Chapter 1
Systems of Measurement

Conceptual Problems

***1 •**

Determine the Concept The fundamental physical quantities in the SI system include mass, length, and time. Force, being the product of mass and acceleration, is not a fundamental quantity. $\boxed{(c)\text{ is correct.}}$

2 •

Picture the Problem We can express and simplify the ratio of m/s to m/s^2 to determine the final units.

Express and simplify the ratio of m/s to m/s^2:

$$\frac{\frac{\text{m}}{\text{s}}}{\frac{\text{m}}{\text{s}^2}} = \frac{\text{m}\cdot\text{s}^2}{\text{m}\cdot\text{s}} = \text{s and } \boxed{(d)\text{ is correct.}}$$

3 •

Determine the Concept Consulting Table 1-1 we note that the prefix giga means 10^9. $\boxed{(c)\text{ is correct.}}$

4 •

Determine the Concept Consulting Table 1-1 we note that the prefix mega means 10^6. $\boxed{(d)\text{ is correct.}}$

***5 •**

Determine the Concept Consulting Table 1-1 we note that the prefix pico means 10^{-12}. $\boxed{(a)\text{ is correct.}}$

6 •

Determine the Concept Counting from left to right and ignoring zeros to the left of the first nonzero digit, the last significant figure is the first digit that is in doubt. Applyin criterion, the three zeros after the decimal point are not significant figures, but the last zero is significant. Hence, there are four significant figures in this number. $\boxed{(c)\text{ is correct.}}$

7 •

Determine the Concept Counting from left to right, the last significant figure is the first digit that is in doubt. Applying this criterion, there are six significant figures in this number. $\boxed{(e)\text{ is correct.}}$

8 •

Determine the Concept The advantage is that the length measure is always with you. The disadvantage is that arm lengths are not uniform; if you wish to purchase a board of ″two arm lengths″ it may be longer or shorter than you wish, or else you may have to physically go to the lumberyard to use your own arm as a measure of length.

9 •

(*a*) True. You cannot add ″apples to oranges″ or a length (distance traveled) to a volume (liters of milk).

(*b*) False. The distance traveled is the product of speed (length/time) multiplied by the time of travel (time).

(*c*) True. Multiplying by any conversion factor is equivalent to multiplying by 1. Doing so does not change the value of a quantity; it changes its units.

Estimation and Approximation

***10** ••

Picture the Problem Because θ is small, we can approximate it by $\theta \approx D/r_m$ provided that it is in radian measure. We can solve this relationship for the diameter of the moon.

Express the moon's diameter D in terms of the angle it subtends at the earth θ and the earth-moon distance r_m:

$$D = \theta r_m$$

Find θ in radians:

$$\theta = 0.524° \times \frac{2\pi\text{ rad}}{360°} = 0.00915\text{ rad}$$

Substitute and evaluate D:

$$D = (0.00915\text{ rad})(384\text{ Mm}) = \boxed{3.51 \times 10^6\text{ m}}$$

***11 ••**

Picture the Problem We'll assume that the sun is made up entirely of hydrogen. Then we can relate the mass of the sun to the number of hydrogen atoms and the mass of each.

Express the mass of the sun M_S as the product of the number of hydrogen atoms N_H and the mass of each atom M_H:

$$M_S = N_H M_H$$

Solve for N_H:

$$N_H = \frac{M_S}{M_H}$$

Substitute numerical values and evaluate N_H:

$$N_H = \frac{1.99\times10^{30}\ \text{kg}}{1.67\times10^{-27}\ \text{kg}} = \boxed{1.19\times10^{57}}$$

12 ••

Picture the Problem Let P represent the population of the United States, r the rate of consumption and N the number of aluminum cans used annually. The population of the United States is roughly 3×10^8 people. Let's assume that, on average, each person drinks one can of soft drink every day. The mass of a soft-drink can is approximately 1.8×10^{-2} kg.

(*a*) Express the number of cans N used annually in terms of the daily rate of consumption of soft drinks r and the population P:

$$N = rP\Delta t$$

Substitute numerical values and approximate N:

$$N = \left(\frac{1\,\text{can}}{\text{person}\cdot\text{d}}\right)\left(3\times10^8\ \text{people}\right)\times(1\,\text{y})\left(365.24\frac{\text{d}}{\text{y}}\right) \approx \boxed{10^{11}\ \text{cans}}$$

(*b*) Express the total mass of aluminum used per year for soft drink cans M as a function of the number of cans consumed and the mass m per can:

$$M = Nm$$

Substitute numerical values and evaluate M:

$$M = \left(10^{11}\ \text{cans/y}\right)\left(1.8\times10^{-2}\ \text{kg/can}\right) \approx \boxed{2\times10^{9}\ \text{kg/y}}$$

(*c*) Express the value of the aluminum as the product of M and the value at recycling centers:

$$\begin{aligned}\text{Value} &= (\$1/\text{kg})M \\ &= (\$1/\text{kg})\left(2\times10^{9}\ \text{kg/y}\right) \\ &= \$2\times10^{9}/\text{y} \\ &= \boxed{2\ \text{billion dollars/y}}\end{aligned}$$

13 ••

Picture the Problem We can estimate the number of words in *Encyclopedia Britannica* by counting the number of volumes, estimating the average number of pages per volume, estimating the number of words per page, and finding the product of these measurements and estimates. Doing so in *Encyclopedia Britannica* leads to an estimate of approximately 200 million for the number of words. If we assume an average word length of five letters, then our estimate of the number of letters in *Encyclopedia Britannica* becomes 10^9.

(*a*) Relate the area available for one letter s^2 and the number of letters N to be written on the pinhead to the area of the pinhead:

$Ns^2 = \dfrac{\pi}{4}d^2$ where d is the diameter of the pinhead.

Solve for s to obtain:

$$s = \sqrt{\frac{\pi d^2}{4N}}$$

Substitute numerical values and evaluate s:

$$s = \sqrt{\frac{\pi\left[\left(\frac{1}{16}\ \text{in}\right)\left(2.54\,\frac{\text{cm}}{\text{in}}\right)\right]^2}{4\left(10^{9}\right)}} \approx \boxed{10^{-8}\ \text{m}}$$

(*b*) Express the number of atoms per letter n in terms of s and the atomic spacing in a metal d_{atomic}:

$$n = \frac{s}{d_{\text{atomic}}}$$

Substitute numerical values and evaluate n:

$$n = \frac{10^{-8}\ \text{m}}{5\times10^{-10}\ \text{atoms/m}} \approx \boxed{20\ \text{atoms}}$$

***14** ••

Picture the Problem The population of the United States is roughly 3×10^{8} people. Assuming that the average family has four people, with an average of two cars per family, there are about 1.5×10^8 cars in the United States. If we double that number to include trucks, cabs, etc., we have 3×10^8 vehicles. Let's assume that each vehicle uses, on average, about 12 gallons of gasoline per week.

(*a*) Find the daily consumption of gasoline *G*:

$$G = \left(3\times10^8 \text{ vehicles}\right)\left(2\,\text{gal/d}\right) = 6\times10^8 \text{ gal/d}$$

Assuming a price per gallon P = \$1.50, find the daily cost C of gasoline:

$$C = GP = \left(6\times10^8 \text{ gal/d}\right)\left(\$1.50/\text{gal}\right) = \$9\times10^8/\text{d} \approx \boxed{\$1\,\text{billion dollars/d}}$$

(*b*) Relate the number of barrels *N* of crude oil required annually to the yearly consumption of gasoline *Y* and the number of gallons of gasoline *n* that can be made from one barrel of crude oil:

$$N = \frac{Y}{n} = \frac{G\Delta t}{n}$$

Substitute numerical values and estimate *N*:

$$N = \frac{\left(6\times10^8 \text{ gal/d}\right)\left(365.24\,\text{d/y}\right)}{19.4\,\text{gal/barrel}} \approx \boxed{10^{10}\text{ barrels/y}}$$

15 ••

Picture the Problem We'll assume a population of 300 million (fairly accurate as of September, 2002) and a life expectancy of 76 y. We'll also assume that a diaper has a volume of about half a liter. In (*c*) we'll assume the disposal site is a rectangular hole in the ground and use the formula for the volume of such an opening to estimate the surface area required.

(*a*) Express the total number *N* of disposable diapers used in the United States per year in terms of the number of children *n* in diapers and the number of diapers *D* used by each child in 2.5 y:

$$N = nD$$

Use the daily consumption, the number of days in a year, and the estimated length of time a child is in diapers to estimate the number of diapers *D* required per child:

$$D = \frac{3\,\text{diapers}}{\text{d}} \times \frac{365.24\,\text{d}}{\text{y}} \times 2.5\,\text{y} \approx 3\times10^3 \text{ diapers/child}$$

Use the assumed life expectancy to estimate the number of children n in diapers:

$$n = \left(\frac{2.5\,\text{y}}{76\,\text{y}}\right)\left(300\times10^{6}\ \text{children}\right)$$
$$\approx 10^{7}\ \text{children}$$

Substitute to obtain:

$$N = \left(10^{7}\ \text{children}\right)$$
$$\times\left(3\times10^{3}\ \text{diapers/child}\right)$$
$$\approx \boxed{3\times10^{10}\ \text{diapers}}$$

(*b*) Express the required landfill volume V in terms of the volume of diapers to be buried:

$$V = NV_{\text{one diaper}}$$

Substitute numerical values and evaluate V:

$$V = \left(3\times10^{10}\ \text{diapers}\right)\left(0.5\,\text{L/diaper}\right)$$
$$\approx \boxed{1.5\times10^{7}\ \text{m}^{3}}$$

(*c*) Express the required volume in terms of the volume of a rectangular parallelepiped:

$$V = Ah$$

Solve and evaluate h:

$$A = \frac{V}{h} = \frac{1.5\times10^{7}\ \text{m}^{3}}{10\,\text{m}} = 1.5\times10^{6}\ \text{m}^{2}$$

Use a conversion factor to express this area in square miles:

$$A = 1.5\times10^{6}\ \text{m}^{2}\times\frac{1\ \text{mi}^{2}}{2.590\,\text{km}^{2}}$$
$$\approx \boxed{0.6\,\text{mi}^{2}}$$

16 •••

Picture the Problem The number of bits that can be stored on the disk can be found from the product of the capacity of the disk and the number of bits per byte. In part (*b*) we'll need to estimate (*i*) the number of bits required for the alphabet, (*ii*) the average number of letters per word, (*iii*) an average number of words per line, (*iv*) an average number of lines per page, and (*v*) a book length in pages.

(*a*) Express the number of bits N_{bits} as a function of the number of bits per byte and the capacity of the hard disk N_{bytes}:

$$N_{\text{bits}} = N_{\text{bytes}}\left(8\,\text{bits/byte}\right)$$
$$= \left(2\times10^{9}\ \text{bytes}\right)\left(8\,\text{bits/byte}\right)$$
$$= \boxed{1.60\times10^{10}\ \text{bits}}$$

(*b*) Assume an average of 8 letters/word and 8 bits/character to estimate the number of bytes required per word:

$$8\,\frac{\text{bits}}{\text{character}}\times 8\,\frac{\text{characters}}{\text{word}} = 64\,\frac{\text{bits}}{\text{word}} = 8\,\frac{\text{bytes}}{\text{word}}$$

Assume 10 words/line and 60 lines/page:

$$600\,\frac{\text{words}}{\text{page}}\times 8\,\frac{\text{bytes}}{\text{word}} = 4800\,\frac{\text{bytes}}{\text{page}}$$

Assume a book length of 300 pages and approximate the number bytes required:

$$300\,\text{pages}\times 4800\,\frac{\text{bytes}}{\text{page}} = 1.44\times 10^{6}\ \text{bytes}$$

Divide the number of bytes per disk by our estimated number of bytes required per book to obtain an estimate of the number of books the 2-gigabyte hard disk can hold:

$$N_{\text{books}} = \frac{2\times 10^{9}\ \text{bytes}}{1.44\times 10^{6}\ \text{bytes/book}} \approx \boxed{1400\,\text{books}}$$

***17 ••**

Picture the Problem Assume that, on average, four cars go through each toll station per minute. Let R represent the yearly revenue from the tolls. We can estimate the yearly revenue from the number of lanes N, the number of cars per minute n, and the \$6 toll per car C.

$$R = NnC = 14\,\text{lanes}\times 4\,\frac{\text{cars}}{\text{min}}\times 60\,\frac{\text{min}}{\text{h}}\times 24\,\frac{\text{h}}{\text{d}}\times 365.24\,\frac{\text{d}}{\text{y}}\times\frac{\$6}{\text{car}} = \boxed{\$177\text{M}}$$

Units

18 •

Picture the Problem We can use the metric prefixes listed in Table 1-1 and the abbreviations on page EP-1 to express each of these quantities.

(*a*)

$$1{,}000{,}000\,\text{watts} = 10^{6}\ \text{watts} = \boxed{1\,\text{MW}}$$

(*b*)

$$0.002\,\text{gram} = 2\times 10^{-3}\ \text{g} = \boxed{2\,\text{mg}}$$

(*c*)

$$3\times 10^{-6}\ \text{meter} = \boxed{3\,\mu\text{m}}$$

(*d*)

$$30{,}000\,\text{seconds} = 30\times 10^{3}\ \text{s} = \boxed{30\,\text{ks}}$$

19 •

Picture the Problem We can use the definitions of the metric prefixes listed in Table 1-1 to express each of these quantities without prefixes.

(*a*)

$$40\,\mu\text{W} = 40\times10^{-6}\text{ W} = \boxed{0.000040\,\text{W}}$$

(*b*)

$$4\,\text{ns} = 4\times10^{-9}\text{ s} = \boxed{0.000000004\,\text{s}}$$

(*c*)

$$3\,\text{MW} = 3\times10^{6}\text{ W} = \boxed{3{,}000{,}000\,\text{W}}$$

(*d*)

$$25\,\text{km} = 25\times10^{3}\text{ m} = \boxed{25{,}000\,\text{m}}$$

***20** •

Picture the Problem We can use the definitions of the metric prefixes listed in Table 1-1 to express each of these quantities without abbreviations.

(*a*) $10^{-12}\text{ boo} = \boxed{1\,\text{picoboo}}$

(*b*) $10^{9}\text{ low} = \boxed{1\,\text{gigalow}}$

(*c*) $10^{-6}\text{ phone} = \boxed{1\,\text{microphone}}$

(*d*) $10^{-18}\text{ boy} = \boxed{1\,\text{attoboy}}$

(*e*) $10^{6}\text{ phone} = \boxed{1\,\text{megaphone}}$

(*f*) $10^{-9}\text{ goat} = \boxed{1\,\text{nanogoat}}$

(*g*) $10^{12}\text{ bull} = \boxed{1\,\text{terabull}}$

21 ••

Picture the Problem We can determine the SI units of each term on the right-hand side of the equations from the units of the physical quantity on the left-hand side.

(*a*) Because x is in meters, C_1 and C_2t must be in meters:

$$\boxed{C_1\text{ is in m; }C_2\text{ is in m/s}}$$

(*b*) Because x is in meters, $\frac{1}{2}C_1t^2$ must be in meters:

$$\boxed{C_1\text{ is in m/s}^2}$$

(*c*) Because v^2 is in m^2/s^2, $2C_1x$ must be in m^2/s^2:

$$\boxed{C_1\text{ is in m/s}^2}$$

(*d*) The argument of trigonometric function must be dimensionless; i.e. without units. Therefore, because x

$$\boxed{C_1\text{ is in m; }C_2\text{ is in s}^{-1}}$$

is in meters:

(*e*) The argument of an exponential function must be dimensionless; i.e. without units. Therefore, because v is in m/s:

$$\boxed{C_1 \text{ is in m/s}; C_2 \text{ is in s}^{-1}}$$

22 ••

Picture the Problem We can determine the US customary units of each term on the right-hand side of the equations from the units of the physical quantity on the left-hand side.

(*a*) Because x is in feet, C_1 and C_2t must be in feet:

$$\boxed{C_1 \text{ is in ft}; C_2 \text{ is in ft/s}}$$

(*b*) Because x is in feet, $\frac{1}{2}C_1t^2$ must be in feet:

$$\boxed{C_1 \text{ is in ft/s}^2}$$

(*c*) Because v^2 is in ft^2/s^2, $2C_1x$ must be in ft^2/s^2:

$$\boxed{C_1 \text{ is in ft/s}^2}$$

(*d*) The argument of trigonometric function must be dimensionless; i.e. without units. Therefore, because x is in feet:

$$\boxed{C_1 \text{ is in ft}; C_2 \text{ is in s}^{-1}}$$

(*e*) The argument of an exponential function must be dimensionless; i.e. without units. Therefore, because v is in ft/s:

$$\boxed{C_1 \text{ is in ft/s}; C_2 \text{ is in s}^{-1}}$$

Conversion of Units

23 •

Picture the Problem We can use the formula for the circumference of a circle to find the radius of the earth and the conversion factor 1 mi = 1.61 km to convert distances in meters into distances in miles.

(*a*) The Pole-Equator distance is one-fourth of the circumference:

$$c = \boxed{4 \times 10^7 \text{ m}}$$

(*b*) Use the formula for the circumference of a circle to obtain:

$$R=\frac{c}{2\pi}=\frac{4\times10^{-7}\ \text{m}}{2\pi}=\boxed{6.37\times10^{6}\ \text{m}}$$

(*c*) Use the conversion factors 1 km = 1000 m and 1 mi = 1.61 km:

$$c=4\times10^{7}\ \text{m}\times\frac{1\,\text{km}}{10^{3}\ \text{m}}\times\frac{1\,\text{mi}}{1.61\,\text{km}}$$

$$=\boxed{2.48\times10^{4}\ \text{mi}}$$

and

$$R=6.37\times10^{6}\ \text{m}\times\frac{1\,\text{km}}{10^{3}\ \text{m}}\times\frac{1\,\text{mi}}{1.61\,\text{km}}$$

$$=\boxed{3.96\times10^{3}\ \text{mi}}$$

24 •

Picture the Problem We can use the conversion factor 1 mi = 1.61 km to convert speeds in km/h into mi/h.

Find the speed of the plane in km/s:

$$v=2(340\,\text{m/s})=680\,\text{m/s}$$

$$=\left(680\frac{\text{m}}{\text{s}}\right)\left(\frac{1\,\text{km}}{10^{3}\ \text{m}}\right)\left(3600\frac{\text{s}}{\text{h}}\right)$$

$$=\boxed{2450\,\text{km/h}}$$

Convert *v* into mi/h:

$$v=\left(2450\frac{\text{km}}{\text{h}}\right)\left(\frac{1\,\text{mi}}{1.61\,\text{km}}\right)$$

$$=\boxed{1520\,\text{mi/h}}$$

***25 •**

Picture the Problem We'll first express his height in inches and then use the conversion factor 1 in = 2.54 cm.

Express the player's height into inches:

$$h=6\,\text{ft}\times\frac{12\,\text{in}}{\text{ft}}+10.5\,\text{in}=82.5\,\text{in}$$

Convert *h* into cm:

$$h=82.5\,\text{in}\times\frac{2.54\,\text{cm}}{\text{in}}=\boxed{210\,\text{cm}}$$

26 •

Picture the Problem We can use the conversion factors 1 mi = 1.61 km, 1 in = 2.54 cm, and 1 m = 1.094 yd to complete these conversions.

(*a*)
$$100\,\frac{\text{km}}{\text{h}} = 100\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{mi}}{1.61\,\text{km}} = \boxed{62.1\,\frac{\text{mi}}{\text{h}}}$$

(*b*)
$$60\,\text{cm} = 60\,\text{cm} \times \frac{1\,\text{in}}{2.54\,\text{cm}} = \boxed{23.6\,\text{in}}$$

(*c*)
$$100\,\text{yd} = 100\,\text{yd} \times \frac{1\,\text{m}}{1.094\,\text{yd}} = \boxed{91.4\,\text{m}}$$

27 •

Picture the Problem We can use the conversion factor 1.609 km = 5280 ft to convert the length of the main span of the Golden Gate Bridge into kilometers.

Convert 4200 ft into km:
$$4200\,\text{ft} = 4200\,\text{ft} \times \frac{1.609\,\text{km}}{5280\,\text{ft}} = \boxed{1.28\,\text{km}}$$

***28** •

Picture the Problem Let v be the speed of an object in mi/h. We can use the conversion factor 1 mi = 1.61 km to convert this speed to km/h.

Multiply v mi/h by 1.61 km/mi to convert v to km/h:
$$v\,\frac{\text{mi}}{\text{h}} = v\,\frac{\text{mi}}{\text{h}} \times \frac{1.61\,\text{km}}{\text{mi}} = \boxed{1.61v\,\text{km/h}}$$

29 •

Picture the Problem Use the conversion factors 1 h = 3600 s, 1.609 km = 1 mi, and 1 mi = 5280 ft to make these conversions.

(*a*) $$1.296 \times 10^5\,\frac{\text{km}}{\text{h}^2} = \left(1.296 \times 10^5\,\frac{\text{km}}{\text{h}^2}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = \boxed{36.0\,\frac{\text{km}}{\text{h}\cdot\text{s}}}$$

(*b*) $$1.296 \times 10^5\,\frac{\text{km}}{\text{h}^2} = \left(1.296 \times 10^5\,\frac{\text{km}}{\text{h}^2}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)^2\left(\frac{10^3\,\text{m}}{\text{km}}\right) = \boxed{10.0\,\frac{\text{m}}{\text{s}^2}}$$

(*c*) $$60\,\frac{\text{mi}}{\text{h}} = \left(60\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{5280\,\text{ft}}{1\,\text{mi}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = \boxed{88.0\,\frac{\text{ft}}{\text{s}}}$$

(*d*) $$60\,\frac{\text{mi}}{\text{h}} = \left(60\,\frac{\text{mi}}{\text{h}}\right)\left(\frac{1.609\,\text{km}}{1\,\text{mi}}\right)\left(\frac{10^3\,\text{m}}{\text{km}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right) = \boxed{26.8\,\frac{\text{m}}{\text{s}}}$$

30 •

Picture the Problem We can use the conversion factor 1 L = 1.057 qt to convert gallons into liters and then use this gallons-to-liters conversion factor to convert barrels into cubic meters.

$$(a)\ 1\,\text{gal} = (1\,\text{gal})\left(\frac{4\,\text{qt}}{\text{gal}}\right)\left(\frac{1\,\text{L}}{1.057\,\text{qt}}\right) = \boxed{3.784\,\text{L}}$$

$$(b)\ 1\,\text{barrel} = (1\,\text{barrel})\left(\frac{42\,\text{gal}}{\text{barrel}}\right)\left(\frac{3.784\,\text{L}}{\text{gal}}\right)\left(\frac{10^{-3}\,\text{m}^3}{\text{L}}\right) = \boxed{0.1589\,\text{m}^3}$$

31 •

Picture the Problem We can use the conversion factor given in the problem statement and the fact that 1 mi = 1.609 km to express the number of square meters in one acre.

Multiply by 1 twice, properly chosen, to convert one acre into square miles, and then into square meters:

$$1\,\text{acre} = (1\,\text{acre})\left(\frac{1\,\text{mi}^2}{640\,\text{acres}}\right)\left(\frac{1609\,\text{m}}{\text{mi}}\right)^2$$
$$= \boxed{4050\,\text{m}^2}$$

32 ••

Picture the Problem The volume of a right circular cylinder is the area of its base multipli its height. Let *d* represent the diameter and *h* the height of the right circular cylinder; use conversion factors to express the volume *V* in the given units.

(*a*) Express the volume of the cylinder:

$$V = \tfrac{1}{4}\pi d^2 h$$

Substitute numerical values and evaluate *V*:

$$V = \tfrac{1}{4}\pi(6.8\,\text{in})^2(2\,\text{ft})$$
$$= \tfrac{1}{4}\pi(6.8\,\text{in})^2(2\,\text{ft})\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)^2$$
$$= \boxed{0.504\,\text{ft}^3}$$

(*b*) Use the fact that 1 m = 3.281 ft to convert the volume in cubic feet into cubic meters:

$$V = \left(0.504\,\text{ft}^3\right)\left(\frac{1\,\text{m}}{3.281\,\text{ft}}\right)^3$$
$$= \boxed{0.0143\,\text{m}^3}$$

(*c*) Because 1 L = 10^{-3} m^3:

$$V = \left(0.0143\,\text{m}^3\right)\left(\frac{1\,\text{L}}{10^{-3}\,\text{m}^3}\right) = \boxed{14.3\,\text{L}}$$

***33 ••**

Picture the Problem We can treat the SI units as though they are algebraic quantities to simplify each of these combinations of physical quantities and constants.

(*a*) Express and simplify the units of v^2/x:

$$\frac{(\mathrm{m/s})^2}{\mathrm{m}} = \frac{\mathrm{m}^2}{\mathrm{m}\cdot\mathrm{s}^2} = \boxed{\frac{\mathrm{m}}{\mathrm{s}^2}}$$

(*b*) Express and simplify the units of $\sqrt{x/a}$:

$$\sqrt{\frac{\mathrm{m}}{\mathrm{m/s}^2}} = \sqrt{\mathrm{s}^2} = \boxed{\mathrm{s}}$$

(*c*) Noting that the constant factor $\frac{1}{2}$ has no units, express and simplify the units of $\frac{1}{2}at^2$:

$$\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)(\mathrm{s})^2 = \left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)(\mathrm{s}^2) = \boxed{\mathrm{m}}$$

Dimensions of Physical Quantities

34 •

Picture the Problem We can use the facts that each term in an equation must have the same dimensions and that the arguments of a trigonometric or exponential function must be dimensionless to determine the dimensions of the constants.

(*a*)

$$\begin{array}{ccccccc} x & = & C_1 & + & C_2 & t \\ L & & \boxed{L} & & \boxed{\frac{L}{T}} & T \end{array}$$

(*b*)

$$\begin{array}{ccccc} x & = & \frac{1}{2} & C_1 & t^2 \\ L & & & \boxed{\frac{L}{T^2}} & T^2 \end{array}$$

(*c*)

$$\begin{array}{ccccc} v^2 & = & 2 & C_1 & x \\ \frac{L^2}{T^2} & & & \boxed{\frac{L}{T^2}} & L \end{array}$$

(*d*)

$$\begin{array}{cccccc} x & = & C_1 & \cos & C_2 & t \\ L & & \boxed{L} & & \boxed{\frac{1}{T}} & T \end{array}$$

(*e*)

$$\begin{array}{cccccc} v & = & C_1 & \exp(& -C_2 & t) \\ \frac{L}{T} & & \boxed{\frac{L}{T}} & & \boxed{\frac{1}{T}} & T \end{array}$$

35 ••

Picture the Problem Because the exponent of the exponential function must be dimension the dimension of λ must be $\boxed{T^{-1}.}$

***36 ••**

Picture the Problem We can solve Newton's law of gravitation for *G* and substitute the dimensions of the variables. Treating them as algebraic quantities will allow us to express the dimensions in their simplest form. Finally, we can substitute the SI units for the dimensions to find the units of *G*.

Solve Newton's law of gravitation for *G* to obtain:

$$G = \frac{Fr^2}{m_1 m_2}$$

Substitute the dimensions of the variables:

$$G = \frac{\frac{ML}{T^2} \times L^2}{M^2} = \boxed{\frac{L^3}{MT^2}}$$

Use the SI units for *L, M,* and *T*:

$$\text{Units of } G \text{ are } \boxed{\frac{\text{m}^3}{\text{kg}\cdot\text{s}^2}}$$

37 ••

Picture the Problem Let *m* represent the mass of the object, *v* its speed, and *r* the radius of the circle in which it moves. We can express the force as the product of *m*, *v*, and *r* (each raised to a power) and then use the dimensions of force *F,* mass *m,* speed *v,* and radius *r* to obtain three equations in the assumed powers. Solving these equations simultaneously will give us the dependence of *F* on *m, v,* and *r.*

Express the force in terms of powers of the variables:

$$F = m^a v^b r^c$$

Substitute the dimensions of the physical quantities:

$$MLT^{-2} = M^a \left(\frac{L}{T}\right)^b L^c$$

Simplify to obtain:

$$MLT^{-2} = M^a L^{b+c} T^{-b}$$

Equate the exponents to obtain:

$a = 1$,
$b + c = 1$, and
$-b = -2$

Solve this system of equations to obtain:

$a = 1$, $b = 2$, and $c = -1$

Substitute in equation (1):

$$F = mv^2 r^{-1} = \boxed{m\frac{v^2}{r}}$$

38 ••

Picture the Problem We note from Table 1-2 that the dimensions of power are ML^2/T^3. The dimensions of mass, acceleration, and speed are M, L/T^2, and L/T respectively.

Express the dimensions of *mav*:

$$[mav] = M \times \frac{L}{T^2} \times \frac{L}{T} = \frac{ML^2}{T^3}$$

From Table 1-2:

$$[P] = \frac{ML^2}{T^3}$$

$$\boxed{\text{Comparing these results, we see that the product of mass, acceleration, and speed has the dimensions of power.}}$$

39 ••

Picture the Problem The dimensions of mass and velocity are M and L/T, respectively. We note from Table 1-2 that the dimensions of force are ML/T^2.

Express the dimensions of momentum:

$$[mv] = M \times \frac{L}{T} = \frac{ML}{T}$$

From Table 1-2:

$$[F] = \frac{ML}{T^2}$$

Express the dimensions of force multiplied by time:

$$[Ft] = \frac{ML}{T^2} \times T = \frac{ML}{T}$$

$$\boxed{\text{Comparing these results, we see that momentum has the dimensions of force multiplied by time.}}$$

40 ••

Picture the Problem Let X represent the physical quantity of interest. Then we can express the dimensional relationship between F, X, and P and solve this relationship for the dimensions of X.

Express the relationship of X to force and power dimensionally:

$$[F][X] = [P]$$

Solve for $[X]$:

$$[X] = \frac{[P]}{[F]}$$

Substitute the dimensions of force and power and simplify to obtain:

$$[X]=\frac{\frac{ML^2}{T^3}}{\frac{ML}{T^2}}=\frac{L}{T}$$

Because the dimensions of velocity are L/T, we can conclude that:

$$\boxed{[P]=[F][v]}$$

Remarks: While it is true that *P* = *Fv*, dimensional analysis does not reveal the presence of dimensionless constants. For example, if $P=\pi Fv$, the analysis shown above would fail to establish the factor of π.

***41 ••**

Picture the Problem We can find the dimensions of C by solving the drag force equation for C and substituting the dimensions of force, area, and velocity.

Solve the drag force equation for the constant C:

$$C=\frac{F_{\text{air}}}{Av^2}$$

Express this equation dimensionally:

$$[C]=\frac{[F_{\text{air}}]}{[A][v]^2}$$

Substitute the dimensions of force, area, and velocity and simplify to obtain:

$$[C]=\frac{\frac{ML}{T^2}}{L^2\left(\frac{L}{T}\right)^2}=\boxed{\frac{M}{L^3}}$$

42 ••

Picture the Problem We can express the period of a planet as the product of these factors (each raised to a power) and then perform dimensional analysis to determine the values of the exponents.

Express the period T of a planet as the product of r^a, G^b, and M_{S}^c:

$$T=Cr^aG^bM_{\text{S}}^c \qquad (1)$$

where C is a dimensionless constant.

Solve the law of gravitation for the constant G:

$$G=\frac{Fr^2}{m_1m_2}$$

Express this equation dimensionally:

$$[G]=\frac{[F][r]^2}{[m_1][m_2]}$$

Substitute the dimensions of F, r, and m:

$$[G]=\frac{\frac{ML}{T^2}\times(L)^2}{M\times M}=\frac{L^3}{MT^2}$$

Noting that the dimension of time is represented by the same letter as is the period of a planet, substitute the dimensions in equation (1) to obtain:

$$T=(L)^a\left(\frac{L^3}{MT^2}\right)^b(M)^c$$

Introduce the product of M^0 and L^0 in the left hand side of the equation and simplify to obtain:

$$M^0L^0T^1=M^{c-b}L^{a+3b}T^{-2b}$$

Equate the exponents on the two sides of the equation to obtain:

$0 = c – b$,
$0 = a + 3b$, and
$1 = –2b$

Solve these equations simultaneously to obtain:

$$a=\tfrac{3}{2},\, b=-\tfrac{1}{2},\text{ and } c=-\tfrac{1}{2}$$

Substitute in equation (1):

$$T=Cr^{3/2}G^{-1/2}M_{\text{S}}^{-1/2}=\boxed{\frac{C}{\sqrt{GM_{\text{S}}}}r^{3/2}}$$

Scientific Notation and Significant Figures

***43** •

Picture the Problem We can use the rules governing scientific notation to express each of these numbers as a decimal number.

(*a*) $3\times10^4=\boxed{30{,}000}$

(*b*) $6.2\times10^{-3}=\boxed{0.0062}$

(*c*) $4\times10^{-6}=\boxed{0.000004}$

(*d*) $2.17\times10^5=\boxed{217{,}000}$

44 •

Picture the Problem We can use the rules governing scientific notation to express each of these measurements in scientific notation.

(*a*) $3.1\,\text{GW}=\boxed{3.1\times10^9\text{ W}}$

(*c*) $2.3\,\text{fs}=\boxed{2.3\times10^{-15}\text{ s}}$

(b) $10\,\text{pm} = 10\times10^{-12}\text{ m} = \boxed{10^{-11}\text{ m}}$ (d) $4\,\mu\text{s} = \boxed{4\times10^{-6}\text{ s}}$

45 •

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(a) The number of significant figures in each factor is three; therefore the result has three significant figures:

$$(1.14)(9.99\times10^{4}) = \boxed{1.14\times10^{5}}$$

(b) Express both terms with the same power of 10. Because the first measurement has only two digits after the decimal point, the result can have only two digits after the decimal point:

$$\begin{aligned}(2.78\times10^{-8})-(5.31\times10^{-9}) &= (2.78-0.531)\times10^{-8}\\ &= \boxed{2.25\times10^{-8}}\end{aligned}$$

(c) We'll assume that 12 is exact. Hence, the answer will have three significant figures:

$$\frac{12\pi}{4.56\times10^{-3}} = \boxed{8.27\times10^{3}}$$

(d) Proceed as in (b):

$$\begin{aligned}27.6+(5.99\times10^{2}) &= 27.6+599\\ &= 627\\ &= \boxed{6.27\times10^{2}}\end{aligned}$$

46 •

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(a) Note that both factors have four significant figures.

$$(200.9)(569.3) = \boxed{1.144\times10^{5}}$$

(b) Express the first factor in scientific notation and note that both factors have three significant figures.

$$\begin{aligned}(0.000000513)(62.3\times10^{7}) &= (5.13\times10^{-7})(62.3\times10^{7})\\ &= \boxed{3.20\times10^{2}}\end{aligned}$$

(*c*) Express both terms in scientific notation and note that the second has only three significant figures. Hence the result will have only three significant figures.

$$\begin{aligned}28401 + \left(5.78\times10^{4}\right) &= \left(2.841\times10^{4}\right) + \left(5.78\times10^{4}\right)\\ &= \left(2.841+5.78\right)\times10^{4}\\ &= \boxed{8.62\times10^{4}}\end{aligned}$$

(*d*) Because the divisor has three significant figures, the result will have three significant figures.

$$\frac{63.25}{4.17\times10^{-3}} = \boxed{1.52\times10^{4}}$$

***47 •**

Picture the Problem Let N represent the required number of membranes and express N in terms of the thickness of each cell membrane.

Express N in terms of the thickness of a single membrane:

$$N = \frac{1\,\text{in}}{7\,\text{nm}}$$

Convert the units into SI units and simplify to obtain:

$$N = \frac{1\,\text{in}}{7\,\text{nm}}\times\frac{2.54\,\text{cm}}{\text{in}}\times\frac{1\,\text{m}}{100\,\text{cm}}\times\frac{1\,\text{nm}}{10^{-9}\,\text{m}} = \boxed{4\times10^{6}}$$

48 •

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) Both factors and the result have three significant figures:

$$\left(2.00\times10^{4}\right)\left(6.10\times10^{-2}\right) = \boxed{1.22\times10^{3}}$$

(*b*) Because the second factor has three significant figures, the result will have three significant figures:

$$\left(3.141592\right)\left(4.00\times10^{5}\right) = \boxed{1.26\times10^{6}}$$

(*c*) Both factors and the result have three significant figures:

$$\frac{2.32\times10^{3}}{1.16\times10^{8}} = \boxed{2.00\times10^{-5}}$$

(*d*) Write both terms using the same power of 10. Note that the result will have only three significant figures:

$$\begin{aligned}\left(5.14\times10^{3}\right) + \left(2.78\times10^{2}\right) &= \left(5.14\times10^{3}\right) + \left(0.278\times10^{3}\right)\\ &= \left(5.14+0.278\right)\times10^{3}\\ &= \boxed{5.42\times10^{3}}\end{aligned}$$

(*e*) Follow the same procedure used in (*d*):

$$\left(1.99\times 10^{2}\right)+\left(9.99\times 10^{-5}\right)$$
$$=\left(1.99\times 10^{2}\right)+\left(0.000000999\times 10^{2}\right)$$
$$=\boxed{1.99\times 10^{2}}$$

***49** •

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) The second factor and the result have three significant figures:

$$3.141592654\times(23.2)^{2}=\boxed{1.69\times 10^{3}}$$

(*b*) We'll assume that 2 is exact. Therefore, the result will have two significant figures:

$$2\times 3.141592654\times 0.76=\boxed{4.8}$$

(*c*) We'll assume that 4/3 is exact. Therefore the result will have two significant figures:

$$\frac{4}{3}\pi\times(1.1)^{3}=\boxed{5.6}$$

(*d*) Because 2.0 has two significant figures, the result has two significant figures:

$$\frac{(2.0)^{5}}{3.141592654}=\boxed{10}$$

General Problems

50 •

Picture the Problem We can use the conversion factor 1 mi = 1.61 km to convert 100 km/h into mi/h.

Multiply 100 km/h by 1 mi/1.61 km to obtain:

$$100\frac{\text{km}}{\text{h}}=100\frac{\text{km}}{\text{h}}\times\frac{1\text{mi}}{1.61\text{km}}$$
$$=\boxed{62.1\text{mi/h}}$$

***51** •

Picture the Problem We can use a series of conversion factors to convert 1 billion seconds into years.

Multiply 1 billion seconds by the appropriate conversion factors to convert into years:

$$10^9\ \text{s} = 10^9\ \text{s} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1\,\text{day}}{24\,\text{h}} \times \frac{1\,\text{y}}{365.24\,\text{days}} = \boxed{31.7\,\text{y}}$$

52 •

Picture the Problem In both the examples cited we can equate expressions for the physical quantities, expressed in different units, and then divide both sides of the equation by one of the expressions to obtain the desired conversion factor.

(*a*) Divide both sides of the equation expressing the speed of light in the two systems of measurement by 186,000 mi/s to obtain:

$$1 = \frac{3\times 10^8\ \text{m/s}}{1.86\times 10^5\ \text{mi/h}} = 1.61\times 10^3\ \text{m/mi}$$
$$= \left(1.61\times 10^3\ \frac{\text{m}}{\text{mi}}\right)\left(\frac{1\,\text{km}}{10^3\ \text{m}}\right)$$
$$= \boxed{1.61\,\text{km/mi}}$$

(*b*) Find the volume of 1.00 kg of water:

Volume of 1.00 kg = 10^3 g is 10^3 cm^3

Express 10^3 cm^3 in ft^3:

$$(10\,\text{cm})^3\left(\frac{1\,\text{in}}{2.54\,\text{cm}}\right)^3\left(\frac{1\,\text{ft}}{12\,\text{in}}\right)^3 = 0.0353\,\text{ft}^3$$

Relate the weight of 1 ft^3 of water to the volume occupied by 1 kg of water:

$$\frac{1.00\,\text{kg}}{0.0353\,\text{ft}^3} = 62.4\,\frac{\text{lb}}{\text{ft}^3}$$

Divide both sides of the equation by the left-hand side to obtain:

$$1 = \frac{62.4\,\dfrac{\text{lb}}{\text{ft}^3}}{\dfrac{1.00\,\text{kg}}{0.0353\,\text{ft}^3}} = \boxed{2.20\,\text{lb/kg}}$$

53 ••

Picture the Problem We can use the given information to equate the ratios of the number of uranium atoms in 8 g of pure uranium and of 1 atom to its mass.

Express the proportion relating the number of uranium atoms N_U in 8 g of pure uranium to the mass of 1 atom:

$$\frac{N_\text{U}}{8\,\text{g}} = \frac{1\,\text{atom}}{4.0\times 10^{-26}\ \text{kg}}$$

Solve for and evaluate N_U:

$$N_U = (8\,\text{g})\left(\frac{1\,\text{atom}}{4.0\times10^{-26}\,\text{kg}}\right) = \boxed{2.0\times10^{23}}$$

54 ••

Picture the Problem We can relate the weight of the water to its weight per unit volume and the volume it occupies.

Express the weight w of water falling on the acre in terms of the weight of one cubic foot of water, the depth d of the water, and the area A over which the rain falls:

$$w = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)Ad$$

Find the area A in ft^2:

$$A = (1\,\text{acre})\left(\frac{1\,\text{mi}^2}{640\,\text{acre}}\right)\left(\frac{5280\,\text{ft}}{\text{mi}}\right)^2 = 4.356\times10^4\,\text{ft}^2$$

Substitute numerical values and evaluate w:

$$w = \left(62.4\frac{\text{lb}}{\text{ft}^3}\right)\left(4.356\times10^4\,\text{ft}^2\right)(1.4\,\text{in})\left(\frac{1\,\text{ft}}{12\,\text{in}}\right) = \boxed{3.17\times10^5\,\text{lb}}$$

55 ••

Picture the Problem We can use the definition of density and the formula for the volume c sphere to find the density of iron. Once we know the density of iron, we can use these same relationships to find what the radius of the earth would be if it had the same mass per unit volume as iron.

(*a*) Using its definition, express the density of iron:

$$\rho = \frac{m}{V}$$

Assuming it to be spherical, express the volume of an iron nucleus as a function of its radius:

$$V = \tfrac{4}{3}\pi r^3$$

Substitute to obtain:

$$\rho = \frac{3m}{4\pi r^3} \qquad (1)$$

Substitute numerical values and evaluate ρ:

$$\rho = \frac{3\left(9.3\times10^{-26}\ \text{kg}\right)}{4\pi\left(5.4\times10^{-15}\ \text{m}\right)^3} = \boxed{1.41\times10^{17}\ \text{kg/m}^3}$$

(*b*) Because equation (1) relates the density of any spherical object to its mass and radius, we can solve for *r* to obtain:

$$r = \sqrt[3]{\frac{3m}{4\pi\rho}}$$

Substitute numerical values and evaluate *r*:

$$r = \sqrt[3]{\frac{3\left(5.98\times10^{24}\ \text{kg}\right)}{4\pi\left(1.41\times10^{17}\ \text{kg/m}^3\right)}} = \boxed{216\,\text{m}}$$

56 ••

Picture the Problem Apply the general rules concerning the multiplication, division, addition, and subtraction of measurements to evaluate each of the given expressions.

(*a*) Because all of the factors have two significant figures, the result will have two significant figures:

$$\frac{\left(5.6\times10^{-5}\right)\left(0.0000075\right)}{2.4\times10^{-12}} = \frac{\left(5.6\times10^{-5}\right)\left(7.5\times10^{-6}\right)}{2.4\times10^{-12}} = \boxed{1.8\times10^{2}}$$

(*b*) Because the factor with the fewest significant figures in the first term has two significant figures, the result will have two significant figures. Because its last significant figure is in the tenth's position, the difference between the first and second term will have its last significant figure in the tenth's position:

$$\left(14.2\right)\left(6.4\times10^{7}\right)\left(8.2\times10^{-9}\right) - 4.06 = 7.8 - 4.06 = \boxed{3.4}$$

(*c*) Because all of the factors have two significant figures, the result will have two significant figures:

$$\frac{\left(6.1\times10^{-6}\right)^2\left(3.6\times10^{4}\right)^3}{\left(3.6\times10^{-11}\right)^{1/2}} = \boxed{2.9\times10^{8}}$$

(*d*) Because the factor with the fewest significant figures has two significant figures, the result will have two significant figures.

$$\frac{(0.000064)^{1/3}}{(12.8\times10^{-3})(490\times10^{-1})^{1/2}} = \frac{(6.4\times10^{-5})^{1/3}}{(12.8\times10^{-3})(490\times10^{-1})^{1/2}} = \boxed{0.45}$$

***57** ••

Picture the Problem We can use the relationship between an angle θ, measured in radians, subtended at the center of a circle, the radius R of the circle, and the length L of the arc to answer these questions concerning the astronomical units of measure.

(*a*) Relate the angle θ subtended by an arc of length S to the distance R:

$$\theta = \frac{S}{R} \qquad (1)$$

Solve for and evaluate S:

$$S = R\theta = (1\,\text{parsec})(1\,\text{s})\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\times\left(\frac{1°}{60\,\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{360°}\right) = \boxed{4.85\times10^{-6}\,\text{parsec}}$$

(*b*) Solve equation (1) for and evaluate R:

$$R = \frac{S}{\theta} = \frac{1.496\times10^{11}\,\text{m}}{(1\,\text{s})\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{1°}{60\,\text{min}}\right)\left(\frac{2\pi\,\text{rad}}{360°}\right)} = \boxed{3.09\times10^{16}\,\text{m}}$$

(*c*) Relate the distance D light travels in a given interval of time Δt to its speed c and evaluate D for $\Delta t = 1$ y:

$$D = c\Delta t = \left(3\times10^{8}\,\frac{\text{m}}{\text{s}}\right)(1\,\text{y})\left(3.156\times10^{7}\,\frac{\text{s}}{\text{y}}\right) = \boxed{9.47\times10^{15}\,\text{m}}$$

(*d*) Use the definition of 1 AU and the result from part (*c*) to obtain:

$$1c\cdot\text{y} = \left(9.47\times10^{15}\text{ m}\right)\left(\frac{1\text{AU}}{1.496\times10^{11}\text{ m}}\right) = \boxed{6.33\times10^{4}\text{ AU}}$$

(*e*) Combine the results of parts (*b*) and (*c*) to obtain:

$$1\text{parsec} = \left(3.08\times10^{16}\text{ m}\right)\times\left(\frac{1c\cdot\text{y}}{9.47\times10^{15}\text{ m}}\right) = \boxed{3.25c\cdot\text{y}}$$

58 ••

Picture the Problem Let N_e and N_p represent the number of electrons and the number of protons, respectively and ρ the critical average density of the universe. We can relate these quantities to the masses of the electron and proton using the definition of density.

(*a*) Using its definition, relate the required density ρ to the electron density N_e/V:

$$\rho = \frac{m}{V} = \frac{N_\text{e}m_\text{e}}{V}$$

Solve for N_e/V:

$$\frac{N_\text{e}}{V} = \frac{\rho}{m_\text{e}} \qquad (1)$$

Substitute numerical values and evaluate N_e/V:

$$\frac{N_\text{e}}{V} = \frac{6\times10^{-27}\text{ kg/m}^3}{9.11\times10^{-31}\text{ kg/electron}} = \boxed{6.59\times10^{3}\text{ electrons/m}^3}$$

(*b*) Express and evaluate the ratio of the masses of an electron and a proton:

$$\frac{m_\text{e}}{m_\text{p}} = \frac{9.11\times10^{-31}\text{ kg}}{1.67\times10^{-27}\text{ kg}} = 5.46\times10^{-4}$$

Rewrite equation (1) in terms of protons:

$$\frac{N_\text{p}}{V} = \frac{\rho}{m_\text{p}} \qquad (2)$$

Divide equation (2) by equation (1) to obtain:

$$\frac{\frac{N_\text{p}}{V}}{\frac{N_\text{e}}{V}} = \frac{m_\text{e}}{m_\text{p}} \text{ or } \frac{N_\text{p}}{V} = \frac{m_\text{e}}{m_\text{p}}\left(\frac{N_\text{e}}{V}\right)$$

Substitute numerical values and use the result from part (*a*) to evaluate N_p/V:

$$\frac{N_p}{V} = \left(5.46\times10^{-4}\right)\times\left(6.59\times10^{3}\ \text{protons/m}^3\right) = \boxed{3.59\ \text{protons/m}^3}$$

***59** ••

Picture the Problem We can use the definition of density to relate the mass of the water in the cylinder to its volume and the formula for the volume of a cylinder to express the volume of water used in the detector's cylinder. To convert our answer in kg to lb, we can use the fact that 1 kg weighs about 2.205 lb.

Relate the mass of water contained in the cylinder to its density and volume:

$$m = \rho V$$

Express the volume of a cylinder in terms of its diameter d and height h:

$$V = A_{\text{base}}h = \frac{\pi}{4}d^2h$$

Substitute to obtain:

$$m = \rho\frac{\pi}{4}d^2h$$

Substitute numerical values and evaluate m:

$$m = \left(10^3\ \text{kg/m}^3\right)\left(\frac{\pi}{4}\right)\left(39.3\ \text{m}\right)^2\left(41.4\ \text{m}\right) = 5.02\times10^7\ \text{kg}$$

Convert 5.02×10^7 kg to tons:

$$m = 5.02\times10^7\ \text{kg}\times\frac{2.205\ \text{lb}}{\text{kg}}\times\frac{1\ \text{ton}}{2000\ \text{lb}} = 55.4\times10^3\ \text{ton}$$

$$\boxed{\text{The 50,000 – ton claim is conservative. The actual weight is closer to 55,000 tons.}}$$

60 •••

Picture the Problem We'll solve this problem two ways. First, we'll substitute two of the ordered pairs in the given equation to obtain two equations in C and n that we can solve simultaneously. Then we'll use a spreadsheet program to create a graph of log T as a function of log m and use its curve-fitting capability to find n and C. Finally, we can identify the data points that deviate the most from a straight-line plot by examination of the graph.

1st Solution for (*a*)

(*a*) To estimate C and n, we can apply the relation $T = Cm^n$ to two arbitrarily selected data points. We'll use the 1st and 6th ordered pairs. This will produce simultaneous equations that can be solved for C and n.

$T_1 = Cm_1^n$

and

$T_6 = Cm_6^n$

Divide the second equation by the first to obtain:

$$\frac{T_6}{T_1} = \frac{Cm_6^n}{Cm_1^2} = \left(\frac{m_6}{m_1}\right)^n$$

Substitute numerical values and solve for n to obtain:

$$\frac{1.75\,\text{s}}{0.56\,\text{s}} = \left(\frac{1\,\text{kg}}{0.1\,\text{kg}}\right)^n$$

or

$$3.125 = 10^n \Rightarrow n = \boxed{0.4948}$$

and so a ″judicial″ guess is that $n = 0.5$.

Substituting this value into the second equation gives:

$T_5 = Cm_5^{0.5}$

so

$1.75\,\text{s} = \text{C}(1\,\text{kg})^{0.5}$

Solving for C gives:

$$C = \boxed{1.75\,\text{s/kg}^{0.5}}$$

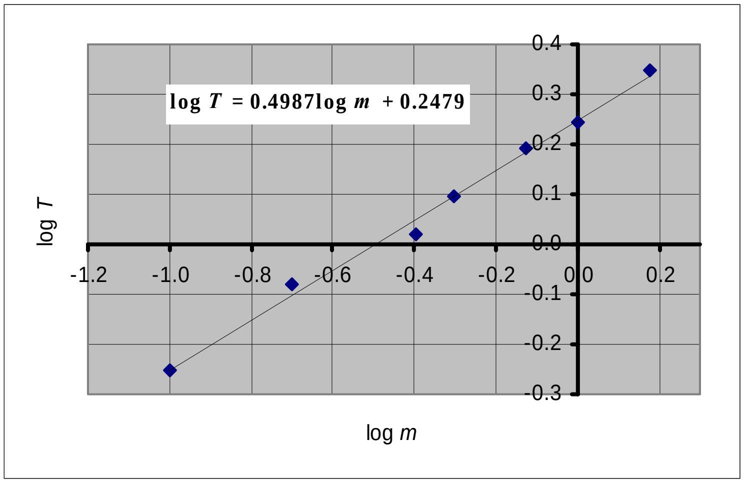
2nd Solution for (*a*)

Take the logarithm (we'll arbitrarily use base 10) of both sides of $T = Cm^n$ and simplify to obtain:

$$\log(T) = \log(Cm^n) = \log C + \log m^n$$
$$= n\log m + \log C$$

which, we note, is of the form $y = mx + b$. Hence a graph of log T vs. log m should be linear with a slope of n and a log T-intercept log C.

The graph of log T vs. log m shown below was created using a spreadsheet program. The equation shown on the graph was obtained using Excel's ″Add Trendline″ function. (Excel's ″Add Trendline″ function uses regression analysis to generate the trendline.)



Comparing the equation on the graph generated by the Add Trendline function to $\log(T) = n \log m + \log C$, we observe:

$$n = \boxed{0.499}$$

and

$$C = 10^{0.2479} = \boxed{1.77\,\text{s/kg}^{1/2}}$$

or

$$T = \boxed{\left(1.77\,\text{s/kg}^{1/2}\right)m^{0.499}}$$

(b) From the graph we see that the data points that deviate the most from a straight-line plot are:

$$\boxed{\begin{array}{l} m = 0.02\text{ kg}, T = 0.471\text{ s}, \\ \text{and} \\ m = 1.50\text{ kg}, T = 2.22\text{ s} \end{array}}$$

(b) $\boxed{\begin{array}{l}\text{From the graph we see that the points generated using the data pairs} \\ (0.02\text{ kg}, 0.471\text{ s})\text{ and }(0.4\text{ kg}, 1.05\text{ s})\text{ deviate the most from the line} \\ \text{representing the best fit to the points plotted on the graph.}\end{array}}$

Remarks: Still another way to find *n* and *C* is to use your graphing calculator to perform regression analysis on the given set of data for log *T* versus log *m*. The slope yields *n* and the *y*-intercept yields log *C*.

61 •••

Picture the Problem We can plot log T versus log r and find the slope of the best-fit line to determine the exponent n. We can then use any of the ordered pairs to evaluate C. Once we know n and C, we can solve $T = Cr^n$ for r as a function of T.

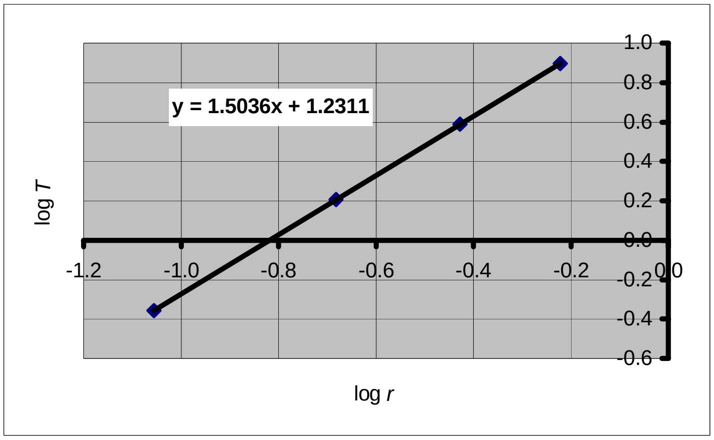
(a) Take the logarithm (we'll arbitrarily use base 10) of both sides of $T = Cr^n$ and simplify to obtain:

$$\log(T) = \log\left(Cr^n\right) = \log C + \log r^n$$
$$= n \log r + \log C$$

Note that this equation is of the form

$y = mx + b$. Hence a graph of log T vs. log r should be linear with a slope of n and a log T-intercept log C.

The graph of log T versus log r shown below was created using a spreadsheet program. The equation shown on the graph was obtained using Excel's ″Add Trendline″ function. (Excel's ″Add Trendline″ function uses regression analysis to generate the trendline.)



From the regression analysis we observe that:

$n = \boxed{1.50}$

and

$C = 10^{1.2311} = \boxed{17.0\,\text{y}/(\text{Gm})^{3/2}}$

or $T = \boxed{\left(17.0\,\text{y}/(\text{Gm})^{3/2}\right) r^{1.50}}$ (1)

(*b*) Solve equation (1) for the radius of the planet's orbit:

$$r = \left(\frac{T}{17.0\,\text{y}/(\text{Gm})^{3/2}}\right)^{2/3}$$

Substitute numerical values and evaluate *r*:

$$r = \left(\frac{6.20\,\text{y}}{17.0\,\text{y}/(\text{Gm})^{3/2}}\right)^{2/3} = \boxed{0.510\ \text{Gm}}$$

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Picture the Problem We can express the relationship between the period T of the pendulum, its length L, and the acceleration of gravity g as $T = CL^a g^b$ and perform dimensional analysis to find the values of a and b and, hence, the function relating these variables. Once we've performed the experiment called for in part (*b*), we can determine an experimental value for C.

(*a*) Express T as the product of L

$$T = CL^a g^b \qquad (1)$$

and g raised to powers a and b: where C is a dimensionless constant.

Write this equation in dimensional form:

$$[T]=[L]^a[g]^b$$

Noting that the symbols for the dimension of the period and length of the pendulum are the same as those representing the physical quantities, substitute the dimensions to obtain:

$$T = L^a\left(\frac{L}{T^2}\right)^b$$

Because L does not appear on the left-hand side of the equation, we can write this equation as:

$$L^0T^1 = L^{a+b}T^{-2b}$$

Equate the exponents to obtain:

$$a+b=0 \text{ and } -2b=1$$

Solve these equations simultaneously to find a and b:

$$a=\tfrac{1}{2} \text{ and } b=-\tfrac{1}{2}$$

Substitute in equation (1) to obtain:

$$T = CL^{1/2}g^{-1/2} = \boxed{C\sqrt{\frac{L}{g}}} \qquad (2)$$

(*b*) If you use pendulums of lengths 1 m and 0.5 m; the periods should be about:

$$T(1\,\text{m}) = \boxed{2\,\text{s}}$$

and

$$T(0.5\,\text{m}) = \boxed{1.4\,\text{s}}$$

(*c*) Solve equation (2) for C:

$$C = T\sqrt{\frac{g}{L}}$$

Evaluate C with L = 1 m and T = 2 s:

$$C = (2\,\text{s})\sqrt{\frac{9.81\,\text{m/s}^2}{1\,\text{m}}} = 6.26 \approx 2\pi$$

Substitute in equation (2) to obtain:

$$T = \boxed{2\pi\sqrt{\frac{L}{g}}}$$

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Picture the Problem The weight of the earth's atmosphere per unit area is known as the atmospheric pressure. We can use this definition to express the weight *w* of the earth's atmosphere as the product of the atmospheric pressure and the surface area of the earth.

Using its definition, relate atmospheric pressure to the weight of the earth's atmosphere:

$$P = \frac{w}{A}$$

Solve for *w*:

$$w = PA$$

Relate the surface area of the earth to its radius *R*:

$$A = 4\pi R^2$$

Substitute to obtain:

$$w = 4\pi R^2 P$$

Substitute numerical values and evaluate *w*:

$$w = 4\pi(6370\,\text{km})^2\left(\frac{10^3\,\text{m}}{\text{km}}\right)^2\left(\frac{39.37\,\text{in}}{\text{m}}\right)^2\left(14.7\,\frac{\text{lb}}{\text{in}^2}\right) = \boxed{1.16\times10^{19}\,\text{lb}}$$