \frac{\text{K}\cdot\text{J}}{\text{T}^2\cdot\text{m}^3}}$$

30.65 At equilibrium, $\Sigma\tau = +|\mu\times\mathbf{B}| - mg\left(\frac{L}{2}\cos 5.00^\circ\right) = 0,$



or $\mu B \sin 5.00^\circ = \frac{mgL}{2}\cos 5.00^\circ$

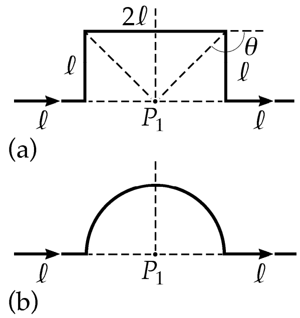
Therefore, $B = \frac{mgL}{2\mu \tan 5.00^\circ} = \frac{(0.0394 \text{ kg})(9.80 \text{ m/s}^2)(0.100 \text{ m})}{2(7.65 \text{ J/T})\tan 5.00^\circ}$

$B = \boxed{28.8 \text{ mT}}$

30.66 The central wire creates field $\mathbf{B} = \mu_0 I_1/2\pi R$ counterclockwise. The curved portions of the loop feels no force since $\boldsymbol{\ell}\times\mathbf{B} = 0$ there. The straight portions both feel $I\boldsymbol{\ell}\times\mathbf{B}$ forces to the right, amounting to

$$\mathbf{F}_B = I_2 2L\frac{\mu_0 I_1}{2\pi R} = \boxed{\frac{\mu_0 I_1 I_2 L}{\pi R} \text{ to the right}}$$

30.67 When the conductor is in the rectangular shape shown in figure (a), the segments carrying current straight toward or away from point P_1 do not contribute to the magnetic field at P_1. Each of the other four setions of length ℓ makes an equal contribution to the total field into the page at P_1. To find the contribution of the horizontal section of current in the upper right, we use



$$B = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2) \quad \text{with} \quad a = \ell,\ \theta_1 = 90°,\ \text{and} \quad \theta_2 = 135°$$

So $$B_1 = \frac{4\mu_0 I}{4\pi\ell}\left(0 - \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I}{\sqrt{2}\,\pi\ell}$$

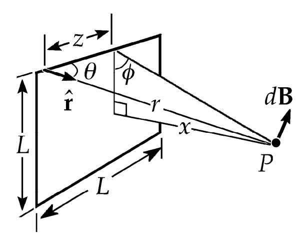
When the conductor is in the shape of a circular arc, the magnitude or the field at the center is given by Equation 30.6, $B = \frac{\mu_0 I}{4\pi R}\theta$. From the geometry in this case, we find $R = \frac{4\ell}{\pi}$ and $\theta = \pi$.

Therefore, $B_2 = \frac{\mu_0 I\pi}{4\pi(4\ell/\pi)} = \frac{\mu_0 I\pi}{16\ell}$; so that $\boxed{\frac{B_1}{B_2} = \frac{8\sqrt{2}}{\pi^2}}$

30.68 $$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(9.00\times10^3\right)\left(1.50\times10^{-8}\right)}{4\pi\times10^{-7}} = \boxed{675\ \text{A}}$$

Flow of $\boxed{\text{positive current is downward } \textit{or} \text{ negative charge flows upward}}$.

30.69 By symmetry of the arrangement, the magnitude of the net magnetic field at point P is $B = 8B_{0x}$ where B_0 is the contribution to the field due to current in an edge length equal to $L/2$. In order to calculate B_0, we use the Biot-Savart law and consider the plane of the square to be the yz-plane with point P on the x-axis. The contribution to the magnetic field at point P due to a current element of length dz and located a distance z along the axis is given by Equation 30.3.



$$\mathbf{B}_0 = \frac{\mu_0 I}{4\pi}\int \frac{d1\times\sim}{r^2}$$

From the figure we see that

$$r = \sqrt{x^2 + (L^2/4) + z^2} \qquad \text{and} \qquad |d1\times\sim| = dz\sin\theta = dz\sqrt{\frac{L^2/4 + x^2}{L^2/4 + x^2 + z^2}}$$

By symmetry all components of the field $\mathbf{B}$ at P cancel except the components along x (perpendicular to the plane of the square); and

$B_{0x} = B_0 \cos\phi$ where $\cos\phi = \dfrac{L/2}{\sqrt{L^2/4 + x^2}}$.

Therefore, $\mathbf{B}_{0x} = \dfrac{\mu_0 I}{4\pi}\displaystyle\int_0^{L/2} \dfrac{\sin\theta\cos\phi\, dz}{r^2}$ and $B = 8B_{0x}$.

Using the expressions given above for sin θ cos ϕ, and r, we find

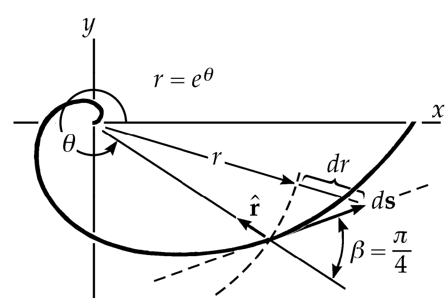
$$B = \frac{\mu_0 I L^2}{2\pi\left(x^2 + \frac{L^2}{4}\right)\sqrt{x^2 + \frac{L^2}{2}}}$$

30.70 (a) From Equation 30.10, the magnetic field produced by one loop at the center of the second loop is given by $B = \dfrac{\mu_0 I R^2}{2x^3} = \dfrac{\mu_0 I\left(\pi R^2\right)}{2\pi x^3} = \dfrac{\mu_0 \mu}{2\pi x^3}$ where the magnetic moment of either loop is $\mu = I\left(\pi R^2\right)$. Therefore,

$$|F_x| = \mu\frac{dB}{dx} = \mu\left(\frac{\mu_0\mu}{2\pi}\right)\left(\frac{3}{x^4}\right) = \frac{3\mu_0\left(\pi R^2 I\right)^2}{2\pi x^4} = \boxed{\frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4}}$$

(b) $$|F_x| = \frac{3\pi}{2}\frac{\mu_0 I^2 R^4}{x^4} = \frac{3\pi}{2}\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(10.0\ \text{A})^2\left(5.00\times10^{-3}\ \text{m}\right)^4}{\left(5.00\times10^{-2}\ \text{m}\right)^4} = \boxed{5.92\times10^{-8}\ \text{N}}$$

30.71 There is no contribution from the straight portion of the wire since $d\mathbf{s}\times\hat{\mathbf{r}} = 0$. For the field of the spiral,



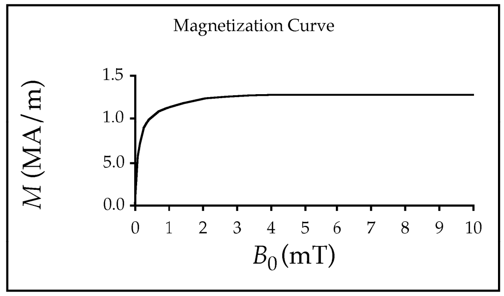
$$d\mathbf{B} = \frac{\mu_0 I}{(4\pi)}\frac{(d\mathbf{s}\times\hat{\mathbf{r}})}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\frac{|d\mathbf{s}|\sin\theta|\hat{\mathbf{r}}|}{r^2} = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi}\left(\sqrt{2}\,dr\right)\left[\sin\left(\frac{3\pi}{4}\right)\right]\frac{1}{r^2}$$

$$B = \frac{\mu_0 I}{4\pi}\int_{\theta=0}^{2\pi} r^{-2}\,dr = -\frac{\mu_0 I}{4\pi}\left(r^{-1}\right)\Big|_{\theta=0}^{2\pi}$$

Substitute $r = e^\theta$: $B = -\dfrac{\mu_0 I}{4\pi}\left[e^{-\theta}\right]_0^{2\pi} = -\dfrac{\mu_0 I}{4\pi}\left[e^{-2\pi} - e^0\right] = \boxed{\dfrac{\mu_0 I}{4\pi}\left(1 - e^{-2\pi}\right)}$ (out of the page)

30.72 (a) $\mathbf{B} = \mathbf{B}_0 + \mu_0 \mathbf{M}$

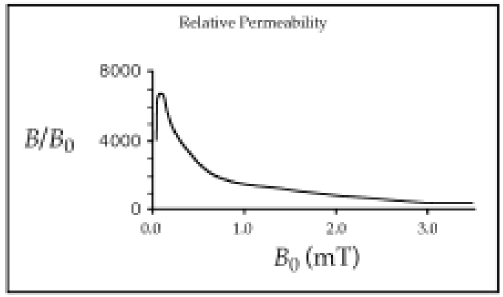


$$\mathbf{M} = \frac{\mathbf{B} - \mathbf{B}_0}{\mu_0} \quad \text{and} \quad M = \frac{|\mathbf{B} - \mathbf{B}_0|}{\mu_0}$$

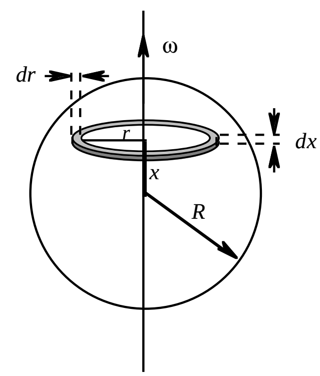
Assuming that $\mathbf{B}$ and $\mathbf{B}_0$ are parallel, this becomes $M = (B - B_0)/\mu_0$

The magnetization curve gives a plot of M versus B_0.

(b) The second graph is a plot of the relative permeability (B/B_0) as a function of the applied field B_0.



30.73 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The contribution to the field from each ring is



$$dB = \frac{\mu_0 r^2\, dI}{2\left(x^2 + r^2\right)^{3/2}} \quad \text{where} \quad dI = \frac{dQ}{t} = \frac{\omega\, dQ}{2\pi}$$

$$dQ = \rho\, dV = \rho(2\pi r\, dr)(dx)$$

$$dB = \frac{\mu_0 \rho \omega r^3\, dr\, dx}{2\left(x^2 + r^2\right)^{3/2}} \quad \text{where} \quad \rho = \frac{Q}{\left(\frac{4}{3}\pi R^3\right)}$$

$$B = \int_{x=-R}^{+R} \int_{r=0}^{\sqrt{R^2 - x^2}} \frac{\mu_0 \rho \omega}{2} \frac{r^3\, dr\, dx}{\left(x^2 + r^2\right)^{3/2}}$$

Let $v = r^2 + x^2$, $\quad dv = 2r\, dr$, and $\quad r^2 = v - x^2$

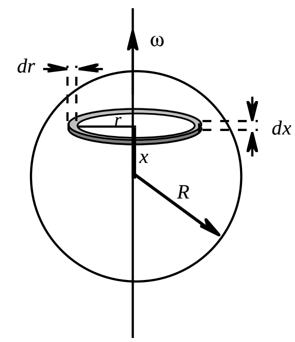
$$B = \int_{x=-R}^{+R} \int_{v=x^2}^{R^2} \frac{\mu_0 \rho \omega}{2} \frac{\left(v - x^2\right) dv}{2 v^{3/2}} dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[\int_{v=x^2}^{R^2} v^{-1/2}\, dv - x^2 \int_{v=x^2}^{R^2} v^{-3/2}\, dv \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[2v^{1/2}\Big|_{x^2}^{R^2} + \left(2x^2\right) v^{-1/2}\Big|_{x^2}^{R^2} \right] dx = \frac{\mu_0 \rho \omega}{4} \int_{x=-R}^{R} \left[2\left(R - |x|\right) + 2x^2 \left(\frac{1}{R} - \frac{1}{|x|} \right) \right] dx$$

$$B = \frac{\mu_0 \rho \omega}{4} \int_{-R}^{R} \left[2\frac{x^2}{R} - 4|x| + 2R \right] dx = \frac{2\mu_0 \rho \omega}{4} \int_0^R \left[2\frac{x^2}{R} - 4x + 2R \right] dx$$

$$B = \frac{2\mu_0 \rho \omega}{4} \left(\frac{2R^3}{3R} - \frac{4R^2}{2} + 2R^2 \right) = \boxed{\frac{\mu_0 \rho \omega R^2}{3}}$$

30.74 Consider the sphere as being built up of little rings of radius r, centered on the rotation axis. The current associated with each rotating ring of charge is



$$dI = \frac{dQ}{t} = \frac{\omega}{2\pi}\left[\rho(2\pi r\,dr)(dx)\right]$$

The magnetic moment contributed by this ring is

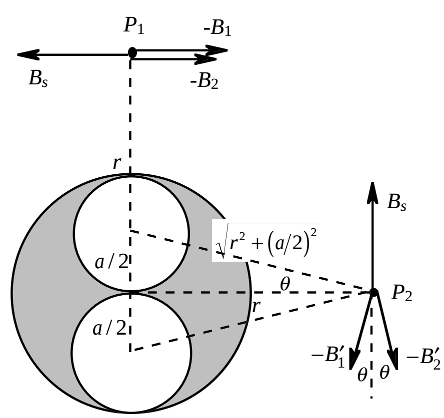
$$d\mu = A(dI) = \pi r^2 \frac{\omega}{2\pi}\left[\rho(2\pi r\,dr)(dx)\right] = \pi\omega\rho r^3\,dr\,dx$$

$$\mu = \pi\omega\rho\int_{x=-R}^{+R}\left[\int_{r=0}^{\sqrt{R^2-x^2}} r^3\,dr\right]dx = \pi\omega\rho\int_{x=-R}^{+R}\frac{\left(\sqrt{R^2-x^2}\right)^4}{4}dx = \pi\omega\rho\int_{x=-R}^{+R}\frac{\left(R^2-x^2\right)^2}{4}dx$$

$$\mu = \frac{\pi\omega\rho}{4}\int_{x=-R}^{+R}\left(R^4 - 2R^2x^2 + x^4\right)dx = \frac{\pi\omega\rho}{4}\left[R^4(2R) - 2R^2\left(\frac{2R^3}{3}\right) + \frac{2R^5}{5}\right]$$

$$\mu = \frac{\pi\omega\rho}{4}R^5\left(2 - \frac{4}{3} + \frac{2}{5}\right) = \frac{\pi\omega\rho R^5}{4}\left(\frac{16}{15}\right) = \boxed{\frac{4\pi\omega\rho R^5}{15}} \quad \text{up}$$

30.75 Note that the current I exists in the conductor with a current density $J = I/A$, where



$$A = \pi\left[a^2 - a^2/4 - a^2/4\right] = \pi a^2/2$$

Therefore, $J = 2I/\pi a^2$.

To find the field at either point P_1 or P_2, find B_s which would exist if the conductor were solid, using Ampère's law. Next, find B_1 and B_2 that *would* be due to the conductors of radius $a/2$ that *could* occupy the void where the holes exist. Then use the superposition principle and subtract the field that would be due to the part of the conductor where the holes exist from the field of the solid conductor.

(a) At point P_1, $B_s = \dfrac{\mu_0 J(\pi a^2)}{2\pi r}$, $B_1 = \dfrac{\mu_0 J\pi(a/2)^2}{2\pi(r-a/2)}$, and $B_2 = \dfrac{\mu_0 J\pi(a/2)^2}{2\pi(r+a/2)}$.

$$B = B_s - B_1 - B_2 = \frac{\mu_0 J\pi a^2}{2\pi}\left[\frac{1}{r} - \frac{1}{4(r-a/2)} - \frac{1}{4(r+a/2)}\right]$$

$$B = \frac{\mu_0(2I)}{2\pi}\left[\frac{4r^2 - a^2 - 2r^2}{4r\left(r^2 - a^2/4\right)}\right] = \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2 - a^2}{4r^2 - a^2}\right] \text{directed to the left}}$$

(b) At point P_2, $B_s = \dfrac{\mu_0 J\left(\pi a^2\right)}{2\pi r}$ and $B_1' = B_2' = \dfrac{\mu_0 J\pi (a/2)^2}{2\pi\sqrt{r^2 + (a/2)^2}}$.

The horizontal components of B_1' and B_2' cancel while their vertical components add.

$$B = B_s - B_1'\cos\theta - B_2'\cos\theta = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r} - 2\left(\frac{\mu_0 J\pi a^2/4}{2\pi\sqrt{r^2 + a^2/4}}\right)\frac{r}{\sqrt{r^2 + a^2/4}}$$

$$B = \frac{\mu_0 J\left(\pi a^2\right)}{2\pi r}\left[1 - \frac{r^2}{2\left(r^2 + a^2/4\right)}\right] = \frac{\mu_0 (2I)}{2\pi r}\left[1 - \frac{2r^2}{4r^2 + a^2}\right] = \boxed{\frac{\mu_0 I}{\pi r}\left[\frac{2r^2 + a^2}{4r^2 + a^2}\right] \text{ directed toward the top of the page}}$$