Chapter 45 Solutions

***45.1** $\Delta m = (m_n + M_{\text{U}}) - (M_{\text{Zr}} + M_{\text{Te}} + 3m_n)$

$\Delta m = (1.008\ 665\ \text{u} + 235.043\ 924\ \text{u}) - (97.912\ 0\ \text{u} + 134.908\ 7\ \text{u} + 3(1.008\ 665\ \text{u}))$

$\Delta m = 0.205\ 89\ \text{u} = 3.418 \times 10^{-28}\ \text{kg}$ so $Q = \Delta mc^2 = 3.076 \times 10^{-11}\ \text{J} = \boxed{192\ \text{MeV}}$

45.2 Three different fission reactions are possible: ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{144}_{54}\text{Xe} + 2\,{}^{1}_{0}\text{n}$ $\boxed{{}^{144}_{54}\text{Xe}}$

${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{143}_{54}\text{Xe} + 3\,{}^{1}_{0}\text{n}$ $\boxed{{}^{143}_{54}\text{Xe}}$ ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{90}_{38}\text{Sr} + {}^{142}_{54}\text{Xe} + 4\,{}^{1}_{0}\text{n}$ $\boxed{{}^{142}_{54}\text{Xe}}$

45.3 ${}^{1}_{0}\text{n} + {}^{232}_{90}\text{Th} \rightarrow {}^{233}_{90}\text{Th} \rightarrow {}^{233}_{91}\text{Pa} + e^- + \bar{\nu}$ ${}^{233}_{91}\text{Pa} \rightarrow {}^{233}_{92}\text{U} + e^- + \bar{\nu}$

45.4 ${}^{1}_{0}\text{n} + {}^{238}_{92}\text{U} \rightarrow {}^{239}_{92}\text{U} \rightarrow {}^{239}_{93}\text{Np} + e^- + \bar{\nu}$ ${}^{239}_{93}\text{Np} \rightarrow {}^{239}_{94}\text{Pu} + e^- + \bar{\nu}$

45.5 (a) $Q = (\Delta m)c^2 = \left[m_n + M_{\text{U235}} - M_{\text{Ba141}} - M_{\text{Kr92}} - 3m_n\right]c^2$

$\Delta m = \left[(1.008\ 665 + 235.043\ 924) - (140.913\ 9 + 91.897\ 3 + 3 \times 1.008\ 665)\right]\text{u} = 0.215\ 39\ \text{u}$

$Q = (0.215\ 39\ \text{u})(931.5\ \text{MeV/u}) = \boxed{201\ \text{MeV}}$

(b) $f = \dfrac{\Delta m}{m_i} = \dfrac{0.215\ 39\ \text{u}}{236.052\ 59\ \text{u}} = 9.13 \times 10^{-4} = \boxed{0.0913\%}$

45.6 If the electrical power output of 1000 MW is 40.0% of the power derived from fission reactions, the power output of the fission process is

$$\frac{1000\ \text{MW}}{0.400} = \left(2.50 \times 10^{9}\ \frac{\text{J}}{\text{s}}\right)\left(\frac{8.64 \times 10^{4}}{\text{d}}\right) = 2.16 \times 10^{14}\ \text{J/d}$$

The number of fissions per day is $\left(2.16 \times 10^{14}\ \frac{\text{J}}{\text{d}}\right)\left(\frac{1\ \text{fission}}{200 \times 10^{6}\ \text{eV}}\right)\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = 6.74 \times 10^{24}\ \text{d}^{-1}$

This also is the number of ${}^{235}\text{U}$ nuclei used, so the mass of ${}^{235}\text{U}$ used per day is

$$\left(6.74 \times 10^{24}\ \frac{\text{nuclei}}{\text{d}}\right)\left(\frac{235\ \text{g/mol}}{6.02 \times 10^{23}\ \text{nuclei/mol}}\right) = 2.63 \times 10^{3}\ \text{g/d} = \boxed{2.63\ \text{kg/d}}$$

In contrast, a coal-burning steam plant producing the same electrical power uses more than 6×10^{6} kg/d of coal.

45.7 The available energy to do work is 0.200 times the energy content of the fuel.

$$(1.00\text{ kg fuel})\left(\frac{0.0340\ {}^{235}\text{U}}{\text{fuel}}\right)\left(\frac{1000\text{ g}}{1\text{ kg}}\right)\left(\frac{1\text{ mol}}{235\text{ g}}\right)\left(\frac{6.02\times10^{23}}{\text{mol}}\right)\left(\frac{(208)(1.60\times10^{-13}\text{ J})}{\text{fission}}\right)$$

$$\left(2.90\times10^{12}\text{ J}\right)(0.200) = 5.80\times10^{11}\text{ J} = \left(1.00\times10^{5}\text{ N}\right)\cdot d$$

$$d = 5.80\times10^{6}\text{ m} = \boxed{5.80\text{ Mm}}$$

Goal Solution

Suppose enriched uranium containing 3.40% of the fissionable isotope ${}^{235}_{92}\text{U}$ is used as fuel for a ship. The water exerts an average frictional drag of 1.00×10^{5} N on the ship. How far can the ship travel per kilogram of fuel? Assume that the energy released per fission event is 208 MeV and that the ship's engine has an efficiency of 20.0%.

G: Nuclear fission is much more efficient for converting mass to energy than burning fossil fuels. However, without knowing the rate of diesel fuel consumption for a comparable ship, it is difficult to estimate the nuclear fuel rate. It seems plausible that a ship could cross the Atlantic ocean with only a few kilograms of nuclear fuel, so a reasonable range of uranium fuel consumption might be 10 km / kg to 10 000 km / kg.

O: The fuel consumption rate can be found from the energy released by the nuclear fuel and the work required to push the ship through the water.

A: One kg of enriched uranium contains 3.40% ${}^{235}_{92}\text{U}$ so $m_{235} = (1000\text{ g})(0.0340) = 34.0\text{ g}$

In terms of number of nuclei, this is equivalent to

$$N_{235} = (34.0\text{ g})\left(\frac{1}{235\text{ g / mol}}\right)\left(6.02\times10^{23}\text{ atoms / mol}\right) = 8.71\times10^{22}\text{ nuclei}$$

If all these nuclei fission, the thermal energy released is equal to

$$\left(8.71\times10^{22}\text{ nuclei}\right)\left(208\ \frac{\text{MeV}}{\text{nucleus}}\right)\left(1.602\times10^{-19}\text{ J/eV}\right) = 2.90\times10^{12}\text{ J}$$

Now, for the engine, $\text{efficiency} = \dfrac{\text{work output}}{\text{heat input}}$ or $e = \dfrac{fd\cos\theta}{Q_h}$

So the distance the ship can travel per kilogram of uranium fuel is

$$d = \frac{eQ_h}{f\cos(0)} = \frac{0.200\left(2.90\times10^{12}\text{ J}\right)}{1.00\times10^{5}\text{ N}} = 5.80\times10^{6}\text{ m}$$

L: The ship can travel 5 800 km / kg of uranium fuel, which is on the high end of our prediction range. The distance between New York and Paris is 5 851 km, so this ship could cross the Atlantic ocean on just one kilogram of uranium fuel.

45.8 (a) For a sphere: $V = \frac{4}{3}\pi r^3$ and $r = \left(\frac{3V}{4\pi}\right)^{1/3}$ so $\frac{A}{V} = \frac{4\pi r^2}{(4/3)\pi r^3} = \boxed{4.84V^{-1/3}}$

(b) For a cube: $V = \ell^3$ and $\ell = V^{1/3}$ so $\frac{A}{V} = \frac{6\ell^2}{\ell^3} = \boxed{6V^{-1/3}}$

(c) For a parallelepiped: $V = 2a^3$ and $a = \left(\frac{V}{2}\right)^{1/3}$ so $\frac{A}{V} = \frac{\left(2a^2 + 8a^2\right)}{2a^3} = \boxed{6.30V^{-1/3}}$

(d) Therefore, the $\boxed{\text{sphere has the least leakage}}$ and the $\boxed{\text{parallelepiped has the greatest leakage}}$ for a given volume.

45.9 mass of ^{235}U available $\approx (0.007)\left(10^9 \text{ metric tons}\right)\left(\frac{10^6 \text{ g}}{1 \text{ metric ton}}\right) = 7 \times 10^{12}$ g

number of nuclei $\sim \left(\frac{7 \times 10^{12} \text{ g}}{235 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \frac{\text{nuclei}}{\text{mol}}\right) = 1.8 \times 10^{34}$ nuclei

The energy available from fission (at 208 MeV/event) is

$$E \sim \left(1.8 \times 10^{34} \text{ events}\right)(208 \text{ MeV / event})\left(1.60 \times 10^{-13} \text{ J / MeV}\right) = 6.0 \times 10^{23} \text{ J}$$

This would last for a time of $t = \frac{E}{\mathcal{P}} \sim \frac{6.0 \times 10^{23} \text{ J}}{7.0 \times 10^{12} \text{ J/s}} = \left(8.6 \times 10^{10} \text{ s}\right)\left(\frac{1 \text{ yr}}{3.16 \times 10^7 \text{ s}}\right) \sim \boxed{3000 \text{ yr}}$

45.10 In one minute there are $\frac{60.0 \text{ s}}{1.20 \text{ ms}} = 5.00 \times 10^4$ fissions.

So the rate increases by a factor of $(1.000\ 25)^{50000} = \boxed{2.68 \times 10^5}$

45.11 $\mathcal{P} = 10.0$ MW $= 1.00 \times 10^7$ J/s

If each decay delivers 1.00 MeV $= 1.60 \times 10^{-13}$ J, then the number of decays/s $= \boxed{6.25 \times 10^{19} \text{ Bq}}$

45.12 (a) The Q value for the D-T reaction is 17.59 MeV.

Heat content in fuel for D-T reaction: $\dfrac{(17.59 \text{ MeV})(1.60\times10^{-13} \text{ J/MeV})}{(5 \text{ u})(1.66\times10^{-27} \text{ kg/u})} = 3.39\times10^{14} \text{ J/kg}$

$$r_{\text{DT}} = \frac{(3.00\times10^{9} \text{ J / s})(3600 \text{ s / hr})}{(3.39\times10^{14} \text{ J / kg})(10^{-3} \text{ kg / g})} = \boxed{31.9 \text{ g/h burning of D and T}}$$

(b) Heat content in fuel for D-D reaction: $Q = \frac{1}{2}(3.27 + 4.03) = 3.65$ MeV average of two Q values

$$\frac{(3.65 \text{ MeV})(1.60\times10^{-13} \text{ J/MeV})}{(4 \text{ u})(1.66\times10^{-27} \text{ kg/u})} = 8.80\times10^{13} \text{ J/kg}$$

$$r_{\text{DD}} = \frac{(3.00\times10^{9} \text{ J / s})(3600 \text{ s / hr})}{(8.80\times10^{12} \text{ J / kg})(10^{-3} \text{ kg / g})} = \boxed{122 \text{ g/h burning of D}}$$

45.13 (a) At closest approach, the electrostatic potential energy equals the total energy E.

$$U_f = \frac{k_e(Z_1e)(Z_2e)}{r_{\min}} = E: \qquad E = \frac{\left(8.99\times10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19} \text{ C}\right)^2 Z_1Z_2}{1.00\times10^{-14} \text{ m}} = \boxed{\left(2.30\times10^{-14} \text{ J}\right)Z_1Z_2}$$

(b) For both the D-D and the D-T reactions, $Z_1 = Z_2 = 1$. Thus, the minimum energy required in both cases is

$$E = \left(2.30\times10^{-14} \text{ J}\right)\left(\frac{1 \text{ MeV}}{1.60\times10^{-13} \text{ J}}\right) = \boxed{0.144 \text{ MeV}}$$

45.14 (a) $r_f = r_{\text{D}} + r_{\text{T}} = \left(1.20\times10^{-15} \text{ m}\right)\left[(2)^{1/3} + (3)^{1/3}\right] = \boxed{3.24\times10^{-15} \text{ m}}$

(b) $U_f = \dfrac{k_e e^2}{r_f} = \dfrac{\left(8.99\times10^{9} \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19} \text{ C}\right)^2}{3.24\times10^{-15} \text{ m}} = 7.10\times10^{-14} \text{ J} = \boxed{444 \text{ keV}}$

(c) Conserving momentum, $m_{\text{D}}v_i = (m_{\text{D}} + m_{\text{T}})v_f$, or $v_f = \left(\dfrac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right)v_i = \boxed{\tfrac{2}{5}v_i}$

(d) $K_i + U_i = K_f + U_f$: $\qquad K_i + 0 = \frac{1}{2}(m_{\text{D}} + m_{\text{T}})v_f^2 + U_f = \frac{1}{2}(m_{\text{D}} + m_{\text{T}})\left(\dfrac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right)^2 v_i^2 + U_f$

$$K_i + 0 = \left(\frac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right)\left(\tfrac{1}{2}m_{\text{D}}v_i^2\right) + U_f = \left(\frac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right)K_i + U_f$$

$\left(1 - \dfrac{m_{\text{D}}}{m_{\text{D}} + m_{\text{T}}}\right)K_i = U_f$: $\qquad K_i = U_f\left(\dfrac{m_{\text{D}} + m_{\text{T}}}{m_{\text{T}}}\right) = \frac{5}{3}(444 \text{ keV}) = \boxed{740 \text{ keV}}$

(e) $\boxed{\text{Possibly by tunneling.}}$

45.15 (a) Average KE per particle is $\frac{3}{2}k_BT = \frac{1}{2}mv^2$.

Therefore, $$v_{\text{rms}} = \sqrt{\frac{3k_BT}{m}} = \sqrt{\frac{3\left(1.38\times10^{-23}\text{ J/K}\right)\left(4.00\times10^{8}\text{ K}\right)}{2\left(1.67\times10^{-27}\text{ kg}\right)}} = \boxed{2.23\times10^{6}\text{ m/s}}$$

(b) $$t = \frac{x}{v} \sim \frac{0.1\text{ m}}{10^{6}\text{ m/s}} \quad \boxed{\sim 10^{-7}\text{ s}}$$

45.16 (a) $$V = \left(317\times10^{6}\text{ mi}^3\right)\left(\frac{1609\text{ m}}{1\text{ mi}}\right)^3 = 1.32\times10^{18}\text{ m}^3$$

$$m_{\text{water}} = \rho V = \left(10^{3}\text{ kg/m}^3\right)\left(1.32\times10^{18}\text{ m}^3\right) = 1.32\times10^{21}\text{ kg}$$

$$m_{H_2} = \left(\frac{M_{H_2}}{M_{H_2O}}\right)m_{H_2O} = \left(\frac{2.016}{18.015}\right)\left(1.32\times10^{21}\text{ kg}\right) = 1.48\times10^{20}\text{ kg}$$

$$m_{\text{Deuterium}} = (0.0300\%)\,m_{H_2} = \left(0.0300\times10^{-2}\right)\left(1.48\times10^{20}\text{ kg}\right) = 4.43\times10^{16}\text{ kg}$$

The number of deuterium nuclei in this mass is

$$N = \frac{m_{\text{Deuterium}}}{m_{\text{Deuteron}}} = \frac{4.43\times10^{16}\text{ kg}}{(2.014\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right)} = 1.33\times10^{43}$$

Since two deuterium nuclei are used per fusion, ${}^2_1\text{H} + {}^2_1\text{H} \rightarrow {}^4_2\text{He} + Q$, the number of events is $N/2 = 6.63\times10^{42}$.

The energy released per event is

$$Q = \left[M_{{}^2\text{H}} + M_{{}^2\text{H}} - M_{{}^4\text{He}}\right]c^2 = \left[2(2.014\,102) - 4.002\,602\right]\text{u}\,\left(931.5\text{ MeV/u}\right) = 23.8\text{ MeV}$$

The total energy available is then

$$E = \left(\frac{N}{2}\right)Q = \left(6.63\times10^{42}\right)(23.8\text{ MeV})\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = \boxed{2.52\times10^{31}\text{ J}}$$

(b) The time this energy could possibly meet world requirements is

$$t = \frac{E}{\mathcal{P}} = \frac{2.52\times10^{31}\text{ J}}{100\left(7.00\times10^{12}\text{ J/s}\right)} = \left(3.61\times10^{16}\text{ s}\right)\left(\frac{1\text{ yr}}{3.16\times10^{7}\text{ s}}\right) = \boxed{1.14\times10^{9}\text{ yr}} \sim 1\text{ billion years.}$$

45.17 (a) Including both ions and electrons, the number of particles in the plasma is $N = 2nV$ where n is the ion density and V is the volume of the container. Application of Equation 21.6 gives the total energy as

$$E = \tfrac{3}{2} N k_B T = 3nVk_B T = 3\left(2.0\times10^{13}\ \text{cm}^{-3}\right)\left[\left(50\ \text{m}^3\right)\left(\frac{10^6\ \text{cm}^3}{1\ \text{m}^3}\right)\right]\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.0\times10^8\ \text{K}\right)$$

$$E = \boxed{1.7\times10^7\ \text{J}}$$

(b) From Table 20.2, the heat of vaporization of water is $L_v = 2.26\times10^6\ \text{J/kg}$. The mass of water that could be boiled away is

$$m = \frac{E}{L_v} = \frac{1.7\times10^7\ \text{J}}{2.26\times10^6\ \text{J/kg}} = \boxed{7.3\ \text{kg}}$$

45.18 (a) Lawson's criterion for the D-T reaction is $n\tau \ge 10^{14}\ \text{s/cm}^3$. For a confinement time of $\tau = 1.00$ s, this requires a minimum ion density of $n = \boxed{10^{14}\ \text{cm}^{-3}}$

(b) At the ignition temperature of $T = 4.5\times10^7$ K and the ion density found above, the plasma pressure is

$$P = 2nk_B T = 2\left[\left(10^{14}\ \text{cm}^{-3}\right)\left(\frac{10^6\text{cm}^3}{1\ \text{m}^3}\right)\right]\left(1.38\times10^{-23}\ \text{J/K}\right)\left(4.5\times10^7\ \text{K}\right) = \boxed{1.24\times10^5\ \text{J/m}^3}$$

(c) The required magnetic energy density is then

$$u_B = \frac{B^2}{2\mu_0} \ge 10P = 10\left(1.24\times10^5\ \text{J/m}^3\right) = 1.24\times10^6\ \text{J/m}^3,$$

$$B \ge \sqrt{2\left(4\pi\times10^{-7}\ \text{N/A}^2\right)\left(1.24\times10^6\ \text{J/m}^3\right)} = \boxed{1.77\ \text{T}}$$

45.19 Let the number of ^{6}Li atoms, each having mass 6.015 u, be N_6 while the number of ^{7}Li atoms, each with mass 7.016 u, is N_7.

Then, $N_6 = 7.50\%$ of $N_{\text{total}} = 0.0750\left(N_6 + N_7\right)$, or $N_7 = \left(\dfrac{0.925}{0.0750}\right)N_6$

Also, total mass $= \left[N_6(6.015\ \text{u}) + N_7(7.016\ \text{u})\right]\left(1.66\times10^{-27}\ \text{kg/u}\right) = 2.00$ kg,

or $$N_6\left[(6.015\ \text{u}) + \left(\frac{0.925}{0.0750}\right)(7.016\ \text{u})\right]\left(1.66\times10^{-27}\ \text{kg/u}\right) = 2.00\ \text{kg}.$$

This yields $N_6 = \boxed{1.30\times10^{25}}$ as the number of ^{6}Li atoms and

$$N_7 = \left(\frac{0.925}{0.0750}\right)\left(1.30\times10^{25}\right) = \boxed{1.61\times10^{26}}$$ as the number of ^{7}Li atoms.

45.20 The number of nuclei in 1.00 metric ton of trash is

$$N = 1000 \text{ kg}(1000 \text{ g / kg})\left(6.02\times10^{23} \text{ nuclei / mol}\right)\Big/(56.0 \text{ g / mol}) = 1.08\times10^{28} \text{ nuclei}$$

At an average charge of 26.0 e / nucleus, $q = \left(1.08\times10^{28}\right)(26.0)\left(1.60\times10^{-19}\right) = 4.47\times10^{10} \text{ C}$

Therefore $t = \dfrac{q}{I} = \dfrac{4.47\times10^{10}}{1.00\times10^{6}} = 4.47\times10^{4} \text{ s} = \boxed{12.4 \text{ h}}$

45.21 $N_0 = \dfrac{\text{mass present}}{\text{mass of nucleus}} = \dfrac{5.00 \text{ kg}}{(89.9077 \text{ u})\left(1.66\times10^{-27} \text{ kg/u}\right)} = 3.35\times10^{25} \text{ nuclei}$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{29.1 \text{ yr}} = 2.38\times10^{-2} \text{ yr}^{-1} = 4.52\times10^{-8} \text{ min}^{-1}$$

$$R_0 = \lambda N_0 = \left(4.52\times10^{-8} \text{ min}^{-1}\right)\left(3.35\times10^{25}\right) = 1.52\times10^{18} \text{ counts/min}$$

$\dfrac{R}{R_0} = e^{-\lambda t} = \dfrac{10.0 \text{ counts/min}}{1.52\times10^{18} \text{ counts/min}} = 6.60\times10^{-18}$ and $\lambda t = -\ln\left(6.60\times10^{-18}\right) = 39.6$

giving $t = \dfrac{39.6}{\lambda} = \dfrac{39.6}{2.38\times10^{-2} \text{ yr}^{-1}} = \boxed{1.66\times10^{3} \text{ yr}}$

45.22 Source: 100 mrad of 2-MeV γ-rays / h at a 1.00-m distance.

(a) For γ-rays, dose in rem = dose in rad.

Thus a person would have to stand $\boxed{10.0 \text{ hours}}$ to receive 1.00 rem from a 100-mrad / h source.

(b) If the γ-radiation is emitted isotropically, the dosage rate falls off as $1 / r^2$.

Thus a dosage 10.0 mrad / h would be received at a distance $r = \sqrt{10.0} \text{ m} = \boxed{3.16 \text{ m}}$.

45.23 (a) The number of x-rays taken per year is

$$n = (8 \text{ x-ray/d})(5 \text{ d/wk})(50 \text{ wk/yr}) = 2.0\times10^{3} \text{ x-ray/yr}$$

The average dose per photograph is $\dfrac{5.0 \text{ rem/yr}}{2.0\times10^{3} \text{ x-ray/yr}} = \boxed{2.5\times10^{-3} \text{ rem/x-ray}}$

(b) The technician receives low-level background radiation at a rate of 0.13 rem/yr. The dose of 5.0 rem/yr received as a result of the job is

$$\frac{5.0\ \text{rem/yr}}{0.13\ \text{rem/yr}} = \boxed{38 \text{ times background levels}}$$

45.24 (a) $I = I_0 e^{-\mu x}$, so $$x = \frac{1}{\mu}\ln\left(\frac{I_0}{I}\right)$$

With $\mu = 1.59\ \text{cm}^{-1}$, the thickness when $I = I_0/2$ is $$x = \frac{1}{1.59\ \text{cm}^{-1}}\ln(2) = \boxed{0.436\ \text{cm}}$$

(b) When $\dfrac{I_0}{I} = 1.00\times 10^4$, $$x = \frac{1}{1.59\ \text{cm}^{-1}}\ln\left(1.00\times 10^4\right) = \boxed{5.79\ \text{cm}}$$

45.25 $1\ \text{rad} = 10^{-2}\ \text{J / kg}$ $\quad Q = mc\Delta T \quad \mathcal{P}t = mc\Delta T$

$$t = \frac{mc\Delta T}{\mathcal{P}} = \frac{m(4186\ \text{J / kg}\cdot{}^\circ\text{C})(50.0\ {}^\circ\text{C})}{(10)\left(10^{-2}\ \text{J / kg}\cdot\text{s}\right)(m)} = \boxed{2.09\times 10^6\ \text{s}} \approx 24\ \text{days!}$$

Note that power is the product of dose rate and mass.

45.26 $$\frac{Q}{m} = \frac{\text{absorbed energy}}{\text{unit mass}} = (1000\ \text{rad})\,\frac{10^{-2}\ \text{J / kg}}{1\ \text{rad}} = 10.0\ \text{J / kg}$$

The rise in body temperature is calculated from $Q = mc\,\Delta T$ where $c = 4186\ \text{J/kg}\cdot{}^\circ\text{C}$ for water and the human body

$$\Delta T = \frac{Q}{mc} = (10.0\ \text{J / kg})\,\frac{1}{4186\ \text{J / kg}\cdot{}^\circ\text{C}} = \boxed{2.39\times 10^{-3}\ {}^\circ\text{C}} \quad \text{(Negligible)}$$

45.27 If half of the 0.140-MeV gamma rays are absorbed by the patient, the total energy absorbed is

$$E = \frac{(0.140\ \text{MeV})}{2}\left[\left(\frac{1.00\times 10^{-8}\ \text{g}}{98.9\ \text{g/mol}}\right)\left(\frac{6.02\times 10^{23}\ \text{nuclei}}{1\ \text{mol}}\right)\right] = 4.26\times 10^{12}\ \text{MeV}$$

$$E = \left(4.26\times 10^{12}\ \text{MeV}\right)\left(1.60\times 10^{-13}\ \text{J/MeV}\right) = 0.682\ \text{J}$$

Thus, the dose received is $$\text{Dose} = \frac{0.682\ \text{J}}{60.0\ \text{kg}}\left(\frac{1\ \text{rad}}{10^{-2}\ \text{J/kg}}\right) = \boxed{1.14\ \text{rad}}$$

45.28 The nuclei initially absorbed are $N_0 = \left(1.00\times10^{-9}\text{ g}\right)\left(\dfrac{6.02\times10^{23}\text{ nuclei/mol}}{89.9\text{ g/mol}}\right) = 6.70\times10^{12}$

The number of decays in time t is $\Delta N = N_0 - N = N_0\left(1-e^{-\lambda t}\right) = N_0\left(1-e^{-(\ln 2)t/T_{1/2}}\right)$

At the end of 1 year, $\dfrac{t}{T_{1/2}} = \dfrac{1.00\text{ yr}}{29.1\text{ yr}} = 0.0344$

and $\Delta N = N_0 - N = \left(6.70\times10^{12}\right)\left(1-e^{-0.0238}\right) = 1.58\times10^{11}$

The energy deposited is $E = \left(1.58\times10^{11}\right)(1.10\text{ MeV})\left(1.60\times10^{-13}\text{ J/MeV}\right) = 0.0277\text{ J}$

Thus, the dose received is $\text{Dose} = \left(\dfrac{0.0277\text{ J}}{70.0\text{ kg}}\right) = \boxed{3.96\times10^{-4}\text{ J/kg}} = 0.0396\text{ rad}$

45.29 (a) $$\frac{E}{E_\beta} = \frac{\frac{1}{2}C(\Delta V)^2}{0.500\text{ MeV}} = \frac{\frac{1}{2}\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)^2}{(0.500\text{ MeV})\left(1.60\times10^{-13}\text{ J / MeV}\right)} = \boxed{3.12\times10^{7}}$$

(b) $$N = \frac{Q}{e} = \frac{C(\Delta V)}{e} = \frac{\left(5.00\times10^{-12}\text{ F}\right)\left(1.00\times10^{3}\text{ V}\right)}{1.60\times10^{-19}\text{ C}} = \boxed{3.12\times10^{10}\text{ electrons}}$$

45.30 (a) $$\text{amplification} = \frac{\text{energy discharged}}{E} = \frac{\frac{1}{2}C(\Delta V)^2}{E} = \boxed{\frac{C(\Delta V)^2}{2E}}$$

(b) $$N = \frac{\text{charge released}}{\text{charge of electron}} = \boxed{\frac{C(\Delta V)}{e}}$$

45.31 (a) $E_I = 10.0\text{ eV}$ is the energy required to liberate an electron from a dynode. Let n_i be the number of electrons incident upon a dynode, each having gained energy $e(\Delta V)$ as it was accelerated to this dynode. The number of electrons that will be freed from this dynode is $N_i = n_i e(\Delta V)/E_I$:

At the first dynode, $n_i = 1$ and $N_1 = \dfrac{(1)e(100\text{ V})}{10.0\text{ eV}} = \boxed{10^1\text{ electrons}}$

(b) For the second dynode, $n_i = N_1 = 10^1$, so $N_2 = \dfrac{(10^1)e(100\text{ V})}{10.0\text{ eV}} = 10^2$.

At the third dynode, $n_i = N_2 = 10^2$ and $N_3 = \dfrac{(10^2)e(100\text{ V})}{10.0\text{ eV}} = 10^3$.

Observing the developing pattern, we see that the number of electrons incident on the seventh and last dynode is $n_7 = N_6 = \boxed{10^6}$.

(c) The number of electrons incident on the last dynode is $n_7 = 10^6$. The total energy these electrons deliver to that dynode is given by

$$E = n_i e(\Delta V) = 10^6\, e(700\text{ V} - 600\text{ V}) = \boxed{10^8\text{ eV}}$$

***45.32** (a) The average time between slams is $60\text{ min}/38 = 1.6\text{ min}$. Sometimes, the actual interval is nearly zero. Perhaps about equally as often, it is $2\times 1.6\text{ min}$. Perhaps about half as often, it is $4\times 1.6\text{ min}$. Somewhere around $5\times 1.6\text{ min} = \boxed{8.0\text{ min}}$, the chances of randomness producing so long a wait get slim, so such a long wait might likely be due to mischief.

(b) The midpoints of the time intervals are separated by 5.00 minutes. We use $R = R_0 e^{-\lambda t}$. Subtracting the background counts,

$$337 - 5(15) = [372 - 5(15)]e^{-(\ln 2/T_{1/2})(5.00\text{ min})}$$

or $$\ln\left(\frac{262}{297}\right) = \ln(0.882) = -3.47\text{ min}/T_{1/2} \quad \text{which yields } T_{1/2} = \boxed{27.6\text{ min}}\ .$$

(c) As in the random events in part (a), we imagine a ± 5 count counting uncertainty. The smallest likely value for the half-life is then given by

$$\ln\left(\frac{262-5}{297+5}\right) = -3.47\text{ min}/T_{1/2},\ \text{or}\ \left(T_{1/2}\right)_{\text{min}} = 21.1\text{ min}$$

The largest credible value is found from

$$\ln\left(\frac{262+5}{297-5}\right) = -3.47\text{ min}/T_{1/2},\ \text{yielding}\ \left(T_{1/2}\right)_{\text{max}} = 38.8\text{ min}$$

Thus, $T_{1/2} = \left(\dfrac{38.8+21.1}{2}\right) \pm \left(\dfrac{38.8-21.1}{2}\right)\text{ min} = (30\pm 9)\text{ min} = \boxed{30\text{ min} \pm 30\%}$

45.33 The initial specific activity of ^{59}Fe in the steel,

$$(R/m)_0 = \frac{20.0\ \mu\text{Ci}}{0.200\ \text{kg}} = \frac{100\ \mu\text{Ci}}{\text{kg}}\left(\frac{3.70\times10^4\ \text{Bq}}{1\ \mu\text{Ci}}\right) = 3.70\times10^6\ \text{Bq/kg}$$

After 1000 h, $$\frac{R}{m} = (R/m)_0 e^{-\lambda t} = (3.70\times10^6\ \text{Bq/kg})e^{-(6.40\times10^{-4}\ \text{h}^{-1})(1000\ \text{h})} = 1.95\times10^6\ \text{Bq/kg}$$

The activity of the oil, $$R_{\text{oil}} = \left(\frac{800}{60.0}\ \text{Bq/liter}\right)(6.50\ \text{liters}) = 86.7\ \text{Bq}$$

Therefore, $$m_{\text{in oil}} = \frac{R_{\text{oil}}}{(R/m)} = \frac{86.7\ \text{Bq}}{1.95\times10^6\ \text{Bq/kg}} = 4.45\times10^{-5}\ \text{kg}$$

So that wear rate is $$\frac{4.45\times10^{-5}\ \text{kg}}{1000\ \text{h}} = \boxed{4.45\times10^{-8}\ \text{kg/h}}$$

***45.34** The half-life of ^{14}O is 70.6 s, so the decay constant is $\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{70.6\ \text{s}} = 0.009\,82\ \text{s}^{-1}$

The ^{14}O nuclei remaining after five min is $N = N_0 e^{-\lambda t} = \left(10^{10}\right)e^{-\left(0.009\,82\ \text{s}^{-1}\right)(300\ \text{s})} = 5.26\times10^8$

The number of these in one cubic centimeter of blood is

$$N' = N\left(\frac{1.00\ \text{cm}^3}{\text{total vol. of blood}}\right) = \left(5.26\times10^8\right)\left(\frac{1.00\ \text{cm}^3}{2000\ \text{cm}^3}\right) = 2.63\times10^5$$

and their activity is $R = \lambda N' = \left(0.009\,82\ \text{s}^{-1}\right)\left(2.63\times10^5\right) = 2.58\times10^3\ \text{Bq}$ $\boxed{\sim 10^3\ \text{Bq}}$

***45.35** (a) The number of photons is $10^4\ \text{MeV}/1.04\ \text{MeV} = 9.62\times10^3$. Since only 50% of the photons are detected, the number of ^{65}Cu nuclei decaying is twice this value, or 1.92×10^4. In two half-lives, three-fourths of the original nuclei decay, so $\frac{3}{4}N_0 = 1.92\times10^4$ and $N_0 = 2.56\times10^4$. This is 1% of the ^{65}Cu, so the number of ^{65}Cu is 2.56×10^6 $\boxed{\sim 10^6}$.

(b) Natural copper is 69.17% ^{63}Cu and 30.83% ^{65}Cu. Thus, if the sample contains N_{Cu} copper atoms, the number of atoms of each isotope is
$N_{63} = 0.6917\,N_{\text{Cu}}$ and $N_{65} = 0.3083\,N_{\text{Cu}}$.

Therefore, $$\frac{N_{63}}{N_{65}} = \frac{0.6917}{0.3083} \quad\text{or}\quad N_{63} = \left(\frac{0.6917}{0.3083}\right)N_{65} = \left(\frac{0.6917}{0.3083}\right)\left(2.56\times10^6\right) = 5.75\times10^6$$

The total mass of copper present is then $m_{\text{Cu}} = (62.93\ \text{u})N_{63} + (64.93\ \text{u})N_{65}$:

$$m_{\text{Cu}} = \left[(62.93)\left(5.75\times10^6\right) + (64.93)\left(2.56\times10^6\right)\right]\text{u}\ \left(1.66\times10^{-24}\ \text{g/u}\right) = 8.77\times10^{-16}\ \text{g}\ \ \boxed{\sim 10^{-15}\ \text{g}}$$

45.36 (a) Starting with $N = 0$ radioactive atoms at $t = 0$, the rate of increase is (production – decay)

$$\frac{dN}{dt} = R - \lambda N \qquad \text{so} \qquad dN = (R - \lambda N)dt$$

The variables are separable. $\int_{N=0}^{N} \frac{dN}{R - \lambda N} = \int_{t=0}^{t} dt$

$$-\frac{1}{\lambda}\ln\left(\frac{R-\lambda N}{R}\right) = t \qquad \text{so} \qquad \ln\left(\frac{R-\lambda N}{R}\right) = -\lambda t$$

$$\left(\frac{R-\lambda N}{R}\right) = e^{-\lambda t} \qquad \text{and} \qquad 1 - \frac{\lambda}{R}N = e^{-\lambda t}$$

Therefore, $N = \boxed{\frac{R}{\lambda}\left(1 - e^{-\lambda t}\right)}$

(b) The maximum number of radioactive nuclei would be $\boxed{R/\lambda}$

45.37 (a) At 6×10^8 K, each carbon nucleus has thermal energy of

$$\tfrac{3}{2}k_B T = (1.5)(8.62 \times 10^{-5}\ \text{eV / K})(6 \times 10^8\ \text{K}) = \boxed{8 \times 10^4\ \text{eV}}$$

(b) The energy released is $E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Ne}^{20}\right) - m\left(\text{He}^{4}\right)\right]c^2$

$$E = (24.000\ 000 - 19.992\ 435 - 4.002\ 602)(931.5)\ \text{MeV} = \boxed{4.62\ \text{MeV}}$$

In the second reaction, $E = \left[2m\left(\text{C}^{12}\right) - m\left(\text{Mg}^{24}\right)\right](931.5)\text{MeV / u}$

$$E = (24.000\ 000 - 23.985\ 042)(931.5)\text{MeV} = \boxed{13.9\ \text{MeV}}$$

(c) The energy released is the energy of reaction of the # of carbon nuclei in a 2.00-kg sample, which corresponds to

$$\Delta E = \left(2.00 \times 10^3\ \text{g}\right)\left(\frac{6.02 \times 10^{23}\ \text{atoms / mol}}{12.0\ \text{g / mol}}\right)\left(\frac{4.62\ \text{MeV / fusion event}}{2\ \text{nuclei / fusion event}}\right)\left(\frac{1\ \text{kWh}}{2.25 \times 10^{19}\ \text{MeV}}\right)$$

$$\Delta E = \frac{\left(1.00 \times 10^{26}\right)(4.62)}{2\left(2.25 \times 10^{19}\right)}\ \text{kWh} = \boxed{1.03 \times 10^7\ \text{kWh}}$$

45.38 (a) Suppose each ^{235}U fission releases 208 MeV of energy. Then, the number of nuclei that must have undergone fission is

$$N = \frac{\text{total release}}{\text{energy per nuclei}} = \frac{5 \times 10^{13}\ \text{J}}{(208\ \text{MeV})\left(1.60 \times 10^{-13}\ \text{J/MeV}\right)} = \boxed{1.5 \times 10^{24}\ \text{nuclei}}$$

(b) $$\text{mass} = \left(\frac{1.5 \times 10^{24}\ \text{nuclei}}{6.02 \times 10^{23}\ \text{nuclei/mol}}\right)(235\ \text{g/mol}) \approx \boxed{0.6\ \text{kg}}$$

45.39 For a typical ^{235}U, $Q = 208$ MeV; and the initial mass is 235 u. Thus, the fractional energy loss is

$$\frac{Q}{mc^2} = \frac{208\ \text{MeV}}{(235\ \text{u})(931.5\ \text{MeV/u})} = 9.50 \times 10^{-4} = \boxed{0.0950\%}$$

For the D-T fusion reaction, $Q = 17.6$ MeV

The initial mass is $m = (2.014\ \text{u}) + (3.016\ \text{u}) = 5.03$ u

The fractional loss in this reaction is $$\frac{Q}{mc^2} = \frac{17.6\ \text{MeV}}{(5.03\ \text{u})(931.5\ \text{MeV/u})} = 3.75 \times 10^{-3} = \boxed{0.375\%}$$

$$\frac{0.375\%}{0.0950\%} = 3.95 \quad \text{or} \quad \boxed{\text{the fractional loss in D} - \text{T fusion is about 4 times that in } ^{235}\text{U fission}}$$

.

45.40 To conserve momentum, the two fragments must move in opposite directions with speeds v_1 and v_2 such that

$$m_1 v_1 = m_2 v_2 \qquad \text{or} \qquad v_2 = \left(\frac{m_1}{m_2}\right) v_1$$

The kinetic energies after the break-up are then

$$K_1 = \tfrac{1}{2} m_1 v_1^2 \qquad \text{and} \qquad K_2 = \tfrac{1}{2} m_2 v_2^2 = \tfrac{1}{2} m_2 \left(\frac{m_1}{m_2}\right)^2 v_1^2 = \left(\frac{m_1}{m_2}\right) K_1$$

The fraction of the total kinetic energy carried off by m_1 is $$\frac{K_1}{K_1 + K_2} = \frac{K_1}{K_1 + (m_1/m_2)K_1} = \boxed{\frac{m_2}{m_1 + m_2}}$$

and the fraction carried off by m_2 is $$1 - \frac{m_2}{m_1 + m_2} = \boxed{\frac{m_1}{m_1 + m_2}}$$

45.41 The decay constant is $\lambda = \dfrac{\ln 2}{T_{1/2}} = \dfrac{\ln 2}{(12.3\ \text{yr})(3.16\times 10^{7}\ \text{s/yr})} = 1.78\times 10^{-9}\ \text{s}^{-1}$

The tritium in the plasma decays at a rate of

$$R = \lambda N = \left(1.78\times 10^{-9}\ \text{s}^{-1}\right)\left[\left(\frac{2.00\times 10^{14}}{\text{cm}^3}\right)\left(\frac{10^{6}\ \text{cm}^3}{1\ \text{m}^3}\right)\left(50.0\ \text{m}^3\right)\right]$$

$$R = 1.78\times 10^{13}\ \text{Bq} = \left(1.78\times 10^{13}\ \text{Bq}\right)\left(\frac{1\ \text{Ci}}{3.70\times 10^{10}\ \text{Bq}}\right) = \boxed{482\ \text{Ci}}$$

$\boxed{\text{The fission inventory is } \dfrac{4\times 10^{10}\ \text{Ci}}{482\ \text{Ci}} \sim 10^{8} \text{ times greater}}$ than this amount.

Goal Solution

The half-life of tritium is 12.3 yr. If the TFTR fusion reactor contained $50.0\ \text{m}^3$ of tritium at a density equal to 2.00×10^{14} ions / cm^3, how many curies of tritium were in the plasma? Compare this value with a fission inventory (the estimated supply of fissionable material) of 4×10^{10} Ci.

G: It is difficult to estimate the activity of the tritium in the fusion reactor without actually calculating it; however, we might expect it to be a small fraction of the fission (not fusion) inventory.

O: The decay rate (activity) can be found by multiplying the decay constant λ by the number of ${}^{3}_{1}\text{H}$ particles. The decay constant can be found from the half-life of tritium, and the number of particles from the density and volume of the plasma.

A: The number of Hydrogen-3 nuclei is

$$N = \left(50.0\ \text{m}^3\right)\left(2.00\times 10^{14}\ \frac{\text{particles}}{\text{m}^3}\right)\left(100\ \frac{\text{cm}}{\text{m}}\right)^3 = 1.00\times 10^{22}\ \text{particles}$$

$$\lambda = \frac{\ln\ 2}{T_{1/2}} = \frac{0.693}{12.3\ \text{yr}}\left(\frac{1\ \text{yr}}{3.16\times 10^{7}\ \text{s}}\right) = 1.78\times 10^{-9}\ \text{s}^{-1}$$

The activity is then

$$R = \lambda N = \left(1.78\times 10^{-9}\ \text{s}^{-1}\right)\left(1.00\times 10^{22}\ \text{nuclei}\right) = 1.78\times 10^{13}\ \text{Bq} = \left(1.78\times 10^{13}\ \text{Bq}\right)\left(\frac{1\ \text{Ci}}{3.70\times 10^{10}\ \text{Bq}}\right) = 482\ \text{Ci}$$

L: Even though 482 Ci is a large amount of radioactivity, it is smaller than 4.00×10^{10} Ci by about a hundred million. Therefore, loss of containment is a smaller hazard for a fusion power reactor than for a fission reactor.

45.42 Momentum conservation: $0 = m_{\text{Li}}\mathbf{v}_{\text{Li}} + m_\alpha \mathbf{v}_\alpha$, or, $m_{\text{Li}}v_{\text{Li}} = m_\alpha v_\alpha$

Thus,
$$K_{\text{Li}} = \frac{1}{2}m_{\text{Li}}v_{\text{Li}}^2 = \frac{1}{2}\frac{(m_{\text{Li}}v_{\text{Li}})^2}{m_{\text{Li}}} = \frac{(m_\alpha v_\alpha)^2}{2m_{\text{Li}}} = \left(\frac{m_\alpha^2}{2m_{\text{Li}}}\right)v_\alpha^2$$

$$K_{\text{Li}} = \left(\frac{(4.002\,6\text{ u})^2}{2(7.016\,9\text{ u})}\right)(9.30\times10^6\text{ m/s})^2 = (1.14\text{ u})(9.30\times10^6\text{ m/s})^2$$

$$K_{\text{Li}} = 1.14(1.66\times10^{-27}\text{ kg})(9.30\times10^6\text{ m/s})^2 = 1.64\times10^{-13}\text{ J} = \boxed{1.02\text{ MeV}}$$

45.43 The complete fissioning of 1.00 gram of U^{235} releases

$$\Delta Q = \frac{(1.00\text{ g})}{235\text{ grams / mol}}\left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(200\ \frac{\text{MeV}}{\text{fission}}\right)\left(1.60\times10^{-13}\ \frac{\text{J}}{\text{MeV}}\right) = 8.20\times10^{10}\text{ J}$$

If all this energy could be utilized to convert m kilograms of 20.0°C water to 400°C steam (see Chapter 20 of text for values), then

$$\Delta Q = mc_w\,\Delta T + mL_v + mc_s\,\Delta T$$

$$\Delta Q = m\left[(4186\text{ J / kg}\cdot{}^\circ\text{C})(80.0\ {}^\circ\text{C}) + 2.26\times10^6\text{ J / kg} + (2010\text{ J / kg}\cdot{}^\circ\text{C})(300\ {}^\circ\text{C})\right]$$

Therefore $m = \dfrac{8.20\times10^{10}\text{ J}}{3.20\times10^6\text{ J / kg}} = \boxed{2.56\times10^4\text{ kg}}$

45.44 When mass m of ^{235}U undergoes complete fission, releasing 200 MeV per fission event, the total energy released is:

$$Q = \left(\frac{m}{235\text{ g/mol}}\right)N_{\text{A}}(200\text{ MeV})$$ where N_{A} is Avogadro's number.

If all this energy could be utilized to convert a mass m_w of liquid water at T_c into steam at T_h, then,

$$Q = m_w\left[c_w(100^\circ\text{C} - T_c) + L_v + c_s(T_h - 100^\circ\text{C})\right]$$

where c_w is the specific heat of liquid water, L_v is the latent heat of vaporization, and c_s is the specific heat of steam. Solving for the mass of water converted gives

$$m_w = \frac{Q}{\left[c_w(100^\circ\text{C} - T_c) + L_v + c_s(T_h - 100^\circ\text{C})\right]} = \boxed{\frac{mN_{\text{A}}(200\text{ MeV})}{(235\text{ g/mol})\left[c_w(100^\circ\text{C} - T_c) + L_v + c_s(T_h - 100^\circ\text{C})\right]}}$$

45.45 (a) The number of molecules in 1.00 liter of water (mass = 1000 g) is

$$N = \left(\frac{1.00 \times 10^3\ \text{g}}{18.0\ \text{g/mol}}\right)\left(6.02 \times 10^{23}\ \text{molecules/mol}\right) = 3.34 \times 10^{25}\ \text{molecules}$$

The number of deuterium nuclei contained in these molecules is

$$N' = \left(3.34 \times 10^{25}\ \text{molecules}\right)\left(\frac{1\ \text{deuteron}}{3300\ \text{molecules}}\right) = 1.01 \times 10^{22}\ \text{deuterons}$$

Since 2 deuterons are consumed per fusion event, the number of events possible is $N'/2 = 5.07 \times 10^{21}$ reactions, and the energy released is

$$E_{\text{fusion}} = \left(5.07 \times 10^{21}\ \text{reactions}\right)\left(3.27\ \text{MeV/reaction}\right) = 1.66 \times 10^{22}\ \text{MeV}$$

$$E_{\text{fusion}} = \left(1.66 \times 10^{22}\ \text{MeV}\right)\left(1.60 \times 10^{-13}\ \text{J/MeV}\right) = \boxed{2.65 \times 10^{9}\ \text{J}}$$

(b) In comparison to burning 1.00 liter of gasoline, the energy from the fusion of deuterium is

$$\frac{E_{\text{fusion}}}{E_{\text{gasoline}}} = \frac{2.65 \times 10^{9}\ \text{J}}{3.40 \times 10^{7}\ \text{J}} = \boxed{78.0\ \text{times larger}}\ .$$

45.46 The number of nuclei in 0.155 kg of ^{210}Po is

$$N_0 = \left(\frac{155\ \text{g}}{209.98\ \text{g/mol}}\right)\left(6.02 \times 10^{23}\ \text{nuclei/g}\right) = 4.44 \times 10^{23}\ \text{nuclei}$$

The half-life of ^{210}Po is 138.38 days, so the decay constant is given by

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{(138.38\ \text{d})\left(8.64 \times 10^{4}\ \text{s/d}\right)} = 5.80 \times 10^{-8}\ \text{s}^{-1}$$

The initial activity is $R_0 = \lambda N_0 = \left(5.80 \times 10^{-8}\ \text{s}^{-1}\right)\left(4.44 \times 10^{23}\ \text{nuclei}\right) = 2.58 \times 10^{16}\ \text{Bq}$

The energy released in each ${}^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + {}^{4}_{2}\text{He}$ reaction is $Q = \left[M_{{}^{210}_{84}\text{Po}} - M_{{}^{206}_{82}\text{Pb}} - M_{{}^{4}_{2}\text{He}}\right]c^2$:

$$Q = \left[209.982\,848 - 205.974\,440 - 4.002\,602\right]\text{u}\left(931.5\ \frac{\text{MeV}}{\text{u}}\right) = 5.41\ \text{MeV}$$

Thus, assuming a conversion efficiency of 1.00%, the initial power output of the battery is

$$\mathcal{P} = (0.0100)R_0Q = (0.0100)\left(2.58 \times 10^{16}\ \frac{\text{decays}}{\text{s}}\right)\left(5.41\ \frac{\text{MeV}}{\text{decay}}\right)\left(1.60 \times 10^{-13}\ \frac{\text{J}}{\text{MeV}}\right) = \boxed{223\ \text{W}}$$

45.47 (a) The thermal power transferred to the water is $\mathcal{P}_w = 0.970(\text{waste heat})$

$$\mathcal{P}_w = 0.970(3065 - 1000)\,\text{MW} = 2.00 \times 10^9 \text{ J/s}$$

r_w is the mass of heated per hour: $r_w = \dfrac{\mathcal{P}_w}{c(\Delta T)} = \dfrac{(2.00 \times 10^9 \text{ J/s})(3600 \text{ s/h})}{(4186 \text{ J/kg}\cdot{}^\circ\text{C})(3.50\ {}^\circ\text{C})} = \boxed{4.91 \times 10^8 \text{ kg/h}}$

The volume used per hour is $\dfrac{4.91 \times 10^8 \text{ kg/h}}{1.00 \times 10^3 \text{ kg/m}^3} = \boxed{4.91 \times 10^5 \text{ m}^3\text{/h}}$

(b) The ^{235}U fuel is consumed at a rate $r_f = \left(\dfrac{3065 \times 10^6 \text{ J/s}}{7.80 \times 10^{10} \text{ J/g}}\right)\left(\dfrac{1 \text{ kg}}{1000 \text{ g}}\right)\left(\dfrac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{0.141 \text{ kg/h}}$

***45.48** (a) $\Delta V = 4\pi r^2(\Delta r) = 4\pi(14.0 \times 10^3 \text{ m})^2(0.05 \text{ m}) = 1.23 \times 10^8 \text{ m}^3$ $\boxed{\sim 10^8 \text{ m}^3}$

(b) The force on the next layer is determined by atmospheric pressure.

$$W = P(\Delta V) = \left(1.013 \times 10^5 \ \frac{\text{N}}{\text{m}^2}\right)(1.23 \times 10^8 \text{ m}^3) = 1.25 \times 10^{13} \text{ J} \quad \boxed{\sim 10^{13} \text{ J}}$$

(c) $1.25 \times 10^{13} \text{ J} = \frac{1}{10}(\text{yield})$, so $\text{yield} = 1.25 \times 10^{14} \text{ J}$ $\boxed{\sim 10^{14} \text{ J}}$

(d) $\dfrac{1.25 \times 10^{14} \text{ J}}{4.2 \times 10^9 \text{ J/ton TNT}} = 2.97 \times 10^4 \text{ ton TNT} \ \sim 10^4 \text{ ton TNT}$ or $\boxed{\sim 10 \text{ kilotons}}$

***45.49** (a) $V = \ell^3 = \dfrac{m}{\rho}$, so $\ell = \left(\dfrac{m}{\rho}\right)^{1/3} = \left(\dfrac{70.0 \text{ kg}}{18.7 \times 10^3 \text{ kg/m}^3}\right)^{1/3} = \boxed{0.155 \text{ m}}$

(b) Add 92 electrons to both sides of the given nuclear reaction. Then it becomes ${}^{238}_{92}\text{U atom} \to 8\,{}^{4}_{2}\text{He atom} + {}^{206}_{82}\text{Pb atom} + Q_{\text{net}}$.

$$Q_{\text{net}} = \left[M_{{}^{238}_{92}\text{U}} - 8M_{{}^{4}_{2}\text{He}} - M_{{}^{206}_{82}\text{Pb}}\right]c^2 = [238.050\,784 - 8(4.002\,602) - 205.974\,440]\,\text{u}\ (931.5 \text{ MeV/u})$$

$$Q_{\text{net}} = \boxed{51.7 \text{ MeV}}$$

(c) If there is a single step of decay, the number of decays per time is the decay rate R and the energy released in each decay is Q. Then the energy released per time is $\boxed{\mathcal{P} = QR}$. If there is a series of decays in steady state, the equation is still true, with Q representing the net decay energy.

(d) The decay rate for all steps in the radioactive series in steady state is set by the parent uranium:

$$N = \left(\frac{7.00 \times 10^4 \text{ g}}{238 \text{ g/mol}}\right)\left(6.02 \times 10^{23} \text{ nuclei/mol}\right) = 1.77 \times 10^{26} \text{ nuclei}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{4.47 \times 10^9 \text{ yr}} = 1.55 \times 10^{-10} \frac{1}{\text{yr}}$$

$$R = \lambda N = \left(1.55 \times 10^{-10} \frac{1}{\text{yr}}\right)\left(1.77 \times 10^{26} \text{ nuclei}\right) = 2.75 \times 10^{16} \text{ decays/yr},$$

so $\mathcal{P} = QR = (51.7 \text{ MeV})\left(2.75 \times 10^{16} \frac{1}{\text{yr}}\right)\left(1.60 \times 10^{-13} \text{ J/MeV}\right) = \boxed{2.27 \times 10^5 \text{ J/yr}}$

(e) dose in rem = dose in rad x RBE

$$5.00 \frac{\text{rem}}{\text{yr}} = \left(\text{dose in } \frac{\text{rad}}{\text{yr}}\right) 1.10, \text{ giving } \left(\text{dose in } \frac{\text{rad}}{\text{yr}}\right) = 4.55 \frac{\text{rad}}{\text{yr}}$$

The allowed whole-body dose is then $(70.0 \text{ kg})\left(4.55 \frac{\text{rad}}{\text{yr}}\right)\left(\frac{10^{-2} \text{ J/kg}}{1 \text{ rad}}\right) = \boxed{3.18 \text{ J/yr}}$

45.50 $E_T \equiv E(\text{thermal}) = \frac{3}{2} k_B T = 0.039 \text{ eV}$

$E_T = \left(\frac{1}{2}\right)^n E$ where $n \equiv$ number of collisions, and $0.039 = \left(\frac{1}{2}\right)^n \left(2.0 \times 10^6\right)$

Therefore, $n = 25.6 = \boxed{26 \text{ collisions}}$

45.51 From conservation of energy: $K_\alpha + K_n = Q$ or $\frac{1}{2} m_\alpha v_\alpha{}^2 + \frac{1}{2} m_n v_n{}^2 = 17.6 \text{ MeV}$

Conservation of momentum: $m_\alpha v_\alpha = m_n v_n$ or $v_\alpha = \left(\frac{m_n}{m_\alpha}\right) v_n.$

The energy equation becomes: $\frac{1}{2} m_\alpha \left(\frac{m_n}{m_\alpha}\right)^2 v_n^2 + \frac{1}{2} m_n v_n^2 = \left(\frac{m_n + m_\alpha}{m_\alpha}\right)\left(\frac{1}{2} m_n v_n^2\right) = 17.6 \text{ MeV}$

Thus, $K_n = \left(\frac{m_\alpha}{m_n + m_\alpha}\right)(17.6 \text{ MeV}) = \left(\frac{4.002\ 602}{1.008\ 665 + 4.002\ 602}\right) = \boxed{14.1 \text{ MeV}}$

Goal Solution

Assuming that a deuteron and a triton are at rest when they fuse according to

$$^2\text{H} + {}^3\text{H} \to {}^4\text{He} + \text{n} + 17.6\ \text{MeV},$$

determine the kinetic energy acquired by the neutron.

G: The products of this nuclear reaction are an alpha particle and a neutron, with total kinetic energy of 17.6 MeV. In order to conserve momentum, the lighter neutron will have a larger velocity than the more massive alpha particle (which consists of two protons and two neutrons). Since the kinetic energy of the particles is proportional to the square of their velocities but only linearly proportional to their mass, the neutron should have the larger kinetic energy, somewhere between 8.8 and 17.6 MeV.

O: Conservation of linear momentum and energy can be applied to find the kinetic energy of the neutron. We first suppose the particles are moving nonrelativistically.

A: The momentum of the alpha particle and that of the neutron must add to zero, so their velocities must be in opposite directions with magnitudes related by

$$m_n\mathbf{v}_n + m_\alpha\mathbf{v}_\alpha = 0 \qquad \text{or} \qquad (1.0087\ \text{u})v_n = (4.0026\ \text{u})v_\alpha$$

At the same time, their kinetic energies must add to 17.6 MeV

$$E = \tfrac{1}{2}m_n v_n^2 + \tfrac{1}{2}m_\alpha v_\alpha^2 = \tfrac{1}{2}(1.0087\ \text{u})v_n^2 + \tfrac{1}{2}(4.0026\ \text{u})v_\alpha^2 = 17.6\ \text{MeV}$$

Substitute $v_\alpha = 0.2520v_n$: $\quad E = (0.50435\ \text{u})v_n^2 + (0.12710\ \text{u})v_n^2 = 17.6\ \text{MeV}\left(\dfrac{1\ \text{u}}{931.494\ \text{MeV}/c^2}\right)$

$$v_n = \sqrt{\frac{0.0189c^2}{0.63145}} = 0.173c = 5.19\times 10^7\ \text{m/s}$$

Since this speed is not too much greater than $0.1c$, we can get a reasonable estimate of the kinetic energy of the neutron from the classical equation,

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.0087\ \text{u})(0.173c)\left(\frac{931.494\ \text{MeV}/c^2}{\text{u}}\right) = 14.1\ \text{MeV}$$

L: The kinetic energy of the neutron is within the range we predicted. For a more accurate calculation of the kinetic energy, we should use relativistic expressions. Conservation of momentum gives

$$\gamma_n m_n\mathbf{v}_n + \gamma_\alpha m_\alpha\mathbf{v}_\alpha = 0 \qquad 1.0087\frac{v_n}{\sqrt{1 - v_n^2/c^2}} = 4.0026\frac{v_\alpha}{\sqrt{1 - v_\alpha^2/c^2}}$$

yielding $$\frac{v_\alpha^2}{c^2} = \frac{v_n^2}{15.746c^2 - 14.746v_n^2}$$

Then $$(\gamma_n - 1)m_nc^2 + (\gamma_\alpha - 1)m_\alpha c^2 = 17.6\ \text{MeV}$$

and $v_n = 0.171c$, implying that $(\gamma_n - 1)m_nc^2 = 14.0\ \text{MeV}$

45.52 From Table A.3, the half-life of ^{32}P is 14.26 d. Thus, the decay constant is

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{\ln 2}{14.26\text{ d}} = 0.0486\text{ d}^{-1} = 5.63\times10^{-7}\text{ s}^{-1}.$$

$$N_0 = \frac{R_0}{\lambda} = \frac{5.22\times10^{6}\text{ decay/s}}{5.63\times10^{-7}\text{ s}^{-1}} = 9.28\times10^{12}\text{ nuclei}$$

At $t = 10.0$ days, the number remaining is

$$N = N_0 e^{-\lambda t} = \left(9.28\times10^{12}\text{ nuclei}\right)e^{-\left(0.0486\text{ d}^{-1}\right)\left(10.0\text{ d}\right)} = 5.71\times10^{12}\text{ nuclei}$$

so the number of decays has been $N_0 - N = 3.57\times10^{12}$ and the energy released is

$$E = \left(3.57\times10^{12}\right)(700\text{ keV})\left(1.60\times10^{-16}\ \frac{\text{J}}{\text{keV}}\right) = 0.400\text{ J}$$

If this energy is absorbed by 100 g of tissue, the absorbed dose is

$$\text{Dose} = \left(\frac{0.400\text{ J}}{0.100\text{ kg}}\right)\left(\frac{1\text{ rad}}{10^{-2}\text{ J/kg}}\right) = \boxed{400\text{ rad}}$$

45.53 (a) The number of Pu nuclei in 1.00 kg = $\dfrac{6.02\times10^{23}\text{ nuclei/mol}}{239.05\text{ g/mol}}(1000\text{ g})$

The total energy = $(25.2\times10^{23}\text{ nuclei})(200\text{ MeV}) = 5.04\times10^{26}\text{ MeV}$

$E = (5.04\times10^{26}\text{ MeV})(4.44\times10^{-20}\text{ kWh/MeV}) = \boxed{2.24\times10^{7}\text{ kWh}}$ or 22 million kWh

(b) $E = \Delta mc^2 = (3.016\,049\text{ u} + 2.014\,102\text{ u} - 4.002\,602\text{ u} - 1.008\,665\text{ u})(931.5\text{ MeV/u})$

$E = \boxed{17.6\text{ MeV for each D-T fusion}}$

(c) E_n = (Total number of D nuclei)(17.6)(4.44×10^{-20})

$E_n = (6.02\times10^{23})(1000/2.014)(17.6)(4.44\times10^{-20}) = \boxed{2.34\times10^{8}\text{ kWh}}$

(d) E_n = the number of C atoms in 1.00 kg × 4.20 eV

$E_n = (6.02\times10^{26}/12.0)(4.20\times10^{-6}\text{ MeV})(4.44\times10^{-20}) = \boxed{9.36\text{ kWh}}$

(e) Coal is cheap at this moment in human history. We hope that safety and waste disposal problems can be solved so that nuclear energy can be affordable before scarcity drives up the price of fossil fuels.

***45.54** Add two electrons to both sides of the given reaction. Then $4\,{}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + Q$

where $Q = (\Delta m)c^2 = \left[4(1.007\ 825) - 4.002\ 602\right]\text{u}\ (931.5\ \text{MeV/u}) = 26.7\ \text{MeV}$

or $Q = (26.7\ \text{MeV})\left(1.60\times10^{-13}\ \text{J/MeV}\right) = 4.28\times10^{-12}\ \text{J}$

The proton fusion rate is then

$$\text{rate} = \frac{\text{power output}}{\text{energy per proton}} = \frac{3.77\times10^{26}\ \text{J/s}}{\left(4.28\times10^{-12}\ \text{J}\right)/(4\ \text{protons})} = \boxed{3.53\times10^{38}\ \text{protons/s}}$$

***45.55** (a) $Q_{\text{I}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}}\right]c^2$, and $Q_{\text{II}} = \left[M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} - M_{\text{C}} - M_{\text{E}} + M_{\text{C}} + M_{\text{D}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

$$Q_{\text{net}} = Q_{\text{I}} + Q_{\text{II}} = \left[M_{\text{A}} + M_{\text{B}} + M_{\text{D}} - M_{\text{E}} - M_{\text{F}} - M_{\text{G}}\right]c^2$$

Thus, reactions may be added. Any product like C used in a subsequent reaction does not contribute to the energy balance.

(b) Adding all five reactions gives ${}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} + {}^1_1\text{H} + {}^1_1\text{H} + {}^{\ 0}_{-1}\text{e} \rightarrow {}^4_2\text{He} + 2\nu + Q_{\text{net}}$

or $4\,{}^1_1\text{H} + 2\,{}^{\ 0}_{-1}\text{e} \rightarrow {}^4_2\text{He} + 2\nu + Q_{\text{net}}$

Adding two electrons to each side $4\,{}^1_1\text{H atom} \rightarrow {}^4_2\text{He atom} + Q_{\text{net}}$

Thus, $Q_{\text{net}} = \left[4M_{{}^1_1\text{H}} - M_{{}^4_2\text{He}}\right]c^2 = \left[4(1.007\ 825) - 4.002\ 602\right]\text{u}\ (931.5\ \text{MeV/u}) = \boxed{26.7\ \text{MeV}}$

45.56 (a) The mass of the pellet is $m = \rho V = \left(0.200\ \frac{\text{g}}{\text{cm}^3}\right)\left[\frac{4\pi}{3}\left(\frac{1.50\times10^{-2}\ \text{cm}}{2}\right)^3\right] = 3.53\times10^{-7}\ \text{g}$

The pellet consists of equal numbers of ${}^2\text{H}$ and ${}^3\text{H}$ atoms, so the average atomic weight is 2.50 and the total number of atoms is

$$N = \left(\frac{3.53\times10^{-7}\ \text{g}}{2.50\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right) = 8.51\times10^{16}\ \text{atoms}$$

When the pellet is vaporized, the plasma will consist of $2N$ particles (N nuclei and N electrons). The total energy delivered to the plasma is 1.00% of 200 kJ or 2.00 kJ. The temperature of the plasma is found from $E = (2N)\left(\frac{3}{2}k_{\text{B}}T\right)$ as

$$T = \frac{E}{3Nk_{\text{B}}} = \frac{2.00\times10^3\ \text{J}}{3\left(8.51\times10^{16}\right)\left(1.38\times10^{-23}\ \text{J/K}\right)} = \boxed{5.68\times10^8\ \text{K}}$$

(b) Each fusion event uses 2 nuclei, so $N/2$ events will occur. The energy released will be

$$E = \left(\frac{N}{2}\right)Q = \left(\frac{8.51\times10^{16}}{2}\right)(17.59\ \text{MeV})\left(1.60\times10^{-13}\ \frac{\text{J}}{\text{MeV}}\right) = 1.20\times10^5\ \text{J} = \boxed{120\ \text{kJ}}$$

***45.57** (a) The solar-core temperature of 15 MK gives particles enough kinetic energy to overcome the Coulomb-repulsion barrier to ${}^1_1\text{H} + {}^3_2\text{He} \to {}^4_2\text{He} + \text{e}^+ + \nu$, estimated as $k_e(e)(2e)/r$. The Coulomb barrier to Bethe's fifth and eight reactions is like $k_e(e)(7e)/r$, larger by $\frac{7}{2}$ times, so the temperature should be like $\frac{7}{2}\left(15\times10^6\ \text{K}\right) \approx \boxed{5\times10^7\ \text{K}}$.

(b) For ${}^{12}\text{C} + {}^1\text{H} \to {}^{13}\text{N} + Q$,

$$Q_1 = (12.000\,000 + 1.007\,825 - 13.005\,738)(931.5\ \text{MeV}) = \boxed{1.94\ \text{MeV}}$$

For the second step, add seven electrons to both sides to have: ${}^{13}\text{N atom} \to {}^{13}\text{C atom} + \text{e}^- + \text{e}^+ + Q$.

$$Q_2 = \left[13.005\,738 - 13.003\,355 - 2(0.000\,549)\right](931.5\ \text{MeV}) = \boxed{1.20\ \text{MeV}}$$

$$Q_3 = Q_7 = 2(0.000\,549)(931.5\ \text{MeV}) = \boxed{1.02\ \text{MeV}}$$

$$Q_4 = \left[13.003\,355 + 1.007\,825 - 14.003\,074\right](931.5\ \text{MeV}) = \boxed{7.55\ \text{MeV}}$$

$$Q_5 = \left[14.003\,074 + 1.007\,825 - 15.003\,065\right](931.5\ \text{MeV}) = \boxed{7.30\ \text{MeV}}$$

$$Q_6 = \left[15.003\,065 - 15.000\,108 - 2(0.000\,549)\right](931.5\ \text{MeV}) = \boxed{1.73\ \text{MeV}}$$

$$Q_8 = \left[15.000\,108 + 1.007\,825 - 12 - 4.002\,602\right](931.5\ \text{MeV}) = \boxed{4.97\ \text{MeV}}$$

The sum is $\boxed{26.7\ \text{MeV}}$, the same as for the proton-proton cycle.

(c) Not all of the energy released heats the star. When a neutrino is created, it will likely fly directly out of the star without interacting with any other particle.

45.58 (a) $\dfrac{I_2}{I_1} = \dfrac{I_0 e^{-\mu_2 x}}{I_0 e^{-\mu_1 x}} = \boxed{e^{-(\mu_2 - \mu_1)x}}$

(b) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40 - 41.0)(0.100)} = e^{3.56} = \boxed{35.2}$

(c) $\dfrac{I_{50}}{I_{100}} = e^{-(5.40 - 41.0)(1.00)} = e^{35.6} = \boxed{2.89\times10^{15}}$

Thus, a 1.00-cm aluminum plate has essentially removed the long-wavelength x-rays from the beam.