CHAPTER 38

38.1 $\sin\theta = \dfrac{\lambda}{a} = \dfrac{6.328\times10^{-7}}{3.00\times10^{-4}} = 2.11\times10^{-3}$

$\dfrac{y}{1.00\text{ m}} = \tan\theta \approx \sin\theta = \theta$ (for small θ)

$2y = \boxed{4.22\text{ mm}}$

38.2 The positions of the first-order minima are $y/L \approx \sin\theta = \pm\lambda/a$. Thus, the spacing between these two minima is $\Delta y = 2(\lambda/a)L$ and the wavelength is

$$\lambda = \left(\frac{\Delta y}{2}\right)\left(\frac{a}{L}\right) = \left(\frac{4.10\times10^{-3}\text{ m}}{2}\right)\left(\frac{0.550\times10^{-3}\text{ m}}{2.06\text{ m}}\right) = \boxed{547\text{ nm}}$$

38.3 $\dfrac{y}{L} = \sin\theta = \dfrac{m\lambda}{a}$ $\quad \Delta y = 3.00\times10^{-3}\text{ m}$ $\quad \Delta m = 3 - 1 = 2$ $\quad$ and $\quad a = \dfrac{\Delta m \lambda L}{\Delta y}$

$$a = \frac{(2)(690\times10^{-9}\text{ m})(0.500\text{ m})}{3.00\times10^{-3}\text{ m}} = \boxed{2.30\times10^{-4}\text{ m}}$$

***38.4** For destructive interference,

$\sin\theta = m\dfrac{\lambda}{a} = \dfrac{\lambda}{a} = \dfrac{5.00\text{ cm}}{36.0\text{ cm}} = 0.139$ $\quad$ and $\quad \theta = 7.98°$

$\dfrac{d}{L} = \tan\theta$ gives $d = L\tan\theta = (6.50\text{ m})\tan 7.98° = 0.912\text{ m}$

$d = \boxed{91.2\text{ cm}}$

***38.5** If the speed of sound is 340 m/s,

$$\lambda = \frac{v}{f} = \frac{340 \text{ m/s}}{650 \text{ /s}} = 0.523 \text{ m}$$

Diffraction minima occur at angles described by $a \sin \theta = m\lambda$

1.10 m sin θ_1 = 1(0.523 m) θ_1 = 28.4°

1.10 m sin θ_2 = 2(0.523 m) θ_2 = 72.0°

1.10 m sin θ_3 = 3(0.523 m) θ_3 nonexistent

Maxima appear straight ahead at $\boxed{0°}$ and left and right at an angle given approximately by

(1.10 m) sin θ_x = 1.5(0.523 m) θ_x $\boxed{\approx 46°}$

There is no solution to $a \sin \theta = 2.5\lambda$, so our answer is already complete, with $\boxed{\text{three}}$ sound maxima.

38.6 (a) $\sin\theta = \frac{y}{L} = \frac{m\lambda}{a}$

Therefore, for first minimum, m = 1 and

$$L = \frac{ay}{m\lambda} = \frac{(7.50\times10^{-4} \text{ m})(8.50\times10^{-4} \text{ m})}{(1)(587.5\times10^{-9} \text{ m})} = \boxed{1.09 \text{ m}}$$

(b) $w = 2y_1$ yields $y_1 = 0.850$ mm

$$w = 2(0.850\times10^{-3} \text{ m}) = \boxed{1.70 \text{ mm}}$$

38.7 $\sin\theta \approx \frac{y}{L} = \frac{4.10\times10^{-3} \text{ m}}{1.20 \text{ m}}$

$$\frac{\beta}{2} = \frac{\pi a \sin\theta}{\lambda} = \frac{\pi(4.00\times10^{-4} \text{ m})}{546.1\times10^{-9} \text{ m}}\left(\frac{4.10\times10^{-3} \text{ m}}{1.20 \text{ m}}\right) = 7.86 \text{ rad}$$

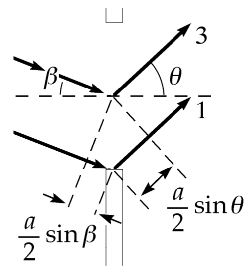
$$\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 = \left[\frac{\sin(7.86)}{7.86}\right]^2 = \boxed{1.62\times10^{-2}}$$

38.8 Bright fringes will be located approximately midway between adjacent dark fringes. Therefore, for the second bright fringe, let $m = 2.5$ and use

$$\sin\theta = m\lambda/a \approx y/L \ .$$

The wavelength will be $\lambda \approx \frac{ay}{mL} = \frac{(0.800\times10^{-3}\text{ m})(1.40\times10^{-3}\text{ m})}{2.5(0.800\text{ m})} = 5.60\times10^{-7}\text{ m} = \boxed{560\text{ nm}}$

38.9 Equation 38.1 states that $\sin\theta = m\lambda/a$, where $m = \pm 1, \pm 2, \pm 3, \ldots$. The requirement for $m = 1$ is from an analysis of the extra path distance traveled by ray 1 compared to ray 3 in Figure 38.5. This extra distance must be equal to $\lambda/2$ for destructive interference. When the source rays approach the slit at an angle β, there is a distance added to the path difference (of ray 1 compared to ray 3) of $(a/2)\sin\beta$ Then, for destructive interference,



$$\frac{a}{2}\sin\beta + \frac{a}{2}\sin\theta = \frac{\lambda}{2} \qquad \text{so} \qquad \sin\theta = \frac{\lambda}{a} - \sin\beta.$$

Dividing the slit into 4 parts leads to the 2nd order minimum: $\sin\theta = \frac{2\lambda}{a} - \sin\beta$

Dividing the slit into 6 parts gives the third order minimum: $\sin\theta = \frac{3\lambda}{a} - \sin\beta$

Generalizing, we obtain the condition for the mth order minimum: $\sin\theta = \frac{m\lambda}{a} - \sin\beta$

***38.10** (a) Double-slit interference maxima are at angles given by $d\sin\theta = m\lambda$.

For $m = 0$, $\theta_0 = \boxed{0°}$

For $m = 1$, $(2.80\ \mu\text{m})\sin\theta = 1(0.5015\ \mu\text{m})$: $\theta_1 = \sin^{-1}(0.179) = \boxed{10.3°}$

Similarly, for $m = 2, 3, 4, 5$ and 6, $\theta_2 = \boxed{21.0°}$, $\theta_3 = \boxed{32.5°}$, $\theta_4 = \boxed{45.8°}$, $\theta_5 = \boxed{63.6°}$, and $\theta_6 = \sin^{-1}(1.07) = \text{nonexistent}$.

Thus, there are $5 + 5 + 1 = \boxed{\text{11 directions for interference maxima}}$.

(b) We check for missing orders by looking for single-slit diffraction minima, at $a\sin\theta = m\lambda$.

For $m = 1$, $(0.700\ \mu\text{m})\sin\theta = 1(0.5015\ \mu\text{m})$ and $\theta_1 = 45.8°$.

Thus, there is no bright fringe at this angle. There are only $\boxed{\text{nine bright fringes}}$, at $\boxed{\theta = 0°, \pm 10.3°, \pm 21.0°, \pm 32.5°, \text{ and } \pm 63.6°}$.

(c) $I = I_{\max}\left[\dfrac{\sin(\pi a \sin\theta/\lambda)}{\pi a \sin\theta/\lambda}\right]^2$

At $\theta = 0°$, $\dfrac{\sin\theta}{\theta} \to 1$ and $\dfrac{I}{I_{\max}} \to \boxed{1.00}$

At $\theta = 10.3°$, $\dfrac{\pi a \sin\theta}{\lambda} = \dfrac{\pi(0.700\ \mu\text{m})\sin 10.3°}{0.5015\ \mu\text{m}} = 0.785\ \text{rad} = 45.0°$

$$\frac{I}{I_{\max}} = \left[\frac{\sin 45.0°}{0.785}\right]^2 = \boxed{0.811}$$

Similarly, at $\theta = 21.0°$, $\dfrac{\pi a \sin\theta}{\lambda} = 1.57\ \text{rad} = 90.0°$ and $\dfrac{I}{I_{\max}} = \boxed{0.405}$

At $\theta = 32.5°$, $\dfrac{\pi a \sin\theta}{\lambda} = 2.36\ \text{rad} = 135°$ and $\dfrac{I}{I_{\max}} = \boxed{0.0901}$

At $\theta = 63.6°$, $\dfrac{\pi a \sin\theta}{\lambda} = 3.93\ \text{rad} = 225°$ and $\dfrac{I}{I_{\max}} = \boxed{0.0324}$

38.11 $\sin\theta = \dfrac{\lambda}{a} = \dfrac{5.00 \times 10^{-7}\ \text{m}}{5.00 \times 10^{-4}} = \boxed{1.00 \times 10^{-3}\ \text{rad}}$

38.12 $\theta_{\min} = \dfrac{y}{L} = 1.22\dfrac{\lambda}{D}$

$y = \dfrac{(1.22)(5.00 \times 10^{-7})(0.0300)}{7.00 \times 10^{-3}} = \boxed{2.61\ \mu\text{m}}$

y = radius of star-image
L = length of eye
λ = 500 nm
D = pupil diameter
θ = half angle

38.13 Following Equation 38.9 for diffraction from a circular opening, the beam spreads into a cone of half-angle

$$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{(632.8 \times 10^{-9}\ \text{m})}{(0.00500\ \text{m})} = 1.54 \times 10^{-4}\ \text{rad}$$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$r_{\text{beam}} = \theta_{\min}(1.00 \times 10^4\ \text{m}) = 1.544\ \text{m}$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = \boxed{3.09\ \text{m}}$

Goal Solution

A helium-neon laser emits light that has a wavelength of 632.8 nm. The circular aperture through which the beam emerges has a diameter of 0.500 cm. Estimate the diameter of the beam 10.0 km from the laser.

G: A typical laser pointer makes a spot about 5 cm in diameter at 100 m, so the spot size at 10 km would be about 100 times bigger, or about 5 m across. Assuming that this HeNe laser is similar, we could expect a comparable beam diameter.

O: We assume that the light is parallel and not diverging as it passes through and fills the circular aperture. However, as the light passes through the circular aperture, it will spread from diffraction according to Equation 38.9.

A: The beam spreads into a cone of half-angle $\theta_{\text{min}} = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{\left(632.8\times10^{-9}\text{ m}\right)}{\left(0.00500\text{ m}\right)} = 1.54\times10^{-4}\text{ rad}$

The radius of the beam ten kilometers away is, from the definition of radian measure,

$$r_{\text{beam}} = \theta_{\text{min}}\left(1.00\times10^{4}\text{ m}\right) = 1.54\text{ m}$$

and its diameter is $d_{\text{beam}} = 2r_{\text{beam}} = 3.09\text{ m}$

L: The beam is several meters across as expected, and is about 600 times larger than the laser aperture. Since most HeNe lasers are low power units in the mW range, the beam at this range would be so spread out that it would be too dim to see on a screen.

38.14 $\theta_{\text{min}} = 1.22\dfrac{\lambda}{D} = \dfrac{d}{L}$ $\quad 1.22\left(\dfrac{5.80\times10^{-7}\text{ m}}{4.00\times10^{-3}\text{ m}}\right) = \dfrac{d}{1.80\text{ mi}}\left(\dfrac{1\text{ mi}}{1609\text{ m}}\right)$ $\quad d = \boxed{0.512\text{ m}}$

The shortening of the wavelength inside the patriot's eye does not change the answer.

38.15 By Rayleigh's criterion, two dots separated center-to-center by 2.00 mm would overlap when

$$\theta_{\text{min}} = \frac{d}{L} = 1.22\frac{\lambda}{D}$$

Thus, $L = \dfrac{dD}{1.22\lambda} = \dfrac{\left(2.00\times10^{-3}\text{ m}\right)\left(4.00\times10^{-3}\text{ m}\right)}{1.22\left(500\times10^{-9}\text{ nm}\right)} = \boxed{13.1\text{ m}}$

38.16 $D = 1.22\dfrac{\lambda}{\theta_{\text{min}}} = \dfrac{1.22(5.00\times10^{-7})}{1.00\times10^{-5}}\text{ m} = \boxed{6.10\text{ cm}}$

38.17 $\theta_{\min} = 1.22\left(\dfrac{\text{wavelength}}{\text{pupil diameter}}\right) = \dfrac{(\text{distance between sources})}{L}$

Thus, $\dfrac{1.22\lambda}{d} = \dfrac{w}{vt}$, or $w = \dfrac{1.22\lambda(vt)}{d}$

Taillights are red. Take $\lambda \approx 650$ nm: $w \approx \dfrac{1.22(650\times10^{-9}\text{ m})(20.0\text{ m/s})(600\text{ s})}{5.00\times10^{-3}\text{ m}} = \boxed{1.90\text{ m}}$

38.18 $\theta_{\min} = 1.22\left(\dfrac{\text{wavelength}}{\text{pupil diameter}}\right) = \dfrac{(\text{distance between sources})}{L}$ so $\dfrac{1.22\lambda}{d} = \dfrac{w}{vt}$

$w = \boxed{\dfrac{1.22\lambda(vt)}{d}}$ where $\lambda \approx 650$ nm is the average wavelength radiated by the red taillights.

38.19 $\dfrac{1.22\lambda}{D} = \dfrac{d}{L}$ $\lambda = \dfrac{c}{f} = 0.0200$ m $D = 2.10$ m $L = 9000$ m

$d = 1.22\dfrac{(0.0200\text{ m})(9000\text{ m})}{2.10\text{ m}} = \boxed{105\text{ m}}$

38.20 Apply Rayleigh's criterion, $\theta_{\min} = \dfrac{x}{D} = 1.22\dfrac{\lambda}{d}$

where $\theta_{\min}$ = half-angle of light cone, x = radius of spot, λ = wavelength of light, d = diameter of telescope, D = distance to Moon.

Then, the diameter of the spot on the Moon is

$$2x = 2\left(1.22\frac{\lambda D}{d}\right) = \frac{2(1.22)(694.3\times10^{-9}\text{ m})(3.84\times10^{8}\text{ m})}{2.70\text{ m}} = \boxed{241\text{ m}}$$

38.21 For 0.100° angular resolution, $1.22\dfrac{(3.00\times10^{-3}\text{ m})}{D} = (0.100°)\left(\dfrac{\pi}{180°}\right)$ $D = \boxed{2.10\text{ m}}$

38.22 $L = 88.6\times10^{9}$ m, $D = 0.300$ m, $\lambda = 590\times10^{-9}$ m

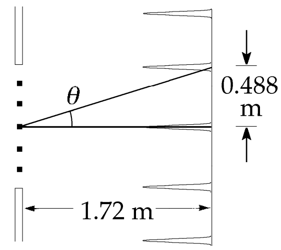
(a) $1.22\dfrac{\lambda}{D} = \theta_{\min} = \boxed{2.40\times10^{-6}\text{ rad}}$

(b) $d = \theta_{\min}L = \boxed{213\text{ km}}$

38.23 $d = \dfrac{1.00\text{ cm}}{2000} = \dfrac{1.00 \times 10^{-2}\text{ m}}{2000} = 5.00\ \mu\text{m}$

$$\sin\theta = \frac{m\lambda}{d} = \frac{1\left(640 \times 10^{-9}\text{ m}\right)}{5.00 \times 10^{-6}\text{ m}} = 0.128 \qquad \theta = \boxed{7.35°}$$

38.24 The principal maxima are defined by



$$d\sin\theta = m\lambda \quad m = 0, 1, 2, \ldots$$

For $m = 1$, $\lambda = d\sin\theta$

where θ is the angle between the central ($m = 0$) and the first order ($m = 1$) maxima. The value of θ can be determined from the information given about the distance between maxima and the grating-to-screen distance. From the figure,

$$\tan\theta = \frac{0.488\text{ m}}{1.72\text{ m}} = 0.284 \quad \text{so} \quad \theta = 15.8° \quad \text{and} \quad \sin\theta = 0.273$$

The distance between grating "slits" equals the reciprocal of the number of grating lines per centimeter

$$d = \frac{1}{5310\text{ cm}^{-1}} = 1.88 \times 10^{-4}\text{ cm} = 1.88 \times 10^{3}\text{ nm}$$

The wavelength is $\lambda = d\sin\theta = (1.88 \times 10^{3}\text{ nm})(0.273) = \boxed{514\text{ nm}}$

38.25 The grating spacing is $d = \dfrac{(1.00 \times 10^{-2}\text{ m})}{4500} = 2.22 \times 10^{-6}\text{ m}$

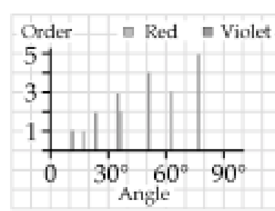


Figure for Goal Solution

In the 1st-order spectrum, diffraction angles are given by

$\sin\theta = \dfrac{\lambda}{d}$: $\sin\theta_1 = \dfrac{656 \times 10^{-9}\text{ m}}{2.22 \times 10^{-6}\text{ m}} = 0.295$

so that for red $\theta_1 = 17.17°$

and for violet $\sin\theta_2 = \dfrac{434 \times 10^{-9}\text{ m}}{2.22 \times 10^{-6}\text{ m}} = 0.195$

so that $\theta_2 = 11.26°$

The angular separation is in first-order, $\Delta\theta = 17.17° - 11.26° = \boxed{5.91°}$

In the second-order spectrum, $\Delta\theta = \sin^{-1}\left(\frac{2\lambda_1}{d}\right) - \sin^{-1}\left(\frac{2\lambda_2}{d}\right) = \boxed{13.2°}$

Again, in the third order, $\Delta\theta = \sin^{-1}\left(\frac{3\lambda_1}{d}\right) - \sin^{-1}\left(\frac{3\lambda_2}{d}\right) = \boxed{26.5°}$

Since the red line does not appear in the fourth-order spectrum, the answer is complete.

Goal Solution

The hydrogen spectrum has a red line at 656 nm and a violet line at 434 nm. What is the angular separation between two spectral lines obtained with a diffraction grating that has 4500 lines/cm?

G: Most diffraction gratings yield several spectral orders within the 180° viewing range, which means that the angle between red and violet lines is probably 10° to 30°.

O: The angular separation is the difference between the angles corresponding to the red and violet wavelengths for each visible spectral order according to the diffraction grating equation, $d\sin\theta = m\lambda$.

A: The grating spacing is $d = \left(1.00\times10^{-2}\text{ m}\right)/4500\text{ lines} = 2.22\times10^{-6}\text{ m}$

In the first-order spectrum (m = 1), the angles of diffraction are given by $\sin\theta = \lambda/d$:

$$\sin\theta_{1r} = \frac{656\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}} = 0.295 \quad \text{so} \quad \theta_{1r} = 17.17°$$

$$\sin\theta_{1v} = \frac{434\times10^{-9}\text{ m}}{2.22\times10^{-6}\text{ m}} = 0.195 \quad \text{so} \quad \theta_{1v} = 11.26°$$

The angular separation is $\Delta\theta_1 = \theta_{1r} - \theta_{1v} = 17.17 - 11.26 = 5.91$

In the 2nd-order ($m = 2$) $\Delta\theta_2 = \sin^{-1}\left(\frac{2\lambda_r}{d}\right) - \sin^{-1}\left(\frac{2\lambda_v}{d}\right) = 13.2°$

In the third order ($m = 3$), $\Delta\theta_3 = \sin^{-1}\left(\frac{3\lambda_r}{d}\right) - \sin^{-1}\left(\frac{3\lambda_v}{d}\right) = 26.5°$

In the fourth order, the red line is not visible: $\theta_{4r} = \sin^{-1}(4\lambda_r / d) = \sin^{-1}(1.18)$ does not exist

L: The full spectrum is visible in the first 3 orders with this diffraction grating, and the fourth is partially visible. We can also see that the pattern is dispersed more for higher spectral orders so that the angular separation between the red and blue lines increases as m increases. It is also worth noting that the spectral orders can overlap (as is the case for the second and third order spectra above), which makes the pattern look confusing if you do not know what you are looking for.

38.26 $\sin\theta = 0.350$: $d = \dfrac{\lambda}{\sin\theta} = \dfrac{632.8 \text{ nm}}{0.350} = 1.81 \times 10^3 \text{ nm}$

Line spacing = $\boxed{1.81\ \mu\text{m}}$

***38.27** (a) $d = \dfrac{1}{3660 \text{ lines/cm}} = 2.732 \times 10^{-4} \text{ cm} = 2.732 \times 10^{-6} \text{ m} = 2732 \text{ nm}$

$\lambda = \dfrac{d\sin\theta}{m}$: At $\theta = 10.09°$ $\lambda = \boxed{478.7 \text{ nm}}$

At $\theta = 13.71°$, $\lambda = \boxed{647.6 \text{ nm}}$

At $\theta = 14.77°$, $\lambda = \boxed{696.6 \text{ nm}}$

(b) $d = \dfrac{\lambda}{\sin\theta_1}$ and $\lambda = d\sin\theta_2$ so $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\lambda}{\left(\dfrac{\lambda}{\sin\theta_1}\right)} = 2\sin\theta_1$

Therefore, if $\theta_1 = 10.09°$ then $\sin\theta_2 = 2\sin(10.09°)$ gives $\theta_2 = \boxed{20.51°}$

Similarly, for $\theta_1 = 13.71°$, $\theta_2 = \boxed{28.30°}$ and for $\theta_1 = 14.77°$, $\theta_2 = \boxed{30.66°}$

38.28 $d = \dfrac{1}{800/\text{mm}} = 1.25 \times 10^{-6} \text{ m}$

The blue light goes off at angles $\sin\theta_m = \dfrac{m\lambda}{d}$: $\theta_1 = \sin^{-1}\left(\dfrac{1 \times 5.00 \times 10^{-7} \text{ m}}{1.25 \times 10^{-6} \text{ m}}\right) = 23.6°$

$\theta_2 = \sin^{-1}(2 \times 0.400) = 53.1°$

$\theta_3 = \sin^{-1}(3 \times 0.400) =$ nonexistent

The red end of the spectrum is at $\theta_1 = \sin^{-1}\left(\dfrac{1 \times 7.00 \times 10^{-7} \text{ m}}{1.25 \times 10^{-6} \text{ m}}\right) = 34.1°$

$\theta_2 = \sin^{-1}(2 \times 0.560) =$ nonexistent

So only the first-order spectrum is complete, and $\boxed{\text{it does not overlap}}$ the second-order spectrum.

38.29 (a) From Equation 38.12, $R = Nm$ where $N = (3000 \text{ lines/cm})(4.00 \text{ cm}) = 1.20 \times 10^4$ lines.

In the 1st order, $R = (1)(1.20 \times 10^4 \text{ lines}) = \boxed{1.20 \times 10^4}$

In the 2nd order, $R = (2)(1.20 \times 10^4 \text{ lines}) = \boxed{2.40 \times 10^4}$

In the 3rd order, $R = (3)(1.20 \times 10^4 \text{ lines}) = \boxed{3.60 \times 10^4}$

(b) From Equation 38.11, $R = \dfrac{\lambda}{\Delta\lambda}$:

In the 3rd order, $\Delta\lambda = \dfrac{\lambda}{R} = \dfrac{400 \text{ nm}}{3.60 \times 10^4} = 0.0111 \text{ nm} = \boxed{11.1 \text{ pm}}$

38.30 $\sin\theta = \dfrac{m\lambda}{d}$

Therefore, taking the ends of the visible spectrum to be $\lambda_v = 400$ nm and $\lambda_r = 750$ nm, the ends the different order spectra are:

End of second order: $\sin\theta_{2r} = \dfrac{2\lambda_r}{d} = \dfrac{1500 \text{ nm}}{d}$

Start of third order: $\sin\theta_{3v} = \dfrac{2\lambda_v}{d} = \dfrac{1200 \text{ nm}}{d}$

Thus, it is seen that $\boxed{\theta_{2r} > \theta_{3v} \text{ and these orders must overlap}}$ regardless of the value of the grating spacing *d*.

38.31 (a) $Nm = \dfrac{\lambda}{\Delta\lambda}$ $N(1) = \dfrac{531.7 \text{ nm}}{0.19 \text{ nm}} = \boxed{2800}$

(b) $\dfrac{1.32 \times 10^{-2} \text{ m}}{2800} = \boxed{4.72\ \mu\text{m}}$

38.32 $d\sin\theta = m\lambda$ and, differentiating, $d(\cos\theta)d\theta = md\lambda$ or $d\sqrt{1-\sin^2\theta}\,\Delta\theta \approx m\Delta\lambda$

$d\sqrt{1 - m^2\lambda^2 / d^2}\,\Delta\theta \approx m\Delta\lambda$ so $\boxed{\Delta\theta \approx \dfrac{\Delta\lambda}{\sqrt{d^2 / m^2 - \lambda^2}}}$

38.33 $d = \dfrac{1.00\times 10^{-3}\ \text{m/mm}}{250\ \text{lines/mm}} = 4.00\times 10^{-6}\ \text{m} = 4000\ \text{nm}$

$d\sin\theta = m\lambda \quad \Rightarrow \quad m = \dfrac{d\sin\theta}{\lambda}$

(a) The number of times a complete order is seen is the same as the number of orders in which the long wavelength limit is visible.

$m_{\max} = \dfrac{d\sin\theta_{\max}}{\lambda} = \dfrac{(4000\ \text{nm})\sin 90.0°}{700\ \text{nm}} = 5.71$ or $\boxed{\text{5 orders is the maximum}}$.

(b) The highest order in which the violet end of the spectrum can be seen is:

$m_{\max} = \dfrac{d\sin\theta_{\max}}{\lambda} = \dfrac{(4000\ \text{nm})\sin 90.0°}{400\ \text{nm}} = 10.0$ or $\boxed{\text{10 orders in the short-wavelength region}}$

38.34 $d = \dfrac{1}{4200/\text{cm}} = 2.38\times 10^{-6}\ \text{m} = 2380\ \text{nm}$

$d\sin\theta = m\lambda$ or $\theta = \sin^{-1}\left(\dfrac{m\lambda}{d}\right)$ and $y = L\tan\theta = L\tan\left[\sin^{-1}\left(\dfrac{m\lambda}{d}\right)\right]$

Thus, $\Delta y = L\left\{\tan\left[\sin^{-1}\left(\dfrac{m\lambda_2}{d}\right)\right] - \tan\left[\sin^{-1}\left(\dfrac{m\lambda_1}{d}\right)\right]\right\}$

For $m = 1$, $\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\dfrac{589.6}{2380}\right)\right] - \tan\left[\sin^{-1}\left(\dfrac{589}{2380}\right)\right]\right\} = 0.554\ \text{mm}$

For $m = 2$, $\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\dfrac{2(589.6)}{2380}\right)\right] - \tan\left[\sin^{-1}\left(\dfrac{2(589)}{2380}\right)\right]\right\} = 1.54\ \text{mm}$

For $m = 3$, $\Delta y = (2.00\ \text{m})\left\{\tan\left[\sin^{-1}\left(\dfrac{3(589.6)}{2380}\right)\right] - \tan\left[\sin^{-1}\left(\dfrac{3(589)}{2380}\right)\right]\right\} = 5.04\ \text{mm}$

Thus, the observed order must be $\boxed{m = 2}$.

38.35 $2d\sin\theta = m\lambda$: $\lambda = \dfrac{2d\sin\theta}{m} = \dfrac{2(0.353\times 10^{-9}\ \text{m})\sin(7.60°)}{(1)} = 9.34\times 10^{-11}\ \text{m} = \boxed{0.0934\ \text{nm}}$

38.36 $2d\sin\theta = m\lambda \Rightarrow d = \dfrac{m\lambda}{2\sin\theta} = \dfrac{(1)(0.129\ \text{nm})}{2\sin(8.15°)} = \boxed{0.455\ \text{nm}}$

38.37 $2d\sin\theta = m\lambda$ so $\sin\theta = \dfrac{m\lambda}{2d} = \dfrac{1(0.140\times10^{-9}\text{ m})}{2(0.281\times10^{-9}\text{ m})} = 0.249$ and $\boxed{\theta = 14.4°}$

38.38 $\sin\theta_m = \dfrac{m\lambda}{2d}$: $\sin 12.6° = \dfrac{1\lambda}{2d} = 0.218$

$\sin\theta_2 = \dfrac{2\lambda}{2d} = 2(0.218)$ so $\theta_2 = 25.9°$

$\boxed{\text{Three}}$ other orders appear: $\theta_3 = \sin^{-1}(3\times0.218) = 40.9°$

$\theta_4 = \sin^{-1}(4\times0.218) = 60.8°$

$\theta_5 = \sin^{-1}(5\times0.218) =$ nonexistent

38.39 $2d\sin\theta = m\lambda$ $\theta = \sin^{-1}\left[\dfrac{m\lambda}{2d}\right] = \sin^{-1}\left[\dfrac{2\times0.166}{2\times0.314}\right] = \boxed{31.9°}$

***38.40** Figure 38.25 of the text shows the situation. $2d\sin\theta = m\lambda$ or $\lambda = \dfrac{2d\sin\theta}{m}$

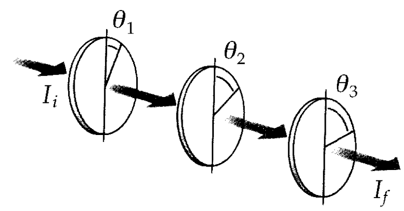
$$m = 1 \quad\Rightarrow\quad \lambda_1 = \frac{2(2.80\text{ m})\sin 80.0°}{1} = \boxed{5.51\text{ m}}$$

$$m = 2 \quad\Rightarrow\quad \lambda_2 = \frac{2(2.80\text{ m})\sin 80.0°}{2} = \boxed{2.76\text{ m}}$$

$$m = 3 \quad\Rightarrow\quad \lambda_3 = \frac{2(2.80\text{ m})\sin 80.0°}{3} = \boxed{1.84\text{ m}}$$

***38.41** The average value of the cosine-squared function is one-half, so the first polarizer transmits $\dfrac{1}{2}$ the light. The second transmits $\cos^2 30.0° = \dfrac{3}{4}$.

$$I_f = \frac{1}{2}\times\frac{3}{4}\,I_i = \boxed{\frac{3}{8}I_i}$$



38.42 (a) $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, $\theta_3 = 60.0°$

$I_f = I_i \cos^2(\theta_1 - 0°)\cos^2(\theta_2 - \theta_1)\cos^2(\theta_3 - \theta_2)$

$I_f = (10.0 \text{ units})\cos^2(20.0°)\cos^2(20.0°)\cos^2(20.0°) = \boxed{6.89 \text{ units}}$

(b) $\theta_1 = 0°$, $\theta_2 = 30.0°$, $\theta_3 = 60.0°$

$I_f = (10.0 \text{ units})\cos^2(0°)\cos^2(30.0°)\cos^2(30.0°) = \boxed{5.63 \text{ units}}$

38.43 $I = I_{max}\cos^2\theta \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{I}{I_{max}}\right)^{1/2}$

(a) $\frac{I}{I_{max}} = \frac{1}{3.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{3.00}\right)^{1/2} = \boxed{54.7°}$

(b) $\frac{I}{I_{max}} = \frac{1}{5.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{5.00}\right)^{1/2} = \boxed{63.4°}$

(c) $\frac{I}{I_{max}} = \frac{1}{10.0} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\frac{1}{10.0}\right)^{1/2} = \boxed{71.6°}$

38.44 By Brewster's law, $n = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$

38.45 $\sin\theta_c = \frac{1}{n}$ or $n = \frac{1}{\sin\theta_c} = \frac{1}{\sin 34.4°} = 1.77$

Also, $\tan\theta_p = n$. Thus, $\theta_p = \tan^{-1}(n) = \tan^{-1}(1.77) = \boxed{60.5°}$

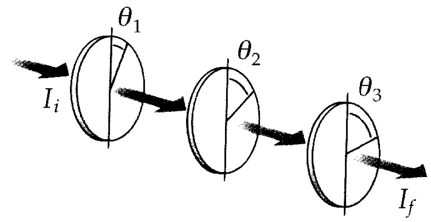
38.46 $\sin\theta_c = \frac{1}{n}$ and $\tan\theta_p = n$

Thus, $\sin\theta_c = \frac{1}{\tan\theta_p}$ or $\boxed{\cot\theta_p = \sin\theta_c}$

38.47 Complete polarization occurs at Brewster's angle $\tan\theta_p = 1.33$ $\theta_p = 53.1°$

Thus, the Moon is $\boxed{36.9°}$ above the horizon.

38.48 For incident unpolarized light of intensity I_{max}:



After transmitting 1st disk: $I = \left(\frac{1}{2}\right)I_{max}$

After transmitting 2nd disk: $I = \left(\frac{1}{2}\right)I_{max}\cos^2\theta$

After transmitting 3rd disk: $I = \left(\frac{1}{2}\right)I_{max}\left(\cos^2\theta\right)\cos^2(90°-\theta)$

where the angle between the first and second disk is $\theta = \omega t$.

Using trigonometric identities $\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$ and $\cos^2(90-\theta) = \sin^2\theta = \frac{1}{2}\left(1-\cos 2\theta\right)$

we have $I = \frac{1}{2}I_{max}\left[\frac{(1+\cos 2\theta)}{2}\right]\left[\frac{(1-\cos 2\theta)}{2}\right] = \frac{1}{8}I_{max}(1-\cos^2 2\theta) = \frac{1}{8}I_{max}\left(\frac{1}{2}\right)(1-\cos 4\theta)$

Since $\theta = \omega t$, the intensity of the emerging beam is given by $\boxed{I = \frac{1}{16}I_{max}(1-\cos 4\omega t)}$

38.49 Let the first sheet have its axis at angle θ to the original plane of polarization, and let each further sheet have its axis turned by the same angle.

The first sheet passes intensity $I_{max}\cos^2\theta.$

The second sheet passes $I_{max}\cos^4\theta,$

nd the nth sheet lets through $I_{max}\cos^{2n}\theta \ge 0.90 I_{max}$ where $\theta = 45°/n$

Try different integers to find $\cos^{2\times 5}\left(\frac{45°}{5}\right) = 0.885,$ $\cos^{2\times 6}\left(\frac{45°}{6}\right) = 0.902,$

(a) So $n = \boxed{6}$

(b) $\theta = \boxed{7.50°}$

***38.50** Consider vocal sound moving at 340 m/s and of frequency 3000 Hz. Its wavelength is

$$\lambda = \frac{v}{f} = \frac{340\ \text{m/s}}{3000\ \text{Hz}} = 0.113\ \text{m}$$

If your mouth, for horizontal dispersion, behaves similarly to a slit 6.00 cm wide, then $a \sin\theta = m\lambda$ predicts no diffraction minima. You are a nearly isotropic source of this sound. It spreads out from you nearly equally in all directions. On the other hand, if you use a megaphone with width 60.0 cm at its wide end, then $a\sin\theta = m\lambda$ predicts the first diffraction minimum at

$$\theta = \sin^{-1}\left(\frac{m\lambda}{a}\right) = \sin^{-1}\left(\frac{0.113\ \text{m}}{0.600\ \text{m}}\right) = 10.9°$$

This suggests that the sound is radiated mostly toward the front into a diverging beam of angular diameter only about 20°. With less sound energy wasted in other directions, more is available for your intended auditors. We could check that a distant observer to the side or behind you receives less sound when a megaphone is used.

38.51 The first minimum is at $a \sin\theta = 1\lambda$.

This has no solution if $\frac{\lambda}{a} > 1$

or if $a < \lambda = \boxed{632.8\ \text{nm}}$

38.52 $x = 1.22\frac{\lambda}{d}D = 1.22\left(\frac{5.00\times10^{-7}\ \text{m}}{5.00\times10^{-3}\ \text{m}}\right)(250\times10^{3}\ \text{m}) = \boxed{30.5\ \text{m}}$

$D = 250\times10^3$ m
$\lambda = 5.00\times10^{-7}$ m
$d = 5.00\times10^{-3}$ m

38.53 $d = \frac{1}{400/\text{mm}} = 2.50\times10^{-6}$ m

(a) $d\sin\theta = m\lambda$ $\theta_a = \sin^{-1}\left(\frac{2\times541\times10^{-9}\ \text{m}}{2.50\times10^{-6}\ \text{m}}\right) = \boxed{25.6°}$

(b) $\lambda = \frac{541\times10^{-9}\ \text{m}}{1.33} = 4.07\times10^{-7}$ m

$\theta_b = \sin^{-1}\left(\frac{2\times4.07\times10^{-7}\ \text{m}}{2.50\times10^{-6}\ \text{m}}\right) = \boxed{19.0°}$

(c) $d\sin\theta_a = 2\lambda$ $\qquad d\sin\theta_b = \frac{2\lambda}{n}$ $\qquad n\sin\theta_b = 1\sin\theta_a$

***38.54** (a) $\lambda = \frac{v}{f} = \frac{3.00\times 10^{8}\ \text{m/s}}{1.40\times 10^{9}\ \text{s}^{-1}} = 0.214\ \text{m}$

$\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{0.214\ \text{m}}{3.60\times 10^{4}\ \text{m}}\right) = \boxed{7.26\ \mu\text{rad}} = 7.26\ \mu\text{rad}\left(\frac{180\times 60\times 60\ \text{s}}{\pi}\right) = \boxed{1.50\ \text{arc seconds}}$

(b) $\theta_{\min} = \frac{d}{L}$: $d = \theta_{\min} L = \left(7.26\times 10^{-6}\ \text{rad}\right)\left(26\ 000\ \text{ly}\right) = \boxed{0.189\ \text{ly}}$

(c) $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{500\times 10^{-9}\ \text{m}}{12.0\times 10^{-3}\ \text{m}}\right) = \boxed{50.8\ \mu\text{rad}}$ (10.5 seconds of arc)

(d) $d = \theta_{\min} L = \left(50.8\times 10^{-6}\ \text{rad}\right)\left(30.0\ \text{m}\right) = 1.52\times 10^{-3}\ \text{m} = \boxed{1.52\ \text{mm}}$

38.55 $\theta_{\min} = 1.22\frac{\lambda}{D} = 1.22\frac{(2.00\ \text{m})}{(10.0\ \text{m})} = \boxed{0.244\ \text{rad} = 14.0^\circ}$

38.56 With a grazing angle of 36.0°, the angle of incidence is 54.0°

$\tan\theta_p = n = \tan 54.0^\circ = 1.38$

In the liquid, $\lambda_n = \lambda / n = 750\ \text{nm} / 1.38 = \boxed{545\ \text{nm}}$

38.57 (a) $d\sin\theta = m\lambda$, or $d = \frac{m\lambda}{\sin\theta} = \frac{3\left(500\times 10^{-9}\ \text{m}\right)}{\sin 32.0^\circ} = 2.83\ \mu\text{m}$

Therefore, lines per unit length = $\frac{1}{d} = \frac{1}{2.83\times 10^{-6}\ \text{m}}$

or lines per unit length $= 3.53\times 10^{5}/\text{m} = \boxed{3.53\times 10^{3}/\text{cm}}$.

(b) $\sin\theta = \frac{m\lambda}{d} = \frac{m\left(500\times 10^{-9}\ \text{m}\right)}{2.83\times 10^{-6}\ \text{m}} = m(0.177)$

For $\sin\theta \le 1.00$, we must have $m(0.177) \le 1.00$ or $m \le 5.65$

Therefore, the highest order observed is $m = 5$

Total number primary maxima observed is $2m + 1 = \boxed{11}$

Goal Solution

Light of wavelength 500 nm is incident normally on a diffraction grating. If the third-order maximum of the diffraction pattern is observed at 32.0°, (a) what is the number of rulings per centimeter for the grating? (b) Determine the total number of primary maxima that can be observed in this situation.

G: The diffraction pattern described in this problem seems to be similar to previous problems that have diffraction gratings with 2 000 to 5 000 lines/mm. With the third-order maximum at 32°, there are probably 5 or 6 maxima on each side of the central bright fringe, for a total of 11 or 13 primary maxima.

O: The diffraction grating equation can be used to find the grating spacing and the angles of the other maxima that should be visible within the 180° viewing range.

A: (a) Use Equation 38.10, $d\sin\theta = m\lambda$

$$d = \frac{m\lambda}{\sin\theta} = \frac{(3)(5.00\times10^{-7}\text{ m})}{\sin(32.0°)} = 2.83\times10^{-6}\text{ m}$$

Thus, the grating gauge is $\frac{1}{d} = 3.534\times10^{5}$ lines / m = 3530 lines / cm ◊

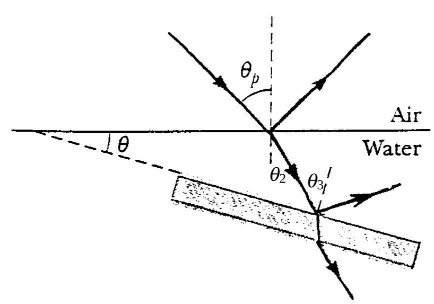
(b) $$\sin\theta = m\left(\frac{\lambda}{d}\right) = \frac{m(5.00\times10^{-7}\text{ m})}{2.83\times10^{-6}\text{ m}} = m(0.177)$$

For $\sin\theta \le 1$, we require that $m(1.77) \le 1$ or $m \le 5.65$. Since m must be an integer, its maximum value is really 5. Therefore, the total number of maxima is $2m + 1 = 11$

L: The results agree with our predictions, and apparently there are 5 maxima on either side of the central maximum. If more maxima were desired, a grating with **fewer** lines/cm would be required; however, this would reduce the ability to resolve the difference between lines that appear close together.

38.58 For the air-to-water interface,

$$\tan\theta_p = \frac{n_{\text{water}}}{n_{\text{air}}} = \frac{1.33}{1.00} \qquad \theta_p = 53.1°$$



and $(1.00)\sin\theta_p = (1.33)\sin\theta_2$

$$\theta_2 = \sin^{-1}\left(\frac{\sin 53.1°}{1.33}\right) = 36.9°$$

For the water-to-glass interface, $\tan\theta_p = \tan\theta_3 = \frac{n_{\text{glass}}}{n_{\text{water}}} = \frac{1.50}{1.33}$ so $\theta_3 = 48.4°$

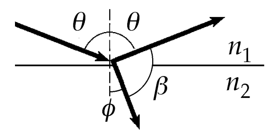
The angle between surfaces is $\theta = \theta_3 - \theta_2 = \boxed{11.5°}$

38.59 The limiting resolution between lines $\theta_{min} = 1.22\dfrac{\lambda}{D} = 1.22\dfrac{\left(550\times10^{-9}\text{ m}\right)}{\left(5.00\times10^{-3}\text{ m}\right)} = 1.34\times10^{-4}$ rad

Assuming a picture screen with vertical dimension l, the minimum viewing distance for no visible lines is found from $\theta_{min} = (l/485)/L$. The desired ratio is then

$$\frac{L}{l} = \frac{1}{485\,\theta_{min}} = \frac{1}{485\left(1.34\times10^{-4}\text{ rad}\right)} = \boxed{15.4}$$

38.60 (a) Applying Snell's law gives $n_2 \sin\phi = n_1 \sin\theta$. From the sketch, we also see that:



$$\theta + \phi + \beta = \pi, \quad \text{or} \quad \phi = \pi - (\theta + \beta)$$

Using the given identity: $\sin\phi = \sin\pi\cos(\theta+\beta) - \cos\pi\sin(\theta+\beta)$,

which reduces to: $\sin\phi = \sin(\theta+\beta)$.

Applying the identity again: $\sin\phi = \sin\theta\cos\beta + \cos\theta\sin\beta$

Snell's law then becomes: $n_2\left(\sin\theta\cos\beta + \cos\theta\sin\beta\right) = n_1\sin\theta$

or (after dividing by $\cos\theta$): $n_2(\tan\theta\cos\beta + \sin\beta) = n_1\tan\theta$.

Solving for $\tan\theta$ gives: $\boxed{\tan\theta = \dfrac{n_2\sin\beta}{n_1 - n_2\cos\beta}}$

(b) If $\beta = 90.0°$, $n_1 = 1.00$, and $n_2 = n$, the above result becomes:

$\tan\theta = \dfrac{n(1.00)}{1.00 - 0}$, or $n = \tan\theta$, which is Brewster's law.

38.61 (a) From Equation 38.1, $\theta = \sin^{-1}\left(\dfrac{m\lambda}{a}\right)$

In this case $m = 1$ and $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{7.50\times10^{9}\text{ Hz}} = 4.00\times10^{-2}$ m

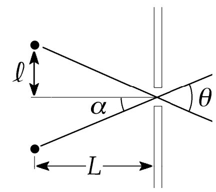
Thus, $\theta = \sin^{-1}\left(\dfrac{4.00\times10^{-2}\text{ m}}{6.00\times10^{-2}\text{ m}}\right) = \boxed{41.8°}$

(b) From Equation 38.4, $\dfrac{I}{I_{max}} = \left[\dfrac{\sin(\beta/2)}{\beta/2}\right]^2$ where $\beta = \dfrac{2\pi a\sin\theta}{\lambda}$

When $\theta = 15.0°$, $\beta = \dfrac{2\pi(0.0600\text{ m})\sin 15.0°}{0.0400\text{ m}} = 2.44\text{ rad}$

and $\dfrac{I}{I_{max}} = \left[\dfrac{\sin(1.22\text{ rad})}{1.22\text{ rad}}\right]^2 = \boxed{0.593}$

(c) $\sin\theta = \dfrac{\lambda}{a}$ so $\theta = 41.8°$:



This is the minimum angle subtended by the two sources at the slit. Let α be the half angle between the sources, each a distance $\ell = 0.100$ m from the center line and a distance L from the slit plane. Then,

$L = \ell\cot\alpha = (0.100\text{ m})\cot(41.8/2) = \boxed{0.262\text{ m}}$

38.62 $\dfrac{I}{I_{max}} = \dfrac{1}{2}(\cos^2 45.0°)(\cos^2 45.0°) = \boxed{\dfrac{1}{8}}$

38.63 (a) The E and O rays, in phase at the surface of the plate, will have a phase difference

$\theta = (2\pi/\lambda)\delta$

after traveling distance d through the plate. Here δ is the difference in the *optical path* lengths of these rays. The optical path length between two points is the product of the actual path length d and the index of refraction. Therefore,

$\delta = |dn_O - dn_E|$

The absolute value is used since n_O/n_E may be more or less than unity. Therefore,

$$\theta = \left(\frac{2\pi}{\lambda}\right)|dn_O - dn_E| = \boxed{\left(\frac{2\pi}{\lambda}\right)d|n_O - n_E|}$$

(b) $d = \dfrac{\lambda\theta}{2\pi|n_O - n_E|} = \dfrac{(550\times10^{-9}\text{ m})(\pi/2)}{2\pi|1.544 - 1.553|} = 1.53\times10^{-5}\text{ m} = \boxed{15.3\ \mu\text{m}}$

***38.64** For a diffraction grating, the locations of the principal maxima for wavelength λ are given by $\sin\theta = m\lambda/d \approx y/L$. The grating spacing may be expressed as $d = a/N$ where a is the width of the grating and N is the number of slits. Thus, the screen locations of the maxima become

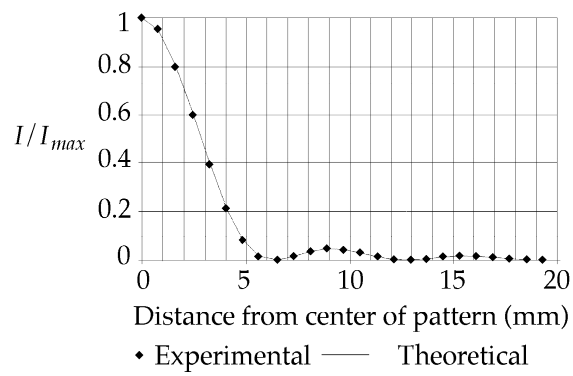
$y = NLm\lambda / a$. If two nearly equal wavelengths are present, the difference in the screen locations of corresponding maxima is

$$\Delta y = \frac{NLm(\Delta\lambda)}{a}$$

For a single slit of width a, the location of the first diffraction minimum is $\sin\theta = \lambda/a \approx y/L$, or $y = (L/a)\lambda$. If the two wavelengths are to be just resolved by Rayleigh's criterion, $y = \Delta y$ from above. Therefore,

$$\left(\frac{L}{a}\right)\lambda = \frac{NLm(\Delta\lambda)}{a}$$ or the resolving power of the grating is $\boxed{R'' \frac{\lambda}{\Delta\lambda} = Nm}$.

38.65 The first minimum in the single-slit diffraction pattern occurs at



$$\sin\theta = \frac{\lambda}{a} » \frac{y_{\min}}{L}$$

Thus, the slit width is given by

$$a = \frac{\lambda L}{y_{\min}}$$

For a minimum located at $y_{\min} = 6.36 \text{ mm} \pm 0.08 \text{ mm}$,

the width is $a = \frac{(632.8 \cdot 10^{-9} \text{ m})(1.00 \text{ m})}{6.36 \cdot 10^{-3} \text{ m}} = \boxed{99.5\ \mu\text{m} \pm 1\%}$

38.66 (a) From Equation 38.4,

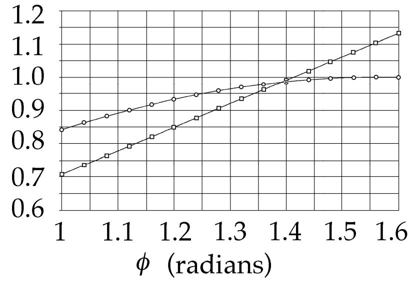
$$\frac{I}{I_{\max}} = \left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2$$

If we define $\phi \equiv \beta/2$ this becomes $\frac{I}{I_{\max}} = \left[\frac{\sin\phi}{\phi}\right]^2$

Therefore, when $\frac{I}{I_{\max}} = \frac{1}{2}$ we must have $\frac{\sin\phi}{\phi} = \frac{1}{\sqrt{2}}$, or $\boxed{\sin\phi = \frac{\phi}{\sqrt{2}}}$

(b) Let $y_1 = \sin\phi$ and $y_2 = \dfrac{\phi}{\sqrt{2}}$.

A plot of y_1 and y_2 in the range $1.00 \le \phi \le \pi/2$ is shown to the right.



The solution to the transcendental equation is found to be $\boxed{\phi = 1.39 \text{ rad}}$.

(c) $$\beta = \frac{2\pi a \sin\theta}{\lambda} = 2\phi$$

gives $$\sin\theta = \left(\frac{\phi}{\pi}\right)\frac{\lambda}{a} = 0.443\frac{\lambda}{a}.$$

If $\dfrac{\lambda}{a}$ is small, then $\theta \approx 0.443\dfrac{\lambda}{a}$.

This gives the half-width, measured away from the maximum at $\theta = 0$. The pattern is symmetric, so the full width is given by

$$\Delta\theta = 0.443\frac{\lambda}{a} - \left(-0.443\frac{\lambda}{a}\right) = \boxed{\frac{0.886\lambda}{a}}$$

38.67

ϕ	$\sqrt{2}\sin\phi$	
1	1.19	bigger than ϕ
2	1.29	smaller than ϕ
1.5	1.41	smaller
1.4	1.394	
1.39	1.391	bigger
1.395	1.392	
1.392	1.3917	smaller
1.3915	1.39154	bigger
1.39152	1.39155	bigger
1.3916	1.391568	smaller
1.39158	1.391563	
1.39157	1.391560	
1.39156	1.391558	
1.391559	1.3915578	
1.391558	1.3915575	
1.391557	1.3915573	
1.3915574	1.3915574	

We get the answer to seven digits after 17 steps. Clever guessing, like using the value of $\sqrt{2}\sin\phi$ as the next guess for ϕ, could reduce this to around 13 steps.

***38.68** In $I = I_{max}\left[\frac{\sin(\beta/2)}{(\beta/2)}\right]^2$ find $\frac{dI}{d\beta} = I_{max}\left(\frac{2\sin(\beta/2)}{(\beta/2)}\right)\left[\frac{(\beta/2)\cos(\beta/2)(1/2) - \sin(\beta/2)(1/2)}{(\beta/2)^2}\right]$

and require that it be zero. The possibility $\sin(\beta/2) = 0$ locates all of the minima and the central maximum, according to

$\beta/2 = 0,\ \pi,\ 2\pi,\ \ldots;$ $\beta = \frac{2\pi a\sin\theta}{\lambda} = 0,\ 2\pi,\ 4\pi,\ \ldots;$ $a\sin\theta = 0,\ \lambda,\ 2\lambda,\ \ldots.$

The side maxima are found from $\frac{\beta}{2}\cos\left(\frac{\beta}{2}\right) - \sin\left(\frac{\beta}{2}\right) = 0$, or $\tan\left(\frac{\beta}{2}\right) = \frac{\beta}{2}$.

This has solutions $\boxed{\frac{\beta}{2} = 4.4934}$, $\boxed{\frac{\beta}{2} = 7.7253}$, and others, giving

(a) $\pi a\sin\theta = 4.4934\lambda$ $\boxed{a\sin\theta = 1.4303\lambda}$

(b) $\pi a\sin\theta = 7.7253\lambda$ $\boxed{a\sin\theta = 2.4590\lambda}$

***38.69** (a) We require $\theta_{min} = 1.22\frac{\lambda}{D} = \frac{\text{radius of diffraction disk}}{L} = \frac{D}{2L}$.

Then $\boxed{D^2 = 2.44\lambda L}$

(b) $D = \sqrt{2.44\left(500\times10^{-9}\text{ m}\right)(0.150\text{ m})} = \boxed{428\ \mu\text{m}}$