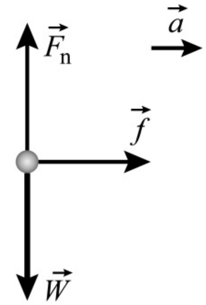
Chapter 5
Applications of Newton's Laws

Conceptual Problems

1 •

Determine the Concept Because the objects are speeding up (accelerating), there must be a net force acting on them. The forces acting on an object are the normal force exerted by the floor of the truck, the weight of the object, and the friction force; also exerted by the floor of the truck.

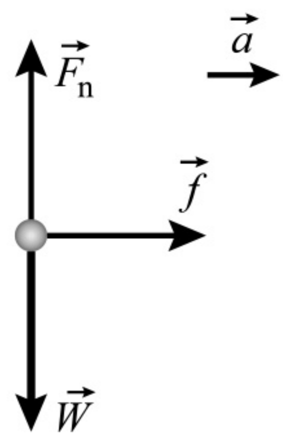


Of these forces, the only one that acts in the direction of the acceleration (chosen to be to the right in the free-body diagram) is the friction force.

The force of friction between the object and the floor of the truck must be the force that causes the object to accelerate.

***2** •

Determine the Concept The forces acting on an object are the normal force exerted by the floor of the truck, the weight of the object, and the friction force; also exerted by the floor of the truck. Of these forces, the only one that acts in the direction of the acceleration (chosen to be to the right in the free-body diagram) is the friction force. Apply Newton's 2^{nd} law to the object to determine how the critical acceleration depends on its weight.



Taking the positive x direction to be to the right, apply $\Sigma F_x = ma_x$ and solve for a_x:

$f = \mu_s w = \mu_s mg = ma_x$

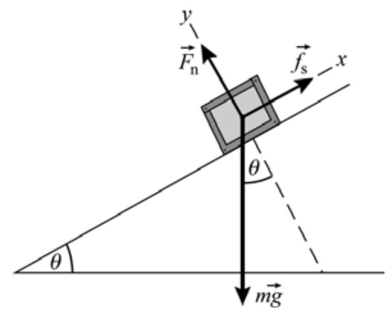
and

$a_x = \mu_s g$

Because a_x is independent of m and w, the critical accelerations are the same.

3 •

Determine the Concept The forces acting on the block are the normal force $\vec{F}_n$ exerted by the incline, the weight of the block $m\vec{g}$ exerted by the earth, and the static friction force $\vec{f}_s$ exerted by an external agent. We can use the definition of μ_s and the conditions for equilibrium to determine the relationship between μ_s and θ.



Apply $\sum F_x = ma_x$ to the block: $\quad f_s - mg\sin\theta = 0 \quad (1)$

Apply $\sum F_y = ma_y$ in the y direction: $\quad F_n - mg\cos\theta = 0 \quad (2)$

Divide equation (1) by equation (2) to obtain:

$$\tan\theta = \frac{f_s}{F_n}$$

Substitute for f_s ($\leq \mu_s F_n$):

$$\tan\theta \leq \frac{\mu_s F_n}{F_n} = \mu_s$$

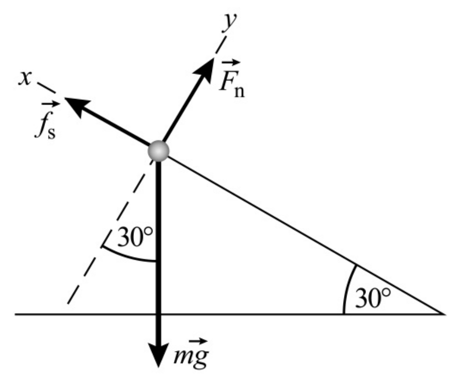
and $\boxed{(d) \text{ is correct.}}$

***4** •

Determine the Concept The block is in equilibrium under the influence of $\vec{F}_n$, $m\vec{g}$, and $\vec{f}_s$; i.e.,

$$\vec{F}_n + m\vec{g} + \vec{f}_s = 0$$

We can apply Newton's 2nd law in the x direction to determine the relationship between f_s and mg.



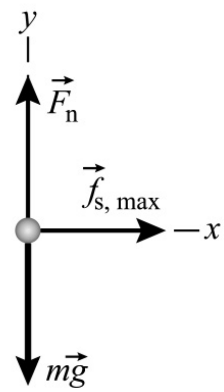
Apply $\sum F_x = 0$ to the block: $\quad f_s - mg\sin\theta = 0$

Solve for f_s: $\quad f_s = mg\sin\theta$

and $\boxed{(d) \text{ is correct.}}$

5 ••

Picture the Problem The forces acting on the car as it rounds a curve of radius R at maximum speed are shown on the free-body diagram to the right. The centripetal force is the static friction force exerted by the roadway on the tires. We can apply Newton's 2nd law to the car to derive an expression for its maximum speed and then compare the speeds under the two friction conditions described.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = f_{s,max} = m\frac{v_{max}^2}{R}$$

and

$$\sum F_y = F_n - mg = 0$$

From the y equation we have:

$$F_n = mg$$

Express $f_{s,max}$ in terms of F_n in the x equation and solve for v_{max}:

$$v_{max} = \sqrt{\mu_s gR}$$

or

$$v_{max} = \text{constant}\sqrt{\mu_s}$$

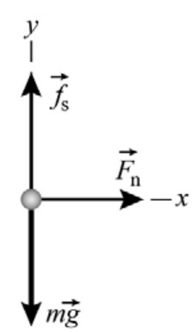
Express v'_{max} for $\mu'_s = \frac{1}{2}\mu_s$:

$$v'_{max} = \text{constant}\sqrt{\frac{\mu_s}{2}} = .707v_{max} \approx 71\%v_{max}$$

and $\boxed{(b)\text{ is correct.}}$

***6** ••

Picture the Problem The normal reaction force F_n provides the centripetal force and the force of static friction, $\mu_s F_n$, keeps the cycle from sliding down the wall. We can apply Newton's 2nd law and the definition of $f_{s,max}$ to derive an expression for v_{min}.



Apply $\sum \vec{F} = m\vec{a}$ to the motorcycle:

$$\sum F_x = F_n = m\frac{v^2}{R}$$

and

$$\sum F_y = f_s - mg = 0$$

For the minimum speed:

$$f_s = f_{s,max} = \mu_s F_n$$

Substitute for f_s, eliminate F_n between the force equations, and solve for v_{min}:

$$v_{min} = \sqrt{\frac{Rg}{\mu_s}}$$

Assume that $R = 6$ m and $\mu_s = 0.8$ and solve for v_{min}:

$$v_{min} = \sqrt{\frac{(6\,\text{m})(9.81\,\text{m/s}^2)}{0.8}}$$

$$= \boxed{8.58\,\text{m/s} = 30.9\,\text{km/h}}$$

7 ••

Determine the Concept As the spring is extended, the force exerted by the spring on the block increases. Once that force is greater than the maximum value of the force of static friction on the block, the block will begin to move. However, as it accelerates, it will shorten the length of the spring, decreasing the force that the spring exerts on the block. As this happens, the force of kinetic friction can then slow the block to a stop, which starts the cycle over again. One interesting application of this to the real world is the bowing of a violin string: The string under tension acts like the spring, while the bow acts as the block, so as the bow is dragged across the string, the string periodically sticks and frees itself from the bow.

8 •

True. The velocity of an object moving in a circle is continually changing independently of whether the object's speed is changing. The change in the velocity vector and the acceleration vector and the net force acting on the object all point toward the center of circle. This center-pointing force is called a centripetal force.

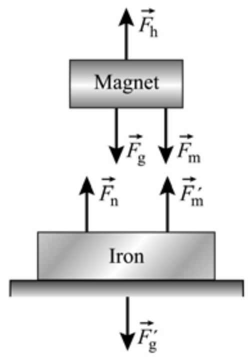
9 •

Determine the Concept A particle traveling in a vertical circle experiences a downward gravitational force plus an additional force that constrains it to move along a circular path. Because the net force acting on the particle will vary with location along its trajectory, neither (*b*), (*c*), nor (*d*) can be correct. Because the velocity of a particle moving along a circular path is continually changing, (*a*) cannot be correct. $\boxed{(e)\text{ is correct.}}$

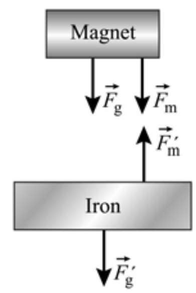
***10** •

Determine the Concept We can analyze these demonstrations by drawing force diagrams for each situation. In both diagrams, h denotes ″hand″, g denotes ″gravitational″, m denotes ″magnetic″, and n denotes ″normal″.

(*a*) Demonstration 1:



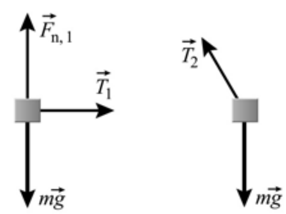
Demonstration 2:



(*b*) Because the magnet doesn't lift the iron in the first demonstration, the force exerted on the iron must be less than its (the iron's) weight. This is still true when the two are falling, but the motion of the iron is not restrained by the table, and the motion of the magnet is not restrained by the hand. Looking at the second diagram, the net force pulling the magnet down is greater than its weight, implying that its acceleration is greater than g. The opposite is true for the iron: the magnetic force acts upwards, slowing it down, so its acceleration will be less than g. Because of this, the magnet will catch up to the iron piece as they fall.

***11 •••**

Picture the Problem The free-body diagrams show the forces acting on the two objects some time after block 2 is dropped. Note that, while $\vec{T}_1 \neq \vec{T}_2$, $T_1 = T_2$.



The only force pulling block 2 to the left is the horizontal component of the tension. Because this force is smaller than the magnitude of the tension, the acceleration of block 1, which is identical to block 2, to the right ($T_1 = T_2$) will always be greater than the acceleration of block 2 to the left.

Because the initial distance from block 1 to the pulley is the same as the initial distance of block 2 to the wall, block 1 will hit the pulley before block 2 hits the wall.

12 •

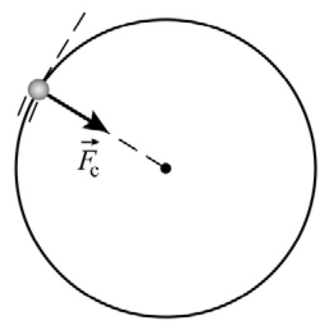
True. The terminal speed of an object is given by $v_t = (mg/b)^{1/n}$, where b depends on the shape and area of the falling object as well as upon the properties of the medium in which the object is falling.

13 •

Determine the Concept The terminal speed of a sky diver is given by $v_t = (mg/b)^{1/n}$, where b depends on the shape and area of the falling object as well as upon the properties of the medium in which the object is falling. The sky diver's orientation as she falls determines the surface area she presents to the air molecules that must be pushed aside. $\boxed{(d) \text{ is correct.}}$

14 ••

Determine the Concept In your frame of reference (the accelerating reference frame of the car), the direction of the force must point toward the center of the circular path along which you are traveling; that is, in the direction of the centripetal force that keeps you moving in a circle. The friction between you and the seat you are sitting on supplies this force. The reason you seem to be "pushed" to the outside of the curve is that your body's inertia "wants" , in accordance with Newton's law of inertia, to keep it moving in a straight line–that is, tangent to the curve.



***15 •**

Determine the Concept The centripetal force that keeps the moon in its orbit around the earth is provided by the gravitational force the earth exerts on the moon. As described by Newton's 3rd law, this force is equal in magnitude to the force the moon exerts on the earth. $\boxed{(d) \text{ is correct.}}$

16 •

Determine the Concept The only forces acting on the block are its weight and the force the surface exerts on it. Because the loop-the-loop surface is frictionless, the force it exerts on the block must be perpendicular to its surface.

Point A: the weight is downward and the normal force is to the right.	Free-body diagram 3
Point B: the weight is downward, the normal force is upward, and the normal force is greater than the weight so that their difference is the centripetal force.	Free-body diagram 4
Point C: the weight is downward and the normal force is to the left.	Free-body diagram 5
Point D: both the weight and the normal forces are downward.	Free-body diagram 2

17 ••

Picture the Problem Assume that the drag force on an object is given by the Newtonian formula $F_D = \frac{1}{2}CA\rho v^2$, where A is the projected surface area, v is the object's speed, ρ is the density of air, and C a dimensionless coefficient.

Express the net force acting on the falling object:

$$F_{net} = mg - F_D = ma$$

Substitute for F_D under terminal speed conditions and solve for the terminal speed:

$$mg - \tfrac{1}{2}CA\rho v_T^2 = 0$$

or

$$v_T = \sqrt{\frac{2mg}{CA\rho}}$$

Thus, the terminal velocity depends on the ratio of the mass of the object to its surface area.

For a rock, which has a relatively small surface area compared to its mass, the terminal speed will be relatively high; for a lightweight, spread-out object like a feather, the opposite is true.

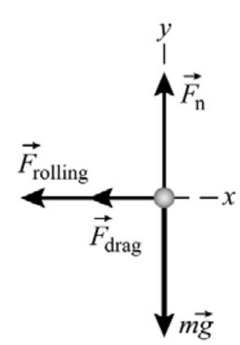
Another issue is that the higher the terminal velocity is, the longer it takes for a falling object to reach terminal velocity. From this, the feather will reach its terminal velocity quickly, and fall at an almost constant speed very soon after being dropped; a rock, if not dropped from a great height, will have almost the same acceleration as if it were in free-fall for the duration of its fall, and thus be continually speeding up as it falls.

An interesting point is that the average drag force acting on the rock will be larger than that acting on the feather precisely *because* the rock's average speed is larger than the feather's, as the drag force increases as v^2. This is another reminder that force is not the same thing as acceleration.

Estimation and Approximation

***18 •**

Picture the Problem The free-body diagram shows the forces on the Tercel as it slows from 60 to 55 mph. We can use Newton's 2nd law to calculate the average force from the rate at which the car's speed decreases and the rolling force from its definition. The drag force can be inferred from the average and rolling friction forces and the drag coefficient from the defining equation for the drag force.



(*a*) Apply $\sum F_x = ma_x$ to the car to relate the average force acting on it to its average velocity:

$$F_{av} = ma_{av} = m\frac{\Delta v}{\Delta t}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = (1020\,\text{kg})\frac{5\frac{\text{mi}}{\text{h}}\times 1.609\frac{\text{km}}{\text{mi}}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1000\,\text{m}}{\text{km}}}{3.92\,\text{s}} = \boxed{581\,\text{N}}$$

(*b*) Using its definition, express and evaluate the force of rolling friction:

$$f_{\text{rolling}} = \mu_{\text{rolling}}F_{\text{n}} = \mu_{\text{rolling}}mg$$
$$= (0.02)(1020\,\text{kg})(9.81\,\text{m/s}^2)$$
$$= \boxed{200\,\text{N}}$$

Assuming that only two forces are acting on the car in the direction of its motion, express their relationship and solve for and evaluate the drag force:

$$F_{\text{av}} = F_{\text{drag}} + F_{\text{rolling}}$$
and
$$F_{\text{drag}} = F_{\text{av}} - F_{\text{rolling}}$$
$$= 581\,\text{N} - 200\,\text{N} = \boxed{381\,\text{N}}$$

(*c*) Convert 57.5 mi/h to m/s:

$$57.5\frac{\text{mi}}{\text{h}} = 57.5\frac{\text{mi}}{\text{h}}\times\frac{1.609\,\text{km}}{\text{mi}}$$
$$\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{10^3\,\text{m}}{\text{km}}$$
$$= 25.7\,\text{m/s}$$

Using the definition of the drag force and its calculated value from (*b*) and the average speed of the car during this 5 mph interval, solve for *C*:

$$F_{\text{drag}} = \tfrac{1}{2}C\rho Av^2 \Rightarrow C = \frac{2F_{\text{drag}}}{\rho Av^2}$$

Substitute numerical values and evaluate *C*:

$$C = \frac{2(381\,\text{N})}{(1.21\,\text{kg/m}^3)(1.91\,\text{m}^2)(25.7\,\text{m/s})^2}$$
$$= \boxed{0.499}$$

19 •

Picture the Problem We can use the dimensions of force and velocity to determine the dimensions of the constant *b* and the dimensions of ρ, r, and v to show that, for $n = 2$, Newton's expression is consistent dimensionally with our result from part (*b*). In parts (*d*) and (*e*), we can apply Newton's 2nd law under terminal velocity conditions to find the terminal velocity of the sky diver near the surface of the earth and at a height of 8 km.

(*a*) Solve the drag force equation for *b* with $n = 1$:

$$b = \frac{F_{\text{d}}}{v}$$

Substitute the dimensions of F_d and v and simplify to obtain:

$$[b]=\frac{\frac{ML}{T^2}}{\frac{L}{T}}=\boxed{\frac{M}{T}}$$

and the units of b are $\boxed{\text{kg/s}}$

(*b*) Solve the drag force equation for b with $n = 2$:

$$b=\frac{F_d}{v^2}$$

Substitute the dimensions of F_d and v and simplify to obtain:

$$[b]=\frac{\frac{ML}{T^2}}{\left(\frac{L}{T}\right)^2}=\boxed{\frac{M}{L}}$$

and the units of b are $\boxed{\text{kg/m}}$

(*c*) Express the dimensions of Newton's expression:

$$[F_d]=\left[\tfrac{1}{2}\rho\pi r^2v^2\right]=\left(\frac{M}{L^3}\right)(L)^2\left(\frac{L}{T}\right)^2$$

$$=\boxed{\frac{ML}{T^2}}$$

From part (*b*) we have:

$$[F_d]=[bv^2]=\left(\frac{M}{L}\right)\left(\frac{L}{T}\right)^2$$

$$=\boxed{\frac{ML}{T^2}}$$

(*d*) Letting the downward direction be the positive y direction, apply $\sum F_y = ma_y$ to the sky diver:

$$mg-\tfrac{1}{2}\rho\pi r^2v_t^2=0$$

Solve for and evaluate v_t:

$$v_t=\sqrt{\frac{2mg}{\rho\pi r^2}}=\sqrt{\frac{2(56\,\text{kg})(9.81\,\text{m/s}^2)}{\pi(1.2\,\text{kg/m}^3)(0.3\,\text{m})^2}}$$

$$=\boxed{56.9\,\text{m/s}}$$

(*e*) Evaluate v_t at a height of 8 km:

$$v_t=\sqrt{\frac{2(56\,\text{kg})(9.81\,\text{m/s}^2)}{\pi(0.514\,\text{kg/m}^3)(0.3\,\text{m})^2}}$$

$$=\boxed{86.9\,\text{m/s}}$$

20 ••

Picture the Problem From Newton's 2nd law, the equation describing the motion of falling raindrops and large hailstones is $mg - F_d = ma$ where $F_d = \frac{1}{2}\rho\pi r^2 v^2 = bv^2$ is the drag force. Under terminal speed conditions ($a = 0$), the drag force is equal to the weight of the falling object. Take the radius of a raindrop r_r to be 0.5 mm and the radius of a golf-ball sized hailstone r_h to be 2 cm.

Using $b = \frac{1}{2}\pi\rho r^2$, evaluate b_r and b_h:

$$b_r = \tfrac{1}{2}\pi\left(1.2\,\text{kg/m}^3\right)\left(0.5\times10^{-3}\,\text{m}\right)^2 = 4.71\times10^{-7}\,\text{kg/m}$$

and

$$b_h = \tfrac{1}{2}\pi\left(1.2\,\text{kg/m}^3\right)\left(2\times10^{-2}\,\text{m}\right)^2 = 7.54\times10^{-4}\,\text{kg/m}$$

Express the mass of a sphere in terms of its volume and density:

$$m = \rho V = \frac{4\pi r^3\rho}{3}$$

Using $\rho_r = 10^3$ kg/m^3 and $\rho_h = 920$ kg/m^3, evaluate m_r and m_h:

$$m_r = \frac{4\pi\left(0.5\times10^{-3}\,\text{m}\right)^3\left(10^3\,\text{kg/m}^3\right)}{3} = 5.24\times10^{-7}\,\text{kg}$$

and

$$m_h = \frac{4\pi\left(2\times10^{-2}\,\text{m}\right)^3\left(920\,\text{kg/m}^3\right)}{3} = 3.08\times10^{-2}\,\text{kg}$$

Express the relationship between v_t and the weight of a falling object under terminal speed conditions and solve for v_t:

$$bv_t^2 = mg \Rightarrow v_t = \sqrt{\frac{mg}{b}}$$

Use numerical values to evaluate $v_{t,r}$ and $v_{t,h}$:

$$v_{t,r} = \sqrt{\frac{\left(5.24\times10^{-7}\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)}{4.71\times10^{-7}\,\text{kg/m}}} = \boxed{3.30\,\text{m/s}}$$

and

$$v_{t,h} = \sqrt{\frac{\left(3.08\times10^{-2}\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)}{7.54\times10^{-4}\,\text{kg/m}}} = \boxed{20.0\,\text{m/s}}$$

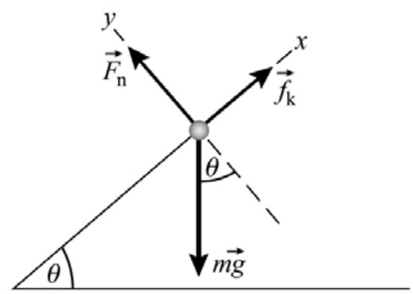
Friction

***21** •

Picture the Problem The block is in equilibrium under the influence of $\vec{\boldsymbol{F}}_{\text{n}}$, $m\vec{\boldsymbol{g}}$, and $\vec{\boldsymbol{f}}_{\text{k}}$; i.e.,

$$\vec{\boldsymbol{F}}_{\text{n}} + m\vec{\boldsymbol{g}} + \vec{\boldsymbol{f}}_{\text{k}} = 0$$

We can apply Newton's 2nd law to determine the relationship between f_k, θ, and mg.



Using its definition, express the coefficient of kinetic friction:

$$\mu_k = \frac{f_k}{F_n} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$f_k - mg\sin\theta = ma_x = 0$ because $a_x = 0$

Solve for f_k:

$f_k = mg\sin\theta$

Apply $\sum F_y = ma_y$ to the block:

$F_n - mg\cos\theta = ma_y = 0$ because $a_y = 0$

Solve for F_n:

$F_n = mg\cos\theta$

Substitute in equation (1) to obtain:

$$\mu_k = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

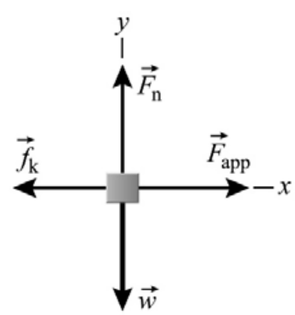
and $\boxed{(b) \text{ is correct.}}$

22 •

Picture the Problem The block is in equilibrium under the influence of $\vec{\boldsymbol{F}}_{\text{n}}$, $m\vec{\boldsymbol{g}}$, $\vec{\boldsymbol{F}}_{\text{app}}$, and $\vec{\boldsymbol{f}}_{\text{k}}$; i.e.,

$$\vec{\boldsymbol{F}}_{\text{n}} + m\vec{\boldsymbol{g}} + \vec{F}_{\text{app}} + \vec{\boldsymbol{f}}_{\text{k}} = 0$$

We can apply Newton's 2nd law to determine f_k.



Apply $\sum F_x = ma_x$ to the block:

$F_{app} - f_k = ma_x = 0$ because $a_x = 0$

Solve for f_k:

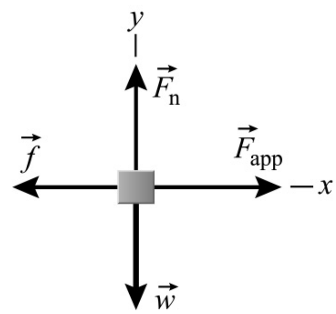
$f_k = F_{app} = 20\text{ N}$

and

$\boxed{(e) \text{ is correct.}}$

***23** •

Picture the Problem Whether the friction force is that due to static friction or kinetic friction depends on whether the applied tension is greater than the maximum static friction force. We can apply the definition of the maximum static friction to decide whether $f_{s,max}$ or T is greater.



Calculate the maximum static friction force:

$$f_{s,max} = \mu_s F_n = \mu_s w = (0.8)(20\text{ N}) = 16\text{ N}$$

(*a*) Because $f_{s,max} > T$:

$$f = f_s = T = \boxed{15.0\text{ N}}$$

(*b*) Because $T > f_{s,max}$:

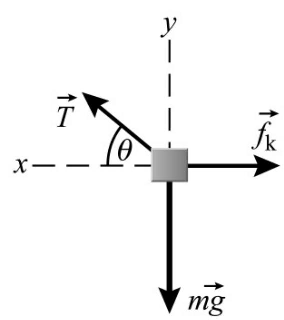
$$f = f_k = \mu_k w = (0.6)(20\text{ N}) = \boxed{12.0\text{ N}}$$

24 •

Picture the Problem The block is in equilibrium under the influence of the forces $\vec{T}$, $\vec{f}_k$, and $m\vec{g}$; i.e.,

$$\vec{T} + \vec{f}_k + m\vec{g} = 0$$

We can apply Newton's 2nd law to determine the relationship between T and f_k.



Apply $\sum F_x = ma_x$ to the block:

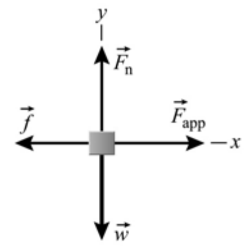
$$T\cos\theta - f_k = ma_x = 0 \text{ because } a_x = 0$$

Solve for f_k:

$$f_k = T\cos\theta \text{ and } \boxed{(b) \text{ is correct.}}$$

25 •

Picture the Problem Whether the friction force is that due to static friction or kinetic friction depends on whether the applied tension is greater than the maximum static friction force.



Calculate the maximum static friction force:

$$f_{s,max} = \mu_s F_n = \mu_s w = (0.6)(100\text{ kg})(9.81\text{ m/s}^2) = 589\text{ N}$$

Because $f_{s,max} > F_{app}$, the box does not move and :

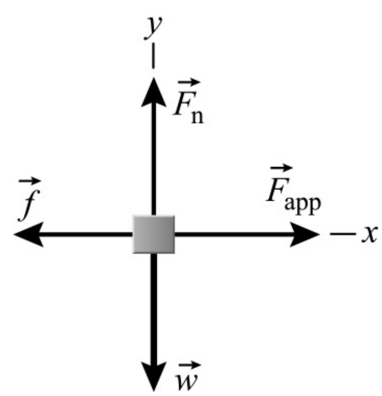
$$F_{app} = f_s = \boxed{500\text{ N}}$$

26 •

Picture the Problem Because the box is moving with constant velocity, its acceleration is zero and it is in equilibrium under the influence of $\vec{F}_{app}$, $\vec{F}_n$, $\vec{w}$, and $\vec{f}$; i.e.,

$$\vec{F}_{app} + \vec{F}_n + \vec{w} + \vec{f} = 0$$

We can apply Newton's 2nd law to determine the relationship between f and mg.



The definition of μ_k is:

$$\mu_k = \frac{f_k}{F_n}$$

Apply $\sum F_y = ma_y$ to the box:

$$F_n - w = ma_y = 0 \text{ because } a_y = 0$$

Solve for F_n:

$$F_n = w = 600\text{ N}$$

Apply $\sum F_x = ma_x$ to the box:

$$\Sigma F_x = F_{app} - f = ma_x = 0 \text{ because } a_x = 0$$

Solve for f_k:

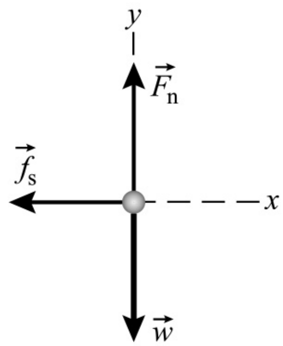
$$F_{app} = f_k = 250\text{ N}$$

Substitute to obtain μ_k:

$$\mu_k = (250\text{ N})/(600\text{ N}) = \boxed{0.417}$$

27 •

Picture the Problem Assume that the car is traveling to the right and let the positive x direction also be to the right. We can use Newton's 2nd law of motion and the definition of μ_s to determine the maximum acceleration of the car. Once we know the car's maximum acceleration, we can use a constant-acceleration equation to determine the least stopping distance.



(*a*) Apply $\sum F_x = ma_x$ to the car:

$$-f_{s,max} = -\mu_s F_n = ma_x \qquad (1)$$

Apply $\sum F_y = ma_y$ to the car and solve for F_n:

$$F_n - w = ma_y = 0$$

or, because $a_y = 0$,

$$F_n = mg \qquad (2)$$

Substitute (2) in (1) and solve for $a_{x,max}$:

$$a_{x,max} = \mu_s g = (0.6)(9.81\,\text{m/s}^2)$$
$$= \boxed{-5.89\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the stopping distance of the car to its initial velocity and its acceleration and solve for its displacement:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

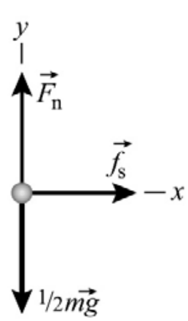
$$\Delta x = \frac{-v_0^2}{2a}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{-(30\,\text{m/s})^2}{2(-5.89\,\text{m/s}^2)} = \boxed{76.4\,\text{m}}$$

***28** •

Picture the Problem The free-body diagram shows the forces acting on the drive wheels, the ones we're assuming support half the weight of the car. We can use the definition of acceleration and apply Newton's 2nd law to the horizontal and vertical components of the forces to determine the minimum coefficient of friction between the road and the tires.



(*a*) $\boxed{\text{Because } \mu_s > \mu_k,\ f \text{ will be greater if the wheels do not slip.}}$

(*b*) Apply $\sum F_x = ma_x$ to the car:

$$f_s = \mu_s F_n = ma_x \qquad (1)$$

Apply $\sum F_y = ma_y$ to the car and solve for F_n:

$$F_n - \tfrac{1}{2}mg = ma_y$$

Because $a_y = 0$,

$$F_n - \tfrac{1}{2}mg = 0 \Rightarrow F_n = \tfrac{1}{2}mg$$

Find the acceleration of the car:

$$a_x = \frac{\Delta v}{\Delta t} = \frac{(90\,\text{km/h})(1000\,\text{m/km})}{12\,\text{s}} = 2.08\,\text{m/s}^2$$

Solve equation (1) for μ_s:

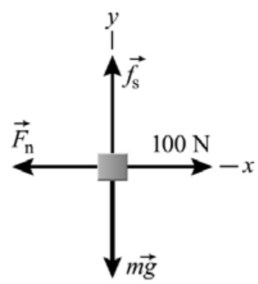
$$\mu_s = \frac{ma_x}{\frac{1}{2}mg} = \frac{2a_x}{g}$$

Substitute numerical values and evaluate a_x:

$$\mu_s = \frac{2(2.08\,\text{m/s}^2)}{9.81\,\text{m/s}^2} = \boxed{0.424}$$

29 •

Picture the Problem The block is in equilibrium under the influence of the forces shown on the free-body diagram. We can use Newton's 2nd law and the definition of μ_s to solve for f_s and F_n.



(*a*) Apply $\sum F_y = ma_y$ to the block and solve for f_s:

$$f_s - mg = ma_y$$

or, because $a_y = 0$,

$$f_s - mg = 0$$

Solve for and evaluate f_s:

$$f_s = mg = (5\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{49.1\,\text{N}}$$

(*b*) Use the definition of μ_s to express F_n:

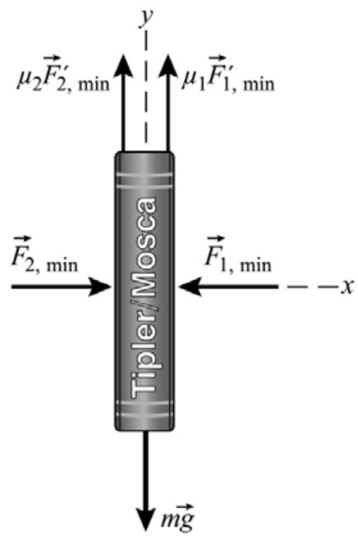
$$F_n = \frac{f_{s,\text{max}}}{\mu_s}$$

Substitute numerical values and evaluate F_n:

$$F_n = \frac{49.1\,\text{N}}{0.4} = \boxed{123\,\text{N}}$$

30 •

Picture the Problem The free-body diagram shows the forces acting on the book. The normal force is the net force the student exerts in squeezing the book. Let the horizontal direction be the x direction and upward the y direction. Note that the normal force is the same on either side of the book because it is not accelerating in the horizontal direction. The book could be accelerating downward. We can apply Newton's 2nd law to relate the minimum force required to hold the book in place to its mass and to the coefficients of static friction. In part (b), we can proceed similarly to relate the acceleration of the book to the coefficients of kinetic friction.



(a) Apply $\sum \vec{F} = m\vec{a}$ to the book:

$$\sum F_x = F_{2,\min} - F_{1,\min} = 0$$

and

$$\sum F_y = \mu_{s,1}F'_{1,\min} + \mu_{s,2}F'_{2,\min} - mg = 0$$

Noting that $F'_{1,\min} = F'_{2,\min}$, solve the y equation for $F_{\min}$:

$$F_{\min} = \frac{mg}{\mu_{,1} + \mu_{s,2}} = \frac{(10.2\,\text{kg})(9.81\,\text{m/s}^2)}{0.32 + 0.16}$$

$$= \boxed{208\,\text{N}}$$

(b) Apply $\sum F_y = ma_y$ with the book accelerating downward, to obtain:

$$\sum F_y = \mu_{k,1}F + \mu_{k,2}F - mg = ma$$

Solve for a to obtain:

$$a = \frac{\mu_{k,} + \mu_{k,2}}{m}F - g$$

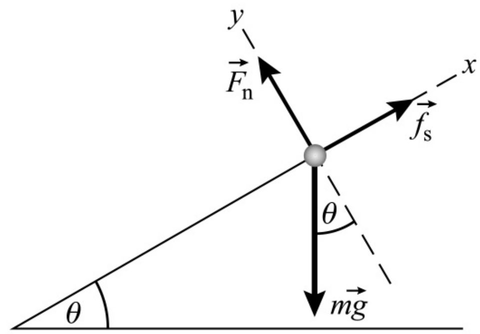
Substitute numerical values and evaluate a:

$$a = \frac{0.2 + 0.09}{10.2\,\text{kg}}(195\,\text{N}) - 9.81\,\text{m/s}^2$$

$$= \boxed{-4.27\,\text{m/s}^2}$$

31 •

Picture the Problem A free-body diagram showing the forces acting on the car is shown to the right. The friction force that the ground exerts on the tires is the force f_s shown acting up the incline. We can use the definition of the coefficient of static friction and Newton's 2nd law to relate the angle of the incline to the forces acting on the car.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = f_s - mg\sin\theta = 0 \qquad (1)$$

and

$$\sum F_y = F_n - mg\cos\theta = 0 \qquad (2)$$

Solve equation (1) for f_s and equation (2) for F_n:

$$f_s = mg\sin\theta$$

and

$$F_n = mg\cos\theta$$

Use the definition of μ_s to relate f_s and F_n:

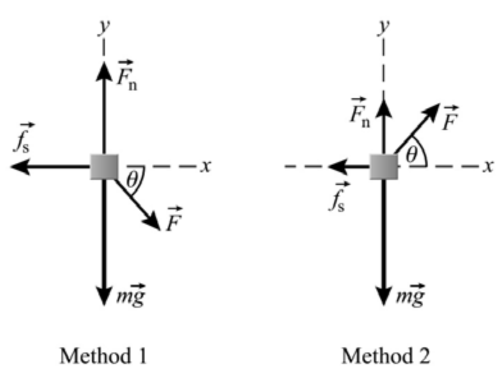
$$\mu_s = \frac{f_s}{F_n} = \frac{mg\sin\theta}{mg\cos\theta} = \tan\theta$$

Solve for and evaluate θ:

$$\theta = \tan^{-1}\mu_s = \tan^{-1}(0.08) = \boxed{4.57°}$$

***32** •

Picture the Problem The free-body diagrams for the two methods are shown to the right. Method 1 results in the box being pushed into the floor, increasing the normal force and the static friction force. Method 2 partially lifts the box,, reducing the normal force and the static friction force. We can apply Newton's 2nd law to obtain expressions that relate the maximum static friction force to the applied force $\vec{F}$.



(*a*) $\boxed{\text{Method 2 is preferable as it reduces } F_n \text{ and, therefore, } f_{s.}}$

(*b*) Apply $\sum F_x = ma_x$ to the box:

$$F\cos\theta - f_s = F\cos\theta - \mu_s F_n = 0$$

Method 1: Apply $\sum F_y = ma_y$ to the block and solve for F_n:

$$F_n - mg - F\sin\theta = 0$$
$$\therefore F_n = mg + F\sin\theta$$

Relate $f_{s,max}$ to F_n:

$$f_{s,max} = \mu_s F_n = \mu_s(mg + F\sin\theta) \qquad (1)$$

Method 2: Apply $\sum F_y = ma_y$ to the forces in the y direction and solve for F_n:

$$F_n - mg + F\sin\theta = 0$$
$$\therefore F_n = mg - F\sin\theta$$

Relate $f_{s,max}$ to F_n:

$$f_{s,max} = \mu_s F_n = \mu_s(mg - F\sin\theta) \qquad (2)$$

Express the condition that must be satisfied to move the box by either method:

$$f_{s,max} = F\cos\theta \qquad (3)$$

Method 1: Substitute (1) in (3) and solve for F:

$$F_1 = \frac{\mu_s mg}{\cos\theta - \mu_s\sin\theta} \qquad (4)$$

Method 2: Substitute (2) in (3) and solve for F:

$$F_2 = \frac{\mu_s mg}{\cos\theta + \mu_s\sin\theta} \qquad (5)$$

Evaluate (4) and (5) with $\theta = 30°$:

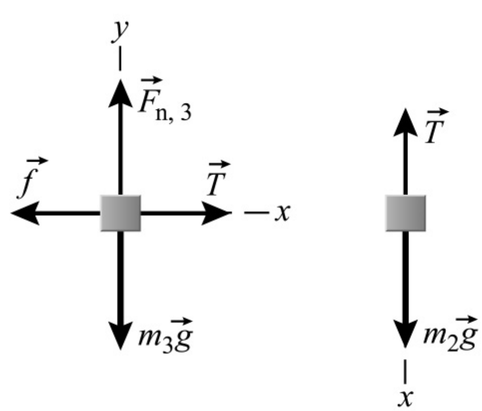
$$F_1(30°) = \boxed{520\ \text{N}}$$
$$F_2(30°) = \boxed{252\ \text{N}}$$

Evaluate (4) and (5) with $\theta = 0°$:

$$F_1(0°) = F_2(0°) = \mu_s mg = \boxed{294\ \text{N}}$$

33 •

Picture the Problem Draw a free-body diagram for each object. In the absence of friction, the 3-kg box will move to the right, and the 2-kg box will move down. The friction force is indicated by $\vec{f}$ without subscript; it is $\vec{f}_s$ for (*a*) and $\vec{f}_k$ for (*b*). For values of μ_s less than the value found in part (a) required for equilibrium, the system will accelerate and the fall time for a given distance can be found using a constant-acceleration equation.



(*a*) Apply $\sum F_x = ma_x$ to the 3-kg

$$T - f_s = 0 \text{ because } a_x = 0 \qquad (1)$$

box:

Apply $\sum F_y = ma_y$ to the 3-kg box, solve for $F_{n,3}$, and substitute in (1):

$F_{n,3} - m_3 g = 0$ because $a_y = 0$
and
$T - \mu_s m_3 g = 0$ (2)

Apply $\sum F_x = ma_x$ to the 2-kg box:

$m_2 g - T = 0$ because $a_x = 0$ (3)

Solve (2) and (3) simultaneously and solve for μ_s:

$$\mu_s = \frac{m_2}{m_3} = \boxed{0.667}$$

(b) The time of fall is related to the acceleration, which is constant:

$$\Delta x = v_0 \Delta t + \tfrac{1}{2} a (\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2} a (\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta x}{a}}$$

Apply $\sum F_x = ma_x$ to each box:

$T - \mu_k m_3 g = m_3 a$ (4)
and
$m_2 g - T = m_2 a$ (5)

Add equations (4) and (5) and solve for a:

$$a = \frac{(m_2 - \mu_k m_3) g}{m_2 + m_3} = \frac{[2\,\text{kg} - 0.3(3\,\text{kg})](9.81\,\text{m/s}^2)}{2\,\text{kg} + 3\,\text{kg}} = \boxed{2.16\,\text{m/s}^2}$$

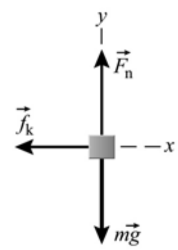
Substitute to obtain:

$$\Delta t = \sqrt{\frac{2(2\,\text{m})}{2.16\,\text{m/s}^2}} = \boxed{1.36\,\text{s}}$$

34 ••

Picture the Problem The application of Newton's 2nd law to the block will allow us to express the coefficient of kinetic friction in terms of the acceleration of the block. We can then use a constant-acceleration equation to determine the block's acceleration. The pictorial representation summarizes what we know about the motion.

A free-body diagram showing the forces acting on the block is shown to the right.



Apply $\sum F_x = ma_x$ to the block: $-f_k = -\mu_k F_n = ma$ (1)

Apply $\sum F_y = ma_y$ to the block and solve for F_n: $F_n - mg = 0$ because $a_y = 0$ and

$F_n = mg$ (2)

Substitute (2) in (1) and solve for μ_k: $\mu_k = -a/g$ (3)

Using a constant-acceleration equation, relate the initial and final velocities of the block to its displacement and acceleration:

$$v_1^2 = v_0^2 + 2a\Delta x$$

or, because $v_1 = 0$, $v_0 = v$, and $\Delta x = d$,

$$0 = v^2 + 2ad$$

Solve for a to obtain:

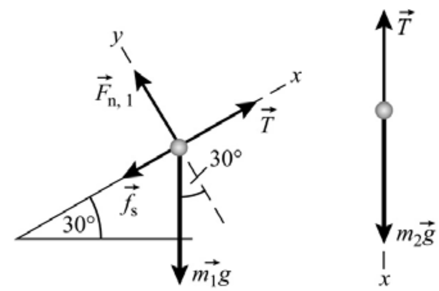
$$a = \frac{-v^2}{2d}$$

Substitute for a in equation (3) to obtain:

$$\mu_k = \boxed{\frac{v^2}{2gd}}$$

***35 ••**

Picture the Problem We can find the speed of the system when it has moved a given distance by using a constant-acceleration equation. Under the influence of the forces shown in the free-body diagrams, the blocks will have a common acceleration a. The application of Newton's 2nd law to each block, followed by the elimination of the tension T and the use of the definition of f_k, will allow us to determine the acceleration of the system.



Using a constant-acceleration equation, relate the speed of the system to its acceleration and displacement; solve for its speed:

$$v^2 = v_0^2 + 2a\Delta x$$

and, because $v_0 = 0$,

$$v = \sqrt{2a\Delta x}$$

Apply $\vec{F}_{net} = m\vec{a}$ to the block whose mass is m_1:

$$\Sigma F_x = T - f_k - m_1 g\sin 30° = m_1 a \qquad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos 30° = 0 \qquad (2)$$

Using $f_k = \mu_k F_n$, substitute (2) in (1) to obtain:

$$T - \mu_k m_1 g\cos 30° - m_1 g\sin 30° = m_1 a$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = m_2 a$$

Add the last two equations to eliminate T and solve for a to obtain:

$$a = \frac{(m_2 - \mu_k m_1 \cos 30° - m_1 \sin 30°)g}{m_1 + m_2}$$

$$= 1.16\,\text{m/s}^2$$

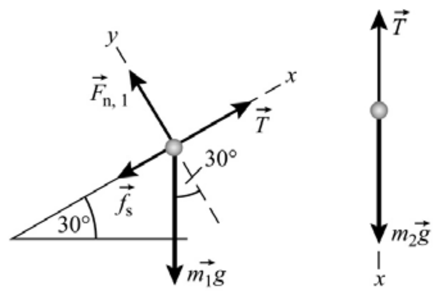
Substitute and evaluate a:

$$v = \sqrt{2(1.16\,\text{m/s}^2)(0.3\,\text{m})} = 0.835\,\text{m/s}$$

and $\boxed{(a)\text{ is correct.}}$

36 ••

Picture the Problem Under the influence of the forces shown in the free-body diagrams, the blocks are in static equilibrium. While f_s can be either up or down the incline, the free-body diagram shows the situation in which motion is impending up the incline. The application of Newton's 2nd law to each block, followed by the elimination of the tension T and the use of the definition of f_s, will allow us to determine the range of values for m_2.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_1:

$$\Sigma F_x = T \pm f_{s,max} - m_1 g \sin 30° = 0 \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g \cos 30° = 0 \quad (2)$$

Using $f_{s,max} = \mu_s F_n$, substitute (2) in (1) to obtain:

$$T \pm \mu_s m_1 g \cos 30° - m_1 g \sin 30° = m_1 a \quad (3)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = 0 \quad (4)$$

Add equations (3) and (4) to eliminate T and solve for m_2:

$$\begin{aligned} m_2 &= m_1(\pm \mu_s \cos 30° + \sin 30°) \\ &= (4\,\text{kg})[\pm(0.4)\cos 30° + \sin 30°] \end{aligned} \quad (5)$$

Evaluate (5) denoting the value of m_2 with the plus sign as $m_{2,+}$ and the value of m_2 with the minus sign as $m_{2,-}$ to determine the range of values of m_2 for which the system is in static equilibrium:

$$m_{2,+} = 3.39\,\text{kg and } \text{m}_{2,-} = 0.614\,\text{kg}$$

$$\therefore \boxed{0.614\,\text{kg} \le m_2 \le 3.39\,\text{kg}}$$

(*b*) With $m_2 = 1$ kg, the impending motion is down the incline and the static friction force is up the incline. Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$$T + f_s - m_1 g \sin 30° = 0 \quad (6)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

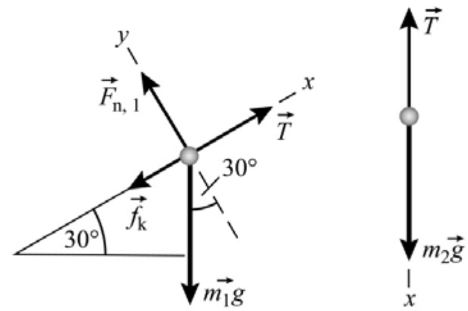
$$m_2 g - T = 0 \quad (7)$$

Add equations (6) and (7) and solve for and evaluate f_s:

$$f_s = (m_1\sin 30° - m_2)g$$
$$= [(4\text{ kg})\sin 30° - 1\text{ kg}](9.81\text{ m/s}^2)$$
$$= \boxed{9.81\,\text{N}}$$

37 ••

Picture the Problem Under the influence of the forces shown in the free-body diagrams, the blocks will have a common acceleration a. The application of Newton's 2nd law to each block, followed by the elimination of the tension T and the use of the definition of f_k, will allow us to determine the acceleration of the system. Finally, we can substitute for the tension in either of the motion equations to determine the acceleration of the masses.



Apply $\sum \vec{F} = m\vec{a}$ to the block whose mass is m_1:

$$\Sigma F_x = T - f_k - m_1 g\sin 30° = m_1 a \qquad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos 30° = 0 \qquad (2)$$

Using $f_k = \mu_k F_n$, substitute (2) in (1) to obtain:

$$T - \mu_k m_1 g\cos 30° - m_1 g\sin 30° = m_1 a \qquad (3)$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$m_2 g - T = m_2 a \qquad (4)$$

Add equations (3) and (4) to eliminate T and solve for and evaluate a to obtain:

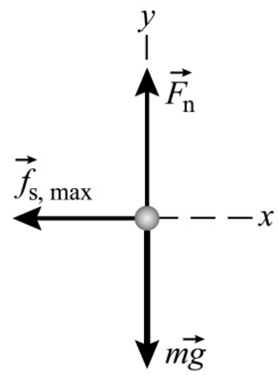
$$a = \frac{(m_2 - \mu_k m_1\cos 30° - m_1\sin 30°)g}{m_1 + m_2}$$
$$= \boxed{2.36\text{ m/s}^2}$$

Substitute for a in equation (3) to obtain:

$$T = \boxed{37.3\,\text{N}}$$

***38 ••**

Picture the Problem The truck will stop in the shortest possible distance when its acceleration is a maximum. The maximum acceleration is, in turn, determined by the maximum value of the static friction force. The free-body diagram shows the forces acting on the box as the truck brakes to a stop. Assume that the truck is moving in the positive x direction and apply Newton's 2nd law and the definition of $f_{s,max}$ to find the shortest stopping distance.



Using a constant-acceleration equation, relate the truck's stopping distance to its acceleration and initial velocity; solve for the stopping distance:

$$v^2 = v_0^2 + 2a\Delta x$$

or, since $v = 0$,

$$\Delta x_{min} = \sqrt{\frac{-v_0^2}{2a_{max}}}$$

Apply $\vec{F}_{net} = m\vec{a}$ to the block:

$$\Sigma F_x = -f_{s,max} = ma_{max} \qquad (1)$$

and

$$\Sigma F_y = F_n - mg = 0 \qquad (2)$$

Using the definition of $f_{s,max}$, solve equations (1) and (2) simultaneously for a:

$$f_{s,max} \equiv \mu_s F_n$$

and

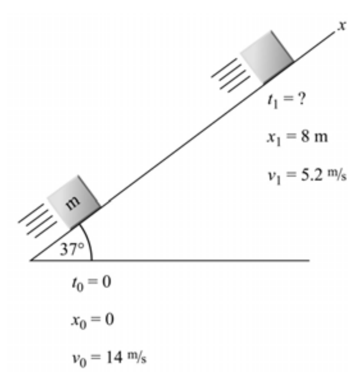
$$a_{max} = -\mu_s g = -(0.3)(9.81 \text{ m/s}^2) = -2.943 \text{ m/s}^2$$

Substitute numerical values and evaluate Δx_{min}:

$$\Delta x_{min} = \sqrt{\frac{-(80\,\text{km/h})^2(1000\,\text{km/m})^2(1\,\text{h}/3600\,\text{s})^2}{2(-2.943\,\text{m/s}^2)}} = \boxed{9.16\,\text{m}}$$

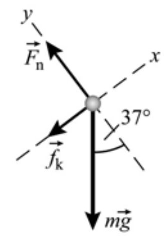
39 ••

Picture the Problem We can find the coefficient of friction by applying Newton's 2nd law and determining the acceleration from the given values of displacement and initial velocity. We can find the displacement and speed of the block by using constant-acceleration equations. During its motion up the incline,



the sum of the kinetic friction force and a component of the object's weight will combine to bring the object to rest. When it is moving down the incline, the difference between the weight component and the friction force will be the net force.

(*a*) Draw a free-body diagram for the block as it travels up the incline:



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = -f_k - mg\sin 37° = ma \quad (1)$$

and

$$\Sigma F_y = F_n - mg\cos 37° = 0 \quad (2)$$

Substitute $f_k = \mu_k F_n$ and F_n from (2) in (1) and solve for μ_k:

$$\mu_k = \frac{-g\sin 37° - a}{g\cos 37°} = -\tan 37° - \frac{a}{g\cos 37°} \quad (3)$$

Using a constant-acceleration equation, relate the final velocity of the block to its initial velocity, acceleration, and displacement; solve for and evaluate a:

$$v_1^2 = v_0^2 + 2a\Delta x$$

$$a = \frac{v_1^2 - v_0^2}{2\Delta x} = \frac{(5.2\,\text{m/s})^2 - (14\,\text{m/s})^2}{2(8\,\text{m})} = -10.6\,\text{m/s}^2$$

Substitute for a in (3) to obtain:

$$\mu_k = -\tan 37° - \frac{-10.6\,\text{m/s}^2}{(9.81\,\text{m/s}^2)\cos 37°} = \boxed{0.599}$$

(*b*) Use the same constant-acceleration equation used above but with $v_1 = 0$, solve for the displacement of the block as it slides to a stop:

$$v_1^2 = v_0^2 + 2a\Delta x \text{ where } v_1 = 0$$

and

$$\Delta x = \frac{-v_0^2}{2a} = \frac{-(14\,\text{m/s})^2}{2(-10.6\,\text{m/s}^2)} = \boxed{9.25\,\text{m}}$$

(*c*) When the block slides down the incline, f_k is in the positive x direction:

$$\Sigma F_x = f_k - mg\sin 37° = ma$$

and

$$\Sigma F_y = F_n - mg\cos 37° = 0$$

Solve for a as in part (a): $a = g(\mu_k \cos 37° - \sin 37°) = -1.21\,\text{m/s}^2$

Use the same constant-acceleration equation used in part (b) to obtain: $v^2 = v_0^2 + 2a\Delta x$

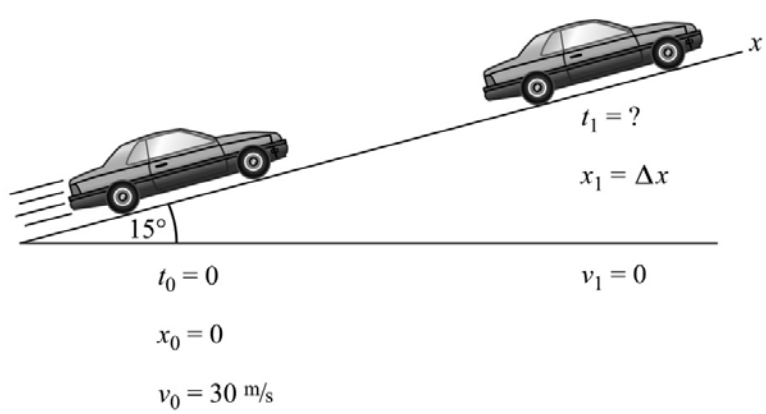
Set $v_0 = 0$ and solve for v: $v = \sqrt{2a\Delta x}$

Substitute numerical values and evaluate v:

$$v = \sqrt{2(-1.21\,\text{m/s}^2)(-9.25\,\text{m})} = \boxed{4.73\,\text{m/s}}$$

40 ••

Picture the Problem We can find the stopping distances by applying Newton's 2nd law to the automobile and then using a constant-acceleration equation. The friction force the road exerts on the tires and the component of the car's weight along the incline combine to provide the net force that stops the car. The pictorial representation summarizes what we know about the motion of the car. We can use Newton's 2nd law to determine the acceleration of the car and a constant-acceleration equation to obtain its stopping distance.



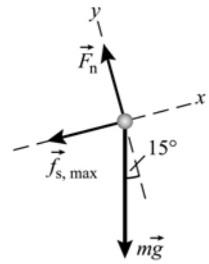
(a) Using a constant-acceleration equation, relate the final speed of the car to its initial speed, acceleration, and displacement; solve for its displacement:

$$v_1^2 = v_0^2 + 2a_{\text{max}}\Delta x_{\text{min}}$$

or, because $v_1 = 0$,

$$\Delta x_{\text{min}} = \frac{-v_0^2}{2a_{\text{max}}}$$

Draw the free-body diagram for the car going up the incline:



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\Sigma F_x = -f_{s,max} - mg\sin 15° = ma \qquad (1)$$

and

$$\Sigma F_y = F_n - mg\cos 15° = 0 \qquad (2)$$

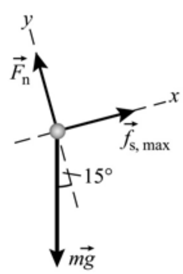
Substitute $f_{s,max} = \mu_s F_n$ and F_n from (2) in (1) and solve for a:

$$a_{max} = -g(\mu_s \cos 15° + \sin 15°) = -9.17\,\text{m/s}^2$$

Substitute to obtain:

$$\Delta x_{min} = \frac{-(30\,\text{m/s})^2}{2(-9.17\,\text{m/s}^2)} = \boxed{49.1\,\text{m}}$$

(*b*) Draw the free-body diagram for the car going down the incline:



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\Sigma F_x = f_{s,max} - mg\sin 15° = ma$$

and

$$\Sigma F_y = F_n - mg\cos 15° = 0$$

Proceed as in (*a*) to obtain a_{max}:

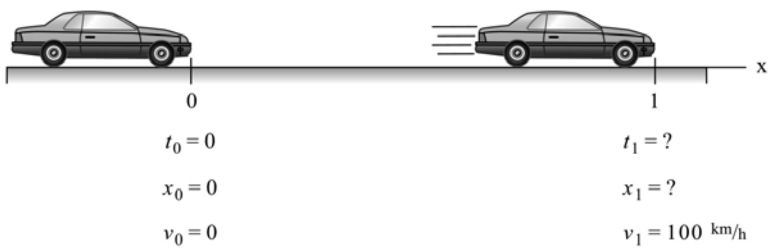
$$a_{max} = g(\mu_s \cos 15° - \sin 15°) = 4.09\,\text{m/s}^2$$

Again, proceed as in (*a*) to obtain the displacement of the car:

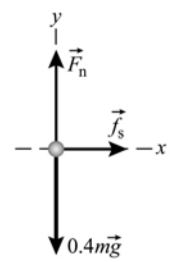
$$\Delta x_{min} = \left|\frac{-v_0^2}{2a_{max}}\right| = \frac{(30\,\text{m/s})^2}{2(4.09\,\text{m/s}^2)} = \boxed{110\,\text{m}}$$

41 ••

Picture the Problem The friction force the road exerts on the tires provides the net force that accelerates the car. The pictorial representation summarizes what we know about the motion of the car. We can use Newton's 2nd law to determine the acceleration of the car and a constant-acceleration equation to calculate how long it takes it to reach 100 km/h.



(*a*) Because 40% of the car's weight is on its two drive wheels and the accelerating friction forces act just on these wheels, the free-body diagram shows just the forces acting on the drive wheels.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\Sigma F_x = f_{s,max} = ma \quad (1)$$

and

$$\Sigma F_y = F_n - 0.4mg = 0 \quad (2)$$

Use the definition of $f_{s,max}$ in equation (1) and eliminate F_n between the two equations to obtain:

$$a = 0.4\mu_s g = 0.4(0.7)(9.81\,\mathrm{m/s^2}) = \boxed{2.75\,\mathrm{m/s^2}}$$

(*b*) Using a constant-acceleration equation, relate the initial and final velocities of the car to its acceleration and the elapsed time; solve for the time:

$$v_1 = v_0 + a\Delta t$$

or, because $v_0 = 0$ and $\Delta t = t_1$,

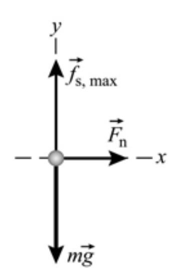
$$t_1 = \frac{v_1}{a}$$

Substitute numerical values and evaluate t_1:

$$t_1 = \frac{(100\,\mathrm{km/h})(1\,\mathrm{h}/3600\,\mathrm{s})(1000\,\mathrm{m/km})}{2.75\,\mathrm{m/s^2}} = \boxed{10.1\,\mathrm{s}}$$

***42** ••

Picture the Problem To hold the box in place, the acceleration of the cart and box must be great enough so that the static friction force acting on the box will equal the weight of the box. We can use Newton's 2nd law to determine the minimum acceleration required.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the box:

$$\Sigma F_x = F_n = ma_{min} \quad (1)$$

and

$$\Sigma F_y = f_{s,max} - mg = 0 \quad (2)$$

Substitute μF_n for $f_{s,max}$ in equation (2), eliminate F_n between the two equations and solve for and evaluate a_{min}:

$$\mu F_n - mg = 0, \quad \mu(ma_{min}) - mg = 0$$

and

$$a_{min} = \frac{g}{\mu_s} = \frac{9.81\,\text{m/s}^2}{0.6} = \boxed{16.4\,\text{m/s}^2}$$

(*b*) Solve equation (2) for $f_{s,max}$, and substitute numerical values and evaluate $f_{s,max}$:

$$f_{s,max} = mg = (2\text{ kg})(9.81\text{ m/s}^2) = \boxed{19.6\,\text{N}}$$

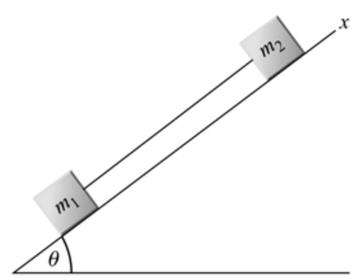
(*c*) If a is twice that required to hold the box in place, f_s will still have its maximum value given by:

$$f_{s,max} = \boxed{19.6\,\text{N}}$$

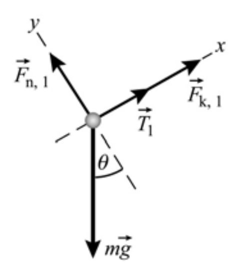
(*d*) $\boxed{\text{Because } g/\mu_s \text{ is } a_{min}\text{, the box will not fall if } a \ge g/\mu_s.}$

43 ••

Picture the Problem The pictorial representation shows the orientation of the two blocks with a common acceleration on the inclined surface. Draw the free-body diagrams for each block and apply Newton's 2nd law of motion and the definition of the kinetic friction force to each block to obtain simultaneous equations in a and T.



Draw the free-body diagram for the lower block:



Apply $\sum \vec{F} = m\vec{a}$ to the lower block:

$$\Sigma F_x = f_{k,1} + T_1 - m_1 g\sin\theta = m_1 a \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos\theta = 0 \quad (2)$$

The relationship between $f_{k,1}$ and $F_{n,1}$ is:

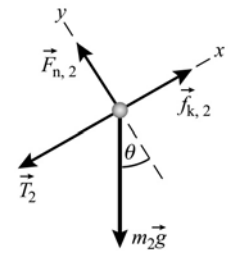
$$f_{k,1} = \mu_{k,1} F_{n,1} \quad (3)$$

Eliminate $f_{k,1}$ and $F_{n,1}$ between (1),

$$\mu_{k,1} m_1 g\cos\theta + T_1 - m_1 g\sin\theta = m_1 a \quad (4)$$

(2), and (3) to obtain:

Draw the free-body diagram for the upper block:



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = f_{k,2} - T_2 - m_2 g\sin\theta = m_2 a \qquad (5)$$

and

$$\Sigma F_y = F_{n,2} - m_2 g\cos\theta = 0 \qquad (6)$$

The relationship between $f_{k,2}$ and $F_{n,2}$ is:

$$f_{k,2} = \mu_{k,2} F_{n,2} \qquad (7)$$

Noting that $T_2 = T_1$, eliminate $f_{k,2}$ and $F_{n,2}$ between (5), (6), and (7) to obtain:

$$\mu_{k,2} m_2 g\cos\theta - T_2 - m_2 g\sin\theta = m_2 a \qquad (8)$$

Add equations (4) and (8) to eliminate T and solve for a:

$$a = g\left[\frac{\mu_{k,1} m_1 + \mu_{k,2} m_2}{m_1 + m_2}\cos\theta - \sin\theta\right]$$

Substitute numerical values and evaluate a to obtain:

$$a = \boxed{0.965\,\text{m/s}^2}$$

(*b*) Eliminate a between equations (4) and (8) and solve for $T = T_1 = T_2$ to obtain:

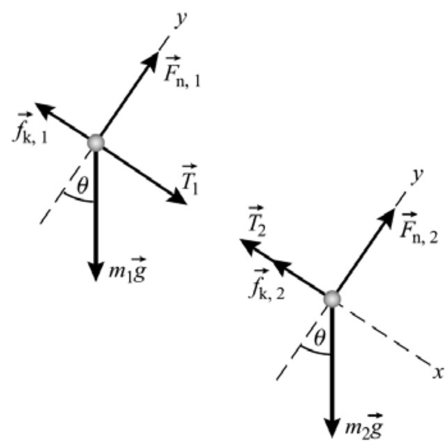
$$T = \frac{m_1 m_2 (\mu_{k,2} - \mu_{k,1}) g\cos\theta}{m_1 + m_2}$$

Substitute numerical values and evaluate T:

$$T = \boxed{0.184\,\text{N}}$$

***44 ••**

Picture the Problem The free-body diagram shows the forces acting on the two blocks as they slide down the incline. Down the incline has been chosen as the positive x direction. T is the force transmitted by the stick; it can be either tensile ($T > 0$) or compressive ($T < 0$). By applying Newton's 2nd law to these blocks, we can obtain equations in T and a from which we can eliminate either by solving them simultaneously. Once we have expressed T, the role of the stick will become apparent.



(a) Apply $\sum \vec{F} = m\vec{a}$ to block 1:

$$\sum F_x = T_1 + m_1 g\sin\theta - f_{k,1} = m_1 a$$

and

$$\sum F_y = F_{n,1} - m_1 g\cos\theta = 0$$

Apply $\sum \vec{F} = m\vec{a}$ to block 2:

$$\sum F_x = m_2 g\sin\theta - T_2 - f_{k,2} = m_2 a$$

and

$$\sum F_y = F_{n,2} - m_2 g\cos\theta = 0$$

Letting $T_1 = T_2 = T$, use the definition of the kinetic friction force to eliminate $f_{k,1}$ and $F_{n,1}$ between the equations for block 1 and $f_{k,2}$ and $F_{n,1}$ between the equations for block 2 to obtain:

$$m_1 a = m_1 g\sin\theta + T - \mu_1 m_1 g\cos\theta \qquad (1)$$

and

$$m_2 a = m_2 g\sin\theta - T - \mu_2 m_2 g\cos\theta \qquad (2)$$

Add equations (1) and (2) to eliminate T and solve for a:

$$a = \boxed{g\left(\sin\theta - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2}\cos\theta\right)}$$

(b) Rewrite equations (1) and (2) by dividing both sides of (1) by m_1 and both sides of (2) by m_2 to obtain.

$$a = g\sin\theta + \frac{T}{m_1} - \mu_1 g\cos\theta \qquad (3)$$

and

$$a = g\sin\theta - \frac{T}{m_2} - \mu_2 g\cos\theta \qquad (4)$$

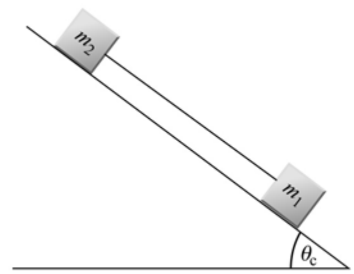
Subtracting (4) from (3) and rearranging yields:

$$T = \boxed{\left(\frac{m_1 m_2}{m_1 - m_2}\right)(\mu_1 - \mu_2)g\cos\theta}$$

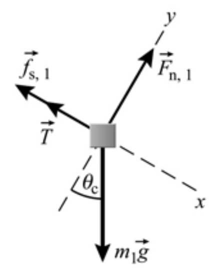
If $\mu_1 = \mu_2, T = 0$ and the blocks move down the incline with the same acceleration of $g(\sin\theta - \mu\cos\theta)$. Inserting a stick between them can't change this; therefore, the stick must exert no force on either block.

45 ••

Picture the Problem The pictorial representation shows the orientation of the two blocks on the inclined surface. Draw the free-body diagrams for each block and apply Newton's 2nd law of motion and the definition of the static friction force to each block to obtain simultaneous equations in θ_c and T.



(a) Draw the free-body diagram for the lower block:



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = m_1 g\sin\theta_c - f_{s,1} - T = 0 \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos\theta_c = 0 \quad (2)$$

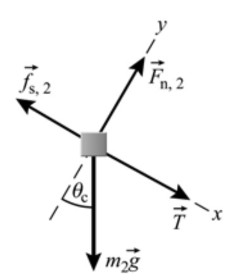
The relationship between $f_{s,1}$ and $F_{n,1}$ is:

$$f_{s,1} = \mu_{s,1} F_{n,1} \quad (3)$$

Eliminate $f_{s,1}$ and $F_{n,1}$ between (1), (2), and (3) to obtain:

$$m_1 g\sin\theta_c - \mu_{s,1} m_1 g\cos\theta_c - T = 0 \quad (4)$$

Draw the free-body diagram for the upper block:



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = T + m_2 g\sin\theta_c - f_{s,2} = 0 \quad (5)$$

and

$$\Sigma F_y = F_{n,2} - m_2 g\cos\theta_c = 0 \quad (6)$$

The relationship between $f_{s,2}$ and $F_{n,2}$ is:

$$f_{s,2} = \mu_{s,2} F_{n,2} \quad (7)$$

Eliminate $f_{s,2}$ and $F_{n,2}$ between (5), (6), and (7) to obtain:

$$T + m_2 g\sin\theta_c - \mu_{s,2} m_2 g\cos\theta_c = 0 \quad (8)$$

Add equations (4) and (8) to eliminate T and solve for θ_c:

$$\theta_c = \tan^{-1}\left[\frac{\mu_{s,1}m_1 + \mu_{s,2}m_2}{m_1 + m_2}\right]$$

$$= \tan^{-1}\left[\frac{(0.4)(0.2\,\text{kg}) + (0.6)(0.1\,\text{kg})}{0.1\,\text{kg} + 0.2\,\text{kg}}\right]$$

$$= \boxed{25.0°}$$

(*b*) Because θ_c is greater than the angle of repose ($\tan^{-1}(\mu_{s,1}) = \tan^{-1}(0.4) = 21.8°$) for the lower block, it would slide if $T = 0$. Solve equation (4) for T:

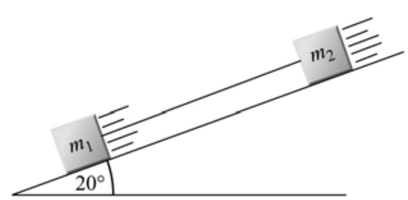
$$T = m_1 g\left(\sin\theta_C - \mu_{s,1}\cos\theta_C\right)$$

Substitute numerical values and evaluate T:

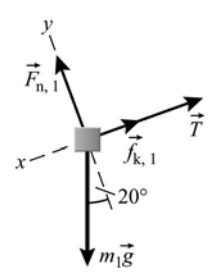
$$T = (0.2\,\text{kg})\left(9.81\,\text{m/s}^2\right)\left[\sin 25° - (0.4)\cos 25°\right] = \boxed{0.118\,\text{N}}$$

46 ••

Picture the Problem The pictorial representation shows the orientation of the two blocks with a common acceleration on the inclined surface. Draw the free-body diagrams for each block and apply Newton's 2nd law and the definition of the kinetic friction force to each block to obtain simultaneous equations in a and T.



(*a*) Draw the free-body diagram for the lower block:



Apply $\sum \vec{F} = m\vec{a}$ to the lower block:

$$\Sigma F_x = m_1 g\sin 20° - f_{k,1} - T = m_1 a \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g\cos 20° = 0 \quad (2)$$

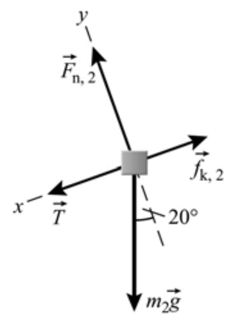
Express the relationship between $f_{k,1}$ and $F_{n,1}$:

$$f_{k,1} = \mu_{k,1} F_{n,1} \quad (3)$$

Eliminate $f_{k,1}$ and $F_{n,1}$ between (1), (2), and (3) to obtain:

$$m_1 g \sin 20° - \mu_{k,1} m_1 g \cos 20° - T = m_1 a \quad (4)$$

Draw the free-body diagram for the upper block:



Apply $\sum \vec{F} = m\vec{a}$ to the upper block:

$$\Sigma F_x = T + m_2 g \sin 20° - f_{k,2} = m_2 a \quad (5)$$

and

$$\Sigma F_y = F_{n,2} - m_2 g \cos 20° = 0 \quad (6)$$

Express the relationship between $f_{k,2}$ and $F_{n,2}$:

$$f_{k,2} = \mu_{k,2} F_{n,2} \quad (7)$$

Eliminate $f_{k,2}$ and $F_{n,2}$ between (5), (2), and (7) to obtain:

$$T + m_2 g \sin 20° - \mu_{k,2} m_2 g \cos 20° = m_2 a \quad (8)$$

Add equations (4) and (8) to eliminate T and solve for a:

$$a = \boxed{g\left(\sin 20° - \frac{\mu_1 m_1 + \mu_2 m_2}{m_1 + m_2} \cos 20°\right)}$$

Substitute the given values and evaluate a:

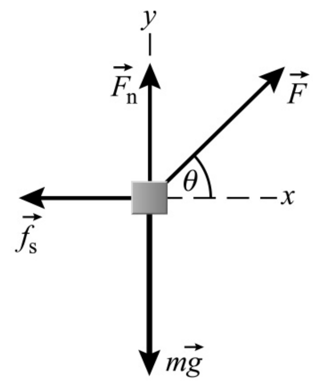
$$a = \boxed{0.944\ \text{m/s}^2}$$

(*b*) Substitute for a in either equation (4) or (8) to obtain:

$T = \boxed{-0.426\ \text{N}}$; i.e., the rod is under compression.

***47** ••

Picture the Problem The vertical component of $\vec{F}$ reduces the normal force; hence, the static friction force between the surface and the block. The horizontal component is responsible for any tendency to move and equals the static friction force until it exceeds its maximum value. We can apply Newton's 2nd law to the box, under equilibrium conditions, to relate F to θ.



(*a*) The static-frictional force opposes the motion of the object, and the maximum value of the static-frictional force is proportional to the normal force F_N. The normal force is equal to the weight minus the vertical component F_V of the force F. Keeping the magnitude F constant while increasing θ from zero results in a decrease in F_V and thus a corresponding decrease in the maximum static-frictional force f_{max}. The object will begin to move if the horizontal component F_H of the force F exceeds f_{max}. An increase in θ results in a decrease in F_H. As θ increases from 0, the decrease in F_N is larger than the decrease in F_H, so the object is more and more likely to slip. However, as θ approaches 90°, F_H approaches zero and no movement will be initiated. If F is large enough and if θ increases from 0, then at some value of θ the block will start to move.

(*b*) Apply $\sum\vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = F\cos\theta - f_s = 0 \quad (1)$$

and

$$\Sigma F_y = F_n + F\sin\theta - mg = 0 \quad (2)$$

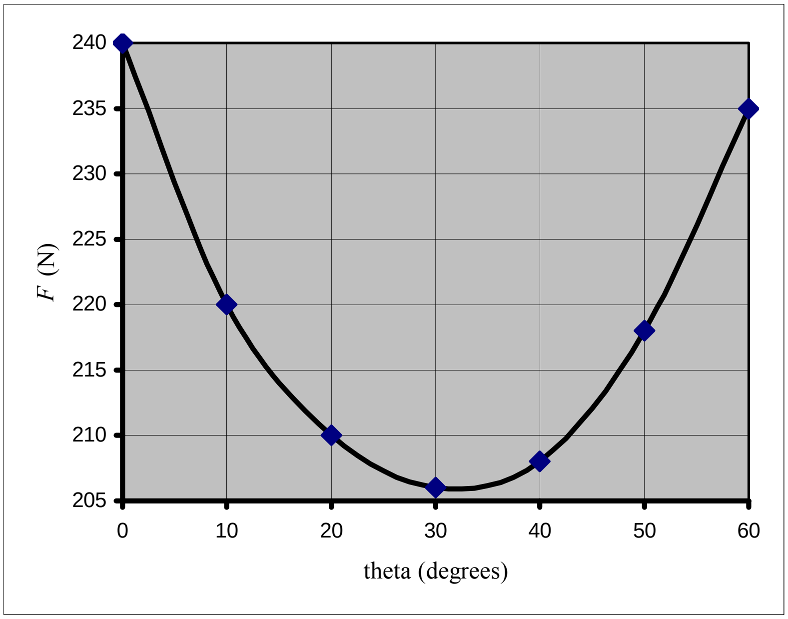
Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta}$$

Use this function with mg = 240 N to generate the table shown below:

θ	(deg)	0	10	20	30	40	50	60
F	(N)	240	220	210	206	208	218	235

The following graph of $F(\theta)$ was plotted using a spreadsheet program.

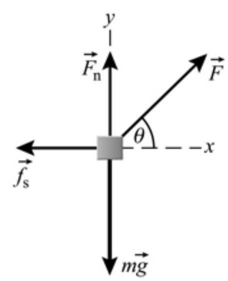


From the graph, we can see that the minimum value for F occurs when $\theta \approx 32°$.

Remarks: An alternative to manually plotting F as a function of θ or using a spreadsheet program is to use a graphing calculator to enter and graph the function.

48 •••

Picture the Problem The free-body diagram shows the forces acting on the block. We can apply Newton's 2nd law, under equilibrium conditions, to relate F to θ and then set its derivative with respect to θ equal to zero to find the value of θ that minimizes F.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = F\cos\theta - f_s = 0 \qquad (1)$$

and

$$\Sigma F_y = F_n + F\sin\theta - mg = 0 \qquad (2)$$

Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta + \mu_s \sin\theta} \qquad (3)$$

To find θ_{min}, differentiate F with respect to θ and set the derivative equal to zero for extrema of the function:

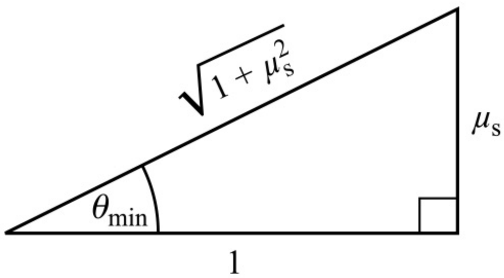
$$\frac{dF}{d\theta} = \frac{(\cos\theta + \mu_s \sin\theta)\frac{d}{d\theta}(\mu_s mg)}{(\cos\theta + \mu_s \sin\theta)^2} - \frac{\mu_s mg \frac{d}{d\theta}(\cos\theta + \mu_s \sin\theta)}{(\cos\theta + \mu_s \sin\theta)^2}$$

$$= \frac{\mu_s mg(-\sin\theta + \mu_s \cos\theta)}{(\cos\theta + \mu_s \sin\theta)^2} = 0 \text{ for extrema}$$

Solve for θ_{min} to obtain:

$$\theta_{min} = \boxed{\tan^{-1}\mu_s}$$

(*b*) Use the reference triangle shown below to substitute for $\cos\theta$ and $\sin\theta$ in equation (3):



$$F_{min} = \frac{\mu_s mg}{\frac{1}{\sqrt{1+\mu_s^2}} + \mu_s \frac{\mu_s}{\sqrt{1+\mu_s^2}}}$$

$$= \frac{\mu_s mg}{\frac{1+\mu_s^2}{\sqrt{1+\mu_s^2}}}$$

$$= \boxed{\frac{\mu_s}{\sqrt{1+\mu_s^2}} mg}$$

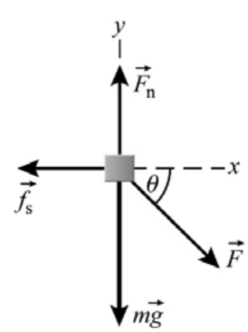
(*c*)

The coefficient of kinetic friction is less than the coefficient of static friction. An analysis identical to the one above shows that the minimum force one should apply to keep the block moving should be applied at an angle given by $\theta_{min} = \tan^{-1}\mu_k$. Therefore, once the block is moving the coefficient of friction will decrease, so the angle can be decreased.

49 ••

Picture the Problem The vertical component of $\vec{F}$ increases the normal force and the static friction force between the surface and the block. The horizontal component is responsible for any tendency to move and equals the static friction force until it exceeds its maximum value. We can apply Newton's 2nd law to the box, under equilibrium conditions, to relate F to θ.

(*a*) As θ increases from zero, F increases the normal force exerted by the surface and the static friction force. As the horizontal component of F decreases with increasing θ, one would expect F to continue to increase.



(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = F\cos\theta - f_s = 0 \quad (1)$$

and

$$\Sigma F_y = F_n - F\sin\theta - mg = 0 \quad (2)$$

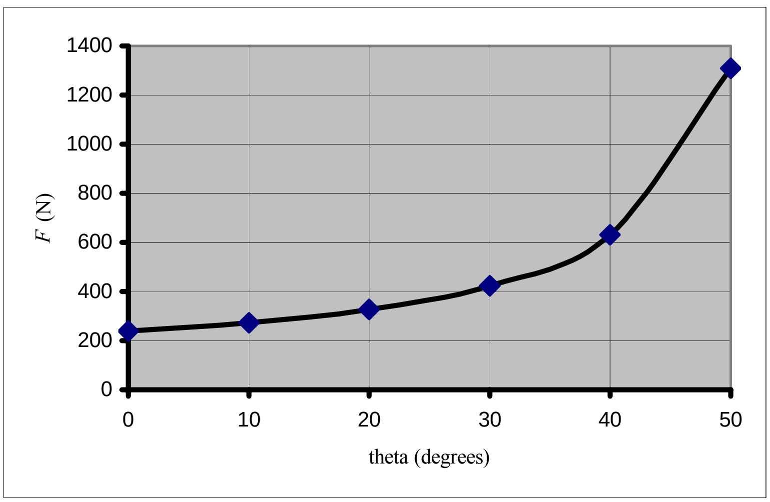
Assuming that $f_s = f_{s,max}$, eliminate f_s and F_n between equations (1) and (2) and solve for F:

$$F = \frac{\mu_s mg}{\cos\theta - \mu_s \sin\theta} \quad (3)$$

Use this function with mg = 240 N to generate the table shown below.

θ	(deg)	0	10	20	30	40	50	60
F	(N)	240	273	327	424	631	1310	very large

The graph of F as a function of θ, plotted using a spreadsheet program, confirms our prediction that F continues to increase with θ.



(*a*) From the graph we see that:

$$\theta_{min} = \boxed{0°}$$

(*b*) Evaluate equation (3) for $\theta = 0°$ to obtain:

$$F = \frac{\mu_s mg}{\cos 0° - \mu_s \sin 0°} = \boxed{\mu_s mg}$$

(*c*)

$$\boxed{\text{You should keep the angle at } 0°.}$$

Remarks: An alternative to the use of a spreadsheet program is to use a graphing calculator to enter and graph the function.

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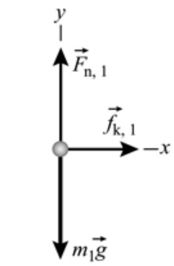
Picture the Problem The forces acting on each of these masses are shown in the free-body diagrams below. m_1 represents the mass of the 20-kg mass and m_2 that of the 100-kg mass. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{k,1}$ ($= f_{k,2}$) act on both masses but in opposite directions. Newton's 2nd law and the definition of kinetic friction forces can be used to determine the various forces and the acceleration called for in this problem.

(*a*) Draw a free-body diagram showing the forces acting on the 20-kg mass:



Apply $\sum \vec{F} = m\vec{a}$ to this mass:

$$\Sigma F_x = f_{k,1} = m_1 a_1 \quad (1)$$

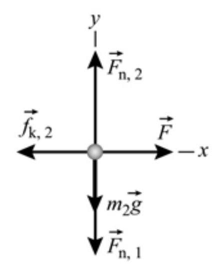
and

$$\Sigma F_y = F_{n,1} - m_1 g = 0 \quad (2)$$

Solve equation (1) for $f_{k,1}$:

$$f_{k,1} = m_1 a_1 = (20\text{ kg})(4\text{ m/s}^2) = \boxed{80.0\text{ N}}$$

(*b*) Draw a free-body diagram showing the forces acting on the 100-kg mass:



Apply $\sum F_x = ma_x$ to the 100-kg object and evaluate F_{net}:

$$F_{net} = m_2 a_2 = (100\text{ kg})(6\text{ m/s}^2) = \boxed{600\text{ N}}$$

Express F in terms of F_{net} and $f_{k,2}$:

$$F = F_{net} + f_{k,2} = 600\text{ N} + 80\text{ N} = \boxed{680\text{ N}}$$

(*c*) When the 20-kg mass falls off, the 680-N force acts just on the 100-kg mass and its acceleration is given by Newton's 2nd law:

$$a = \frac{F_{net}}{m} = \frac{680\text{ N}}{100\text{ kg}} = \boxed{6.80\text{ m/s}^2}$$

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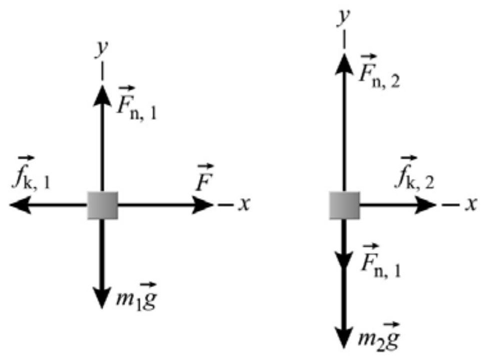
Picture the Problem The forces acting on each of these blocks are shown in the free-body diagrams to the right. m_1 represents the mass of the 60-kg block and m_2 that of the 100-kg block. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{k,1}$ ($=f_{k,2}$) act on both objects but in opposite directions. Newton's 2nd law and the definition of kinetic friction forces can be used to determine the coefficient of kinetic friction and acceleration of the 100-kg block.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the 60-kg block:

$$\Sigma F_x = F - f_{k,1} = m_1 a_1 \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g = 0 \quad (2)$$

Apply $\sum F_x = ma_x$ to the 100-kg block:

$$f_{k,2} = m_2 a_2 \quad (3)$$

Using equation (2), express the relationship between the kinetic friction forces $\vec{f}_{k,1}$ and $\vec{f}_{k,2}$:

$$f_{k,1} = f_{k,2} = f_k = \mu_k F_{n,1} = \mu_k m_1 g \quad (4)$$

Substitute equation (4) into equation (1) and solve for μ_k:

$$\mu_k = \frac{F - m_1 a_1}{m_1 g}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{320\,\text{N} - (60\,\text{kg})(3\,\text{m/s}^2)}{(60\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{0.238}$$

(*b*) Substitute equation (4) into equation (3) and solve for a_2:

$$a_2 = \frac{\mu_k m_1 g}{m_2}$$

Substitute numerical values and evaluate a_2:

$$a_2 = \frac{(0.238)(60\,\text{kg})(9.81\,\text{m/s}^2)}{100\,\text{kg}} = \boxed{1.40\,\text{m/s}^2}$$

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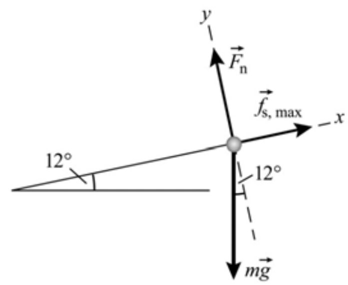
Picture the Problem The accelerations of the truck can be found by applying Newton's 2nd law of motion. The free-body diagram for the truck climbing the incline with maximum acceleration is shown to the right.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the truck when it is climbing the incline:

$$\Sigma F_x = f_{s,max} - mg\sin 12° = ma \quad (1)$$

and

$$\Sigma F_y = F_n - mg\cos 12° = 0 \quad (2)$$

Solve equation (2) for F_n and use the definition of $f_{s,max}$ to obtain:

$$f_{s,max} = \mu_s mg\cos 12° \quad (3)$$

Substitute equation (3) into equation (1) and solve for *a*:

$$a = g(\mu_s \cos 12° - \sin 12°)$$

Substitute numerical values and evaluate *a*:

$$a = (9.81\,\text{m/s}^2)[(0.85)\cos 12° - \sin 12°] = \boxed{6.12\,\text{m/s}^2}$$

(*b*) When the truck is descending the incline with maximum acceleration, the static friction force points down the incline; i.e., its direction is reversed on the FBD. Apply $\sum F_x = ma_x$ to the truck under these conditions:

$$-f_{s,max} - mg\sin 12° = ma \quad (4)$$

Substitute equation (3) into equation (4) and solve for *a*:

$$a = -g(\mu_s \cos 12° + \sin 12°)$$

Substitute numerical values and evaluate *a*:

$$a = (-9.81\,\text{m/s}^2)[(0.85)\cos 12° + \sin 12°] = \boxed{-10.2\,\text{m/s}^2}$$

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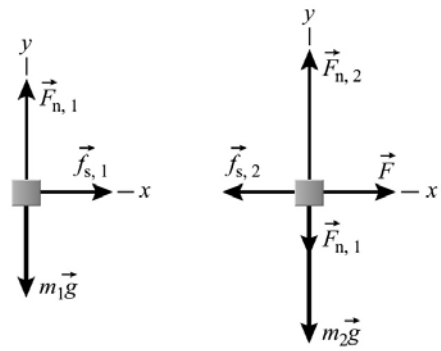
Picture the Problem The forces acting on each of the blocks are shown in the free-body diagrams to the right. m_1 represents the mass of the 2-kg block and m_2 that of the 4-kg block. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{s,1}$ $(= f_{s,2})$ act on both objects but in opposite directions. Newton's 2nd law and the definition of the maximum static friction force can be used to determine the maximum force acting on the 4-kg block for which the 2-kg block does not slide.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the 2-kg block:

$$\Sigma F_x = f_{s,1,\max} = m_1 a_{\max} \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g = 0 \quad (2)$$

Apply $\sum \vec{F} = m\vec{a}$ to the 4-kg block:

$$\Sigma F_x = F - f_{s,2,\max} = m_2 a_{\max} \quad (3)$$

and

$$\Sigma F_y = F_{n,2} - F_{n,1} - m_2 g = 0 \quad (4)$$

Using equation (2), express the relationship between the static friction forces $\vec{f}_{s,1,\max}$ and $\vec{f}_{s,2,\max}$:

$$f_{s,1,\max} = f_{s,2,\max} = \mu_s m_1 g \quad (5)$$

Substitute (5) in (1) and solve for $a_{\max}$:

$$a_{\max} = \mu_s g = (0.3)g = 2.94\ \text{m/s}^2$$

Solve equation (3) for $F = F_{\max}$:

$$F_{\max} = m_2 a_{\max} + \mu_s m_1 g$$

Substitute numerical values and evaluate $F_{\max}$:

$$F_{\max} = (4\,\text{kg})(2.94\,\text{m/s}^2) + (0.3)(2\,\text{kg}) \times (9.81\,\text{m/s}^2) = \boxed{17.7\,\text{N}}$$

(*b*) Use Newton's 2nd law to express the acceleration of the blocks moving as a unit:

$$a = \frac{F}{m_1 + m_2}$$

Substitute numerical values and evaluate a:

$$a = \frac{\frac{1}{2}(17.7\,\text{N})}{2\,\text{kg} + 4\,\text{kg}} = \boxed{1.47\,\text{m/s}^2}$$

Because the friction forces are an action-reaction pair, the friction force acting on each block is given by:

$$f_s = m_1 a = (2\text{ kg})(1.47\text{ m/s}^2) = \boxed{2.94\text{ N}}$$

(*c*) If $F = 2F_{max}$, then m_1 slips on m_2 and the friction force (now kinetic) is given by:

$$f = f_k = \mu_k m_1 g$$

Use $\sum F_x = ma_x$ to relate the acceleration of the 2-kg block to the net force acting on it and solve for a_1:

$$f_k = \mu_k m_1 g = m_1 a_1$$

and

$$a_1 = \mu_k g = (0.2)g = \boxed{1.96\text{ m/s}^2}$$

Use $\sum F_x = ma_x$ to relate the acceleration of the 4-kg block to the net force acting on it and solve for a_2:

$$F - \mu_k m_1 g = m_2 a_2$$

$$a_2 = \frac{F - \mu_k m_1 g}{m_2} = \frac{2(17.7\text{ N}) - (0.2)(2\text{ kg})(9.81\text{ m/s}^2)}{4\text{ kg}} = \boxed{7.87\text{ m/s}^2}$$

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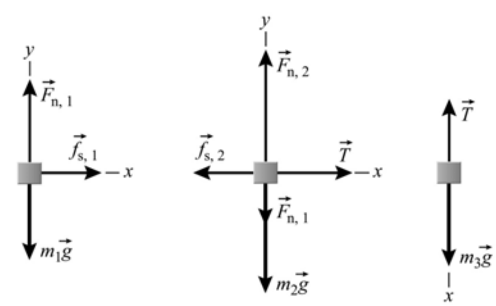
Picture the Problem Let the positive *x* direction be the direction of motion of these blocks. The forces acting on each of the blocks are shown, for the static friction case, on the free-body diagrams to the right. As described by Newton's 3rd law, the normal reaction force $F_{n,1}$ and the friction force $f_{s,1}$ ($= f_{s,2}$) act on both objects but in opposite directions. Newton's 2nd law and the definition of the maximum static friction force can be used to determine the maximum acceleration of the block whose mass is m_1.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the 2-kg block:

$$\Sigma F_x = f_{s,1,max} = m_1 a_{max} \quad (1)$$

and

$$\Sigma F_y = F_{n,1} - m_1 g = 0 \quad (2)$$

Apply $\sum \vec{F} = m\vec{a}$ to the 4-kg block:

$$\Sigma F_x = T - f_{s,2,max} = m_2 a_{max} \quad (3)$$

and

$$\Sigma F_y = F_{n,2} - F_{n,1} - m_2 g = 0 \quad (4)$$

Using equation (2), express the relationship between the static friction forces $\vec{f}_{s,1,\max}$ and $\vec{f}_{s,2,\max}$:

$$f_{\mathrm{s},1,\max} = f_{\mathrm{s},2,\max} = \mu_{\mathrm{s}}\, m_1 g \qquad (5)$$

Substitute (5) in (1) and solve for $a_{\max}$:

$$a_{\max} = \mu_{\mathrm{s}} g = (0.6)g = \boxed{5.89\,\mathrm{m/s^2}}$$

(*b*) Use $\sum F_x = ma_x$ to express the acceleration of the blocks moving as a unit:

$$T = (m_1 + m_2)\, a_{\max} \qquad (6)$$

Apply $\sum F_x = ma_x$ to the object whose mass is m_3:

$$m_3 g - T = m_3\, a_{\max} \qquad (7)$$

Add equations (6) and (7) to eliminate T and then solve for and evaluate m_3:

$$m_3 = \frac{\mu_{\mathrm{s}}(m_1 + m_2)}{1 - \mu_{\mathrm{s}}} = \frac{(0.6)(10\,\mathrm{kg} + 5\,\mathrm{kg})}{1 - 0.6} = \boxed{22.5\,\mathrm{kg}}$$

(*c*) If $m_3 = 30$ kg, then m_1 will slide on m_2 and the friction force (now kinetic) is given by:

$$f = f_{\mathrm{k}} = \mu_{\mathrm{k}} m_1 g$$

Use $\sum F_x = ma_x$ to relate the acceleration of the 30-kg block to the net force acting on it:

$$m_3 g - T = m_3 a_3 \qquad (8)$$

Noting that $a_2 = a_3$ and that the friction force on the body whose mass is m_2 is due to kinetic friction, add equations (3) and (8) and solve for and evaluate the common acceleration:

$$a_2 = a_3 = \frac{g(m_3 - \mu_{\mathrm{k}} m_1)}{m_2 + m_3} = \frac{(9.81\,\mathrm{m/s^2})[30\,\mathrm{kg} - (0.4)(5\,\mathrm{kg})]}{10\,\mathrm{kg} + 30\,\mathrm{kg}} = \boxed{6.87\,\mathrm{m/s^2}}$$

With block 1 sliding on block 2, the friction force acting on each is kinetic and equations (1) and (3) become:

$$f_{\mathrm{k}} = \mu_{\mathrm{k}} m_1 g = m_1 a_1 \qquad (1')$$

$$T - f_{\mathrm{k}} = T - \mu_{\mathrm{k}} m_1 g = m_2 a_2 \qquad (3')$$

Solve equation (1′) for and evaluate a_1:

$$a_1 = \mu_k g = (0.4)(9.81\,\text{m/s}^2) = \boxed{3.92\,\text{m/s}^2}$$

Solve equation (3′) for T:

$$T = m_2 a_2 + \mu_k m_1 g$$

Substitute numerical values and evaluate T:

$$T = (10\,\text{kg})(6.87\,\text{m/s}^2) + (0.4)(5\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{88.3\,\text{N}}$$

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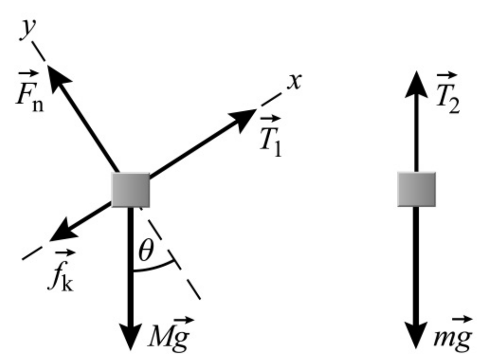
Picture the Problem Let the direction of motion be the positive x direction. The free-body diagrams show the forces acting on both the block (M) and the counterweight (m). While $\vec{T}_1 \neq \vec{T}_2$, $T_1 = T_2$. By applying Newton's 2nd law to these blocks, we can obtain equations in T and a from which we can eliminate the tension. Once we know the acceleration of the block, we can use constant-acceleration equations to determine how far it moves in coming to a momentary stop.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block on the incline:

$$\sum F_x = T_1 - Mg\sin\theta - f_k = Ma$$

and

$$\sum F_y = F_n - Mg\cos\theta = 0$$

Apply $\sum \vec{F} = m\vec{a}$ to the counterweight:

$$\sum F_x = mg - T_2 = ma \qquad (1)$$

Letting $T_1 = T_2 = T$ and using the definition of the kinetic friction force, eliminate f_k and F_n between the equations for the block on the incline to obtain:

$$T - Mg\sin\theta - \mu_k Mg\cos\theta = Ma \qquad (2)$$

Eliminate T from equations (1) and (2) by adding them and solve for a:

$$a = \frac{m - M(\sin\theta + \mu_k\cos\theta)}{m+M}g$$

Substitute numerical values and evaluate a:

$$a = \frac{550\,\text{kg} - (1600\,\text{kg})(\sin 10^\circ + 0.15\cos 10^\circ)}{550\,\text{kg} + 1600\,\text{kg}}(9.81\,\text{m/s}^2) = \boxed{0.163\,\text{m/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the speed of the block at the instant the rope breaks to its acceleration and displacement as it slides to a stop. Solve for its displacement:

$$v_f^2 = v_i^2 + 2a\Delta x$$

or, because $v_f = 0$,

$$\Delta x = \frac{-v_i^2}{2a} \qquad (3)$$

The block had been accelerating up the incline for 3 s before the rope broke, so it has an initial speed of :

$(0.163\text{ m/s}^2)(3\text{ s}) = 0.489\text{ m/s}$

From equation (2) we can see that, when the rope breaks ($T = 0$) and:

$$\begin{aligned} a &= -g(\sin\theta + \mu_k \cos\theta) \\ &= -(9.81\,\text{m/s}^2)[\sin 10° + (0.15)\cos 10°] \\ &= -3.15\,\text{m/s}^2 \end{aligned}$$

where the minus sign indicates that the block is being accelerated down the incline, although it is still sliding up the incline.

Substitute in equation (3) and evaluate Δx:

$$\Delta x = \frac{-(0.489\,\text{m/s})^2}{2(-3.15\,\text{m/s}^2)} = \boxed{0.0380\,\text{m}}$$

(*c*) When the block is sliding down the incline, the kinetic friction force will be up the incline. Express the block's acceleration:

$$\begin{aligned} a &= -g(\sin\theta - \mu_k \cos\theta) \\ &= -(9.81\,\text{m/s}^2)[\sin 10° - (0.15)\cos 10°] \\ &= \boxed{-0.254\,\text{m/s}^2} \end{aligned}$$

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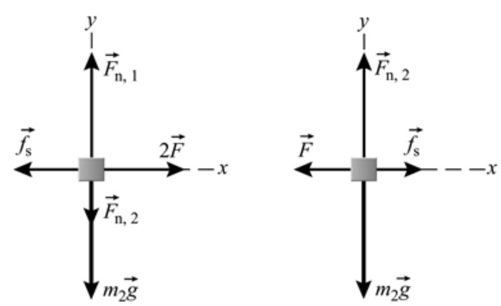
Picture the Problem If the 10-kg block is not to slide on the bracket, the maximum value for $\vec{F}$ must be equal to the maximum value of f_s and will produce the maximum acceleration of this block and the bracket. We can apply Newton's 2nd law and the definition of $f_{s,max}$ to first calculate the maximum acceleration and then the maximum value of F.



(*a*) and (*b*) Apply $\sum \vec{F} = m\vec{a}$ to the 10-kg block when it is experiencing its maximum acceleration:

$$\Sigma F_x = f_{s,max} - F = m_2 a_{2,max} \qquad (1)$$

and

$$\Sigma F_y = F_{n,2} - m_2 g = 0 \qquad (2)$$

Express the static friction force acting on the 10-kg block:

$$f_{s,max} = \mu_s F_{n,2} \qquad (3)$$

Eliminate $f_{s,max}$ and $F_{n,2}$ from

$$\mu_s m_2 g - F = m_2 a_{2,max} \qquad (4)$$

equations (1), (2) and (3) to obtain:

Apply $\sum F_x = ma_x$ to the bracket to obtain:

$$2F - \mu_s m_2 g = m_1 a_{1,\max} \qquad (5)$$

Because $a_{1,\max} = a_{2,\max}$, denote this acceleration by $a_{\max}$. Eliminate F from equations (4) and (5) and solve for $a_{\max}$:

$$a_{\max} = \frac{\mu_s m_2 g}{m_1 + 2m_2}$$

Substitute numerical values and evaluate $a_{\max}$:

$$a_{\max} = \frac{(0.4)(10\,\text{kg})(9.81\,\text{m/s}^2)}{5\,\text{kg} + 2(10\,\text{kg})} = \boxed{1.57\,\text{m/s}^2}$$

Solve equation (4) for $F = F_{max}$:

$$F = \mu_s m_2 g - m_2 a_{\max} = m_2(\mu_s g - a_{\max})$$

Substitute numerical values and evaluate F:

$$F = (10\,\text{kg})\left[(0.4)(9.81\,\text{m/s}^2) - 1.57\,\text{m/s}^2\right] = \boxed{23.5\,\text{N}}$$

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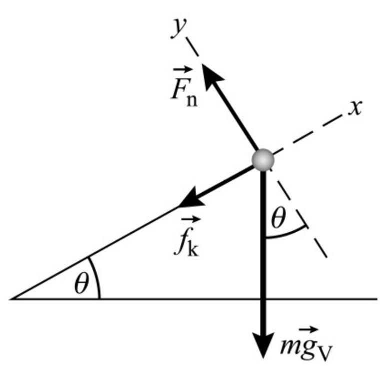
Picture the Problem The free-body diagram shows the forces acting on the block as it is moving up the incline. By applying Newton's 2nd law, we can obtain expressions for the accelerations of the block up and down the incline. Adding and subtracting these equations, together with the data found in the notebook, will lead to values for g_V and μ_k.



Apply $\sum_i \vec{F}_i = m\vec{a}$ to the block when it is moving up the incline:

$$\sum F_x = -f_k - mg_V \sin\theta = ma_{up}$$

and

$$\sum F_y = F_n - mg_V \cos\theta = 0$$

Using the definition of f_k, eliminate F_n between the two equations to obtain:

$$a_{up} = -\mu_k g_V \cos\theta - g_V \sin\theta \qquad (1)$$

When the block is moving down the incline, f_k is in the positive x direction, and its acceleration is:

$$a_{down} = \mu_k g_V \cos\theta - g_V \sin\theta \qquad (2)$$

Add equations (1) and (2) to obtain:

$$a_{\text{up}} + a_{\text{down}} = -2g_{\text{V}} \sin\theta \quad (3)$$

Solve equation (3) for g_{V}:

$$g_{\text{V}} = \frac{a_{\text{up}} + a_{\text{down}}}{-2\sin\theta} \quad (4)$$

Determine θ from the figure:

$$\theta = \tan^{-1}\left[\frac{0.73\,\text{glapp}}{3.82\,\text{glapp}}\right] = 10.8°$$

Substitute the data from the notebook in equation (4) to obtain:

$$g_{\text{V}} = \frac{1.73\,\text{glapp/plipp}^2 + 1.42\,\text{glapp/plipp}^2}{-2\sin 10.8°} = \boxed{-8.41\,\text{glapp/plipp}^2}$$

Subtract equation (1) from equation (2) to obtain:

$$a_{\text{down}} - a_{\text{up}} = 2\mu_{\text{k}} g_{\text{V}} \cos\theta$$

Solve for μ_{k}:

$$\mu_{\text{k}} = \frac{a_{\text{down}} - a_{\text{up}}}{2g_{\text{V}}\cos\theta}$$

Substitute numerical values and evaluate μ_{k}:

$$\mu_{\text{k}} = \frac{-1.42\,\text{glapp/plipp}^2 - 1.73\,\text{glapp/plipp}^2}{2\left(-8.41\,\text{glapp/plipp}^2\right)\cos 10.8°} = \boxed{0.191}$$

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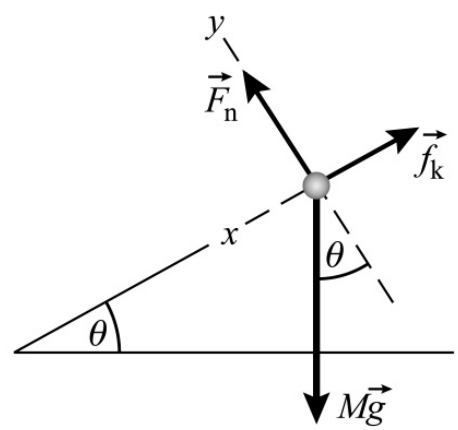
Picture the Problem The free-body diagram shows the block sliding down the incline under the influence of a friction force, its weight, and the normal force exerted on it by the inclined surface. We can find the range of values for m for the two situations described in the problem statement by applying Newton's 2nd law of motion to, first, the conditions under which the block will not move or slide if pushed, and secondly, if pushed, the block will move up the incline.



(*a*) Assume that the block is sliding down the incline with a constant velocity and with no hanging weight ($m = 0$) and apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\sum F_x = -f_{\text{k}} + Mg\sin\theta = 0$$

and

$$\sum F_y = F_{\text{n}} - Mg\cos\theta = 0$$

Using $f_{\text{k}} = \mu_{\text{k}} F_{\text{n}}$, eliminate F_{n}

$$F_{\text{net}} = -\mu_{\text{k}} Mg\cos\theta + Mg\sin\theta$$

between the two equations and solve for the net force acting on the block:

If the block is moving, this net force must be nonnegative and:

$$(-\mu_k \cos\theta + \sin\theta)Mg \geq 0$$

This condition requires that:

$$\mu_k \leq \tan\theta = \tan 18° = 0.325$$

Because $\mu_k = 0.2$, this condition is satisfied and:

$$m_{min} = 0$$

To find the maximum value, note that the maximum possible value for the tension in the rope is mg. For the block to move down the incline, the component of the block's weight parallel to the incline minus the frictional force must be greater than or equal to the tension in the rope:

$$Mg\sin\theta - \mu_k Mg\cos\theta \geq mg$$

Solve for m_{max}:

$$m_{max} \leq M(\sin\theta - \mu_k \cos\theta)$$

Substitute numerical values and evaluate m_{max}:

$$m_{max} \leq (100\,\text{kg})[\sin 18° - (0.2)\cos 18°] = 11.9\,\text{kg}$$

The range of values for m is:

$$\boxed{0 \leq m \leq 11.9\,\text{kg}}$$

(*b*) If the block is being dragged up the incline, the frictional force will point down the incline, and:

$$Mg\sin\theta + \mu_k Mg\cos\theta < mg$$

Solve for and evaluate m_{min}:

$$m_{min} > M(\sin\theta + \mu_k\cos\theta) = (100\text{ kg})[\sin 18° + (0.2)\cos 18°] = 49.9\,\text{kg}$$

If the block is not to move unless pushed:

$$Mg\sin\theta + \mu_s Mg\cos\theta > mg$$

Solve for and evaluate m_{max}:

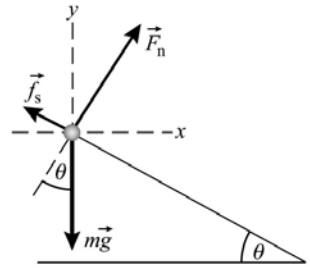
$$m_{max} < M(\sin\theta + \mu_s\cos\theta) = (100\text{ kg})[\sin 18° + (0.4)\cos 18°] = 68.9\,\text{kg}$$

The range of values for m is:

$$\boxed{49.9\,\text{kg} \leq m \leq 68.9\,\text{kg}}$$

59 •••

Picture the Problem The free-body diagram shows the forces acting on the 0.5 kg block when the acceleration is a minimum. Note the choice of coordinate system is consistent with the direction of $\vec{F}$. Apply Newton's 2nd law to the block and solve the resulting equations for a_{min} and a_{max}.



(*a*) Apply $\sum\vec{F} = m\vec{a}$ to the 0.5-kg block:

$$\Sigma F_x = F_n\sin\theta - f_s\cos\theta = ma \quad (1)$$

and

$$\Sigma F_y = F_n\cos\theta + f_s\sin\theta - mg = 0 \quad (2)$$

Under minimum acceleration, $f_s = f_{s,max}$. Express the relationship between $f_{s,max}$ and F_n:

$$f_{s,max} = \mu_s F_n \quad (3)$$

Substitute $f_{s,max}$ for f_s in equation (2) and solve for F_n:

$$F_n = \frac{mg}{\cos\theta + \mu_s\sin\theta}$$

Substitute for F_n in equation (1) and solve for $a = a_{min}$:

$$a_{min} = g\frac{\sin\theta - \mu_s\cos\theta}{\cos\theta + \mu_s\sin\theta}$$

Substitute numerical values and evaluate a_{min}:

$$a_{min} = \left(9.81\,\text{m/s}^2\right)\frac{\sin 35° - (0.8)\cos 35°}{\cos 35° + (0.8)\sin 35°} = -0.627\,\text{m/s}^2$$

Treat the block and incline as a single object to determine F_{min}:

$$F_{min} = m_{tot}a_{min} = (2.5\text{ kg})(-0.627\text{ m/s}^2) = \boxed{-1.57\,\text{N}}$$

To find the maximum acceleration, reverse the direction of $\vec{f}_s$ and apply $\sum\vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = F_n\sin\theta + f_s\cos\theta = ma \quad (4)$$

and

$$\Sigma F_y = F_n\cos\theta - f_s\sin\theta - mg = 0 \quad (5)$$

Proceed as above to obtain:

$$a_{max} = g\frac{\sin\theta + \mu_s\cos\theta}{\cos\theta - \mu_s\sin\theta}$$

Substitute numerical values and evaluate a_{max}:

$$a_{max} = \left(9.81\,\text{m/s}^2\right)\frac{\sin 35° + (0.8)\cos 35°}{\cos 35° - (0.8)\sin 35°} = 33.5\,\text{m/s}^2$$

Treat the block and incline as a single object to determine F_{max}:

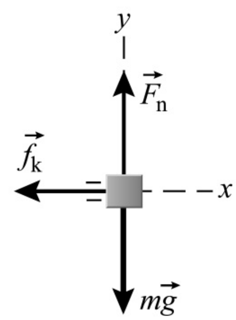
$$F_{max} = m_{tot}a_{max} = (2.5\ \text{kg})(33.5\ \text{m/s}^2) = \boxed{83.8\ \text{N}}$$

(*b*) Repeat (*a*) with $\mu_s = 0.4$ to obtain:

$$F_{min} = \boxed{5.75\ \text{N}}\ \text{and}\ F_{max} = \boxed{37.5\ \text{N}}$$

60 •

Picture the Problem The kinetic friction force f_k is the product of the coefficient of sliding friction μ_k and the normal force F_n the surface exerts on the sliding object. By applying Newton's 2nd law in the vertical direction, we can see that, on a horizontal surface, the normal force is the weight of the sliding object. Note that the acceleration of the block is opposite its direction of motion.



(*a*) Relate the force of kinetic friction to μ_k and the normal force acting on the sliding wooden object:

$$f_k = \mu_k F_n = \frac{0.11}{\left(1+2.3\times10^{-4}v^2\right)^2}mg$$

Substitute v = 10 m/s and evaluate f_k:

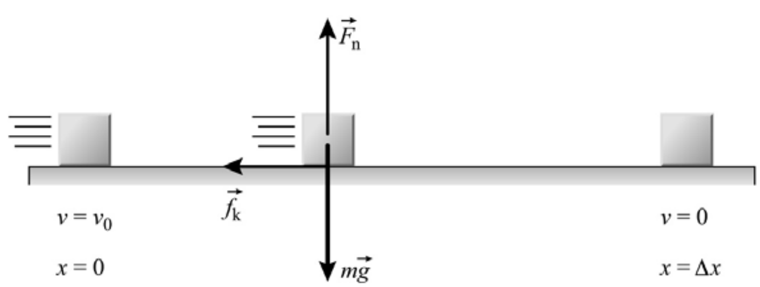
$$f_k = \frac{0.11(100\ \text{kg})(9.81\ \text{m/s}^2)}{\left(1+2.3\times10^{-4}(10\ \text{m/s})^2\right)^2} = \boxed{103\ \text{N}}$$

(*b*) Substitute v = 20 m/s and evaluate f_k:

$$f_k = \frac{0.11(100\ \text{kg})(9.81\ \text{m/s}^2)}{\left(1+2.3\times10^{-4}(20\ \text{m/s})^2\right)^2} = \boxed{90.5\ \text{N}}$$

61 ••

Picture the Problem The pictorial representation shows the block sliding from left to right and coming to rest when it has traveled a distance Δ*x*. Note that the direction of the motion is opposite that of the block's acceleration. The acceleration and stopping distance of the blocks can be found from constant-acceleration equations. Let the direction of motion of the sliding blocks be the positive *x* direction. Because the surface is horizontal, the normal force acting on the sliding block is the block's weight.



(*a*) Using a constant-acceleration equation, relate the block's stopping distance to its initial speed and acceleration; solve for the stopping distance:

$$v^2 = v_0^2 + 2a\Delta x$$

or, because $v = 0$,

$$\Delta x = \frac{-v_0^2}{2a} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the sliding block, introduce Konecny's empirical expression, and solve for the block's acceleration:

$$a = \frac{F_{\text{net},x}}{m} = \frac{-f_k}{m} = -\frac{0.4F_n^{0.91}}{m}$$

$$= -\frac{0.4(mg)^{0.91}}{m}$$

Evaluate a with m = 10 kg:

$$a = -\frac{(0.4)\left[(10\,\text{kg})(9.81\,\text{m/s}^2)\right]^{0.91}}{10\,\text{kg}}$$

$$= \boxed{-2.60\,\text{m/s}^2}$$

Substitute in equation (1) and evaluate the stopping distance when v_0 = 10 m/s:

$$\Delta x = \frac{-(10\,\text{m/s})^2}{2\left(-2.60\,\text{m/s}^2\right)} = \boxed{19.2\,\text{m}}$$

(*b*) Proceed as in (*a*) with m = 100 kg to obtain:

$$a = -\frac{(0.4)\left[(100\,\text{kg})(9.81\,\text{m/s}^2)\right]^{0.91}}{100\,\text{kg}}$$

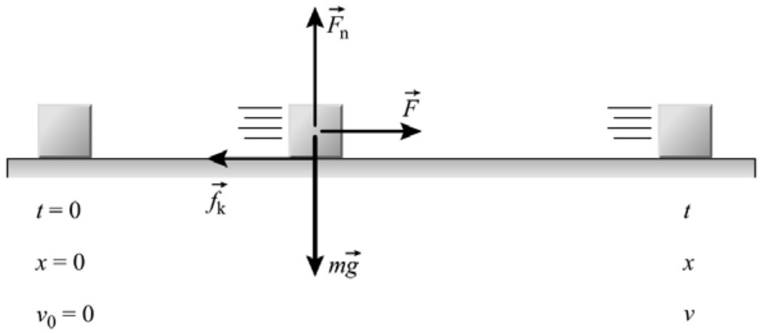
$$= \boxed{-2.11\,\text{m/s}^2}$$

Find the stopping distance as in (*a*):

$$\Delta x = \frac{-(10\,\text{m/s})^2}{2\left(-2.11\,\text{m/s}^2\right)} = \boxed{23.7\,\text{m}}$$

***62 •••**

Picture the Problem The kinetic friction force f_k is the product of the coefficient of sliding friction μ_k and the normal force F_n the surface exerts on the sliding object. By applying Newton's 2nd law in the vertical direction, we can see that, on a horizontal surface, the normal force is the weight of the sliding object. We can apply Newton's 2nd law in the horizontal (x) direction to relate the block's acceleration to the net force acting on it. In the spreadsheet program, we'll find the acceleration of the block from this net force (which is velocity dependent), calculate the increase in the block's speed from its acceleration and the elapsed time and add this increase to its speed at end of the previous time interval, determine how far it has moved in this time interval, and add this distance to its previous position to find its current position. We'll also calculate the position of the block x_2, under the assumption that $\mu_k = 0.11$, using a constant-acceleration equation.



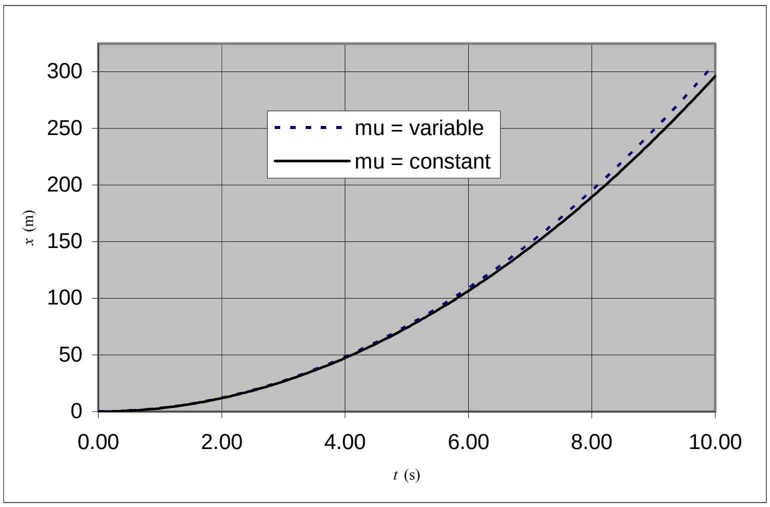
The spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
C9	C8+$B$6	$t + \Delta t$
D9	D8+F9*$B$6	$v + a\Delta t$
E9	$B$5–($B$3)*($B$2)*$B$5/ (1+$B$4*D9^2)^2	$F - \dfrac{\mu_k mg}{\left(1 + 2.34 \times 10^{-4} v^2\right)^2}$
F9	E10/$B$5	F_{net} / m
G9	G9+D10*$B$6	$x + v\Delta t$
K9	0.5*5.922*I10^2	$\frac{1}{2} at^2$
L9	J10-K10	$x - x_2$

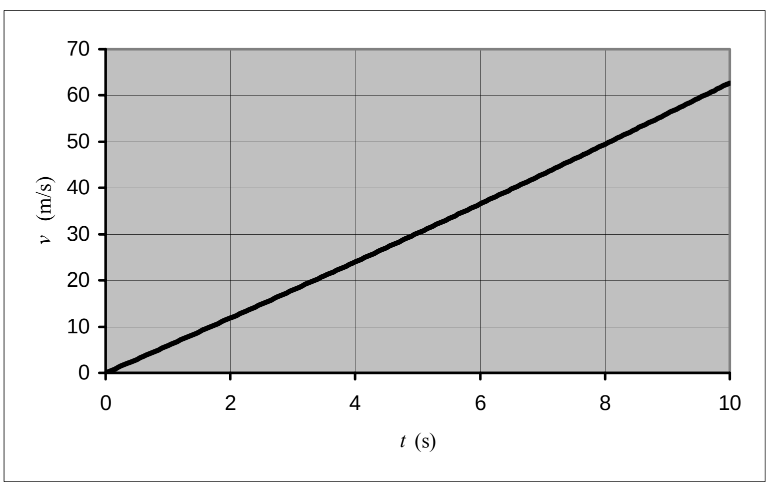
	A	B	C	D	E	F	G	H	I	J
1	g=	9.81	m/s^2							
2	Coeff1=	0.11								
3	Coeff2=	2.30E-04								
4	Mass=	10	kg							
5	Applied Force=	70	N							
6	Time step=	0.05	s				t	x	x2	x–x2
7										
8										
9	t	v	Net force	a	x			mu=variable	mu=constant	
10	0.00	0.00			0.00		0.00	0.00	0.00	0.00
11	0.05	0.30	59.22	5.92	0.01		0.05	0.01	0.01	0.01
12	0.10	0.59	59.22	5.92	0.04		0.10	0.04	0.03	0.01
13	0.15	0.89	59.22	5.92	0.09		0.15	0.09	0.07	0.02
14	0.20	1.18	59.22	5.92	0.15		0.20	0.15	0.12	0.03
15	0.25	1.48	59.23	5.92	0.22		0.25	0.22	0.19	0.04
205	9.75	61.06	66.84	6.68	292.37		9.75	292.37	281.48	10.89
206	9.80	61.40	66.88	6.69	295.44		9.80	295.44	284.37	11.07

207	9.85	61.73	66.91	6.69	298.53		9.85	298.53	287.28	11.25
208	9.90	62.07	66.94	6.69	301.63		9.90	301.63	290.21	11.42
209	9.95	62.40	66.97	6.70	304.75		9.95	304.75	293.15	11.61
210	10.00	62.74	67.00	6.70	307.89		10.00	307.89	296.10	11.79

The displacement of the block as a function of time, for a constant coefficient of friction ($\mu_k = 0.11$) is shown as a solid line on the graph and for a variable coefficient of friction, is shown as a dotted line. Because the coefficient of friction decreases with increasing particle speed, the particle travels slightly farther when the coefficient of friction is variable.

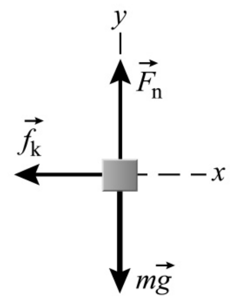


The velocity of the block, with variable coefficient of kinetic friction, is shown below.



63 ••

Picture the Problem The free-body diagram shows the forces acting on the block as it moves to the right. The kinetic friction force will slow the block and, eventually, bring it to rest. We can relate the coefficient of kinetic friction to the stopping time and distance by applying Newton's 2$^{\text{nd}}$ law and then using constant-acceleration equations.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block of wood:

$$\sum F_x = -f_\text{k} = ma$$

and

$$\sum F_y = F_\text{n} - mg = 0$$

Using the definition of f_k, eliminate F_n between the two equations to obtain:

$$a = -\mu_\text{k} g \qquad (1)$$

Use a constant-acceleration equation to relate the acceleration of the block to its displacement and its stopping time:

$$\Delta x = v_0 \Delta t + \tfrac{1}{2} a (\Delta t)^2 \qquad (2)$$

Relate the initial speed of the block, v_0, to its displacement and stopping distance:

$$\Delta x = v_\text{av} \Delta t = \frac{v_0 + v}{2} \Delta t = \tfrac{1}{2} v_0 \Delta t \text{ since } v = 0. \qquad (3)$$

Use this result to eliminate v_0 in equation (2):

$$\Delta x = -\tfrac{1}{2} a (\Delta t)^2 \qquad (4)$$

Substitute equation (1) in equation (4) and solve for μ_k:

$$\mu_k = \frac{2\Delta x}{g(\Delta t)^2}$$

Substitute for Δx = 1.37 m and Δt = 0.97 s to obtain:

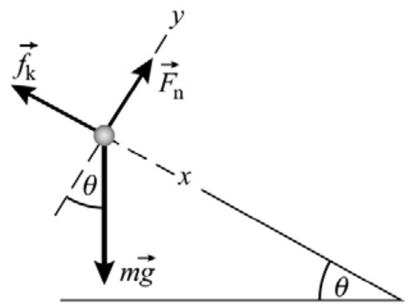
$$\mu_k = \frac{2(1.37\,\text{m})}{(9.81\,\text{m/s}^2)(0.97\,\text{s})^2} = \boxed{0.297}$$

(*b*) Use equation (3) to find v_0:

$$v_0 = \frac{2\Delta x}{\Delta t} = \frac{2(1.37\,\text{m})}{0.97\,\text{s}} = \boxed{2.82\,\text{m/s}}$$

***64** ••

Picture the Problem The free-body diagram shows the forces acting on the block as it slides down an incline. We can apply Newton's 2nd law to these forces to obtain the acceleration of the block and then manipulate this expression algebraically to show that a graph of $a/\cos\theta$ versus $\tan\theta$ will be linear with a slope equal to the acceleration due to gravity and an intercept whose absolute value is the coefficient of kinetic friction.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block as it slides down the incline:

$$\sum F_x = mg\sin\theta - f_k = ma$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Substitute $\mu_k F_n$ for f_k and eliminate F_n between the two equations to obtain:

$$a = g(\sin\theta - \mu_k\cos\theta)$$

Divide both sides of this equation by $\cos\theta$ to obtain:

$$\frac{a}{\cos\theta} = g\tan\theta - g\mu_k$$

Note that this equation is of the form $y = mx + b$:

Thus, if we graph $a/\cos\theta$ versus $\tan\theta$, we should get a straight line with slope g and y-intercept $-g\mu_k$.

(*b*) A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
C7	θ	
D7	a	
E7	TAN(C7*PI()/180)	$\tan\left(\theta\times\frac{\pi}{180}\right)$
F7	D7/COS(C7*PI()/180)	$\frac{a}{\cos\left(\theta\times\frac{\pi}{180}\right)}$

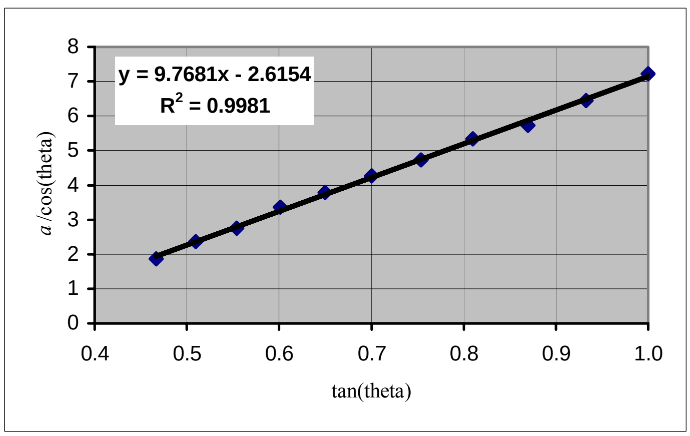
	C	D	E	F
6	theta	a	tan(theta)	a/cos(theta)
7	25	1.691	0.466	1.866
8	27	2.104	0.510	2.362
9	29	2.406	0.554	2.751

10	31	2.888	0.601	3.370
11	33	3.175	0.649	3.786
12	35	3.489	0.700	4.259
13	37	3.781	0.754	4.735
14	39	4.149	0.810	5.338
15	41	4.326	0.869	5.732
16	43	4.718	0.933	6.451
17	45	5.106	1.000	7.220

A graph of $a/\cos\theta$ versus $\tan\theta$ is shown below. From the curve fit (Excel's Trendline was used), $g = 9.77\text{ m/s}^2$ and $\mu_k = \dfrac{2.62\text{ m/s}^2}{9.77\text{ m/s}^2} = 0.268.$

The percentage error in g from the commonly accepted value of 9.81 m/s^2 is

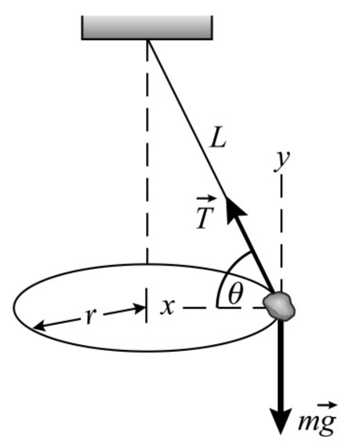
$$100\left(\frac{9.81\text{m/s}^2 - 9.77\text{ m/s}^2}{9.81\text{m/s}^2}\right) = \boxed{0.408\%}$$



Motion Along a Curved Path

65 •

Picture the Problem The free-body diagram showing the forces acting on the stone is superimposed on a sketch of the stone rotating in a horizontal circle. The only forces acting on the stone are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the horizontal is θ by applying Newton's 2nd law of motion to the forces acting on the stone.



Apply $\sum \vec{F} = m\vec{a}$ to the stone:

$\Sigma F_x = T\cos\theta = ma_c = mv^2/r$ (1)

and

$\Sigma F_y = T\sin\theta - mg = 0$ (2)

Use the right triangle in the diagram to relate r, L, and θ:

$r = L\cos\theta$ (3)

Eliminate T and r between equations (1), (2) and (3) and solve for v_2:

$$v^2 = gL\cot\theta\cos\theta \quad (4)$$

Express the velocity of the stone in terms of its period:

$$v = \frac{2\pi r}{t_{1\,\text{rev}}} \quad (5)$$

Eliminate v between equations (4) and (5) and solve for θ:

$$\theta = \sin^{-1}\frac{gt_{1\,\text{rev}}^2}{4\pi^2 L}$$

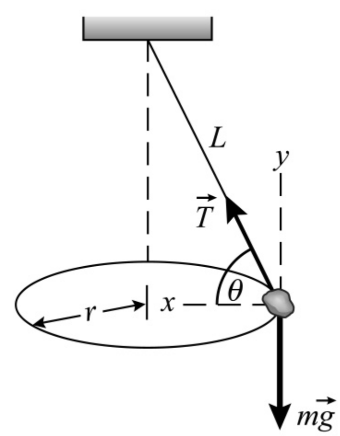
Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\frac{(9.81\,\text{m/s}^2)(1.22\,\text{s})^2}{4\pi^2(0.85\,\text{m})} = 25.8°$$

and $\boxed{(c)\text{ is correct.}}$

66 •

Picture the Problem The free-body diagram showing the forces acting on the stone is superimposed on a sketch of the stone rotating in a horizontal circle. The only forces acting on the stone are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the horizontal is θ by applying Newton's 2^{nd} law of motion to the forces acting on the stone.



Apply $\sum \vec{F} = m\vec{a}$ to the stone:

$\Sigma F_x = T\cos\theta = ma_c = mv^2/r$ (1)

and

$\Sigma F_y = T\sin\theta - mg = 0$ (2)

Use the right triangle in the diagram to relate r, L, and θ:

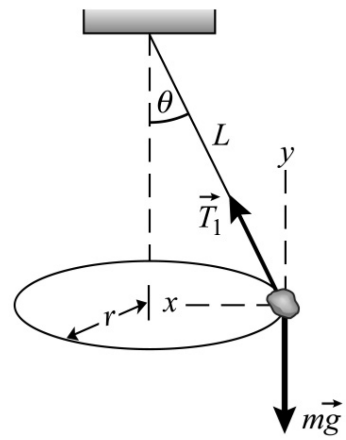
$r = L\cos\theta$ (3)

Eliminate T and r between equations (1), (2), and (3) and solve for v:

$$v = \sqrt{gL\cot\theta\cos\theta}$$
$$= \sqrt{(9.81\,\text{m/s}^2)(0.8\,\text{m})\cot 20°\cos 20°}$$
$$= \boxed{4.50\,\text{m/s}}$$

67 •

Picture the Problem The free-body diagram showing the forces acting on the stone is superimposed on a sketch of the stone rotating in a horizontal circle. The only forces acting on the stone are the tension in the string and the gravitational force. The centripetal force required to maintain the circular motion is a component of the tension. We'll solve the problem for the general case in which the angle with the vertical is θ by applying Newton's 2nd law of motion to the forces acting on the stone.



(*a*) Apply $\sum\vec{F} = m\vec{a}$ to the stone:

$$\Sigma F_x = T\sin\theta = ma_c = mv^2/r \qquad (1)$$
and
$$\Sigma F_y = T\cos\theta - mg = 0 \qquad (2)$$

Eliminate T between equations (1) and (2) and solve for v:

$$v = \sqrt{rg\tan\theta}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{(0.35\,\text{m})(9.81\,\text{m/s}^2)\tan 30°}$$
$$= \boxed{1.41\,\text{m/s}}$$

(*b*) Solve equation (2) for T:

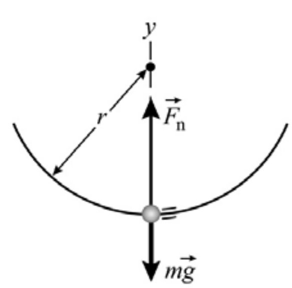
$$T = \frac{mg}{\cos\theta}$$

Substitute numerical values and evaluate T:

$$T = \frac{(0.75\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 30°} = \boxed{8.50\,\text{N}}$$

***68** ••

Picture the Problem The sketch shows the forces acting on the pilot when her plane is at the lowest point of its dive. $\vec{F}_n$ is the force the airplane seat exerts on her. We'll apply Newton's 2nd law for circular motion to determine F_n and the radius of the circular path followed by the airplane.



(*a*) Apply $\sum F_y = ma_y$ to the pilot: $F_n - mg = ma_c$

Solve for and evaluate F_n:

$$F_n = mg + ma_c = m(g + a_c)$$
$$= m(g + 8.5g) = 9.5mg$$
$$= (9.5)(50\text{ kg})(9.81\text{ m/s}^2)$$
$$= \boxed{4.66\text{ kN}}$$

(*b*) Relate her acceleration to her velocity and the radius of the circular arc and solve for the radius:

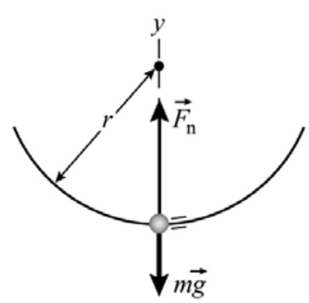
$$a_c = \frac{v^2}{r} \Rightarrow r = \frac{v^2}{a_c}$$

Substitute numerical values and evaluate r:

$$r = \frac{[(345\text{ km/h})(1\text{ h/3600 s})(1000\text{ m/km})]^2}{8.5(9.81\text{ m/s}^2)} = \boxed{110\text{ m}}$$

69 ••

Picture the Problem The diagram shows the forces acting on the pilot when her plane is at the lowest point of its dive. $\vec{F}_n$ is the force the airplane seat exerts on her. We'll use the definitions of centripetal acceleration and centripetal force and apply Newton's 2nd law to calculate these quantities and the normal force acting on her.



(*a*) Her acceleration is centripetal and given by:

$$a_c = \frac{v^2}{r}\text{, upward}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{[(180\text{ km/h})(1\text{ h/3600 s})(10^3\text{ /km})]^2}{300\text{ m}}$$
$$= \boxed{8.33\text{ m/s}^2\text{, upward}}$$

(*b*) The net force acting on her at the bottom of the circle is the force responsible for her centripetal acceleration:

$$F_{net} = ma_c = (65\text{ kg})(8.33\text{ m/s}^2)$$
$$= \boxed{541\text{ N, upward}}$$

(*c*) Apply $\sum F_y = ma_y$ to the pilot: $F_n - mg = ma_c$

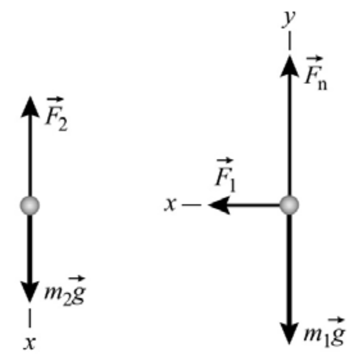
Solve for F_n: $F_n = mg + ma_c = m(g + a_c)$

Substitute numerical values and evaluate F_n:

$$F_n = (65\text{ kg})(9.81\text{ m/s}^2 + 8.33\text{ m/s}^2) = \boxed{1.18\text{ kN, upward}}$$

70 ••

Picture the Problem The free-body diagrams for the two objects are shown to the right. The hole in the table changes the direction the tension in the string (which provides the centripetal force required to keep the object moving in a circular path) acts. The application of Newton's 2nd law and the definition of centripetal force will lead us to an expression for r as a function of m_1, m_2, and the time T for one revolution.



Apply $\sum F_x = ma_x$ to both objects and use the definition of centripetal acceleration to obtain:

$m_2g - F_2 = 0$
and
$F_1 = m_1a_c = m_1v^2/r$

Because $F_1 = F_2$ we can eliminate both of them between these equations to obtain:

$$m_2g - m_1\frac{v^2}{r} = 0$$

Express the speed v of the object in terms of the distance it travels each revolution and the time T for one revolution:

$$v = \frac{2\pi r}{T}$$

Substitute to obtain:

$$m_2g - m_1\frac{4\pi^2r^2}{rT^2} = 0$$

or

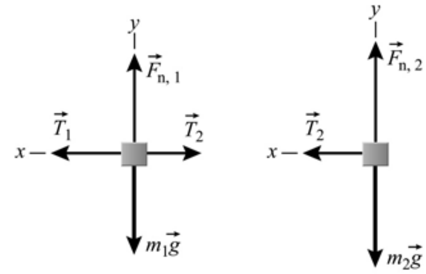
$$m_2g - m_1\frac{4\pi^2r}{T^2} = 0$$

Solve for r:

$$r = \boxed{\frac{m_2gT^2}{4\pi^2m_1}}$$

***71 ••**

Picture the Problem The free-body diagrams show the forces acting on each block. We can use Newton's 2nd law to relate these forces to each other and to the masses and accelerations of the blocks.



Apply $\sum F_x = ma_x$ to the block whose mass is m_1:

$$T_1 - T_2 = m_1 \frac{v_1^2}{L_1}$$

Apply $\sum F_x = ma_x$ to the block whose mass is m_2:

$$T_2 = m_2 \frac{v_2^2}{L_1 + L_2}$$

Relate the speeds of each block to their common period and their distance from the center of the circle:

$$v_1 = \frac{2\pi L_1}{T} \text{ and } v_2 = \frac{2\pi(L_1 + L_2)}{T}$$

Solve the first force equation for T_2, substitute for v_2, and simplify to obtain:

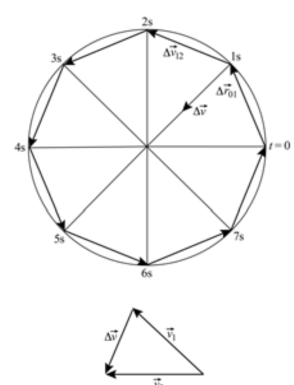
$$T_2 = \boxed{[m_2(L_1 + L_2)]\left(\frac{2\pi}{T}\right)^2}$$

Substitute for T_2 and v_1 in the first force equation to obtain:

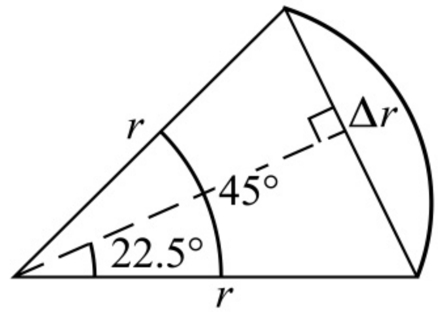
$$T_1 = \boxed{[m_2(L_1 + L_2) + m_1 L_1]\left(\frac{2\pi}{T}\right)^2}$$

***72 ••**

Picture the Problem The path of the particle and its position at 1-s intervals are shown. The displacement vectors are also shown. The velocity vectors for the average velocities in the first and second intervals are along $\vec{r}_{01}$ and $\vec{r}_{12}$, respectively, and are shown in the lower diagram. $\Delta\vec{v}$ points toward the center of the circle.



Use the diagram to the right to find Δr:



$\Delta r = 2r\sin 22.5° = 2(4\text{ cm})\sin 22.5°$
$= 3.06\text{ cm}$

Find the average velocity of the particle along the chords:

$v_{av} = \Delta r/\Delta t = (3.06\text{ cm})/(1\text{ s})$
$= 3.06\text{ cm/s}$

Using the lower diagram and the fact that the angle between $\vec{v}_1$ and $\vec{v}_2$ is 45°, express Δv in terms of v_1 $(= v_2)$:

$\Delta v = 2v_1\sin 22.5°$

Evaluate Δv using v_{av} as v_1:

$\Delta v = 2(3.06\text{ cm/s})\sin 22.5° = 2.34\text{ cm/s}$

Now we can determine $a = \Delta v/\Delta t$:

$$a = \frac{2.34\text{ cm/s}}{1\text{ s}} = \boxed{2.34\text{ cm/s}^2}$$

Find the speed v $(= v_1 = v_2 \ldots)$ of the particle along its circular path:

$$v = \frac{2\pi r}{T} = \frac{2\pi(4\text{ cm})}{8\text{ s}} = 3.14\text{ cm/s}$$

Calculate the radial acceleration of the particle:

$$a_c = \frac{v^2}{r} = \frac{(3.14\text{ cm/s})^2}{4\text{ cm}} = \boxed{2.46\text{ cm/s}^2}$$

Compare a_c and a by taking their ratio:

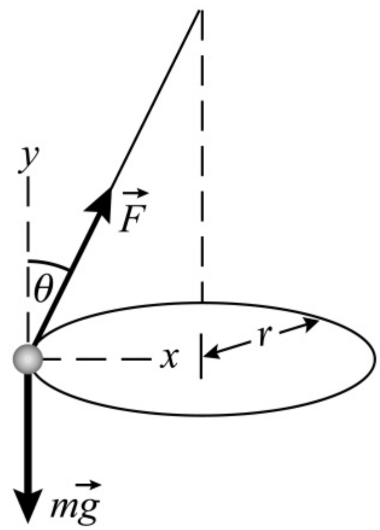
$$\frac{a_c}{a} = \frac{2.46\text{ cm/s}^2}{2.34\text{ cm/s}^2} = 1.05$$

or

$$\boxed{a_c = 1.05a}$$

73 ••

Picture the Problem The diagram to the right has the free-body diagram for the child superimposed on a pictorial representation of her motion. The force her father exerts is $\vec{F}$ and the angle it makes with respect to the direction we've chosen as the positive y direction is θ. We can infer her speed from the given information concerning the radius of her path and the period of her motion. Applying Newton's 2nd law as it describes circular motion will allow us to find both the direction and magnitude of $\vec{F}$.



Apply $\sum \vec{F} = m\vec{a}$ to the child:

$$\Sigma F_x = F\sin\theta = mv^2/r$$

and

$$\Sigma F_y = F\cos\theta - mg = 0$$

Eliminate F between these equations and solve for θ:

$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Express v in terms of the radius and period of the child's motion:

$$v = \frac{2\pi r}{T}$$

Substitute for v in the expression for θ to obtain:

$$\theta = \tan^{-1}\left[\frac{4\pi^2 r}{gT^2}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{4\pi^2(0.75\,\text{m})}{(9.81\,\text{m/s}^2)(1.5\,\text{s})^2}\right] = \boxed{53.3^\circ}$$

Solve the y equation for F:

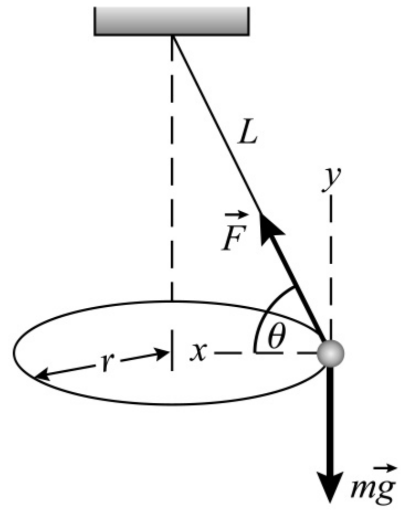
$$F = \frac{mg}{\cos\theta}$$

Substitute numerical values and evaluate F:

$$F = \frac{(25\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 53.3^\circ} = \boxed{410\,\text{N}}$$

74 ••

Picture the Problem The diagram to the right has the free-body diagram for the bob of the conical pendulum superimposed on a pictorial representation of its motion. The tension in the string is $\vec{F}$ and the angle it makes with respect to the direction we've chosen as the positive x direction is θ. We can find θ from the y equation and the information provided about the tension. Then, by using the definition of the speed of the bob in its orbit and applying Newton's 2nd law as it describes circular motion, we can find the period T of the motion.



Apply $\sum \vec{F} = m\vec{a}$ to the pendulum bob:

$\Sigma F_x = F\cos\theta = mv^2/r$

and

$\Sigma F_y = F\sin\theta - mg = 0$

Using the given information that $F = 6mg$, solve the y equation for θ:

$$\theta = \sin^{-1}\frac{mg}{F} = \sin^{-1}\frac{mg}{6mg} = 9.59°$$

With $F = 6mg$, solve the x equation for v:

$$v = \sqrt{6rg\cos\theta}$$

Relate the period T of the motion to the speed of the bob and the radius of the circle in which it moves:

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{6rg\cos\theta}}$$

From the diagram, one can see that:

$r = L\cos\theta$

Substitute for r in the expression for the period to obtain:

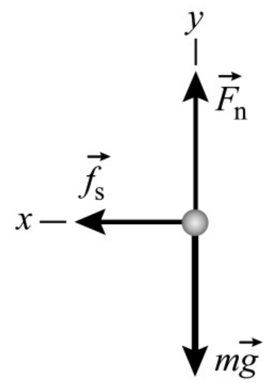
$$T = 2\pi\sqrt{\frac{L}{6g}}$$

Substitute numerical values and evaluate T:

$$T = 2\pi\sqrt{\frac{0.5\,\text{m}}{6(9.81\,\text{m/s}^2)}} = \boxed{0.579\,\text{s}}$$

75 ••

Picture the Problem The static friction force f_s is responsible for keeping the coin from sliding on the turntable. Using Newton's 2nd law of motion, the definition of the period of the coin's motion, and the definition of the maximum static friction force, we can find the magnitude of the friction force and the value of the coefficient of static friction for the two surfaces.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the coin:

$$\sum F_x = f_s = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n - mg = 0$$

If T is the period of the coin's motion, its speed is given by:

$$v = \frac{2\pi r}{T}$$

Substitute for v in the force equation and simplify to obtain:

$$f_s = \frac{4\pi^2 mr}{T^2}$$

Substitute numerical values and evaluate f_s:

$$f_s = \frac{4\pi^2\left((0.1\,\text{kg})(0.1\,\text{m})\right)}{(1\,\text{s})^2} = \boxed{0.395\,\text{N}}$$

(*b*) Determine F_n from the y equation:

$$F_n = mg$$

If the coin is about to slide at $r = 16$ cm, $f_s = f_{s,\max}$. Solve for μ_s in terms of $f_{s,\max}$ and F_n:

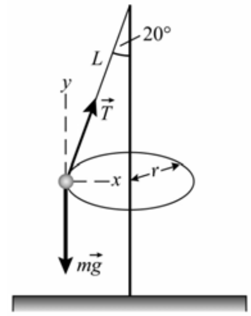
$$\mu_s = \frac{f_{s,\max}}{F_n} = \frac{\frac{4\pi^2 mr}{T^2}}{mg} = \frac{4\pi^2 r}{gT^2}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \frac{4\pi^2(0.16\,\text{m})}{(9.81\,\text{m/s}^2)(1\,\text{s})^2} = \boxed{0.644}$$

76 ••

Picture the Problem The forces acting on the tetherball are shown superimposed on a pictorial representation of the motion. The horizontal component of $\vec{T}$ is the centripetal force. Applying Newton's 2nd law of motion and solving the resulting equations will yield both the tension in the cord and the speed of the ball.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the tetherball:

$$\sum F_x = T\sin 20° = m\frac{v^2}{r}$$

and

$$\sum F_y = T\cos 20° - mg = 0$$

Solve the *y* equation for *T*:

$$T = \frac{mg}{\cos 20°}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{(0.25\,\text{kg})(9.81\,\text{m/s}^2)}{\cos 20°} = \boxed{2.61\,\text{N}}$$

(*b*) Eliminate *T* between the force equations and solve for *v*:

$$v = \sqrt{rg\tan 20°}$$

Note from the diagram that:

$$r = L\sin 20°$$

Substitute for *r* in the expression for *v* to obtain:

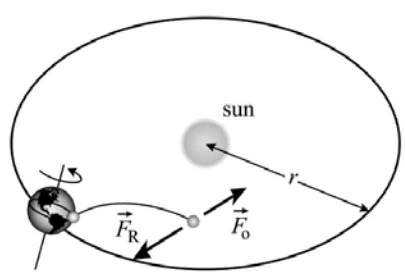
$$v = \sqrt{gL\sin 20°\tan 20°}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{(9.81\,\text{m/s}^2)(1.2\,\text{m})\sin 20°\tan 20°} = \boxed{1.21\,\text{m/s}}$$

***77** ••

Picture the Problem The diagram includes a pictorial representation of the earth in its orbit about the sun and a force diagram showing the force on an object at the equator that is due to the earth's rotation, $\vec{F}_R$, and the force on the object due to the orbital motion of the earth about



the sun, $\vec{\boldsymbol{F}}_\mathrm{o}$. Because these are centripetal forces, we can calculate the accelerations they require from the speeds and radii associated with the two circular motions.

Express the radial acceleration due to the rotation of the earth:

$$a_\mathrm{R} = \frac{v_\mathrm{R}^2}{R}$$

Express the speed of the object on the equator in terms of the radius of the earth R and the period of the earth's rotation T_R:

$$v_\mathrm{R} = \frac{2\pi R}{T_\mathrm{R}}$$

Substitute for v_R in the expression for a_R to obtain:

$$a_\mathrm{R} = \frac{4\pi^2 R}{T_\mathrm{R}^2} = \frac{4\pi^2(6370\,\mathrm{km})(1000\,\mathrm{m/km})}{\left[(24\,\mathrm{h})\left(\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right)\right]^2}$$
$$= 3.37\times10^{-2}\,\mathrm{m/s^2}$$
$$= \boxed{3.44\times10^{-3}\,g}$$

Express the radial acceleration due to the orbital motion of the earth:

$$a_\mathrm{o} = \frac{v_\mathrm{o}^2}{r}$$

Express the speed of the object on the equator in terms of the earth-sun distance r and the period of the earth's motion about the sun T_o:

$$v_\mathrm{o} = \frac{2\pi r}{T_\mathrm{o}}$$

Substitute for v_o in the expression for a_o to obtain:

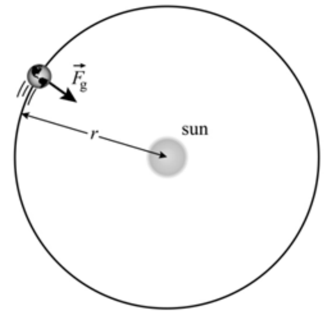
$$a_\mathrm{o} = \frac{4\pi^2 r}{T_\mathrm{o}^2}$$

Substitute numerical values and evaluate a_c:

$$a_\mathrm{o} = \frac{4\pi^2(1.5\times10^{11}\,\mathrm{m})}{\left[(365\,\mathrm{d})\left(\frac{24\,\mathrm{h}}{1\,\mathrm{d}}\right)\left(\frac{3600\,\mathrm{s}}{1\,\mathrm{h}}\right)\right]^2}$$
$$= 5.95\times10^{-3}\,\mathrm{m/s^2} = \boxed{6.07\times10^{-4}\,g}$$

78 •

Picture the Problem The most significant force acting on the earth is the gravitational force exerted by the sun. More distant or less massive objects exert forces on the earth as well, but we can calculate the net force by considering the radial acceleration of the earth in its orbit. Similarly, we can calculate the net force acting on the moon by considering its radial acceleration in its orbit about the earth.



(*a*) Apply $\sum F_r = ma_r$ to the earth:

$$F_{\text{on earth}} = m\frac{v^2}{r}$$

Express the orbital speed of the earth in terms of the time it takes to make one trip around the sun (i.e., its period) and its average distance from the sun:

$$v = \frac{2\pi r}{T}$$

Substitute for v to obtain:

$$F_{\text{on earth}} = \frac{4\pi^2 mr}{T^2}$$

Substitute numerical values and evaluate $F_{\text{on earth}}$:

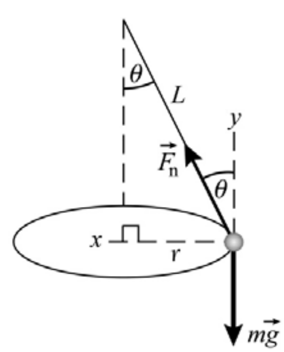
$$F_{\text{on earth}} = \frac{4\pi^2\left(5.98\times10^{24}\text{ kg}\right)\left(1.496\times10^{11}\text{ m}\right)}{\left(365.24\text{ d}\times\frac{24\text{ h}}{\text{d}}\times\frac{3600\text{ s}}{\text{h}}\right)^2} = \boxed{3.55\times10^{22}\text{ N}}$$

(*b*) Proceed as in (*a*) to obtain:

$$F_{\text{on moon}} = \frac{4\pi^2\left(7.35\times10^{22}\text{ kg}\right)\left(3.844\times10^{8}\text{ m}\right)}{\left(27.32\text{ d}\times\frac{24\text{ h}}{\text{d}}\times\frac{3600\text{ s}}{\text{h}}\right)^2} = \boxed{2.00\times10^{20}\text{ N}}$$

79 ••

Picture the Problem The semicircular wire of radius 10 cm limits the motion of the bead in the same manner as would a 10-cm string attached to the bead and fixed at the center of the semicircle. The horizontal component of the normal force the wire exerts on the bead is the centripetal force. The application of Newton's 2nd law, the definition of the



speed of the bead in its orbit, and the relationship of the frequency of a circular motion to its period will yield the angle at which the bead will remain stationary relative to the rotating wire.

Apply $\sum \vec{F} = m\vec{a}$ to the bead:

$$\sum F_x = F_n \sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - mg = 0$$

Eliminate F_n from the force equations to obtain:

$$\tan\theta = \frac{v^2}{rg}$$

The frequency of the motion is the reciprocal of its period T. Express the speed of the bead as a function of the radius of its path and its period:

$$v = \frac{2\pi r}{T}$$

Using the diagram, relate r to L and θ:

$$r = L\sin\theta$$

Substitute for r and v in the expression for $\tan\theta$ and solve for θ:

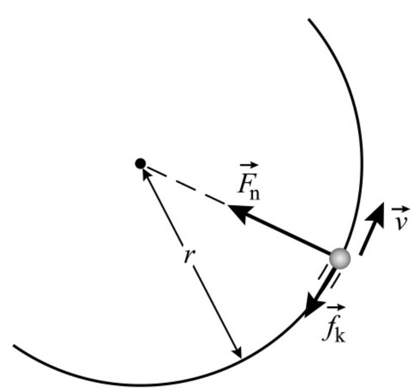
$$\theta = \cos^{-1}\left[\frac{gT^2}{4\pi^2 L}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left[\frac{(9.81\,\text{m/s}^2)(0.5\,\text{s})^2}{4\pi^2(0.1\,\text{m})}\right] = \boxed{51.6°}$$

80 •••

Picture the Problem Note that the acceleration of the bead has two components, the radial component perpendicular to $\vec{v}$, and a tangential component due to friction that is opposite to $\vec{v}$. The application of Newton's 2nd law will result in a differential equation with separable variables. Its integration will lead to an expression for the speed of the bead as a function of time.



Apply $\sum \vec{F} = m\vec{a}$ to the bead in the radial and tangential directions:

$$\sum F_r = F_\text{n} = m\frac{v^2}{r}$$

and

$$\sum F_t = -f_\text{k} = ma_t = m\frac{dv}{dt}$$

Express f_k in terms of μ_k and F_n:

$$f_\text{k} = \mu_\text{k} F_\text{n}$$

Substitute for F_n and f_k in the tangential equation to obtain the differential equation:

$$\frac{dv}{dt} = -\frac{\mu_\text{k}}{r}v^2$$

Separate the variables to obtain:

$$\frac{dv}{v^2} = -\frac{\mu_\text{k}}{r}dt$$

Express the integral of this equation with the limits of integration being from v_0 to v on the left-hand side and from 0 to t on the right-hand side:

$$\int_{v_0}^{v} \frac{1}{v'^2}dv' = -\frac{\mu_\text{k}}{r}\int_0^t dt'$$

Evaluate these integrals to obtain:

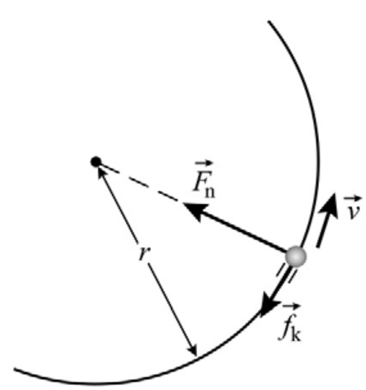
$$-\left(\frac{1}{v} - \frac{1}{v_0}\right) = -\left(\frac{\mu_\text{k}}{r}\right)t$$

Solve this equation for v:

$$\boxed{v = v_0\left(\frac{1}{1+\left(\frac{\mu_\text{k} v_0}{r}\right)t}\right)}$$

81 •••

Picture the Problem Note that the acceleration of the bead has two components–the radial component perpendicular to $\vec{v}$, and a tangential component due to friction that is opposite to $\vec{v}$. The application of Newton's 2nd law will result in a differential equation with separable variables. Its integration will lead to an expression for the speed of the bead as a function of time.



(*a*) In Problem 81 it was shown that:

$$v = v_0\left(\frac{1}{1+\left(\frac{\mu_k v_0}{r}\right)t}\right)$$

Express the centripetal acceleration of the bead:

$$a_c = \frac{v^2}{r} = \boxed{\frac{v_0^2}{r}\left(\frac{1}{1+\left(\frac{\mu_k v_0}{r}\right)t}\right)^2}$$

(*b*) Apply Newton's 2nd law to the bead:

$$\sum F_r = F_n = m\frac{v^2}{r}$$

and

$$\sum F_t = -f_k = ma_t = m\frac{dv}{dt}$$

Eliminate F_n and f_k to rewrite the radial force equation and solve for a_t:

$$a_t = -\mu_k\frac{v^2}{r} = \boxed{-\mu_k a_c}$$

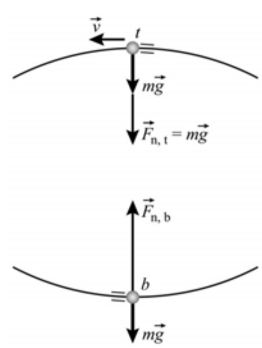
(*c*) Express the resultant acceleration in terms of its radial and tangential components:

$$a = \sqrt{a_t^2 + a_c^2} = \sqrt{(-\mu_k a_c)^2 + a_c^2}$$
$$= \boxed{a_c\sqrt{1+\mu_k^2}}$$

Concepts of Centripetal Force

***82 •**

Picture the Problem The diagram depicts a seat at its highest and lowest points. Let ″t″ denote the top of the loop and ″b″ the bottom of the loop. Applying Newton's 2nd law to the seat at the top of the loop will establish the value of mv^2/r; this can then be used at the bottom of the loop to determine $F_{n,b}$.



Apply $\sum F_r = ma_r$ to the seat at the top of the loop:

$$mg + F_{n,t} = 2mg = ma_r = mv^2/r$$

Apply $\sum F_r = ma_r$ to the seat at the

$$F_{n,b} - mg = mv^2/r$$

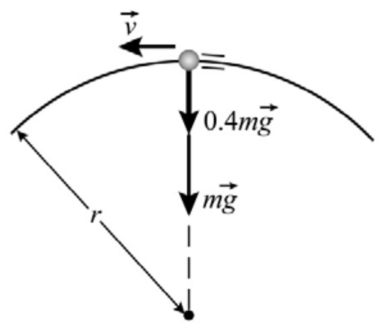
bottom of the loop:

Solve for $F_{n,b}$ and substitute for mv^2/r to obtain:

$F_{n,b} = 3mg$ and $\boxed{(d)\text{ is correct.}}$

83 •

Picture the Problem The speed of the roller coaster is imbedded in the expression for its radial acceleration. The radial acceleration is determined by the net radial force acting on the passenger. We can use Newton's 2nd law to relate the net force on the passenger to the speed of the roller coaster.



Apply $\sum F_{radial} = ma_{radial}$ to the passenger:

$mg + 0.4mg = mv^2/r$

Solve for v:

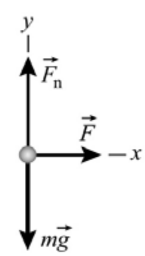
$v = \sqrt{1.4gr}$

Substitute numerical values and evaluate v:

$$v = \sqrt{1.4(9.81\,\text{m/s}^2)(12.0\,\text{m})} = \boxed{12.8\,\text{m/s}}$$

84 •

Picture the Problem The force F the passenger exerts on the armrest of the car door is the radial force required to maintain the passenger's speed around the curve and is related to that speed through Newton's 2nd law of motion.



Apply $\sum F_x = ma_x$ to the forces acting on the passenger:

$$F = m\frac{v^2}{r}$$

Solve this equation for v:

$$v = \sqrt{\frac{rF}{m}}$$

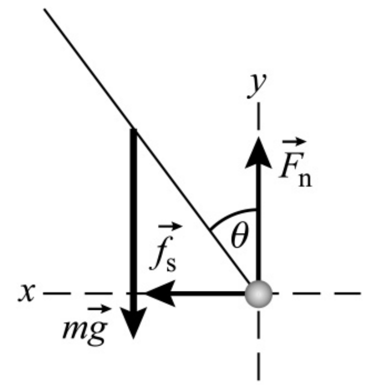
Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{(80\,\text{m})(220\,\text{N})}{70\,\text{kg}}} = 15.9\,\text{m/s}$$

and $\boxed{(a)\text{ is correct.}}$

***85 •••**

Picture the Problem The forces acting on the bicycle are shown in the force diagram. The static friction force is the centripetal force exerted by the surface on the bicycle that allows it to move in a circular path. $\vec{F}_n + \vec{f}_s$ makes an angle θ with the vertical direction. The application of Newton's 2nd law will allow us to relate this angle to the speed of the bicycle and the coefficient of static friction.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the bicycle:

$$\sum F_x = f_s = \frac{mv^2}{r}$$

and

$$\sum F_y = F_n - mg = 0$$

Relate F_n and f_s to θ:

$$\tan\theta = \frac{f_s}{F_n} = \frac{\frac{mv^2}{r}}{mg} = \frac{v^2}{rg}$$

Solve for *v*:

$$v = \sqrt{rg\tan\theta}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{(20\,\text{m})(9.81\,\text{m/s}^2)\tan 15°} = \boxed{7.25\,\text{m/s}}$$

(*b*) Relate f_s to μ_s and F_n:

$$f_s = \tfrac{1}{2} f_{s,\max} = \tfrac{1}{2}\mu_s mg$$

Solve for μ_s and substitute for f_s to obtain:

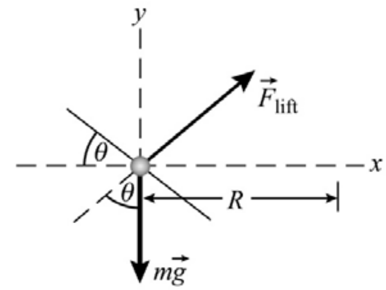
$$\mu_s = \frac{2f_s}{mg} = \frac{2v^2}{rg}$$

Substitute numerical values and evaluate μ_s

$$\mu_s = \frac{2(7.25\,\text{m/s})^2}{(20\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{0.536}$$

86 ••

Picture the Problem The diagram shows the forces acting on the plane as it flies in a horizontal circle of radius *R*. We can apply Newton's 2nd law to the plane and eliminate the lift force in order to obtain an expression for *R* as a function of *v* and θ.



Apply $\sum \vec{F} = m\vec{a}$ to the plane:

$$\sum F_x = F_{\text{lift}} \sin\theta = m\frac{v^2}{R}$$

and

$$\sum F_y = F_{\text{lift}} \cos\theta - mg = 0$$

Eliminate F_{lift} between these equations to obtain:

$$\tan\theta = \frac{v^2}{Rg}$$

Solve for *R*:

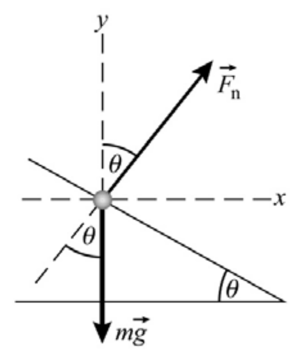
$$R = \frac{v^2}{g\tan\theta}$$

Substitute numerical values and evaluate *R*:

$$R = \frac{\left(480\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}}\right)^2}{\left(9.81\,\text{m/s}^2\right)\tan 40^\circ} = \boxed{2.16\,\text{km}}$$

87 •

Picture the Problem Under the conditions described in the problem statement, the only forces acting on the car are the normal force exerted by the road and the gravitational force exerted by the earth. The horizontal component of the normal force is the centripetal force. The application of Newton's 2nd law will allow us to express θ in terms of *v*, *r*, and *g*.



Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_{\text{n}} \sin\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_{\text{n}} \cos\theta - mg = 0$$

Eliminate F_n from the force equations to obtain:

$$\tan\theta = \frac{v^2}{rg}$$

Solve for θ:

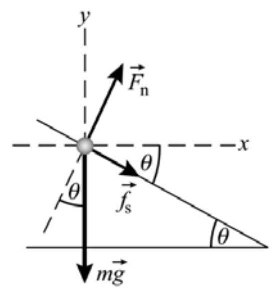
$$\theta = \tan^{-1}\left[\frac{v^2}{rg}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left\{\frac{[(90\,\text{km/h})(1\,\text{h}/3600\,\text{s})(1000\,\text{m/km})]^2}{(160\,\text{m})(9.81\,\text{m/s}^2)}\right\} = \boxed{21.7°}$$

***88 ••**

Picture the Problem Both the normal force and the static friction force contribute to the centripetal force in the situation described in this problem. We can apply Newton's 2nd law to relate f_s and F_n and then solve these equations simultaneously to determine each of these quantities.



(*a*) Apply $\sum\vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_n \sin\theta + f_s\cos\theta = m\frac{v^2}{r}$$

and

$$\sum F_y = F_n\cos\theta - f_s\sin\theta - mg = 0$$

Multiply the *x* equation by $\sin\theta$ and the *y* equation by $\cos\theta$ to obtain:

$$f_s\sin\theta\cos\theta + F_n\sin^2\theta = m\frac{v^2}{r}\sin\theta$$

and

$$F_n\cos^2\theta - f_s\sin\theta\cos\theta - mg\cos\theta = 0$$

Add these equations to eliminate f_s:

$$F_n - mg\cos\theta = m\frac{v^2}{r}\sin\theta$$

Solve for F_n:

$$F_n = mg\cos\theta + m\frac{v^2}{r}\sin\theta$$
$$= m\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)$$

Substitute numerical values and evaluate F_n:

$$F_n = (800\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos 10° + \frac{(85\,\text{km/h})^2(1000\,\text{m/km})^2(1\,\text{h/3600\,s})^2}{150\,\text{m}}\sin 10°\right]$$
$$= \boxed{8.25\,\text{kN}}$$

(*b*) Solve the *y* equation for f_s:

$$f_s = \frac{F_n \cos\theta - mg}{\sin\theta}$$

Substitute numerical values and evaluate f_s:

$$f_s = \frac{(8.25\,\text{kN})\cos 10° - (800\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 10°} = \boxed{1.59\,\text{kN}}$$

(*c*) Express $\mu_{\text{s,min}}$ in terms of f_s and F_n:

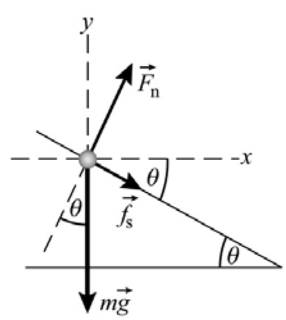
$$\mu_{\text{s,min}} = \frac{f_s}{F_n}$$

Substitute numerical values and evaluate $\mu_{\text{s,min}}$:

$$\mu_{\text{s,min}} = \frac{1.59\text{ kN}}{8.25\,\text{kN}} = \boxed{0.193}$$

89 ••

Picture the Problem Both the normal force and the static friction force contribute to the centripetal force in the situation described in this problem. We can apply Newton's 2nd law to relate f_s and F_n and then solve these equations simultaneously to determine each of these quantities.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the car:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v^2}{r}$$
$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Multiply the *x* equation by $\sin\theta$ and the *y* equation by $\cos\theta$:

$$f_s \sin\theta\cos\theta + F_n \sin^2\theta = m\frac{v^2}{r}\sin\theta$$
$$F_n \cos^2\theta - f_s \sin\theta\cos\theta - mg\cos\theta = 0$$

Add these equations to eliminate f_s:

$$F_n - mg\cos\theta = m\frac{v^2}{r}\sin\theta$$

Solve for F_n:

$$F_n = mg\cos\theta + m\frac{v^2}{r}\sin\theta = m\left(g\cos\theta + \frac{v^2}{r}\sin\theta\right)$$

Substitute numerical values and evaluate F_n:

$$F_n = (800\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos 10° + \frac{(38\,\text{km/h})^2(1000\,\text{m/km})^2(1\,\text{h}/3600\,\text{s})^2}{150\,\text{m}}\sin 10°\right] = \boxed{7.832\,\text{kN}}$$

(*b*) Solve the *y* equation for f_s:

$$f_s = \frac{F_n\cos\theta - mg}{\sin\theta} = F_n\cot\theta - \frac{mg}{\sin\theta}$$

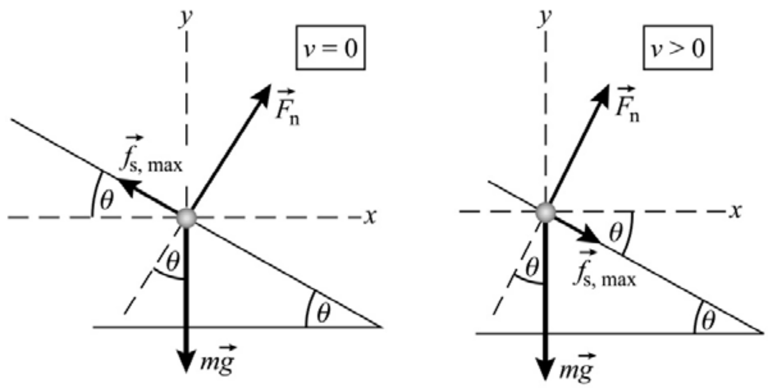
Substitute numerical values and evaluate f_s:

$$f_s = (7.832\,\text{kN})\cot 10° - \frac{(800\,\text{kg})(9.81\,\text{m/s}^2)}{\sin 10°} = \boxed{-777\,\text{N}}$$

The negative sign tells us that f_s points upward along the inclined plane rather than as shown in the force diagram.

***90 •••**

Picture the Problem The free-body diagram to the left is for the car at rest. The static friction force up the incline balances the downward component of the car's weight and prevents it from sliding. In the free-body diagram to the right, the static friction force points in the opposite direction as the tendency of the moving car is to slide toward the outside of the curve.



Apply $\sum \vec{F} = m\vec{a}$ to the car that is at rest:

$$\sum F_y = F_n \cos\theta + f_s \sin\theta - mg = 0 \quad (1)$$

and

$$\sum F_x = F_n \sin\theta - f_s \cos\theta = 0 \quad (2)$$

Substitute $f_s = f_{s,max} = \mu_s F_n$ in equation (2) and solve for and evaluate the maximum allowable value of θ:

$$\theta = \tan^{-1}\mu_s = \tan^{-1} 0.08 = \boxed{4.57°}$$

Apply $\sum \vec{F} = m\vec{a}$ to the car that is moving with speed v:

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0 \quad (3)$$

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v^2}{r} \quad (4)$$

Substitute $f_s = \mu_s F_n$ in equations (3) and (4) and simplify to obtain:

$$F_n(\cos\theta - \mu_s \sin\theta) = mg \quad (5)$$

$$F_n(\mu_s \cos\theta + \sin\theta) = m\frac{v^2}{r} \quad (6)$$

Substitute numerical values into (5) and (6) to obtain:

$$0.9904F_n = mg$$

and

$$0.1595F_n = m\frac{v^2}{r}$$

Eliminate F_n and solve for r:

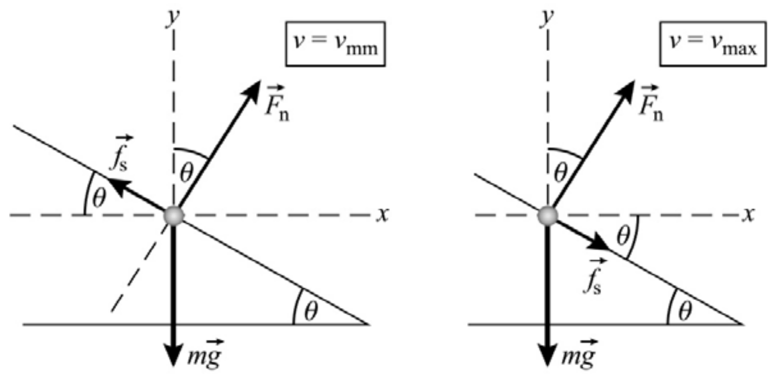
$$r = \frac{v^2}{0.1610g}$$

Substitute numerical values and evaluate r:

$$r = \frac{(60\,\text{km/h} \times 1\,\text{h}/3600\,\text{s} \times 1000\,\text{m/km})^2}{0.1610(9.81\,\text{m/s}^2)} = \boxed{176\,\text{m}}$$

91 •••

Picture the Problem The free-body diagram to the left is for the car rounding the curve at the minimum (nonsliding down the incline) speed. The static friction force up the incline balances the downward component of the car's weight and prevents it from sliding. In the free-body diagram to the right, the static friction force points in the opposite direction as the tendency of the car moving with the maximum safe speed is to slide toward the outside of the curve. Application of Newton's 2nd law and the simultaneous solution of the force equations will yield v_{min} and v_{max}.



Apply $\sum \vec{F} = m\vec{a}$ to a car traveling around the curve when the coefficient of static friction is zero:

$$\sum F_x = F_n \sin\theta = m\frac{v_{min}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - mg = 0$$

Divide the first of these equations by the second to obtain:

$$\tan\theta = \frac{v^2}{rg} \text{ or } \theta = \tan^{-1}\left(\frac{v^2}{rg}\right)$$

Substitute numerical values and evaluate the banking angle:

$$\theta = \tan^{-1}\left[\frac{(40\,\text{km/h})^2(1000\,\text{m/km})^2(1\text{h}/3600\,\text{s}^2)}{(30\,\text{m})(9.81\,\text{m/s}^2)}\right] = 22.8°$$

Apply $\sum \vec{F} = m\vec{a}$ to the car traveling around the curve at minimum speed:

$$\sum F_x = F_n \sin\theta - f_s \cos\theta = m\frac{v_{min}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta + f_s \sin\theta - mg = 0$$

Substitute $f_s = f_{s,max} = \mu_s F_n$ in the force equations and simplify to obtain:

$$F_n(\mu_s \cos\theta - \sin\theta) = m\frac{v_{min}^2}{r}$$

and

$$F_n(\cos\theta + \mu_s \sin\theta) = mg$$

Evaluate these equations for $\theta = 22.8°$ and $\mu_s = 0.3$:

$$0.1102 F_n = m\frac{v_{min}^2}{r}$$

and

$$1.038 F_n = mg$$

Eliminate F_n between these two equations and solve for v_{min}:

$$v_{min} = \sqrt{0.106 rg}$$

Substitute numerical values and evaluate v_{min}:

$$v_{\text{min}} = \sqrt{0.106(30\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{5.59\,\text{m/s} = 20.1\,\text{km/h}}$$

Apply $\sum \vec{F} = m\vec{a}$ to the car traveling around the curve at maximum speed:

$$\sum F_x = F_n \sin\theta + f_s \cos\theta = m\frac{v_{\text{max}}^2}{r}$$

and

$$\sum F_y = F_n \cos\theta - f_s \sin\theta - mg = 0$$

Substitute $f_s = f_{s,\text{max}} = \mu_s F_n$ in the force equations and simplify to obtain:

$$F_n(\mu_s \cos\theta + \sin\theta) = m\frac{v_{\text{max}}^2}{r}$$

and

$$F_n(\cos\theta - \mu_s \sin\theta) = mg$$

Evaluate these equations for $\theta = 22.8°$ and $\mu_s = 0.3$:

$$0.6641 F_n = m\frac{v_{\text{max}}^2}{r}$$

and

$$0.8056 F_n = mg$$

Eliminate F_n between these two equations and solve for v_{max}:

$$v_{\text{max}} = \sqrt{0.8243 rg}$$

Substitute numerical values and evaluate v_{max}:

$$v_{\text{max}} = \sqrt{(0.8243)(30\,\text{m})(9.81\,\text{m/s}^2)} = \boxed{15.6\,\text{m/s} = 56.1\,\text{km/h}}$$

Drag Forces

92 •

Picture the Problem We can apply Newton's 2nd law to the particle to obtain its equation of motion. Applying terminal speed conditions will yield an expression for b that we can evaluate using the given numerical values.

Apply $\sum F_y = ma_y$ to the particle:

$$mg - bv = ma_y$$

When the particle reaches its terminal speed $v = v_t$ and $a_y = 0$:

$$mg - bv_t = 0$$

Solve for b to obtain:

$$b = \frac{mg}{v_t}$$

Substitute numerical values and evaluate b:

$$b=\frac{(10^{-13}\,\text{kg})(9.81\,\text{m/s}^2)}{3\times10^{-4}\,\text{m/s}}=\boxed{3.27\times10^{-9}\,\text{kg/s}}$$

93 •

Picture the Problem We can apply Newton's 2nd law to the Ping-Pong ball to obtain its equation of motion. Applying terminal speed conditions will yield an expression for b that we can evaluate using the given numerical values.

Apply $\sum F_y = ma_y$ to the Ping-Pong ball:

$$mg-bv^2=ma_y$$

When the Ping-Pong ball reaches its terminal speed $v = v_t$ and $a_y = 0$:

$$mg-bv_t^2=0$$

Solve for b to obtain:

$$b=\frac{mg}{v_t^2}$$

Substitute numerical values and evaluate b:

$$b=\frac{(2.3\times10^{-3}\,\text{kg})(9.81\,\text{m/s}^2)}{(9\,\text{m/s})^2}=\boxed{2.79\times10^{-4}\,\text{kg/m}}$$

***94** •

Picture the Problem Let the upward direction be the positive y direction and apply Newton's 2nd law to the sky diver.

(*a*) Apply $\sum F_y = ma_y$ to the sky diver:

$$F_d-mg=ma_y$$

or, because $a_y = 0$,

$$F_d=mg \qquad (1)$$

Substitute numerical values and evaluate F_d:

$$F_d=(60\,\text{kg})(9.81\,\text{m/s}^2)=\boxed{589\,\text{N}}$$

(*b*) Substitute $F_d = bv_t^2$ in equation (1) to obtain:

$$bv_t^2=mg$$

Solve for b:

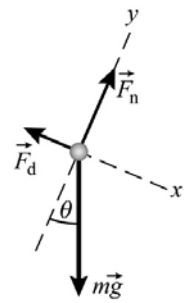
$$b=\frac{mg}{v_t^2}=\frac{F_d}{v_t^2}$$

Substitute numerical values and evaluate b:

$$b = \frac{589\,\text{N}}{(25\,\text{m/s})^2} = \boxed{0.942\,\text{kg/m}}$$

95 ••

Picture the Problem The free-body diagram shows the forces acting on the car as it descends the grade with its terminal velocity. The application of Newton's 2nd law with $a = 0$ and F_d equal to the given function will allow us to solve for the terminal velocity of the car.



Apply $\sum F_x = ma_x$ to the car:

$$mg\sin\theta - F_\text{d} = ma_x$$

or, because $v = v_\text{t}$ and $a_x = 0$,

$$mg\sin\theta - F_\text{d} = 0$$

Substitute for F_d to obtain:

$$mg\sin\theta - 100\,\text{N} - \left(1.2\,\text{N}\cdot\text{s}^2/\text{m}^2\right)v_\text{t}^2 = 0$$

Solve for v_t:

$$v_\text{t} = \sqrt{\frac{mg\sin\theta - 100\,\text{N}}{1.2\,\text{N}\cdot\text{s}^2/\text{m}^2}}$$

Substitute numerical values and evaluate v_t:

$$v_\text{t} = \sqrt{\frac{(800\,\text{kg})\left(9.81\,\text{m/s}^2\right)\sin 6° - 100\,\text{N}}{1.2\,\text{N}\cdot\text{s}^2/\text{m}^2}}$$

$$= 24.5\,\text{m/s} = \boxed{88.2\,\text{km/h}}$$

96 •••

Picture the Problem Let the upward direction be the positive y direction and apply Newton's 2nd law to the particle to obtain an equation from which we can find the particle's terminal speed.

(*a*) Apply $\sum F_y = ma_y$ to a pollution particle:

$$mg - 6\pi\eta r v = ma_y$$

or, because $a_y = 0$,

$$mg - 6\pi\eta r v_\text{t} = 0$$

Solve for v_t to obtain:

$$v_\text{t} = \frac{mg}{6\pi\eta r}$$

Express the mass of a sphere in terms of its volume:

$$m = \rho V = \rho\left(\frac{4\pi r^3}{3}\right)$$

Substitute for m to obtain:

$$v_t = \frac{2r^2\rho g}{9\eta}$$

Substitute numerical values and evaluate v_t:

$$v_t = \frac{2\left(10^{-5}\text{ m}\right)^2\left(2000\text{ kg/m}^3\right)\left(9.81\text{ m/s}^2\right)}{9\left(1.8\times10^{-5}\text{ N}\cdot\text{s/m}^2\right)} = \boxed{2.42\text{ cm/s}}$$

(*b*) Use distance equals average speed times the fall time to find the time to fall 100 m at 2.42 cm/s:

$$t = \frac{10^4\text{ cm}}{2.42\text{ cm/s}} = 4.13\times10^3\text{ s} = \boxed{1.15\text{ h}}$$

***97 •••**

Picture the Problem The motion of the centrifuge will cause the pollution particles to migrate to the end of the test tube. We can apply Newton's 2nd law and Stokes' law to derive an expression for the terminal speed of the sedimentation particles. We can then use this terminal speed to calculate the sedimentation time. We'll use the 12 cm distance from the center of the centrifuge as the average radius of the pollution particles as they settle in the test tube. Let R represent the radius of a particle and r the radius of the particle's circular path in the centrifuge.

Express the sedimentation time in terms of the sedimentation speed v_t:

$$\Delta t_{\text{sediment}} = \frac{\Delta x}{v_t}$$

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to a pollution particle:

$$6\pi\eta R v_t = ma_c$$

Express the mass of the particle in terms of its radius R and density ρ:

$$m = \rho V = \tfrac{4}{3}\pi R^3\rho$$

Express the acceleration of the pollution particles due to the motion of the centrifuge in terms of their orbital radius r and period T:

$$a_c = \frac{v^2}{r} = \frac{\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{4\pi^2 r}{T^2}$$

Substitute for m and a_c and simplify to obtain:

$$6\pi\eta R v_t = \tfrac{4}{3}\pi R^3\rho\left(\frac{4\pi^2 r}{T^2}\right) = \frac{16\pi^3\rho r R^3}{3T^2}$$

Solve for v_t:

$$v_{\text{t}} = \frac{8\pi^2 \rho r R^2}{9\eta T^2}$$

Find the period T of the motion from the number of revolutions the centrifuge makes in 1 second:

$$T = \frac{1}{800\,\text{rev/min}} = 1.25\times10^{-3}\ \text{min/rev}$$
$$= 1.25\times10^{-3}\ \text{min/rev}\times 60\,\text{s/min}$$
$$= 75.0\times10^{-3}\ \text{s/rev}$$

Substitute numerical values and evaluate v_t:

$$v_{\text{t}} = \frac{8\pi^2\left(2000\,\text{kg/m}^3\right)\left(0.12\,\text{m}\right)\left(10^{-5}\,\text{m}\right)^2}{9\left(1.8\times10^{-5}\,\text{N}\cdot\text{s/m}^2\right)\left(75\times10^{-3}\,\text{s}\right)^2}$$
$$= 2.08\,\text{m/s}$$

Find the time it takes the particles to move 8 cm as they settle in the test tube:

$$\Delta t_{\text{sediment}} = \frac{\Delta x}{v} = \frac{8\,\text{cm}}{208\,\text{cm/s}} = \boxed{38.5\,\text{ms}}$$

In Problem 96 it was shown that the rate of fall of the particles in air is 2.42 cm/s. Find the time required to fall 8 cm in air under the influence of gravity:

$$\Delta t_{\text{air}} = \frac{\Delta x}{v} = \frac{8\,\text{cm}}{2.42\,\text{cm/s}} = \boxed{3.31\,\text{s}}$$

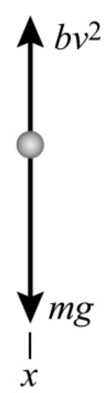
Find the ratio of the two times:

$$\Delta t_{\text{air}}/\Delta t_{\text{sediment}} \approx \boxed{100}$$

Euler's Method

98 ••

Picture the Problem The free-body diagram shows the forces acting on the baseball sometime after it has been thrown downward but before it has reached its terminal speed. In order to use Euler's method, we'll need to determine how the acceleration of the ball varies with its speed. We can do this by applying Newton's 2nd law to the ball and using its terminal speed to express the constant in the acceleration equation in terms of the ball's terminal speed. We can then use $v_{n+1} = v_n + a_n\Delta t$ to find the speed of the ball at any given time.



Apply Newton's 2nd law to the ball to obtain:

$$mg - bv^2 = m\frac{dv}{dt}$$

Solve for dv/dt to obtain:

$$\frac{dv}{dt} = g - \frac{b}{m}v^2$$

When the ball reaches its terminal speed:

$$0 = g - \frac{b}{m}v_t^2 \Rightarrow \frac{b}{m} = \frac{g}{v_t^2}$$

Substitute to obtain:

$$\frac{dv}{dt} = g\left(1 - \frac{v^2}{v_t^2}\right)$$

Express the position of the ball to obtain:

$$x_{n+1} = x_n + \frac{v_{n+1} + v_n}{2}\Delta t$$

Letting a_n be the acceleration of the ball at time t_n, express its speed when $t = t_n + 1$:

$$v_{n+1} = v_n + a_n \Delta t$$

where

$$a_n = g\left(1 - \frac{v_n^2}{v_t^2}\right)$$

and Δt is an arbitrarily small interval of time.

A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

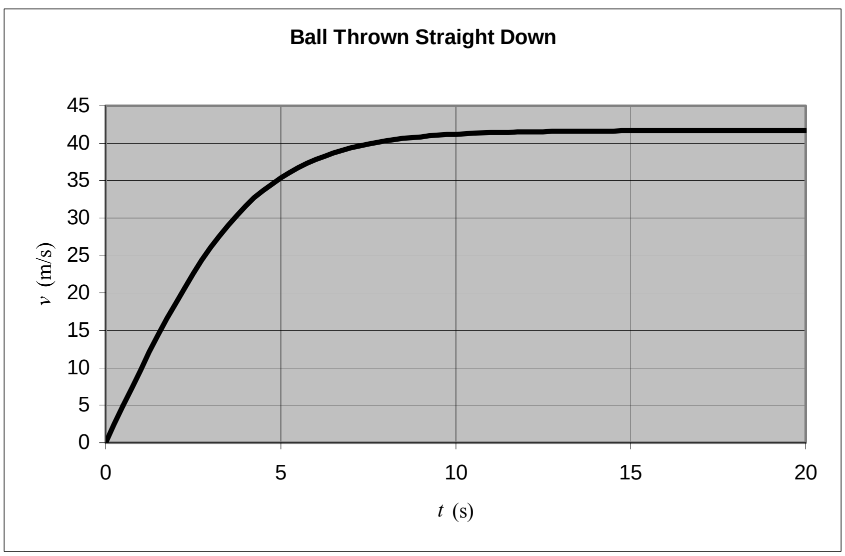
Cell	Formula/Content	Algebraic Form
A10	B9+$B$1	$t + \Delta t$
B10	B9+0.5*(C9+C10)*$B$1	$x_{n+1} = x_n + \frac{v_{n+1} + v_n}{2}\Delta t$
C10	C9+D9*$B$1	$v_{n+1} = v_n + a_n\Delta t$
D10	$B$4*(1–C10^2/$B$5^2)	$a_n = g\left(1 - \frac{v_n^2}{v_t^2}\right)$

	A	B	C	D
1	Δt=	0.5	s	
2	x0=	0	m	
3	v0=	9.722	m/s	
4	a0=	9.81	m/s^2	
5	vt=	41.67	m/s	
6				
7	t	x	v	a
8	(s)	(m)	(m/s)	(m/s^2)
9	0.0	0	9.7	9.28
10	0.5	6	14.4	8.64

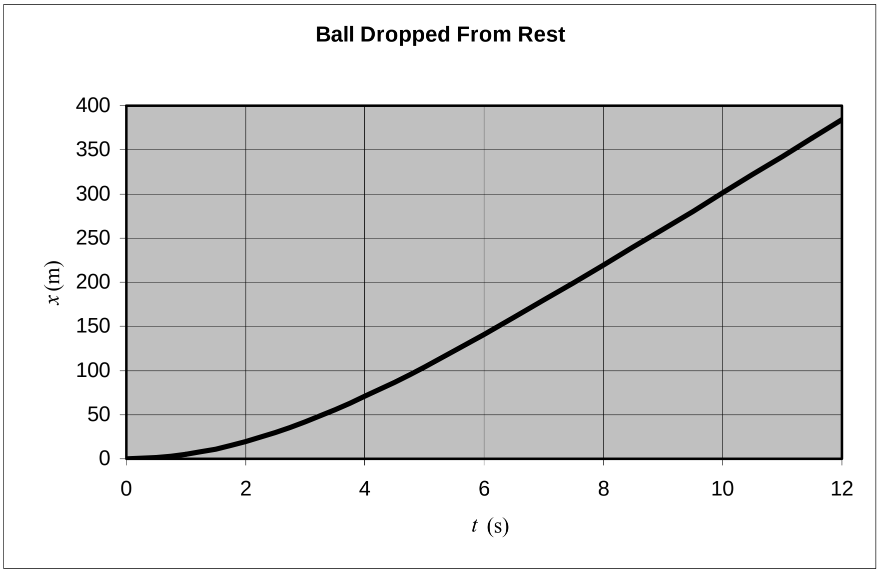
11	1.0	14	18.7	7.84
12	1.5	25	22.6	6.92
28	9.5	317	41.3	0.17
29	10.0	337	41.4	0.13
30	10.5	358	41.5	0.10
38	14.5	524	41.6	0.01
39	15.0	545	41.7	0.01
40	15.5	566	41.7	0.01
41	16.0	587	41.7	0.01
42	16.5	608	41.7	0.00

From the table we can see that the speed of the ball after 10 s is approximately $\boxed{41.4\,\text{m/s.}}$ We can estimate the uncertainty in this result by halving Δt and recalculating the speed of the ball at $t = 10$ s. Doing so yields $v(10\text{ s}) \approx 41.3$ m/s, a difference of about $\boxed{0.02\%.}$

The graph shows the velocity of the ball thrown straight down as a function of time.

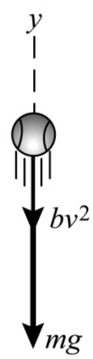


Reset Δt to 0.5 s and set $v_0 = 0$. Ninety-nine percent of 41.67 m/s is approximately 41.3 m/s. Note that the ball will reach this speed in about $\boxed{10.5\,\text{s}}$ and that the distance it travels in this time is about $\boxed{322\,\text{m.}}$ The following graph shows the distance traveled by the ball dropped from rest as a function of time.



***99** ••

Picture the Problem The free-body diagram shows the forces acting on the baseball after it has left your hand. In order to use Euler's method, we'll need to determine how the acceleration of the ball varies with its speed. We can do this by applying Newton's 2nd law to the baseball. We can then use $v_{n+1} = v_n + a_n\Delta t$ and $x_{n+1} = x_n + v_n\Delta t$ to find the speed and position of the ball.



Apply $\sum F_y = ma_y$ to the baseball:

$$-bv|v| - mg = m\frac{dv}{dt}$$

where $|v| = v$ for the upward part of the flight of the ball and $|v| = -v$ for the downward part of the flight.

Solve for dv/dt:

$$\frac{dv}{dt} = -g - \frac{b}{m}v|v|$$

Under terminal speed conditions ($|v| = -v_{\text{t}}$):

$$0 = -g + \frac{b}{m}v_{\text{t}}^2$$

and

$$\frac{b}{m} = \frac{g}{v_{\text{t}}^2}$$

Substitute to obtain:

$$\frac{dv}{dt} = -g - \frac{g}{v_t^2} v|v| = -g\left(1 + \frac{v|v|}{v_t^2}\right)$$

Letting a_n be the acceleration of the ball at time t_n, express its position and speed when $t = t_n + 1$:

$$y_{n+1} = y_n + \tfrac{1}{2}(v_n + v_{n-1})\Delta t$$

and

$$v_{n+1} = v_n + a_n \Delta t$$

where

$$a_n = -g\left(1 + \frac{v_n|v_n|}{v_t^2}\right)$$

and Δt is an arbitrarily small interval of time.

A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

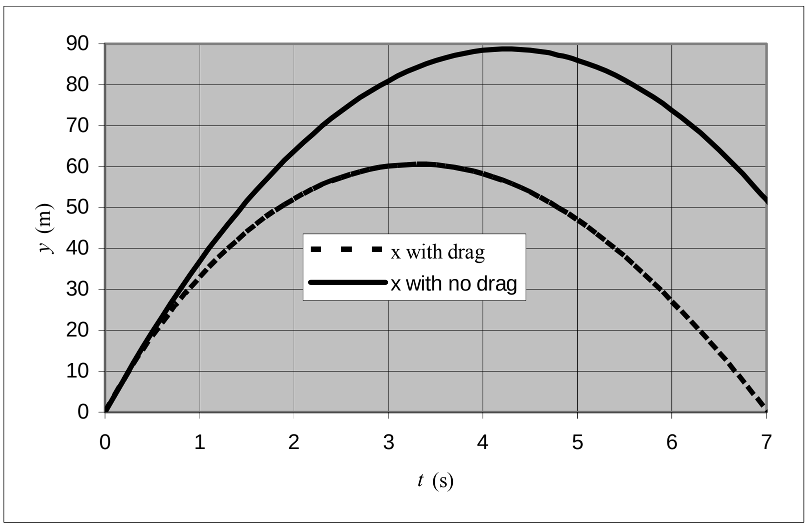
Cell	Formula/Content	Algebraic Form
D11	D10+$B$6	$t + \Delta t$
E10	41.7	v_0
E11	E10–$B$4* (1+E10*ABS(E10)/($B$5^2))*$B$6	$v_{n+1} = v_n + a_n \Delta t$
F10	0	y_0
F11	F10+0.5*(E10+E11)*$B$6	$y_{n+1} = y_n + \frac{1}{2}(v_n + v_{n-1})\Delta t$
G10	0	y_0
G11	$E$10*D11–0.5*$B$4*D11^2	$v_0 t - \frac{1}{2} g t^2$

	A	B	C	D	E	F	G
4	g=	9.81	m/s^2				
5	vt=	41.7	m/s				
6	Δt=	0.1	s				
7							
8							
9				t	v	y	y no drag
10				0.0	41.70	0.00	0.00
11				0.1	39.74	4.07	4.12
12				0.2	37.87	7.95	8.14
40				3.0	3.01	60.13	81.00
41				3.1	2.03	60.39	82.18
42				3.2	1.05	60.54	83.26
43				3.3	0.07	60.60	84.25
44				3.4	–0.91	60.55	85.14
45				3.5	–1.89	60.41	85.93
46				3.6	–2.87	60.17	86.62

78				6.8	–28.34	6.26	56.98
79				6.9	–28.86	3.41	54.44
80				7.0	–29.37	0.49	51.80
81				7.1	–29.87	–2.47	49.06

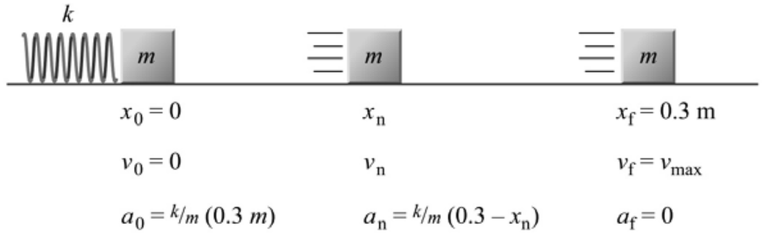
From the table we can see that, after 3.5 s, the ball reaches a height of about $\boxed{60.4\,\text{m.}}$ It reaches its peak a little earlier–at about $\boxed{3.3\,\text{s,}}$ and its height at $t = 3.3$ s is $\boxed{60.6\,\text{m.}}$ The ball hits the ground at about $t = \boxed{7\,\text{s}}$ –so it spends a little longer coming down than going up.

The solid curve on the following graph shows $y(t)$ when there is no drag on the baseball and the dotted curve shows $y(t)$ under the conditions modeled in this problem.



100 ••

Picture the Problem The pictorial representation shows the block in its initial position against the compressed spring, later as the spring accelerates it to the right, and finally when it has reached its maximum speed at $x_f = 0$. In order to use Euler's method, we'll need to determine how the acceleration of the block varies with its position. We can do this by applying Newton's 2nd law to the box. We can then use $v_{n+1} = v_n + a_n \Delta t$ and $x_{n+1} = x_n + v_n \Delta t$ to find the speed and position of the block.



Apply $\sum F_x = ma_x$ to the block: $k(0.3\,\text{m} - x_n) = ma_n$

Solve for a_n:

$$a_n = \frac{k}{m}(0.3\,\text{m} - x_n)$$

Express the position and speed of the block when $t = t_n + 1$:

$$x_{n+1} = x_n + v_n \Delta t$$

and

$$v_{n+1} = v_n + a_n \Delta t$$

where

$$a_n = \frac{k}{m}(0.3\,\text{m} - x_n)$$

and Δt is an arbitrarily small interval of time.

A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
A10	A9+$B$1	$t + \Delta t$
B10	B9+C10*$B$1	$x_n + v_n \Delta t$
C10	C9+D9*$B$1	$v_n + a_n \Delta t$
D10	($B$4/$B$5)*(0.3–B10)	$\frac{k}{m}(0.3 - x_n)$

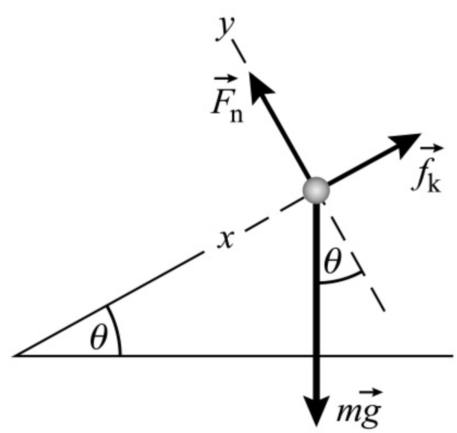
	A	B	C	D
1	Δt=	0.005	s	
2	x0=	0	m	
3	v0=	0	m/s	
4	k =	50	N/m	
5	m =	0.8	kg	
6				
7	t	x	v	a
8	(s)	(m)	(m/s)	(m/s^2)
9	0.000	0.00	0.00	18.75
10	0.005	0.00	0.09	18.72
11	0.010	0.00	0.19	18.69
12	0.015	0.00	0.28	18.63
45	0.180	0.25	2.41	2.85
46	0.185	0.27	2.42	2.10
47	0.190	0.28	2.43	1.34
48	0.195	0.29	2.44	0.58
49	0.200	0.30	2.44	–0.19

From the table we can see that it took about $\boxed{0.200\,\text{s}}$ for the spring to push the block 30 cm and that it was traveling about $\boxed{2.44\,\text{m/s}}$ at that time. We can estimate the uncertainty in this result by halving Δt and recalculating the speed of the ball at $t = 10$ s. Doing so yields $v(0.200\text{ s}) \approx 2.41$ m/s, a difference of about $\boxed{1.2\%.}$

General Problems

101 •

Picture the Problem The forces that act on the block as it slides down the incline are shown on the free-body diagram to the right. The acceleration of the block can be determined from the distance-and-time information given in the problem. The application of Newton's 2nd law to the block will lead to an expression for the coefficient of kinetic friction as a function of the block's acceleration and the angle of the incline.



Apply $\sum \vec{F} = m\vec{a}$ to the block:

$$\Sigma F_x = mg\sin\theta - f_k = ma$$
and
$$\Sigma F_y = F_n - mg = 0$$

Set $f_k = \mu_k F_n$, F_n between the two equations, and solve for μ_k:

$$\mu_k = \frac{g\sin\theta - a}{g\cos\theta}$$

Using a constant-acceleration equation, relate the distance the block slides to its sliding time:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2 \text{ where } v_0 = 0$$

Solve for a:

$$a = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate a:

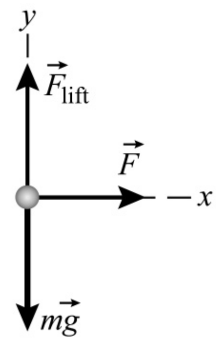
$$a = \frac{2(2.4\,\text{m})}{(5.2\,\text{s})^2} = 0.1775\,\text{m/s}^2$$

Find μ_k for $a = 0.1775$ m/s^2 and $\theta = 28°$:

$$\mu_k = \frac{(9.81\,\text{m/s}^2)\sin 28° - 0.1775\,\text{m/s}^2}{(9.81\,\text{m/s}^2)\cos 28°} = \boxed{0.511}$$

102 •

Picture the Problem The free-body diagram shows the forces acting on the model airplane. The speed of the plane can be calculated from the data concerning the radius of its path and the time it takes to make one revolution. The application of Newton's 2nd law will give us the tension F in the string.



(*a*) Express the speed of the airplane in terms of the circumference of the circle in which it is flying and its period:

$$v = \frac{2\pi r}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi(5.7\,\text{m})}{\frac{4}{1.2}\text{s}} = \boxed{10.7\,\text{m/s}}$$

(*b*) Apply $\sum F_x = ma_x$ to the model airplane:

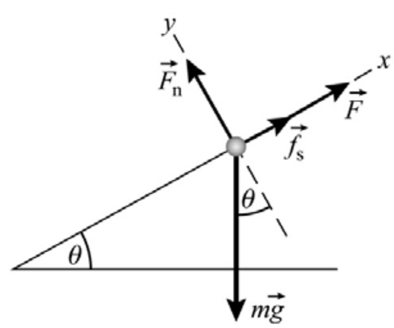
$$F = m\frac{v^2}{r}$$

Substitute numerical values and evaluate F:

$$F = (0.4\,\text{kg})\frac{(10.7\,\text{m/s})^2}{5.7\,\text{m}} = \boxed{8.03\,\text{N}}$$

***103** ••

Picture the Problem The free-body diagram shows the forces acting on the box. If the student is pushing with a force of 200 N and the box is on the verge of moving, the static friction force must be at its maximum value. In part (*b*), the motion is impending up the incline; therefore the direction of $f_{\text{s,max}}$ is down the incline.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the box:

$$\sum F_x = f_{\text{s}} + F - mg\sin\theta = 0$$

and

$$\sum F_y = F_{\text{n}} - mg\cos\theta = 0$$

Substitute $f_s = f_{s,max} = \mu_s F_n$, eliminate F_n between the two equations, and solve for μ_s:

$$\mu_s = \tan\theta - \frac{F}{mg\cos\theta}$$

Substitute numerical values and evaluate μ_s:

$$\mu_s = \tan 30° - \frac{200\text{ N}}{(800\text{ N})\cos 30°} = \boxed{0.289}$$

(*b*) Find $f_{s,max}$ from the *x*-direction force equation:

$$f_{s,max} = mg\sin\theta - F$$

Substitute numerical values and evaluate $f_{s,max}$:

$$f_{s,max} = (800\text{ N})\sin 30° - 200\text{ N} = 200\text{ N}$$

If the block is on the verge of sliding up the incline, $f_{s,max}$ must act down the incline. The *x*-direction force equation becomes:

$$-f_{s,max} + F - mg\sin\theta = 0$$

Solve the *x*-direction force equation for *F*:

$$F = mg\sin\theta + f_{s,max}$$

Substitute numerical values and evaluate *F*:

$$F = (800\text{ N})\sin 30° + 200\text{ N} = \boxed{600\text{ N}}$$

104 •

Picture the Problem The path of the particle is a circle if *r* is a constant. Once we have shown that it is, we can calculate its value from its components. The direction of the particle's motion can be determined by examining two positions of the particle at times that are close to each other.

(*a*) and (*b*) Express the magnitude of $\vec{r}$ in terms of its components:

$$r = \sqrt{r_x^2 + r_y^2}$$

Evaluate *r* with $r_x = -10\text{ m}\cos\omega t$ and $r_y = 10\text{ m}\sin\omega t$:

$$r = \sqrt{[(-10\text{ m})\cos\omega t]^2 + [(10\text{ m})\sin\omega t]^2} = \sqrt{100(\cos^2\omega t + \sin^2\omega t)}\text{ m} = \boxed{10.0\text{ m}}$$

(*c*) Evaluate r_x and r_y at $t = 0$ s:

$$r_x = -(10\,\text{m})\cos 0° = -10\,\text{m}$$
$$r_y = (10\,\text{m})\sin 0° = 0$$

Evaluate r_x and r_y at $t = \Delta t$, where Δt is small:

$$r_x = -(10\,\text{m})\cos\omega\Delta t \approx -(10\,\text{m})\cos 0°$$
$$= -10\,\text{m}$$
$$r_y = (10\,\text{m})\sin\omega\Delta t$$
$$= \Delta y \text{ where } \Delta y \text{ is positive}$$

and $\boxed{\text{the motion is clockwise}}$

(*d*) Differentiate $\vec{r}$ with respect to time to obtain $\vec{v}$:

$$\vec{v} = d\vec{r}/dt$$
$$= [(10\omega\sin\omega t)\,m]\,\hat{i} + [(10\omega\cos\omega t)m]\,\hat{j}$$

Use the components of $\vec{v}$ to find its speed:

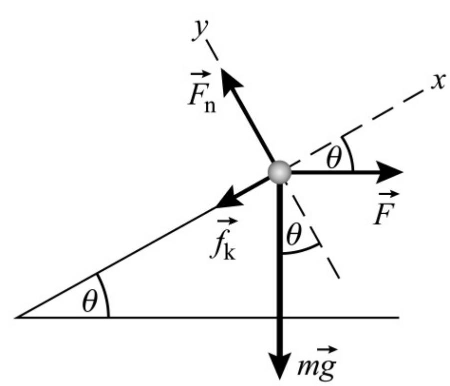
$$v = \sqrt{v_x^2 + v_y^2}$$
$$= \sqrt{[(10\omega\sin\omega t)\text{m}]^2 + [(10\omega\cos\omega t)\text{m}]^2}$$
$$= (10\,\text{m})\omega = (10\,\text{m})(2\,\text{s}^{-1})$$
$$= \boxed{20.0\,\text{m/s}}$$

(*e*) Relate the period of the particle's motion to the radius of its path and its speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi(10\,\text{m})}{20\,\text{m/s}} = \boxed{\pi\,\text{s}}$$

105 ••

Picture the Problem The free-body diagram shows the forces acting on the crate of books. The kinetic friction force opposes the motion of the crate up the incline. Because the crate is moving at constant speed in a straight line, its acceleration is zero. We can determine F by applying Newton's 2nd law to the crate, substituting for f_k, eliminating the normal force, and solving for the required force.



Apply $\sum\vec{F} = m\vec{a}$ to the crate, with both a_x and a_y equal to zero, to the crate:

$$\sum F_x = F\cos\theta - f_k - mg\sin\theta = 0$$

and

$$\sum F_y = F_n - F\sin\theta - mg\cos\theta = 0$$

Substitute $\mu_s F_n$ for f_k and eliminate F_n to obtain:

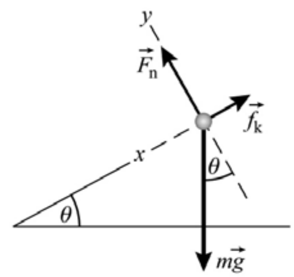
$$F = \frac{mg(\sin\theta + \mu_k \cos\theta)}{\cos\theta - \mu_k \sin\theta}$$

Substitute numerical values and evaluate F:

$$F = \frac{(100\,\text{kg})(9.81\,\text{m/s}^2)(\sin 30° + (0.5)\cos 30°)}{\cos 30° - (0.5)\sin 30°} = \boxed{1.49\,\text{kN}}$$

106 ••

Picture the Problem The free-body diagram shows the forces acting on the object as it slides down the inclined plane. We can calculate its speed at the bottom of the incline from its acceleration and displacement and find its acceleration from Newton's 2nd law.



Using a constant-acceleration equation, relate the initial and final velocities of the object to its acceleration and displacement: solve for the final velocity:

$$v^2 = v_0^2 + 2a\Delta x$$

Because $v_0 = 0$, $v = \sqrt{2a\Delta x}$ (1)

Apply $\sum \vec{F} = m\vec{a}$ to the sliding object:

$$\sum F_x = -f_k + mg\sin\theta = ma$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Solve the y equation for F_n and using $f_k = \mu_k F_n$, eliminate both F_n and f_k from the x equation and solve for a:

$$a = g(\sin\theta - \mu_k \cos\theta) \qquad (2)$$

Substitute equation (2) in equation (1) and solve for v:

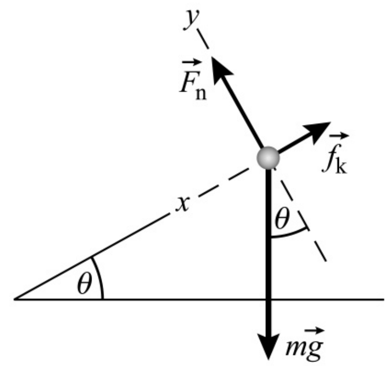
$$v = \sqrt{2g(\sin\theta - \mu_k \cos\theta)\Delta x}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(\sin 30° - (0.35)\cos 30°)(72\,\text{m})} = 16.7\,\text{m/s} \text{ and } \boxed{(d)\text{ is correct.}}$$

***107 ••**

Picture the Problem The free-body diagram shows the forces acting on the brick as it slides down the inclined plane. We'll apply Newton's 2nd law to the brick when it is sliding down the incline with constant speed to derive an expression for μ_k in terms of θ_0. We'll apply Newton's 2nd law a second time for $\theta = \theta_1$ and solve the equations simultaneously to obtain an expression for a as a function of θ_0 and θ_1.



Apply $\sum \vec{F} = m\vec{a}$ to the brick when it is sliding with constant speed:

$$\sum F_x = -f_k + mg\sin\theta_0 = 0$$

and

$$\sum F_y = F_n - mg\cos\theta_0 = 0$$

Solve the y equation for F_n and using $f_k = \mu_k F_n$, eliminate both F_n and f_k from the x equation and solve for μ_k:

$$\mu_k = \tan\theta_0$$

Apply $\sum \vec{F} = m\vec{a}$ to the brick when $\theta = \theta_1$:

$$\sum F_x = -f_k + mg\sin\theta_1 = ma$$

and

$$\sum F_y = F_n - mg\cos\theta_1 = 0$$

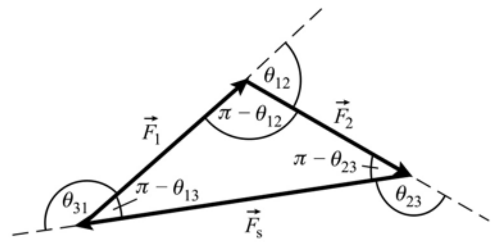
Solve the y equation for F_n, use $f_k = \mu_k F_n$ to eliminate both F_n and f_k from the x equation, and use the expression for μ_k obtained above to obtain:

$$\boxed{a = g(\sin\theta_1 - \tan\theta_0\cos\theta_1)}$$

108 ••

Picture the Problem The fact that the object is in static equilibrium under the influence of the three forces means that $\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = 0$. Drawing the corresponding force triangle will allow us to relate the forces to the angles between them through the law of sines and the law of cosines.

(a) Using the fact that the object is in static equilibrium, redraw the force diagram connecting the forces head-to-tail:



Apply the law of sines to the triangle:

$$\frac{F_1}{\sin(\pi-\theta_{23})}=\frac{F_2}{\sin(\pi-\theta_{13})}=\frac{F_3}{\sin(\pi-\theta_{12})}$$

Use the trigonometric identity $\sin(\pi-\alpha)=\sin\alpha$ to obtain:

$$\boxed{\frac{F_1}{\sin\theta_{23}}=\frac{F_2}{\sin\theta_{13}}=\frac{F_3}{\sin\theta_{12}}}$$

(b) Apply the law of cosines to the triangle:

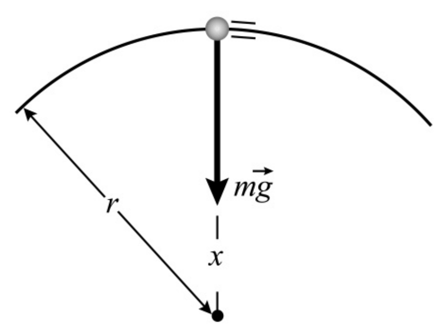
$$F_1^2=F_2^2+F_3^2-2F_2F_3\cos(\pi-\theta_{23})$$

Use the trigonometric identity $\cos(\pi-\alpha)=-\cos\alpha$ to obtain:

$$\boxed{F_1^2=F_2^2+F_3^2+2F_2F_3\cos\theta_{23}}$$

109 ••

Picture the Problem We can calculate the acceleration of the passenger from his/her speed that, in turn, is a function of the period of the motion. To determine the longest period of the motion, we focus our attention on the situation at the very top of the ride when the seat belt is exerting no force on the rider. We can use Newton's 2nd law to relate the period of the motion to the acceleration and speed of the rider.



(a) Because the motion is at constant speed, the acceleration is entirely radial and is given by:

$$a_c=\frac{v^2}{r}$$

Express the speed of the motion of the ride as a function of the radius of the circle and the period of its motion:

$$v=\frac{2\pi r}{T}$$

Substitute in the expression for a_c to obtain:

$$a_c = \frac{4\pi^2 r}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(5\,\text{m})}{(2\,\text{s})^2} = \boxed{49.3\,\text{m/s}^2}$$

(*b*) Apply $\sum \vec{F} = m\vec{a}$ to the passenger when he/she is at the top of the circular path and solve for a_c:

$$\sum F_r = mg = ma_c$$

and

$a_c = g$

Relate the acceleration of the motion to its radius and speed and solve for v:

$$g = \frac{v^2}{r} \Rightarrow v = \sqrt{gr}$$

Express the period of the motion as a function of the radius of the circle and the speed of the passenger and solve for T_m:

$$T_m = \frac{2\pi r}{v} = 2\pi\sqrt{\frac{r}{g}}$$

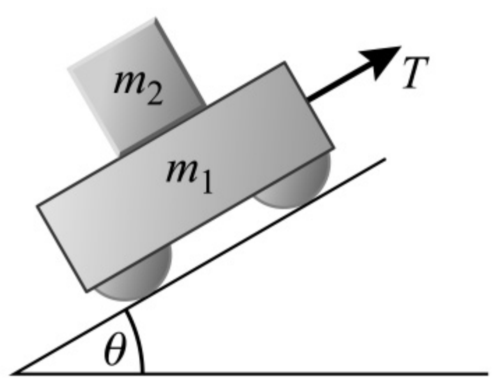
Substitute numerical values and evaluate T_m:

$$T_m = 2\pi\sqrt{\frac{5\,\text{m}}{9.81\,\text{m/s}^2}} = \boxed{4.49\,\text{s}}$$

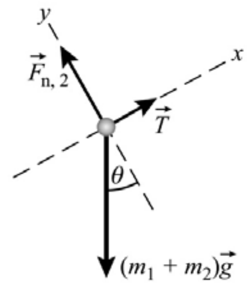
Remarks: The rider is "weightless" under the conditions described in part (*b*).

***110** ••

Picture the Problem The pictorial representation to the right shows the cart and its load on the inclined plane. The load will not slip provided its maximum acceleration is not exceeded. We can find that maximum acceleration by applying Newton's 2nd law to the load. We can then apply Newton's 2nd law to the cart-plus-load system to determine the tension in the rope when the system is experiencing its maximum acceleration.



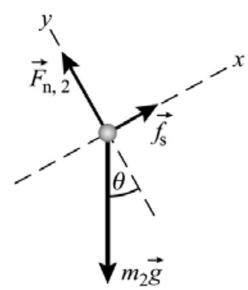
Draw the free-body diagram for the cart and its load:



Apply $\sum F_x = ma_x$ to the cart plus its load:

$$T - (m_1 + m_2)g\sin\theta = (m_1 + m_2)a_{max} \quad (1)$$

Draw the free-body diagram for the load of mass m_2 on top of the cart:



Apply $\sum \vec{F} = m\vec{a}$ to the load on top of the cart:

$$\sum F_x = f_{s,max} - m_2 g\sin\theta = m_2 a_{max}$$

and

$$\sum F_y = F_{n,2} - m_2 g\cos\theta = 0$$

Using $f_{s,max} = \mu_s F_{n,2}$, eliminate $F_{n,2}$ between the two equations and solve for the maximum acceleration of the load:

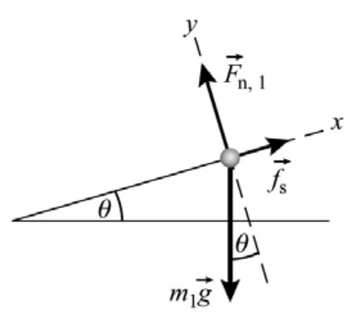
$$a_{max} = g(\mu_s \cos\theta - \sin\theta) \quad (2)$$

Substitute equation (2) in equation (1) and solve for T:

$$T = \boxed{(m_1 + m_2)g\mu_s \cos\theta}$$

111 ••

Picture the Problem The free-body diagram for the sled while it is held stationary by the static friction force is shown to the right. We can solve this problem by repeatedly applying Newton's 2nd law under the conditions specified in each part of the problem.



(*a*) Apply $\sum F_y = ma_y$ to the sled:

$$F_{n,1} - m_1 g\cos\theta = 0$$

Solve for $F_{n,1}$: $F_{n,1} = m_1 g \cos\theta$

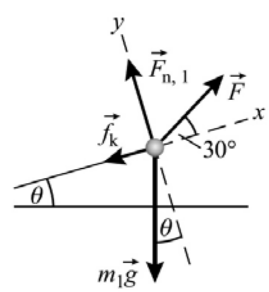
Substitute numerical values and evaluate $F_{n,1}$: $F_{n,1} = (200\,\text{N})\cos 15° = \boxed{193\,\text{N}}$

(*b*) Apply $\sum F_x = ma_x$ to the sled: $f_s - m_1 g \sin\theta = 0$

Solve for f_s: $f_s = m_1 g \sin\theta$

Substitute numerical values and evaluate f_s: $f_s = (200\,\text{N})\sin 15° = \boxed{51.8\,\text{N}}$

(*c*) Draw the free-body diagram for the sled when the child is pulling on the rope:



Apply $\sum \vec{F} = m\vec{a}$ to the sled to determine whether it moves:

$$\sum F_x = F_{net} = F\cos 30° - m_1 g \sin\theta - f_{s,max}$$

and

$$\sum F_y = F_{n,1} + F\sin 30° - m_1 g\cos\theta = 0$$

Solve the y-direction equation for $F_{n,1}$: $F_{n,1} = -F\sin 30° + m_1 g\cos\theta$

Substitute numerical values and evaluate $F_{n,1}$:

$$F_{n,1} = -(100\,\text{N})\sin 30° + (200\,\text{N})\cos 15° = 143\,\text{N}$$

Express $f_{s,max}$:

$$f_{s,max} = \mu_s F_{n,1} = (0.5)(143\text{ N}) = 71.5\text{ N}$$

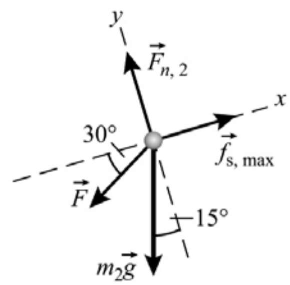
Use the x-direction force equation to evaluate F_{net}:

$$F_{net} = (100\text{ N})\cos 30° - (200\text{ N})\sin 15° - 71.5\text{ N} = -36.7\text{ N}$$

Because the net force is negative, the sled does not move: $\boxed{f_k \text{ is undetermined}}$

(*d*) Because the sled does not move: $\boxed{\mu_k \text{ is undetermined}}$

(*e*) Draw the FBD for the child:



Express the net force F_c exerted on the child by the incline:

$$F_c = \sqrt{F_{n2}^2 + f_{s,\max}^2} \qquad (1)$$

Noting that the child is stationary, apply $\sum \vec{F} = m\vec{a}$ to the child:

$$\sum F_x = f_{s,\max} - F\cos 30° - m_2 g \sin 15° = 0$$

and

$$\sum F_y = F_{n2} - m_2 g \sin 15° - F \sin 30° = 0$$

Solve the x equation for $f_{s,max}$ and the y equation for F_{n2}:

$$f_{s,\max} = F\cos 30° + m_2 g \sin 15°$$

and

$$F_{n2} = m_2 g \sin 15° + F \sin 30°$$

Substitute numerical values and evaluate F_x and F_{n2}:

$$f_{s,\max} = (500\text{ N})\cos 30° + (100\text{ N})\sin 15° = 459\text{ N}$$

and

$$F_{n2} = (100\text{ N})\sin 15° + (500\text{ N})\sin 30° = 276\text{ N}$$

Substitute numerical values in equation (1) and evaluate F:

$$F_c = \sqrt{(276\text{ N})^2 + (459\text{ N})^2} = \boxed{536\text{ N}}$$

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Picture the Problem Let v represent the speed of rotation of the station, and r the distance from the center of the station. Because the O'Neill colony is, presumably, in deep space, the only acceleration one would experience in it would be that due to its rotation.

(*a*) Express the acceleration of anyone who is standing inside the station:

$a = v^2/r$

This acceleration is directed toward the axis of rotation. If someone inside the station drops an apple, the apple will not have any forces acting on it once released, but will move along a straight line at constant speed. However, from the point of view of our observer inside the station, if he views himself as unmoving, the apple is perceived to have an acceleration of mv^2/r directed away from the axis of rotation (a "centrifugal" force).

(*b*) Each deck must rotate the central axis with the same period T. Relate the speed of a person on a particular deck to his/her distance r from the center:

$$v = \frac{2\pi r}{T}$$

Express the "acceleration of gravity" perceived by someone a distance r from the center:

$$\frac{v^2}{r} = \frac{4\pi^2 r}{T^2}$$

$$\boxed{\text{i.e., the "acceleration due to gravity" decreases as } r \text{ decreases.}}$$

(*c*) Relate the desired acceleration to the radius of Babylon 5 and its period:

$$a = \frac{4\pi^2 r}{T^2}$$

Solve for T:

$$T = \sqrt{\frac{4\pi^2 r}{a}}$$

Substitute numerical values and evaluate T:

$$T = \sqrt{\frac{4\pi^2\left(0.3\,\text{mi} \times \frac{1.609\,\text{km}}{\text{mi}}\right)}{9.8\,\text{m/s}^2}}$$

$$= 44.1\,\text{s} = 0.735\,\text{min}$$

Take the reciprocal of this time to find the number of revolutions per minute Babylon 5 has to make in order to provide this "earth-like" acceleration:

$$T^{-1} = \boxed{1.36\,\text{rev/min}}$$

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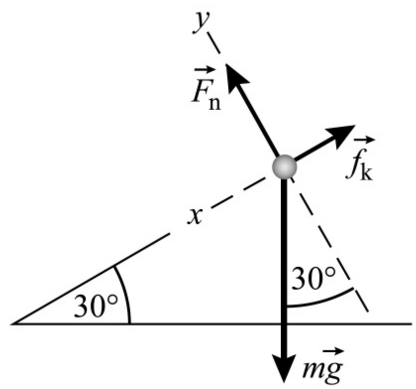
Picture the Problem The free-body diagram shows the forces acting on the child as she slides down the incline. We'll first use Newton's 2nd law to derive an expression for μ_k in terms of her acceleration and then use Newton's 2nd law to find her acceleration when riding the frictionless cart. Using a constant-acceleration equation, we'll relate these two accelerations to her descent times and solve for her acceleration when sliding. Finally, we can use this acceleration in the expression for μ_k.



Apply $\sum \vec{F} = m\vec{a}$ to the child as she slides down the incline:

$$\sum F_x = mg\sin 30° - f_k = ma_1$$

and

$$\sum F_y = F_n - mg\cos 30° = 0$$

Using $f_k = \mu_k F_n$, eliminate f_k and F_n between the two equations and solve for μ_k:

$$\mu_k = \tan 30° - \frac{a_1}{g\cos 30°} \qquad (1)$$

Apply $\sum F_x = ma_x$ to the child as she rides the frictionless cart down the incline and solve for her acceleration a_2:

$$mg\sin 30° = ma_2$$

and

$$a_2 = g\sin 30° = 4.91\,\text{m/s}^2$$

Letting s represent the distance she slides down the incline, use a constant-acceleration equation to relate her sliding times to her accelerations and distance traveled down the slide :

$$s = v_0 t_1 + \tfrac{1}{2}a_1 t_1^2 \text{ where } v_0 = 0$$

and

$$s = v_0 t_2 + \tfrac{1}{2}a_2 t_2^2 \text{ where } v_0 = 0$$

Equate these expressions, substitute $t_2 = \frac{1}{2}t_1$ and solve for a_1:

$$a_1 = \tfrac{1}{4}a_2 = \tfrac{1}{4}g\sin 30° = 1.23\,\text{m/s}^2$$

Evaluate equation (1) with $a_1 = 1.23\ \text{m/s}^2$:

$$\mu_k = \tan 30° - \frac{1.23\,\text{m/s}^2}{(9.81\,\text{m/s}^2)\cos 30°} = \boxed{0.433}$$

***114 ••**

Picture the Problem The path of the particle is a circle if r is a constant. Once we have shown that it is, we can calculate its value from its components and determine the particle's velocity and acceleration by differentiation. The direction of the net force acting on the particle can be determined from the direction of its acceleration.

(*a*) Express the magnitude of $\vec{r}$ in terms of its components:

$$r = \sqrt{r_x^2 + r_y^2}$$

Evaluate r with $r_x = R\sin\omega t$ and $r_y = R\cos\omega t$:

$$r = \sqrt{[R\sin\omega t]^2 + [R\cos\omega t]^2} = \sqrt{R^2(\sin^2\omega t + \cos^2\omega t)} = R = 4.0\,\text{m}$$

$\boxed{\therefore \text{the path of the particle is a circle centered at the origin.}}$

(*b*) Differentiate $\vec{r}$ with respect to time to obtain $\vec{v}$:

$$\vec{v} = d\vec{r}/dt = [R\omega\cos\omega t]\hat{i} + [-R\omega\sin\omega t]\hat{j} = \boxed{[(8\pi\cos 2\pi t)\text{m/s}]\hat{i} - [(8\pi\sin 2\pi t)\text{m/s}]\hat{j}}$$

Express the ratio $\frac{v_x}{v_y}$:

$$\frac{v_x}{v_y} = \frac{8\pi\cos\omega t}{-8\pi\sin\omega t} = -\cot\omega t$$

Express the ratio $-\frac{y}{x}$:

$$-\frac{y}{x} = -\frac{R\cos\omega t}{R\sin\omega t} = -\cot\omega t$$

$$\therefore \boxed{\frac{v_x}{v_y} = -\frac{y}{x}}$$

(*c*) Differentiate $\vec{v}$ with respect to time to obtain $\vec{a}$:

$$\vec{a} = d\vec{v}/dt = \boxed{[(-16\pi^2\text{m/s}^2)\sin\omega t]\hat{i} + [(-16\pi^2\text{m/s}^2)\cos\omega t]\hat{j}}$$

Factor $-4\pi^2/\text{s}^2$ from $\vec{a}$ to obtain:

$$\vec{a} = \left(-4\pi^2/\text{s}^2\right)\left[(4\sin\omega t)\hat{i} + (4\cos\omega t)\hat{j}\right] = \boxed{\left(-4\pi^2/\text{s}^2\right)\vec{r}}$$

Because $\vec{a}$ is in the opposite direction from $\vec{r}$, it is directed toward the center of the circle in which the particle is traveling.

Find the ratio $\dfrac{v^2}{r}$:

$$\frac{v^2}{r} = \frac{(8\pi\ \text{m/s})^2}{4\ \text{m}} = \boxed{16\pi^2\ \text{m/s}^2 = a}$$

(*d*) Apply $\sum\vec{F} = m\vec{a}$ to the particle:

$$F_{\text{net}} = ma = (0.8\ \text{kg})\left(16\pi^2\ \text{m/s}^2\right) = \boxed{12.8\pi^2\ \text{N}}$$

Because the direction of $\vec{F}_{\text{net}}$ is the same as that of $\vec{a}$:

$$\boxed{\vec{F}_{\text{net}}\ \text{is toward the center of the circle.}}$$

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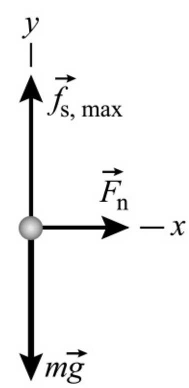
Picture the Problem The free-body diagram showing the forces acting on a rider being held in place by the maximum static friction force is shown to the right. The application of Newton's 2nd law and the definition of the maximum static friction force will be used to determine the period T of the motion. The reciprocal of the period will give us the minimum number of revolutions required per unit time to hold the riders in place.



Apply $\sum\vec{F} = m\vec{a}$ to the riders while they are held in place by friction:

$$\sum F_x = F_{\text{n}} = m\frac{v^2}{r}$$

and

$$\sum F_y = f_{\text{s,max}} - mg = 0$$

Using $f_{\text{s,max}} = \mu_{\text{s}}F_{\text{n}}$ and $v = \dfrac{2\pi r}{T}$, eliminate F_{n} between the force equations and solve for the period of the motion:

$$T = 2\pi\sqrt{\frac{\mu_{\text{s}} r}{g}} = 2\pi\sqrt{\frac{(0.4)(4\ \text{m})}{9.81\ \text{m/s}^2}} = 2.54\ \text{s} = 0.00423\ \text{min}$$

The number of revolutions per minute is the reciprocal of the period in minutes:

$$\boxed{23.6\,\text{rev/min}}$$

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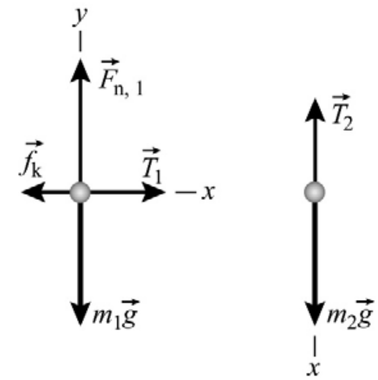
Picture the Problem The free-body diagrams to the right show the forces acting on the blocks whose masses are m_1 and m_2. The application of Newton's 2nd law and the use of a constant-acceleration equation will allow us to find a relationship between the coefficient of kinetic friction and m_1. The repetition of this procedure with the additional object on top of the object whose mass is m_1 will lead us to a second equation that, when solved simultaneously with the former equation, leads to a quadratic equation in m_1. Finally, its solution will allow us to substitute in an expression for μ_k and determine its value.



Using a constant-acceleration equation, relate the displacement of the system in its first configuration as a function of its acceleration and fall time:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a_1(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a_1(\Delta t)^2$$

Solve for a_1:

$$a_1 = \frac{2\Delta x}{(\Delta t)^2}$$

Substitute numerical values and evaluate a_1:

$$a_1 = \frac{2(1.5\,\text{m})}{(0.82\,\text{s})^2} = 4.46\,\text{m/s}^2$$

Apply $\sum F_x = ma_x$ to the object whose mass is m_2 and solve for T_1:

$$m_2g - T_1 = m_2a_1$$

and

$$\begin{aligned}T_1 &= m_2(g-a)\\ &= (2.5\,\text{kg})(9.81\,\text{m/s}^2 - 4.46\,\text{m/s}^2)\\ &= 13.375\,\text{N}\end{aligned}$$

Apply $\sum \vec{F} = m\vec{a}$ to the object whose mass is m_1:

$$\sum F_x = T_1 - f_k = m_1a_1$$

and

$$\sum F_y = F_{n,1} - m_1 g = 0$$

Using $f_k = \mu_k F_n$, eliminate F_n between the two equations to obtain:

$$T_1 - \mu_k m_1 g = m_1 a_1 \quad (1)$$

Find the acceleration a_2 for the second run:

$$a_2 = \frac{2\Delta x}{(\Delta t)^2} = \frac{2(1.5\,\text{m})}{(1.3\,\text{s})^2} = 1.775\,\text{m/s}^2$$

Evaluate T_2:

$$T_2 = m_2(g - a) = (2.5\,\text{kg})(9.81\,\text{m/s}^2 - 1.775\,\text{m/s}^2) = 20.1\,\text{N}$$

Apply $\sum F_x = ma_x$ to the 1.2-kg object in place:

$$T_2 - \mu_k(m_1 + 1.2\,\text{kg})g = (m_1 + 1.2\,\text{kg})a_2 \quad (2)$$

Solve equation (1) for μ_k:

$$\mu_k = \frac{T_1 - m_1 a_1}{m_1 g} \quad (3)$$

Substitute for μ_k in equation (2) and simplify to obtain the quadratic equation in m_1:

$$2.685 m_1^2 + 9.947 m_1 - 16.05 = 0$$

Solve the quadratic equation to obtain:

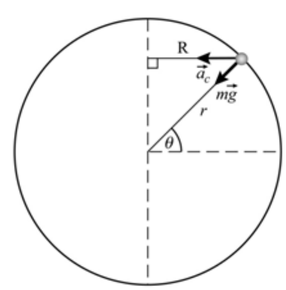
$$m_1 = (-1.85 \pm 3.07)\,\text{kg} \Rightarrow m_1 = \boxed{1.22\,\text{kg}}$$

Substitute numerical values in equation (3) and evaluate μ_k:

$$\mu_k = \frac{13.375\,\text{N} - (1.22\,\text{kg})(4.66\,\text{m/s}^2)}{(1.22\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{0.643}$$

***117** •••

Picture the Problem The diagram shows a point on the surface of the earth at latitude θ. The distance R to the axis of rotation is given by $R = r\cos\theta$. We can use the definition of centripetal acceleration to express the centripetal acceleration of a point on the surface of the earth due to the rotation of the earth.



(*a*) Referring to the figure, express a_c for a point on the surface of the earth at latitude θ:

$$a_c = \frac{v^2}{R} \text{ where } R = r\cos\theta$$

Express the speed of the point due to the rotation of the earth:

$$v = \frac{2\pi R}{T}$$

where T is the time for one revolution.

Substitute for v in the expression for a_c and simplify to obtain:

$$a_c = \frac{4\pi^2 r\cos\theta}{T^2}$$

Substitute numerical values and evaluate a_c:

$$a_c = \frac{4\pi^2(6370\,\text{km})\cos\theta}{[(24\,\text{h})(3600\,\text{s/h})]^2}$$

$$= \boxed{(3.37\,\text{cm/s}^2)\cos\theta\text{, toward the earth's axis.}}$$

(*b*)

A stone dropped from a hand at a location on earth. The effective weight of the stone is equal to $m\vec{a}_{\text{st, surf}}$, where $\vec{a}_{\text{st, surf}}$ is the acceleration of the falling stone (neglecting air resistance) relative to the local surface of the earth. The gravitational force on the stone is equal to $m\vec{a}_{\text{st,iner}}$, where $\vec{a}_{\text{st, iner}}$ is the acceleration of the local surface of the earth relative to the inertial frame (the acceleration of the surface due to the rotation of the earth). Multiplying through this equation by m and rearranging gives $m\vec{a}_{\text{st, surf}} = m\vec{a}_{\text{st, iner}} - m\vec{a}_{\text{surf, iner}}$, which relates the apparent weight to the acceleration due to gravity and the acceleration due to the earth's rotation. A vector addition diagram can be used to show that the magnitude of $m\vec{a}_{\text{st, surf}}$ is slightly less than that of $m\vec{a}_{\text{st, iner}}$.

(*c*) At the equator, the gravitational acceleration and the radial acceleration are both directed toward the center of the earth. Therefore:

$$g = g_{\text{eff}} + a_c$$
$$= 978\,\text{cm/s}^2 + (3.37\,\text{cm/s}^2)\cos 0°$$
$$= \boxed{981.4\,\text{cm/s}^2}$$

At latitude θ the gravitational acceleration points toward the center of the earth whereas the centripetal acceleration points toward the axis of rotation. Use the

$$g_{\text{eff}}^2 = g^2 + a_c^2 - 2ga_c\cos\theta$$

law of cosines to relate g_{eff}, g, and a_c:

Substitute for θ, g_{eff}, and a_c and simplify to obtain the quadratic equation:

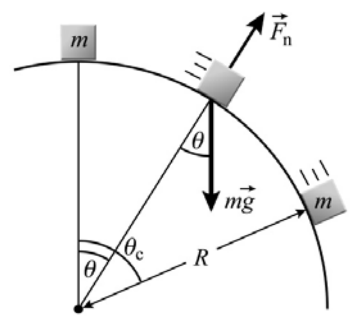
$$g^2 - \left(4.75\,\text{cm/s}^2\right)g - 962350\,\text{cm}^2/\text{s}^4 = 0$$

Solve for the physically meaningful (i.e., positive) root to obtain:

$$g = \boxed{983\,\text{cm/s}^2}$$

***118 •••**

Picture the Problem The diagram shows the block in its initial position, an intermediate position, and as it is separating from the sphere. Because the sphere is frictionless, the only forces acting on the block are the normal and gravitational forces. We'll apply Newton's 2nd law and set F_n equal to zero to determine the angle θ_c at which the block leaves the surface.



Taking the inward direction to be positive, apply $\sum F_r = ma_r$ to the block:

$$mg\cos\theta - F_n = m\frac{v^2}{R}$$

Apply the separation condition to obtain:

$$mg\cos\theta_c = m\frac{v^2}{R}$$

Solve for $\cos\theta_c$:

$$\cos\theta_c = \frac{v^2}{gR} \qquad (1)$$

Apply $\sum F_t = ma_t$ to the block:

$$mg\sin\theta = ma_t$$

or

$$a_t = \frac{dv}{dt} = g\sin\theta$$

Note that a is not constant and, hence, we cannot use constant-acceleration equations.

Multiply the left-hand side of the equation by one in the form of $d\theta/d\theta$ and rearrange to obtain:

$$\frac{dv}{dt}\frac{d\theta}{d\theta} = g\sin\theta$$

and

$$\frac{d\theta}{dt}\frac{dv}{d\theta} = g\sin\theta$$

Relate the arc distance s the block travels to the angle θ and the radius R of the sphere:

$$\theta = \frac{s}{R} \text{ and } \frac{d\theta}{dt} = \frac{1}{R}\frac{ds}{dt} = \frac{v}{R}$$

where v is the block's instantaneous speed.

Substitute to obtain:

$$\frac{v}{R}\frac{dv}{d\theta} = g\sin\theta$$

Separate the variables and integrate from $v' = 0$ to v and $\theta = 0$ to θ_c:

$$\int_0^v v'dv' = gR\int_0^{\theta_c} \sin\theta d\theta$$

or

$$v^2 = 2gR(1 - \cos\theta_c)$$

Substitute in equation (1) to obtain:

$$\cos\theta_c = \frac{2gR(1-\cos\theta_c)}{gR} = 2(1-\cos\theta_c)$$

Solve for and evaluate θ_c:

$$\theta_c = \cos^{-1}\left(\frac{2}{3}\right) = \boxed{48.2°}$$