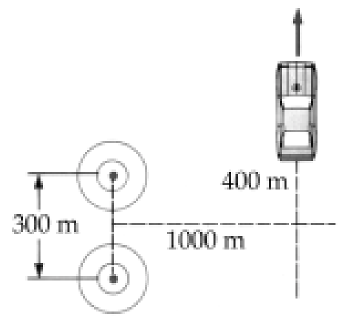
Chapter 37 Solutions

37.1 $\Delta y_{\text{bright}} = \frac{\lambda L}{d} = \frac{(632.8 \times 10^{-9})(5.00)}{2.00 \times 10^{-4}}$ m = $\boxed{1.58\text{ cm}}$

37.2 $y_{\text{bright}} = \frac{\lambda L}{d} m$ For $m = 1$, $\lambda = \frac{yd}{L} = \frac{(3.40 \times 10^{-3}\text{ m})(5.00 \times 10^{-4}\text{ m})}{3.30\text{ m}} = \boxed{515\text{ nm}}$

37.3 Note, with the conditions given, the small angle approximation **does not work well**. That is, $\sin\theta$, $\tan\theta$, and θ are significantly different. The approach to be used is outlined below.



(a) At the $m = 2$ maximum, $\tan\theta = \frac{400\text{ m}}{1000\text{ m}} = 0.400$

$\theta = 21.8°$ so $\lambda = \frac{d\sin\theta}{m} = \frac{(300\text{ m})\sin 21.8°}{2} = \boxed{55.7\text{ m}}$

(b) The next minimum encountered is the $m = 2$ minimum; and at that point,

$d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ which becomes $d\sin\theta = \frac{5}{2}\lambda$

or $\sin\theta = \frac{5\lambda}{2d} = \frac{5(55.7\text{ m})}{2(300\text{ m})} = 0.464$ and $\theta = 27.7°$

so $y = (1000\text{ m})\tan 27.7° = 524$ m

Therefore, the car must travel an additional $\boxed{124\text{ m}}$.

37.4 $\lambda = \frac{v}{f} = \frac{354\text{ m/s}}{2000/\text{s}} = 0.177$ m

(a) $d\sin\theta = m\lambda$ so $(0.300\text{ m})\sin\theta = 1(0.177\text{ m})$ and $\theta = \boxed{36.2°}$

(b) $d\sin\theta = m\lambda$ so $d\sin 36.2° = 1(0.0300\text{ m})$ and $d = \boxed{5.08\text{ cm}}$

(c) $(1.00 \times 10^{-6}\text{ m})\sin 36.2° = 1\lambda$ so $\lambda = 590$ nm

$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\text{ m/s}}{5.90 \times 10^{-7}\text{ m}} = \boxed{508\text{ THz}}$

37.5 For the tenth minimum, $m = 9$. Using Equation 37.3, $\sin\theta = \dfrac{\lambda}{d}\left(9 + \dfrac{1}{2}\right)$

Also, $\tan\theta = \dfrac{y}{L}$. For small θ, $\sin\theta \approx \tan\theta$. Thus,

$$d = \frac{9.5\lambda}{\sin\theta} = \frac{9.5\lambda L}{y} = \frac{9.5(589\times10^{-9}\text{ m})(2.00\text{ m})}{7.26\times10^{-3}\text{ m}} = 1.54\times10^{-3}\text{ m} = \boxed{1.54\text{ mm}}$$

<u>Goal Solution</u>

Young's double-slit experiment is performed with 589-nm light and a slit-to-screen distance of 2.00 m. The tenth interference minimum is observed 7.26 mm from the central maximum. Determine the spacing of the slits.

G: For the situation described, the observed interference pattern is very narrow, (the minima are less than 1 mm apart when the screen is 2 m away). In fact, the minima and maxima are so close together that it would probably be difficult to resolve adjacent maxima, so the pattern might look like a solid blur to the naked eye. Since the angular spacing of the pattern is inversely proportional to the slit width, we should expect that for this narrow pattern, the space between the slits will be larger than the typical fraction of a millimeter, and certainly much greater than the wavelength of the light ($d >> \lambda = 589$ nm).

O: Since we are given the location of the tenth minimum for this interference pattern, we should use the equation for **destructive interference** from a double slit. The figure for Problem 7 shows the critical variables for this problem.

A: In the equation $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$,

The first minimum is described by $m = 0$ and the tenth by $m = 9$: $\sin\theta = \dfrac{\lambda}{d}\left(9 + \frac{1}{2}\right)$

Also, $\tan\theta = y/L$, but for small θ, $\sin\theta \approx \tan\theta$. Thus, $d = \dfrac{9.5\lambda}{\sin\theta} = \dfrac{9.5\lambda L}{y}$

$$d = \frac{9.5(5890\cdot10^{-10}\text{ m})(2.00\text{ m})}{7.26\cdot10^{-3}\text{ m}} = 1.54\cdot10^{-3}\text{ m} = 1.54\text{ mm} = 1.54\text{ mm}$$

L: The spacing between the slits is relatively large, as we expected (about 3 000 times greater than the wavelength of the light). In order to more clearly distinguish between maxima and minima, the pattern could be expanded by increasing the distance to the screen. However, as L is increased, the overall pattern would be less bright as the light expands over a larger area, so that beyond some distance, the light would be too dim to see.

***37.6** $\lambda = \dfrac{340\text{ m/s}}{2000\text{ Hz}} = 0.170\text{ m}$

Maxima are at $d\sin\theta = m\lambda$:

$m = 0$ gives $\theta = 0°$

$m = 1$ gives $\sin\theta = \dfrac{\lambda}{d} = \dfrac{0.170\text{ m}}{0.350\text{ m}}$ $\theta = 29.1°$

$m = 2$ gives $\sin\theta = \dfrac{2\lambda}{d} = 0.971$ $\theta = 76.3°$

$m = 3$ gives $\sin\theta = 1.46$ No solution.

Minima at $d\sin\theta = \left(m + \frac{1}{2}\right)\lambda$:

$m = 0$ gives $\sin\theta = \dfrac{\lambda}{2d} = 0.243$ $\theta = 14.1°$

$m = 1$ gives $\sin\theta = \dfrac{3\lambda}{2d} = 0.729$ $\theta = 46.8°$

$m = 2$ gives No solution.

$\boxed{\text{So we have maxima at } 0°, 29.1°, \text{ and } 76.3° \text{ and minima at } 14.1° \text{ and } 46.8°.}$

37.7 (a) For the bright fringe,

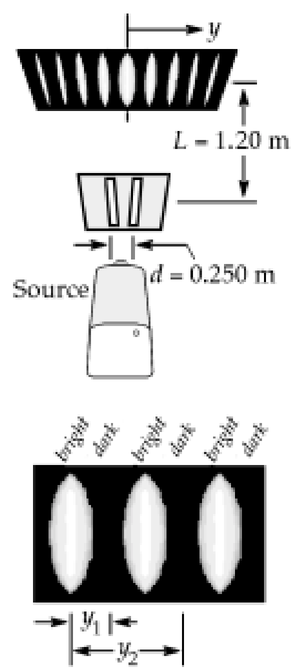
$y_{\text{bright}} = \dfrac{m\lambda L}{d}$ where $m = 1$

$$y = \frac{(546.1\times10^{-9}\text{ m})(1.20\text{ m})}{0.250\times10^{-3}\text{ m}} = 2.62\times10^{-3}\text{ m} = \boxed{2.62\text{ mm}}$$

(b) For the dark bands, $y_{\text{dark}} = \dfrac{\lambda L}{d}\left(m + \dfrac{1}{2}\right)$; $m = 0, 1, 2, 3, \ldots$

$$y_2 - y_1 = \frac{\lambda L}{d}\left[\left(1+\frac{1}{2}\right) - \left(0+\frac{1}{2}\right)\right] = \frac{\lambda L}{d}(1) = \frac{(546.1\times10^{-9}\text{ m})(1.20\text{ m})}{0.250\times10^{-3}\text{ m}}$$

$\Delta y = \boxed{2.62\text{ mm}}$



Figures for Goal Solution

Goal Solution

A pair of narrow, parallel slits separated by 0.250 mm is illuminated by green light (λ = 546.1 nm). The interference pattern is observed on a screen 1.20 m away from the plane of the slits. Calculate the distance (a) from the central maximum to the first bright region on either side of the central maximum and (b) between the first and second dark bands.

G: The spacing between adjacent maxima and minima should be fairly uniform across the pattern as long as the width of the pattern is much less than the distance to the screen (so that the small angle approximation is valid). The separation between fringes should be at least a millimeter if the pattern can be easily observed with a naked eye.

O: The bright regions are areas of constructive interference and the dark bands are destructive interference, so the corresponding double-slit equations will be used to find the y distances.

It can be confusing to keep track of four different symbols for distances. Three are shown in the drawing to the right. Note that:

y is the unknown distance from the bright central maximum (m = 0) to another maximum or minimum on either side of the center of the interference pattern.

λ is the wavelength of the light, determined by the source.

A: (a) For **very small** θ $\sin\theta \approx \tan\theta$ and $\tan\theta = y/L$

and the equation for constructive interference $\sin\theta = m\lambda/d$ (Eq. 37.2)

becomes $y_{\text{bright}} \approx (\lambda L/d)m$ (Eq. 37.5)

Substituting values, $y_{\text{bright}} = \dfrac{(546\times 10^{-9}\text{ m})(1.20\text{ m})}{0.250\times 10^{-3}\text{ m}}(1) = 2.62\text{ mm}$

(b) If you have trouble remembering whether Equation 37.5 or Eq. 37.6 applies to a given situation, you can instead remember that the first bright band is in the center, and dark bands are halfway between bright bands. Thus, Eq. 37.5 describes them all, with $m = 0,\ 1,\ 2\ldots$ for bright bands, and with $m = 0.5,\ 1.5,\ 2.5\ldots$ for dark bands. The dark band version of Eq. 37.5 is simply Eq. 37.6:

$$y_{\text{dark}} = \frac{\lambda L}{d}\left(m + \tfrac{1}{2}\right)$$

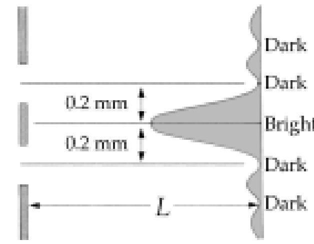
$$\Delta y_{\text{dark}} = \left(1 + \tfrac{1}{2}\right)\frac{\lambda L}{d} - \left(0 + \tfrac{1}{2}\right)\frac{\lambda L}{d} = \frac{\lambda L}{d} = 2.62\text{ mm}$$

L: This spacing is large enough for easy resolution of adjacent fringes. The distance between minima is the same as the distance between maxima. We expected this equality since the angles are small:

$$\theta = (2.62\text{ mm})/(1.20\text{ m}) = 0.00218\text{ rad} = 0.125°$$

When the angular spacing exceeds about 3°, then $\sin\theta$ differs from $\tan\theta$ when written to three significant figures.

37.8 Taking $m = 0$ and $y = 0.200$ mm in Equation 37.6 gives



$$L \approx \frac{2dy}{\lambda} = \frac{2(0.400\times10^{-3}\text{ m})(0.200\times10^{-3}\text{ m})}{442\times10^{-9}\text{ m}} = 0.362\text{ m}$$

$$L \approx \boxed{36.2\text{ cm}}$$

Geometric optics incorrectly predicts bright regions opposite the slits and darkness in between. But, as this example shows, interference can produce just the opposite.

37.9 Location of A = central maximum,

Location of B = first minimum.

So, $$\Delta y = [y_{\text{min}} - y_{\text{max}}] = \frac{\lambda L}{d}\left(0+\frac{1}{2}\right) - 0 = \frac{1}{2}\frac{\lambda L}{d} = 20.0\text{ m}$$

Thus, $$d = \frac{\lambda L}{2(20.0\text{ m})} = \frac{(3.00\text{ m})(150\text{ m})}{40.0\text{ m}} = \boxed{11.3\text{ m}}$$

37.10 At 30.0°, $d\sin\theta = m\lambda$

$(3.20\times10^{-4}\text{ m})\sin 30.0° = m(500\times10^{-9}\text{ m})$ so $m = 320$

There are 320 maxima to the right, 320 to the left, and one for $m = 0$ straight ahead.

There are $\boxed{641\text{ maxima}}$.

37.11 $$\phi = \frac{2\pi}{\lambda}d\sin\theta \approx \frac{2\pi}{\lambda}d\left(\frac{y}{L}\right)$$

(a) $$\phi = \frac{2\pi}{(5.00\times10^{-7}\text{ m})}(1.20\times10^{-4}\text{ m})\sin\left(0.500^{\circ}\right) = \boxed{13.2\text{ rad}}$$

(b) $$\phi = \frac{2\pi}{(5.00\times10^{-7}\text{ m})}(1.20\times10^{-4}\text{ m})\left(\frac{5.00\times10^{-3}\text{ m}}{1.20\text{ m}}\right) = \boxed{6.28\text{ rad}}$$

(c) If $\phi = 0.333 \text{ rad} = \dfrac{2\pi d \sin\theta}{\lambda}$, $\theta = \sin^{-1}\left(\dfrac{\lambda\phi}{2\pi d}\right) = \sin^{-1}\left[\dfrac{(5.00\times10^{-7}\text{ m})(0.333\text{ rad})}{2\pi(1.20\times10^{-4}\text{ m})}\right]$

$$\theta = \boxed{1.27\times10^{-2}\text{ deg}}$$

(d) If $d\sin\theta = \dfrac{\lambda}{4}$, $\theta = \sin^{-1}\left(\dfrac{\lambda}{4d}\right) = \sin^{-1}\left[\dfrac{5\times10^{-7}\text{ m}}{4(1.20\times10^{-4}\text{ m})}\right]$

$$\theta = \boxed{5.97\times10^{-2}\text{ deg}}$$

37.12 The path difference between rays 1 and 2 is: $\delta = d\sin\theta_1 - d\sin\theta_2$

For constructive interference, this path difference must be equal to an integral number of wavelengths: $d\sin\theta_1 - d\sin\theta_2 = m\lambda$, or

$$\boxed{d(\sin\theta_1 - \sin\theta_2) = m\lambda}$$

37.13 (a) The path difference $\delta = d\sin\theta$ and when $L >> y$

$$\delta \approx \frac{yd}{L} = \frac{(1.80\times10^{-2}\text{ m})(1.50\times10^{-4}\text{ m})}{1.40\text{ m}} = 1.93\times10^{-6}\text{ m} = \boxed{1.93\ \mu\text{m}}$$

(b) $\dfrac{\delta}{\lambda} = \dfrac{1.93\times10^{-6}\text{ m}}{6.43\times10^{-7}\text{ m}} = 3.00$, or $\boxed{\delta = 3.00\ \lambda}$

(c) Point P will be a $\boxed{\text{maximum}}$ since the path difference is an integer multiple of the wavelength.

37.14 (a) $\dfrac{I}{I_{max}} = \cos^2\left(\dfrac{\phi}{2}\right)$ (Equation 37.11)

Therefore, $\phi = 2\cos^{-1}\left(\dfrac{I}{I_{max}}\right)^{1/2} = 2\cos^{-1}(0.640)^{1/2} = \boxed{1.29\text{ rad}}$

(b) $\delta = \dfrac{\lambda\phi}{2\pi} = \dfrac{(486\text{ nm})(1.29\text{ rad})}{2\pi} = \boxed{99.8\text{ nm}}$

37.15 $I_{av} = I_{max}\cos^2\left(\dfrac{\pi d\sin\theta}{\lambda}\right)$

For small θ, $\sin\theta = \dfrac{y}{L}$ and $I_{av} = 0.750 I_{max}$

$$y = \frac{\lambda L}{\pi d}\cos^{-1}\left(\frac{I_{av}}{I_{max}}\right)^{1/2}$$

$$y = \frac{(6.00\times10^{-7})(1.20\text{ m})}{\pi(2.50\times10^{-3}\text{ m})}\cos^{-1}\left(\frac{0.750 I_{max}}{I_{max}}\right)^{1/2} = \boxed{48.0\ \mu\text{m}}$$

37.16 $I = I_{max}\cos^2\left(\dfrac{\pi yd}{\lambda L}\right)$

$$\frac{I}{I_{max}} = \cos^2\left[\frac{\pi\left(6.00\times10^{-3}\text{ m}\right)\left(1.80\times10^{-4}\text{ m}\right)}{\left(656.3\times10^{-9}\text{ m}\right)(0.800\text{ m})}\right] = \boxed{0.987}$$

37.17 (a) From Equation 37.8,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\cdot\frac{y}{\sqrt{y^2+D^2}}$$

$$\phi \approx \frac{2\pi yd}{\lambda D} = \frac{2\pi\left(0.850\times10^{-3}\text{ m}\right)\left(2.50\times10^{-3}\text{ m}\right)}{\left(600\times10^{-9}\text{ m}\right)(2.80\text{ m})} = \boxed{7.95\text{ rad}}$$

(b) $$\frac{I}{I_{max}} = \frac{\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta\right)}{\cos^2\left(\dfrac{\pi d}{\lambda}\sin\theta_{max}\right)} = \frac{\cos^2\dfrac{\phi}{2}}{\cos^2 m\pi}$$

$$\frac{I}{I_{max}} = \cos^2\frac{\phi}{2} = \cos^2\left(\frac{7.95\text{ rad}}{2}\right) = \boxed{0.453}$$

Goal Solution

Two narrow parallel slits separated by 0.850 mm are illuminated by 600-nm light, and the viewing screen is 2.80 m away from the slits. (a) What is the phase difference between the two interfering waves on a screen at a point 2.50 mm from the central bright fringe? (b) What is the ratio of the intensity at this point to the intensity at the center of a bright fringe?

G: It is difficult to accurately predict the relative intensity at the point of interest without actually doing the calculation. The waves from each slit could meet in phase ($\phi = 0$) to produce a bright spot of **constructive interference**, out of phase ($\phi = 180°$) to produce a dark region of **destructive interference**, or most likely the phase difference will be somewhere between these extremes, $0 < \phi < 180°$, so that the relative intensity will be $0 < I/I_{\max} < 1$.

O: The phase angle depends on the path difference of the waves according to Equation 37.8. This phase difference is used to find the average intensity at the point of interest. Then the relative intensity is simply this intensity divided by the maximum intensity.

A: (a) Using the variables shown in the diagram for problem 7 we have,

$$\phi = \frac{2\pi d}{\lambda}\sin\theta = \frac{2\pi d}{\lambda}\left(\frac{y}{\sqrt{y^2+L^2}}\right) \cong \frac{2\pi y d}{\lambda L} = \frac{2\pi\left(0.850\cdot 10^{-3}\text{ m}\right)\left(0.00250\text{ m}\right)}{\left(600\cdot 10^{-9}\text{ m}\right)\left(2.80\text{ m}\right)} = 7.95\text{ rad} = 2\pi + 1.66\text{ rad} = 95.5$$

(b) $$\frac{I}{I_{\max}} = \frac{\cos^2\left(\frac{\pi d}{\lambda}\sin\theta\right)}{\cos^2\left(\frac{\pi d}{\lambda}\sin\theta_{\max}\right)} = \frac{\cos^2\left(\frac{\phi}{2}\right)}{\cos^2(m\pi)} = \cos^2\left(\frac{\phi}{2}\right) = \cos^2\left(\frac{95.5}{2}\right) = 0.452$$

L: It appears that at this point, the waves show **partial interference** so that the combination is about half the brightness found at the central maximum. We should remember that the equations used in this solution do not account for the diffraction caused by the finite width of each slit. This diffraction effect creates an "envelope" that diminishes in intensity away from the central maximum as shown by the dotted line in Figures 37.13 and P37.60. Therefore, the relative intensity at $y = 2.50$ mm will actually be slightly less than 0.452.

37.18 (a) The resultant amplitude is

$$E_r = E_0\sin\omega t + E_0\sin(\omega t+\phi) + E_0\sin(\omega t + 2\phi), \qquad \text{where} \qquad \phi = \frac{2\pi}{\lambda}d\sin\theta.$$

$$E_r = E_0\left(\sin\omega t + \sin\omega t\cos\phi + \cos\omega t\sin\phi + \sin\omega t\cos 2\phi + \cos\omega t\sin 2\phi\right)$$

$$E_r = E_0(\sin\omega t)\left(1+\cos\phi + 2\cos^2\phi - 1\right) + E_0(\cos\omega t)(\sin\phi + 2\sin\phi\cos\phi)$$

$$E_r = E_0(1+2\cos\phi)(\sin\omega t\cos\phi + \cos\omega t\sin\phi) = E_0(1+2\cos\phi)\sin(\omega t+\phi)$$

Then the intensity is $I \propto E_r^2 = E_0^2 (1 + 2\cos\phi)^2 \left(\frac{1}{2}\right)$

where the time average of $\sin^2(\omega t + \phi)$ is 1/2.

From one slit alone we would get intensity $I_{\max} \propto E_0^2 \left(\frac{1}{2}\right)$ so $\boxed{I = I_{\max}\left[1 + 2\cos\left(\frac{2\pi d \sin\theta}{\lambda}\right)\right]^2}$

(b) Look at the $N = 3$ graph in Figure 37.13. Minimum intensity is zero, attained where $\cos\phi = -1/2$. One relative maximum occurs at $\cos\phi = -1.00$, where $I = I_{\max}$.

The larger local maximum happens where $\cos\phi = +1.00$, giving $I = 9.00\, I_0$.

The ratio of intensities at primary versus secondary maxima is $\boxed{9.00}$.

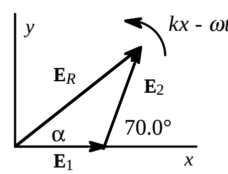
***37.19** (a) We can use $\sin A + \sin B = 2\sin(A/2 + B/2)\cos(A/2 - B/2)$ to find the sum of the two sine functions to be

$$E_1 + E_2 = (24.0\ \text{kN/C})\sin(15x - 4.5t + 35.0°)\cos 35.0°$$

$$E_1 + E_2 = (19.7\ \text{kN/C})\sin(15x - 4.5t + 35.0°)$$

Thus, the total wave has amplitude $\boxed{19.7\ \text{kN/C}}$ and has a constant phase difference of $\boxed{35.0°}$ from the first wave.

(b) In units of kN/C, the resultant phasor is



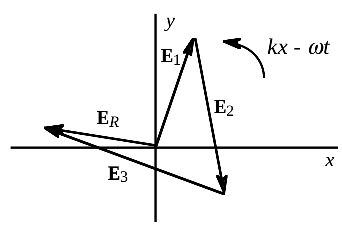
$$\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 = (12.0\,\mathbf{i}) + (12.0\cos 70.0°\,\mathbf{i} + 12.0\sin 70.0°\,\mathbf{j}) = 16.1\,\mathbf{i} + 11.3\,\mathbf{j}$$

$$\mathbf{E}_R = \sqrt{(16.1)^2 + (11.3)^2}\ \text{at}\ \tan^{-1}(11.3/16.1) = \boxed{19.7\ \text{kN/C at } 35.0°}$$

(c) $\mathbf{E}_R = 12.0\cos 70.0°\,\mathbf{i} + 12.0\sin 70.0°\,\mathbf{j}$

$+15.5\cos 80.0°\,\mathbf{i} - 15.5\sin 80.0°\,\mathbf{j}$

$+17.0\cos 160°\,\mathbf{i} + 17.0\sin 160°\,\mathbf{j}$



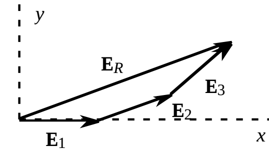
$$\mathbf{E}_R = -9.18\,\mathbf{i} + 1.83\,\mathbf{j} = \boxed{9.36\ \text{kN/C at } 169°}$$

The wave function of the total wave is $E_P = (9.36\ \text{kN/C})\sin(15x - 4.5t + 169°)$

37.20 (a) $\mathbf{E}_R = E_0\left[\mathbf{i} + \left(\mathbf{i}\cos 20.0° + \mathbf{j}\sin 20.0°\right) + \left(\mathbf{i}\cos 40.0° + \mathbf{j}\sin 40.0°\right)\right]$

$\mathbf{E}_R = E_0\left[2.71\mathbf{i} + 0.985\mathbf{j}\right] = 2.88E_0$ at $20.0° =$ $\boxed{2.88E_0 \text{ at } 0.349 \text{ rad}}$

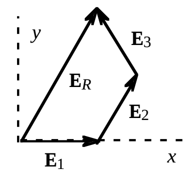
$E_P = 2.88E_0 \sin\left(\omega t + 0.349\right)$



(b) $\mathbf{E}_R = E_0\left[\mathbf{i} + \left(\mathbf{i}\cos 60.0° + \mathbf{j}\sin 60.0°\right) + \left(\mathbf{i}\cos 120° + \mathbf{j}\sin 120°\right)\right]$

$\mathbf{E}_R = E_0\left[1.00\mathbf{i} + 1.73\mathbf{j}\right] = 2.00E_0$ at $60.0° =$ $\boxed{2.00E_0 \text{ at } \pi/3 \text{ rad}}$

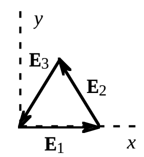
$E_P = 2.00E_0 \sin\left(\omega t + \pi/3\right)$



(c) $\mathbf{E}_R = E_0\left[\mathbf{i} + \left(\mathbf{i}\cos 120° + \mathbf{j}\sin 120°\right) + \left(\mathbf{i}\cos 240° + \mathbf{j}\sin 240°\right)\right]$

$\mathbf{E}_R = E_0\left[0\mathbf{i} + 0\mathbf{j}\right] = \boxed{0}$

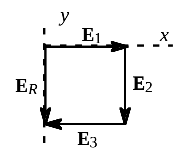
$E_P = 0$



(d) $\mathbf{E}_R = E_0\left[\mathbf{i} + \left(\mathbf{i}\cos 3\pi/2 + \mathbf{j}\sin 3\pi/2\right) + \left(\mathbf{i}\cos 3\pi + \mathbf{j}\sin 3\pi\right)\right]$

$\mathbf{E}_R = E_0\left[0\mathbf{i} - 1.00\mathbf{j}\right] = E_0$ at $270° =$ $\boxed{E_0 \text{ at } 3\pi/2 \text{ rad}}$

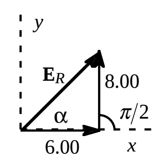
$E_P = E_0 \sin\left(\omega t + 3\pi/2\right)$



37.21 $\mathbf{E}_R = 6.00\mathbf{i} + 8.00\mathbf{j} = \sqrt{(6.00)^2 + (8.00)^2}$ at $\tan^{-1}\left(8.00/6.00\right)$

$\mathbf{E}_R = 10.0$ at $53.1° = 10.0$ at 0.927 rad

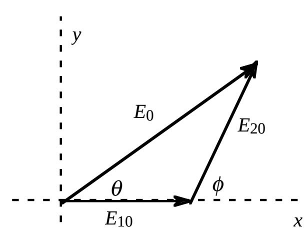
$E_P = \boxed{10.0\sin\left(100\pi t + 0.927\right)}$



37.22 If $E_1 = E_{10}\sin\omega t$ and $E_2 = E_{20}\sin\left(\omega t + \phi\right)$, then by phasor addition, the amplitude of $\mathbf{E}$ is

$$E_0 = \sqrt{\left(E_{10} + E_{20}\cos\phi\right)^2 + \left(E_{20}\sin\phi\right)^2} = \boxed{\sqrt{E_{10}^2 + 2E_{10}E_{20}\cos\phi + E_{20}^2}}$$

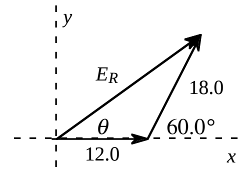
and the phase angle is found from $\boxed{\sin\theta = \dfrac{E_{20}\sin\phi}{E_0}}$



37.23 $\mathbf{E}_R = 12.0\,\mathbf{i} + \left(18.0\cos 60.0°\,\mathbf{i} + 18.0\sin 60.0°\,\mathbf{j}\right)$

$\mathbf{E}_R = 21.0\,\mathbf{i} + 15.6\,\mathbf{j} = 26.2$ at $36.6°$

$E_P = \boxed{26.2\sin(\omega t + 36.6°)}$



37.24 Constructive interference occurs where $m = 0, 1, 2, 3, \ldots$, for

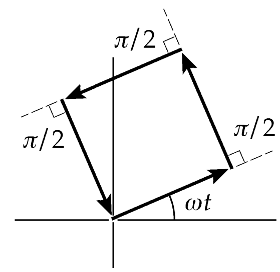
$$\left(\frac{2\pi x_1}{\lambda} - 2\pi f t + \frac{\pi}{6}\right) - \left(\frac{2\pi x_2}{\lambda} - 2\pi f t + \frac{\pi}{8}\right) = 2\pi m \quad \frac{2\pi(x_1 - x_2)}{\lambda} + \left(\frac{\pi}{6} - \frac{\pi}{8}\right) = 2\pi m$$

$$\frac{(x_1 - x_2)}{\lambda} + \frac{1}{12} - \frac{1}{16} = m$$

$$\boxed{x_1 - x_2 = \left(m - \frac{1}{48}\right)\lambda \qquad m = 0, 1, 2, 3, \ldots}$$

37.25 See the figure to the right:

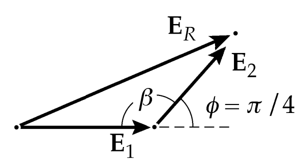
$\boxed{\phi = \pi/2}$



37.26 $E_R^2 = E_1^2 + E_2^2 - 2E_1E_2\cos\beta$, where $\beta = 180 - \phi$.

Since $I \propto E^2$,

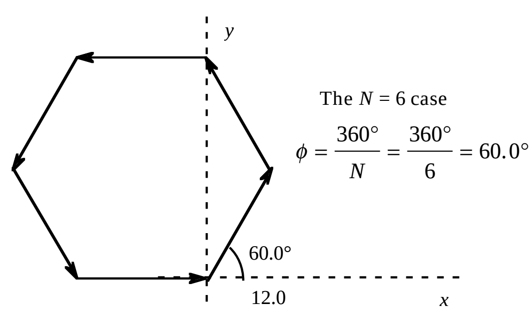
$$I_R = \boxed{I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi}$$



37.27 Take $\boxed{\phi = 360°/N}$ where N defines the number of coherent sources. Then,

$$E_R = \sum_{m=1}^{N} E_0 \sin(\omega t + m\phi) = 0$$

In essence, the set of N electric field components complete a full circle and return to zero.



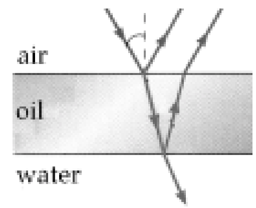
***37.28** Light reflecting from the first surface suffers phase reversal. Light reflecting from the second surface does not, but passes twice through the thickness t of the film. So, for constructive interference, we require

$$\frac{\lambda_n}{2} + 2t = \lambda_n$$

where $\lambda_n = \frac{\lambda}{n}$ is the wavelength in the material. Then $$2t = \frac{\lambda_n}{2} = \frac{\lambda}{2n}$$

$\lambda = 4nt = 4 \times 1.33 \times 115 \text{ nm} = \boxed{612 \text{ nm}}$

37.29 (a) The light reflected from the top of the oil film undergoes phase reversal. Since 1.45 > 1.33, the light reflected from the bottom undergoes no reversal. For constructive interference of reflected light, we then have



$$2nt = \left(m + \frac{1}{2}\right)\lambda \quad \text{or} \quad \lambda_m = \frac{2nt}{\left(m + \frac{1}{2}\right)} = \frac{2(1.45)(280 \text{ nm})}{\left(m + \frac{1}{2}\right)}$$

Substituting for m, we have $m = 0$: $\lambda_0 = 1620$ nm (infrared)

$m = 1$: $\lambda_1 = 541$ nm (green)

$m = 2$: $\lambda_2 = 325$ nm (ultraviolet)

Both infrared and ultraviolet light are invisible to the human eye, so the dominant color in the reflected light is $\boxed{\text{green}}$.

(b) The dominant wavelengths in the transmitted light are those that produce destructive interference in the reflected light. The condition for destructive interference upon reflection is

$$2nt = m\lambda \quad \text{or} \quad \lambda_m = \frac{2nt}{m} = \frac{812 \text{ nm}}{m}$$

Substituting for m gives: $m = 1$, $\lambda_1 = 812$ nm (near infrared)

$m = 2$, $\lambda_2 = 406$ nm (violet)

$m = 3$, $\lambda_3 = 271$ nm (ultraviolet)

Of these. the only wavelength visible to the human eye (and hence the dominate wavelength observed in the transmitted light) is 406 nm. Thus, the dominant color in the transmitted light is $\boxed{\text{violet}}$.

37.30 Since 1 < 1.25 < 1.33, light reflected both from the top and from the bottom surface of the oil suffers phase reversal.

For constructive interference we require $2t = \frac{m\lambda_{\text{cons}}}{n}$

and for destructive interference, $2t = \frac{\left(m + \frac{1}{2}\right)\lambda_{\text{des}}}{n}$

Then $\frac{\lambda_{\text{cons}}}{\lambda_{\text{dest}}} = 1 + \frac{1}{2m} = \frac{640 \text{ nm}}{512 \text{ nm}} = 1.25$ and $m =$ 2

Therefore, $t = \frac{2(640 \text{ nm})}{2(1.25)} = \boxed{512 \text{ nm}}$

37.31 Treating the anti-reflectance coating like a camera-lens coating, $2t = \left(m + \frac{1}{2}\right)\frac{\lambda}{n}$

Let $m = 0$: $t = \frac{\lambda}{4n} = \frac{3.00 \text{ cm}}{4(1.50)} = \boxed{0.500 \text{ cm}}$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection!

37.32 $2nt = \left(m + \frac{1}{2}\right)\lambda$ so $t = \left(m + \frac{1}{2}\right)\frac{\lambda}{2n}$

Minimum $t = \left(\frac{1}{2}\right)\frac{(500 \text{ nm})}{2(1.30)} = \boxed{96.2 \text{ nm}}$

37.33 Since the light undergoes a 180° phase change at each surface of the film, the condition for *constructive* interference is $2nt = m\lambda$, or $\lambda = 2nt/m$. The film thickness is $t = 1.00 \times 10^{-5} \text{ cm} = 1.00 \times 10^{-7} \text{ m} = 100 \text{ nm}$. Therefore, the wavelengths intensified in the reflected light are

$$\lambda = \frac{2(1.38)(100 \text{ nm})}{m} = \frac{276 \text{ nm}}{m} \quad \text{where} \quad m = 1, 2, 3, \ldots$$

or $\lambda_1 = 276 \text{ nm}$, $\lambda_2 = 138 \text{ nm}$, ... All reflection maxima are in the ultraviolet and beyond. $\boxed{\text{No visible wavelengths are intensified.}}$

***37.34** (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase reversal and light reflected from the back does undergo phase reversal. This effect by itself would produce destructive interference, so we want the distance down and back to be one whole wavelength in the film: $2t = \lambda/n$.

$$t = \frac{\lambda}{2n} = \frac{656.3\text{ nm}}{2(1.378)} = \boxed{238\text{ nm}}$$

(b) The filter will expand. As t increases in $2nt = \lambda$, so does $\boxed{\lambda\text{ increase}}$.

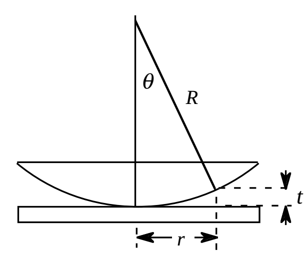
(c) Destructive interference for reflected light happens also for λ in $2nt = 2\lambda$,

or $\lambda = 1.378(238\text{ nm}) = \boxed{328\text{ nm}}$ (near ultraviolet).

37.35 If the path length $\Delta = \lambda$, the transmitted light will be bright. Since $\Delta = 2d = \lambda$,

$$d_{\min} = \frac{\lambda}{2} = \frac{580\text{ nm}}{2} = \boxed{290\text{ nm}}$$

37.36 The condition for bright fringes is



$$2t + \frac{\lambda}{2n} = m\frac{\lambda}{n} \qquad m = 1,\ 2,\ 3,\ \ldots$$

From the sketch, observe that

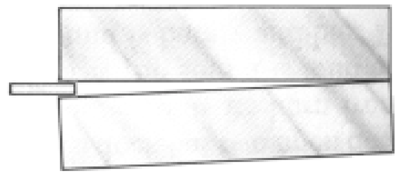
$$t = R(1 - \cos\theta) \approx R\left(1 - 1 + \frac{\theta^2}{2}\right) = \frac{R}{2}\left(\frac{r}{R}\right)^2 = \frac{r^2}{2R}$$

The condition for a bright fringe becomes $\dfrac{r^2}{R} = \left(m - \dfrac{1}{2}\right)\dfrac{\lambda}{n}$.

Thus, for fixed m and λ, $nr^2 =$ constant.

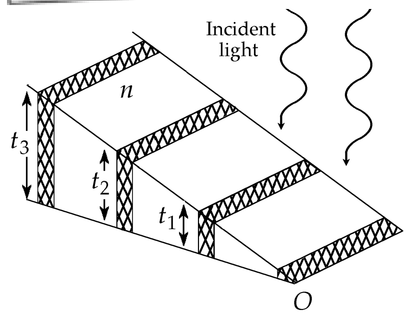
Therefore, $n_{\text{liquid}} r_f^2 = n_{\text{air}} r_i^2$ and $n_{\text{liquid}} = (1.00)\dfrac{(1.50\text{ cm})^2}{(1.31\text{ cm})^2} = \boxed{1.31}$

37.37 For destructive interference in the air, $2t = m\lambda$.



For 30 dark fringes, including the one where the plates meet,

$$t = \frac{29(600\text{ nm})}{2} = 8.70 \times 10^{-6}\text{ m}$$



Therefore, the <u>radius</u> of the wire is

$$r = \frac{d}{2} = \frac{8.70\ \mu\text{m}}{2} = \boxed{4.35\ \mu\text{m}}$$

<u>Goal Solution</u>

An air wedge is formed between two glass plates separated at one edge by a very fine wire as shown in Figure P37.37. When the wedge is illuminated from above by 600-nm light, 30 dark fringes are observed. Calculate the radius of the wire.

G: The radius of the wire is probably less than 0.1 mm since it is described as a “very fine wire.”

O: Light reflecting from the bottom surface of the top plate undergoes no phase shift, while light reflecting from the top surface of the bottom plate is shifted by π, and also has to travel an extra distance $2t$, where t is the thickness of the air wedge.

For destructive interference, $2t = m\lambda \quad (m = 0, 1, 2, 3, \ldots)$

The first dark fringe appears where $m = 0$ at the line of contact between the plates. The 30th dark fringe gives for the diameter of the wire $2t = 29\lambda$, and $t = 14.5\lambda$.

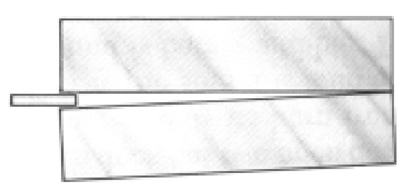
A: $r = \frac{t}{2} = 7.25\lambda = 7.25\left(600 \times 10^{-9}\text{ m}\right) = 4.35\ \mu\text{m}$

L: This wire is not only less than 0.1 mm; it is even thinner than a typical human hair (~ 50 μm).

37.38 For destructive interference, $2t = \frac{\lambda m}{n}$.

At the position of the maximum thickness of the air film,

$$m = \frac{2tn}{\lambda} = \frac{2(4.00 \times 10^{-5}\text{ m})(1.00)}{5.461 \times 10^{-7}\text{ m}} = 146.5$$



The greatest integer value is m = 146.

Therefore, including the dark band at zero thickness, there are $\boxed{\text{147 dark fringes}}$.

***37.39** For total darkness, we want destructive interference for reflected light for both 400 nm and 600 nm. With phase reversal at just one reflecting surface, the condition for destructive interference is

$$2n_{\text{air}}t = m\lambda \qquad m = 0,\ 1,\ 2,\ \ldots$$

The least common multiple of these two wavelengths is 1200 nm, so we get no reflected light at $2(1.00)t = 3(400\text{ nm}) = 2(600\text{ nm}) = 1200\text{ nm}$, so t = 600 nm at this second dark fringe.

By similar triangles, $\dfrac{600\text{ nm}}{x} = \dfrac{0.0500\text{ mm}}{10.0\text{ cm}}$,

or the distance from the contact point is $x = \left(600\times10^{-9}\text{ m}\right)\left(\dfrac{0.100\text{ m}}{5.00\times10^{-5}\text{ m}}\right) = \boxed{1.20\text{ mm}}$

37.40 $2t = m\lambda \quad \Rightarrow \quad m = \dfrac{2t}{\lambda} = \dfrac{2(1.80\times10^{-4}\text{ m})}{550.5\times10^{-9}\text{ m}} = \boxed{654\text{ dark fringes}}$

37.41 When the mirror on one arm is displaced by Δl, the path difference increases by $2\Delta l$. A shift resulting in the formation of successive dark (or bright) fringes requires a path length change of one-half wavelength. Therefore, $2\Delta l = m\lambda/2$, where in this case, m = 250.

$$\Delta l = m\frac{\lambda}{4} = \frac{(250)\left(6.328\times10^{-7}\text{ m}\right)}{4} = \boxed{39.6\ \mu\text{m}}$$

37.42 Distance = $2(3.82\times10^{-4}\text{ m}) = 1700\lambda \qquad \lambda = 4.49\times10^{-7}\text{ m} = \boxed{449\text{ nm}}$

The light is $\boxed{\text{blue}}$

37.43 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \dfrac{2L}{\lambda}$. It changes to $m_2 = \dfrac{2L}{\lambda/n} = \dfrac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \dfrac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = 1 + \frac{N\lambda}{2L} = 1 + \frac{35\left(633\times10^{-9}\text{ m}\right)}{2(0.0300\text{ m})} = \boxed{1.000\ 369}$$

37.44 Counting light going both directions, the number of wavelengths originally in the cylinder is $m_1 = \frac{2L}{\lambda}$. It changes to $m_2 = \frac{2L}{\lambda/n} = \frac{2nL}{\lambda}$ as the cylinder is filled with gas. If N is the number of bright fringes passing, $N = m_2 - m_1 = \frac{2L}{\lambda}(n-1)$, or the index of refraction of the gas is

$$n = \boxed{1 + \frac{N\lambda}{2L}}$$

37.45 The wavelength is $\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{60.0 \times 10^6 \text{ s}^{-1}} = 5.00 \text{ m}$.

Along the line *AB* the two traveling waves going in opposite directions add to give a standing wave. The two transmitters are exactly 2.00 wavelengths apart and the signal from *B*, when it arrives at *A*, will always be in phase with transmitter *B*. Since *B* is 180° out of phase with *A*, the two signals always interfere destructively at the position of *A*.

The first antinode (point of constructive interference) is located at distance

$\frac{\lambda}{4} = \frac{5.00 \text{ m}}{4} = \boxed{1.25 \text{ m}}$ from the node at *A*.

***37.46** My middle finger has width $d = 2$ cm.

(a) Two adjacent directions of constructive interference for 600-nm light are described by

$d \sin\theta = m\lambda$ $\theta_0 = 0$ $(2 \times 10^{-2} \text{ m}) \sin\theta_1 = 1\,(6 \times 10^{-7} \text{ m})$

Thus, $\theta_1 = 2 \times 10^{-3}$ degree

and $\theta_1 - \theta_0 \boxed{\sim 10^{-3} \text{ degree}}$

(b) Choose $\theta_1 = 20°$ $2 \times 10^{-2} \text{ m} \sin 20° = 1\lambda$ $\lambda = 7$ mm

Millimeter waves are $\boxed{\text{microwaves}}$

$f = \frac{c}{\lambda} = \frac{3 \times 10^8 \text{ m/s}}{7 \times 10^{-3} \text{ m}} \boxed{\sim 10^{11} \text{ Hz}}$

37.47 If the center point on the screen is to be a dark spot rather than bright, passage through the plastic must delay the light by one-half wavelength. Calling the thickness of the plastic *t*.

$\frac{t}{\lambda} + \frac{1}{2} = \frac{t}{(\lambda/n)} = \frac{nt}{\lambda}$ or $t = \boxed{\frac{\lambda}{2(n-1)}}$ where n is the index of refraction for the plastic.

***37.48** No phase shift upon reflection from the upper surface (glass to air) of the film, but there will be a shift of $\lambda/2$ due to the reflection at the lower surface of the film (air to metal). The total phase difference in the two reflected beams is then $\delta = 2nt + \lambda/2$. For constructive interference, $\delta = m\lambda$, or $2(1.00)t + \lambda/2 = m\lambda$. Thus, the film thickness for the mth order bright fringe is:

$$t_m = \left(m - \frac{1}{2}\right)\frac{\lambda}{2} = m\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4},$$

and the thickness for the $m - 1$ bright fringe is: $$t_{m-1} = (m-1)\left(\frac{\lambda}{2}\right) - \frac{\lambda}{4}.$$

Therefore, the change in thickness required to go from one bright fringe to the next is $\Delta t = t_m - t_{m-1} = \lambda/2$. To go through 200 bright fringes, the change in thickness of the air film must be: $200(\lambda/2) = 100\lambda$. Thus, the increase in the length of the rod is

$$\Delta L = 100\lambda = 100\left(5.00 \times 10^{-7}\ \text{m}\right) = 5.00 \times 10^{-5}\ \text{m},$$

From $\Delta L = L_i \alpha(\Delta T)$, we have: $$\alpha = \frac{\Delta L}{L_i(\Delta T)} = \frac{5.00 \times 10^{-5}\ \text{m}}{(0.100\ \text{m})(25.0°\text{C})} = \boxed{20.0 \times 10^{-6}\ °\text{C}^{-1}}$$

***37.49** Since 1 < 1.25 < 1.34, light reflected from top and bottom surfaces of the oil undergoes phase reversal. The path difference is then $2t$, which must be equal to

$$m\lambda_n = \frac{m\lambda}{n}$$

for maximum reflection, with $m = 1$ for the given first-order condition and $n = 1.25$. So

$$t = \frac{m\lambda}{2n} = \frac{1(500\ \text{nm})}{2(1.25)} = 200\ \text{nm}$$

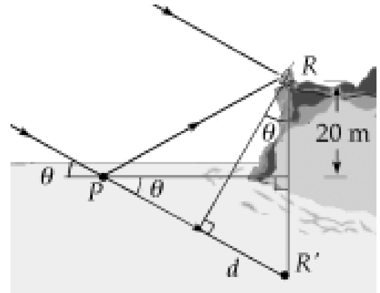
The volume we assume to be constant: $1.00\ \text{m}^3 = (200\ \text{nm})A$

$$A = \frac{1.00\ \text{m}^3}{200(10^{-9}\ \text{m})} = 5.00 \times 10^6\ \text{m}^2 = \boxed{5.00\ \text{km}^2}$$

37.50 For destructive interference, the path length must differ by $m\lambda$. We may treat this problem as a double slit experiment if we remember the light undergoes a $\pi/2$-phase shift at the mirror. The second slit is the mirror image of the source, 1.00 cm below the mirror plane. Using Equation 37.5,

$$y_{\text{dark}} = \frac{m\lambda L}{d} = \frac{1(5.00 \times 10^{-7}\ \text{m})(100\ \text{m})}{(2.00 \times 10^{-2}\ \text{m})} = \boxed{2.50\ \text{mm}}$$

37.51 One radio wave reaches the receiver R directly from the distant source at an angle θ above the horizontal. The other wave undergoes phase reversal as it reflects from the water at P.



Constructive interference first occurs for a path difference of

$$d = \frac{\lambda}{2} \qquad (1)$$

The angles θ in the figure are equal because they each form part of a right triangle with a shared angle at R'.

It is equally far from P to R as from P to R', the mirror image of the telescope.

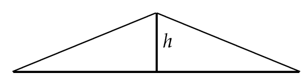
So the path difference is $d = 2(20.0\text{ m})\sin\theta = (40.0\text{ m})\sin\theta$

The wavelength is $\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{60.0\times10^{6}\text{ Hz}} = 5.00\text{ m}$

Substituting for d and λ in Equation (1), $(40.0\text{ m})\sin\theta = \frac{5.00\text{ m}}{2}$

Solving for the angle θ, $\sin\theta = \frac{5.00\text{ m}}{80.0\text{ m}}$ and $\boxed{\theta = 3.58°}$

37.52 $2\sqrt{(15.0\text{ km})^2 + h^2} = 30.175\text{ km}$



$(15.0\text{ km})^2 + h^2 = 227.63$ $\qquad h = \boxed{1.62\text{ km}}$

37.53 From Equation 37.13, $\frac{I}{I_{max}} = \cos^2\left(\frac{\pi y d}{\lambda L}\right)$

Let λ_2 equal the wavelength for which $\frac{I}{I_{max}} \to \frac{I_2}{I_{max}} = 0.640$

Then $\lambda_2 = \frac{\pi y d / L}{\cos^{-1}(I_2/I_{max})^{1/2}}$

But $\frac{\pi y d}{L} = \lambda_1 \cos^{-1}\left(\frac{I_1}{I_{max}}\right)^{1/2} = (600\text{ nm})\cos^{-1}(0.900) = 271\text{ nm}$

Substituting this value into the expression for λ_2, $\lambda_2 = \frac{271\text{ nm}}{\cos^{-1}(0.640^{1/2})} = \boxed{421\text{ nm}}$

Note that in this problem, $\cos^{-1}\left(\frac{I}{I_{max}}\right)^{1/2}$ must be expressed in radians.

37.54 For Young's experiment, use $\delta = d\sin\theta = m\lambda$. Then, at the point where the two bright lines coincide,

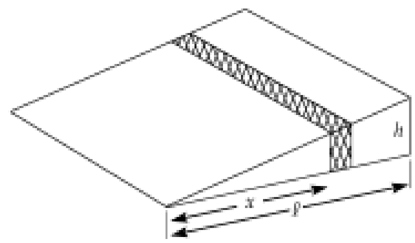
$d\sin\theta = m_1\lambda_1 = m_2\lambda_2$ so $\dfrac{\lambda_1}{\lambda_2} = \dfrac{540}{450} = \dfrac{m_2}{m_1} = \dfrac{6}{5}$

$$\sin\theta = \frac{6\lambda_2}{d} = \frac{6(450\text{ nm})}{0.150\text{ mm}} = 0.0180$$

Since $\sin\theta \approx \theta$ and $L = 1.40$ m, $x = \theta L = (0.0180)(1.40\text{ m}) = \boxed{2.52\text{ cm}}$

37.55 For dark fringes, $2nt = m\lambda$

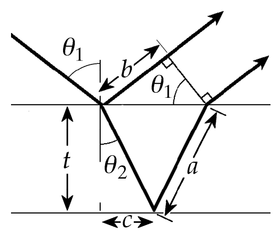
and at the edge of the wedge, $t = \dfrac{84(500\text{ nm})}{2}$.



When submerged in water, $2nt = m\lambda$

$m = \dfrac{2(1.33)(42)(500\text{ nm})}{500\text{ nm}}$ so $m + 1 = \boxed{113\text{ dark fringes}}$

***37.56** At entrance, $1.00\sin 30.0° = 1.38\sin\theta_2$ $\theta_2 = 21.2°$



Call t the unknown thickness. Then

$\cos 21.2° = \dfrac{t}{a}$ $a = \dfrac{t}{\cos 21.2°}$

$\tan 21.2° = \dfrac{c}{t}$ $c = t\tan 21.2°$

$\sin\theta_1 = \dfrac{b}{2c}$ $b = 2t\tan 21.2°\sin 30.0°$

The net shift for the second ray, including the phase reversal on reflection of the first, is

$$2an - b - \frac{\lambda}{2}$$

where the factor n accounts for the shorter wavelength in the film. For constructive interference, we require

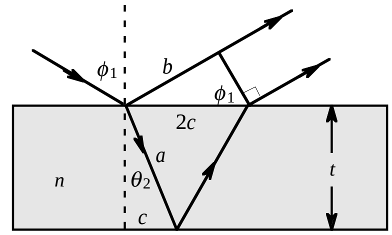
$$2an - b - \frac{\lambda}{2} = m\lambda$$

The minimum thickness will be given by $2an - b - \dfrac{\lambda}{2} = 0.$

$$\frac{\lambda}{2} = 2an - b = 2\frac{nt}{\cos 21.2°} - 2t(\tan 21.2°)\sin 30.0°$$

$$\frac{590\text{ nm}}{2} = \left(\frac{2\times 1.38}{\cos 21.2°} - 2\tan 21.2°\sin 30.0°\right)t = 2.57t$$ $t = \boxed{115\text{ nm}}$

37.57 The shift between the two reflected waves is $\delta = 2na - b - \lambda/2$ where a and b are as shown in the ray diagram, n is the index of refraction, and the factor of $\lambda/2$ is due to phase reversal at the top surface. For constructive interference, $\delta = m\lambda$ where m has integer values. This condition becomes



$$2na - b = \left(m + \frac{1}{2}\right)\lambda \qquad (1)$$

From the figure's geometry, $a = \dfrac{t}{\cos\theta_2}$, $c = a\sin\theta_2 = \dfrac{t\sin\theta_2}{\cos\theta_2}$, $b = 2c\sin\phi_1 = \dfrac{2t\sin\theta_2}{\cos\theta_2}\sin\phi_1$

Also, from Snell's law, $\sin\phi_1 = n\sin\theta_2$. Thus, $b = \dfrac{2nt\sin^2\theta_2}{\cos\theta_2}$

With these results, the condition for constructive interference given in Equation (1) becomes:

$$2n\left(\frac{t}{\cos\theta_2}\right) - \frac{2nt\sin^2\theta_2}{\cos\theta_2} = \frac{2nt}{\cos\theta_2}\left(1 - \sin^2\theta_2\right) = \left(m + \frac{1}{2}\right)\lambda \quad \text{or} \quad \boxed{2nt\cos\theta_2 = \left(m + \frac{1}{2}\right)\lambda}$$

37.58 (a) Minimum: $2nt = m\lambda_2$ $\quad m = 0, 1, 2, \ldots$

Maximum: $2nt = \left(m' + \frac{1}{2}\right)\lambda_1$ $\quad m' = 0, 1, 2, \ldots$

for $\lambda_1 > \lambda_2$, $\left(m' + \frac{1}{2}\right) < m$ so $m' = m - 1$

Then $2nt = m\lambda_2 = \left(m - \frac{1}{2}\right)\lambda_1$

$2m\lambda_2 = 2m\lambda_1 - \lambda_1$ so $\boxed{m = \dfrac{\lambda_1}{2(\lambda_1 - \lambda_2)}}$

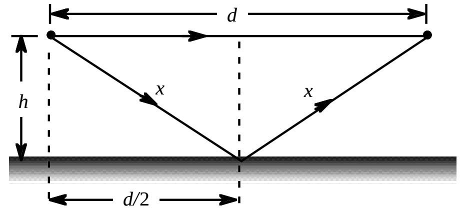
(b) $m = \dfrac{500}{2(500 - 370)} = 1.92 \rightarrow 2$ (wavelengths measured to ± 5 nm)

[Minimum]: $2nt = m\lambda_2$ $\quad 2(1.40)t = 2(370\text{ nm})$ $\quad t = 264\text{ nm}$

[Maximum]: $2nt = \left(m - 1 + \frac{1}{2}\right)\lambda = 1.5\lambda$ $\quad 2(1.40)t = (1.5)500\text{ nm}$ $\quad t = 268\text{ nm}$

Film thickness = $\boxed{266\text{ nm}}$

37.59 From the sketch, observe that



$$x = \sqrt{h^2 + (d/2)^2} = \frac{\sqrt{4h^2 + d^2}}{2}$$

Including the phase reversal due to reflection from the ground, the total shift between the two waves is $\delta = 2x - d - \lambda/2$.

(a) For constructive interference, the total shift must be an integral number of wavelengths, or $\delta = m\lambda$ where m = 0, 1, 2, 3, ...

Thus, $2x - d = \left(m + \frac{1}{2}\right)\lambda$ or $\lambda = \frac{4x - 2d}{2m + 1}$

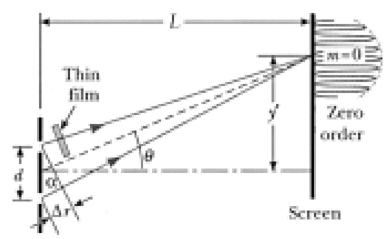
For the longest wavelength, m = 0, giving $\lambda = 4x - 2d = \boxed{2\sqrt{4h^2 + d^2} - 2d}$

(b) For destructive interference, $\delta = \left(m - \frac{1}{2}\right)\lambda$ where m = 1, 2, 3, ...

Thus, $2x - d = m\lambda$ or $\lambda = \frac{2x - d}{m}$.

For the longest wavelength, m = 1 giving $\lambda = 2x - d = \boxed{\sqrt{4h^2 + d^2} - d}$

37.60 Call t the thickness of the sheet. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. As light advances through distance t in air, the number of cycles it goes through is t / λ_a.



The number of cycles in the sheet is $\frac{t}{(\lambda_a / n)} = \frac{nt}{\lambda_a}$

Thus, the sheet introduces phase difference $\phi = 2\pi\left(\frac{nt}{\lambda_a} - \frac{t}{\lambda_a}\right)$

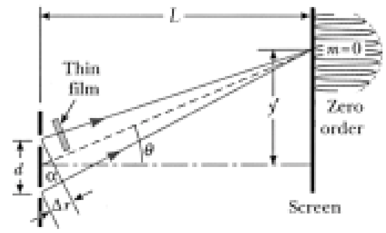
The corresponding difference in **path length** is $\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = \frac{2\pi}{\lambda_a}(nt - t)\frac{\lambda_a}{2\pi} = (n - 1)t$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel, so the angle θ may be expressed as $\tan\theta = \Delta r/d = y'/L$.

Substituting for Δr and solving for y' gives

$$y' = \Delta r\left(\frac{L}{d}\right) = \frac{t(n-1)L}{d} = \frac{(5.00 \times 10^{-5}\text{ m})(1.50 - 1)(1.00\text{ m})}{(3.00 \times 10^{-4}\text{ m})} = 0.0833\text{ m} = \boxed{8.33\text{ cm}}$$

37.61 Call t the thickness of the film. The central maximum corresponds to zero phase difference. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film. The phase difference ϕ is



$$\phi = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)$$

The corresponding difference in **path length** Δr is $\Delta r = \phi\left(\frac{\lambda_a}{2\pi}\right) = 2\pi\left(\frac{t}{\lambda_a}\right)(n-1)\left(\frac{\lambda_a}{2\pi}\right) = t(n-1)$

Note that the wavelength of the light does not appear in this equation. In the figure, the two rays from the slits are essentially parallel.

Thus the angle θ may be expressed as $\tan\theta = \frac{\Delta r}{d} = \frac{y'}{L}$

Eliminating Δr by substitution, $\frac{y'}{L} = \frac{t(n-1)}{d}$ gives $\boxed{y' = \frac{t(n-1)L}{d}}$

Goal Solution

Consider the double-slit arrangement shown in Figure P37.60, where the slit separation is d and the slit to screen distance is L. A sheet of transparent plastic having an index of refraction n and thickness t is placed over the upper slit. As a result, the central maximum of the interference pattern moves upward a distance y'. Find y'.

G: Since the film shifts the pattern upward, we should expect y' to be proportional to n, t, and L.

O: The film increases the optical path length of the light passing through the upper slit, so the physical distance of this path must be shorter for the waves to meet in phase ($\phi = 0$) to produce the central maximum. Thus, the added distance Δr traveled by the light from the lower slit must introduce a phase difference equal to that introduced by the plastic film.

A: First calculate the additional phase difference due to the plastic. Recall that the relation between phase difference and path difference is $\phi = 2\pi\delta/\lambda$. The presence of plastic affects this by changing the wavelength of the light, so that the phase change of the light in air and plastic, as it travels over the thickness t is

$$\phi_{air} = \frac{2\pi t}{\lambda_{air}} \quad \text{and} \quad \phi_{plastic} = \frac{2\pi t}{\lambda_{air} / n}$$

Thus, plastic causes an additional phase change of $\Delta\phi = \frac{2\pi t}{\lambda_{air}}(n-1)$

Next, in order to interfere constructively, we must calculate the additional distance that the light from the bottom slit must travel.

$$\Delta r = \frac{\Delta\phi\,\lambda_{air}}{2\pi} = t(n-1)$$

In the small angle approximation we can write $\Delta r = y'd/L$, so $y' = \frac{t(n-1)L}{dL}$

L: As expected, y' is proportional to t and L. It increases with increasing n, being proportional to $(n-1)$. It is also inversely proportional to the slit separation d, which makes sense since slits that are closer together make a wider interference pattern.

37.62 $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8 \text{ m/s}}{1.50 \times 10^6 \text{ Hz}} = 200 \text{ m}$

For destructive interference, the path difference is one-half wavelength.

Thus, $$\frac{\lambda}{2} = 100 \text{ m} = x + \sqrt{x^2 + \left(2.00 \times 10^4 \text{ m}\right)^2} - 2.00 \times 10^4 \text{ m},$$

or $$2.01 \times 10^4 \text{ m} - x = \sqrt{x^2 + \left(2.00 \times 10^4 \text{ m}\right)^2}$$

Squaring and solving, $x = \boxed{99.8 \text{ m}}$

37.63 (a) Constructive interference in the reflected light requires $2t = \left(m + \frac{1}{2}\right)\lambda$. The first bright ring has $m = 0$ and the 55th has $m = 54$, so at the edge of the lens

$$t = \frac{54.5(650 \times 10^{-9} \text{ m})}{2} = 17.7 \ \mu\text{m}$$

Now from the geometry in Figure 37.18, the distance from the center of curvature down to the flat side of the lens is

$$\sqrt{R^2 - r^2} = R - t \qquad \text{or} \qquad R^2 - r^2 = R^2 - 2Rt + t^2$$

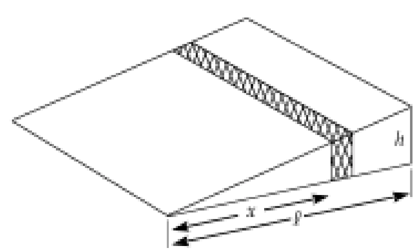
$$R = \frac{r^2 + t^2}{2t} = \frac{(5.00 \times 10^{-2} \text{ m})^2 + (1.77 \times 10^{-5} \text{ m})^2}{2(1.77 \times 10^{-5} \text{ m})} = \boxed{70.6 \text{ m}}$$

(b) $\dfrac{1}{f} = (n-1)\left(\dfrac{1}{R_2} - \dfrac{1}{R_2}\right) = 0.520\left(\dfrac{1}{\infty} - \dfrac{1}{-70.6 \text{ m}}\right)$ so $f = \boxed{136 \text{ m}}$

37.64 Bright fringes occur when $2t = \dfrac{\lambda}{n}\left(m + \dfrac{1}{2}\right)$

and dark fringes occur when $2t = \left(\dfrac{\lambda}{n}\right)m$

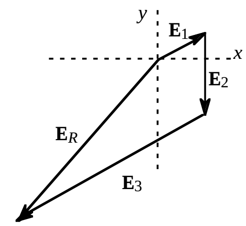
The thickness of the film at x is $t = \left(\dfrac{h}{\ell}\right)x$.



Therefore, $\boxed{x_{\text{bright}} = \dfrac{\lambda \ell}{2hn}\left(m + \dfrac{1}{2}\right)}$ and $\boxed{x_{\text{dark}} = \dfrac{\lambda \ell m}{2hn}}$

37.65 $\mathbf{E}_R = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 = \left[\cos\frac{\pi}{6} + 3.00\cos\frac{7\pi}{2} + 6.00\cos\frac{4\pi}{3}\right]\mathbf{i} + \left[\sin\frac{\pi}{6} + 3.00\sin\frac{7\pi}{2} + 6.00\sin\frac{4\pi}{3}\right]\mathbf{j}$

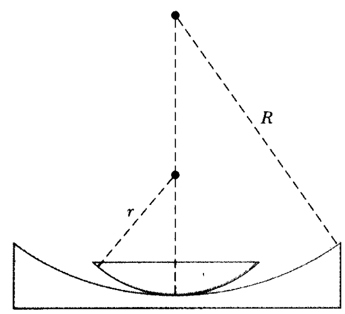
$\mathbf{E}_R = -2.13\,\mathbf{i} - 7.70\,\mathbf{j}$



$\mathbf{E}_R = \sqrt{(-2.13)^2 + (-7.70)^2}$ at $\tan^{-1}\left(\frac{-7.70}{-2.13}\right) = 7.99$ at 4.44 rad

Thus, $E_P = \boxed{7.99\sin(\omega t + 4.44\text{ rad})}$

37.66 For bright rings the gap t between surfaces is given by $2t = \left(m + \frac{1}{2}\right)\lambda$ The first bright ring has $m = 0$ and the hundredth has $m = 99$.



So, $t = \frac{1}{2}(99.5)\left(500 \times 10^{-9}\text{ m}\right) = 24.9\ \mu\text{m}$.

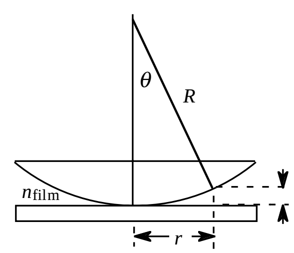
Call r_b the ring radius. From the geometry of the figure at the right,

$$t = r - \sqrt{r^2 - r_b^2} - \left(R - \sqrt{R^2 - r_b^2}\right)$$

Since $r_b << r$, we can expand in series: $t = r - r\left(1 - \frac{1}{2}\frac{r_b^2}{r^2}\right) - R + R\left(1 - \frac{1}{2}\frac{r_b^2}{R^2}\right) = \frac{1}{2}\frac{r_b^2}{r} - \frac{1}{2}\frac{r_b^2}{R}$

$$r_b = \left[\frac{2t}{1/r - 1/R}\right]^{1/2} = \left[\frac{2\left(24.9 \times 10^{-6}\text{ m}\right)}{1/4.00\text{ m} - 1/12.0\text{ m}}\right]^{1/2} = \boxed{1.73\text{ cm}}$$

***37.67** The shift between the waves reflecting from the top and bottom surfaces of the film at the point where the film has thickness t is $\delta = 2t\,n_{\text{film}} + (\lambda/2)$, with the factor of $\lambda/2$ being due to a phase reversal at *one* of the surfaces.



For the dark rings (destructive interference), the total shift should be $\delta = \left(m + \frac{1}{2}\right)\lambda$ with $m = 0, 1, 2, 3, \ldots$. This requires that $t = m\lambda/2n_{\text{film}}$.

To find t in terms of r and R, $R^2 = r^2 + (R - t)^2$ so $r^2 = 2Rt + t^2$

Since t is much smaller than R, $t^2 << 2Rt$ and $r^2 \approx 2Rt = 2R\left(\frac{m\lambda}{2n_{\text{film}}}\right)$.

Thus, where m is an integer, $\boxed{r \approx \sqrt{\frac{m\lambda R}{n_{\text{film}}}}}$

37.68 (a) Bright bands are observed when $$2nt = \left(m + \frac{1}{2}\right)\lambda$$

Hence, the first bright band (m = 0) corresponds to $nt = \lambda/4$.

Since $\frac{x_1}{x_2} = \frac{t_1}{t_2}$, we have $$x_2 = x_1\left(\frac{t_2}{t_1}\right) = x_1\left(\frac{\lambda_2}{\lambda_1}\right) = (3.00\text{ cm})\left(\frac{680\text{ nm}}{420\text{ nm}}\right) = \boxed{4.86\text{ cm}}$$

(b) $t_1 = \frac{\lambda_1}{4n} = \frac{420\text{ nm}}{4(1.33)} = \boxed{78.9\text{ nm}}$ $t_2 = \frac{\lambda_2}{4n} = \frac{680\text{ nm}}{4(1.33)} = \boxed{128\text{ nm}}$

(c) $\theta \approx \tan\theta = \frac{t_1}{x_1} = \frac{78.9\text{ nm}}{3.00\text{ cm}} = \boxed{2.63\times 10^{-6}\text{ rad}}$

37.69 $2h\sin\theta = \left(m + \frac{1}{2}\right)\lambda$ bright

$2h\left(\frac{\Delta y}{2L}\right) = \frac{1}{2}\lambda$ so $h = \frac{L\lambda}{2\Delta y} = \frac{(2.00\text{ m})(606\times 10^{-9}\text{ m})}{2(1.2\times 10^{-3}\text{ m})} = \boxed{0.505\text{ mm}}$

37.70 Superposing the two vectors, $E_R = |\mathbf{E}_1 + \mathbf{E}_2|$

$$E_R = |\mathbf{E}_1 + \mathbf{E}_2| = \sqrt{\left(E_0 + \frac{E_0}{3}\cos\phi\right)^2 + \left(\frac{E_0}{3}\sin\phi\right)^2} = \sqrt{E_0^2 + \frac{2}{3}E_0^2\cos\phi + \frac{E_0^2}{9}\cos^2\phi + \frac{E_0^2}{9}\sin^2\phi}$$

$$E_R = \sqrt{\frac{10}{9}E_0^2 + \frac{2}{3}E_0^2\cos\phi}$$

Since intensity is proportional to the square of the amplitude,

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\cos\phi$$

Using the trigonometric identity $\cos\phi = 2\cos^2\frac{\phi}{2} - 1$, this becomes

$$I = \frac{10}{9}I_{\max} + \frac{2}{3}I_{\max}\left(2\cos^2\frac{\phi}{2} - 1\right) = \frac{4}{9}I_{\max} + \frac{4}{3}I_{\max}\cos^2\frac{\phi}{2},$$

or $$\boxed{I = \frac{4}{9}I_{\max}\left(1 + 3\cos^2\frac{\phi}{2}\right)}$$