Chapter 6
Work and Energy

Conceptual Problems

***1** •

Determine the Concept A force does work on an object when its point of application moves through some distance and there is a component of the force along the line of motion.

(*a*) False. The *net* force acting on an object is the vector sum of all the forces acting on the object and is responsible for displacing the object. Any or all of the forces contributing to the net force may do work.

(*b*) True. The object could be at rest in one reference frame and moving in another. If we consider only the frame in which the object is at rest, then, because it must undergo a displacement in order for work to be done on it, we would conclude that the statement is true.

(*c*) True. A force that is always perpendicular to the velocity of a particle changes neither it's kinetic nor potential energy and, hence, does no work on the particle.

2 •

Determine the Concept If we ignore the work that you do in initiating the horizontal motion of the box and the work that you do in bringing it to rest when you reach the second table, then neither the kinetic nor the potential energy of the system changed as you moved the box across the room. Neither did any forces acting on the box produce displacements. Hence, we must conclude that the minimum work you did on the box is zero.

3 •

False. While it is true that the person's kinetic energy is not changing due to the fact that she is moving at a constant speed, her gravitational potential energy is continuously changing and so we must conclude that the force exerted by the seat on which she is sitting is doing work on her.

***4** •

Determine the Concept The kinetic energy of any object is proportional to the square of its speed. Because $K = \frac{1}{2}mv^2$, replacing v by $2v$ yields

$K' = \frac{1}{2}m(2v)^2 = 4\left(\frac{1}{2}mv^2\right) = 4K.$ Thus doubling the speed of a car quadruples its kinetic energy.

5 •

Determine the Concept No. The work done on any object by any force $\vec{F}$ is defined as $dW = \vec{F} \cdot d\vec{r}$. The direction of $\vec{F}_{\text{net}}$ is toward the center of the circle in which the object is traveling and $d\vec{r}$ is tangent to the circle. No work is done by the net force because $\vec{F}_{\text{net}}$ and $d\vec{r}$ are perpendicular so the dot product is zero.

6 •

Determine the Concept The kinetic energy of any object is proportional to the square of its speed and is always positive. Because $K = \frac{1}{2}mv^2$, replacing v by $3v$ yields $K' = \frac{1}{2}m(3v)^2 = 9\left(\frac{1}{2}mv^2\right) = 9K$. Hence tripling the speed of an object increases its kinetic energy by a factor of 9 and $\boxed{(d) \text{ is correct.}}$

***7** •

Determine the Concept The work required to stretch or compress a spring a distance x is given by $W = \frac{1}{2}kx^2$ where k is the spring's stiffness constant. Because $W \propto x^2$, doubling the distance the spring is stretched will require four times as much work.

8 •

Determine the Concept No. We know that if a *net* force is acting on a particle, the particle must be accelerated. If the *net* force does no work on the particle, then we must conclude that the kinetic energy of the particle is constant and that the *net* force is acting perpendicular to the direction of the motion and will cause a departure from straight-line motion.

9 •

Determine the Concept We can use the definition of power as the scalar product of force and velocity to express the dimension of power.

Power is defined as:	$P \equiv \vec{F} \cdot \vec{v}$
Express the dimension of force:	$[M][L/T^2]$
Express the dimension of velocity:	$[L/T]$
Express the dimension of power in terms of those of force and velocity:	$[M][L/T^2][L/T] = [M][L]^2/[T]^3$ and $\boxed{(d) \text{ is correct.}}$

10 •

Determine the Concept The change in gravitational potential energy, over elevation changes that are small enough so that the gravitational field can be considered constant, is $mg\Delta h$, where Δh is the elevation change. Because Δh is the same for both Sal and Joe, their gains in gravitational potential energy are the same. $\boxed{(c)\text{ is correct.}}$

11 •

(*a*) False. The definition of work is not limited to displacements caused by conservative forces.

(*b*) False. Consider the work done by the gravitational force on an object in freefall.

(*c*) True. This is the definition of work done by a conservative force.

***12** ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x; i.e., $F_x = -dU/dx$.

(*a*) Examine the slopes of the curve at each of the lettered points, remembering that F_x is the negative of the slope of the potential energy graph, to complete the table:

Point	dU/dx	F_x
A	+	–
B	0	0
C	–	+
D	0	0
E	+	–
F	0	0

(*b*) Find the point where the slope is steepest:

$\boxed{\text{At point } C\ \left|F_x\right| \text{ is greatest.}}$

(*c*) If $d^2U/dx^2 < 0$, then the curve is concave downward and the equilibrium is *unstable*.

$\boxed{\text{At point } B \text{ the equilibrium is unstable.}}$

If $d^2U/dx^2 > 0$, then the curve is concave upward and the equilibrium is *stable*.

$\boxed{\text{At point } D \text{ the equilibrium is stable.}}$

Remarks: At point *F*, $d^2U/dx^2 = 0$ and the equilibrium is neither *stable* nor *unstable*; it is said to be *neutral*.

13 •

(*a*) False. Any force acting on an object may do work depending on whether the force produces a displacement … or is displaced as a consequence of the object's motion.

(*b*) False. Consider an element of area under a force-versus-time graph. Its units are N·s whereas the units of work are N·m.

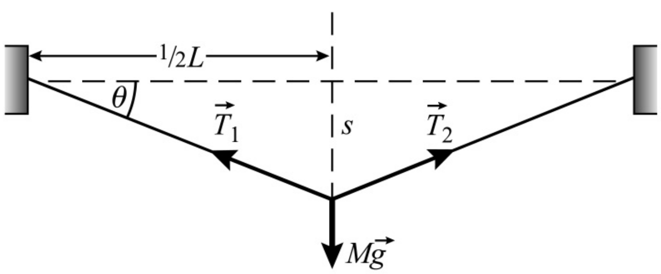
14 •

Determine the Concept Work $dW\left(=\vec{F}\cdot d\vec{s}\right)$ is done when a force $\vec{F}$ produces a displacement $d\vec{s}$. Because $\vec{F}\cdot d\vec{s} \equiv Fds\cos\theta = \left(F\cos\theta\right)ds$, W will be negative if the value of θ is such that $F\cos\theta$ is negative. $\boxed{(d)\text{ is correct.}}$

Estimation and Approximation

***15** ••

Picture the Problem The diagram depicts the situation when the tightrope walker is at the center of rope. M represents her mass and the vertical components of tensions $\vec{T}_1$ and $\vec{T}_2$, equal in magnitude, support her weight. We can apply a condition for static equilibrium in the vertical direction to relate the tension in the rope to the angle θ and use trigonometry to find s as a function of θ.



(*a*) Use trigonometry to relate the sag s in the rope to its length L and θ:

$$\tan\theta = \frac{s}{\frac{1}{2}L} \text{ and } s = \frac{L}{2}\tan\theta$$

Apply $\sum F_y = 0$ to the tightrope walker when she is at the center of the rope to obtain:

$2T\sin\theta - Mg = 0$ where T is the magnitude of $\vec{T}_1$ and $\vec{T}_2$.

Solve for θ to obtain:

$$\theta = \sin^{-1}\left(\frac{Mg}{2T}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{(50\,\text{kg})(9.81\,\text{m/s}^2)}{2(5000\,\text{N})}\right] = 2.81°$$

Substitute to obtain:

$$s = \frac{10\,\text{m}}{2}\tan 2.81° = \boxed{0.245\,\text{m}}$$

(*b*) Express the change in the tightrope walker's gravitational potential energy as the rope sags:

$$\Delta U = U_{\text{at center}} - U_{\text{end}} = Mg\Delta y$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = (50\,\text{kg})(9.81\,\text{m/s}^2)(-0.245\,\text{m}) = \boxed{-120\,\text{J}}$$

16 •

Picture the Problem You can estimate your change in potential energy due to this change in elevation from the definition of ΔU. You'll also need to estimate the height of one story of the Empire State building. We'll assume your mass is 70 kg and the height of one story to be 3.5 m. This approximation gives us a height of 1170 ft (357 m), a height that agrees to within 7% with the actual height of 1250 ft from the ground floor to the observation deck. We'll also assume that it takes 3 min to ride non-stop to the top floor in one of the high-speed elevators.

(*a*) Express the change in your gravitational potential energy as you ride the elevator to the 102nd floor:

$$\Delta U = mg\Delta h$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = (70\,\text{kg})(9.81\,\text{m/s}^2)(357\,\text{m}) = \boxed{245\,\text{kJ}}$$

(*b*) Ignoring the acceleration intervals at the beginning and the end of your ride, express the work done on you by the elevator in terms of the change in your gravitational potential energy:

$$W = Fh = \Delta U$$

Solve for and evaluate F:

$$F = \frac{\Delta U}{h} = \frac{245\,\text{kJ}}{357\,\text{m}} = \boxed{686\,\text{N}}$$

(*c*) Assuming a 3 minute ride to the top, express and evaluate the average power delivered to the elevator:

$$P = \frac{\Delta U}{\Delta t} = \frac{245\,\text{kJ}}{(3\,\text{min})(60\,\text{s/min})} = \boxed{1.36\,\text{kW}}$$

17 •

Picture the Problem We can find the kinetic energy K of the spacecraft from its definition and compare its energy to the annual consumption in the U.S. W by examining the ratio K/W.

Using its definition, express and evaluate the kinetic energy of the spacecraft:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(10000\,\text{kg})(3\times10^7\ \text{m/s})^2 = 4.50\times10^{18}\ \text{J}$$

Express this amount of energy as a percentage of the annual consumption in the United States:

$$\frac{K}{E} \approx \frac{4.50\times10^{18}\ \text{J}}{5\times10^{20}\ \text{J}} \approx \boxed{1\%}$$

***18** ••

Picture the Problem We can find the orbital speed of the Shuttle from the radius of its orbit and its period and its kinetic energy from $K = \tfrac{1}{2}mv^2$. We'll ignore the variation in the acceleration due to gravity to estimate the change in the potential energy of the orbiter between its value at the surface of the earth and its orbital value.

(*a*) Express the kinetic energy of the orbiter:

$$K = \tfrac{1}{2}mv^2$$

Relate the orbital speed of the orbiter to its radius r and period T:

$$v = \frac{2\pi r}{T}$$

Substitute and simplify to obtain:

$$K = \tfrac{1}{2}m\left(\frac{2\pi r}{T}\right)^2 = \frac{2\pi^2 mr^2}{T^2}$$

Substitute numerical values and evaluate K:

$$K = \frac{2\pi^2(8\times10^4\ \text{kg})[(200\,\text{mi}+3960\,\text{mi})(1.609\,\text{km/mi})]^2}{[(90\,\text{min})(60\,\text{s/min})]^2} = \boxed{2.43\,\text{TJ}}$$

(*b*) Assuming the acceleration due to gravity to be constant over the 200 mi and equal to its value at the surface of the earth (actually, it is closer to 9 m/s^2 at an elevation of 200 mi), express the change in gravitational potential energy of the orbiter, relative to the surface of the earth, as the Shuttle goes into orbit:

$$\Delta U = mgh$$

Substitute numerical values and evaluate ΔU:

$$\Delta U = (8\times10^4\text{ kg})(9.81\text{ m/s}^2)\times(200\text{ mi})(1.609\text{ km/mi}) = \boxed{0.253\text{ TJ}}$$

(*c*) No, they shouldn't be equal because there is more than just the force of gravity to consider here. When the shuttle is resting on the surface of the earth, it is supported against the force of gravity by the normal force the earth exerts upward on it. We would need to take into consideration the change in potential energy of the surface of earth in its deformation under the weight of the shuttle to find the actual change in potential energy.

19 •

Picture the Problem Let's assume that the width of the driveway is 18 ft. We'll also assume that you lift each shovel full of snow to a height of 1 m, carry it to the edge of the driveway, and drop it. We'll ignore the fact that you must slightly accelerate each shovel full as you pick it up and as you carry it to the edge of the driveway. While the density of snow depends on the extent to which it has been compacted, one liter of freshly fallen snow is approximately equivalent to 100 mL of water.

Express the work you do in lifting the snow a distance h:

$$W = \Delta U = mgh = \rho_{\text{snow}}V_{\text{snow}}gh$$

where ρ is the density of the snow.

Using its definition, express the densities of water and snow:

$$\rho_{\text{snow}} = \frac{m_{\text{snow}}}{V_{\text{snow}}} \text{ and } \rho_{\text{water}} = \frac{m_{\text{water}}}{V_{\text{water}}}$$

Divide the first of these equations by the second to obtain:

$$\frac{\rho_{\text{snow}}}{\rho_{\text{water}}} = \frac{V_{\text{water}}}{V_{\text{snow}}} \text{ or } \rho_{\text{snow}} = \rho_{\text{water}}\frac{V_{\text{water}}}{V_{\text{snow}}}$$

Substitute and evaluate the ρ_{snow}:

$$\rho_{\text{snow}} = (10^3\text{ kg/m}^3)\frac{100\text{ mL}}{\text{L}} = 100\text{ kg/m}^3$$

Calculate the volume of snow covering the driveway:

$$V_{\text{snow}} = (50\text{ ft})(18\text{ ft})\left(\frac{10}{12}\text{ ft}\right) = 750\text{ ft}^3\times\frac{28.32\text{ L}}{\text{ft}^3}\times\frac{10^{-3}\text{ m}^3}{\text{L}} = 21.2\text{ m}^3$$

Substitute numerical values in the expression for W to obtain an estimate (a lower bound) for the work you would do on the snow in removing it:

$$W = (100\text{ kg/m}^3)(21.2\text{ m}^3)(9.81\text{ m/s}^2)(1\text{ m}) = \boxed{20.8\text{ kJ}}$$

Work and Kinetic Energy

***20 •**

Picture the Problem We can use $\frac{1}{2}mv^2$ to find the kinetic energy of the bullet.

(*a*) Use the definition of *K*:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.015\,\text{kg})(1.2\times10^3\ \text{m/s})^2 = \boxed{10.8\,\text{kJ}}$$

(*b*) Because $K \propto v^2$:

$$K' = \tfrac{1}{4}K = \boxed{2.70\,\text{kJ}}$$

(*c*) Because $K \propto v^2$:

$$K' = 4K = \boxed{43.2\,\text{kJ}}$$

21 •

Picture the Problem We can use $\frac{1}{2}mv^2$ to find the kinetic energy of the baseball and the jogger.

(*a*) Use the definition of *K*:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(0.145\,\text{kg})(45\,\text{m/s})^2 = \boxed{147\,\text{J}}$$

(*b*) Convert the jogger's pace of 9 min/mi into a speed:

$$v = \left(\frac{1\,\text{mi}}{9\,\text{min}}\right)\left(\frac{1\,\text{min}}{60\,\text{s}}\right)\left(\frac{1609\,\text{m}}{1\,\text{mi}}\right) = 2.98\,\text{m/s}$$

Use the definition of *K*:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(60\,\text{kg})(2.98\,\text{m/s})^2 = \boxed{266\,\text{J}}$$

22 •

Picture the Problem The work done in raising an object a given distance is the product of the force producing the displacement and the displacement of the object. Because the weight of an object is the gravitational force acting on it and this force acts downward, the work done by gravity is the negative of the weight of the object multiplied by its displacement. The change in kinetic energy of an object is equal to the work done by the *net* force acting on it.

(*a*) Use the definition of *W*:

$$W = \vec{\boldsymbol{F}}\cdot\Delta\vec{\boldsymbol{y}} = F\Delta y$$

$= (80\text{ N})(3\text{ m}) = \boxed{240\text{ J}}$

(*b*) Use the definition of *W*:

$W = \vec{F}\cdot\Delta\vec{y} = -mg\Delta y$, because $\vec{F}$ and $\Delta\vec{y}$ are in opposite directions.

$\therefore\ W = -(6\text{ kg})(9.81\text{ m/s}^2)(3\text{ m})$
$= \boxed{-177\text{ J}}$

(*c*) According to the work-kinetic energy theorem:

$K = W + W_g = 240\text{ J} + (-177\text{ J})$
$= \boxed{63.0\text{ J}}$

23 •

Picture the Problem The constant force of 80 N is the net force acting on the box and the work it does is equal to the *change* in the kinetic energy of the box.

Using the work-kinetic energy theorem, relate the work done by the constant force to the *change* in the kinetic energy of the box:

$$W = K_\text{f} - K_\text{i} = \tfrac{1}{2}m\left(v_\text{f}^2 - v_\text{i}^2\right)$$

Substitute numerical values and evaluate *W*:

$$W = \tfrac{1}{2}(5\,\text{kg})\left[(68\,\text{m/s})^2 - (20\,\text{m/s})^2\right] = \boxed{10.6\,\text{kJ}}$$

***24** ••

Picture the Problem We can use the definition of kinetic energy to find the mass of your friend.

Using the definition of kinetic energy and letting ″1″ denote your mass and speed and ″2″ your girlfriend’s, express the equality of your kinetic energies and solve for your girlfriend’s mass as a function of both your masses and speeds:

$$\tfrac{1}{2}m_1v_1^2 = \tfrac{1}{2}m_2v_2^2$$

and

$$m_2 = m_1\left(\frac{v_1}{v_2}\right)^2 \qquad (1)$$

Express the condition on your speed that enables you to run at the same speed as your girlfriend:

$$v_2 = 1.25v_1 \qquad (2)$$

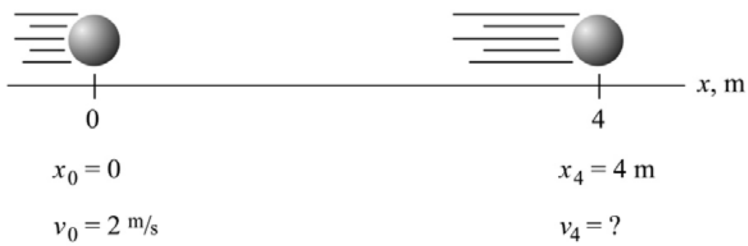
Substitute equation (2) in equation (1) to obtain:

$$m_2 = m_1\left(\frac{v_1}{v_2}\right)^2 = (85\,\text{kg})\left(\frac{1}{1.25}\right)^2 = \boxed{54.4\,\text{kg}}$$

Work Done by a Variable Force

25 ••

Picture the Problem The pictorial representation shows the particle as it moves along the positive x axis. The particle's kinetic energy increases because work is done on it. We can calculate the work done on it from the graph of F_x vs. x and relate its kinetic energy when it is at $x = 4$ m to its kinetic energy when it was at the origin and the work done on it by using the work-kinetic energy theorem.



(*a*) Calculate the kinetic energy of the particle when it is at $x = 0$:

$$K_0 = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(3\,\text{kg})(2\,\text{m/s})^2 = \boxed{6.00\,\text{J}}$$

(*b*) Because the force and displacement are parallel, the work done is the area under the curve. Use the formula for the area of a triangle to calculate the area under the F as a function of x graph:

$$W_{0\to4} = \tfrac{1}{2}(\text{base})(\text{altitude}) = \tfrac{1}{2}(4\,\text{m})(6\,\text{N}) = \boxed{12.0\,\text{J}}$$

(*c*) Express the kinetic energy of the particle at $x = 4$ m in terms of its speed and mass and solve for its speed:

$$v_4 = \sqrt{\frac{2K_4}{m}} \quad (1)$$

Using the work-kinetic energy theorem, relate the work done on the particle to its *change* in kinetic energy and solve for the particle's kinetic energy at $x = 4$ m:

$$W_{0\to4} = K_4 - K_0$$

$$K_4 = K_0 + W_{0\to4} = 6.00\,\text{J} + 12.0\,\text{J} = 18.0\,\text{J}$$

Substitute numerical values in equation (1) and evaluate v_4:

$$v_4 = \sqrt{\frac{2(18.0\,\text{J})}{3\,\text{kg}}} = \boxed{3.46\,\text{m/s}}$$

***26 ••**

Picture the Problem The work done by this force as it displaces the particle is the area under the curve of *F* as a function of *x*. Note that the constant *C* has units of N/m^3.

Because *F* varies with position non-linearly, express the work it does as an integral and evaluate the integral between the limits *x* = 1.5 m and *x* = 3 m:

$$\begin{aligned} W &= \left(C\,\text{N/m}^3\right)\int_{1.5\,\text{m}}^{3\,\text{m}} x'^3\,dx' \\ &= \left(C\,\text{N/m}^3\right)\left[\tfrac{1}{4}x'^4\right]_{1.5\,\text{m}}^{3\,\text{m}} \\ &= \frac{\left(C\,\text{N/m}^3\right)}{4}\left[(3\,\text{m})^4-(1.5\,\text{m})^4\right] \\ &= \boxed{19C\,\text{J}} \end{aligned}$$

27 ••

Picture the Problem The work done on the dog by the leash as it stretches is the area under the curve of *F* as a function of *x*. We can find this area (the work Lou does holding the leash) by integrating the force function.

Because *F* varies with position non-linearly, express the work it does as an integral and evaluate the integral between the limits *x* = 0 and *x* = x_1:

$$\begin{aligned} W &= \int_0^{x_1}\left(-kx'-ax'^2\right)dx' \\ &= \left[-\tfrac{1}{2}kx'^2-\tfrac{1}{3}ax'^3\right]_0^{x_1} \\ &= \boxed{-\tfrac{1}{2}kx_1^2-\tfrac{1}{3}ax_1^3} \end{aligned}$$

28 ••

Picture the Problem The work done on an object can be determined by finding the area bounded by its graph of F_x as a function of *x* and the *x* axis. We can find the kinetic energy and the speed of the particle at any point by using the work-kinetic energy theorem.

(*a*) Express *W*, the area under the curve, in terms of the area of one square, A_{square}, and the number of squares *n*:

$$W = n\,A_{\text{square}}$$

Determine the work equivalent of one square:

$$W = (0.5\text{ N})(0.25\text{ m}) = 0.125\text{ J}$$

Estimate the number of squares under the curve between $x = 0$ and $x = 2$ m:

$$n \approx 22$$

Substitute to determine W:

$$W = 22(0.125\text{ J}) = \boxed{2.75\text{ J}}$$

(b) Relate the kinetic energy of the object at $x = 2$ m, K_2, to its initial kinetic energy, K_0, and the work that was done on it between $x = 0$ and $x = 2$ m:

$$\begin{aligned} K_2 &= K_0 + W_{0\to2} \\ &= \tfrac{1}{2}(3\,\text{kg})(2.40\,\text{m/s})^2 + 2.75\,\text{J} \\ &= \boxed{11.4\,\text{J}} \end{aligned}$$

(c) Calculate the speed of the object at $x = 2$ m from its kinetic energy at the same location:

$$v = \sqrt{\frac{2K_2}{m}} = \sqrt{\frac{2(11.4\,\text{J})}{3\,\text{kg}}} = \boxed{2.76\,\text{m/s}}$$

(d) Estimate the number of squares under the curve between $x = 0$ and $x = 4$ m:

$$n \approx 26$$

Substitute to determine W:

$$W = 26(0.125\,\text{J}) = \boxed{3.25\,\text{J}}$$

(e) Relate the kinetic energy of the object at $x = 4$ m, K_4, to its initial kinetic energy, K_0, and the work that was done on it between $x = 0$ and $x = 4$ m:

$$\begin{aligned} K_4 &= K_0 + W_{0\to4} \\ &= \tfrac{1}{2}(3\,\text{kg})(2.40\,\text{m/s})^2 + 3.25\,\text{J} \\ &= 11.9\,\text{J} \end{aligned}$$

Calculate the speed of the object at $x = 4$ m from its kinetic energy at the same location:

$$v = \sqrt{\frac{2K_4}{m}} = \sqrt{\frac{2(11.9\,\text{J})}{3\,\text{kg}}} = \boxed{2.82\,\text{m/s}}$$

***29** ••

Picture the Problem We can express the mass of the water in Margaret's bucket as the difference between its initial mass and the product of the rate at which it loses water and her position during her climb. Because Margaret must do work against gravity in lifting and carrying the bucket, the work she does is the integral of the product of the gravitational field and the mass of the bucket as a function of its position.

(a) Express the mass of the bucket and the water in it as a function of

$$m(y) = 40\,\text{kg} - ry$$

its initial mass, the rate at which it is losing water, and Margaret's position, y, during her climb:

Find the rate, $r = \frac{\Delta m}{\Delta y}$, at which Margaret's bucket loses water:

$$r = \frac{\Delta m}{\Delta y} = \frac{20\,\text{kg}}{20\,\text{m}} = 1\,\text{kg/m}$$

Substitute to obtain:

$$m(y) = 40\,\text{kg} - ry = \boxed{40\,\text{kg} - \frac{1\,\text{kg}}{\text{m}}\,y}$$

(b) Integrate the force Margaret exerts on the bucket, $m(y)g$, between the limits of $y = 0$ and $y = 20$ m:

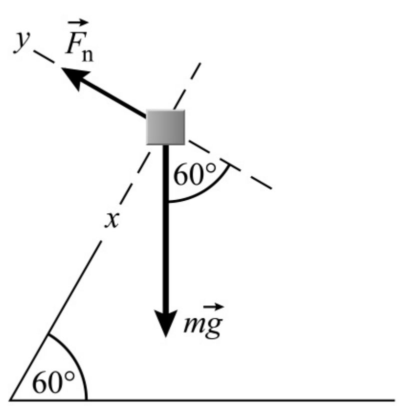
$$W = g\int_0^{20\,\text{m}}\left(40\,\text{kg} - \frac{1\,\text{kg}}{\text{m}}\,y'\right)dy' = \left(9.81\,\text{m/s}^2\right)\left[(40\,\text{kg})y' - \tfrac{1}{2}(1\,\text{kg/m})y'^2\right]_0^{20\,\text{m}} = \boxed{5.89\,\text{kJ}}$$

Remarks: We could also find the work Margaret did on the bucket, at least approximately, by plotting a graph of $m(y)g$ and finding the area under this curve between $y = 0$ and $y = 20$ m.

Work, Energy, and Simple Machines

30 •

Picture the Problem The free-body diagram shows the forces that act on the block as it slides down the frictionless incline. We can find the work done by these forces as the block slides 2 m by finding their components in the direction of, or opposite to, the motion. When we have determined the work done on the block, we can use the work-kinetic energy theorem or a constant-acceleration equation to calculate its kinetic energy and its speed at any given location.



(*a*) $\boxed{\text{From the free-body diagram, we see that the forces acting on the block are a gravitational force that acts downward and the normal force that the incline exerts perpendicularly to the incline.}}$

Identify the component of *mg* that acts down the incline and calculate the work done by it:

$F_x = mg\sin 60°$

Express the work done by this force:

$W = F_x\Delta x = mg\Delta x\sin 60°$

Substitute numerical values and evaluate *W*:

$$W = (6\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m})\sin 60° = \boxed{102\text{ J}}$$

Remarks: F_n and mgcos60°, being perpendicular to the motion, do no work on the block

(*b*) The total work done on the block is the work done by the net force:

$$W = F_{\text{net}}\Delta x = mg\Delta x\sin 60° = (6\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m})\sin 60° = \boxed{102\text{ J}}$$

(*c*) Express the change in the kinetic energy of the block in terms of the distance, Δx, it has moved down the incline:

$\Delta K = K_f - K_i = W = (mg\sin 60°)\Delta x$

or, because $K_i = 0$,

$K_f = W = (mg\sin 60°)\Delta x$

Relate the speed of the block when it has moved a distance Δx down the incline to its kinetic energy at that location:

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2mg\Delta x\sin 60°}{m}} = \sqrt{2g\Delta x\sin 60°}$$

Determine this speed when $\Delta x = 1.5$ m:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(1.5\,\text{m})\sin 60°} = \boxed{5.05\,\text{m/s}}$$

(*d*) As in part (*c*), express the change in the kinetic energy of the block in terms of the distance, Δx, it has moved down the incline and

$\Delta K = K_f - K_i$

$= W$

$= (mg\sin 60°)\Delta x$

and

solve for K_f: $K_f = (mg\sin 60°)\Delta x + K_i$

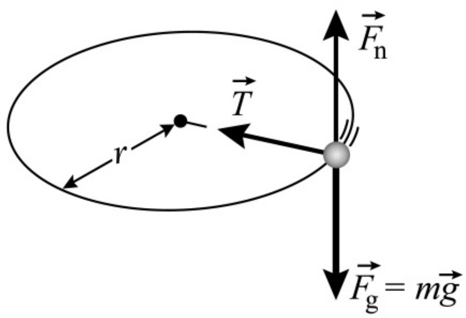
Substitute for the kinetic energy terms and solve for v_f to obtain: $v_f = \sqrt{2g\sin 60°\Delta x + v_i^2}$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(1.5\,\text{m})\sin 60° + (2\,\text{m/s})^2} = \boxed{5.43\,\text{m/s}}$$

31 •

Picture the Problem The free-body diagram shows the forces acting on the object as in moves along its circular path on a frictionless horizontal surface. We can use Newton's 2nd law to obtain an expression for the tension in the string and the definition of work to determine the amount of work done by each force during one revolution.



(*a*) Apply $\sum F_r = ma_r$ to the 2-kg object and solve for the tension:

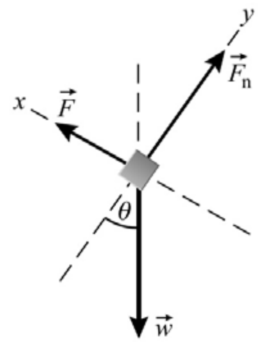
$$T = m\frac{v^2}{r} = (2\,\text{kg})\frac{(2.5\,\text{m/s})^2}{3\,\text{m}} = \boxed{4.17\,\text{N}}$$

(*b*) From the FBD we can see that the forces acting on the object are: $\boxed{\vec{\boldsymbol{T}}, \vec{\boldsymbol{F}}_g, \text{ and } \vec{\boldsymbol{F}}_n}$

Because all of these forces act perpendicularly to the direction of motion of the object, none of them do any work.

***32** •

Picture the Problem The free-body diagram, with $\vec{F}$ representing the force required to move the block at constant speed, shows the forces acting on the block. We can apply Newton's 2nd law to the block to relate F to its weight w and then use the definition of the mechanical advantage of an inclined plane. In the second part of the problem we'll use the definition of work.



(*a*) Express the mechanical advantage M of the inclined plane:

$$M = \frac{w}{F}$$

Apply $\sum F_x = ma_x$ to the block:

$$F - w\sin\theta = 0 \text{ because } a_x = 0.$$

Solve for F and substitute to obtain:

$$M = \frac{w}{w\sin\theta} = \frac{1}{\sin\theta}$$

Refer to the figure to obtain:

$$\sin\theta = \frac{H}{L}$$

Substitute to obtain:

$$M = \boxed{\frac{1}{\sin\theta} = \frac{L}{H}}$$

(*b*) Express the work done pushing the block up the ramp:

$$W_{\text{ramp}} = FL = mgL\sin\theta$$

Express the work done lifting the block into the truck:

$$W_{\text{lifting}} = mgH = mgL\sin\theta$$

and

$$\boxed{W_{\text{ramp}} = W_{\text{lifting}}}$$

33 •

Picture the Problem We can find the work done per revolution in lifting the weight and the work done in each revolution of the handle and then use the definition of mechanical advantage.

Express the mechanical advantage of the jack:

$$M = \frac{W}{F}$$

Express the work done by the jack in one complete revolution (the weight W is raised a distance p):

$$W_{\text{lifting}} = Wp$$

Express the work done by the force F in one complete revolution:

$$W_{\text{turning}} = 2\pi RF$$

Equate these expressions to obtain: $Wp = 2\pi RF$

Solve for the ratio of W to F: $M = \frac{W}{F} = \boxed{\frac{2\pi R}{p}}$

Remarks: One does the same amount of work turning as lifting; exerting a smaller force over a greater distance.

34 •

Picture the Problem The object whose weight is $\vec{w}$ is supported by two portions of the rope resulting in what is known as a *mechanical advantage* of 2. The work that is done in each instance is the product of the force doing the work and the displacement of the object on which it does the work.

(*a*) If w moves through a distance h: F moves a distance $\boxed{2h}$

(*b*) Assuming that the kinetic energy of the weight does not change, relate the work done on the object to the change in its potential energy to obtain: $W = \Delta U = wh\cos\theta = \boxed{wh}$

(*c*) Because the force you exert on the rope and its displacement are in the same direction: $W = F(2h)\cos\theta = F(2h)$

Determine the tension in the ropes supporting the object:

$$\sum F_{\text{vertical}} = 2F - w = 0$$

and

$$F = \tfrac{1}{2}w$$

Substitute for F: $W = F(2h) = \tfrac{1}{2}w(2h) = \boxed{wh}$

(*d*) The mechanical advantage of the inclined plane is the ratio of the weight that is lifted to the force required to lift it, i.e.: $M = \frac{w}{F} = \frac{w}{\frac{1}{2}w} = \boxed{2}$

Remarks: Note that the mechanical advantage is also equal to the number of ropes supporting the load.

Dot Products

***35 •**

Picture the Problem Because $\vec{A}\cdot\vec{B} \equiv AB\cos\theta$ we can solve for $\cos\theta$ and use the fact that $\vec{A}\cdot\vec{B} = -AB$ to find θ.

Solve for θ: $\theta = \cos^{-1}\dfrac{\vec{A}\cdot\vec{B}}{AB}$

Substitute for $\vec{A}\cdot\vec{B}$ and evaluate θ: $\theta = \cos^{-1}(-1) = \boxed{180°}$

36 •

Picture the Problem We can use its definition to evaluate $\vec{A}\cdot\vec{B}$.

Express the definition of $\vec{A}\cdot\vec{B}$: $\vec{A}\cdot\vec{B} = AB\cos\theta$

Substitute numerical values and evaluate $\vec{A}\cdot\vec{B}$:

$$\vec{A}\cdot\vec{B} = (6\,\text{m})(6\,\text{m})\cos 60° = \boxed{18.0\,\text{m}^2}$$

37 •

Picture the Problem The scalar product of two-dimensional vectors $\vec{A}$ and $\vec{B}$ is $A_xB_x + A_yB_y$.

(*a*) For $\vec{A} = 3\hat{i} - 6\hat{j}$ and $\vec{B} = -4\hat{i} + 2\hat{j}$: $\vec{A}\cdot\vec{B} = (3)(-4) + (-6)(2) = \boxed{-24}$

(*b*) For $\vec{A} = 5\hat{i} + 5\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{j}$: $\vec{A}\cdot\vec{B} = (5)(2) + (5)(-4) = \boxed{-10}$

(*c*) For $\vec{A} = 6\hat{i} + 4\hat{j}$ and $\vec{B} = 4\hat{i} - 6\hat{j}$: $\vec{A}\cdot\vec{B} = (6)(4) + (4)(-6) = \boxed{0}$

38 •

Picture the Problem The scalar product of two-dimensional vectors $\vec{A}$ and $\vec{B}$ is $AB\cos\theta = A_xB_x + A_yB_y$. Hence the angle between vectors $\vec{A}$ and $\vec{B}$ is given by

$$\theta = \cos^{-1}\frac{A_xB_x + A_yB_y}{AB}.$$

(*a*) For $\vec{A} = 3\hat{i} - 6\hat{j}$ and $\vec{B} = -4\hat{i} + 2\hat{j}$:

$$\vec{A}\cdot\vec{B} = (3)(-4) + (-6)(2) = -24$$

$$A = \sqrt{(3)^2 + (-6)^2} = \sqrt{45}$$

$$B = \sqrt{(-4)^2 + (2)^2} = \sqrt{20}$$

and

$$\theta = \cos^{-1}\frac{-24}{\sqrt{45}\sqrt{20}} = \boxed{143°}$$

(*b*) For $\vec{A} = 5\hat{i} + 5\hat{j}$ and $\vec{B} = 2\hat{i} - 4\hat{j}$:

$$\vec{A}\cdot\vec{B} = (5)(2) + (5)(-4) = -10$$

$$A = \sqrt{(5)^2 + (5)^2} = \sqrt{50}$$

$$B = \sqrt{(2)^2 + (-4)^2} = \sqrt{20}$$

and

$$\theta = \cos^{-1}\frac{-10}{\sqrt{50}\sqrt{20}} = \boxed{108°}$$

(*c*) For $\vec{A} = 6\hat{i} + 4\hat{j}$ and $\vec{B} = 4\hat{i} - 6\hat{j}$:

$$\vec{A}\cdot\vec{B} = (6)(4) + (4)(-6) = \boxed{0}$$

$$A = \sqrt{(6)^2 + (4)^2} = \sqrt{52}$$

$$B = \sqrt{(4)^2 + (-6)^2} = \sqrt{52}$$

and

$$\theta = \cos^{-1}\frac{0}{\sqrt{52}\sqrt{52}} = \boxed{90.0°}$$

39 •

Picture the Problem The work W done by a force $\vec{F}$ during a displacement $\Delta\vec{s}$ for which it is responsible is given by $\vec{F}\cdot\Delta\vec{s}$.

(*a*) Using the definitions of work and the scalar product, calculate the work done by the given force during the specified displacement:

$$\begin{aligned}W &= \vec{F}\cdot\Delta\vec{s}\\ &= \left(2\,\text{N}\,\hat{i} - 1\,\text{N}\,\hat{j} + 1\,\text{N}\,\hat{k}\right)\\ &\quad\cdot\left(3\,\text{m}\,\hat{i} + 3\,\text{m}\,\hat{j} - 2\,\text{m}\,\hat{k}\right)\\ &= \left[(2)(3) + (-1)(3) + (1)\ (-2)\right]\text{N}\cdot\text{m}\\ &= \boxed{1.00\,\text{J}}\end{aligned}$$

(*b*) Using the definition of work that includes the angle between the force and displacement vectors, solve for the component of $\vec{F}$ in the direction of $\Delta\vec{s}$:

$$W = F\Delta s\cos\theta = (F\cos\theta)\Delta s$$

and

$$F\cos\theta = \frac{W}{\Delta s}$$

Substitute numerical values and evaluate $F\cos\theta$:

$$\begin{aligned}F\cos\theta &= \frac{1\,\text{J}}{\sqrt{(3\,\text{m})^2 + (3\,\text{m})^2 + (-2\,\text{m})^2}}\\ &= \boxed{0.213\,\text{N}}\end{aligned}$$

40 ••

Picture the Problem The component of a vector that is along another vector is the scalar product of the former vector and a unit vector that is parallel to the latter vector.

(*a*) By definition, the unit vector that is parallel to the vector $\vec{A}$ is:

$$\hat{u}_A = \boxed{\frac{\vec{A}}{A} = \frac{A_x\hat{i} + A_y\hat{j} + A_z\hat{k}}{\sqrt{A_x^2 + A_y^2 + A_z^2}}}$$

(*b*) Find the unit vector parallel to $\vec{B}$:

$$\hat{u}_B = \frac{\vec{B}}{B} = \frac{3\hat{i} + 4\hat{j}}{\sqrt{(3)^2 + (4)^2}} = \frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$$

The component of $\vec{A}$ along $\vec{B}$ is:

$$\begin{aligned}\vec{A}\cdot\hat{u}_B &= \left(2\hat{i} - \hat{j} - \hat{k}\right)\cdot\left(\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}\right)\\ &= (2)\left(\frac{3}{5}\right) + (-1)\left(\frac{4}{5}\right) + (-1)(0)\\ &= \boxed{0.400}\end{aligned}$$

***41** ••

Picture the Problem We can use the definitions of the magnitude of a vector and the dot product to show that if $\left|\vec{A} + \vec{B}\right| = \left|\vec{A} - \vec{B}\right|$, then $\vec{A} \perp \vec{B}$.

Express $\left|\vec{A}+\vec{B}\right|^2$: $\left|\vec{A}+\vec{B}\right|^2 = \left(\vec{A}+\vec{B}\right)^2$

Express $\left|\vec{A}-\vec{B}\right|$: $\left|\vec{A}-\vec{B}\right|^2 = \left(\vec{A}-\vec{B}\right)^2$

Equate these expressions to obtain: $\left(\vec{A}+\vec{B}\right)^2 = \left(\vec{A}-\vec{B}\right)^2$

Expand both sides of the equation to obtain: $A^2 + 2\vec{A}\cdot\vec{B} + B^2 = A^2 - 2\vec{A}\cdot\vec{B} + B^2$

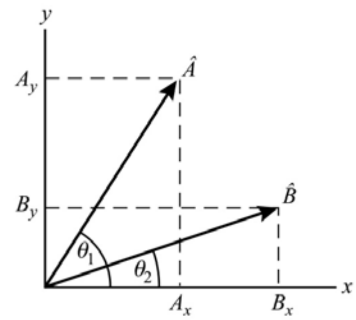
Simplify to obtain: $4\vec{A}\cdot\vec{B} = 0$
or
$\vec{A}\cdot\vec{B} = 0$

From the definition of the dot product we have: $\vec{A}\cdot\vec{B} = AB\cos\theta$
where θ is the angle between $\vec{A}$ and $\vec{B}$.

Because neither $\vec{A}$ nor $\vec{B}$ is the zero vector: $\cos\theta = 0 \Rightarrow \theta = 90°$ and $\vec{A} \perp \vec{B}$.

42 ••

Picture the Problem The diagram shows the unit vectors $\hat{A}$ and $\hat{B}$ arbitrarily located in the 1st quadrant. We can express these vectors in terms of the unit vectors $\hat{i}$ and $\hat{j}$ and their x and y components. We can then form the dot product of $\hat{A}$ and $\hat{B}$ to show that $\cos(\theta_1 - \theta_2) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$.



(*a*) Express $\hat{A}$ in terms of the unit vectors $\hat{i}$ and $\hat{j}$: $\hat{A} = A_x\hat{i} + A_y\hat{j}$
where
$A_x = \boxed{\cos\theta_1}$ and $A_y = \boxed{\sin\theta_1}$

Proceed as above to obtain: $\hat{B} = B_x\hat{i} + B_y\hat{j}$
where
$B_x = \boxed{\cos\theta_2}$ and $B_y = \boxed{\sin\theta_2}$

(*b*) Evaluate $\hat{A}\cdot\hat{B}$:

$$\hat{A}\cdot\hat{B} = \left(\cos\theta_1\hat{i} + \sin\theta_1\hat{j}\right)\cdot\left(\cos\theta_2\hat{i} + \sin\theta_2\hat{j}\right) = \cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2$$

From the diagram we note that: $\hat{A}\cdot\hat{B} = \cos\left(\theta_1 - \theta_2\right)$

Substitute to obtain:

$$\boxed{\cos(\theta_1 - \theta_2) = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2}$$

43 •

Picture the Problem In (*a*) we'll show that it does not follow that $\vec{B} = \vec{C}$ by giving a counterexample.

Let $\vec{A} = \hat{i}$, $\vec{B} = 3\hat{i} + 4\hat{j}$ and $\vec{C} = 3\hat{i} - 4\hat{j}$. Form $\vec{A}\cdot\vec{B}$ and $\vec{A}\cdot\vec{C}$:

$$\vec{A}\cdot\vec{B} = \hat{i}\cdot\left(3\hat{i} + 4\hat{j}\right) = 3$$

and

$$\vec{A}\cdot\vec{C} = \hat{i}\cdot\left(3\hat{i} - 4\hat{j}\right) = 3$$

$$\boxed{\text{No. We've shown by a counter-example that } \vec{B} \text{ is not necessarily equal to } \vec{C}.}$$

44 ••

Picture the Problem We can form the dot product of $\vec{A}$ and $\vec{r}$ and require that $\vec{A}\cdot\vec{r} = 1$ to show that the points at the head of all such vectors $\vec{r}$ lie on a straight line. We can use the equation of this line and the components of $\vec{A}$ to find the slope and intercept of the line.

(*a*) Let $\vec{A} = a_x\hat{i} + a_y\hat{j}$. Then:

$$\vec{A}\cdot\vec{r} = \left(a_x\hat{i} + a_y\hat{j}\right)\cdot\left(x\hat{i} + y\hat{j}\right) = a_x x + a_y y = 1$$

Solve for y to obtain:

$$y = \boxed{-\frac{a_x}{a_y}x + \frac{1}{a_y}}$$

which is of the form $y = mx + b$ and hence is the equation of a line.

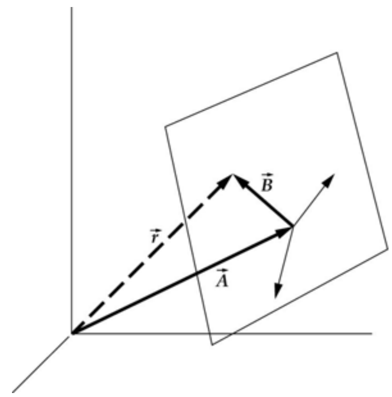
(*b*) Given that $\vec{A} = 2\hat{i} - 3\hat{j}$:

$$m = -\frac{a_x}{a_y} = -\frac{2}{-3} = \boxed{\frac{2}{3}}$$

and

$$b = \frac{1}{a_y} = \frac{1}{-3} = \boxed{-\frac{1}{3}}$$

(*c*) The equation we obtained in (*a*) specifies all vectors whose component parallel to $\vec{A}$ has constant magnitude; therefore, we can write such a vector as $\vec{r} = \dfrac{\vec{A}}{|\vec{A}|^2} + \vec{B}$, where $\vec{B}$ is *any* vector perpendicular to $\vec{A}$. This is shown graphically to the right.



Because all possible vectors $\vec{B}$ lie in a plane, the resultant $\vec{r}$ must lie in a plane as well, as is shown above.

***45 ••**

Picture the Problem The rules for the differentiation of vectors are the same as those for the differentiation of scalars and scalar multiplication is commutative.

(*a*) Differentiate $\vec{r} \cdot \vec{r} = r^2$ = constant:

$$\frac{d}{dt}(\vec{r} \cdot \vec{r}) = \vec{r} \cdot \frac{d\vec{r}}{dt} + \frac{d\vec{r}}{dt} \cdot \vec{r} = 2\vec{v} \cdot \vec{r}$$
$$= \frac{d}{dt}(\text{constant}) = 0$$

Because $\vec{v} \cdot \vec{r} = 0$:

$$\boxed{\vec{v} \perp \vec{r}}$$

(*b*) Differentiate $\vec{v} \cdot \vec{v} = v^2$ = constant with respect to time:

$$\frac{d}{dt}(\vec{v} \cdot \vec{v}) = \vec{v} \cdot \frac{d\vec{v}}{dt} + \frac{d\vec{v}}{dt} \cdot \vec{v} = 2\vec{a} \cdot \vec{v}$$
$$= \frac{d}{dt}(\text{constant}) = 0$$

Because $\vec{a} \cdot \vec{v} = 0$:

$$\boxed{\vec{a} \perp \vec{v}}$$

The results of (*a*) and (*b*) tell us that $\vec{a}$ is perpendicular to $\vec{r}$ and and parallel (or antiparallel) to $\vec{r}$.

(*c*) Differentiate $\vec{v} \cdot \vec{r} = 0$ with respect to time:

$$\frac{d}{dt}(\vec{v} \cdot \vec{r}) = \vec{v} \cdot \frac{d\vec{r}}{dt} + \vec{r} \cdot \frac{d\vec{v}}{dt}$$
$$= v^2 + \vec{r} \cdot \vec{a} = \frac{d}{dt}(0) = 0$$

Because $v^2 + \vec{r}\cdot\vec{a} = 0$: $\boxed{\vec{r}\cdot\vec{a} = -v^2}$ (1)

Express a_r in terms of θ, where θ is the angle between $\vec{r}$ and $\vec{a}$: $a_r = a\cos\theta$

Express $\vec{r}\cdot\vec{a}$: $\vec{r}\cdot\vec{a} = ra\cos\theta = ra_r$

Substitute in equation (1) to obtain: $ra_r = -v^2$

Solve for a_r: $a_r = \boxed{-\frac{v^2}{r}}$

Power

46 ••

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt}$.

Calculate the rate at which force A does work: $P_A = \frac{5\,\text{J}}{10\,\text{s}} = 0.5\,\text{W}$

Calculate the rate at which force B does work: $P_B = \frac{3\,\text{J}}{5\,\text{s}} = 0.6\,\text{W}$ and $\boxed{P_B > P_A}$

47 •

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt} = \vec{F}\cdot\vec{v}$.

(*a*) If the box moves upward with a constant velocity, the net force acting it must be zero and the force that is doing work on the box is: $F = mg$

The power input of the force is: $P = Fv = mgv$

Substitute numerical values and evaluate P: $P = (5\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m/s}) = \boxed{98.1\,\text{W}}$

(*b*) Express the work done by the force in terms of the rate at which energy is delivered:

$$W = Pt = (98.1\text{ W})\,(4\text{ s}) = \boxed{392\,\text{J}}$$

48 •

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \dfrac{dW}{dt} = \vec{F}\cdot\vec{v}$.

(*a*) Using the definition of power, express Fluffy's speed in terms of the rate at which he does work and the force he exerts in doing the work:

$$v = \frac{P}{F} = \frac{6\,\text{W}}{3\,\text{N}} = \boxed{2\,\text{m/s}}$$

(*b*) Express the work done by the force in terms of the rate at which energy is delivered:

$$W = Pt = (6\text{ W})\,(4\text{ s}) = \boxed{24.0\,\text{J}}$$

49 •

Picture the Problem We can use Newton's 2nd law and the definition of acceleration to express the velocity of this object as a function of time. The power input of the force accelerating the object is defined to be the rate at which it does work; i.e., $P = dW/dt = \vec{F}\cdot\vec{v}$.

(*a*) Express the velocity of the object as a function of its acceleration and time:

$$v = at$$

Apply $\sum\vec{F} = m\vec{a}$ to the object:

$$a = F/m$$

Substitute for *a* in the expression for *v*:

$$v = \frac{F}{m}t = \frac{5\,\text{N}}{8\,\text{kg}}t = \boxed{\left(\tfrac{5}{8}\,\text{m/s}^2\right)t}$$

(*b*) Express the power input as a function of *F* and *v* and evaluate *P*:

$$P = Fv = (5\,\text{N})\left(\tfrac{5}{8}\,\text{m/s}^2\right)t = \boxed{3.13t\,\text{W/s}}$$

(*c*) Substitute *t* = 3 s:

$$P = (3.13\,\text{W/s})(3\,\text{s}) = \boxed{9.38\,\text{W}}$$

50 •

Picture the Problem The power delivered by a force is defined as the rate at which the force does work; i.e., $P = \frac{dW}{dt} = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}}$.

(*a*) For $\vec{\boldsymbol{F}}$ = 4 N $\hat{\boldsymbol{i}}$ + 3 N $\hat{\boldsymbol{k}}$ and $\vec{\boldsymbol{v}}$ = 6 m/s $\hat{\boldsymbol{i}}$:

$$P = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}} = \left(4\,\text{N}\,\hat{\boldsymbol{i}} + 3\,\text{N}\,\hat{\boldsymbol{k}}\right)\cdot\left(6\,\text{m/s}\,\hat{\boldsymbol{i}}\right) = \boxed{24.0\,\text{W}}$$

(*b*) For $\vec{\boldsymbol{F}}$ = 6 N $\hat{\boldsymbol{i}}$ – 5 N $\hat{\boldsymbol{j}}$ and $\vec{\boldsymbol{v}}$ = – 5 m/s $\hat{\boldsymbol{i}}$ + 4 m/s $\hat{\boldsymbol{j}}$:

$$P = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}} = \left(6\,\text{N}\,\hat{\boldsymbol{i}} - 5\,\text{N}\,\hat{\boldsymbol{j}}\right)\cdot\left(-5\,\text{m/s}\,\hat{\boldsymbol{i}} + 4\,\text{m/s}\,\hat{\boldsymbol{j}}\right) = \boxed{-50.0\,\text{W}}$$

(*c*) For $\vec{\boldsymbol{F}}$ = 3 N $\hat{\boldsymbol{i}}$ + 6 N $\hat{\boldsymbol{j}}$ and $\vec{\boldsymbol{v}}$ = 2 m/s $\hat{\boldsymbol{i}}$ + 3 m/s $\hat{\boldsymbol{j}}$:

$$P = \vec{\boldsymbol{F}} \cdot \vec{\boldsymbol{v}} = \left(3\,\text{N}\,\hat{\boldsymbol{i}} + 6\,\text{N}\,\hat{\boldsymbol{j}}\right)\cdot\left(2\,\text{m/s}\,\hat{\boldsymbol{i}} + 3\,\text{m/s}\,\hat{\boldsymbol{j}}\right) = \boxed{24.0\,\text{W}}$$

***51** •

Picture the Problem Choose a coordinate system in which upward is the positive *y* direction. We can find P_{in} from the given information that $P_{\text{out}} = 0.27P_{\text{in}}$. We can express P_{out} as the product of the tension in the cable *T* and the constant speed *v* of the dumbwaiter. We can apply Newton's 2nd law to the dumbwaiter to express *T* in terms of its mass *m* and the gravitational field *g*.

Express the relationship between the motor's input and output power:

$$P_{\text{out}} = 0.27P_{\text{in}}$$
or
$$P_{\text{in}} = 3.7P_{\text{out}}$$

Express the power required to move the dumbwaiter at a constant speed *v*:

$$P_{\text{out}} = Tv$$

Apply $\sum F_y = ma_y$ to the dumbwaiter:

$$T - mg = ma_y$$
or, because $a_y = 0$,
$$T = mg$$

Substitute to obtain:

$$P_{\text{in}} = 3.7Tv = 3.7mgv$$

Substitute numerical values and evaluate P_{in}:

$$P_{\text{in}} = 3.7(35\,\text{kg})(9.81\,\text{m/s}^2)(0.35\,\text{m/s}) = \boxed{445\,\text{W}}$$

52 ••

Picture the Problem Choose a coordinate system in which upward is the positive y direction. We can express P_{drag} as the product of the drag force F_{drag} acting on the skydiver and her terminal velocity v_t. We can apply Newton's 2nd law to the skydiver to express F_{drag} in terms of her mass m and the gravitational field g.

(a) Express the power due to drag force acting on the skydiver as she falls at her terminal velocity v_t:

$$\vec{P}_{\text{drag}} = \vec{F}_{\text{drag}} \cdot \vec{v}_t$$

or, because $\vec{F}_{\text{drag}}$ and $\vec{v}_t$ are antiparallel,

$$P_{\text{drag}} = -F_{\text{drag}} v_t$$

Apply $\sum F_y = ma_y$ to the skydiver:

$$F_{\text{drag}} - mg = ma_y$$

or, because $a_y = 0$,

$$F_{\text{drag}} = mg$$

Substitute to obtain, for the magnitude of P_{drag}:

$$P_{\text{drag}} = \left| -mgv_t \right| \quad (1)$$

Substitute numerical values and evaluate P:

$$P_{\text{drag}} = \left| -(55\,\text{kg})(9.81\,\text{m/s}^2)\left(120\,\frac{\text{mi}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1.609\,\text{km}}{\text{mi}}\right) \right| = \boxed{2.89\times10^4\ \text{W}}$$

(b) Evaluate equation (1) with v = 15 mi/h:

$$P_{\text{drag}} = \left| -(55\,\text{kg})(9.81\,\text{m/s}^2)\left(15\,\frac{\text{mi}}{\text{h}}\right) \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1.609\,\text{km}}{\text{mi}} \right| = \boxed{3.62\,\text{kW}}$$

***53** ••

Picture the Problem Because, in the absence of air resistance, the acceleration of the cannonball is constant, we can use a constant-acceleration equation to relate its velocity to the time it has been in flight. We can apply Newton's 2nd law to the cannonball to find the net force acting on it and then form the dot product of $\vec{F}$ and $\vec{v}$ to express the rate at which the gravitational field does work on the cannonball. Integrating this expression over the time-of-flight T of the ball will yield the desired result.

Express the velocity of the cannonball as a function of time while it is in the air:

$$\vec{v}(t) = 0\hat{i} + (v_0 - gt)\hat{j}$$

Apply $\sum \vec{F} = m\vec{a}$ to the cannonball to express the force acting on it while it is in the air:

$$\vec{F} = -mg\,\hat{j}$$

Evaluate $\vec{F} \cdot \vec{v}$:

$$\vec{F} \cdot \vec{v} = -mg\,\hat{j} \cdot (v_0 - gt)\hat{j} = -mgv_0 + mg^2 t$$

Relate $\vec{F}\cdot\vec{v}$ to the rate at which work is being done on the cannonball:

$$\frac{dW}{dt}=\vec{F}\cdot\vec{v}=-mgv_0+mg^2t$$

Separate the variables and integrate over the time T that the cannonball is in the air:

$$W=\int_0^T\left(-mgv_0+mg^2t\right)dt=\tfrac{1}{2}mg^2T^2-mgv_0T \qquad (1)$$

Using a constant-acceleration equation, relate the speed v of the cannonball when it lands at the bottom of the cliff to its initial speed v_0 and the height of the cliff H:

$$v^2=v_0^2+2a\Delta y$$

or, because $a=g$ and $\Delta y=H$,

$$v^2=v_0^2+2gH$$

Solve for v to obtain:

$$v=\boxed{\sqrt{{v_0}^2+2gH}}$$

Using a constant-acceleration equation, relate the time-of-flight T to the initial and impact speeds of the cannonball:

$$v=v_0-gT$$

Solve for T to obtain:

$$T=\frac{v_0-v}{g}$$

Substitute for T in equation (1) and simplify to evaluate W:

$$W=\tfrac{1}{2}mg^2\frac{v_0^2-2vv_0+v^2}{g^2}-mgv_0\left(\frac{v_0-v}{g}\right)=\tfrac{1}{2}mv^2-\tfrac{1}{2}mv_0^2=\boxed{\Delta K}$$

54 ••

Picture the Problem If the particle is acted on by a *single* force, that force is the *net* force acting on the particle and is responsible for its acceleration. The rate at which energy is delivered by the force is $P=\vec{F}\cdot\vec{v}$.

Express the rate at which this force does work in terms of $\vec{F}$ and $\vec{v}$:

$$P=\vec{F}\cdot\vec{v}$$

The velocity of the particle, in terms of its acceleration and the time that the force has acted is:

$$\vec{v}=\vec{a}t$$

Using Newton's 2nd law, substitute for $\vec{a}$:

$$\vec{v} = \frac{\vec{F}}{m}t$$

Substitute for $\vec{v}$ in the expression for *P* and simplify to obtain:

$$P = \vec{F}\cdot\frac{\vec{F}}{m}t = \frac{\vec{F}\cdot\vec{F}}{m}t = \boxed{\frac{F^2}{m}t}$$

Potential Energy

55 •

Picture the Problem The change in the gravitational potential energy of the earth-man system, near the surface of the earth, is given by $\Delta U = mg\Delta h$, where Δh is measured relative to an arbitrarily chosen reference position.

Express the change in the man's gravitational potential energy in terms of his change in elevation:

$$\Delta U = mg\Delta h$$

Substitute for *m*, *g* and Δh and evaluate ΔU:

$$\Delta U = (80\,\text{kg})(9.81\,\text{m/s}^2)(6\,\text{m}) = \boxed{4.71\,\text{kJ}}$$

56 •

Picture the Problem The water going over the falls has gravitational potential energy relative to the base of the falls. As the water falls, the falling water acquires kinetic energy until, at the base of the falls; its energy is entirely kinetic. The rate at which energy is delivered to the base of the falls is given by $P = dW/dt = -dU/dt$.

Express the rate at which energy is being delivered to the base of the falls; remembering that half the potential energy of the water is converted to electric energy:

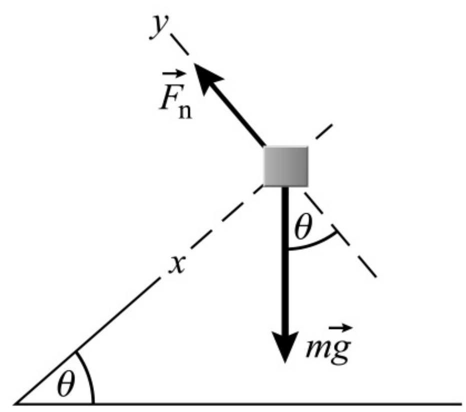
$$P = \frac{dW}{dt} = -\frac{dU}{dt} = -\tfrac{1}{2}\frac{d}{dt}(mgh) = -\tfrac{1}{2}gh\frac{dm}{dt}$$

Substitute numerical values and evaluate *P*:

$$P = -\tfrac{1}{2}(9.81\,\text{m/s}^2)(-128\,\text{m}) \times (1.4\times10^6\,\text{kg/s}) = \boxed{879\,\text{MW}}$$

57 •

Picture the Problem In the absence of friction, the sum of the potential and kinetic energies of the box remains constant as it slides down the incline. We can use the conservation of the mechanical energy of the system to calculate where the box will be and how fast it will be moving at any given time. We can also use Newton's 2nd law to show that the acceleration of the box is constant and constant-acceleration equations to calculate where the box will be and how fast it will be moving at any given time.



(*a*) Express and evaluate the gravitational potential energy of the box, relative to the ground, at the top of the incline:

$U_i = mgh = (2\text{ kg})(9.81\text{ m/s}^2)(20\text{ m}) = \boxed{392\text{ J}}$

(*b*) Using a constant-acceleration equation, relate the displacement of the box to its initial speed, acceleration and time-of-travel:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta x = \tfrac{1}{2}a(\Delta t)^2$$

Apply $\sum F_x = ma_x$ to the box as it slides down the incline and solve for its acceleration:

$$mg\sin\theta = ma \Rightarrow a = g\sin\theta$$

Substitute for a and evaluate $\Delta x(t = 1\text{ s})$:

$$\Delta x(1\text{s}) = \tfrac{1}{2}(g\sin\theta)(\Delta t)^2 = \tfrac{1}{2}(9.81\text{m/s}^2)(\sin 30°)(1\text{s})^2 = \boxed{2.45\text{m}}$$

Using a constant-acceleration equation, relate the speed of the box at any time to its initial speed and acceleration and solve for its speed when $t = 1$ s:

$v = v_0 + at$ where $v_0 = 0$

and

$$v(1\text{s}) = a\Delta t = (g\sin\theta)\Delta t = (9.81\text{m/s}^2)(\sin 30°)(1\text{s}) = \boxed{4.91\text{m/s}}$$

(*c*) Calculate the kinetic energy of the box when it has traveled for 1 s:

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(2\,\text{kg})(4.91\,\text{m/s})^2 = \boxed{24.1\,\text{J}}$$

Express the potential energy of the box after it has traveled for 1 s in terms of its initial potential energy and its kinetic energy:

$$U = U_\text{i} - K = 392\,\text{J} - 24.1\,\text{J} = \boxed{368\,\text{J}}$$

(*d*) Express the kinetic energy of the box at the bottom of the incline in terms of its initial potential energy and solve for its speed at the bottom of the incline:

$$K = U_i = \tfrac{1}{2}mv^2 = \boxed{392\,\text{J}}$$

and

$$v = \sqrt{\frac{2U_i}{m}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2(392\,\text{J})}{2\,\text{kg}}} = \boxed{19.8\,\text{m/s}}$$

58 •

Picture the Problem The potential energy function $U(x)$ is defined by the equation $U(x) - U(x_0) = -\int_{x_0}^{x} F dx.$ We can use the given force function to determine $U(x)$ and then the conditions on U to determine the potential functions that satisfy the given conditions.

(*a*) Use the definition of the potential energy function to find the potential energy function associated with F_x:

$$U(x) = U(x_0) - \int_{x_0}^{x} F_x dx = U(x_0) - \int_{x_0}^{x} (6\,\text{N})dx' = \boxed{-(6\,\text{N})(x - x_0)}$$

because $U(x_0) = 0$.

(*b*) Use the result obtained in (*a*) to find $U(x)$ that satisfies the condition that $U(4\text{ m}) = 0$:

$$U(4\,\text{m}) = -(6\,\text{N})(4\,\text{m} - x_0) = 0 \Rightarrow x_0 = 4\,\text{m}$$

and

$$U(x) = -(6\,\text{N})(x - 4\,\text{m}) = \boxed{24\,\text{J} - (6\,\text{N})x}$$

(*c*) Use the result obtained in (*a*) to find U that satisfies the condition that U(6 m) = 14 J:

$$U(6\,\text{m}) = -(6\,\text{N})(6\,\text{m} - x_0) = 14\,\text{J} \Rightarrow x_0 = 50\,\text{m}$$

and

$$U(x) = -(6\,\text{N})\left(x - \frac{25}{3}\,\text{m}\right) = \boxed{50\,\text{J} - (6\,\text{N})x}$$

59 •

Picture the Problem The potential energy of a stretched or compressed ideal spring U_s is related to its force (stiffness) constant k and stretch or compression Δx by $U_s = \frac{1}{2}kx^2$.

(*a*) Relate the potential energy stored in the spring to the distance it has been stretched:

$$U_s = \tfrac{1}{2}kx^2$$

Solve for x:

$$x = \sqrt{\frac{2U_s}{k}}$$

Substitute numerical values and evaluate x:

$$x = \sqrt{\frac{2(50\,\text{J})}{10^4\ \text{N/m}}} = \boxed{0.100\,\text{m}}$$

(*b*) Proceed as in (*a*) with U_s = 100 J:

$$x = \sqrt{\frac{2(100\,\text{J})}{10^4\ \text{N/m}}} = \boxed{0.141\,\text{m}}$$

***60** ••

Picture the Problem In a simple Atwood's machine, the only effect of the pulley is to connect the motions of the two objects on either side of it; i.e., it could be replaced by a piece of polished pipe. We can relate the kinetic energy of the rising and falling objects to the mass of the system and to their common speed and relate their accelerations to the sum and difference of their masses … leading to simultaneous equations in m_1 and m_2.

Use the definition of the kinetic energy of the system to determine the total mass being accelerated:

$$K = \tfrac{1}{2}(m_1 + m_2)v^2$$

and

$$m_1 + m_2 = \frac{2K}{v^2} = \frac{2(80\,\text{J})}{(4\,\text{m/s})^2} = 10.0\,\text{kg} \quad (1)$$

In Chapter 4, the acceleration of the masses was shown to be:

$$a = \frac{m_1 - m_2}{m_1 + m_2}g$$

Because $v(t) = at$, we can eliminate a in the previous equation to obtain:

$$v(t)=\frac{m_1-m_2}{m_1+m_2}gt$$

Solve for m_1-m_2:

$$m_1-m_2=\frac{(m_1+m_2)v(t)}{gt}$$

Substitute numerical values and evaluate m_1-m_2:

$$m_1-m_2=\frac{(10\,\text{kg})(4\,\text{m/s})}{(9.81\,\text{m/s}^2)(3\,\text{s})}=1.36\,\text{kg} \quad (2)$$

Solve equations (1) and (2) simultaneously to obtain:

$$m_1=\boxed{5.68\,\text{kg}}\text{ and }m_2=\boxed{4.32\,\text{kg}}$$

61 ••

Picture the Problem The gravitational potential energy of this system of two objects is the sum of their individual potential energies and is dependent on an arbitrary choice of where, or under what condition(s), the gravitational potential energy is zero. The best choice is one that simplifies the mathematical details of the expression of *U*. In this problem let's choose $U = 0$ where $\theta = 0$.

(*a*) Express *U* for the 2-object system as the sum of their gravitational potential energies; noting that because the object whose mass is m_2 is above the position we have chosen for $U = 0$, its potential energy is positive while that of the object whose mass is m_1 is negative:

$$\begin{aligned}U(\theta)&=U_1+U_2\\&=m_2g\ell_2\sin\theta-m_1g\ell_1\sin\theta\\&=\boxed{(m_2\ell_2-m_1\ell_1)g\sin\theta}\end{aligned}$$

(*b*) Differentiate *U* with respect to θ and set this derivative equal to zero to identify extreme values:

$$\frac{dU}{d\theta}=(m_2\ell_2-m_1\ell_1)g\cos\theta=0$$

from which we can conclude that $\cos\theta = 0$ and $\theta = \cos^{-1}0$.

To be physically meaningful, $-\pi/2\le\theta\le\pi/2$:

$$\therefore\theta=\pm\pi/2$$

Express the 2nd derivative of *U* with respect to θ and evaluate this derivative at $\theta=\pm\pi/2$:

$$\frac{d^2U}{d\theta^2}=-(m_2\ell_2-m_1\ell_1)g\sin\theta$$

If we assume, in the expression for U that we derived in (a), that $m_2\ell_2 - m_1\ell_1 > 0$, then $U(\theta)$ is a sine function and, in the interval of interest, $-\pi/2 \le \theta \le \pi/2$, takes on its minimum value when $\theta = -\pi/2$:

$$\left.\frac{d^2U}{d\theta^2}\right|_{-\pi/2} > 0$$

and $\boxed{U \text{ is a minimum at } \theta = -\pi/2}$

$$\left.\frac{d^2U}{d\theta^2}\right|_{\pi/2} < 0$$

and $\boxed{U \text{ is a maximum at } \theta = \pi/2}$

(c) If $m_1\ell_1 = m_2\ell_2$, then

$(m_2\ell_2 - m_1\ell_1) = 0$

and $\boxed{U = 0 \text{ independently of } \theta.}$

Remarks: An alternative approach to establishing the *U* is a maximum at $\theta = \pi/2$ is to plot its graph and note that, in the interval of interest, *U* is concave downward with its maximum value at $\theta = \pi/2$.

Force, Potential Energy, and Equilibrium

62 •

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is, $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x.

(a) Evaluate $F_x = -\dfrac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(Ax^4\right) = \boxed{-4Ax^3}$$

(b) Set $F_x = 0$ and solve for x:

$$F_x = 0 \Rightarrow \boxed{x = 0}$$

63 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x.

(a) Evaluate $F_x = -\dfrac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(\frac{C}{x}\right) = \boxed{\frac{C}{x^2}}$$

(b) Because $C > 0$:

$\boxed{F_x \text{ is positive for } x \neq 0 \text{ and therefore } \vec{F} \text{ is directed away from the origin.}}$

(*c*) Because U is inversely proportional to x and $C > 0$:

$$\boxed{U(x) \text{ decreases with increasing } x.}$$

(*d*) With $C < 0$:

$$\boxed{\begin{array}{l} F_x \text{ is negative for } x \neq 0 \text{ and therefore} \\ \vec{F} \text{ is directed toward from the origin.} \end{array}}$$

Because U is inversely proportional to x and $C < 0$, $U(x)$ becomes less negative as x increases:

$$\boxed{U(x) \text{ increases with increasing } x.}$$

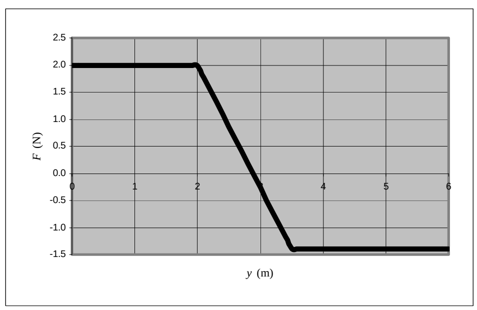
***64 ••**

Picture the Problem F_y is defined to be the negative of the derivative of the potential function with respect to y, i.e. $F_y = -dU/dy$. Consequently, we can obtain F_y by examining the slopes of the graph of U as a function of y.

The table to the right summarizes the information we can obtain from Figure 6-40:

	Slope	F_y
Interval	(N)	(N)
$A \to B$	-2	2
$B \to C$	transitional	$-2 \to 1.4$
$C \to D$	1.4	-1.4

The graph of F as a function of y is shown to the right:



65 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, i.e. $F_x = -dU/dx$. Consequently, given F as a function of x, we can find U by integrating F_x with respect to x.

Evaluate the integral of F_x with respect to x:

$$U(x) = -\int F(x)dx = -\int \frac{a}{x^2}dx$$

$$= \boxed{\frac{a}{x} + U_0}$$

where U_0 is a constant determined by whatever conditions apply to U.

66 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to *x*, that is, $F_x = -dU/dx$. Consequently, given *U* as a function of *x*, we can find F_x by differentiating *U* with respect to *x*. To determine whether the object is in stable or unstable equilibrium at a given point, we'll evaluate d^2U/dx^2 at the point of interest.

(*a*) Evaluate $F_x = -\frac{dU}{dx}$:

$$F_x = -\frac{d}{dx}\left(3x^2 - 2x^3\right) = \boxed{6x(x-1)}$$

(*b*) We know that, at equilibrium, $F_x = 0$:

When $F_x = 0$, $6x(x - 1) = 0$. Therefore, the object is in equilibrium at $\boxed{x = 0 \text{ and } x = 1\,\text{m.}}$

(*c*) To decide whether the equilibrium at a particular point is stable or unstable, evaluate the 2nd derivative of the potential energy function at the point of interest:

$$\frac{dU}{dx} = \frac{d}{dx}\left(3x^2 - 2x^3\right) = 6x - 6x^2$$

and

$$\frac{d^2U}{dx^2} = 6 - 12x$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 0$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = 6 > 0$$

$$\Rightarrow \boxed{\text{stable equilibrium at } x = 0}$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 1$ m:

$$\left.\frac{d^2U}{dx^2}\right|_{x=1\,\text{m}} = 6 - 12 < 0$$

$$\Rightarrow \boxed{\text{unstable equilibrium at } x = 1\,\text{m}}$$

67 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to *x*, i.e. $F_x = -dU/dx$. Consequently, given *U* as a function of *x*, we can find F_x by differentiating *U* with respect to *x*. To determine whether the object is in stable or unstable equilibrium at a given point, we'll evaluate d^2U/dx^2 at the point of interest.

(*a*) Evaluate the negative of the derivative of *U* with respect to *x*:

$$F_x = -\frac{dU}{dx}$$
$$= -\frac{d}{dx}\left(8x^2 - x^4\right) = 4x^3 - 16x$$
$$= \boxed{4x(x+2)(x-2)}$$

(*b*) The object is in equilibrium wherever $F_{\text{net}} = F_x = 0$:

$4x(x+2)(x-2) = 0 \Rightarrow$ the equilibrium points are $\boxed{x = -2\text{ m}, 0, \text{and } 2\text{ m.}}$

(*c*) To decide whether the equilibrium at a particular point is stable or unstable, evaluate the 2nd derivative of the potential energy function at the point of interest:

$$\frac{d^2U}{dx^2} = \frac{d}{dx}\left(16x - 4x^3\right) = 16 - 12x^2$$

Evaluate $\dfrac{d^2U}{dx^2}$ at $x = -2$ m:

$$\left.\frac{d^2U}{dx^2}\right|_{x=-2\text{ m}} = -32 < 0$$
$$\Rightarrow \boxed{\text{unstable equilibrium at } x = -2\text{ m}}$$

Evaluate $\dfrac{d^2U}{dx^2}$ at $x = 0$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = 16 > 0$$
$$\Rightarrow \boxed{\text{stable equilibrium at } x = 0}$$

Evaluate $\dfrac{d^2U}{dx^2}$ at $x = 2$ m:

$$\left.\frac{d^2U}{dx^2}\right|_{x=2\text{ m}} = -32 < 0$$
$$\Rightarrow \boxed{\text{unstable equilibrium at } x = 2\text{ m}}$$

Remarks: You could also decide whether the equilibrium positions are stable or unstable by plotting *F*(*x*) and examining the curve at the equilibrium positions.

68 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to *x*, i.e. $F_x = -dU/dx$. Consequently, given *F* as a function of *x*, we can find *U* by integrating F_x with respect to *x*. Examination of d^2U/dx^2 at extreme points will determine the nature of the stability at these locations.

Determine the equilibrium locations by setting $F_{net} = F(x) = 0$:

$F(x) = x^3 - 4x = x(x^2 - 4) = 0$

$\therefore$ the positions of stable and unstable equilibrium are at $\boxed{x = -2, 0 \text{ and } 2}$.

Evaluate the negative of the integral of $F(x)$ with respect to x:

$$U(x) = -\int F(x)$$
$$= -\int (x^3 - 4x)dx$$
$$= -\frac{x^4}{4} + 2x^2 + U_0$$

where U_0 is a constant whose value is determined by conditions on $U(x)$.

Differentiate $U(x)$ twice:

$$\frac{dU}{dx} = -F_x = -x^3 + 4x$$

and

$$\frac{d^2U}{dx^2} = -3x^2 + 4$$

Evaluate $\frac{d^2U}{dx^2}$ at $x = -2$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=-2} = -8 < 0$$

$\therefore$ $\boxed{\text{the equilibrium is unstable at } x = -2}$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 0$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=0} = 4 > 0$$

$\therefore$ $\boxed{\text{the equilibrium is stable at } x = 0}$

Evaluate $\frac{d^2U}{dx^2}$ at $x = 2$:

$$\left.\frac{d^2U}{dx^2}\right|_{x=2} = -8 < 0$$

$\therefore$ $\boxed{\text{the equilibrium is unstable at } x = 2}$

Thus $U(x)$ has a local minimum at $x = 0$ and local maxima at $x = \pm 2$.

69 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, i.e. $F_x = -dU/dx$. Consequently, given U as a function of x, we can find F_x by differentiating U with respect to x. To determine whether the object is in stable or unstable equilibrium at a given point, we can examine the graph of U.

(a) Evaluate $F_x = -\frac{dU}{dx}$ for $x \leq 3$ m:

$$F_x = -\frac{d}{dx}\left(3x^2 - x^3\right) = 3x\left(2 - x\right)$$

Set $F_x = 0$ to identify those values of x for which the 4-kg object is in equilibrium:

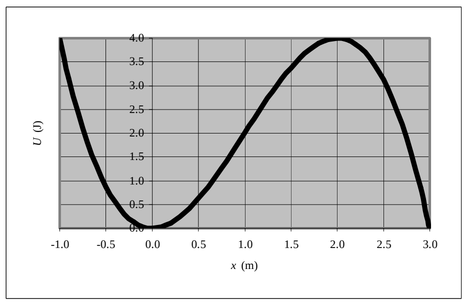
When $F_x = 0$, $3x(2 - x) = 0$.
Therefore, the object is in equilibrium at $\boxed{x = 0 \text{ and } x = 2\,\text{m.}}$

Evaluate $F_x = -\frac{dU}{dx}$ for $x > 3$ m:

$F_x = 0$
because $U = 0$.

$\boxed{\text{Therefore, the object is in neutral equilibrium for } x > 3\,\text{m.}}$

(b) A graph of $U(x)$ in the interval $-1\text{ m} \leq x \leq 3\text{ m}$ is shown to the right:



(c) From the graph, $U(x)$ is a minimum at $x = 0$:

$\therefore$ $\boxed{\text{stable equilibrium at } x = 0}$

From the graph, $U(x)$ is a maximum at $x = 2$ m:

$\therefore$ $\boxed{\text{unstable equilibrium at } x = 2\,\text{m}}$

(d) Relate the kinetic energy of the object to its total energy and its potential energy:

$$K = \tfrac{1}{2}mv^2 = E - U$$

Solve for v:

$$v = \sqrt{\frac{2\left(E - U\right)}{m}}$$

Evaluate $U(x = 2\text{ m})$:

$$U\left(x = 2\,\text{m}\right) = 3\left(2\right)^2 - \left(2\right)^3 = 4\,\text{J}$$

Substitute in the equation for v to obtain:

$$v = \sqrt{\frac{2\left(12\,\text{J} - 4\,\text{J}\right)}{4\,\text{kg}}} = \boxed{2.00\,\text{m/s}}$$

70 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is $F_x = -dU/dx$. Consequently, given F as a function of x, we can find U by integrating F_x with respect to x.

(*a*) Evaluate the negative of the integral of $F(x)$ with respect to x:

$$U(x) = -\int F(x) = -\int Ax^{-3}dx$$
$$= \frac{1}{2}\frac{A}{x^2} + U_0$$

where U_0 is a constant whose value is determined by conditions on $U(x)$.

For $x > 0$:

$$\boxed{U \text{ decreases as } x \text{ increases}}$$

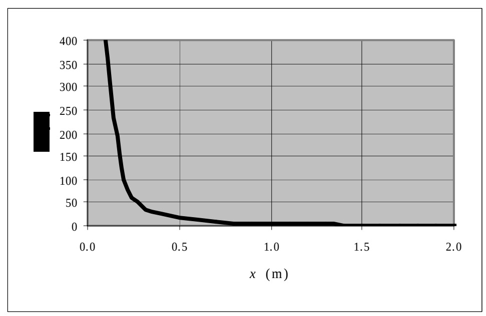
(*b*) As $x \to \infty$, $\frac{1}{2}\frac{A}{x^2} \to 0$:

$\therefore\ U_0 = 0$

and

$$U(x) = \frac{1}{2}\frac{A}{x^2} = \frac{1}{2}\frac{8\,\text{N}\cdot\text{m}^3}{x^2} = \boxed{\frac{4}{x^2}\,\text{N}\cdot\text{m}^3}$$

(*c*) The graph of $U(x)$ is shown to the right:



***71** •••

Picture the Problem Let L be the total length of one cable and the zero of gravitational potential energy be at the top of the pulleys. We can find the value of y for which the potential energy of the system is an extremum by differentiating $U(y)$ with respect to y and setting this derivative equal to zero. We can establish that this value corresponds to a minimum by evaluating the second derivative of $U(y)$ at the point identified by the first derivative. We can apply Newton's 2nd law to the clock to confirm the result we obtain by examining the derivatives of $U(y)$.

(*a*) Express the potential energy of the system as the sum of the potential energies of the clock and counterweights:

$$U(y) = U_{\text{clock}}(y) + U_{\text{weights}}(y)$$

Substitute to obtain:

$$U(y) = \boxed{-mgy - 2Mg\left(L - \sqrt{y^2 + d^2}\right)}$$

(*b*) Differentiate *U*(*y*) with respect to *y*:

$$\frac{dU(y)}{dy} = -\frac{d}{dy}\left[mgy + 2Mg\left(L - \sqrt{y^2 + d^2}\right)\right]$$

$$= -\left[mg - 2Mg\frac{y}{\sqrt{y^2 + d^2}}\right]$$

or

$$mg - 2Mg\frac{y'}{\sqrt{y'^2 + d^2}} = 0 \text{ for extrema}$$

Solve for y' to obtain:

$$y' = d\sqrt{\frac{m^2}{4M^2 - m^2}}$$

Find $\frac{d^2U(y)}{dy^2}$:

$$\frac{d^2U(y)}{dy^2} = -\frac{d}{dy}\left[mg - 2Mg\frac{y}{\sqrt{y^2 + d^2}}\right]$$

$$= \frac{2Mgd^2}{\left(y^2 + d^2\right)^{3/2}}$$

Evaluate $\frac{d^2U(y)}{dy^2}$ at $y = y'$:

$$\left.\frac{d^2U(y)}{dy^2}\right|_{y'} = \left.\frac{2Mgd^2}{\left(y^2 + d^2\right)^{3/2}}\right|_{y'}$$

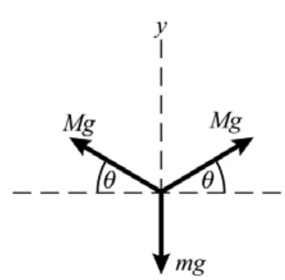
$$= \frac{2Mgd}{\left(\frac{m^2}{4M^2 - m^2} + 1\right)^{3/2}}$$

$$> 0$$

and the potential energy is a minimum at

$$y = \boxed{d\sqrt{\frac{m^2}{4M^2 - m^2}}}$$

(*c*) The FBD for the clock is shown to the right:



Apply $\sum F_y = 0$ to the clock:

$$2Mg\sin\theta - mg = 0$$

and

$$\sin\theta = \frac{m}{2M}$$

Express $\sin\theta$ in terms of y and d:

$$\sin\theta = \frac{y}{\sqrt{y^2 + d^2}}$$

Substitute to obtain:

$$\boxed{\frac{m}{2M} = \frac{y}{\sqrt{y^2 + d^2}}}$$

which is equivalent to the first equation in part (b).

This is a point of stable equilibrium. If the clock is displaced downward, θ increases, leading to a larger upward force on the clock. Similarly, if the clock is displaced upward, the net force from the cables decreases. Because of this, the clock will be pulled back toward the equilibrium point if it is displaced away from it.

Remarks: Because we've shown that the potential energy of the system is a minimum at $y = y'$ (i.e., $U(y)$ is concave upward at that point), we can conclude that this point is one of stable equilibrium.

General Problems

***72** •

Picture the Problem 25 percent of the electrical energy generated is to be diverted to do the work required to change the potential energy of the American people. We can calculate the height to which they can be lifted by equating the change in potential energy to the available energy.

Express the change in potential energy of the population of the United States in this process:

$$\Delta U = Nmgh$$

Letting E represent the total energy generated in February 2002, relate the change in potential to the energy available to operate the elevator:

$$Nmgh = 0.25E$$

Solve for h:

$$h = \frac{0.25E}{Nmg}$$

Substitute numerical values and evaluate h:

$$h = \frac{(0.25)(60.7\times10^9\ \text{kW}\cdot\text{h})\left(\frac{3600\,\text{s}}{1\,\text{h}}\right)}{(287\times10^6)(60\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{323\,\text{km}}$$

73 •

Picture the Problem We can use the definition of the work done in changing the potential energy of a system and the definition of power to solve this problem.

(a) Find the work done by the crane in changing the potential energy of its load:

$$W = mgh = (6\times10^6\ \text{kg})\ (9.81\ \text{m/s}^2)\ (12\ \text{m}) = \boxed{706\,\text{MJ}}$$

(b) Use the definition of power to find the power developed by the crane:

$$P \equiv \frac{dW}{dt} = \frac{706\,\text{MJ}}{60\,\text{s}} = \boxed{11.8\,\text{MW}}$$

74 •

Picture the Problem The power P of the engine needed to operate this ski lift is related to the rate at which it changes the potential energy U of the cargo of the gondolas according to $P = \Delta U/\Delta t$. Because as many empty gondolas are descending as are ascending, we do not need to know their mass.

Express the rate at which work is done as the cars are lifted:

$$P = \frac{\Delta U}{\Delta t}$$

Letting N represent the number of gondola cars and M the mass of each, express the change in U as they are lifted a vertical displacement Δh:

$$\Delta U = NMg\Delta h$$

Substitute to obtain:

$$P \equiv \frac{\Delta U}{\Delta t} = \frac{NMg\Delta h}{\Delta t}$$

Relate Δh to the angle of ascent θ and the length L of the ski lift:

$$\Delta h = L\sin\theta$$

Substitute for Δh in the expression for P:

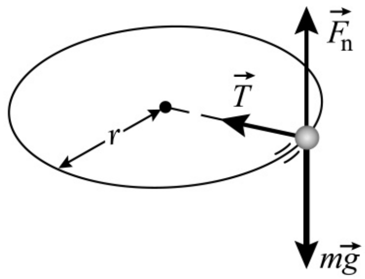
$$P = \frac{NMgL\sin\theta}{\Delta t}$$

Substitute numerical values and evaluate P:

$$P = \frac{12(550\,\text{kg})(9.81\,\text{m/s}^2)(5.6\,\text{km})\sin 30°}{(60\,\text{min})(60\,\text{s/min})} = \boxed{50.4\,\text{kW}}$$

75 •

Picture the Problem The application of Newton's 2nd law to the forces shown in the free-body diagram will allow us to relate R to T. The unknown mass and speed of the object can be eliminated by introducing its kinetic energy.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ the object and solve for R:

$$T = \frac{mv^2}{R} \text{ and } R = \frac{mv^2}{T}$$

Express the kinetic energy of the object:

$$K = \tfrac{1}{2}mv^2$$

Eliminate mv^2 between the two equations to obtain:

$$R = \frac{2K}{T}$$

Substitute numerical values and evaluate R:

$$R = \frac{2(90\,\text{J})}{360\,\text{N}} = \boxed{0.500\,\text{m}}$$

***76** •

Picture the Problem We can solve this problem by equating the expression for the gravitational potential energy of the elevated car and its kinetic energy when it hits the ground.

Express the gravitational potential energy of the car when it is at a distance h above the ground:

$$U = mgh$$

Express the kinetic energy of the car when it is about to hit the ground:

$$K = \tfrac{1}{2}mv^2$$

Equate these two expressions (because at impact, all the potential energy has been converted to kinetic energy) and solve for h:

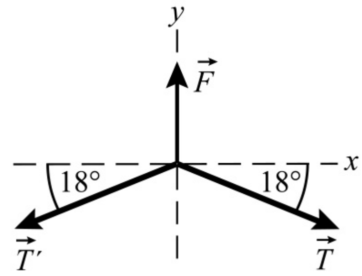
$$h = \frac{v^2}{2g}$$

Substitute numerical values and evaluate h:

$$h = \frac{[(100\,\text{km/h})(1\,\text{h}/3600\,\text{s})]^2}{2(9.81\,\text{m/s}^2)} = \boxed{39.3\,\text{m}}$$

77 •••

Picture the Problem The free-body diagram shows the forces acting on one of the strings at the bridge. The force whose magnitude is F is one-fourth of the force (103 N) the bridge exerts on the strings. We can apply the condition for equilibrium in the y direction to find the tension in each string. Repeating this procedure at the site of the plucking will yield the restoring force acting on the string. We can find the work done on the string as it returns to equilibrium from the product of the average force acting on it and its displacement.



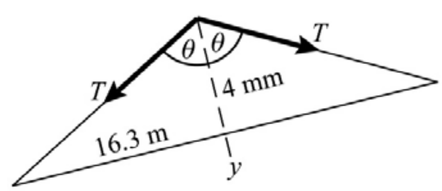
(a) Noting that, due to symmetry, $T' = T$, apply $\sum F_y = 0$ to the string at the point of contact with the bridge:

$$F - 2T\sin 18° = 0$$

Solve for and evaluate T:

$$T = \frac{F}{2\sin 18°} = \frac{\frac{1}{4}(103\,\text{N})}{2\sin 18°} = \boxed{41.7\,\text{N}}$$

(b) A free-body diagram showing the forces restoring the string to its equilibrium position just after it has been plucked is shown to the right:



Express the net force acting on the string immediately after it is released:

$$F_{\text{net}} = 2T\cos\theta$$

Use trigonometry to find θ:

$$\theta = \tan^{-1}\left(\frac{16.3\,\text{cm}}{4\,\text{mm}} \times \frac{10\,\text{mm}}{\text{cm}}\right) = 88.6°$$

Substitute and evaluate F_{net}:

$$F_{\text{net}} = 2(34.4\,\text{N})\cos 88.6° = \boxed{1.68\,\text{N}}$$

(*c*) Express the work done on the string in displacing it a distance dx':

$$dW = F dx'$$

If we pull the string out a distance x', the magnitude of the force pulling it down is approximately:

$$F = (2T)\frac{x'}{L/2} = \frac{4T}{L}x'$$

Substitute to obtain:

$$dW = \frac{4T}{L}x'\,dx'$$

Integrate to obtain:

$$W = \frac{4T}{L}\int_0^x x'dx' = \frac{2T}{L}x^2$$

where x is the final displacement of the string.

Substitute numerical values to obtain:

$$W = \frac{2(41.7\text{ N})}{32.6\times10^{-2}\text{ m}}\left(4\times10^{-3}\text{ m}\right)^2$$

$$= \boxed{4.09\text{ mJ}}$$

78 ••

Picture the Problem F_x is defined to be the negative of the derivative of the potential function with respect to x, that is $F_x = -dU/dx$. Consequently, given F as a function of x, we can find U by integrating F_x with respect to x.

Evaluate the integral of F_x with respect to x:

$$U(x) = -\int F(x)dx = -\int\left(-ax^2\right)dx$$

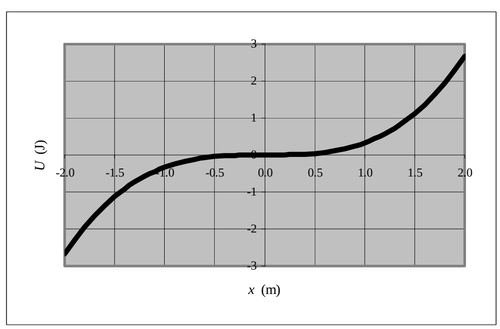
$$= \tfrac{1}{3}ax^3 + U_0$$

Apply the condition that $U(0) = 0$ to determine U_0:

$U(0) = 0 + U_0 = 0 \Rightarrow U_0 = 0$

$$\therefore U(x) = \boxed{\tfrac{1}{3}ax^3}$$

The graph of $U(x)$ is shown to the right:



***79 ••**

Picture the Problem We can use the definition of work to obtain an expression for the position-dependent force acting on the cart. The work done on the cart can be calculated from its change in kinetic energy.

(*a*) Express the force acting on the cart in terms of the work done on it:

$$F(x)=\frac{dW}{dx}$$

Because U is constant:

$$F(x)=\frac{d}{dx}\left(\tfrac{1}{2}mv^2\right)=\frac{d}{dx}\left[\tfrac{1}{2}m(Cx)^2\right]$$
$$=\boxed{mC^2x}$$

(*b*) The work done by this force changes the kinetic energy of the cart:

$$W=\Delta K=\tfrac{1}{2}mv_1^2-\tfrac{1}{2}mv_0^2$$
$$=\tfrac{1}{2}mv_1^2-0=\tfrac{1}{2}m(Cx_1)^2$$
$$=\boxed{\tfrac{1}{2}mC^2x_1^2}$$

80 ••

Picture the Problem The work done by $\vec{F}$ depends on whether it causes a displacement in the direction it acts.

(*a*) Because $\vec{F}$ is along x-axis and the displacement is along y-axis:

$$W=\int\vec{F}\cdot d\vec{s}=\boxed{0}$$

(*b*) Calculate the work done by $\vec{F}$ during the displacement from x = 2 m to 5 m:

$$W=\int\vec{F}\cdot d\vec{s}=\int_{2\,\text{m}}^{5\,\text{m}}\left(2\,\text{N/m}^2\right)x^2dx$$
$$=\left(2\,\text{N/m}^2\right)\left[\frac{x^3}{3}\right]_{2\,\text{m}}^{5\,\text{m}}=\boxed{78.0\,\text{J}}$$

81 ••

Picture the Problem The velocity and acceleration of the particle can be found by differentiation. The power delivered to the particle can be expressed as the product of its velocity and the net force acting on it, and the work done by the force and can be found from the change in kinetic energy this work causes.

In the following, if t is in seconds and m is in kilograms, then v is in m/s, a is in m/s^2, P is in W, and W is in J.

(*a*) The velocity of the particle is given by:

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(2t^3 - 4t^2\right)$$
$$= \boxed{\left(6t^2 - 8t\right)}$$

The acceleration of the particle is given by:

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(6t^2 - 8t\right)$$
$$= \boxed{\left(12t - 8\right)}$$

(*b*) Express and evaluate the rate at which energy is delivered to this particle as it accelerates:

$$P = Fv = mav$$
$$= m(12t - 8)\left(6t^2 - 8t\right)$$
$$= \boxed{8mt\left(9t^2 - 18t + 8\right)}$$

(*c*) Because the particle is moving in such a way that its potential energy is not changing, the work done by the force acting on the particle equals the change in its kinetic energy:

$$W = \Delta K = K_1 - K_0$$
$$= \tfrac{1}{2}m\left[\left(v(t_1)\right)^2 - \left(v(0)\right)^2\right]$$
$$= \tfrac{1}{2}m\left[\left(6t_1^2 - 8t_1\right)\right]^2 - 0$$
$$= \boxed{2mt_1^2\left(3t_1 - 4\right)^2}$$

Remarks: We could also find *W* by integrating *P*(*t*) with respect to time.

82 ••

Picture the Problem We can calculate the work done by the given force from its definition. The power can be determined from $P = \vec{F}\cdot\vec{v}$ and *v* from the change in kinetic energy of the particle produced by the work done on it.

(*a*) Calculate the work done from its definition:

$$W = \int \vec{F}\cdot d\vec{s} = \int_0^{3\,\text{m}} \left(6 + 4x - 3x^2\right)dx$$
$$= \left[6x + \frac{4x^2}{2} - \frac{3x^3}{3}\right]_0^{3\,\text{m}} = \boxed{9.00\ \text{J}}$$

(*b*) Express the power delivered to the particle in terms of $F_{x=3\,\text{m}}$ and its velocity:

$$P = \vec{F}\cdot\vec{v} = F_{x=3\,\text{m}}v$$

Relate the work done on the particle to its kinetic energy and solve for its velocity:

$$W = \Delta K = K_{\text{final}} = \tfrac{1}{2}mv^2 \text{ since } v_0 = 0$$

Solve for and evaluate v:

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(9\,\text{J})}{3\,\text{kg}}} = 2.45\,\text{m/s}$$

Evaluate $F_{x=3\text{ m}}$:

$$F_{x=3\text{m}} = 6 + 4(3) - 3(3)^2 = -9\,\text{N}$$

Substitute for $F_{x=3\text{ m}}$ and v:

$$P = (-9\,\text{N})(2.45\,\text{m/s}) = \boxed{-22.1\,\text{W}}$$

***83** ••

Picture the Problem We'll assume that the firing height is negligible and that the bullet lands at the same elevation from which it was fired. We can use the equation $R = (v_0^2/g)\sin 2\theta$ to find the range of the bullet and constant-acceleration equations to find its maximum height. The bullet's initial speed can be determined from its initial kinetic energy.

Express the range of the bullet as a function of its firing speed and angle of firing:

$$R = \frac{v_0^2}{g}\sin 2\theta$$

Rewrite the range equation using the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$:

$$R = \frac{v_0^2\sin 2\theta}{g} = \frac{2v_0^2\sin\theta\cos\theta}{g}$$

Express the position coordinates of the projectile along its flight path in terms of the parameter t:

$$x = (v_0\cos\theta)t$$

and

$$y = (v_0\sin\theta)t - \tfrac{1}{2}gt^2$$

Eliminate the parameter t and make use of the fact that the maximum height occurs when the projectile is at half the range to obtain:

$$h = \frac{(v_0\sin\theta)^2}{2g}$$

Equate R and h and solve the resulting equation for θ:

$$\tan\theta = 4 \Rightarrow \theta = \tan^{-1}4 = 76.0°$$

Relate the bullet's kinetic energy to its mass and speed and solve for the square of its speed:

$$K = \tfrac{1}{2}mv_0^2 \text{ and } v_0^2 = \frac{2K}{m}$$

Substitute for v_0^2 and θ and evaluate R:

$$R = \frac{2(1200\,\text{J})}{(0.02\,\text{kg})(9.81\,\text{m/s}^2)}\sin 2(76°)$$
$$= \boxed{5.74\,\text{km}}$$

84 ••

Picture the Problem The work done on the particle is the area under the force-versus-displacement curve. Note that for negative displacements, F is positive, so W is negative for $x < 0$.

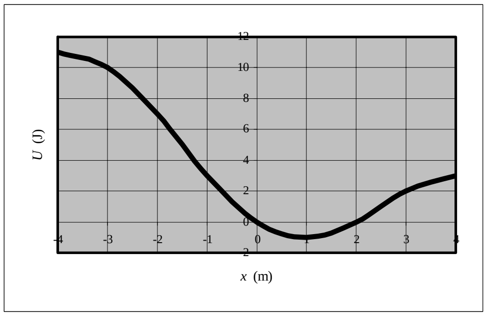
(*a*) Use either the formulas for the areas of simple geometric figures *or* counting squares and multiplying by the work represented by one square to complete the table to the right:

x	W
(m)	(J)
–4	–11
–3	–10
–2	–7
–1	–3
0	0
1	1
2	0
3	–2
4	–3

(*b*) Choosing $U(0) = 0$, and using the definition of $\Delta U = -W$, complete the third column of the table to the right:

x	W	ΔU
(m)	(J)	(J)
–4	–11	11
–3	–10	10
–2	–7	7
–1	–3	3
0	0	0
1	1	–1
2	0	0
3	–2	2
4	–3	3

The graph of U as a function of x is shown to the right:



85 ••

Picture the Problem The work done on the particle is the area under the force-versus-displacement curve. Note that for negative displacements, *F* is negative, so *W* is positive for $x < 0$.

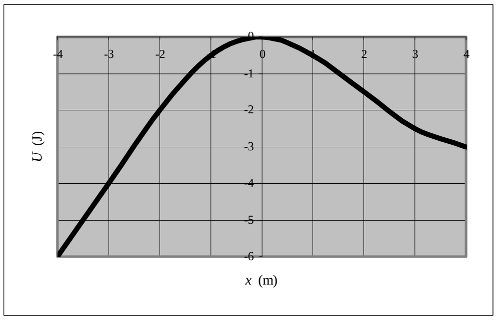
(*a*) Use either the formulas for the areas of simple geometric figures *or* counting squares and multiplying by the work represented by one square to complete the table to the right:

x	*W*
(m)	(J)
–4	6
–3	4
–2	2
–1	0.5
0	0
1	0.5
2	1.5
3	2.5
4	3

(*b*) Choosing $U(0) = 0$, and using the definition of $\Delta U = -W$, complete the third column of the table to the right:

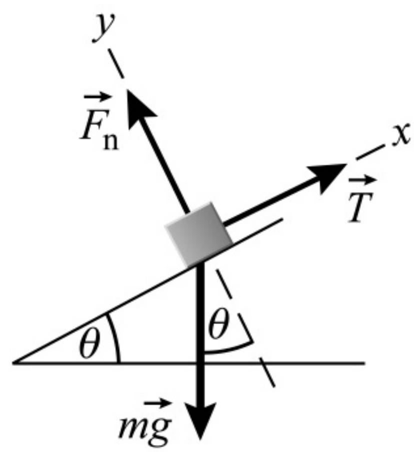
x	*W*	ΔU
(m)	(J)	(J)
–4	6	–6
–3	4	–4
–2	2	–2
–1	0.5	–0.5
0	0	0
1	0.5	–0.5
2	1.5	–1.5
3	2.5	–2.5
4	3	–3

The graph of *U* as a function of *x* is shown to the right:



86 ••

Picture the Problem The pictorial representation shows the box at its initial position 0 at the bottom of the inclined plane and later at position 1. We'll assume that the block is at position 0. Because the surface is frictionless, the work done by the tension will change both the potential and kinetic energy of the block. We'll use Newton's 2nd law to find the acceleration of the block up the incline and a constant-acceleration equation to express *v* in terms of *T*, *x*, *M*, and *θ*. Finally, we can express the power produced by the tension in terms of the tension and the speed of the box.



(*a*) Use the definition of work to express the work the tension *T* does moving the box a distance *x* up the incline:

$$W = \boxed{Tx}$$

(*b*) Apply $\sum F_x = Ma_x$ to the box:

$$T - Mg\sin\theta = Ma_x$$

Solve for a_x:

$$a_x = \frac{T - Mg\sin\theta}{M} = \frac{T}{M} - g\sin\theta$$

Using a constant-acceleration equation, express the speed of the box in terms of its acceleration and the distance *x* it has moved up the incline:

$$v^2 = v_0^2 + 2a_x x$$

or, because $v_0 = 0$,

$$v = \sqrt{2a_x x}$$

Substitute for a_x to obtain:

$$v = \boxed{\sqrt{2\left(\frac{T}{M} - g\sin\theta\right)x}}$$

(*c*) The power produced by the tension in the string is given by:

$$P = Tv = \boxed{T\sqrt{2\left(\frac{T}{M} - g\sin\theta\right)x}}$$

87 •••

Picture the Problem We can use the definition of the magnitude of vector to show that the magnitude of $\vec{F}$ is F_0 and the definition of the scalar product to show that its direction is perpendicular to $\vec{r}$. The work done as the particle moves in a circular path can be found from its definition.

(*a*) Express the magnitude of $\vec{F}$:

$$\left|\vec{F}\right| = \sqrt{F_x^2 + F_y^2} = \sqrt{\left(\frac{F_0}{r}y\right)^2 + \left(-\frac{F_0}{r}x\right)^2} = \frac{F_0}{r}\sqrt{x^2+y^2}$$

Because $r = \sqrt{x^2+y^2}$:

$$\left|\vec{F}\right| = \frac{F_0}{r}\sqrt{x^2+y^2} = \frac{F_0}{r}r = \boxed{F_0}$$

Form the scalar product of $\vec{F}$ and $\vec{r}$:

$$\vec{F}\cdot\vec{r} = \left(\frac{F_0}{r}\right)\left(y\,\hat{i} - x\,\hat{j}\right)\cdot\left(x\,\hat{i} + y\,\hat{j}\right) = \left(\frac{F_0}{r}\right)(xy - xy) = 0$$

Because $\vec{F}\cdot\vec{r} = 0$, $\boxed{\vec{F} \perp \vec{r}}$

(*b*) Because $\vec{F} \perp \vec{r}$, $\vec{F}$ is tangential to the circle and constant. At (5 m, 0), $\vec{F}$ points in the $-\hat{j}$ direction. If $d\vec{s}$ is in the $-\hat{j}$ direction, $dW > 0$. The work it does in one revolution is:

$$W = F_0(2\pi r) = 2\pi(5\,\text{m})F_0 = (10\pi\,\text{m})F_0 \text{ if the rotation is clockwise}$$

and

$$W = (-10\pi\,\text{m})F_0 \text{ if the rotation is counterclockwise.}$$

$$\boxed{W = (10\pi\,\text{m})F_0 \text{ if the rotation is clockwise}, -(10\pi\,\text{m})F_0 \text{ if the rotation is counterclockwise. Because } W \neq 0 \text{ for a complete circuit, } \vec{F} \text{ is not conservative.}}$$

***88** •••

Picture the Problem We can substitute for *r* and $x\hat{i} + y\hat{j}$ in $\vec{F}$ to show that the magnitude of the force varies as the inverse of the square of the distance to the origin, and that its direction is opposite to the radius vector. We can find the work done by this force by evaluating the integral of *F* with respect to *x* from an initial position $x = 2$ m, $y = 0$ m to a final position $x = 5$ m, $y = 0$ m. Finally, we can apply Newton's 2nd law to the particle to relate its speed to its radius, mass, and the constant *b*.

(*a*) Substitute for *r* and $x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$ in $\vec{\boldsymbol{F}}$ to obtain:

$$\vec{\boldsymbol{F}} = -\left(\frac{b}{\left(x^2+y^2\right)^{3/2}}\right)\sqrt{x^2+y^2}\,\hat{\boldsymbol{r}}$$

where $\hat{\boldsymbol{r}}$ is a unit vector pointing from the origin toward the point of application of $\vec{\boldsymbol{F}}$.

Simplify to obtain:

$$\vec{\boldsymbol{F}} = -b\left(\frac{1}{x^2+y^2}\right)\hat{\boldsymbol{r}} = \boxed{-\frac{b}{r^2}\hat{\boldsymbol{r}}}$$

i.e., the magnitude of the force varies as the inverse of the square of the distance to the origin, and its direction is antiparallel (opposite) to the radius vector $\vec{\boldsymbol{r}} = x\hat{\boldsymbol{i}} + y\hat{\boldsymbol{j}}$.

(*b*) Find the work done by this force by evaluating the integral of *F* with respect to *x* from an initial position *x* = 2 m, *y* = 0 m to a final position *x* = 5 m, *y* = 0 m:

$$W = -\int_{2\,\text{m}}^{5\,\text{m}} \frac{b}{x'^2}dx' = b\left[\frac{1}{x'}\right]_{2\,\text{m}}^{5\,\text{m}}$$

$$= 3\,\text{N}\cdot\text{m}^2\left(\frac{1}{5\,\text{m}} - \frac{1}{2\,\text{m}}\right) = \boxed{-0.900\,\text{J}}$$

(*c*) $\boxed{\text{No work is done as the force is perpendicular to the velocity.}}$

(*d*) Because the particle is moving in a circle, the force on the particle must be supplying the centripetal acceleration keeping it moving in the circle. Apply $\sum F_r = ma_c$ to the particle:

$$\frac{b}{r^2} = m\frac{v^2}{r}$$

Solve for *v*:

$$v = \sqrt{\frac{b}{mr}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{3\,\text{N}\cdot\text{m}^2}{(2\,\text{kg})(7\,\text{m})}} = \boxed{0.463\,\text{m/s}}$$

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Picture the Problem A spreadsheet program to calculate the potential is shown below. The constants used in the potential function and the formula used to calculate the ″6-12″ potential are as follows:

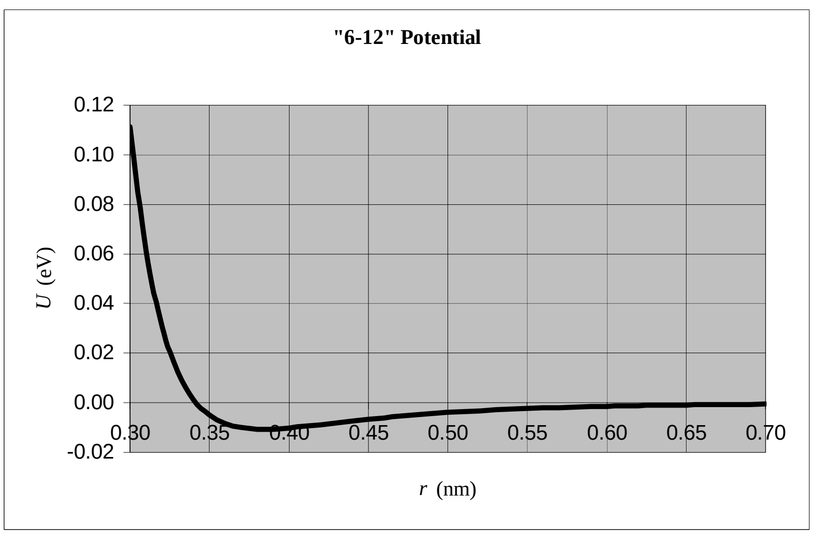
Cell	Content/Formula	Algebraic Form
B2	1.09×10^{-7}	*a*
B3	6.84×10^{-5}	*b*

D8	\$B\$2/C8^12–\$B\$3/C8^6	$\frac{a}{r^{12}}-\frac{b}{r^{6}}$
C9	C8+0.1	$r+\Delta r$

(*a*)

	A	B	C	D
1				
2	a =	1.09E-07		
3	b =	6.84E-05		
4				
5				
6				
7			*r*	*U*
8			3.00E-01	1.11E-01
9			3.10E-01	6.13E-02
10			3.20E-01	3.08E-02
11			3.30E-01	1.24E-02
12			3.40E-01	1.40E-03
13			3.50E-01	–4.95E-03
45			6.70E-01	–7.43E-04
46			6.80E-01	–6.81E-04
47			6.90E-01	–6.24E-04
48			7.00E-01	–5.74E-04

The graph shown below was generated from the data in the table shown above. Because the force between the atomic nuclei is given by $F=-(dU/dr)$, we can conclude that the shape of the potential energy function supports Feynman's claim.



(*b*) The minimum value is about –0.0107 eV, occurring at a separation of approximately 0.380 nm. Because the function is concave upward (a potential ″well″) at this separation, this separation is one of stable equilibrium, although very shallow.

(*c*) Relate the force of attraction between two argon atoms to the slope of the potential energy function:

$$F=-\frac{dU}{dr}=-\frac{d}{dr}\left[\frac{a}{r^{12}}-\frac{b}{r^{6}}\right]$$
$$=\frac{12a}{r^{13}}-\frac{6b}{r^{7}}$$

Substitute numerical values and evaluate $F(5\ \text{Å})$:

$$F=\frac{12\left(1.09\times10^{-7}\right)}{(0.5\,\text{nm})^{13}}-\frac{6\left(6.84\times10^{-5}\right)}{(0.5\,\text{nm})^{7}}=-4.18\times10^{-2}\ \frac{\text{eV}}{\text{nm}}\times\frac{1.6\times10^{-19}\ \text{J}}{\text{eV}}\times\frac{1\,\text{nm}}{10^{-9}\ \text{m}}$$
$$=\boxed{-6.69\times10^{-12}\ \text{N}}$$

where the minus sign means that the force is attractive.

Substitute numerical values and evaluate $F(3.5\ \text{Å})$:

$$F=\frac{12\left(1.09\times10^{-7}\right)}{(0.35\,\text{nm})^{13}}-\frac{6\left(6.84\times10^{-5}\right)}{(0.35\,\text{nm})^{7}}=4.68\times10^{-1}\ \frac{\text{eV}}{\text{nm}}\times\frac{1.6\times10^{-19}\ \text{J}}{\text{eV}}\times\frac{1\,\text{nm}}{10^{-9}\ \text{m}}$$
$$=\boxed{7.49\times10^{-11}\ \text{N}}$$

where the plus sign means that the force is repulsive.

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Picture the Problem A spreadsheet program to plot the Yukawa potential is shown below. The constants used in the potential function and the formula used to calculate the Yukawa potential are as follows:

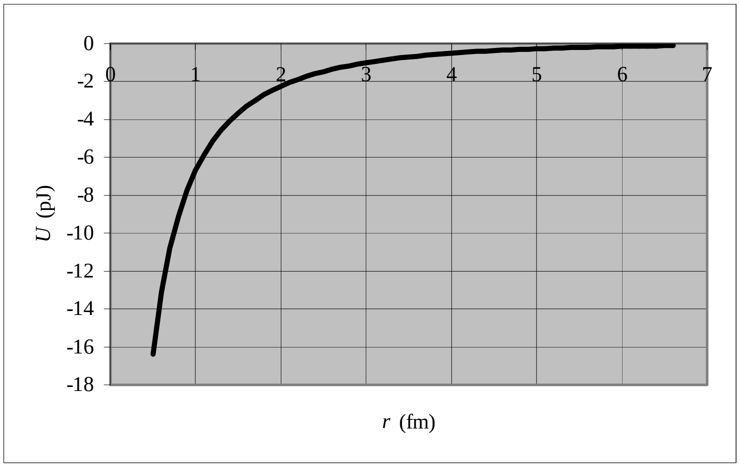
Cell	Content/Formula	Algebraic Form
B1	4	U_0
B2	2.5	a
D8	–$B$1*($B$2/C9)*EXP(–C9/$B$2)	$-U_0\left(\frac{a}{r}\right)e^{-r/a}$
C10	C9+0.1	$r+\Delta r$

(*a*)

	A	B	C	D
1	U0=	4	pJ	
2	a=	2.5	fm	
3				
7				
8			r	U
9			0.5	–16.37
10			0.6	–13.11

11			0.7	–10.80
12			0.8	–9.08
13			0.9	–7.75
14			1	–6.70
64			6	–0.15
65			6.1	–0.14
66			6.2	–0.14
67			6.3	–0.13
68			6.4	–0.12
69			6.5	–0.11
70			6.6	–0.11

U as a function of *r* is shown below.



(*b*) Relate the force between the nucleons to the slope of the potential energy function:

$$F(r) = -\frac{dU(r)}{dr}$$

$$= -\frac{d}{dr}\left[-U_0\left(\frac{a}{r}\right)e^{-r/a}\right]$$

$$= \boxed{-U_0 e^{-r/a}\left(\frac{a}{r^2} + \frac{1}{r}\right)}$$

(*c*) Evaluate *F*(2*a*):

$$F(2a) = -U_0 e^{-2a/a}\left(\frac{a}{(2a)^2} + \frac{1}{2a}\right)$$

$$= -U_0 e^{-2}\left(\frac{3}{4a}\right)$$

Evaluate $F(a)$:

$$F(a) = -U_0 e^{-a/a}\left(\frac{a}{(a)^2} + \frac{1}{a}\right)$$
$$= -U_0 e^{-1}\left(\frac{1}{a} + \frac{1}{a}\right) = -U_0 e^{-1}\left(\frac{2}{a}\right)$$

Express the ratio $F(2a)/F(a)$:

$$\frac{F(2a)}{F(a)} = \frac{-U_0 e^{-2}\left(\frac{3}{4a}\right)}{-U_0 e^{-1}\left(\frac{2}{a}\right)} = \frac{3}{8}e^{-1}$$
$$= \boxed{0.138}$$

(*d*) Evaluate $F(5a)$:

$$F(5a) = -U_0 e^{-5a/a}\left(\frac{a}{(5a)^2} + \frac{1}{5a}\right)$$
$$= -U_0 e^{-5}\left(\frac{6}{25a}\right)$$

Express the ratio $F(5a)/F(a)$:

$$\frac{F(5a)}{F(a)} = \frac{-U_0 e^{-5}\left(\frac{6}{25a}\right)}{-U_0 e^{-1}\left(\frac{2}{a}\right)} = \frac{3}{25}e^{-4}$$
$$= \boxed{2.20\times10^{-3}}$$