Chapter 43 Solutions

43.1 (a) $F = \dfrac{q^2}{4\pi e_0 r^2} = \dfrac{(1.60\times10^{-19})^2(8.99\times10^9)}{(5.00\times10^{-10})^2}\text{ N} = \boxed{0.921\times10^{-9}\text{ N}}$ toward the other ion.

(b) $U = \dfrac{-q^2}{4\pi e_0 r} = -\dfrac{(1.60\times10^{-19})^2(8.99\times10^9)}{5.00\times10^{-10}}\text{ J} \approx \boxed{-2.88\text{ eV}}$

***43.2** We are told $\mathrm{K + Cl + 0.7\ eV \rightarrow K^+ + Cl^-}$

and $\mathrm{Cl + e^- \rightarrow Cl^- + 3.6\ eV}$

or $\mathrm{Cl^- \rightarrow Cl + e^- - 3.6\ eV}$

By substitution, $\mathrm{K + Cl + 0.7\ eV \rightarrow K^+ + Cl + e^- - 3.6\ eV}$

$\mathrm{K + 4.3\ eV \rightarrow K^+ + e^-}$

or the ionization energy of potassium is $\boxed{4.3\text{ eV}}$

43.3 (a) Minimum energy of the molecule is found from

$$\frac{dU}{dr} = -12Ar^{-13} + 6Br^{-7} = 0, \quad \text{yielding} \quad \boxed{r_0 = \left[\frac{2A}{B}\right]^{1/6}}$$

(b) $E = U|_{r=\infty} - U|_{r=r_0} = 0 - \left[\dfrac{A}{4A^2/B^2} - \dfrac{B}{2A/B}\right] = -\left[\dfrac{1}{4} - \dfrac{1}{2}\right]\dfrac{B^2}{A} = \boxed{\dfrac{B^2}{4A}}$

This is also the equal to the binding energy, the amount of energy given up by the two atoms as they come together to form a molecule.

(c) $r_0 = \left[\dfrac{2\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)}{1.488\times10^{-60}\text{ eV}\cdot\text{m}^6}\right]^{1/6} = 7.42\times10^{-11}\text{ m} = \boxed{74.2\text{ pm}}$

$$E = \frac{\left(1.488\times10^{-60}\text{ eV}\cdot\text{m}^6\right)^2}{4\left(0.124\times10^{-120}\text{ eV}\cdot\text{m}^{12}\right)} = \boxed{4.46\text{ eV}}$$

***43.4** At the boiling or condensation temperature, $k_B T \approx 10^{-3}\text{ eV} = 10^{-3}\left(1.6\times10^{-19}\text{ J}\right)$

$$T \approx \frac{1.6\times10^{-22}\text{ J}}{1.38\times10^{-23}\text{ J/K}} \quad \boxed{\sim 10\text{ K}}$$

43.5

$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \frac{132.9(126.9)}{132.9 + 126.9}\left(1.66 \times 10^{-27}\ \text{kg}\right) = 1.08 \times 10^{-25}\ \text{kg}$$

$$I = \mu r^2 = (1.08 \times 10^{-25}\ \text{kg})(0.127 \times 10^{-9}\ \text{m})^2 = 1.74 \times 10^{-45}\ \text{kg} \cdot \text{m}^2$$

(a) $E = \frac{1}{2} I \omega^2 = \dfrac{(I\omega)^2}{2I} = \dfrac{J(J+1)\hbar^2}{2I}$

$J = 0$ gives $E = 0$

$$J = 1 \text{ gives } E = \frac{\hbar^2}{I} = \frac{(6.626 \times 10^{-34}\ \text{J} \cdot \text{s})^2}{4\pi^2(1.74 \times 10^{-45}\ \text{kg} \cdot \text{m}^2)} = 6.41 \times 10^{-24}\ \text{J} = \boxed{40.0\ \mu\text{eV}}$$

$hf = 6.41 \times 10^{-24}\ \text{J} - 0$ to $f = \boxed{9.66 \times 10^{9}\ \text{Hz}}$

(b) $f = \dfrac{E_1}{h} = \dfrac{\hbar^2}{hI} = \dfrac{h}{4\pi^2 \mu r^2} \propto r^{-2}$ $\boxed{\text{If } r \text{ is 10\% too small, } f \text{ is 20\% too large.}}$

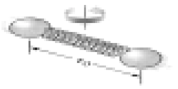
43.6

$$hf = \Delta E = \frac{\hbar^2}{2I}\left[2(2+1)\right] - \frac{\hbar^2}{2I}\left[1(1+1)\right] = \frac{\hbar^2}{2I}(4)$$

$$I = \frac{4\left(h/2\pi\right)^2}{2hf} = \frac{h}{2\pi^2 f} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{2\pi^2\left(2.30 \times 10^{11}\ \text{Hz}\right)} = \boxed{1.46 \times 10^{-46}\ \text{kg} \cdot \text{m}^2}$$

43.7

For the HCl molecule in the $J = 1$ rotational energy level, we are given $r_0 = 0.1275\ \text{nm}$.



$$E_{\text{rot}} = \frac{\hbar^2}{2I} J(J+1)$$

Taking $J = 1$, we have $E_{\text{rot}} = \dfrac{\hbar^2}{I} = \frac{1}{2} I \omega^2$ or $\omega = \sqrt{\dfrac{2\hbar^2}{I^2}} = \sqrt{2}\dfrac{\hbar}{I}$

The moment of inertia of the molecule is given by Equation 43.3. $I = \mu r_0^2 = \left(\dfrac{m_1 m_2}{m_1 + m_2}\right) r_0^2$

$$I = \left[\frac{(1\ \text{u})(35\ \text{u})}{1\ \text{u} + 35\ \text{u}}\right] r_0^2 = (0.972\ \text{u})(1.66 \times 10^{-27}\ \text{kg / u})(1.275 \times 10^{-10}\ \text{m})^2 = 2.62 \times 10^{-47}\ \text{kg} \cdot \text{m}^2$$

Therefore, $\omega = \sqrt{2}\dfrac{\hbar}{I} = \sqrt{2}\dfrac{1.055 \times 10^{-34}\ \text{J} \cdot \text{s}}{2.62 \times 10^{-47}\ \text{kg} \cdot \text{m}^2} = \boxed{5.69 \times 10^{12}\ \text{rad / s}}$

Goal Solution

An HCl molecule is excited to its first rotational-energy level, corresponding to $J = 1$. If the distance between its nuclei is 0.1275 nm, what is the angular speed of the molecule about its center of mass?

G: For a system as small as a molecule, we can expect the angular speed to be much faster than the few rad/s typical of everyday objects we encounter.

O: The rotational energy is given by the angular momentum quantum number, J. The angular speed can be calculated from this kinetic rotational energy and the moment of inertia of this one-dimensional molecule.

A: For the HCl molecule in the $J = 1$ rotational energy level, we are given $r_0 = 0.1275\text{ nm}$.

$E_{rot} = \frac{\hbar}{2I}J(J+1)$ so with $J = 1$, $E_{rot} = \frac{\hbar^2}{I} = \frac{1}{2}I\omega^2$ and $\omega = \sqrt{\frac{2\hbar^2}{I^2}} = \frac{\hbar\sqrt{2}}{I}$

The moment of inertia of the molecule is given by: $I = \mu r_0{}^2 = \left(\frac{m_1 m_2}{m_1 + m_2}\right) r_0{}^2 = \left[\frac{(1\text{ u})(35\text{ u})}{1\text{ u} + 35\text{ u}}\right] r_0{}^2$

$$I = (0.972\text{ u})(1.66\times10^{-27}\text{ kg / u})(1.275\times10^{-10}\text{ m})^2 = 2.62\times10^{-47}\text{ kg}\cdot\text{m}^2$$

Therefore, $\omega = \sqrt{2}\frac{\hbar}{I} = \sqrt{2}\left(\frac{1.055\times10^{-34}\text{ J}\cdot\text{s}}{2.62\times10^{-47}\text{ kg}\cdot\text{m}^2}\right) = 5.69\times10^{12}\text{ rad / s}$

L: This angular speed is more than a billion times faster than the spin rate of a music CD, which rotates at 200 to 500 revolutions per minute, or $\omega = 20\text{ rad / s}$ to 50 rad / s.

43.8 $I = m_1 r_1^2 + m_2 r_2^2$ where $m_1 r_1 = m_2 r_2$ and $r_1 + r_2 = r$

Then $r_1 = \frac{m_2 r_2}{m_1}$ so $\frac{m_2 r_2}{m_1} + r_2 = r$ and $r_2 = \frac{m_1 r}{m_1 + m_2}$

Also, $r_2 = \frac{m_1 r_1}{m_2}$ Thus, $r_1 + \frac{m_1 r_1}{m_2} = r$ and $r_1 = \frac{m_2 r}{m_1 + m_2}$

$$I = m_1\frac{m_2^2 r^2}{(m_1+m_2)^2} + \frac{m_2 m_1^2 r^2}{(m_1+m_2)^2} = \frac{m_1 m_2 r^2(m_2 + m_1)}{(m_1+m_2)^2} = \frac{m_1 m_2 r^2}{m_1 + m_2} = \boxed{\mu r^2}$$

43.9 (a) $\mu = \frac{22.99(35.45)}{(22.99 + 35.45)}\left(1.66\times10^{-27}\text{ kg}\right) = 2.32\times10^{-26}\text{ kg}$

$$I = \mu r^2 = \left(2.32\times10^{-26}\text{ kg}\right)\left(0.280\times10^{-9}\text{ m}\right)^2 = \boxed{1.81\times10^{-45}\text{ kg}\cdot\text{m}^2}$$

(b) $\frac{hc}{\lambda} = \frac{\hbar^2}{2I}2(2+1) - \frac{\hbar^2}{2I}1(1+1) = \frac{3\hbar^2}{I} - \frac{\hbar^2}{I} = \frac{2\hbar^2}{I} = \frac{2h^2}{4\pi^2 I}$

$$\lambda = \frac{c4\pi^2 I}{2h} = \frac{\left(3.00\times10^{8}\text{ m / s}\right)4\pi^2\left(1.81\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}{2\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)} = \boxed{1.62\text{ cm}}$$

43.10 The energy of a rotational transition is $\Delta E = \left(\hbar^2/I\right)J$ where J is the rotational quantum number of the higher energy state (see Equation 43.7). We do not know J from the data. However,

$$\Delta E = \frac{hc}{\lambda} = \frac{\left(6.626\times10^{-34}\ \mathrm{J\cdot s}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{\lambda}\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right)$$

For each observed wavelength,

λ (mm)	ΔE (eV)
0.1204	0.01032
0.0964	0.01288
0.0804	0.01544
0.0690	0.01800
0.0604	0.02056

The $\Delta E's$ consistently increase by 0.00256 eV. $E_1 = \hbar^2/I = 0.00256\ \mathrm{eV}$

and $$I = \frac{\hbar^2}{E_1} = \frac{\left(1.055\times10^{-34}\ \mathrm{J\cdot s}\right)^2}{(0.00256\ \mathrm{eV})}\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right) = \boxed{2.72\times10^{-47}\ \mathrm{kg\cdot m^2}}$$

For the HCl molecule, the internuclear radius is $r = \sqrt{\dfrac{I}{\mu}} = \sqrt{\dfrac{2.72\times10^{-47}}{1.62\times10^{-27}}}\ \mathrm{m} = 0.130\ \mathrm{nm}$

43.11 $$\mu = \frac{m_1 m_2}{m_1+m_2} = \frac{35}{36}\times1.66\times10^{-27}\ \mathrm{kg} = 1.61\times10^{-27}\ \mathrm{kg}$$

$$\Delta E_{\mathrm{vib}} = \hbar\sqrt{\frac{k}{\mu}} = \left(1.055\times10^{-34}\right)\sqrt{\frac{480}{1.61\times10^{-27}}} = 5.74\times10^{-20}\ \mathrm{J} = \boxed{0.358\ \mathrm{eV}}$$

43.12 (a) Minimum amplitude of vibration of HI is

$$\tfrac{1}{2}kA^2 = \tfrac{1}{2}hf:\ A = \sqrt{\frac{hf}{k}} = \sqrt{\frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(6.69\times10^{13}\ /\ \mathrm{s})}{320\ \mathrm{N\ /\ m}}} = 1.18\times10^{-11}\ \mathrm{m} = \boxed{0.0118\ \mathrm{nm}}$$

(b) For HF, $$A = \sqrt{\frac{(6.626\times10^{-34}\ \mathrm{J\cdot s})(8.72\times10^{13}\ /\ \mathrm{s})}{970\ \mathrm{N\ /\ m}}} = 7.72\times10^{-12}\ \mathrm{m} = \boxed{0.00772\ \mathrm{nm}}$$

Since HI has the smaller k, it is more weakly bound.

43.13 (a) The reduced mass of the O_2 is $\mu = \dfrac{(16\text{ u})(16\text{ u})}{(16\text{ u})+(16\text{ u})} = 8\text{ u} = 8\left(1.66\times10^{-27}\text{ kg}\right) = 1.33\times10^{-26}\text{ kg}$

The moment of inertia is then $I = \mu r^2 = \left(1.33\times10^{-26}\text{ kg}\right)\left(1.20\times10^{-10}\text{m}\right)^2 = 1.91\times10^{-46}\text{ kg}\cdot\text{m}^2$

The rotational energies are $E_{\text{rot}} = \dfrac{\hbar^2}{2I}J(J+1) = \dfrac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.91\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}J(J+1)$

Thus $E_{\text{rot}} = \left(2.91\times10^{-23}\text{ J}\right)J(J+1)$

And for $J = 0, 1, 2,$ $E_{\text{rot}} = \boxed{0,\ 3.64\times10^{-4}\text{ eV},\ 1.09\times10^{-3}\text{ eV}}$

(b) $E_{\text{vib}} = \left(v+\dfrac{1}{2}\right)\hbar\sqrt{\dfrac{k}{\mu}} = \left(v+\dfrac{1}{2}\right)\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)\sqrt{\dfrac{1177\text{ N / m}}{8(1.66\times10^{-27}\text{kg})}}$

$E_{\text{vib}} = \left(v+\dfrac{1}{2}\right)\left(3.14\times10^{-20}\text{ J}\right)\left(\dfrac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = \left(v+\dfrac{1}{2}\right)(0.196\text{ eV})$

For $v = 0, 1, 2,$ $\boxed{E_{\text{vib}} = 0.0982\text{ eV},\ 0.295\text{ eV},\ 0.491\text{ eV}}$

43.14 In Benzene, the carbon atoms are each 0.110 nm from the axis and each hydrogen atom is $(0.110 + 0.100\text{ nm}) = 0.210\text{ nm}$ from the axis. Thus, $I = \Sigma mr^2$:

$$I = 6\left(1.99\times10^{-26}\text{ kg}\right)\left(0.110\times10^{-9}\text{ m}\right)^2 + 6\left(1.67\times10^{-27}\text{ kg}\right)\left(0.210\times10^{-9}\text{ m}\right)^2 = 1.89\times10^{-45}\text{ kg}\cdot\text{m}^2$$

The allowed rotational energies are then

$$E_{\text{rot}} = \frac{\hbar^2}{2I}J(J+1) = \frac{\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)^2}{2\left(1.89\times10^{-45}\text{ kg}\cdot\text{m}^2\right)}J(J+1) = \left(2.95\times10^{-24}\text{ J}\right)J(J+1) = \left(18.4\times10^{-6}\text{ eV}\right)J(J+1)$$

$$E_{\text{rot}} = \boxed{(18.4\ \mu\text{eV})J(J+1)\quad\text{where } J = 0, 1, 2, 3, \ldots}$$

The first five of these allowed energies are: $E_{\text{rot}} = 0,\ 36.9\ \mu\text{eV},\ 111\ \mu\text{eV},\ 221\ \mu\text{eV},\text{ and }369\ \mu\text{eV}$

43.15 $hf = \dfrac{h^2}{4\pi^2 I}J$ where the rotational transition is from $J-1$ to J,

where $f = 6.42\times10^{13}\text{ Hz}$ and $I = 1.46\times10^{-46}\text{ kg}\cdot\text{m}^2$ from Example 43.1.

$$J = \frac{4\pi^2 If}{h} = \frac{4\pi^2(1.46\times10^{-46}\text{ kg}\cdot\text{m}^2)(6.42\times10^{13}\text{ / s})}{6.626\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{558}$$

***43.16** The emission energies are the same as the absorption energies, but the final state must be below $(v=1,\ J=0)$. The transition must satisfy $\Delta J = \pm 1$, so it must end with $J=1$. To be lower in energy, it must be $(v=0,\ J=1)$. The emitted photon energy is therefore

$$hf_{\text{photon}} = \left(E_{\text{vib}}\big|_{v=1} + E_{\text{rot}}\big|_{J=0}\right) - \left(E_{\text{vib}}\big|_{v=0} + E_{\text{rot}}\big|_{J=1}\right) = \left(E_{\text{vib}}\big|_{v=1} - E_{\text{vib}}\big|_{v=0}\right) - \left(E_{\text{rot}}\big|_{J=1} - E_{\text{rot}}\big|_{J=0}\right)$$

$$hf_{\text{photon}} = hf_{\text{vib}} - hf_{\text{rot}}$$

Thus, $f_{\text{photon}} = f_{\text{vib}} - f_{\text{rot}} = 6.42 \times 10^{13}\ \text{Hz} - 1.15 \times 10^{11}\ \text{Hz} = \boxed{6.41 \times 10^{13}\ \text{Hz}}$

***43.17** The moment of inertia about the molecular axis is $I_x = \frac{2}{5}mr^2 + \frac{2}{5}mr^2 = \frac{4}{5}m\left(2.00 \times 10^{-15}\ \text{m}\right)^2$

The moment of inertia about a perpendicular axis is $I_y = m\left(\frac{R}{2}\right)^2 + m\left(\frac{R}{2}\right)^2 = \frac{m}{2}\left(2.00 \times 10^{-10}\ \text{m}\right)^2$

The allowed rotational energies are $E_{\text{rot}} = \left(\hbar^2/2I\right)J(J+1)$, so the energy of the first excited state is $E_1 = \hbar^2/I$. The ratio is therefore

$$\frac{E_{1,x}}{E_{1,y}} = \frac{\left(\hbar^2/I_x\right)}{\left(\hbar^2/I_y\right)} = \frac{I_y}{I_x} = \frac{\frac{1}{2}m\left(2.00 \times 10^{-10}\ \text{m}\right)^2}{\frac{4}{5}m\left(2.00 \times 10^{-15}\ \text{m}\right)} = \frac{5}{8}\left(10^5\right)^2 = \boxed{6.25 \times 10^9}$$

***43.18** Consider a cubical salt crystal of edge length 0.1 mm.

The number of atoms is $\left(\frac{10^{-4}\ \text{m}}{0.261 \times 10^{-9}\ \text{m}}\right)^3$ $\boxed{\sim 10^{17}}$

This number of salt crystals would have volume $\left(10^{-4}\ \text{m}\right)^3\left(\frac{10^{-4}\ \text{m}}{0.261 \times 10^{-9}\ \text{m}}\right)^3$ $\boxed{\sim 10^5\ \text{m}^3}$

If it is cubic, it has edge length 40 m.

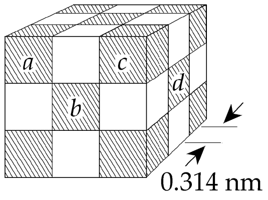
43.19 $$U = -\frac{\alpha k_e e^2}{r_0}\left(1 - \frac{1}{m}\right) = -(1.7476)(8.99 \times 10^9)\frac{(1.60 \times 10^{-19})^2}{(0.281 \times 10^{-9})}\left(1 - \frac{1}{8}\right) = -1.25 \times 10^{-18}\ \text{J} = \boxed{-7.84\ \text{eV}}$$

43.20 Visualize a K^+ ion at the center of each shaded cube, a Cl^- ion at the center of each white one.

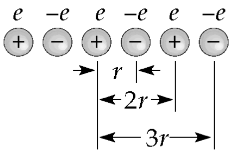
The distance *ab* is $\sqrt{2}(0.314\ \text{nm}) = \boxed{0.444\ \text{nm}}$

Distance *ac* is $2(0.314\ \text{nm}) = \boxed{0.628\ \text{nm}}$

Distance *ad* is $\sqrt{2^2 + (\sqrt{2})^2}(0.314\ \text{nm}) = \boxed{0.769\ \text{nm}}$



43.21 $$U = -\frac{k_e e^2}{r} - \frac{k_e e^2}{r} + \frac{k_e e^2}{2r} + \frac{k_e e^2}{2r} - \frac{k_e e^2}{3r} - \frac{k_e e^2}{3r} + \frac{k_e e^2}{4r} + \frac{k_e e^2}{4r} - \ . \ . \ . \ = -\frac{2k_e e^2}{r}\left(1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ . \ . \ . \right)$$



But, $\ln(1+x) = 1 - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ . \ . \ .$

so, $U = -\frac{2k_e e^2}{r}\ln 2$, or $\boxed{U = -k_e \alpha \frac{e^2}{r} \quad \text{where} \quad \alpha = 2\ln 2}$

43.22 $$E_{\text{F}} = \frac{h^2}{2m}\left(\frac{3n_e}{8\pi}\right)^{2/3} = \left[\frac{(6.625\times10^{-34}\ \text{J}\cdot\text{s})^2}{2(9.11\times10^{-31}\ \text{kg})(1.60\times10^{-19}\ \text{J / eV})}\right](3\ /\ 8\pi)^{2/3} n^{2/3}$$

$E_{\text{F}} = (3.65\times10^{-19})n^{2/3}$ eV with n measured in electrons / m^3

43.23 The density of conduction electrons n is given by $E_{\text{F}} = \frac{h^2}{2m}\left(\frac{3n_e}{8\pi}\right)^{2/3}$

or $$n_e = \frac{8\pi}{3}\left(\frac{2mE_{\text{F}}}{h^2}\right)^{3/2} = \frac{8\pi}{3}\frac{\left[2(9.11\times10^{-31}\ \text{kg})(5.48)(1.60\times10^{-19}\ \text{J})\right]^{3/2}}{(6.626\times10^{-34}\ \text{J}\cdot\text{s})^3} = 5.80\times10^{28}\ \text{m}^{-3}$$

The number-density of silver atoms is

$$n_{Ag} = \left(10.6\times10^3\ \text{kg / m}^3\right)\left(\frac{1\ \text{atom}}{108\ \text{u}}\right)\left(\frac{1\ \text{u}}{1.66\times10^{-27}\ \text{kg}}\right) = 5.91\times10^{28}\ \text{m}^{-3}$$

So an average atom contributes $\frac{5.80}{5.91} = \boxed{0.981 \text{ electron to the conduction band}}$

43.24 (a) $\frac{1}{2}mv^2 = 7.05$ eV

$$v = \sqrt{\frac{2(7.05\ \text{eV})(1.60\times10^{-19}\ \text{J / eV})}{9.11\times10^{-31}\ \text{kg}}} = \boxed{1.57\times10^6\ \text{m / s}}$$

(b) $\boxed{\text{Larger than } 10^{-4}\ \text{m / s by ten orders of magnitude.}}$ However, the energy of an electron at room temperature is typically $k_{\text{B}}T = \frac{1}{40}$ eV.

43.25 For sodium, $M = 23.0 \text{ g/mol}$ and $\rho = 0.971 \text{ g/cm}^3$.

(a) $$n_e = \frac{N_A \rho}{M} = \frac{\left(6.02 \times 10^{23} \text{ electrons/mol}\right)\left(0.971 \text{ g/cm}^3\right)}{23.0 \text{ g/mol}}$$

$$n_e = 2.54 \times 10^{22} \text{ electrons/cm}^3 = \boxed{2.54 \times 10^{28} \text{ electrons/m}^3}$$

(b) $$E_F = \left(\frac{h^2}{2m}\right)\left(\frac{3n_e}{8\pi}\right)^{2/3} = \frac{\left(6.626 \times 10^{-34} \text{ J}\cdot\text{s}\right)^2}{2\left(9.11 \times 10^{-31} \text{ kg}\right)}\left[\frac{3\left(2.54 \times 10^{28} \text{ m}^{-3}\right)}{8\pi}\right]^{2/3} = 5.05 \times 10^{-19} \text{ J} = \boxed{3.15 \text{ eV}}$$

(c) $$v_F = \sqrt{\frac{2E_F}{m}} = \sqrt{\frac{2\left(5.05 \times 10^{-19} \text{ J}\right)}{9.11 \times 10^{-31} \text{ kg}}} = \boxed{1.05 \times 10^6 \text{ m/s}}$$

***43.26** The melting point of silver is 1234 K. Its Fermi energy at 300 K is 5.48 eV. The approximate fraction of electrons excited is

$$\frac{k_B T}{E_F} = \frac{\left(1.38 \times 10^{-23} \text{ J/K}\right)(1234 \text{ K})}{(5.48 \text{ eV})\left(1.60 \times 10^{-19} \text{ J/eV}\right)} \approx \boxed{2\%}$$

43.27 Taking $E_F = 5.48 \text{ eV}$ for sodium at 800 K,

$$f = \left[e^{(E - E_F)/k_B T} + 1\right]^{-1} = 0.950$$

$$e^{(E - E_F)/k_B T} = (1 / 0.950) - 1 = 0.0526$$

$$\frac{E - E_F}{k_B T} = \ln(0.0526) = -2.94$$

$$E - E_F = -2.94 \frac{(1.38 \times 10^{-23})(800) \text{J}}{1.60 \times 10^{-19} \text{ J / eV}} = -0.203 \text{ eV} \qquad \text{or} \qquad \boxed{E = 5.28 \text{ eV}}$$

Goal Solution

Calculate the energy of a conduction electron in silver at 800 K if the probability of finding an electron in that state is 0.950. The Fermi energy is 5.48 eV at this temperature.

G: Since there is a 95% probability of finding the electron in this state, its energy should be slightly less than the Fermi energy, as indicated by the graph in Figure 43.21.

O: The electron energy can be found from the Fermi-Dirac distribution function.

A: Taking $E_F = 5.48$ eV for silver at 800 K, and given $f(E) = 0.950$, we find

$$f(E) = \frac{1}{e^{(E-E_F)/k_BT}+1} = 0.950 \qquad \text{or} \qquad e^{(E-E_F)/k_BT} = \frac{1}{0.950} - 1 = 0.05263$$

$$\frac{E-E_F}{k_BT} = \ln(0.05263) = -2.944 \qquad \text{so} \qquad E - E_F = -2.944k_BT = -2.944\left(1.38\times10^{-23}\text{ J / K}\right)(800\text{ K})$$

$$E = E_F - 3.25\times10^{-20}\text{ J} = 5.48\text{ eV} - 0.203\text{ eV} = 5.28\text{ eV}$$

L: As expected, the energy of the electron is slightly less than the Fermi energy, which is about 5 eV for most metals. There is very little probability of finding an electron significantly above the Fermi energy in a metal.

43.28 $d = 1.00$ mm, so $V = \left(1.00\times10^{-3}\text{ m}\right)^3 = 1.00\times10^{-9}\text{ m}^3$

The **density of states** is $g(E) = CE^{1/2} = \dfrac{8\sqrt{2}\,\pi m^{3/2}}{h^3}E^{1/2}$

or

$$g(E) = \frac{8\sqrt{2}\,\pi\left(9.11\times10^{-31}\text{ kg}\right)^{3/2}}{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)^3}\sqrt{(4.00\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$$

$$g(E) = 8.50\times10^{46}\text{ m}^{-3}\cdot\text{J}^{-1} = 1.36\times10^{28}\text{ m}^{-3}\cdot\text{ eV}^{-1}$$

So, the total number of electrons is

$$N = [g(E)](\Delta E)V = \left(1.36\times10^{28}\text{ m}^{-3}\cdot\text{ eV}^{-1}\right)(0.0250\text{ eV})\left(1.00\times10^{-9}\text{ m}^3\right) = \boxed{3.40\times10^{17}\text{ electrons}}$$

43.29 $E_{\text{av}} = \dfrac{1}{n_e}\displaystyle\int_0^\infty EN(E)\,dE$

At $T = 0$, $N(E) = 0$ for $E > E_{\text{F}}$;

Since $f(E) = 1$ for $E < E_{\text{F}}$ and $f(E) = 0$ for $E > E_{\text{F}}$, we can take $N(E) = CE^{1/2}$

$$E_{\text{av}} = \frac{1}{n_e}\int_0^{E_{\text{F}}} CE^{3/2}\,dE = \frac{C}{n_e}\int_0^{E_{\text{F}}} E^{3/2}\,dE = \frac{2C}{5n_e}E^{5/2}$$

But from Equation 43.24, $\dfrac{C}{n_e} = \dfrac{3}{2}E_{\text{F}}{}^{-3/2}$, so that $E_{\text{av}} = \left(\dfrac{2}{5}\right)\left(\dfrac{3}{2}E_{\text{F}}{}^{-3/2}\right)E_{\text{F}}{}^{5/2} = \boxed{\dfrac{3}{5}E_{\text{F}}}$

43.30 Consider first the wave function in x. At $x = 0$ and $x = L$, $\psi = 0$.

Therefore, $\sin k_xL = 0$ and $k_xL = \pi, 2\pi, 3\pi, \ldots$

Similarly, $\sin k_yL = 0$ and $k_yL = \pi, 2\pi, 3\pi, \ldots$

$\sin k_zL = 0$ and $k_zL = \pi, 2\pi, 3\pi, \ldots$

$$\psi = A\sin\left(\frac{n_x\pi x}{L}\right)\sin\left(\frac{n_y\pi y}{L}\right)\sin\left(\frac{n_z\pi z}{L}\right)$$

From $\dfrac{d^2\psi}{dx^2} + \dfrac{d^2\psi}{dy^2} + \dfrac{d^2\psi}{dz^2} = \dfrac{2m_e}{h^2}(U - E)\psi$, we have inside the box, where $U = 0$,

$$\left(-\frac{n_x^2\pi^2}{L^2} - \frac{n_y^2\pi^2}{L^2} - \frac{n_z^2\pi^2}{L^2}\right)\psi = \frac{2m_e}{h^2}(-E)\psi$$

$$\boxed{E = \frac{h^2\pi^2}{2m_eL^2}(n_x^2 + n_y^2 + n_z^2) \qquad n_x, n_y, n_z = 1, 2, 3, \ldots}$$

Outside the box we require $\psi = 0$.

The minimum energy state inside the box is $n_x = n_y = n_z = 1$, with $E = \dfrac{3h^2\pi^2}{2m_eL^2}$

43.31 (a) The density of states at energy E is $g(E) = CE^{1/2}$

Hence, the required ratio is $\dfrac{g(8.50\text{ eV})}{g(7.00\text{ eV})} = \dfrac{C(8.50)^{1/2}}{C(7.00)^{1/2}} = \boxed{1.10}$

(b) From Eq. 43.22, the number of occupied states having energy E is $N(E) = \dfrac{CE^{1/2}}{e^{(E-E_{\text{F}})/k_{\text{B}}T} + 1}$

Hence, the required ratio is $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{e^{(7.00-7.00)/k_{\text{B}}T} + 1}{e^{(8.50-7.00)/k_{\text{B}}T} + 1}\right]$

At $T = 300\text{ K}$, $k_{\text{B}}T = 4.14 \times 10^{-21}\text{ J} = 0.0259\text{ eV}$, $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \dfrac{(8.50)^{1/2}}{(7.00)^{1/2}}\left[\dfrac{2.00}{e^{(1.50)/0.0259} + 1}\right]$

And $\dfrac{N(8.50\text{ eV})}{N(7.00\text{ eV})} = \boxed{1.55 \times 10^{-25}}$

Comparing this result with that from part (a), we conclude that very few states with $E > E_{\text{F}}$ are occupied.

43.32 (a) $E_g = 1.14\text{ eV}$ for Si

$hf = 1.14\text{ eV} = (1.14\text{ eV})(1.60 \times 10^{-19}\text{ J/eV}) = 1.82 \times 10^{-19}\text{ J}$ so $f \geq \boxed{2.75 \times 10^{14}\text{ Hz}}$

(b) $c = \lambda f$; $\lambda = \dfrac{c}{f} = \dfrac{3.00 \times 10^8\text{ m/s}}{2.75 \times 10^{14}\text{ Hz}} = 1.09 \times 10^{-6}\text{ m} = \boxed{1.09\ \mu\text{m}}$ (in the infrared region)

43.33 Photons of energy greater than 2.42 eV will be absorbed. This means wavelength shorter than

$$\lambda = \frac{hc}{E} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{2.42\times1.60\times10^{-19}\text{ J}} = 514\text{ nm}$$

All the hydrogen Balmer lines except for the red line at 656 nm will be absorbed.

43.34 $$E_g = \frac{hc}{\lambda} = \frac{(6.626\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m / s})}{650\times10^{-9}\text{ m}}\text{ J} \approx \boxed{1.91\text{ eV}}$$

43.35 If $\lambda \le 1.00\times10^{-6}$ m, then photons of sunlight have energy

$$E \ge \frac{hc}{\lambda_{\max}} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{1.00\times10^{-6}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 1.24\text{ eV}$$

Thus, the energy gap for the collector material should be $\boxed{E_g \le 1.24\text{ eV}}$. Since Si has an energy gap $E_g \approx 1.14$ eV, it will absorb radiation of this energy and greater. Therefore, $\boxed{\text{Si is acceptable}}$ as a material for a solar collector.

Goal Solution

Most solar radiation has a wavelength of 1 μm or less. What energy gap should the material in a solar cell have in order to absorb this radiation? Is silicon appropriate (see Table 43.5)?

G: Since most photovoltaic solar cells are made of silicon, this semiconductor seems to be an appropriate material for these devices.

O: To absorb the longest-wavelength photons, the energy gap should be no larger than the photon energy.

A: The minimum photon energy is

$$hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m / s}\right)}{10^{-6}\text{ m}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right) = 1.24\text{ eV}$$

Therefore, the energy gap in the absorbing material should be smaller than 1.24 eV.

L: So silicon, with gap of 1.14 eV < 1.24 eV, is an appropriate material for absorbing solar radiation.

***43.36** If the photon energy is 5.5 eV or higher, the diamond window will absorb. Here,

$$(hf)_{\max} = \frac{hc}{\lambda_{\min}} = 5.50\text{ eV}: \qquad \lambda_{\min} = \frac{hc}{5.5\text{ eV}} = \frac{\left(6.626\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{(5.5\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}$$

$$\lambda_{\min} = 2.26\times10^{-7}\text{ m} = \boxed{226\text{ nm}}$$

43.37 $I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right)$ Thus, $e^{e(\Delta V)/k_BT} = 1 + I/I_0$

and $$\Delta V = \frac{k_BT}{e}\ln\left(1 + I/I_0\right)$$

At $T = 300\text{ K}$, $$\Delta V = \frac{\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})}{1.60\times10^{-19}\text{ C}}\ln\left(1+\frac{I}{I_0}\right) = (25.9\text{ mV})\ln\left(1+\frac{I}{I_0}\right)$$

(a) If $I = 9.00I_0$, $\Delta V = (25.9\text{ mV})\ln(10.0) = \boxed{59.5\text{ mV}}$

(b) If $I = -0.900I_0$, $\Delta V = (25.9\text{ mV})\ln(0.100) = \boxed{-59.5\text{ mV}}$

The basic idea behind a semiconductor device is that a large current or charge can be controlled by a small control voltage.

43.38 The voltage across the diode is about 0.6 V. The voltage drop across the resistor is $(0.025\text{ A})(150\ \Omega) = 3.75\text{ V}$. Thus, $\mathcal{E} - 0.6\text{ V} - 3.8\text{ V} = 0$ and $\mathcal{E} = \boxed{4.4\text{ V}}$

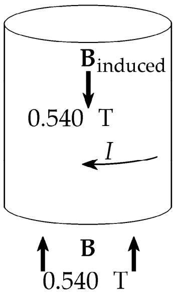
***43.39** First, we evaluate I_0 in $I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right)$, given that $I = 200\text{ mA}$ when $\Delta V = 100\text{ mV}$ and $T = 300\text{ K}$.

$$\frac{e(\Delta V)}{k_BT} = \frac{\left(1.60\times10^{-19}\text{ C}\right)(0.100\text{ V})}{\left(1.38\times10^{-23}\text{ J/K}\right)(300\text{ K})} = 3.86 \text{ so} \qquad I_0 = \frac{I}{e^{e(\Delta V)/k_BT} - 1} = \frac{200\text{ mA}}{e^{3.86} - 1} = 4.28\text{ mA}$$

If $\Delta V = -100\text{ mV}$, $\dfrac{e(\Delta V)}{k_BT} = -3.86$; and the current will be

$$I = I_0\left(e^{e(\Delta V)/k_BT} - 1\right) = (4.28\text{ mA})\left(e^{-3.86} - 1\right) = \boxed{-4.19\text{ mA}}$$

43.40 (a) $\boxed{\text{See the figure at right.}}$



(b) For a surface current around the outside of the cylinder as shown,

$$B = \frac{N\mu_0 I}{\ell} \qquad \text{or} \qquad NI = \frac{B\ell}{\mu_0} = \frac{(0.540\text{ T})(2.50\times10^{-2}\text{ m})}{(4\pi\times10^{-7})\text{ T}\cdot\text{m / A}} = \boxed{10.7\text{ kA}}$$

43.41 By Faraday's law (Equation 32.1), $\dfrac{\Delta\Phi_B}{\Delta t} = L\dfrac{\Delta I}{\Delta t} = A\dfrac{\Delta B}{\Delta t}$.

Thus, $$\Delta I = \frac{A(\Delta B)}{L} = \frac{\pi(0.0100\text{ m})^2(0.0200\text{ T})}{3.10\times10^{-8}\text{ H}} = \boxed{203\text{ A}}$$

The direction of the induced current is such as to maintain the B–field through the ring.

Goal Solution

Determine the current generated in a superconducting ring of niobium metal 2.00 cm in diameter if a 0.0200-T magnetic field in a direction perpendicular to the ring is suddenly decreased to zero. The inductance of the ring is 3.10×10^{-8} H.

G: The resistance of a superconductor is zero, so the current is limited only by the change in magnetic flux and self-inductance. Therefore, unusually large currents (greater than 100 A) are possible.

O: The change in magnetic field through the ring will induce an emf according to Faraday's law of induction. Since we do not know how fast the magnetic field is changing, we must use the ring's inductance and the geometry of the ring to calculate the magnetic flux, which can then be used to find the current.

A: From Faraday's law (Eq. 31.1), we have

$$|\mathcal{E}| = \frac{\Delta\Phi_B}{\Delta t} = A\frac{\Delta B}{\Delta t} = L\frac{\Delta I}{\Delta t} \quad \text{or} \quad \Delta I = \frac{A\Delta B}{L} = \frac{\pi(0.0100\text{ m})^2(0.0200\text{ T})}{3.10\times10^{-8}\text{ H}} = 203\text{ A}$$

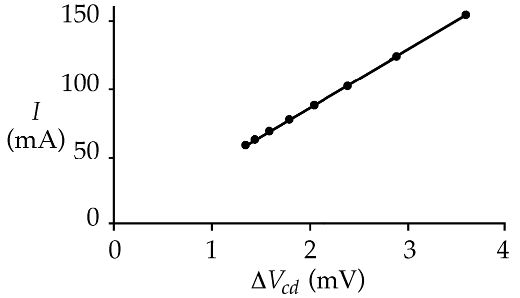
The current is directed so as to produce its own magnetic field in the direction of the original field.

L: This induced current should remain constant as long as the ring is superconducting. If the ring failed to be a superconductor (e.g. if it warmed above the critical temperature), the metal would have a non-zero resistance, and the current would quickly drop to zero. It is interesting to note that we were able to calculate the current in the ring without knowing the emf. In order to calculate the emf, we would need to know how quickly the magnetic field goes to zero.

43.42 (a) $\Delta V = IR$

If $R = 0$, then $\Delta V = 0$, even when $I \neq 0$.

(b) The graph shows a direct proportionality.



$$\text{Slope} = \frac{1}{R} = \frac{\Delta I}{\Delta V} = \frac{(155-57.8)\text{ mA}}{(3.61-1.356)\text{ mV}} = 43.1\ \Omega^{-1}$$

$$R = \boxed{0.0232\ \Omega}$$

(c) Expulsion of magnetic flux and therefore fewer current-carrying paths could explain the decrease in current.

***43.43** (a) Since the interatomic potential is the same for both molecules, the spring constant is the same.

Then $f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}}$ where $\mu_{12} = \frac{(12\text{ u})(16\text{ u})}{12\text{ u} + 16\text{ u}} = 6.86\text{ u}$ and $\mu_{14} = \frac{(14\text{ u})(16\text{ u})}{14\text{ u} + 16\text{ u}} = 7.47\text{ u}$

Therefore,

$$f_{14} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{14}}} = \frac{1}{2\pi}\sqrt{\frac{k}{\mu_{12}}\left(\frac{\mu_{12}}{\mu_{14}}\right)} = f_{12}\sqrt{\frac{\mu_{12}}{\mu_{14}}} = \left(6.42\times10^{13}\text{ Hz}\right)\sqrt{\frac{6.86\text{ u}}{7.47\text{ u}}} = \boxed{6.15\times10^{13}\text{ Hz}}$$

(b) The equilibrium distance is the same for both molecules.

$$I_{14} = \mu_{14}r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)\mu_{12}r^2 = \left(\frac{\mu_{14}}{\mu_{12}}\right)I_{12}$$

$$I_{14} = \left(\frac{7.47\text{ u}}{6.86\text{ u}}\right)\left(1.46\times10^{-46}\text{ kg}\cdot\text{m}^2\right) = \boxed{1.59\times10^{-46}\text{ kg}\cdot\text{m}^2}$$

(c) The molecule can move to the $(v = 1, J = 9)$ state or to the $(v = 1, J = 11)$ state. The energy it can absorb is either

$$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\tfrac{1}{2}\right)hf_{14} + 9(9+1)\frac{\mathrm{h}^2}{2I_{14}}\right] - \left[\left(0+\tfrac{1}{2}\right)hf_{14} + 10(10+1)\frac{\mathrm{h}^2}{2I_{14}}\right],$$

or $$\Delta E = \frac{hc}{\lambda} = \left[\left(1+\tfrac{1}{2}\right)hf_{14} + 11(11+1)\frac{\mathrm{h}^2}{2I_{14}}\right] - \left[\left(0+\tfrac{1}{2}\right)hf_{14} + 10(10+1)\frac{\mathrm{h}^2}{2I_{14}}\right].$$

The wavelengths it can absorb are then

$$\lambda = \frac{c}{f_{14} - 10\mathrm{h}/(2\pi I_{14})} \quad\text{or}\quad \lambda = \frac{c}{f_{14} + 11\mathrm{h}/(2\pi I_{14})}$$

These are: $$\lambda = \frac{3.00\times10^{8}\text{ m/s}}{6.15\times10^{13}\text{ Hz} - \dfrac{10\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)}{2\pi\left(1.59\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}} = \boxed{4.96\ \mu\text{m}}$$

and $$\lambda = \frac{3.00\times10^{8}\text{ m/s}}{6.15\times10^{13}\text{ Hz} + \dfrac{11\left(1.055\times10^{-34}\text{ J}\cdot\text{s}\right)}{2\pi\left(1.59\times10^{-46}\text{ kg}\cdot\text{m}^2\right)}} = \boxed{4.79\ \mu\text{m}}$$

43.44 For the N_2 molecule, $k = 2297 \text{ N / m}$, $m = 2.32\times10^{-26} \text{ kg}$, $r = 1.20\times10^{-10} \text{ m}$, $\mu = m / 2$

$\omega = \sqrt{k/\mu} = 4.45\times10^{14} \text{ rad / s}$, $I = \mu r^2 = (1.16\times10^{-26} \text{ kg})(1.20\times10^{-10} \text{ m})^2 = 1.67\times10^{-46} \text{ kg}\cdot\text{m}^2$

For a rotational state sufficient to allow a transition to the first exited vibrational state,

$$\frac{\hbar^2}{2I}J(J+1) = \hbar\omega \qquad \text{so} \qquad J(J+1) = \frac{2I\omega}{\hbar} = \frac{2(1.67\times10^{-46})(4.45\times10^{14})}{1.055\times10^{-34}} = 1410$$

Thus $\boxed{J = 37}$

43.45 $\Delta E_{\max} = 4.5 \text{ eV} = \left(v+\frac{1}{2}\right)\hbar\omega$ so $\dfrac{(4.5 \text{ eV})(1.6\times10^{-19} \text{ J / eV})}{(1.055\times10^{-34} \text{ J}\cdot\text{s})(8.28\times10^{14} \text{ s}^{-1})} \ge \left(v+\frac{1}{2}\right)$

$8.25 > 7.5$ $\boxed{v = 7}$

43.46 With 4 van der Waal bonds per atom pair or 2 electrons per atom, the total energy of the solid is

$$E = 2(1.74\times10^{-23} \text{ J / atom})\left(\frac{6.02\times10^{23} \text{ atoms}}{4.00 \text{ g}}\right) = \boxed{5.23 \text{ J/g}}$$

43.47 The total potential energy is given by Equation 43.16: $U_{\text{total}} = -\alpha\dfrac{k_e e^2}{r} + \dfrac{B}{r^m}$

The total potential energy has its minimum value U_0 at the equilibrium spacing, $r = r_0$. At this point, $dU/dr\big|_{r=r_0} = 0$,

or $$\left.\frac{dU}{dr}\right|_{r=r_0} = \frac{d}{dr}\left.\left(-\alpha\frac{k_e e^2}{r} + \frac{B}{r^m}\right)\right|_{r=r_0} = \alpha\frac{k_e e^2}{r_0^2} - \frac{mB}{r_0^{m+1}} = 0$$

Thus, $$B = \alpha\frac{k_e e^2}{m}r_0^{m-1}$$

Substituting this value of B into U_{total}, $$U_0 = -\alpha\frac{k_e e^2}{r_0} + \alpha\frac{k_e e^2}{m}r_0^{m-1}\left(\frac{1}{r_0^m}\right) = \boxed{-\alpha\frac{k_e e^2}{r_0}\left(1-\frac{1}{m}\right)}$$

***43.48** Suppose it is a harmonic-oscillator potential well. Then, $\frac{1}{2}hf + 4.48 \text{ eV} = \frac{3}{2}hf + 3.96 \text{ eV}$ is the depth of the well below the dissociation point. We see $hf = 0.520 \text{ eV}$, so the depth of the well is

$$\tfrac{1}{2}hf + 4.48 \text{ eV} = \tfrac{1}{2}(0.520 \text{ eV}) + 4.48 \text{ eV} = \boxed{4.74 \text{ eV}}$$

***43.49** (a) For equilibrium, $\frac{dU}{dx}=0$: $\frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at $3Ax_0^{-2}=B$.

$$x_0=\sqrt{\frac{3A}{B}}=\sqrt{\frac{3\left(0.150\text{ eV}\cdot\text{nm}^3\right)}{3.68\text{ eV}\cdot\text{nm}}}=\boxed{0.350\text{ nm}}$$

(b) The depth of the well is given by $U_0=U|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}$

$$U_0=U|_{x=x_0}=-\frac{2B^{3/2}}{3^{3/2}A^{1/2}}=-\frac{2(3.68\text{ eV}\cdot\text{nm})^{3/2}}{3^{3/2}\left(0.150\text{ eV}\cdot\text{nm}^3\right)^{1/2}}=\boxed{-7.02\text{ eV}}$$

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite x_m such that $\left.\frac{dF_x}{dx}\right|_{x=x_m}=0$

Thus, $\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_0}=0$ so that $x_m=\left(\frac{6A}{B}\right)^{1/2}$

Then $F_{\text{max}}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=-\frac{B^2}{12A}=-\frac{(3.68\text{ eV}\cdot\text{nm})^2}{12\left(0.150\text{ eV}\cdot\text{nm}^3\right)}$

or $F_{\text{max}}=-7.52\,\frac{\text{eV}}{\text{nm}}\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)=-1.20\times10^{-9}\text{ N}=\boxed{-1.20\text{ nN}}$

43.50 (a) For equilibrium, $\frac{dU}{dx}=0$: $\frac{d}{dx}\left(Ax^{-3}-Bx^{-1}\right)=-3Ax^{-4}+Bx^{-2}=0$

$x\to\infty$ describes one equilibrium position, but the stable equilibrium position is at

$3Ax_0^{-2}=B$ or $\boxed{x_0=\sqrt{3A/B}}$

(b) The depth of the well is given by $U_0=U|_{x=x_0}=\frac{A}{x_0^3}-\frac{B}{x_0}=\frac{AB^{3/2}}{3^{3/2}A^{3/2}}-\frac{BB^{1/2}}{3^{1/2}A^{1/2}}=\boxed{-2\sqrt{\frac{B^3}{27A}}}$

(c) $F_x=-\frac{dU}{dx}=3Ax^{-4}-Bx^{-2}$

To find the maximum force, we determine finite x_m such that

$\left.\frac{dF_x}{dx}\right|_{x=x_m}=\left[-12Ax^{-5}+2Bx^{-3}\right]_{x=x_0}=0$ then $F_{\text{max}}=3A\left(\frac{B}{6A}\right)^2-B\left(\frac{B}{6A}\right)=\boxed{-\frac{B^2}{12A}}$

***43.51** (a) At equilibrium separation, $r = r_e$, $$\left.\frac{dU}{dr}\right|_{r=r_e} = -2aB\left[e^{-a\left(r_e - r_0\right)} - 1\right]e^{-a\left(r_e - r_0\right)} = 0$$

We have neutral equilibrium as $r_e \to \infty$ and stable equilibrium at $$e^{-a\left(r_e - r_0\right)} = 1,$$

or $$r_e = \boxed{r_0}$$

(b) At $r = r_0$, $U = 0$. As $r \to \infty$, $U \to B$. The depth of the well is $\boxed{B}$.

(c) We expand the potential in a Taylor series about the equilibrium point:

$$U(r) \approx U\left(r_0\right) + \left.\frac{dU}{dr}\right|_{r=r_0}\left(r - r_0\right) + \frac{1}{2}\left.\frac{d^2U}{dr^2}\right|_{r=r_0}\left(r - r_0\right)^2$$

$$U(r) \approx 0 + 0 + \tfrac{1}{2}(-2Ba)\left[-ae^{-2\left(r-r_0\right)} - ae^{-\left(r-r_0\right)}\left(e^{-2\left(r-r_0\right)} - 1\right)\right]_{r=r_0}\left(r - r_0\right)^2 \approx Ba^2\left(r - r_0\right)^2$$

This is of the form $$\tfrac{1}{2}kx^2 = \tfrac{1}{2}k\left(r - r_0\right)^2$$

for a simple harmonic oscillator with $$k = 2Ba^2$$

Then the molecule vibrates with frequency $$f = \frac{1}{2\pi}\sqrt{\frac{k}{\mu}} = \frac{a}{2\pi}\sqrt{\frac{2B}{\mu}} = \boxed{\frac{a}{\pi}\sqrt{\frac{B}{2\mu}}}$$

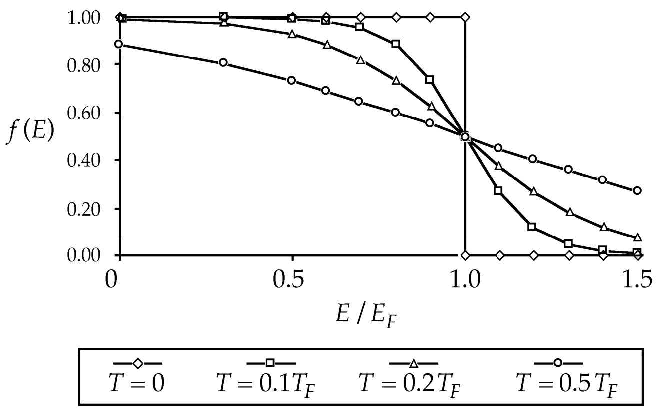
(d) The zero-point energy is $$\tfrac{1}{2}\hbar\omega = \tfrac{1}{2}hf = \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}$$

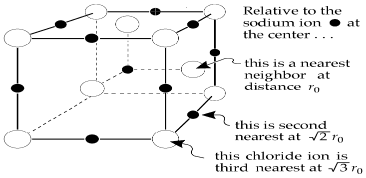
Therefore, to dissociate the molecule in its ground state requires energy $$\boxed{B - \frac{ha}{\pi}\sqrt{\frac{B}{8\mu}}}$$

43.52

	$T=0$		$T=0.1T_F$		$T=0.2T_F$		$T=0.5T_F$	
E/E_F	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$	$e^{\left(\frac{E}{E_F}-1\right)\frac{T_F}{T}}$	$f(E)$
0	$e^{-\infty}$	1.00	$e^{-10.0}$	1.000	$e^{-5.00}$	0.993	$e^{-2.00}$	0.881
0.500	$e^{-\infty}$	1.00	$e^{-5.00}$	0.993	$e^{-2.50}$	0.924	$e^{-1.00}$	0.731
0.600	$e^{-\infty}$	1.00	$e^{-4.00}$	0.982	$e^{-2.00}$	0.881	$e^{-0.800}$	0.690
0.700	$e^{-\infty}$	1.00	$e^{-3.00}$	0.953	$e^{-1.50}$	0.818	$e^{-0.600}$	0.646
0.800	$e^{-\infty}$	1.00	$e^{-2.00}$	0.881	$e^{-1.00}$	0.731	$e^{-0.400}$	0.599
0.900	$e^{-\infty}$	1.00	$e^{-1.00}$	0.731	$e^{-0.500}$	0.622	$e^{-0.200}$	0.550
1.00	e^{0}	0.500	e^{0}	0.500	e^{0}	0.500	e^{0}	0.500
1.10	$e^{+\infty}$	0.00	$e^{1.00}$	0.269	$e^{0.500}$	0.378	$e^{0.200}$	0.450
1.20	$e^{+\infty}$	0.00	$e^{2.00}$	0.119	$e^{1.00}$	0.269	$e^{0.400}$	0.401
1.30	$e^{+\infty}$	0.00	$e^{3.00}$	0.0474	$e^{1.50}$	0.182	$e^{0.600}$	0.354
1.40	$e^{+\infty}$	0.00	$e^{4.00}$	0.0180	$e^{2.00}$	0.119	$e^{0.800}$	0.310
1.50	$e^{+\infty}$	0.00	$e^{5.00}$	0.00669	$e^{2.50}$	0.0759	$e^{1.00}$	0.269

Fermi − Dirac Distribution Function





43.53 (a) There are 6 Cl^- ions at distance $r = r_0$. The contribution of these ions to the electrostatic potential energy is $-6k_e e^2/r_0$.

There are 12 Na^+ ions at distance $r = \sqrt{2}\,r_0$. Their contribution to the electrostatic potential energy is $+12k_e e^2/\sqrt{2}\,r_0$. Next, there are 8 Cl^- ions at distance $r = \sqrt{3}\,r_0$. These contribute a term of $-8k_e e^2/\sqrt{3}\,r_0$ to the electrostatic potential energy.

To three terms, the electrostatic potential energy is:

$$U = \left(-6 + \frac{12}{\sqrt{2}} - \frac{8}{\sqrt{3}}\right)\frac{k_e e^2}{r_0} = -2.13\,\frac{k_e e^2}{r_0} \quad \text{or} \quad \boxed{U = -\alpha\frac{k_e e^2}{r_0} \text{ with } \alpha = 2.13}$$

(b) The fourth term consists of 6 Na^+ at distance $r = 2r_0$. Thus, to four terms,

$$U = (-2.13 + 3)\frac{k_e e^2}{r_0} = 0.866\frac{k_e e^2}{r_0}$$

So we see that the electrostatic potential energy is not even attractive to 4 terms, and that the infinite series $\boxed{\text{does not converge rapidly}}$ when groups of atoms corresponding to nearest neighbors, next-nearest neighbors, etc. are added together.