Chapter 7
Conservation of Energy

Conceptual Problems

***1** •

Determine the Concept Because the peg is frictionless, mechanical energy is conserved as this system evolves from one state to another. The system moves and so we know that $\Delta K > 0$. Because $\Delta K + \Delta U = constant$, $\Delta U < 0$. $\boxed{(a)\text{ is correct.}}$

2 •

Determine the Concept Choose the zero of gravitational potential energy to be at ground level. The two stones have the same initial energy because they are thrown from the same height with the same initial speeds. Therefore, they will have the same total energy at all times during their fall. When they strike the ground, their gravitational potential energies will be zero and their kinetic energies will be equal. Thus, their speeds at impact will be equal. The stone that is thrown at an angle of 30° above the horizontal has a longer flight time due to its initial upward velocity and so they do not strike the ground at the same time. $\boxed{(c)\text{ is correct.}}$

3 •

(*a*) False. Forces that are external to a system can do work on the system to change its energy.

(*b*) False. In order for some object to do work, it must exert a force *over some distance*. The chemical energy stored in the muscles of your legs allows your muscles to do the work that launches you into the air.

4 •

Determine the Concept Your kinetic energy increases at the expense of chemical energy.

***5** •

Determine the Concept As she starts pedaling, chemical energy inside her body is converted into kinetic energy as the bike picks up speed. As she rides it up the hill, chemical energy is converted into gravitational potential and thermal energy. While freewheeling down the hill, potential energy is converted to kinetic energy, and while braking to a stop, kinetic energy is converted into thermal energy (a more random form of kinetic energy) by the frictional forces acting on the bike.

***6** •

Determine the Concept If we define the system to include the falling body and the earth, then no work is done by an external agent and $\Delta K + \Delta U_g + \Delta E_{therm} = 0$. Solving for the change in the gravitational potential energy we find $\Delta U_g = -(\Delta K + \text{friction energy})$.

$\boxed{(b)\text{ is correct.}}$

7 ••

Picture the Problem Because the constant friction force is responsible for a constant acceleration, we can apply the constant-acceleration equations to the analysis of these statements. We can also apply the work-energy theorem with friction to obtain expressions for the kinetic energy of the car and the rate at which it is changing. Choose the system to include the earth and car and assume that the car is moving on a horizontal surface so that $\Delta U = 0$.

(*a*) A constant frictional force causes a constant acceleration. The stopping distance of the car is related to its speed before the brakes were applied through a constant-acceleration equation.

$$v^2 = v_0^2 + 2a\Delta s \text{ where } v = 0.$$

$$\therefore \Delta s = \frac{-v_0^2}{2a} \text{ where } a < 0.$$

Thus, $\Delta s \propto v_0^2$ and statement (*a*) is *false*.

(*b*) Apply the work-energy theorem with friction to obtain:

$$\Delta K = -W_{\mathrm{f}} = -\mu_{\mathrm{k}} mg\Delta s$$

Express the rate at which K is dissipated:

$$\frac{\Delta K}{\Delta t} = -\mu_{\mathrm{k}} mg \frac{\Delta s}{\Delta t}$$

Thus, $\dfrac{\Delta K}{\Delta t} \propto v$ and therefore not constant.

Statement (*b*) is *false*.

(*c*) In part (*b*) we saw that: $K \propto \Delta s$

Because $\Delta s \propto \Delta t$: $K \propto \Delta t$ and statement (*c*) is *false*.

Because none of the above are correct: $\boxed{(d)\text{ is correct.}}$

8 •

Picture the Problem We'll let the zero of potential energy be at the bottom of each ramp and the mass of the block be m. We can use conservation of energy to predict the speed of the block at the foot of each ramp. We'll consider the distance the block travels on each ramp, as well as its speed at the foot of the ramp, in deciding its descent times.

Use conservation of energy to find the speed of the blocks at the bottom of each ramp:

$$\Delta K + \Delta U = 0$$

or

$$K_{\text{bot}} - K_{\text{top}} + U_{\text{bot}} - U_{\text{top}} = 0$$

Because $K_{top} = U_{bot} = 0$:

$$K_{bot} - U_{top} = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_{bot}^2 - mgH = 0$$

Solve for v_{bot}:

$v_{bot} = \sqrt{2gH}$ independently of the shape of the ramp.

Because the block sliding down the circular arc travels a greater distance (an arc length is greater than the length of the chord it defines) but arrives at the bottom of the ramp with the same speed that it had at the bottom of the inclined plane, it will require more time to arrive at the bottom of the arc. $\boxed{(b)\text{ is correct.}}$

9 ••

Determine the Concept No. From the work-kinetic energy theorem, no total work is being done on the rock, as its kinetic energy is constant. However, the rod must exert a tangential force on the rock to keep the speed constant. The effect of this force is to cancel the component of the force of gravity that is tangential to the trajectory of the rock.

Estimation and Approximation

***10** ••

Picture the Problem We'll use the data for the "typical male" described above and assume that he spends 8 hours per day sleeping, 2 hours walking, 8 hours sitting, 1 hour in aerobic exercise, and 5 hours doing moderate physical activity. We can approximate his energy utilization using $E_{activity} = AP_{activity}\Delta t_{activity}$, where A is the surface area of his body, $P_{activity}$ is the rate of energy consumption in a given activity, and $\Delta t_{activity}$ is the time spent in the given activity. His total energy consumption will be the sum of the five terms corresponding to his daily activities.

(*a*) Express the energy consumption of the hypothetical male:

$$E = E_{sleeping} + E_{walking} + E_{sitting} + E_{mod.\,act.} + E_{aerobic\,act.}$$

Evaluate $E_{sleeping}$:

$$\begin{aligned} E_{sleeping} &= AP_{sleeping}\Delta t_{sleeping} \\ &= \left(2\,\text{m}^2\right)\left(40\,\text{W/m}^2\right)\left(8\,\text{h}\right)\left(3600\,\text{s/h}\right) \\ &= 2.30\times10^6\,\text{J} \end{aligned}$$

Evaluate $E_{walking}$:

$$\begin{aligned} E_{walking} &= AP_{walking}\Delta t_{walking} \\ &= \left(2\,\text{m}^2\right)\left(160\,\text{W/m}^2\right)\left(2\,\text{h}\right)\left(3600\,\text{s/h}\right) \\ &= 2.30\times10^6\,\text{J} \end{aligned}$$

Evaluate E_{sitting}:

$$E_{\text{sitting}} = AP_{\text{sitting}}\Delta t_{\text{sitting}} = (2\,\text{m}^2)(60\,\text{W/m}^2)(8\,\text{h})(3600\,\text{s/h}) = 3.46\times10^6\ \text{J}$$

Evaluate $E_{\text{mod. act.}}$:

$$E_{\text{mod. act.}} = AP_{\text{mod. act.}}\Delta t_{\text{mod. act.}} = (2\,\text{m}^2)(175\,\text{W/m}^2)(5\,\text{h})(3600\,\text{s/h}) = 6.30\times10^6\ \text{J}$$

Evaluate $E_{\text{aerobic act.}}$:

$$E_{\text{aerobic act.}} = AP_{\text{aerobic act.}}\Delta t_{\text{aerobic act.}} = (2\,\text{m}^2)(300\,\text{W/m}^2)(1\,\text{h})(3600\,\text{s/h}) = 2.16\times10^6\ \text{J}$$

Substitute to obtain:

$$E = 2.30\times10^6\ \text{J} + 2.30\times10^6\ \text{J} + 3.46\times10^6\ \text{J} + 6.30\times10^6\ \text{J} + 2.16\times10^6\ \text{J} = \boxed{16.5\times10^6\ \text{J}}$$

Express the average metabolic rate represented by this energy consumption:

$$P_{\text{av}} = \frac{E}{\Delta t} = \frac{16.5\times10^6\ \text{J}}{(24\,\text{h})(3600\,\text{s/h})} = \boxed{191\,\text{W}}$$

or about twice that of a 100 W light bulb.

(*b*) Express his average energy consumption in terms of kcal/day:

$$E = \frac{16.5\times10^6\ \text{J/day}}{4190\,\text{J/kcal}} = \boxed{3940\,\text{kcal/day}}$$

(*c*)

$\frac{3940\,\text{kcal}}{175\,\text{lb}} = 22.5\,\text{kcal/lb}$ is higher than the estimate given in the statement of the problem. However, by adjusting the day's activities, the metabolic rate can vary by more than a factor of 2.

11 •

Picture the Problem The rate at which you expend energy, i.e., do work, is defined as *power* and is the ratio of the work done to the time required to do the work.

Relate the rate at which you can expend energy to the work done in running up the four flights of stairs and solve for your running time:

$$P = \frac{\Delta W}{\Delta t} \Rightarrow \Delta t = \frac{\Delta W}{P}$$

Express the work done in climbing the stairs:

$$\Delta W = mgh$$

Substitute for ΔW to obtain:

$$\Delta t = \frac{mgh}{P}$$

Assuming that your weight is 600 N, evaluate Δt:

$$\Delta t = \frac{(600\,\text{N})(4\times 3.5\,\text{m})}{250\,\text{W}} = \boxed{33.6\,\text{s}}$$

12 •

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

(*a*) Relate the rest mass consumed to the energy produced and solve for and evaluate m:

$$E_0 = mc^2 \Rightarrow m = \frac{E_0}{c^2} \qquad (1)$$

$$m = \frac{1\,\text{J}}{(2.998\times 10^8\,\text{m/s})^2} = \boxed{1.11\times 10^{-17}\,\text{kg}}$$

(*b*) Express the energy required as a function of the power of the light bulb and evaluate E:

$$\begin{aligned} E &= 3Pt \\ &= 3(100\,\text{W})(10\,\text{y}) \\ &\quad \times \left(\frac{365.24\,\text{d}}{\text{y}}\right)\left(\frac{24\,\text{h}}{\text{d}}\right)\left(\frac{3600\,\text{s}}{\text{h}}\right) \\ &= 9.47\times 10^{10}\,\text{J} \end{aligned}$$

Substitute in equation (1) to obtain:

$$m = \frac{9.47\times 10^{10}\,\text{J}}{(2.998\times 10^8\,\text{m/s})^2} = \boxed{1.05\,\mu\text{g}}$$

***13** •

Picture the Problem There are about 3×10^8 people in the United States. On the assumption that the average family has 4 people in it and that they own two cars, we have a total of 1.5×10^8 automobiles on the road (excluding those used for industry). We'll assume that each car uses about 15 gal of fuel per week.

Calculate, based on the assumptions identified above, the total annual consumption of energy derived from gasoline:

$$(1.5\times 10^8\,\text{auto})\left(15\frac{\text{gal}}{\text{auto}\cdot\text{week}}\right)\left(52\frac{\text{weeks}}{y}\right)\left(2.6\times 10^8\,\frac{\text{J}}{\text{gal}}\right) = \boxed{3.04\times 10^{19}\,\text{J/y}}$$

Express this rate of energy use as a fraction of the total annual energy use by the US:

$$\frac{3.04\times 10^{19}\,\text{J/y}}{5\times 10^{20}\,\text{J/y}} \approx \boxed{6\%}$$

Remarks: This is an average power expenditure of roughly 9x10^{11} watt, and a total cost (assuming \$1.15 per gallon) of about 140 billion dollars per year.

14 •

Picture the Problem The energy consumption of the U.S. works out to an average power consumption of about 1.6×10^{13} watt. The solar constant is roughly 10^3 W/m^2 (reaching the ground), or about 120 W/m^2 of useful power with a 12% conversion efficiency. Letting P represent the daily rate of energy consumption, we can relate the power available at the surface of the earth to the required area of the solar panels using $P = IA$.

Relate the required area to the electrical energy to be generated by the solar panels:

$$P = IA$$

where I is the solar intensity that reaches the surface of the Earth.

Solve for and evaluate A:

$$A = \frac{P}{I} = \frac{2\left(1.6\times10^{13}\ \text{W}\right)}{120\ \text{W/m}^2}$$
$$= 2.67\times10^{11}\ \text{m}^2$$

where the factor of 2 comes from the fact that the sun is only up for roughly half the day.

Find the side of a square with this area:

$$s = \sqrt{2.67\times10^{11}\ \text{m}^2} = \boxed{516\,\text{km}}$$

Remarks: A more realistic estimate that would include the variation of sunlight over the day and account for latitude and weather variations might very well increase the area required by an order of magnitude.

15 •

Picture the Problem We can relate the energy available from the water in terms of its mass, the vertical distance it has fallen, and the efficiency of the process. Differentiation of this expression with respect to time will yield the rate at which water must pass through its turbines to generate Hoover Dam's annual energy output.

Assuming a total efficiency η, use the definition of gravitational potential energy to express the energy available from the water when it has fallen a distance h:

$$E = \eta mgh$$

Differentiate this expression with respect to time to obtain:

$$P = \frac{d}{dt}\left[\eta mgh\right] = \eta gh\frac{dm}{dt} = \eta\rho gh\frac{dV}{dt}$$

Solve for dV/dt:

$$\frac{dV}{dt} = \frac{P}{\eta\rho gh} \qquad (1)$$

Using its definition, relate the dam's annual power output to the energy produced:

$$P = \frac{\Delta E}{\Delta t}$$

Substitute numerical values to obtain:

$$P=\frac{4\times10^{9}\ \text{kW}\cdot\text{h}}{(365.24\,\text{d})(24\,\text{h/d})}=4.57\times10^{8}\ \text{W}$$

Substitute in equation (1) and evaluate dV/dt:

$$\frac{dV}{dt}=\frac{4.57\times10^{8}\ \text{W}}{0.2(1\,\text{kg/L})(9.81\,\text{m/s}^2)(211\,\text{m})}=\boxed{1.10\times10^{6}\ \text{L/s}}$$

The Conservation of Mechanical Energy

16 •

Picture the Problem The work done in compressing the spring is stored in the spring as potential energy. When the block is released, the energy stored in the spring is transformed into the kinetic energy of the block. Equating these energies will give us a relationship between the compressions of the spring and the speeds of the blocks.

Let the numeral 1 refer to the first case and the numeral 2 to the second case. Relate the compression of the spring in the second case to its potential energy, which equals its initial kinetic energy when released:

$$\tfrac{1}{2}kx_2^2=\tfrac{1}{2}m_2v_2^2=\tfrac{1}{2}(4m_1)(3v_1)^2=18m_1v_1^2$$

Relate the compression of the spring in the first case to its potential energy, which equals its initial kinetic energy when released:

$$\tfrac{1}{2}kx_1^2=\tfrac{1}{2}m_1v_1^2$$

or

$$m_1v_1^2=kx_1^2$$

Substitute to obtain:

$$\tfrac{1}{2}kx_2^2=18kx_1^2$$

Solve for x_2:

$$x_2=\boxed{6x_1}$$

17 •

Picture the Problem Choose the zero of gravitational potential energy to be at the foot of the hill. Then the kinetic energy of the woman on her bicycle at the foot of the hill is equal to her gravitational potential energy when she has reached her highest point on the hill.

Equate the kinetic energy of the rider at the foot of the incline and her gravitational potential energy when she has reached her highest

$$\tfrac{1}{2}mv^2=mgh\Rightarrow h=\frac{v^2}{2g}$$

point on the hill and solve for h:

Relate her displacement along the incline d to h and the angle of the incline:

$d = h/\sin\theta$

Substitute for h to obtain:

$$d\sin\theta = \frac{v^2}{2g}$$

Solve for d:

$$d = \frac{v^2}{2g\sin\theta}$$

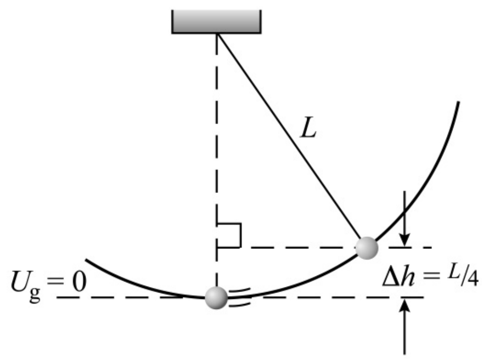
Substitute numerical values and evaluate d:

$$d = \frac{(10\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)\sin 3°} = 97.4\,\text{m}$$

and $\boxed{(c)\text{ is correct.}}$

***18 •**

Picture the Problem The diagram shows the pendulum bob in its initial position. Let the zero of gravitational potential energy be at the low point of the pendulum's swing, the equilibrium position. We can find the speed of the bob at it passes through the equilibrium position by equating its initial potential energy to its kinetic energy as it passes through its lowest point.



Equate the initial gravitational potential energy and the kinetic energy of the bob as it passes through its lowest point and solve for v:

$$mg\Delta h = \tfrac{1}{2}mv^2$$

and

$$v = \sqrt{2g\Delta h}$$

Express Δh in terms of the length L of the pendulum:

$$\Delta h = \frac{L}{4}$$

Substitute and simplify:

$$v = \boxed{\sqrt{\frac{gL}{2}}}$$

19 •

Picture the Problem Choose the zero of gravitational potential energy to be at the foot of the ramp. Let the system consist of the block, the earth, and the ramp. Then there are no external forces acting on the system to change its energy and the kinetic energy of the block at the foot of the ramp is equal to its gravitational potential energy when it has reached its highest point.

Relate the gravitational potential energy of the block when it has reached *h*, its highest point on the ramp, to its kinetic energy at the foot of the ramp:

$$mgh = \tfrac{1}{2}mv^2$$

Solve for *h*:

$$h = \frac{v^2}{2g}$$

Relate the displacement *d* of the block along the ramp to *h* and the angle the ramp makes with the horizontal:

$$d = h/\sin\theta$$

Substitute for *h*:

$$d\sin\theta = \frac{v^2}{2g}$$

Solve for *d*:

$$d = \frac{v^2}{2g\sin\theta}$$

Substitute numerical values and evaluate *d*:

$$d = \frac{(7\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)\sin 40^\circ} = \boxed{3.89\,\text{m}}$$

20 •

Picture the Problem Let the system consist of the earth, the block, and the spring. With this choice there are no external forces doing work to change the energy of the system. Let $U_g = 0$ at the elevation of the spring. Then the initial gravitational potential energy of the 3-kg object is transformed into kinetic energy as it slides down the ramp and then, as it compresses the spring, into potential energy stored in the spring.

(*a*) Apply conservation of energy to relate the distance the spring is compressed to the initial potential energy of the block:

$$W_{ext} = \Delta K + \Delta U = 0$$

and, because $\Delta K = 0$,

$$-mgh + \tfrac{1}{2}kx^2 = 0$$

Solve for *x*:

$$x = \sqrt{\frac{2mgh}{k}}$$

Substitute numerical values and evaluate *x*:

$$x = \sqrt{\frac{2(3\,\text{kg})(9.81\,\text{m/s}^2)(5\,\text{m})}{400\,\text{N/m}}} = \boxed{0.858\,\text{m}}$$

(*b*) The energy stored in the compressed spring will accelerate the block, launching it back up the incline:

$$\boxed{\text{The block will retrace its path, rising to a height of } 5\,\text{m.}}$$

21 •

Picture the Problem With U_g chosen to be zero at the uncompressed level of the spring, the ball's initial gravitational potential energy is negative. The difference between the initial potential energy of the spring and the gravitational potential energy of the ball is first converted into the kinetic energy of the ball and then into gravitational potential energy as the ball rises and slows … eventually coming momentarily to rest.

Apply the conservation of energy to the system as it evolves from its initial to its final state:

$$-mgx + \tfrac{1}{2}kx^2 = mgh$$

Solve for *h*:

$$h = \frac{kx^2}{2mg} - x$$

Substitute numerical values and evaluate *h*:

$$h = \frac{(600\,\text{N/m})(0.05\,\text{m})^2}{2(0.015\,\text{kg})(9.81\,\text{m/s}^2)} - 0.05\,\text{m} = \boxed{5.05\,\text{m}}$$

22 •

Picture the Problem Let the system include the earth and the container. Then the work done by the crane is done by an external force and this work changes the energy of the system. Because the initial and final speeds of the container are zero, the initial and final kinetic energies are zero and the work done by the crane equals the change in the gravitational potential energy of the container. Choose $U_g = 0$ to be at the level of the deck of the freighter.

Apply conservation of energy to the system:

$$W_{\text{ext}} = \Delta E_{\text{sys}} = \Delta K + \Delta U$$

Because $\Delta K = 0$:

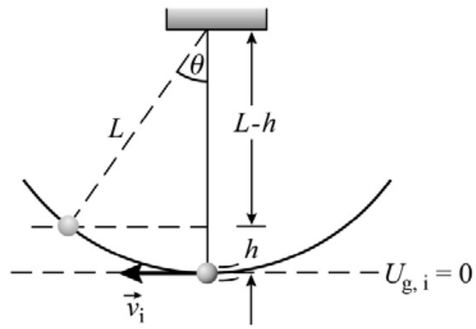
$$W_{ext} = \Delta U = mg\Delta h$$

Evaluate the work done by the crane:

$$W_{ext} = mg\Delta h = (4000\,\text{kg})(9.81\,\text{m/s}^2)(-8\,\text{m}) = \boxed{-314\,\text{kJ}}$$

23 •

Picture the Problem Let the system consist of the earth and the child. Then $W_{ext} = 0$. Choose $U_{g,i} = 0$ at the child's lowest point as shown in the diagram to the right. Then the child's initial energy is entirely kinetic and its energy when it is at its highest point is entirely gravitational potential. We can determine h from energy conservation and then use trigonometry to determine θ.



Using the diagram, relate θ to h and L:

$$\theta = \cos^{-1}\frac{L-h}{L} = \cos^{-1}\left(1-\frac{h}{L}\right)$$

Apply conservation of energy to the system to obtain:

$$\tfrac{1}{2}mv_i^2 - mgh = 0$$

Solve for h:

$$h = \frac{v_i^2}{2g}$$

Substitute to obtain:

$$\theta = \cos^{-1}\left(1-\frac{v_i^2}{2gL}\right)$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left(1-\frac{(3.4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(6\,\text{m})}\right) = \boxed{25.6°}$$

***24** ••

Picture the Problem Let the system include the two objects and the earth. Then $W_{ext} = 0$. Choose $U_g = 0$ at the elevation at which the two objects meet. With this choice, the initial potential energy of the 3-kg object is positive and that of the 2-kg object is negative. Their sum, however, is positive. Given our choice for $U_g = 0$, this initial potential energy is transformed entirely into kinetic energy.

Apply conservation of energy:

$$W_{ext} = \Delta K + \Delta U_g = 0$$

or, because $W_{ext} = 0$,

$$\Delta K = -\Delta U_g$$

Substitute for ΔK and solve for v_f; noting that m represents the sum of the masses of the objects as they are both moving in the final state:

$$\tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = -\Delta U_g$$

or, because $v_i = 0$,

$$v_f = \sqrt{\frac{-2\Delta U_g}{m}}$$

Express and evaluate ΔU_g:

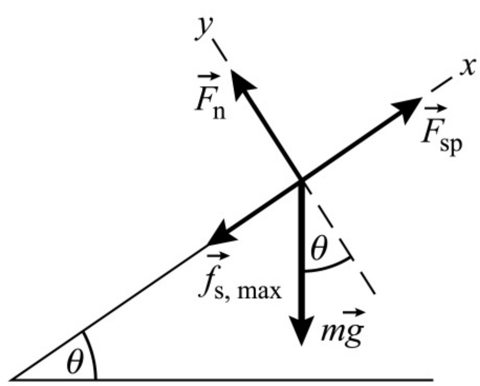
$$\begin{aligned}\Delta U_g &= U_{g,f} - U_{g,i} \\ &= 0 - (3\,\text{kg} - 2\,\text{kg})(0.5\,\text{m}) \\ &\quad \times (9.81\,\text{m/s}^2) \\ &= -4.91\,\text{J}\end{aligned}$$

Substitute and evaluate v_f:

$$v_f = \sqrt{\frac{-2(-4.91\,\text{J})}{5\,\text{kg}}} = \boxed{1.40\,\text{m/s}}$$

25 ••

Picture the Problem The free-body diagram shows the forces acting on the block when it is about to move. F_{sp} is the force exerted by the spring and, because the block is on the verge of sliding, $f_s = f_{s,max}$. We can use Newton's 2nd law, under equilibrium conditions, to express the elongation of the spring as a function of m, k and θ and then substitute in the expression for the potential energy stored in a stretched or compressed spring.



Express the potential energy of the spring when the block is about to move:

$$U = \tfrac{1}{2}kx^2$$

Apply $\sum \vec{F} = m\vec{a}$, under equilibrium conditions, to the block:

$$\sum F_x = F_{sp} - f_{s,max} - mg\sin\theta = 0$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Using $f_{s,max} = \mu_s F_n$ and $F_{sp} = kx$, eliminate $f_{s,max}$ and F_{sp} from the x equation and solve for x:

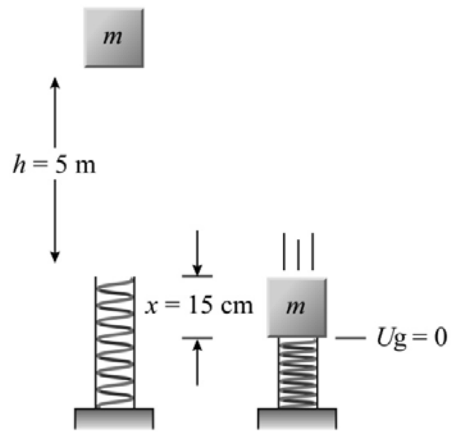
$$x = \frac{mg(\sin\theta + \mu_s \cos\theta)}{k}$$

Substitute for x in the expression for U:

$$U = \tfrac{1}{2}k\left[\frac{mg(\sin\theta + \mu_s \cos\theta)}{k}\right]^2 = \boxed{\frac{[mg(\sin\theta + \mu_s \cos\theta)]^2}{2k}}$$

26 ••

Picture the Problem The mechanical energy of the system, consisting of the block, the spring, and the earth, is initially entirely gravitational potential energy. Let $U_g = 0$ where the spring is compressed 15 cm. Then the mechanical energy when the compression of the spring is 15 cm will be partially kinetic and partially stored in the spring. We can use conservation of energy to relate the initial potential energy of the system to the energy stored in the spring and the kinetic energy of block when it has compressed the spring 15 cm.



Apply conservation of energy to the system:

$$\Delta U + \Delta K = 0$$

or

$$U_{g,f} - U_{g,i} + U_{s,f} - U_{s,i} + K_f - K_i = 0$$

Because $U_{g,f} = U_{s,I} = K_i = 0$:

$$-U_{g,i} + U_{s,f} + K_f = 0$$

Substitute to obtain:

$$-mg(h + x) + \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2 = 0$$

Solve for v:

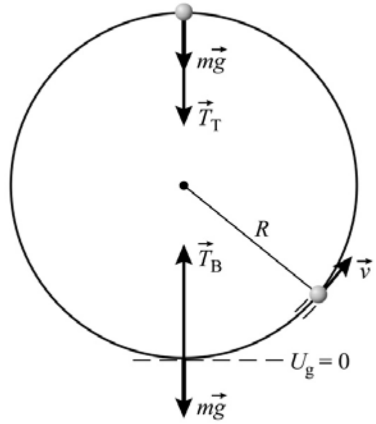
$$v = \sqrt{2g(h + x) - \frac{kx^2}{m}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{2(9.81\,\text{m/s}^2)(5\,\text{m} + 0.15\,\text{m}) - \frac{(3955\,\text{N/m})(0.15\,\text{m})^2}{2.4\,\text{kg}}} = \boxed{8.00\,\text{m/s}}$$

***27 ••**

Picture the Problem The diagram represents the ball traveling in a circular path with constant energy. U_g has been chosen to be zero at the lowest point on the circle and the superimposed free-body diagrams show the forces acting on the ball at the top and bottom of the circular path. We'll apply Newton's 2nd law to the ball at the top and bottom of its path to obtain a relationship between T_T and T_B and the conservation of mechanical energy to relate the speeds of the ball at these two locations.



Apply $\sum F_{radial} = ma_{radial}$ to the ball at the bottom of the circle and solve for T_B:

$$T_B - mg = m\frac{v_B^2}{R}$$

and

$$T_B = mg + m\frac{v_B^2}{R} \qquad (1)$$

Apply $\sum F_{radial} = ma_{radial}$ to the ball at the top of the circle and solve for T_T:

$$T_T + mg = m\frac{v_T^2}{R}$$

and

$$T_T = -mg + m\frac{v_T^2}{R} \qquad (2)$$

Subtract equation (2) from equation (1) to obtain:

$$T_B - T_T = mg + m\frac{v_B^2}{R} - \left(-mg + m\frac{v_T^2}{R}\right) = m\frac{v_B^2}{R} - m\frac{v_T^2}{R} + 2mg \qquad (3)$$

Using conservation of energy, relate the mechanical energy of the ball at the bottom of its path to its mechanical energy at the top of the circle and solve for $m\frac{v_B^2}{R} - m\frac{v_T^2}{R}$:

$$\tfrac{1}{2}mv_B^2 = \tfrac{1}{2}mv_T^2 + mg(2R)$$

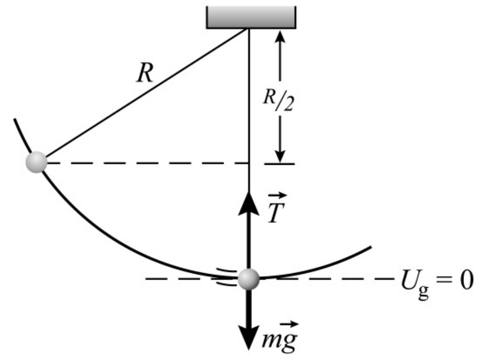
$$m\frac{v_B^2}{R} - m\frac{v_T^2}{R} = 4mg$$

Substitute in equation (3) to obtain:

$$T_B - T_T = \boxed{6mg}$$

28 ••

Picture the Problem Let $U_g = 0$ at the lowest point in the girl's swing. Then we can equate her initial potential energy to her kinetic energy as she passes through the low point on her swing to relate her speed v to R. The FBD show the forces acting on the girl at the low point of her swing. Applying Newton's 2nd law to her will allow us to establish the relationship between the tension T and her speed.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the girl at her lowest point and solve for T:

$$T - mg = m\frac{v^2}{R}$$

and

$$T = mg + m\frac{v^2}{R}$$

Equate the girl's initial potential energy to her final kinetic energy and solve for $\frac{v^2}{R}$:

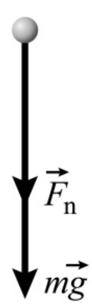
$$mg\frac{R}{2} = \tfrac{1}{2}mv^2 \Rightarrow \frac{v^2}{R} = g$$

Substitute for v^2/R^2 and simplify to obtain:

$$T = mg + mg = \boxed{2mg}$$

29 ••

Picture the Problem The free-body diagram shows the forces acting on the car when it is upside down at the top of the loop. Choose $U_g = 0$ at the bottom of the loop. We can express F_n in terms of v and R by apply Newton's 2nd law to the car and then obtain a second expression in these same variables by applying the conservation of mechanical energy. The simultaneous solution of these equations will yield an expression for F_n in terms of known quantities.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the car at the top of the circle and solve for F_n:

$$F_n + mg = m\frac{v^2}{R}$$

and

$$F_n = m\frac{v^2}{R} - mg \qquad (1)$$

Using conservation of energy, relate the energy of the car at the beginning of its motion to its energy when it is at the top of the loop:

$$mgH = \tfrac{1}{2}mv^2 + mg(2R)$$

Solve for $m\dfrac{v^2}{R}$:

$$m\frac{v^2}{R} = 2mg\left(\frac{H}{R} - 2\right) \qquad (2)$$

Substitute equation (2) in equation (1) to obtain:

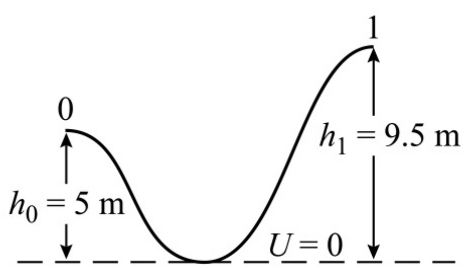
$$F_\text{n} = 2mg\left(\frac{H}{R} - 2\right) - mg$$
$$= mg\left(\frac{2H}{R} - 5\right)$$

Substitute numerical values and evaluate F_n:

$$F_\text{n} = (1500\,\text{kg})(9.81\,\text{m/s}^2)\left[\frac{2(23\,\text{m})}{7.5\,\text{m}} - 5\right] = 1.67\times10^4\ \text{N} \text{ and } \boxed{(c)\text{ is correct.}}$$

30 •

Picture the Problem Let the system include the roller coaster, the track, and the earth and denote the starting position with the numeral 0 and the top of the second hill with the numeral 1. We can use the work-energy theorem to relate the energies of the coaster at its initial and final positions.



(*a*) Use conservation of energy to relate the work done by external forces to the change in the energy of the system:

$$W_\text{ext} = \Delta E_\text{sys} = \Delta K + \Delta U$$

Because the track is frictionless, $W_\text{ext} = 0$:

$$\Delta K + \Delta U = 0$$
and
$$K_1 - K_0 + U_1 - U_0 = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgh_1 - mgh_0 = 0$$

Solve for v_0:

$$v_0 = \sqrt{v_1^2 + 2g(h_1 - h_0)}$$

If the coaster just makes it to the top of the second hill, $v_1 = 0$ and:

$$v_0 = \sqrt{2g(h_1 - h_0)}$$

Substitute numerical values and evaluate v_0:

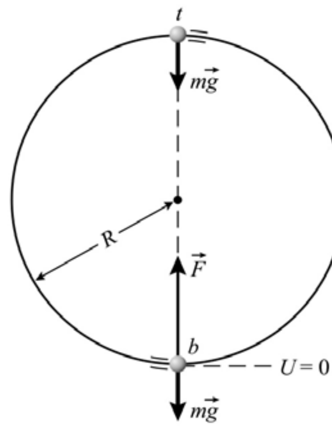
$$v_0 = \sqrt{2\left(9.81\,\text{m/s}^2\right)\left(9.5\,\text{m} - 5\,\text{m}\right)} = \boxed{9.40\,\text{m/s}}$$

(*b*)

No. Note that the required speed depends only on the difference in the heights of the two hills.

31 ••

Picture the Problem Let the radius of the loop be *R* and the mass of one of the riders be *m*. At the top of the loop, the centripetal force on her is her weight (the force of gravity). The two forces acting on her at the bottom of the loop are the normal force exerted by the seat of the car, pushing up, and the force of gravity, pulling down. We can apply Newton's 2nd law to her at both the top and bottom of the loop to relate the speeds at those locations to *m* and *R* and, at *b*, to *F*, and then use conservation of energy to relate v_t and v_b.



Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the rider at the bottom of the circular arc:

$$F - mg = m\frac{v_b^2}{R}$$

Solve for *F* to obtain:

$$F = mg + m\frac{v_b^2}{R} \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the rider at the top of the circular arc:

$$mg = m\frac{v_t^2}{R}$$

Solve for v_t^2:

$$v_t^2 = gR$$

Use conservation of energy to relate the energies of the rider at the top and bottom of the arc:

$$K_b - K_t + U_b - U_t = 0$$

or, because $U_b = 0$,

$$K_b - K_t - U_t = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_b^2 - \tfrac{1}{2}mv_t^2 - 2mgR = 0$$

Solve for v_b^2:

$$v_b^2 = 5gR$$

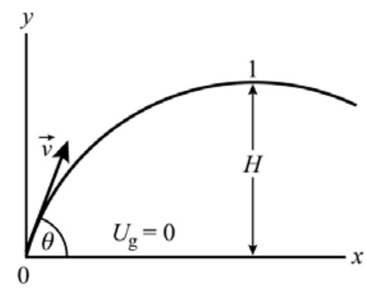
Substitute in equation (1) to obtain:

$$F = mg + m\frac{5gR}{R} = \boxed{6mg}$$

i.e., the rider will feel six times heavier than her normal weight.

***32** ••

Picture the Problem Let the system consist of the stone and the earth and ignore the influence of air resistance. Then $W_{ext} = 0$. Choose $U_g = 0$ as shown in the figure. Apply the law of the conservation of mechanical energy to describe the energy transformations as the stone rises to the highest point of its trajectory.



Apply conservation of energy:

$$W_{ext} = \Delta K + \Delta U = 0$$

and

$$K_1 - K_0 + U_1 - U_0 = 0$$

Because $U_0 = 0$:

$$K_1 - K_0 + U_1 = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_x^2 - \tfrac{1}{2}mv^2 + mgH = 0$$

In the absence of air resistance, the horizontal component of $\vec{v}$ is constant and equal to $v_x = v\cos\theta$. Hence:

$$\tfrac{1}{2}m(v\cos\theta)^2 - \tfrac{1}{2}mv^2 + mgH = 0$$

Solve for v:

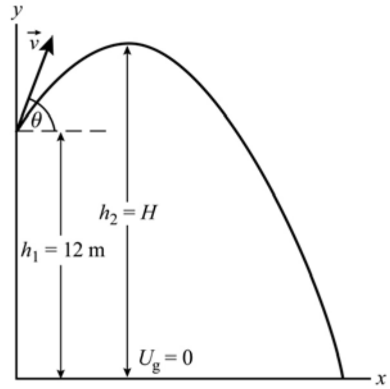
$$v = \sqrt{\frac{2gH}{1-\cos^2\theta}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(9.81\,\text{m/s}^2)(24\,\text{m})}{1-\cos^2 53°}} = \boxed{27.2\,\text{m/s}}$$

33 ••

Picture the Problem Let the system consist of the ball and the earth. Then $W_{ext} = 0$. The figure shows the ball being thrown from the roof of a building. Choose $U_g = 0$ at ground level. We can use the conservation of mechanical energy to determine the maximum height of the ball and its speed at impact with the ground. We can use the definition of the work done by gravity to calculate how much work was done by gravity as the ball rose to its maximum height.



(*a*) Apply conservation of energy:

$$W_{ext} = \Delta K + \Delta U = 0$$

or

$$K_2 - K_1 + U_2 - U_1 = 0$$

Substitute for the energies to obtain:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgh_2 - mgh_1 = 0$$

Note that, at point 2, the ball is moving horizontally and:

$$v_2 = v_1 \cos\theta$$

Substitute for v_2 and h_2:

$$\tfrac{1}{2}m(v_1\cos\theta)^2 - \tfrac{1}{2}mv_1^2 + mgH - mgh_1 = 0$$

Solve for *H*:

$$H = h_1 - \frac{v^2}{2g}\left(\cos^2\theta - 1\right)$$

Substitute numerical values and evaluate *H*:

$$H = 12\,\text{m} - \frac{(30\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)}\left(\cos^2 40° - 1\right) = \boxed{31.0\,\text{m}}$$

(*b*) Using its definition, express the work done by gravity:

$$W_g = -\Delta U = -\left(U_H - U_{h_i}\right) = -\left(mgH - mgh_i\right) = -mg\left(H - h_i\right)$$

Substitute numerical values and evaluate W_g:

$$W_g = -(0.17\,\text{kg})(9.81\,\text{m/s}^2)(31\,\text{m} - 12\,\text{m}) = \boxed{-31.7\,\text{J}}$$

(*c*) Relate the initial mechanical

$$\tfrac{1}{2}mv_i^2 + mgh_i = \tfrac{1}{2}mv_f^2$$

energy of the ball to its just-before-impact energy:

Solve for v_f:

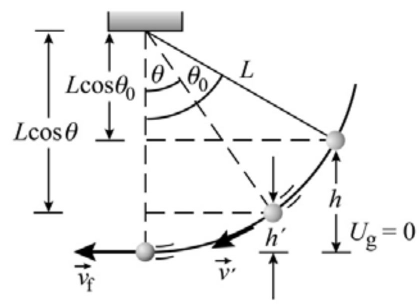
$$v_f = \sqrt{v_i^2 + 2gh_i}$$

Substitute numerical values and evaluate v_f

$$v_f = \sqrt{(30\,\text{m/s})^2 + 2(9.81\,\text{m/s}^2)(12\,\text{m})} = \boxed{33.7\,\text{m/s}}$$

34 ••

Picture the Problem The figure shows the pendulum bob in its release position and in the two positions in which it is in motion with the given speeds. Choose $U_g = 0$ at the low point of the swing. We can apply the conservation of mechanical energy to relate the two angles of interest to the speeds of the bob at the intermediate and low points of its trajectory.



(*a*) Apply conservation of energy:

$$W_{ext} = \Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

where U_f and K_i equal zero.

$$\therefore K_f - U_i = 0$$

Express U_i:

$$U_i = mgh = mgL(1 - \cos\theta_0)$$

Substitute for K_f and U_i:

$$\tfrac{1}{2}mv_f^2 - mgL(1 - \cos\theta_0) = 0$$

Solve for θ_0:

$$\theta_0 = \cos^{-1}\left(1 - \frac{v^2}{2gL}\right)$$

Substitute numerical values and evaluate θ_0:

$$\theta_0 = \cos^{-1}\left[1 - \frac{(2.8\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(0.8\,\text{m})}\right] = \boxed{60.0^\circ}$$

(*b*) Letting primed quantities describe the indicated location, use the law of the conservation of mechanical energy to relate the

$$K_f' - K_i + U_f' - U_i = 0$$

where $K_i = 0$.

$$\therefore K_f' + U_f' - U_i = 0$$

speed of the bob at this point to θ:

Express U_f':

$$U_f' = mgh' = mgL(1-\cos\theta)$$

Substitute for K_f', U_f' and U_i:

$$\tfrac{1}{2}m(v_f')^2 + mgL(1-\cos\theta) - mgL(1-\cos\theta_0) = 0$$

Solve for θ:

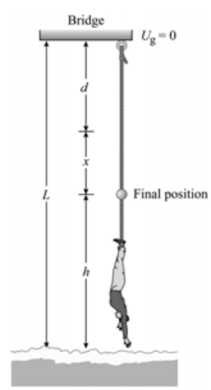
$$\theta = \cos^{-1}\left[\frac{(v_f')^2}{2gL} + \cos\theta_0\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \cos^{-1}\left[\frac{(1.4\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(0.8\,\text{m})} + \cos 60°\right] = \boxed{51.3°}$$

***35 ••**

Picture the Problem Choose $U_g = 0$ at the bridge, and let the system be the earth, the jumper and the bungee cord. Then $W_{ext} = 0$. Use the conservation of mechanical energy to relate to relate her initial and final gravitational potential energies to the energy stored in the stretched bungee, U_s cord. In part (*b*), we'll use a similar strategy but include a kinetic energy term because we are interested in finding her maximum speed.



(*a*) Express her final height h above the water in terms of L, d and the distance x the bungee cord has stretched:

$$h = L - d - x \qquad (1)$$

Use the conservation of mechanical energy to relate her gravitational potential energy as she just touches the water to the energy stored in the stretched bungee cord:

$$W_{ext} = \Delta K + \Delta U = 0$$

Because $\Delta K = 0$ and $\Delta U = \Delta U_g + \Delta U_s$,

$$-mgL + \tfrac{1}{2}kx^2 = 0,$$

where x is the maximum distance the bungee cord has stretched.

Solve for k:

$$k = \frac{2mgL}{x^2}$$

Find the maximum distance the bungee cord stretches:

x = 310 m – 50 m = 260 m.

Evaluate k:

$$k = \frac{2(60\,\text{kg})(9.81\,\text{m/s}^2)(310\,\text{m})}{(260\,\text{m})^2} = 5.40\,\text{N/m}$$

Express the relationship between the forces acting on her when she has finally come to rest and solve for x:

$$F_{\text{net}} = kx - mg = 0$$

and

$$x = \frac{mg}{k}$$

Evaluate x:

$$x = \frac{(60\,\text{kg})(9.81\,\text{m/s}^2)}{5.40\,\text{N/m}} = 109\,\text{m}$$

Substitute in equation (1) and evaluate h:

$$h = 310\,\text{m} - 50\,\text{m} - 109\,\text{m} = \boxed{151\,\text{m}}$$

(b) Using conservation of energy, express her total energy E:

$$E = K + U_{\text{g}} + U_{\text{s}} = E_{\text{i}} = 0$$

Because v is a maximum when K is a maximum, solve for K and set its derivative with respect to x equal to zero:

$$K = -U_{\text{g}} - U_{\text{s}} = mg(d + x) - \tfrac{1}{2}kx^2 \qquad (1)$$

$$\frac{dK}{dx} = mg - kx = 0 \text{ for extreme values}$$

Solve for and evaluate x:

$$x = \frac{mg}{k} = \frac{(60\,\text{kg})(9.81\,\text{m/s}^2)}{5.40\,\text{N/m}} = 109\,\text{m}$$

From equation (1) we have:

$$\tfrac{1}{2}mv^2 = mg(d + x) - \tfrac{1}{2}kx^2$$

Solve for v to obtain:

$$v = \sqrt{2g(d + x) - \frac{kx^2}{m}}$$

Substitute numerical values and evaluate v for x = 109 m:

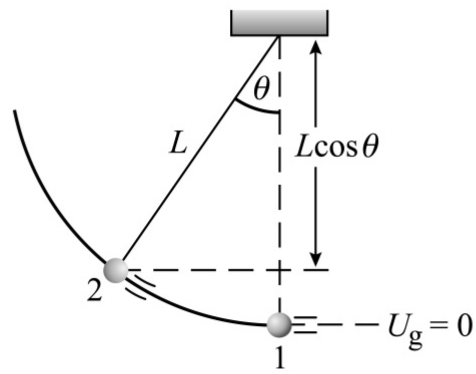
$$v = \sqrt{2(9.81\,\text{m/s}^2)(50\,\text{m} + 109\,\text{m}) - \frac{(5.4\,\text{N/m})(109\,\text{m})^2}{60\,\text{kg}}} = \boxed{45.3\,\text{m/s}}$$

Because $\frac{d^2K}{dx^2} = -k < 0$:

x = 109 m corresponds to K_{max} and so v is a maximum.

36 ••

Picture the Problem Let the system be the earth and pendulum bob. Then $W_{ext} = 0$. Choose $U_g = 0$ at the low point of the bob's swing and apply the law of the conservation of mechanical energy to its motion. When the bob reaches the 30° position its energy will be partially kinetic and partially potential. When it reaches its maximum height, its energy will be entirely potential. Applying Newton's 2nd law will allow us to express the tension in the string as a function of the bob's speed and its angular position.



(*a*) Apply conservation of energy to relate the energies of the bob at points 1 and 2:

$$W_{ext} = \Delta K + \Delta U = 0$$

or

$$K_2 - K_1 + U_2 - U_1 = 0$$

Because $U_1 = 0$,

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + U_2 = 0$$

Express U_2:

$$U_2 = mgL(1-\cos\theta)$$

Substitute for U_2 to obtain:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}mv_1^2 + mgL(1-\cos\theta) = 0$$

Solve for v_2:

$$v_2 = \sqrt{v_1^2 - 2gL(1-\cos\theta)}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{(4.5\,\text{m/s})^2 - 2(9.81\,\text{m/s}^2)(3\,\text{m})(1-\cos30°)} = \boxed{3.52\,\text{m/s}}$$

(*b*) Use the definition of gravitational potential energy to obtain:

$$U_2 = mgL(1-\cos\theta)$$

Substitute numerical values and evaluate U_2:

$$U_2 = (2\,\text{kg})(9.81\,\text{m/s}^2)(3\,\text{m})(1-\cos30°) = \boxed{7.89\,\text{J}}$$

(*c*) Apply $\sum F_{radial} = ma_{radial}$ to the bob to obtain:

$$T - mg\cos\theta = m\frac{v_2^2}{L}$$

Solve for T:

$$T = m\left(g\cos\theta + \frac{v_2^2}{L}\right)$$

Substitute numerical values and evaluate T:

$$T = (2\,\text{kg})\left[(9.81\,\text{m/s}^2)\cos 30° + \frac{(3.52\,\text{m/s})^2}{3\,\text{m}}\right] = \boxed{25.3\,\text{N}}$$

(d) When the bob reaches its greatest height:

$$U = U_{\max} = mgL(1-\cos\theta_{\max})$$

and

$$K_1 + U_{\max} = 0$$

Substitute for K_1 and $U_{\max}$

$$-\tfrac{1}{2}mv_1^2 + mgL(1-\cos\theta_{\max}) = 0$$

Solve for $\theta_{\max}$:

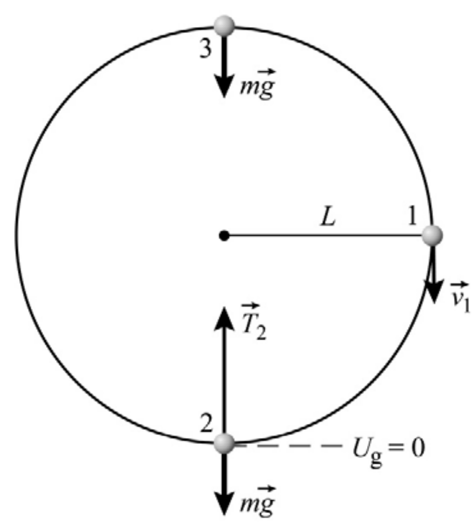
$$\theta_{\max} = \cos^{-1}\left(1 - \frac{v_1^2}{2gL}\right)$$

Substitute numerical values and evaluate $\theta_{\max}$:

$$\theta_{\max} = \cos^{-1}\left[1 - \frac{(4.5\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)(3\,\text{m})}\right] = \boxed{49.0°}$$

37 ••

Picture the Problem Let the system consist of the earth and pendulum bob. Then $W_{ext} = 0$. Choose $U_g = 0$ at the bottom of the circle and let points 1, 2 and 3 represent the bob's initial point, lowest point and highest point, respectively. The bob will gain speed and kinetic energy until it reaches point 2 and slow down until it reaches point 3; so it has its maximum kinetic energy when it is at point 2. We can use Newton's 2nd law at points 2 and 3 in conjunction with the law of the conservation of mechanical energy to find the maximum kinetic energy of the bob and the tension in the string when the bob has its maximum kinetic energy.



(a) Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ to the bob at the top of the circle and solve

$$mg = m\frac{v_3^2}{L}$$

for v_3^2:

and
$v_3^2 = gL$

Use conservation of energy to express the relationship between K_2, K_3 and U_3 and solve for K_2:

$K_3 - K_2 + U_3 - U_2 = 0$ where $U_2 = 0$

Therefore,

$$K_2 = K_{max} = K_3 + U_3 = \tfrac{1}{2}mv_3^2 + mg(2L)$$

Substitute for v_3^2 and simplify to obtain:

$$K_{max} = \tfrac{1}{2}m(gL) + 2mgL = \boxed{\tfrac{5}{2}mgL}$$

(*b*) Apply $\sum F_{radial} = ma_{radial}$ to the bob at the bottom of the circle and solve for T_2:

$$F_{net} = T_2 - mg = m\frac{v_2^2}{L}$$

and

$$T_2 = mg + m\frac{v_2^2}{L} \qquad (1)$$

Use conservation of energy to relate the energies of the bob at points 2 and 3 and solve for K_2:

$K_3 - K_2 + U_3 - U_2 = 0$ where $U_2 = 0$

$$K_2 = K_3 + U_3 = \tfrac{1}{2}mv_3^2 + mg(2L)$$

Substitute for v_3^2 and K_2 and solve for v_2^2:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}m(gL) + mg(2L)$$

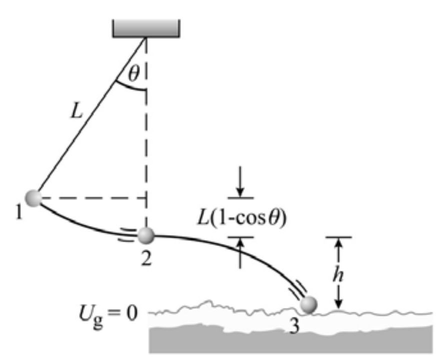
and
$v_2^2 = 5gL$

Substitute in equation (1) to obtain:

$$T_2 = \boxed{6mg}$$

38 ••

Picture the Problem Let the system consist of the earth and child. Then $W_{ext} = 0$. In the figure, the child's initial position is designated with the numeral 1; the point at which the child releases the rope and begins to fall with a 2, and its point of impact with the water is identified with a 3. Choose $U_g = 0$ at the water level. While one could use the law of the conservation of energy between points 1 and 2 and then between points 2 and 3, it is more direct to consider the energy transformations between points 1 and 3. Given our choice of the zero of



gravitational potential energy, the initial potential energy at point 1 is transformed into kinetic energy at point 3.

Apply conservation of energy to the energy transformations between points 1 and 3:

$$W_{ext} = \Delta K + \Delta U = 0$$
$$K_3 - K_1 + U_3 - U_1 = 0$$
where U_3 and K_1 are zero.

Substitute for K_3 and U_1;

$$\tfrac{1}{2}mv_3^2 - mg[h + L(1-\cos\theta)] = 0$$

Solve for v_3:

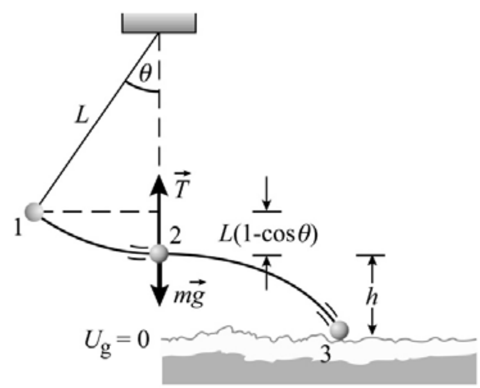
$$v_3 = \sqrt{2g[h + L(1-\cos\theta)]}$$

Substitute numerical values and evaluate v_3:

$$v_3 = \sqrt{2(9.81\,\text{m/s}^2)[3.2\,\text{m} + (10.6\,\text{m})(1-\cos 23°)]} = \boxed{8.91\,\text{m/s}}$$

***39 ••**

Picture the Problem Let the system consist of you and the earth. Then there are no external forces to do work on the system and $W_{ext} = 0$. In the figure, your initial position is designated with the numeral 1, the point at which you release the rope and begin to fall with a 2, and your point of impact with the water is identified with a 3. Choose $U_g = 0$ at the water level. We can apply Newton's 2nd law to the forces acting on you at point 2 and apply conservation of energy between points 1 and 2 to determine the maximum angle at which you can begin your swing and then between points 1 and 3 to determine the speed with which you will hit the water.



(*a*) Use conservation of energy to relate your speed at point 2 to your potential energy there and at point 1:

$$W_{ext} = \Delta K + \Delta U = 0$$
$$K_2 - K_1 + U_2 - U_1 = 0$$
Because $K_1 = 0$,
$$\tfrac{1}{2}mv_2^2 + mgh - [mgL(1-\cos\theta) + mgh] = 0$$

Solve this equation for θ:

$$\theta = \cos^{-1}\left[1 - \frac{v_2^2}{2gL}\right] \qquad (1)$$

Apply $\sum F_{\text{radial}} = ma_{\text{radial}}$ yourself at point 2 and solve for T:

$$T - mg = m\frac{v_2^2}{L}$$

and

$$T = mg + m\frac{v_2^2}{L}$$

Because you've estimated that the rope might break if the tension in it exceeds your weight by 80 N, it must be that:

$$m\frac{v_2^2}{L} = 80\,\text{N}$$

or

$$v_2^2 = \frac{(80\,\text{N})L}{m}$$

Let's assume your weight is 650 N. Then your mass is 66.3 kg and:

$$v_2^2 = \frac{(80\,\text{N})(4.6\,\text{m})}{66.3\text{kg}} = 5.55\,\text{m}^2/\text{s}^2$$

Substitute numerical values in equation (1) to obtain:

$$\theta = \cos^{-1}\left[1 - \frac{5.55\,\text{m}^2/\text{s}^2}{2(9.81\text{m/s}^2)(4.6\,\text{m})}\right] = \boxed{20.2°}$$

(*b*) Apply conservation of energy to the energy transformations between points 1 and 3:

$$W_{\text{ext}} = \Delta K + \Delta U = 0$$

$K_3 - K_1 + U_3 - U_1 = 0$ where U_3 and K_1 are zero

Substitute for K_3 and U_1 to obtain:

$$\tfrac{1}{2}mv_3^2 - mg[h + L(1 - \cos\theta)] = 0$$

Solve for v_3:

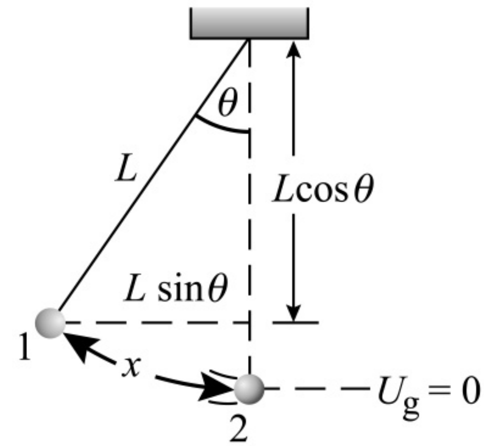
$$v_3 = \sqrt{2g[h + L(1 - \cos\theta)]}$$

Substitute numerical values and evaluate v_3:

$$v_3 = \sqrt{2(9.81\text{m/s}^2)[1.8\,\text{m} + (4.6\,\text{m})(1 - \cos 20.2°)]} = \boxed{6.39\,\text{m/s}}$$

40 ••

Picture the Problem Choose $U_g = 0$ at point 2, the lowest point of the bob's trajectory and let the system consist of the bob and the earth. Given this choice, there are no external forces doing work on the system. Because $\theta << 1$, we can use the trigonometric series for the sine and cosine functions to approximate these functions. The bob's initial energy is partially gravitational potential and partially potential energy stored in the stretched spring. As the bob swings down to point 2 this energy is transformed into kinetic energy. By equating these energies, we can derive an expression for the speed of the bob at point 2.



Apply conservation of energy to the system as the pendulum bob swings from point 1 to point 2:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}kx^2 + mgL(1-\cos\theta)$$

Note, from the figure, that $x \approx L\sin\theta$ when $\theta << 1$:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}k(L\sin\theta)^2 + mgL(1-\cos\theta)$$

Also, when $\theta << 1$:

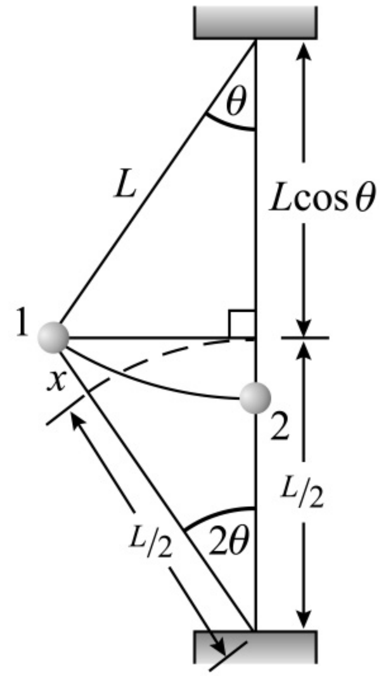
$$\sin\theta \approx \theta \text{ and } \cos\theta \approx 1-\tfrac{1}{2}\theta^2$$

Substitute, simplify and solve for v_2:

$$v_2 = \boxed{L\theta\sqrt{\frac{k}{m}+\frac{g}{L}}}$$

41 •••

Picture the Problem Choose $U_g = 0$ at point 2, the lowest point of the bob's trajectory and let the system consist of the earth, ceiling, spring, and pendulum bob. Given this choice, there are no external forces doing work to change the energy of the system. Because $\theta << 1$, we can use the trigonometric series for the secant and cosine functions to approximate these functions. The bob's initial energy is partially gravitational potential and partially potential energy stored in the stretched spring. As the bob swings down to point 2 this energy is transformed into kinetic energy. By equating these energies, we can derive an expression for the speed of the bob at point 2.



Apply conservation of energy to the system as the pendulum bob swings from point 1 to point 2:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}kx^2 + mgL(1-\cos\theta)$$

Note, from the figure, that $x = \frac{L}{2}(\sec\theta - 1)$ and that, for $\theta << 1$, $x \approx L\sin\theta$:

$$\tfrac{1}{2}mv_2^2 = \tfrac{1}{2}k\left[\frac{L}{2}(\sec\theta - 1)\right]^2 + mgL(1-\cos\theta)$$

Also, when $\theta << 1$:

$$\sec 2\theta \approx 1 + 2\theta^2 \text{ and } \cos\theta \approx 1 - \tfrac{1}{2}\theta^2$$

Substitute, simplify and solve for v_2:

$$v_2 = \boxed{L\theta\sqrt{\frac{g}{L} + \frac{k}{m}\theta^2}}$$

The Conservation of Energy

42 •

Picture the Problem The energy of the eruption is initially in the form of the kinetic energy of the material it thrusts into the air. This energy is then transformed into gravitational potential energy as the material rises.

(*a*) Express the energy of the

$$E = mg\Delta h$$

eruption in terms of the height Δh to which the debris rises:

Relate the density of the material to its mass and volume:

$$\rho = \frac{m}{V}$$

Substitute for m to obtain:

$$E = \rho V g \Delta h$$

Substitute numerical values and evaluate E:

$$E = \left(1600\,\text{kg/m}^3\right)\left(4\,\text{km}^3\right)\left(9.81\,\text{m/s}^2\right)\times\left(500\,\text{m}\right) = \boxed{3.14\times10^{16}\,\text{J}}$$

(*b*) Convert 3.13×10^{16} J to megatons of TNT:

$$3.14\times10^{16}\,\text{J} = 3.14\times10^{16}\,\text{J}\times\frac{1\,\text{Mton TNT}}{4.2\times10^{15}\,\text{J}} = \boxed{7.48\,\text{Mton TNT}}$$

43 ••

Picture the Problem The work done by the student equals the change in his/her gravitational potential energy and is done as a result of the transformation of metabolic energy in the climber's muscles.

(*a*) The increase in gravitational potential energy is:

$$\Delta U = mg\Delta h = \left(80\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)\left(120\,\text{m}\right) = \boxed{94.2\,\text{kJ}}$$

(*b*)

The energy required to do this work comes from chemical energy stored in the body.

(*c*) Relate the chemical energy expended by the student to the change in his/her potential energy and solve for E:

$$0.2E = \Delta U$$

and

$$E = 5\Delta U = 5\left(94.2\,\text{kJ}\right) = \boxed{471\,\text{kJ}}$$

Kinetic Friction

44 •

Picture the Problem As the car skids to a stop on a horizontal road, its kinetic energy is transformed into internal (i.e., thermal) energy. Knowing that energy is transformed into heat by friction, we can use the definition of the coefficient of kinetic friction to calculate its value.

(*a*) Relate the energy dissipated by friction to the change in kinetic energy of the car:

$$W_f = \Delta U_{therm} = \Delta K$$

Because $K_f = 0$, the friction force will transform all the car's initial kinetic energy:

$$W_f = K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(2000\,\text{kg})(25\,\text{m/s})^2 = \boxed{625\,\text{kJ}}$$

(*b*) Relate the kinetic friction force to the coefficient of kinetic friction and the weight of the car and solve for the coefficient of kinetic friction:

$$f_k = \mu_k mg \Rightarrow \mu_k = \frac{f_k}{mg}$$

Express the relationship between the work done by friction and the kinetic friction force and solve f_k:

$$W_f = f_k\Delta s \Rightarrow f_k = \frac{W_f}{\Delta s}$$

Substitute to obtain:

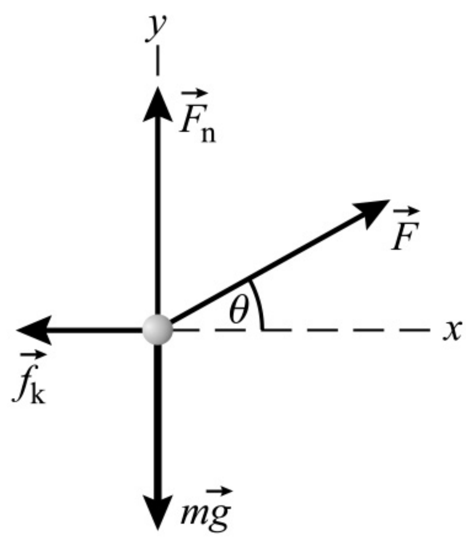
$$\mu_k = \frac{W_f}{mg\Delta s}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{625\,\text{kJ}}{(2000\,\text{kg})(9.81\,\text{m/s}^2)(60\,\text{m})} = \boxed{0.531}$$

45 •

Picture the Problem The free-body diagram shows the forces acting on the sled as it is pulled along a horizontal road. The work done by the applied force can be found using the definition of work. To find the energy dissipated by friction, we'll use Newton's 2nd law to determine f_k and then use it in the definition of work. The change in the kinetic energy of the sled is equal to the net work done on it. Finally, knowing the kinetic energy of the sled after it has traveled 3 m will allow us to solve for its speed at that location.



(*a*) Use the definition of work to calculate the work done by the applied force:

$$W = \vec{F} \cdot \vec{s} = Fs\cos\theta$$
$$= (40\,\text{N})(3\,\text{m})\cos 30° = \boxed{104\,\text{J}}$$

(*b*) Express the energy dissipated by friction as the sled is dragged along the surface:

$$W_f = \mu_k F_n \Delta x$$

Apply $\sum F_y = ma_y$ to the sled and solve for F_n:

$$F_n + F\sin\theta - mg = 0$$
and
$$F_n = mg - F\sin\theta$$

Substitute to obtain:

$$W_f = \mu_k \Delta x (mg - F\sin\theta)$$

Substitute numerical values and evaluate W_f:

$$W_f = (0.4)(3\,\text{m})[(8\,\text{kg})(9.81\,\text{m/s}^2) - (40\,\text{N})\sin 30°]$$
$$= \boxed{70.2\,\text{J}}$$

(*c*) Because $\Delta U = 0$:

$$\Delta K = W - W_f = 104\,\text{J} - 70.2\,\text{J}$$
$$= \boxed{33.8\,\text{J}}$$

(*d*) Because $K_i = 0$:

$$K_f = \Delta K = \tfrac{1}{2}mv^2$$

Solve for v:

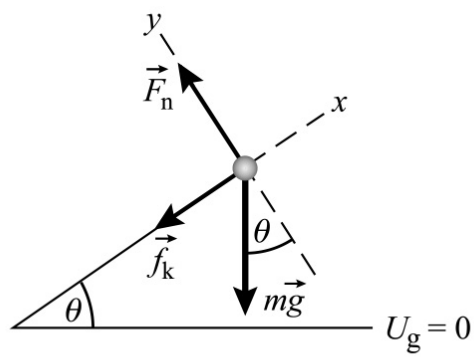
$$v=\sqrt{\frac{2\Delta K}{m}}$$

Substitute numerical values and evaluate v:

$$v=\sqrt{\frac{2(33.8\,\text{J})}{8\,\text{kg}}}=\boxed{2.91\,\text{m/s}}$$

***46** •

Picture the Problem Choose $U_g = 0$ at the foot of the ramp and let the system consist of the block, ramp, and the earth. Then the kinetic energy of the block at the foot of the ramp is equal to its initial kinetic energy less the energy dissipated by friction. The block's kinetic energy at the foot of the incline is partially converted to gravitational potential energy and partially dissipated by friction as the block slides up the incline. The free-body diagram shows the forces acting on the block as it slides up the incline. Applying Newton's 2^{nd} law to the block will allow us to determine f_k and express the energy dissipated by friction.



(*a*) Apply conservation of energy to the system while the block is moving horizontally:

$$W_{ext}=\Delta K+\Delta U$$

or, because $\Delta U = 0$,

$$W_{ext}=\Delta K=K_f-K_i$$

Solve for K_f:

$$K_f=K_i+W_{ext}$$

Express the work done by the friction force:

$$W_{ext}=\vec{\boldsymbol{F}}\cdot\vec{\boldsymbol{s}}=-f_k\Delta x=-\mu_k mg\Delta x$$

Substitute for W_{ext} to obtain:

$$\tfrac{1}{2}mv_f^2=\tfrac{1}{2}mv_i^2-\mu_k mg\Delta x$$

Solve for v_f:

$$v_f=\sqrt{v_i^2-2\mu_k g\Delta x}$$

Substitute numerical values and evaluate v_f:

$$v_f=\sqrt{(7\,\text{m/s})^2-2(0.3)(9.81\,\text{m/s}^2)(2\,\text{m})}=\boxed{6.10\,\text{m/s}}$$

(*b*) Relate the initial kinetic energy of the block to its final potential energy and the energy dissipated by friction:

$$K_i=U_f+W_{ext}$$

Apply $\sum F_y = ma_y$ to the block: $F_n - mg\cos\theta = 0 \Rightarrow F_n = mg\cos\theta$

Express W_{ext}: $W_{ext} = f_k L = \mu_k F_n L = \mu_k mgL\cos\theta$

Express the final potential energy of the block: $U_f = mgL\sin\theta$

Substitute for U_f and W_{ext} to obtain: $K_f = mgL\sin\theta + \mu_k mgL\cos\theta$

Solve for L:

$$L = \frac{K_f}{mg(\sin\theta + \mu_k\cos\theta)}$$

Substitute numerical values and evaluate L:

$$L = \frac{\frac{1}{2}(6.10\,\text{m/s})^2}{(9.81\,\text{m/s}^2)(\sin 40° + (0.3)\cos 40°)} = \boxed{2.17\,\text{m}}$$

47 •

Picture the Problem Let the system include the block, the ramp and horizontal surface, and the earth. Given this choice, there are no external forces acting that will change the energy of the system. Because the curved ramp is frictionless, mechanical energy is conserved as the block slides down it. We can calculate its speed at the bottom of the ramp by using the law of the conservation of energy. The potential energy of the block at the top of the ramp or, equivalently, its kinetic energy at the bottom of the ramp is converted into thermal energy during its slide along the horizontal surface.

(*a*) Choosing $U_g = 0$ at point 2 and letting the numeral 1 designate the initial position of the block and the numeral 2 its position at the foot of the ramp, use conservation of energy to relate the block's potential energy at the top of the ramp to its kinetic energy at the bottom:

$$\tfrac{1}{2}mv_2^2 - mg\Delta h = 0$$

Solve for v_2:

$$v_2 = \sqrt{2g\Delta h}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{2(9.81\,\text{m/s}^2)(3\,\text{m})} = \boxed{7.67\,\text{m/s}}$$

(*b*) The energy dissipated by friction

$$W_f + \Delta K + \Delta U = \Delta E_{therm} + \Delta K + \Delta U = 0$$

is responsible for changing the thermal energy of the system:

Because $\Delta K = 0$ for the slide:

$$W_f = -\Delta U = -(U_2 - U_1) = U_1 = mg\Delta h$$

Substitute numerical values and evaluate W_f:

$$W_f = (2\,\text{kg})(9.81\,\text{m/s}^2)(3\,\text{m}) = \boxed{58.9\,\text{J}}$$

(*c*) Express the energy dissipated by friction in terms of the distance over which it acts, the normal force acting on the block, and the coefficient of kinetic friction:

$$W_f = f_k\Delta x = \mu_k mg\Delta x$$

Solve for μ_k:

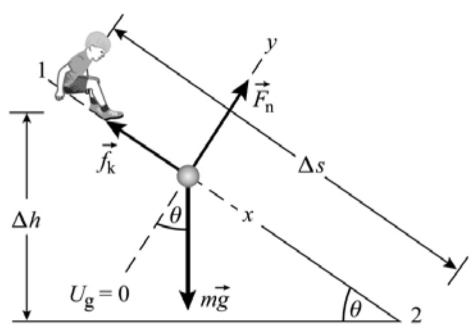
$$\mu_k = \frac{W_f}{mg\Delta x}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{58.9\,\text{J}}{(2\,\text{kg})(9.81\,\text{m/s}^2)(9\,\text{m})} = \boxed{0.333}$$

48 ••

Picture the Problem Let the system consist of the earth, the girl, and the slide. Given this choice, there are no external forces doing work that changes the energy of the system**.** By the time she reaches the bottom of the slide, her potential energy at the top of the slide has been converted into kinetic and thermal energy. Choose $U_g = 0$ at the bottom of the slide and denote the top and bottom of the slide as shown in the figure. We'll use the work-energy theorem with friction to relate these quantities and the forces acting on her during her slide to determine the friction force that transforms some of her initial potential energy into thermal energy.



(*a*) Express the work-energy theorem:

$$W_{ext} = \Delta K + \Delta U + f\Delta s = 0$$

Because $U_2 = K_1 = 0$:

$$K_2 - U_1 + f\Delta s = 0$$

or

$$W_f = U_1 - K_2 = mg\Delta h - \tfrac{1}{2}mv_2^2$$

Substitute numerical values and evaluate W_f:

$$W_f = (20\,\text{kg})(9.81\,\text{m/s}^2)(3.2\,\text{m}) - \tfrac{1}{2}(20\,\text{kg})(1.3\,\text{m/s})^2 = \boxed{611\,\text{J}}$$

(*b*) Relate the energy dissipated by friction to the kinetic friction force and the distance over which this force acts and solve for μ_k:

$$W_f = f_k\Delta s = \mu_k F_n \Delta s$$

and

$$\mu_k = \frac{W_f}{F_n \Delta s}$$

Apply $\sum F_y = ma_y$ to the girl and solve for F_n:

$$F_n - mg\cos\theta = 0$$

and

$$F_n = mg\cos\theta$$

Referring to the figure, relate Δh to Δs and θ:

$$\Delta s = \frac{\Delta h}{\sin\theta}$$

Substitute for Δs and F_n to obtain:

$$\mu_k = \frac{W_f}{mg\dfrac{\Delta h}{\sin\theta}\cos\theta} = \frac{W_f\tan\theta}{mg\Delta h}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{(611\,\text{J})\tan 20°}{(20\,\text{kg})(9.81\,\text{m/s}^2)(3.2\,\text{m})} = \boxed{0.354}$$

49 ••

Picture the Problem Let the system consist of the two blocks, the shelf, and the earth. Given this choice, there are no external forces doing work to change the energy of the system. Due to the friction between the 4-kg block and the surface on which it slides, not all of the energy transformed during the fall of the 2-kg block is realized in the form of kinetic energy. We can find the work done by friction (energy transformed into E_{therm}) from its definition and then use this result in the calculation of the speed of the system when it has moved a given distance.

(*a*) Express the energy dissipated by friction in terms of the coefficient of

$$W_f = f_k\Delta s = \mu_k m_1 g\Delta s$$

kinetic friction, the mass of the sliding block, and the displacement of the block ($\Delta s = y$):

Substitute numerical values and evaluate W_f:

$$W_f = (0.35)(4\,\text{kg})(9.81\,\text{m/s}^2)y = \boxed{(13.7\,\text{N})y}$$

(*b*) Express the total mechanical energy of the system:

$$E_{mech} = \Delta E = -W_f = \boxed{-(13.7\,\text{N})y}$$

(*c*) Express the total mechanical energy of the system:

$$\tfrac{1}{2}(m_1 + m_2)v^2 = m_2 gy - W_f$$

Solve for *v* to obtain:

$$v = \sqrt{\frac{2(m_2 gy - W_f)}{m_1 + m_2}} \quad (1)$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{2\left[(2\,\text{kg})\left(9.81\,\text{m/s}^2\right)(2\,\text{m}) - (13.73\,\text{N})(2\,\text{m})\right]}{4\,\text{kg} + 2\,\text{kg}}} = \boxed{1.98\,\text{m/s}}$$

***50 ••**

Picture the Problem Let the system consist of the particle and the earth. Then the friction force is external to the system and does work to change the energy of the system. The energy dissipated by friction during one revolution is the work done by the friction force.

(*a*) Relate the work done by friction to the change in energy of the system:

$$W_{ext} = W_f = \Delta K + \Delta U = K_f - K_i, \text{ since } \Delta U = 0$$

Substitute for K_f and K_i and simplify to obtain:

$$W_f = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}m\left(\tfrac{1}{2}v_0\right)^2 - \tfrac{1}{2}m(v_0)^2 = \boxed{\tfrac{3}{8}mv_0^2}$$

(*b*) Relate the work done by friction to the distance traveled and the coefficient of kinetic friction and solve for the latter:

$$W_f = \mu_k mg\Delta s = \mu_k mg(2\pi r)$$

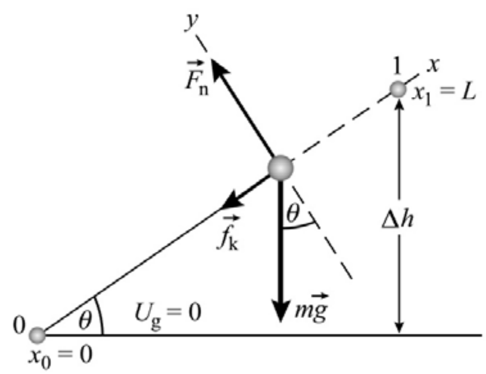
and

$$\mu_k = \frac{W_f}{2\pi mgr} = \frac{\frac{3}{8}mv_0^2}{2\pi mgr} = \boxed{\frac{3v_0^2}{16\pi gr}}$$

(*c*) Because in one revolution it lost $\frac{3}{4}K_i$, it will only require another 1/3 revolution to lose the remaining $\frac{1}{4}K_i$.

51 ••

Picture the Problem The box will slow down and stop due to the work the friction force does on it. Let the system be the earth, the box, and the inclined plane and apply the work-energy theorem with friction. With this choice of the system, there are no external forces doing work to change the energy of the system. The free-body diagram shows the forces acting on the box when it is moving up the incline.



Apply the work-energy theorem with friction to the system:

$$W_{ext} = \Delta E_{sys} = \Delta K + \Delta U + W_f = 0$$

Substitute to obtain:

$$\therefore W_f + \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mg\Delta h = 0 \quad (1)$$

Express the work done by friction as the box moves a distance L up the incline:

$$W_f = f_k L = \mu_k F_n L$$

Referring to the FBD, relate the normal force to the weight of the box and the angle of the incline:

$$F_n = mg\cos\theta$$

Substitute in the expression for W_f to obtain:

$$W_f = \mu_k mgL\cos\theta$$

Relate Δh to the distance L along the incline:

$$\Delta h = L\sin\theta$$

Substitute for W_f and Δh in equation (1) to obtain:

$$\mu_k mgL\cos\theta + \tfrac{1}{2}mv_1^2 - \tfrac{1}{2}mv_0^2 + mgL\sin\theta = 0 \quad (2)$$

Solve equation (2) for L to obtain:

$$L = \frac{v_0^2}{2g(\mu_k \cos\theta + \sin\theta)}$$

Substitute numerical values and evaluate L:

$$L = \frac{(3.8\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)[(0.3)\cos 37° + \sin 37°]} = \boxed{0.875\,\text{m}}$$

Let v_f represent the box's speed as it passes its starting point on the way down the incline. For the block's descent, equation (2) becomes:

$$\mu_k mgL\cos\theta + \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_1^2 - mgL\sin\theta = 0$$

Set $v_1 = 0$ (the block starts from rest at the top of the incline) and solve for v_f :

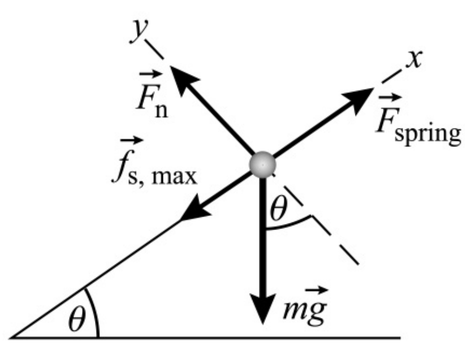
$$v_f = \sqrt{2gL(\sin\theta - \mu_k\cos\theta)}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(0.875\,\text{m})[\sin 37° - (0.3)\cos 37°]} = \boxed{2.49\,\text{m/s}}$$

52 •••

Picture the Problem Let the system consist of the earth, the block, the incline, and the spring. With this choice of the system, there are no external forces doing work to change the energy of the system. The free-body diagram shows the forces acting on the block just before it begins to move. We can apply Newton's 2nd law to the block to obtain an expression for the extension of the spring at this instant. We'll apply the work-energy theorem with friction to the second part of the problem.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the block when it is on the verge of sliding:

$$\sum F_x = F_{spring} - f_{s,max} - mg\sin\theta = 0$$

and

$$\sum F_y = F_n - mg\cos\theta = 0$$

Eliminate F_n, $f_{s,max}$, and F_{spring}

$$kd - \mu_s mg\cos\theta - mg\sin\theta = 0$$

between the two equations to obtain:

Solve for and evaluate d:

$$d = \boxed{\frac{mg}{k}\left(\sin\theta + \mu_s \cos\theta\right)}$$

(b) Begin with the work-energy theorem for problems with friction and no work being done by an external force:

$$\Delta E_{sys} = \Delta K + \Delta U_g + \Delta U_s + W_f = 0$$

Because the block is at rest in both its initial and final states, $\Delta K = 0$ and:

$$\Delta U_g + \Delta U_s + W_f = 0 \qquad (1)$$

Let $U_g = 0$ at the initial position of the block. Then:

$$\Delta U_g = U_{g,final} - U_{g,initial} = mgh - 0 = mgd\sin\theta$$

Express the change in the energy stored in the spring as it relaxes to its unstretched length:

$$\Delta U_s = U_{s,final} - U_{s,initial} = 0 - \tfrac{1}{2}kd^2 = -\tfrac{1}{2}kd^2$$

Express W_f:

$$W_f = f\Delta s = -f_k d = -\mu_k F_n d = -\mu_k mgd\cos\theta$$

Substitute in equation (1) to obtain:

$$mgd\sin\theta - \tfrac{1}{2}kd^2 - \mu_k mgd\cos\theta = 0$$

Finally, solve for μ_k:

$$\mu_k = \boxed{\tfrac{1}{2}\left(\tan\theta - \mu_s\right)}$$

Mass and Energy

53 •

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

(a) Relate the rest mass consumed to the energy produced and solve for and evaluate m:

$$E_0 = mc^2 = \left(1\times10^{-3}\text{ kg}\right)\left(3\times10^{8}\text{ m/s}\right)^2 = \boxed{9.00\times10^{13}\text{ J}}$$

(*b*) Express kW·h in joules:

$$1\,\text{kW}\cdot\text{h} = \left(1\times10^{3}\ \text{J/s}\right)\left(1\,\text{h}\right)\left(3600\,\text{s/h}\right) = 3.60\times10^{6}\ \text{J}$$

Convert 9×10^{13} J to kW·h:

$$9\times10^{13}\ \text{J} = \left(9\times10^{13}\ \text{J}\right)\left(\frac{1\,\text{kW}\cdot\text{h}}{3.60\times10^{6}\ \text{J}}\right) = 2.50\times10^{7}\ \text{kW}\cdot\text{h}$$

Determine the price of the electrical energy:

$$\text{Price} = \left(2.50\times10^{7}\ \text{kW}\cdot\text{h}\right)\left(\frac{\$0.10}{\text{kW}\cdot\text{h}}\right) = \boxed{\$2.5\times10^{6}}$$

(*c*) Relate the energy consumed to its rate of consumption and the time and solve for the latter:

$$E = Pt$$

and

$$t = \frac{E}{P} = \frac{9\times10^{13}\ \text{J}}{100\,\text{W}} = \boxed{9\times10^{11}\,\text{s} = 28{,}500\,\text{y}}$$

54 •

Picture the Problem We can use the equation expressing the equivalence of energy and matter, $E = mc^2$, to find the mass equivalent of the energy from the explosion.

Solve $E = mc^2$ for m:

$$m = \frac{E}{c^2}$$

Substitute numerical values and evaluate m:

$$m = \frac{5\times10^{12}\ \text{J}}{\left(2.998\times10^{8}\ \text{m/s}\right)^2} = \boxed{5.56\times10^{-5}\ \text{kg}}$$

55 •

Picture the Problem The intrinsic rest energy in matter is related to the mass of matter through Einstein's equation $E_0 = mc^2$.

Relate the rest mass of a muon to its rest energy:

$$m_0 = \frac{E}{c^2}$$

Express 1 MeV in joules:

$$1\ \text{MeV} = 1.6\times10^{-13}\ \text{J}$$

Substitute numerical values and evaluate m_0:

$$m_0 = \frac{(105.7\,\text{MeV})(1.6\times10^{-13}\ \text{J/MeV})}{(3\times10^8\ \text{m/s})^2} = \boxed{1.88\times10^{-28}\ \text{kg}}$$

***56 •**

Picture the Problem We can differentiate the mass-energy equation to obtain an expression for the rate at which the black hole gains energy.

Using the mass-energy relationship, express the energy radiated by the black hole:

$$E = 0.01mc^2$$

Differentiate this expression to obtain an expression for the rate at which the black hole is radiating energy:

$$\frac{dE}{dt} = \frac{d}{dt}\left[0.01mc^2\right] = 0.01c^2\frac{dm}{dt}$$

Solve for dm/dt:

$$\frac{dm}{dt} = \frac{dE/dt}{0.01c^2}$$

Substitute numerical values and evaluate dm/dt:

$$\frac{dm}{dt} = \frac{4\times10^{31}\ \text{watt}}{(0.01)(2.998\times10^8\ \text{m/s})^2} = \boxed{4.45\times10^{16}\ \text{kg/s}}$$

57 •

Picture the Problem The number of reactions per second is given by the ratio of the power generated to the energy released per reaction. The number of reactions that must take place to produce a given amount of energy is the ratio of the energy per second (power) to the energy released per second.

In Example 7-15 it is shown that the energy per reaction is 17.59 MeV. Convert this energy to joules:

$$17.59\,\text{MeV} = (17.59\,\text{MeV})\times(1.6\times10^{-19}\ \text{J/eV}) = 28.1\times10^{-13}\ \text{J}$$

The number of reactions per second is:

$$\frac{1000\,\text{J/s}}{28.1\times10^{-13}\ \text{J/reaction}} = \boxed{3.56\times10^{14}\ \text{reactions/s}}$$

58 •

Picture the Problem The energy required for this reaction is the difference between the rest energy of ^{4}He and the sum of the rest energies of ^{3}He and a neutron.

Express the reaction: $^4\text{He} \rightarrow {}^3\text{He} + \text{n}$

The rest energy of a neutron (Table 7-1) is: 939.573 MeV

The rest energy of ^{4}He (Example 7-15) is: 3727.409 MeV

The rest energy of ^{3}He is: 2808.432 MeV

Substitute numerical values to find the difference in the rest energy of ^{4}He and the sum of the rest energies of ^{3}He and n:

$$E = [3727.409 - (2808.41 + 939.573)]\ \text{MeV} = \boxed{20.574\,\text{MeV}}$$

59 •

Picture the Problem The energy required for this reaction is the difference between the rest energy of a neutron and the sum of the rest energies of a proton and an electron.

The rest energy of a proton (Table 7-1) is: 938.280 MeV

The rest energy of an electron (Table 7-1) is: 0.511 MeV

The rest energy of a neutron (Table 7-1) is: 939.573 MeV

Substitute numerical values to find the difference in the rest energy of a neutron and the sum of the rest energies of a positron and an electron:

$$E = [939.573 - (938.280 + 0.511)]\ \text{MeV} = \boxed{0.782\,\text{MeV}}$$

60 ••

Picture the Problem The reaction is $^2\text{H} + {}^2\text{H} \rightarrow {}^4\text{He} + E$. The energy released in this reaction is the difference between twice the rest energy of ^{2}H and the rest energy of ^{4}He. The number of reactions that must take place to produce a given amount of energy is the ratio of the energy per second (power) to the energy released per reaction.

(*a*) The rest energy of ^{4}He

(Example 7-14) is: 3727.409 MeV

The rest energy of a deuteron, ^{2}H, (Table 7-1) is: 1875.628 MeV

The energy released in the reaction is:

$$E = [2(1875.628) - 3727.409]\text{ MeV} = \boxed{23.847\text{ MeV} = 3.816\times10^{-12}\text{ J}}$$

(*b*) The number of reactions per second is:

$$\frac{1000\text{ J/s}}{3.816\times10^{-12}\text{ J/reaction}} = \boxed{2.62\times10^{14}\text{ reactions/s}}$$

61 ••

Picture the Problem The annual consumption of matter by the fission plant is the ratio of its annual energy output to the square of the speed of light. The annual consumption of coal in a coal-burning power plant is the ratio of its annual energy output to energy per unit mass of the coal.

(*a*) Express *m* in terms of *E*:

$$m = \frac{E}{c^2}$$

Assuming an efficiency of 33 percent, find the energy produced annually:

$$\begin{aligned} E &= 3P\Delta t = 3(3\times10^9\text{ J/s})(1\text{ y}) \\ &= 3(3\times10^9\text{ J/s})(3600\text{ s/h}) \\ &\quad\times(24\text{ h/d})(365.24\text{ d}) \\ &= 2.84\times10^{17}\text{ J} \end{aligned}$$

Substitute to obtain:

$$m = \frac{2.84\times10^{17}\text{ J}}{(3\times10^8\text{ m/s})^2} = \boxed{3.16\text{ kg}}$$

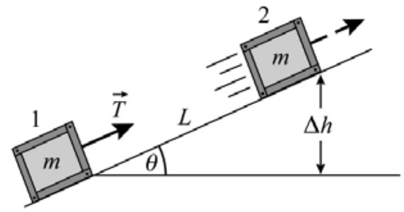
(*b*) Assuming an efficiency of 38 percent, express the mass of coal required in terms of the annual energy production and the energy released per kilogram:

$$m_{\text{coal}} = \frac{E_{\text{annual}}}{0.38(E/m)} = \frac{9.47\times10^{16}\text{ J}}{0.38(3.1\times10^7\text{ J/kg})} = \boxed{8.04\times10^9\text{ kg}}$$

General Problems

***62 ••**

Picture the Problem Let the system consist of the block, the earth, and the incline. Then the tension in the string is an external force that will do work to change the energy of the system. Because the incline is frictionless; the work done by the tension in the string as it displaces the block on the incline is equal to the sum of the changes in the kinetic and gravitational potential energies.



Relate the work done by the tension force to the changes in the kinetic and gravitational potential energies of the block:

$$W_{\text{tension force}} = W_{\text{ext}} = \Delta U + \Delta K$$

Referring to the figure, express the change in the potential energy of the block as it moves from position 1 to position 2:

$$\Delta U = mg\Delta h = mgL\sin\theta$$

Because the block starts from rest:

$$\Delta K = K_2 = \tfrac{1}{2}mv^2$$

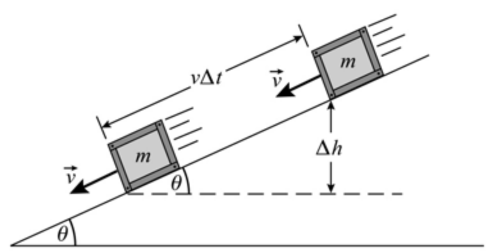
Substitute to obtain:

$$W_{\text{tension force}} = mgL\sin\theta + \tfrac{1}{2}mv^2$$

and $\boxed{\text{(c) is correct.}}$

63 ••

Picture the Problem Let the system include the earth, the block, and the inclined plane. Then there are no external forces to do work on the system and W_{ext} = 0. Apply the work-energy theorem with friction to find an expression for the energy dissipated by friction.



Express the work-energy theorem with friction:

$$W_{\text{ext}} = \Delta K + \Delta U + W_{\text{f}} = 0$$

Because the velocity of the block is constant, $\Delta K = 0$ and:

$$W_f = -\Delta U = -mg\Delta h$$

In time Δt the block slides a distance $v\Delta t$. From the figure:

$$\Delta h = v\Delta t \sin\theta$$

Substitute to obtain:

$$W_f = -mgv\Delta t \sin\theta$$

and $\boxed{(b) \text{ is correct.}}$

64 •

Picture the Problem Let the system include the earth and the box. Then the applied force is external to the system and does work on the system in compressing the spring. This work is stored in the spring as potential energy.

Express the work-energy theorem:

$$W_{ext} = \Delta K + \Delta U_g + \Delta U_s + \Delta E_{therm}$$

Because $\Delta K = \Delta U_g = \Delta E_{therm} = 0$:

$$W_{ext} = \Delta U_s$$

Substitute for W_{ext} and ΔU_s:

$$Fx = \tfrac{1}{2}kx^2$$

Solve for x:

$$x = \frac{2F}{k}$$

Substitute numerical values and evaluate x:

$$x = \frac{2(70\,\text{N})}{6800\,\text{N/m}} = \boxed{2.06\,\text{cm}}$$

***65** •

Picture the Problem The solar constant is the average energy per unit area and per unit time reaching the upper atmosphere. This physical quantity can be thought of as the power per unit area and is known as *intensity*.

Letting $I_{surface}$ represent the intensity of the solar radiation at the surface of the earth, express $I_{surface}$ as a function of power and the area on which this energy is incident:

$$I_{surface} = \frac{P}{A} = \frac{\Delta E / \Delta t}{A}$$

Solve for ΔE:

$$\Delta E = I_{surface} A \Delta t$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \left(1\,\text{kW/m}^2\right)\left(2\,\text{m}^2\right)\left(8\,\text{h}\right)\left(3600\,\text{s/h}\right) = \boxed{57.6\,\text{MJ}}$$

66 ••

Picture the Problem The luminosity of the sun (or of any other object) is the product of the power it radiates per unit area and its surface area. If we let L represent the sun's luminosity, I the power it radiates per unit area (also known as the solar constant or the intensity of its radiation), and A its surface area, then
$L = IA$. We can estimate the solar lifetime by dividing the number of hydrogen nuclei in the sun by the rate at which they are being transformed into energy.

(*a*) Express the total energy the sun radiates every second in terms of the solar constant:

$$L = IA$$

Letting R represent its radius, express the surface area of the sun:

$$A = 4\pi R^2$$

Substitute to obtain:

$$L = 4\pi R^2 I$$

Substitute numerical values and evaluate L:

$$L = 4\pi\left(1.5\times10^{11}\,\text{m}\right)^2\left(1.35\,\text{kW/m}^2\right) = \boxed{3.82\times10^{26}\,\text{watt}}$$

Note that this result is in good agreement with the value given in the text of 3.9×10^{26} watt.

(*b*) Express the solar lifetime in terms of the mass of the sun and the rate at which its mass is being converted to energy:

$$t_{\text{solar}} = \frac{N_{\text{H nuclei}}}{\Delta n/\Delta t} = \frac{M/m}{\Delta n/\Delta t}$$

where M is the mass of the sun, m the mass of a hydrogen nucleus, and n is the number of nuclei used up.

Substitute numerical values to obtain:

$$t_{\text{solar}} = \frac{\dfrac{1.99\times10^{30}\,\text{kg}}{1.67\times10^{-27}\,\text{kg/H nucleus}}}{\Delta n/\Delta t} = \frac{1.19\times10^{57}\,\text{H nuclei}}{\Delta n/\Delta t}$$

For each reaction, 4 hydrogen nuclei are "used up"; so:

$$\frac{\Delta n}{\Delta t} = \frac{4\left(3.82\times10^{26}\,\text{J/s}\right)}{4.27\times10^{-12}\,\text{J}} = 3.57\times10^{38}\,\text{s}^{-1}$$

Because we've assumed that the sun will continue burning until roughly 10% of its hydrogen fuel is used up, the total solar lifetime should be:

$$t_{\text{solar}} = 0.1\left(\frac{1.19\times10^{57}\ \text{H nuclei}}{3.57\times10^{38}\ \text{s}^{-1}}\right) = 3.33\times10^{17}\ \text{s} = \boxed{1.06\times10^{10}\ \text{y}}$$

67 •

Picture the Problem Let the system include the earth and the *Spirit of America*. Then there are no external forces to do work on the car and $W_{\text{ext}} = 0$. We can use the work-energy theorem to relate the coefficient of kinetic friction to the given information. A constant-acceleration equation will yield the car's velocity when 60 s have elapsed.

(*a*) Apply the work-energy theorem with friction to relate the coefficient of kinetic friction μ_k to the initial and final kinetic energies of the car:

$$\tfrac{1}{2}mv^2 - \tfrac{1}{2}mv_0^2 + \mu_k mg\Delta s = 0$$

or, because $v = 0$,

$$-\tfrac{1}{2}mv_0^2 + \mu_k mg\Delta s = 0$$

Solve for μ_k:

$$\mu_k = \frac{v^2}{2g\Delta s}$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{[(708\,\text{km/h})(1\,\text{h}/3600\,\text{s})]^2}{2(9.81\,\text{m/s}^2)(9.5\,\text{km})} = \boxed{0.208}$$

(*b*) Express the kinetic energy of the car:

$$K = \tfrac{1}{2}mv^2 \qquad (1)$$

Using a constant-acceleration equation, relate the speed of the car to its acceleration, initial speed, and the elapsed time:

$$v = v_0 + a\Delta t$$

Express the braking force acting on the car:

$$F_{\text{net}} = -f_k = -\mu_k mg = ma$$

Solve for a:

$$a = -\mu_k g$$

Substitute for a to obtain:

$$v = v_0 - \mu_k g\Delta t$$

Substitute in equation (1) to obtain:

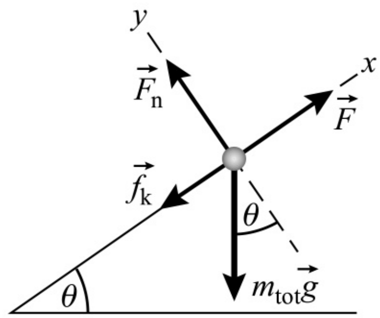
$$K = \tfrac{1}{2}m(v_0 - \mu_k g\Delta t)^2$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{2}(1250\,\text{kg})\left[708\times10^3\,\text{m/h} - (0.208)(9.81\,\text{m/s}^2)(60\,\text{s})\right]^2 = \boxed{3.45\,\text{MJ}}$$

68 ••

Picture the Problem The free-body diagram shows the forces acting on the skiers as they are towed up the slope at constant speed. Because the power required to move them is $\vec{F}\cdot\vec{v}$, we need to find F as a function of m_{tot}, θ, and μ_k. We can apply Newton's 2nd law to obtain such a function.



Express the power required as a function of force on the skiers and their speed:

$$P = Fv \qquad (1)$$

Apply $\sum\vec{F} = m\vec{a}$ to the skiers:

$$\sum F_x = F - f_k - m_{tot}g\sin\theta = 0$$

and

$$\sum F_y = F_n - m_{tot}g\cos\theta = 0$$

Eliminate $f_k = \mu_k F_n$ and F_n between the two equations and solve for F:

$$F = m_{tot}g\sin\theta + \mu_k m_{tot}g\cos\theta$$

Substitute in equation (1) to obtain:

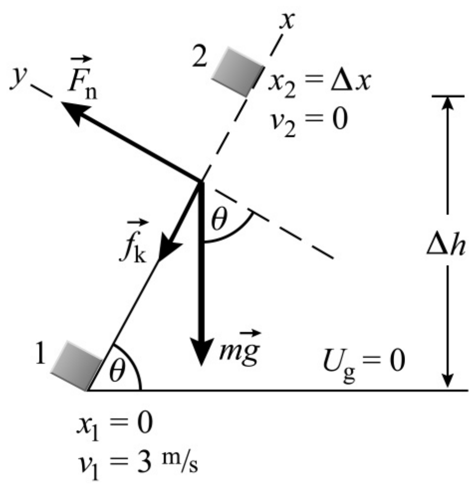
$$P = (m_{tot}g\sin\theta + \mu_k m_{tot}g\cos\theta)v$$
$$= m_{tot}gv(\sin\theta + \mu_k\cos\theta)$$

Substitute numerical values and evaluate P:

$$P = 80(75\,\text{kg})(9.81\,\text{m/s}^2)(2.5\,\text{m/s})[\sin15° + (0.06)\cos15°] = \boxed{46.6\,\text{kW}}$$

69 ••

Picture the Problem The pictorial representation has the free-body diagram for the box superimposed on it. The work done by friction slows and momentarily stops the box as it slides up the incline. The box's speed when it returns to bottom of the incline will be less than its speed when it started up the incline due to the energy dissipated by friction while it was in motion. Let the system include the box, the earth, and the incline. Then $W_{ext} = 0$. We can use the work-energy theorem with friction to solve the several parts of this problem.



(*a*) From the FBD we can see that the forces acting on the box are the normal force exerted by the inclined plane, a kinetic friction force, and the gravitational force (the weight of the box) exerted by the earth.

(*b*) Apply the work-energy theorem with friction to relate the distance Δx the box slides up the incline to its initial kinetic energy, its final potential energy, and the work done against friction:

$$-\tfrac{1}{2}mv_1^2 + mg\Delta h + \mu_k mg\Delta x\cos\theta = 0$$

Referring to the figure, relate Δh to Δx to obtain:

$$\Delta h = \Delta x\sin\theta$$

Substitute for Δh to obtain:

$$-\tfrac{1}{2}mv_1^2 + mg\Delta x\sin\theta + \mu_k mg\Delta x\cos\theta = 0$$

Solve for Δx:

$$\Delta x = \frac{v_1^2}{2g(\sin\theta + \mu_k\cos\theta)}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{(3\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)[\sin 60° + (0.3)\cos 60°]} = \boxed{0.451\,\text{m}}$$

(*c*) Express the energy dissipated by friction:

$$W_f = f_k \Delta x = \mu_k mg\Delta x \cos\theta$$

Substitute numerical values and evaluate W_f:

$$W_f = (0.3)(2\,\text{kg})(9.81\,\text{m/s}^2)\times(0.451\,\text{m})\cos 60° = \boxed{1.33\,\text{J}}$$

(*d*) Use the work-energy theorem with friction:

$$W_{ext} = \Delta K + \Delta U + W_f = 0$$

or

$$K_1 - K_2 + U_1 - U_2 + W_f = 0$$

Because $K_2 = U_1 = 0$ we have:

$$K_1 - U_2 + W_f = 0$$

or

$$\tfrac{1}{2}mv_1^2 - mg\Delta x\sin\theta + \mu_k mg\Delta x\cos\theta = 0$$

Solve for v_1:

$$v_1 = \sqrt{2g\Delta x(\sin\theta - \mu_k\cos\theta)}$$

Substitute numerical values and evaluate v_1:

$$v_1 = \sqrt{2(9.81\,\text{m/s}^2)(0.451\,\text{m})[\sin 60° - (0.3)\cos 60°]} = \boxed{2.52\,\text{m/s}}$$

***70 •**

Picture the Problem The power provided by a motor that is delivering sufficient energy to exert a force *F* on a load which it is moving at a speed *v* is *Fv*.

The power provided by the motor is given by:

$$P = Fv$$

Because the elevator is ascending with constant speed, the tension in the support cable(s) is:

$$F = (m_{elev} + m_{load})g$$

Substitute for *F* to obtain:

$$P = (m_{elev} + m_{load})gv$$

Substitute numerical values and evaluate *P*:

$$P = (2000\,\text{kg})(9.81\,\text{m/s}^2)(2.3\,\text{m/s}) = \boxed{45.1\,\text{kW}}$$

71 ••

Picture the Problem The power a motor must provide to exert a force F on a load that it is moving at a speed v is Fv. The counterweight does negative work and the power of the motor is reduced from that required with no counterbalance.

The power provided by the motor is given by:

$$P = Fv$$

Because the elevator is counterbalanced and ascending with constant speed, the tension in the support cable(s) is:

$$F = \left(m_{\text{elev}} + m_{\text{load}} - m_{\text{cw}}\right)g$$

Substitute and evaluate P:

$$P = \left(m_{\text{elev}} + m_{\text{load}} - m_{\text{cw}}\right)gv$$

Substitute numerical values and evaluate P:

$$P = (500\,\text{kg})\left(9.81\,\text{m/s}^2\right)(2.3\,\text{m/s}) = \boxed{11.3\,\text{kW}}$$

Without a load:

$$F = \left(m_{\text{elev}} - m_{\text{cw}}\right)g$$

and

$$P = \left(m_{\text{elev}} - m_{\text{cw}}\right)gv = (-300\,\text{kg})\left(9.81\,\text{m/s}^2\right)(2.3\,\text{m/s}) = \boxed{-6.77\,\text{kW}}$$

72 ••

Picture the Problem We can use the work-energy theorem with friction to describe the energy transformation within the dart-spring-air-earth system. With this choice of the system, there are no external forces to do work on the system, i.e., $W_{\text{ext}} = 0$. Choose $U_{\text{g}} = 0$ at the elevation of the dart on the compressed spring. The energy initially stored in the spring is transformed into gravitational potential energy and thermal energy. During the dart's descent, its gravitational potential energy is transformed into kinetic energy and thermal energy.

Apply conservation of energy during the dart's ascent:

$$W_{\text{ext}} = \Delta K + \Delta U + W_{\text{f}} = 0$$

or

$$U_{\text{g,f}} - U_{\text{g,i}} + U_{\text{s,f}} - U_{\text{s,i}} + W_{\text{f}} = 0$$

because $\Delta K = 0$

Because $U_{\text{g,i}} = U_{\text{s,f}} = 0$:

$$U_{\text{g,f}} - U_{\text{s,i}} + W_{\text{f}} = 0$$

Substitute for $U_{g,i}$ and $U_{s,f}$ and solve for W_f:

$$W_f = U_{s,i} - U_{g,f} = \tfrac{1}{2}kx^2 - mgh$$

Substitute numerical values and evaluate W_f:

$$W_f = \tfrac{1}{2}(5000\,\text{N/m})(0.03\,\text{m})^2 - (0.007\,\text{kg})(9.81\,\text{m/s}^2)(24\,\text{m}) = \boxed{0.602\,\text{J}}$$

Apply conservation of energy during the dart's descent:

$$W_{ext} = \Delta K + \Delta U + W_f = 0$$

or, because $K_i = U_{g,f} = 0$,

$$K_f - U_{g,i} + W_f = 0$$

Substitute for K_f and $U_{g,i}$ to obtain:

$$\tfrac{1}{2}mv_f^2 - mgh + W_f = 0$$

Solve for v_f:

$$v_f = \sqrt{\frac{2(mgh - W_f)}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{\frac{2\left[(0.007\,\text{kg})(9.81\,\text{m/s}^2)(24\,\text{m}) - 0.602\,\text{J}\right]}{0.007\,\text{kg}}} = \boxed{17.3\,\text{m/s}}$$

***73 ••**

Picture the Problem Let the system consist of the earth, rock, and air. Given this choice, there are no external forces to do work on the system and $W_{ext} = 0$. Choose $U_g = 0$ to be where the rock begins its upward motion. The initial kinetic energy of the rock is partially transformed into potential energy and partially dissipated by air resistance as the rock ascends. During its descent, its potential energy is partially transformed into kinetic energy and partially dissipated by air resistance.

(*a*) Using the definition of kinetic energy, calculate the initial kinetic energy of the rock:

$$K_i = \tfrac{1}{2}mv_i^2 = \tfrac{1}{2}(2\,\text{kg})(40\,\text{m/s})^2 = \boxed{1.60\,\text{kJ}}$$

(*b*) Apply the work-energy theorem with friction to relate the energies of the system as the rock ascends:

$$\Delta K + \Delta U + W_f = 0$$

Because $K_f = 0$:

$$-K_i + \Delta U + W_f = 0$$

and

$$W_f = K_i - \Delta U$$

Substitute numerical values and evaluate W_f:

$$W_f = 1600\,\text{J} - (2\,\text{kg})(9.81\,\text{m/s}^2)(50\,\text{m}) = \boxed{619\,\text{J}}$$

(*c*) Apply the work-energy theorem with friction to relate the energies of the system as the rock descends:

$$\Delta K + \Delta U + W_f = 0$$

Because $K_i = U_f = 0$:

$$K_f - U_i + W_f' = 0$$

where $W_f' = 0.7W_f$.

Substitute for the energies to obtain:

$$\tfrac{1}{2}mv_f^2 - mgh + 0.7W_f = 0$$

Solve for v_f:

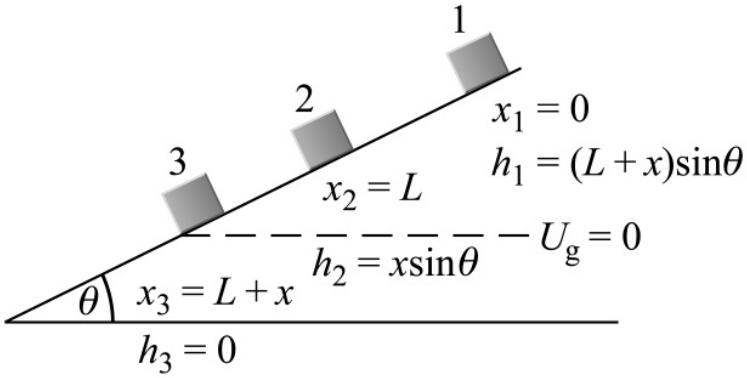
$$v_f = \sqrt{2gh - \frac{1.4W_f}{m}}$$

Substitute numerical values and evaluate v_f:

$$v_f = \sqrt{2(9.81\,\text{m/s}^2)(50\,\text{m}) - \frac{(1.4)(619\,\text{J})}{2\,\text{kg}}} = \boxed{23.4\,\text{m/s}}$$

74 ••

Picture the Problem Let the distance the block slides before striking the spring be *L*. The pictorial representation shows the block at the top of the incline (1), just as it strikes the spring (2), and the block against the fully compressed spring (3). Let the block, spring, and the earth comprise the system. Then $W_{ext} = 0$. Let $U_g = 0$ where the spring is at maximum compression. We can apply the work-energy theorem to relate the energies of the system as it evolves from state 1 to state 3.



Express the work-energy theorem:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$\Delta K + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $\Delta K = U_{g,3} = U_{s,1} = 0$:

$$-U_{g,1} + U_{s,3} = 0$$

Substitute for each of these energy terms to obtain:

$$-mgh_1 + \tfrac{1}{2}kx^2 = 0$$

Substitute for h_3 and h_1:

$$-mg(L + x)\sin\theta + \tfrac{1}{2}kx^2 = 0$$

Rewrite this equation explicitly as a quadratic equation:

$$x^2 - \frac{2mg\sin\theta}{k}x - \frac{2mgL\sin\theta}{k} = 0$$

Solve this quadratic equation to obtain:

$$x = \boxed{\frac{mg}{k}\sin\theta + \sqrt{\left(\frac{mg}{k}\right)^2 \sin^2\theta + \frac{2mgL}{k}\sin\theta}}$$

Note that the negative sign between the two terms leads to a non-physical solution.

***75** •

Picture the Problem We can find the work done by the girder on the slab by calculating the change in the potential energy of the slab.

(*a*) Relate the work the girder does on the slab to the change in potential energy of the slab:

$$W = \Delta U = mg\Delta h$$

Substitute numerical values and evaluate W:

$$W = (1.5\times10^4\ \text{kg})(9.81\ \text{m/s}^2)(0.001\ \text{m}) = \boxed{147\ \text{J}}$$

(*b*) The energy is transferred to the girder from its surroundings, which are warmer than the girder. As the temperature of the girder rises, the atoms in the girder vibrate with a greater average kinetic energy, leading to a larger average separation, which causes the girder's expansion.

76 ••

Picture the Problem The average power delivered by the car's engine is the rate at which it changes the car's energy. Because the car is slowing down as it climbs the hill, its potential energy increases and its kinetic energy decreases.

Express the average power delivered by the car's engine:

$$P_{av} = \frac{\Delta E}{\Delta t}$$

Express the increase in the car's mechanical energy:

$$\Delta E = \Delta K + \Delta U = K_{\text{top}} - K_{\text{bot}} + U_{\text{top}} - U_{\text{bot}} = \tfrac{1}{2}mv_{\text{top}}^2 - \tfrac{1}{2}mv_{\text{bot}}^2 + mg\Delta h = \tfrac{1}{2}m\left(v_{\text{top}}^2 - v_{\text{bot}}^2 + 2g\Delta h\right)$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \tfrac{1}{2}(1500\,\text{kg})\left[(10\,\text{m/s})^2 - (24\,\text{m/s})^2 + 2\left(9.81\,\text{m/s}^2\right)(120\,\text{m})\right] = 1.41\,\text{MJ}$$

Assuming that the acceleration of the car is constant, find its average speed during this climb:

$$v_{\text{av}} = \frac{v_{\text{top}} + v_{\text{bot}}}{2} = 17\,\text{m/s}$$

Using the v_{av}, find the time it takes the car to climb the hill:

$$\Delta t = \frac{\Delta s}{v_{\text{av}}} = \frac{2000\,\text{m}}{17\,\text{m/s}} = 118\,\text{s}$$

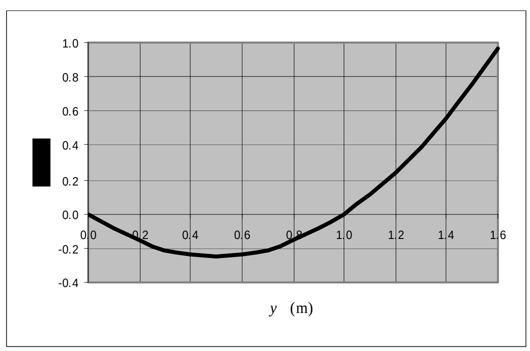
Substitute to determine P_{av}:

$$P_{\text{av}} = \frac{1.41\,\text{MJ}}{118\,\text{s}} = \boxed{11.9\,\text{kW}}$$

***77 ••**

Picture the Problem Given the potential energy function as a function of *y*, we can find the net force acting on a given system from $F = -dU/dy$. The maximum extension of the spring; i.e., the lowest position of the mass on its end, can be found by applying the work-energy theorem. The equilibrium position of the system can be found by applying the work-energy theorem with friction … as can the amount of thermal energy produced as the system oscillates to its equilibrium position.

(*a*) The graph of *U* as a function of *y* is shown to the right. Because *k* and *m* are not specified, *k* has been set equal to 2 and *mg* to 1. The spring is unstretched when $y = y_0 = 0$. Note that the minimum value of *U* (a position of stable equilibrium) occurs near $y = 0.5$ m.



(*b*) Evaluate the negative of the derivative of *U* with respect to *y*:

$$F = -\frac{dU}{dy} = -\frac{d}{dy}\left(\tfrac{1}{2}ky^2 - mgy\right) = \boxed{-ky + mg}$$

(*c*) Apply conservation of energy to the movement of the mass from $y = 0$ to $y = y_{max}$:

$$\Delta K + \Delta U + W_f = 0$$

Because $\Delta K = 0$ (the object starts from rest and is momentarily at rest at $y = y_{max}$) and $W_f = 0$ (no friction), it follows that:

$$\Delta U = U(y_{max}) - U(0) = 0$$

Because $U(0) = 0$, it follows that:

$$U(y_{max}) = 0 \Rightarrow \tfrac{1}{2}ky_{max}^2 - mgy_{max} = 0$$

Solve for y_{max}:

$$y_{max} = \boxed{\frac{2mg}{k}}$$

(*d*) Express the condition of F at equilibrium and solve for y_{eq}:

$$F_{eq} = 0 \Rightarrow -ky_{eq} + mg = 0$$

and

$$y_{eq} = \boxed{\frac{mg}{k}}$$

(*e*) Apply the conservation of energy to the movement of the mass from $y = 0$ to $y = y_{eq}$ and solve for W_f:

$$\Delta K + \Delta U + W_f = 0$$

or, because $\Delta K = 0$,

$$W_f = -\Delta U = U_i - U_f$$

Because $U_i = U(0) = 0$:

$$W_f = -U_f = -\left(\tfrac{1}{2}ky_{eq}^2 - mgy_{eq}\right)$$

Substitute for y_{eq} and simplify to obtain:

$$W_f = \boxed{\frac{m^2g^2}{2k}}$$

78 ••

Picture the Problem The energy stored in the compressed spring is initially transformed into the kinetic energy of the signal flare and then into gravitational potential energy and thermal energy as the flare climbs to its maximum height. Let the system contain the earth, the air, and the flare so that $W_{ext} = 0$. We can use the work-energy theorem with friction in the analysis of the energy transformations during the motion of the flare.

(*a*) The work done on the spring in compressing it is equal to the kinetic energy of the flare at launch.

$$W_s = K_{i,\,flare} = \boxed{\tfrac{1}{2}mv_0^2}$$

Therefore:

(*b*) Ignoring changes in gravitational potential energy (i.e., assume that the compression of the spring is small compared to the maximum elevation of the flare), apply the conservation of energy to the transformation that takes place as the spring decompresses and gives the flare its launch speed:

$$\Delta K + \Delta U_s = 0$$

or

$$K_f - K_i + U_{s,f} - U_{s,i} = 0$$

Because $K_i = \Delta U_g = U_{s,f}$:

$$K_f - U_{s,i} = 0$$

Substitute for K_f and $U_{s,i}$:

$$\tfrac{1}{2}mv_0^2 - \tfrac{1}{2}kd^2 = 0$$

Solve for k to obtain:

$$k = \boxed{\frac{mv_0^2}{d^2}}$$

(*c*) Apply conservation of energy to the upward trajectory of the flare:

$$\Delta K + \Delta U_g + W_f = 0$$

Solve for W_f:

$$\begin{aligned} W_f &= -\Delta K - \Delta U_g \\ &= K_i - K_f + U_i - U_f \end{aligned}$$

Because $K_f = U_i = 0$:

$$W_f = \boxed{\tfrac{1}{2}mv_0^2 - mgh}$$

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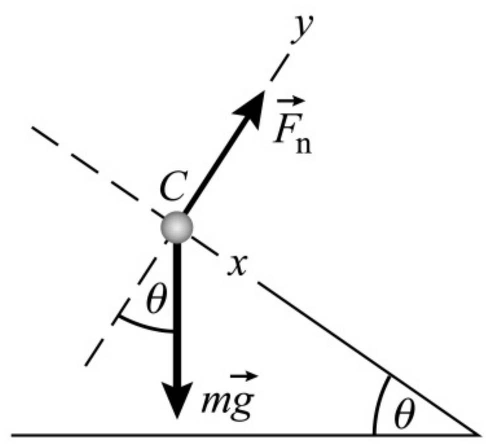
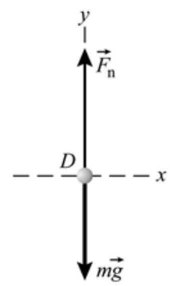
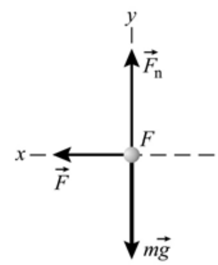
Picture the Problem Let $U_D = 0$. Choose the system to include the earth, the track, and the car. Then there are no external forces to do work on the system and change its energy and we can use Newton's 2nd law and the work-energy theorem to describe the system's energy transformations to point G … and then the work-energy theorem with friction to determine the braking force that brings the car to a stop. The free-body diagram for point C is shown to the right.



The free-body diagram for point D is shown to the right.



The free-body diagram for point F is shown to the right.



(*a*) Apply the work-energy theorem to the system's energy transformations between A and B:

$$\Delta K + \Delta U = 0$$
or
$$K_B - K_A + U_B - U_A = 0$$

If we assume that the car arrives at point B with $v_B = 0$, then:

$$-\tfrac{1}{2}mv_A^2 + mg\Delta h = 0$$
where Δh is the difference in elevation between A and B.

Solve for and evaluate Δh:

$$\Delta h = \frac{v_A^2}{2g}$$

Substitute numerical values and evaluate Δh:

$$\Delta h = \frac{(12\,\text{m/s})^2}{2(9.81\,\text{m/s}^2)} = 7.34\,\text{m}$$

Express the height above the ground:

$$h + \Delta h = 10\,\text{m} + 7.34\,\text{m} = \boxed{17.3\,\text{m}}$$

(*b*) If the car just makes it to point B, i.e., if it gets there with $v_B = 0$, then the force exerted by the track on the car will be the normal force:

$$F_{\text{track on car}} = F_n = mg = (500\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{4.91\,\text{kN}}$$

(*c*) Apply $\sum F_x = ma_x$ to the car at point C (see the FBD) and solve for and evaluate a:

$$mg\sin\theta = ma$$

and

$$a = g\sin\theta = (9.81\,\text{m/s}^2)\sin 30° = \boxed{4.91\,\text{m/s}^2}$$

(*d*) Apply $\sum F_y = ma_y$ to the car at point D (see the FBD) and solve for F_n:

$$F_n - mg = m\frac{v_D^2}{R}$$

and

$$F_n = mg + m\frac{v_D^2}{\text{R}}$$

Apply the work-energy theorem to the system's energy transformations between B and D:

$$\Delta K + \Delta U = 0$$

or

$$K_D - K_B + U_D - U_B = 0$$

Because $K_B = U_D = 0$:

$$K_D - U_B = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_D^2 - mg(h + \Delta h) = 0$$

Solve for v_D^2:

$$v_D^2 = 2g(h + \Delta h)$$

Substitute to find F_n:

$$F_n = mg + m\frac{v_D^2}{\text{R}} = mg + m\frac{2g(h + \Delta h)}{R} = mg\left[1 + \frac{2(h + \Delta h)}{R}\right]$$

Substitute numerical values and evaluate F_n:

$$F_n = (500\,\text{kg})(9.81\,\text{m/s}^2)\left[1 + \frac{2(17.3\,\text{m})}{20\,\text{m}}\right] = \boxed{13.4\,\text{kN, directed upward.}}$$

(*e*) F has two components at point F; one horizontal (the inward force that the track exerts) and the other vertical (the normal force). Apply $\sum \vec{F} = m\vec{a}$ to the car at point F:

$$\sum F_y = F_n - mg = 0 \Rightarrow F_n = mg$$

and

$$\sum F_x = F_c = m\frac{v_F^2}{R}$$

Express the resultant of these two forces:

$$F = \sqrt{F_c^2 + F_n^2} = \sqrt{\left(m\frac{v_F^2}{R}\right)^2 + (mg)^2} = m\sqrt{\frac{v_F^4}{R^2} + g^2}$$

Substitute numerical values and evaluate F:

$$F = (500\,\text{kg})\sqrt{\frac{(12\,\text{m/s})^4}{(30\,\text{m})^2} + \left(9.81\,\text{m/s}^2\right)^2} = \boxed{5.46\,\text{kN}}$$

Express the angle the resultant makes with the x axis:

$$\theta = \tan^{-1}\left[\frac{F_n}{F_c}\right] = \tan^{-1}\left[\frac{gR}{v_F^2}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \tan^{-1}\left[\frac{\left(9.81\,\text{m/s}^2\right)(30\,\text{m})}{(12\,\text{m/s})^2}\right] = \boxed{63.9°}$$

(*f*) Apply the work-energy theorem with friction to the system's energy transformations between F and the car's stopping position:

$$-K_G + W_f = 0$$

and

$$W_f = K_G = \tfrac{1}{2}mv_G^2$$

The work done by friction is also given by:

$$W_f = F_{brake}d$$

where d is the stopping distance.

Equate the two expressions for W_f:

$$F_{brake}d = \tfrac{1}{2}mv_F^2$$

Solve for F_{brake}

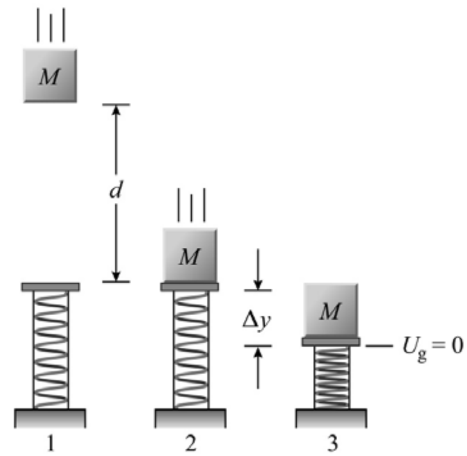
$$F_{brake} = \frac{mv_F^2}{2d}$$

Substitute numerical values and evaluate F_{brake}

$$F_{brake} = \frac{(500\,\text{kg})(12\,\text{m/s})^2}{2(25\,\text{m})} = \boxed{1.44\,\text{kN}}$$

***80** •

Picture the Problem The rate of conversion of mechanical energy can be determined from $P = \vec{F} \cdot \vec{v}$. The pictorial representation shows the elevator moving downward just as it goes into freefall as state 1. In state 2 the elevator is moving faster and is about to strike the relaxed spring. The momentarily at rest elevator on the compressed spring is shown as state 3. Let $U_g = 0$ where the spring has its maximum compression and the system consist of the earth, the elevator, and the spring. Then $W_{ext} = 0$ and we can apply the conservation of mechanical energy to the analysis of the falling elevator and compressing spring.



(*a*) Express the rate of conversion of mechanical energy to thermal energy as a function of the speed of the elevator and braking force acting on it:

$$P = F_{braking} v_0$$

Because the elevator is moving with constant speed, the net force acting on it is zero and:

$$F_{braking} = Mg$$

Substitute for $F_{braking}$ and evaluate P:

$$P = Mgv_0 = (2000\,\text{kg})(9.81\,\text{m/s}^2)(1.5\,\text{m/s}) = \boxed{29.4\,\text{kW}}$$

(*b*) Apply the conservation of energy to the falling elevator and compressing spring:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$K_3 - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_3 = U_{g,3} = U_{s,1} = 0$:

$$-\tfrac{1}{2}Mv_0^2 - Mg(d + \Delta y) + \tfrac{1}{2}k(\Delta y)^2 = 0$$

Rewrite this equation as a quadratic equation in Δy, the maximum compression of the spring:

$$(\Delta y)^2 - \left(\frac{2Mg}{k}\right)\Delta y - \frac{M}{k}\left(2gd + v_0^2\right) = 0$$

Solve for Δy to obtain:

$$\Delta y = \frac{Mg}{k} \pm \sqrt{\frac{M^2 g^2}{k^2} + \frac{M}{k}\left(2gd + v_0^2\right)}$$

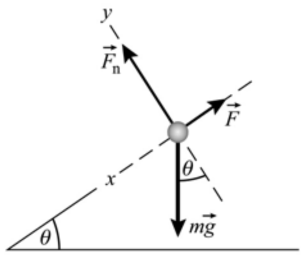
Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{(2000\,\text{kg})(9.81\,\text{m/s}^2)}{1.5\times10^4\ \text{N/m}} + \sqrt{\frac{(2000\,\text{kg})^2(9.81\,\text{m/s}^2)^2}{(1.5\times10^4\ \text{N/m})^2} + \frac{2000\,\text{kg}}{1.5\times10^4\ \text{N/m}}\left[2(9.81\,\text{m/s}^2)(5\,\text{m}) + (1.5\,\text{m/s})^2\right]}$$
$$= \boxed{5.19\,\text{m}}$$

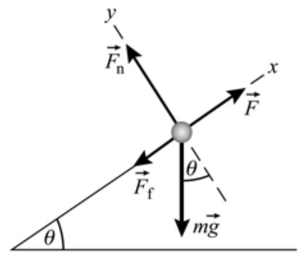
81 •

Picture the Problem We can use Newton's 2nd law to determine the force of friction as a function of the angle of the hill for a given constant speed. The power output of the engine is given by $P = \vec{F}_\text{f} \cdot \vec{v}$.

FBD for (*a*):



FBD for (*b*):



(*a*) Apply $\sum F_x = ma_x$ to the car:

$$mg\sin\theta - F_\text{f} = 0 \Rightarrow F_\text{f} = mg\sin\theta$$

Evaluate F_f for the two speeds:

$$F_{20} = (1000\,\text{kg})(9.81\,\text{m/s}^2)\sin 2.87° = \boxed{491\,\text{N}}$$

and

$$F_{30} = (1000\,\text{kg})(9.81\,\text{m/s}^2)\sin 5.74° = \boxed{981\,\text{N}}$$

(*b*) Express the power an engine must deliver on a level road in order to overcome friction loss and evaluate this expression for v = 20 m/s and 30 m/s:

$$P = F_f v$$

$$P_{20} = (491\,\text{N})(20\,\text{m/s}) = \boxed{9.82\,\text{kW}}$$

and

$$P_{30} = (981\,\text{N})(30\,\text{m/s}) = \boxed{29.4\,\text{kW}}$$

(*c*) Apply $\sum F_x = ma_x$ to the car:

$$\sum F_x = F - mg\sin\theta - F_f = 0$$

Relate F to the power output of the engine and the speed of the car:

$$\text{Since } P = Fv,\ F = \frac{P}{v}$$

Substitute for F and solve for θ:

$$\theta = \sin^{-1}\frac{\frac{P}{v} - F_{20}}{mg}$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\frac{\frac{40\,\text{kW}}{20\,\text{m/s}} - 491\,\text{N}}{(1000\,\text{kg})(9.81\,\text{m/s}^2)} = \boxed{8.85°}$$

(*d*) Express the equivalence of the work done by the engine in driving the car at the two speeds:

$$W_{\text{engine}} = F_{20}(\Delta s)_{20} = F_{30}(\Delta s)_{30}$$

Let ΔV represent the volume of fuel consumed by the engine driving the car on a level road and divide both sides of the work equation by ΔV to obtain:

$$F_{20}\frac{(\Delta s)_{20}}{\Delta V} = F_{30}\frac{(\Delta s)_{30}}{\Delta V}$$

Solve for $\frac{(\Delta s)_{30}}{\Delta V}$:

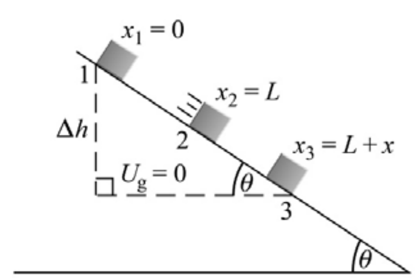
$$\frac{(\Delta s)_{30}}{\Delta V} = \frac{F_{20}}{F_{30}}\frac{(\Delta s)_{20}}{\Delta V}$$

Substitute numerical values and evaluate $\frac{(\Delta s)_{30}}{\Delta V}$:

$$\frac{(\Delta s)_{30}}{\Delta V} = \frac{491\,\text{N}}{981\,\text{N}}(12.7\,\text{km/L}) = \boxed{6.36\,\text{km/L}}$$

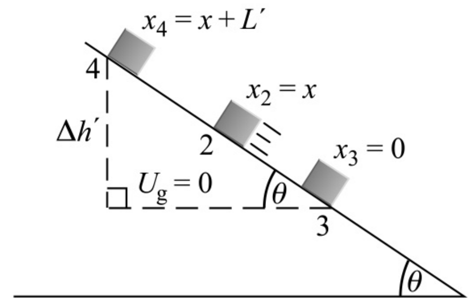
82 ••

Picture the Problem Let the system include the earth, block, spring, and incline. Then $W_{\text{ext}} = 0$. The top pictorial representation shows the block sliding down the incline and compressing the spring. Choose $U_g = 0$ at the elevation at which the spring is fully compressed. We can use the conservation of mechanical



energy to determine the maximum compression of the spring.

The pictorial representation to the right shows the block sliding up the rough incline after being accelerated by the fully compressed spring. We can use the work-energy theorem with friction to determine how far up the incline the block slides before stopping.



(*a*) Apply conservation of mechanical energy to the system as it evolves from state 1 to state 3:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$K_3 - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_3 = K_1 = U_{g,3} = U_{s,1} = 0$:

$$-U_{g,1} + U_{s,3} = 0$$

or

$$-mg\Delta h + \tfrac{1}{2}kx^2 = 0$$

Relate Δh to $L + x$ and θ and substitute to obtain:

$$\Delta h = (L + x)\sin\theta$$

$$\therefore \tfrac{1}{2}kx^2 - mg(L + x)\sin\theta = 0$$

Rewrite this equation in the form of an explicit quadratic equation:

$$\tfrac{1}{2}kx^2 - (mg\sin\theta)x - mgL\sin\theta = 0$$

Substitute for *k*, *m*, *g*, θ, and *L* to obtain:

$$\left(50\,\frac{\text{N}}{\text{m}}\right)x^2 - (9.81\,\text{N})x - 39.24\,\text{J} = 0$$

Solve for the physically meaningful (i.e., positive) root:

$$x = \boxed{0.989\,\text{m}}$$

(*b*) Proceed as in (*a*) but include work done by friction:

$$-U_{g,1} + U_{s,3} + W_f = 0$$

Express the mechanical energy transformed to thermal energy:

$$W_f = F_f(L + x) = \mu_k F_n(L + x) = \mu_k mg\cos\theta(L + x)$$

Substitute for Δh and W_f to obtain:

$$-mg(L+x)\sin\theta+\tfrac{1}{2}kx^2+\mu_k mg\cos\theta(L+x)=0$$

Substitute for k, m, g, θ, μ_k, and L to obtain:

$$\left(50\,\frac{\text{N}}{\text{m}}\right)x^2-(6.41\,\text{N})x-25.65\,\text{J}=0$$

Solve for the positive root:

$$x=\boxed{0.783\,\text{m}}$$

(*c*) Apply the work-energy theorem with friction to the system as it evolves from state 3 to state 4:

$$K_4-K_3+U_{g,4}-U_{g,3}+U_{s,4}-U_{s,3}+W_f=0$$

Because $K_4=K_1=U_{g,3}=U_{s,4}=0$:

$$U_{g,4}-U_{s,3}+W_f=0$$

or

$$-mg\Delta h'+\tfrac{1}{2}kx^2+W_f=0$$

Substitute for $\Delta h'$ and W_f to obtain:

$$-mg(L'+x)\sin\theta+\tfrac{1}{2}kx^2+\mu_k mg\cos\theta(L'+x)=0$$

Solve for L' with x = 0.783 m:

$$L'=\boxed{1.54\,\text{m}}$$

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Picture the Problem The work done by the engines maintains the kinetic energy of the cars and overcomes the work done by frictional forces. Let the system include the earth, track, and the cars *but not the engines*. Then the engines will do external work on the system and we can use this work to find the power output of the train's engines.

(*a*) Use the definition of kinetic energy to evaluate K:

$$K=\tfrac{1}{2}mv^2=\tfrac{1}{2}(2\times10^6\,\text{kg})\left(15\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\right)^2=\boxed{17.4\,\text{MJ}}$$

(*b*) Use the definition of potential energy to express and evaluate the change in potential energy of the train:

$$\Delta U=mg\Delta h=(2\times10^6\,\text{kg})(9.81\,\text{m/s}^2)(707\,\text{m})=\boxed{1.39\times10^{10}\,\text{J}}$$

(*c*) Express the energy dissipated by kinetic friction:

$$W_f=F_f\Delta s$$

Express the frictional force:

$$F_f = 0.008mg$$

Substitute for F_f and evaluate W_f:

$$W_f = 0.008mg\Delta s = 0.008(2\times10^6\text{ kg})\times(9.81\text{ m/s}^2)(62\text{ km}) = \boxed{9.73\times10^9\text{ J}}$$

(*d*) Express the power output of the train's engines in terms of the work done by them:

$$P = \frac{\Delta W}{\Delta t}$$

Use the work-energy theorem with friction to find the work done by the train's engines:

$$W_{ext} = \Delta K + \Delta U + W_f = \Delta U + W_f\text{, since } \Delta K = 0.$$

Find the time during which the engines do this work:

$$\Delta t = \frac{\Delta s}{v}$$

Substitute in the expression for P to obtain:

$$P = \frac{(\Delta U + W_f)v}{\Delta s}$$

Substitute numerical values and evaluate P:

$$P = \frac{(1.39\times10^{10}\text{ J} + 9.73\times10^9\text{ J})\left(15\frac{\text{km}}{\text{h}}\times\frac{1\text{ h}}{3600\text{ s}}\right)}{62\text{ km}} = \boxed{1.59\text{ MW}}$$

***84 ••**

Picture the Problem While on a horizontal surface, the work done by an automobile engine changes the kinetic energy of the car and does work against friction. These energy transformations are described by the work-energy theorem with friction. Let the system include the earth, the roadway, and the car *but not the car's engine*.

(*a*) The required energy equals the change in the kinetic energy of the car:

$$\Delta K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1200\text{ kg})\left(50\frac{\text{km}}{\text{h}}\times\frac{1\text{ h}}{3600\text{ s}}\right)^2 = \boxed{116\text{ kJ}}$$

(*b*) The required energy equals the work done against friction:

$$W_f = F_f\Delta s$$

Substitute numerical values and evaluate W_f:

$$W_f = (300\,\text{N})(300\,\text{m}) = \boxed{90.0\,\text{kJ}}$$

(*c*) Apply the work-energy theorem with friction to express the required energy:

$$E' = W_{ext} = \Delta K + W_f = \Delta K + 0.75E$$

Divide both sides of the equation by *E* to express the ratio of the two energies:

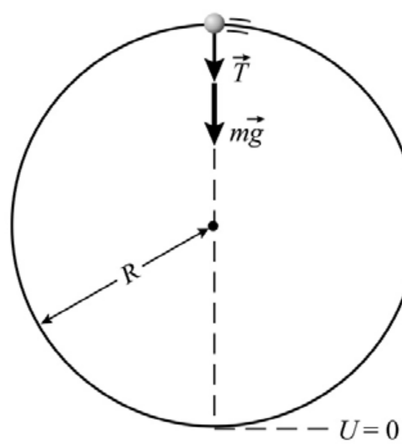
$$\frac{E'}{E} = \frac{\Delta K}{E} + 0.75$$

Substitute numerical values and evaluate E'/E:

$$\frac{E'}{E} = \frac{116\,\text{kJ}}{90\,\text{kJ}} + 0.75 = \boxed{2.04}$$

***85** •••

Picture the Problem Assume that the bob is moving with speed *v* as it passes the top vertical point when looping around the peg. There are two forces acting on the bob: the tension in the string (if any) and the force of gravity, *Mg*; both point downward when the ball is in the topmost position. The minimum possible speed for the bob to pass the vertical occurs when the tension is 0; from this, gravity must supply the centripetal force required to keep the ball moving in a circle. We can use conservation of energy to relate *v* to *L* and *R*.



Express the condition that the bob swings around the peg in a full circle:

$$M\frac{v^2}{R} > Mg$$

Simplify to obtain:

$$\frac{v^2}{R} > g$$

Use conservation of energy to relate the kinetic energy of the bob at the bottom of the loop to its potential energy at the top of its swing:

$$\tfrac{1}{2}Mv^2 = Mg(L - 2R)$$

Solve for v^2:

$$v^2 = 2g(L - 2R)$$

Substitute to obtain:

$$\frac{2g(L - 2R)}{R} > g$$

Solve for R:

$$R < \boxed{\frac{2}{5}L}$$

86 ••

Picture the Problem If the wood exerts an average force F on the bullet, the work it does has magnitude FD. This must be equal to the change in the kinetic energy of the bullet, or because the final kinetic energy of the bullet is zero, to the negative of the initial kinetic energy. We'll let m be the mass of the bullet and v its initial speed and apply the work-kinetic energy theorem to relate the penetration depth to v.

Apply the work-kinetic energy theorem to relate the penetration depth to the change in the kinetic energy of the bullet:

$$W_{\text{total}} = \Delta K = K_{\text{f}} - K_{\text{i}}$$

or, because $K_{\text{f}} = 0$,

$$W_{\text{total}} = -K_{\text{i}}$$

Substitute for W_{total} and K_{i} to obtain:

$$FD = -\tfrac{1}{2}mv^2$$

Solve for D to obtain:

$$D = -\frac{mv^2}{2F}$$

For an identical bullet with twice the speed we have:

$$FD' = -\tfrac{1}{2}m(2v)^2$$

Solve for D' to obtain:

$$D' = 4\left(-\frac{mv^2}{2F}\right) = 4D$$

and $\boxed{(c)\text{ is correct.}}$

87 ••

Picture the Problem For part (a), we'll let the system include the glider, track, weight, and the earth. The speeds of the glider and the falling weight will be the same while they are in motion. Let their common speed when they have moved a distance Y be v and let the zero of potential energy be at the elevation of the weight when it has fallen the distance Y. We can use conservation of energy to relate the speed of the glider (and the weight) to the distance the weight has fallen. In part (b), we'll let the direction of motion be the x direction, the tension in the connecting string be T, and apply Newton's 2nd law to the glider and the weight to find their common acceleration. Because this acceleration is constant, we can use a constant-acceleration equation to find their common speed when they have moved a distance Y.

(a) Use conservation of energy to relate the kinetic and potential energies of the system:

$$\Delta K + \Delta U = 0$$

or

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

Because the system starts from rest and $U_{\text{f}} = 0$:

$$K_{\text{f}} - U_{\text{i}} = 0$$

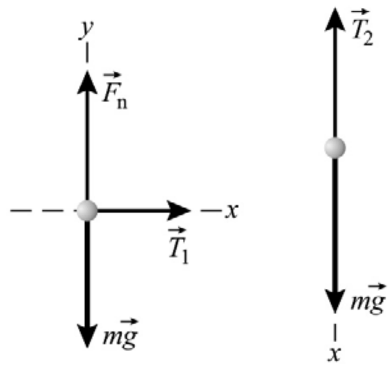
Substitute to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}Mv^2 - mgY = 0$$

Solve for v:

$$v = \boxed{\sqrt{\frac{2mgY}{M+m}}}$$

(b) The free-body diagrams for the glider and the weight are shown to the right:



Apply Newton's 3[rd] law to obtain:

$$\left|\vec{T}_1\right| = \left|\vec{T}_2\right| = T$$

Apply $\sum F_x = ma$ to the glider:

$$T = Ma$$

Apply $\sum F_x = ma$ to the weight:

$$mg - T = ma$$

Add these equations to eliminate T and obtain:

$$mg = Ma + ma$$

Solve for a to obtain:

$$a = g\frac{m}{m+M}$$

Using a constant-acceleration equation, relate the speed of the glider to its initial speed and to the distance that the weight has fallen:

$$v^2 = v_0^2 + 2aY$$

or, because $v_0 = 0$,

$$v^2 = 2aY$$

Substitute for a and solve for v to obtain:

$$v = \boxed{\sqrt{\frac{2mgY}{M+m}}}$$, the same result we obtained in part (a).

***88 ••**

Picture the Problem We're given $P = dW/dt$ and are asked to evaluate it under the assumed conditions.

Express the rate of energy expenditure by the man:

$$P = 3mv^2 = 3(10\,\text{kg})(3\,\text{m/s})^2 = 270\,\text{W}$$

Express the rate of energy expenditure P' assuming that his muscles have an efficiency of 20%:

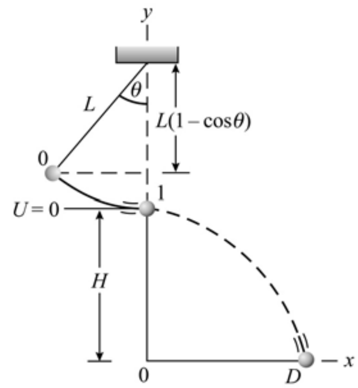
$$P = \tfrac{1}{5}P'$$

Solve for and evaluate P':

$$P' = 5P = 5(270\,\text{W}) = \boxed{1.35\,\text{kW}}$$

89 ••

Picture the Problem The pictorial representation shows the bob swinging through an angle θ before the thread is cut and it is launched horizontally. Let its speed at position 1 be v. We can use conservation of energy to relate v to the change in the potential energy of the bob as it swings through the angle θ. We can find its flight time Δt from a constant-acceleration equation and then express D as the product of v and Δt.



Relate the distance D traveled horizontally by the bob to its launch speed v and time of flight Δt:

$$D = v\Delta t \qquad (1)$$

Use conservation of energy to relate its launch speed v to the length of the pendulum L and the angle θ:

$$K_1 - K_0 + U_1 - U_0 = 0$$

or, because $U_1 = K_0 = 0$,

$$K_1 - U_0 = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv^2 - mgL(1-\cos\theta) = 0$$

Solving for v yields:

$$v = \sqrt{2gL(1-\cos\theta)}$$

In the absence of air resistance, the horizontal and vertical motions of the bob are independent of each other and we can use a constant-acceleration equation to express the time of flight (the time to fall a distance H):

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $\Delta y = -H$, $a_y = -g$, and $v_{0y} = 0$,

$$-H = -\tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt to obtain:

$$\Delta t = \sqrt{2H/g}$$

Substitute in equation (1) and simplify to obtain:

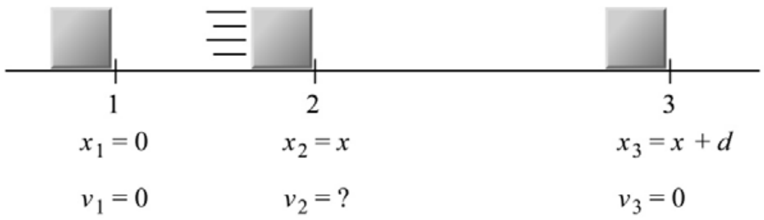
$$D = \sqrt{2gL(1-\cos\theta)}\sqrt{\frac{2H}{g}}$$

$$= \boxed{2\sqrt{HL(1-\cos\theta)}}$$

which shows that, while D depends on θ, it is independent of g.

90 ••

Picture the Problem The pictorial representation depicts the block in its initial position against the compressed spring (1), as it separates from the spring with its maximum kinetic energy (2), and when it has come to rest after moving a distance $x + d$. Let the system consist of the earth, the block, and the surface on which the block slides. With this choice, $W_{ext} = 0$. We can use the work-energy theorem with friction to determine how far the block will slide before coming to rest.



(*a*) Express the work done by the spring on the block:

$$W_{spring} = \Delta U_{spring} = \tfrac{1}{2}kx^2$$

Substitute numerical values and evaluate W_{spring}:

$$W_{spring} = \tfrac{1}{2}(20\text{ N/cm})(3\text{ cm})^2 = \boxed{0.900\text{ J}}$$

(*b*) Relate the energy dissipated by friction to the friction force and the displacement of the block:

$$W_f = F_f\Delta x = \mu_k F_n \Delta x = \mu_k mg\Delta x$$

Substitute numerical values and evaluate W_f:

$$W_f = (0.2)(5\text{ kg})(9.81\text{ m/s}^2)(0.03\text{ m}) = \boxed{0.294\text{ J}}$$

(*c*) Apply the conservation of energy between points 1 and 2:

$$K_2 - K_1 + U_{s,2} - U_{s,1} + W_f = 0$$

Because $K_1 = U_{s,2} = 0$:

$$K_2 - U_{s,1} + W_f = 0$$

Substitute to obtain:

$$\tfrac{1}{2}mv_2^2 - \tfrac{1}{2}kx^2 + W_f = 0$$

Solve for v_2:

$$v_2 = \sqrt{\frac{kx^2 - 2W_f}{m}}$$

Substitute numerical values and evaluate v_2:

$$v_2 = \sqrt{\frac{(20\,\text{N/cm})(3\,\text{cm})^2 - 2(0.294\,\text{J})}{5\,\text{kg}}}$$

$$= \boxed{0.492\,\text{m/s}}$$

(*d*) Apply the conservation of energy between points 1 and 3:

$$\Delta K + U_{s,3} - U_{s,1} + W_f = 0$$

Because $\Delta K = U_{s,3} = 0$:

$$-U_{s,1} + W_f = 0$$

or

$$-\tfrac{1}{2}kx^2 + \mu_k mg(x + d) = 0$$

Solve for *d*:

$$d = \frac{kx^2}{2\mu_k mg} - x$$

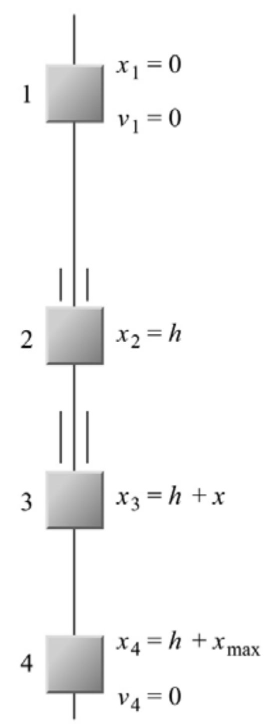
Substitute numerical values and evaluate *d*:

$$d = \frac{(20\,\text{N/cm})(3\,\text{cm})^2}{2(0.2)(5\,\text{kg})(9.81\,\text{m/s}^2)} - 0.03\,\text{m}$$

$$= \boxed{6.17\,\text{cm}}$$

91 ••

Picture the Problem The pictorial representation shows the block initially at rest at point 1, falling under the influence of gravity to point 2, partially compressing the spring as it continues to gain kinetic energy at point 3, and finally coming to rest at point 4 with the spring fully compressed. Let the system consist of the earth, the block, and the spring so that $W_{ext} = 0$. Let $U_g = 0$ at point 3 for part (*a*) and at point 4 for part (*b*). We can use the work-energy theorem to express the kinetic energy of the system as a function of the block's position and then use this function to maximize *K* as well as determine the maximum compression of the spring and the location of the block when the system has half its maximum kinetic energy.



(*a*) Apply conservation of

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

mechanical energy to describe the energy transformations between state 1 and state 3:

or

$$K_3 - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_1 = U_{g,3} = U_{s,1} = 0$:

$$K_3 - U_{g,1} + U_{s,3} = 0$$

and

$$K_3 = K = mg(h+x) - \tfrac{1}{2}kx^2$$

Differentiate K with respect to x and set this derivative equal to zero to identify extreme values:

$$\frac{dK}{dx} = mg - kx = 0 \text{ for extreme values.}$$

Solve for x:

$$x = \frac{mg}{k}$$

Evaluate the second derivative of K with respect to x:

$$\frac{d^2K}{dt^2} = -k < 0$$

$$\Rightarrow x = \frac{mg}{k} \text{ maximizes } K.$$

Evaluate K for $x = mg/k$:

$$K_{max} = mgh + mg\left(\frac{mg}{k}\right) - \tfrac{1}{2}k\left(\frac{mg}{k}\right)^2$$

$$= \boxed{mgh + \frac{m^2g^2}{2k}}$$

(*b*) The spring will have its maximum compression at point 4 where $K = 0$:

$$mg(h + x_{max}) - \tfrac{1}{2}kx_{max}^2 = 0$$

or

$$x_{max}^2 - \frac{2mg}{k}x_{max} - \frac{2mgh}{k} = 0$$

Solve for x and keep the physically meaningful root:

$$x_{max} = \boxed{\frac{mg}{k} + \sqrt{\frac{m^2g^2}{k^2} + \frac{2mgh}{k}}}$$

(*c*) Apply conservation of mechanical energy to the system as it evolves from state 1 to the state in which $K = \tfrac{1}{2}K_{max}$:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

or

$$K - K_1 + U_{g,3} - U_{g,1} + U_{s,3} - U_{s,1} = 0$$

Because $K_1 = U_{g,3} = U_{s,1} = 0$:

$$K - U_{g,1} + U_{s,3} = 0$$

and

$K = mg(h + x) - \frac{1}{2}kx^2$

Substitute for K to obtain:

$$\frac{1}{2}\left(mgh + \frac{m^2g^2}{2k}\right) = mg(h + x) - \frac{1}{2}kx^2$$

Express this equation in quadratic form:

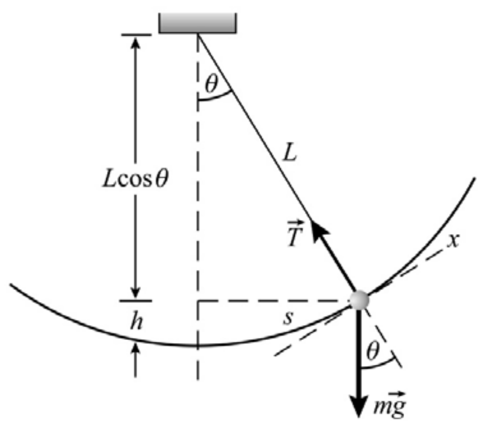
$$x^2 - \frac{2mg}{k}x + \left(\frac{m^2g^2}{2k^2} - \frac{mgh}{k}\right) = 0$$

Solve for the positive value of x:

$$\boxed{x = \frac{mg}{k} + \sqrt{\frac{2m^2g^2}{k^2} + \frac{4mgh}{k}}}$$

92 •••

Picture the Problem The free-body diagram shows the forces acting on the pendulum bob. The application of Newton's 2nd law leads directly to the required expression for the tangential acceleration. Recall that, provided θ is in radian measure, $s = L\theta$. Differentiation with respect to time produces the result called for in part (*b*). The remaining parts of the problem simply require following the directions for each part.



(*a*) Apply $\sum F_x = ma_x$ to the bob:

$$F_{\tan} = -mg\sin\theta = ma_{\tan}$$

Solve for $a_{\tan}$:

$$\boxed{a_{\tan} = dv/dt = -g\sin\theta}$$

(*b*) Relate the arc distance s to the length of the pendulum L and the angle θ:

$$s = L\theta$$

Differentiate with respect to time:

$$ds/dt = \boxed{v = L\,d\theta/dt}$$

(*c*) Multiply $\frac{dv}{dt}$ by $\frac{d\theta}{d\theta}$ and substitute for $\frac{d\theta}{dt}$ from part (*b*):

$$\frac{dv}{dt}=\frac{dv}{dt}\frac{d\theta}{d\theta}=\frac{dv}{d\theta}\frac{d\theta}{dt}$$
$$=\boxed{\frac{dv}{d\theta}\left(\frac{v}{L}\right)}$$

(*d*) Equate the expressions for *dv*/*dt* from (*a*) and (*c*):

$$\frac{dv}{d\theta}\left(\frac{v}{L}\right)=-g\sin\theta$$

Separate the variables to obtain:

$$\boxed{vdv=-gL\sin\theta\, d\theta}$$

(*e*) Integrate the left side of the equation in part (*d*) from $v = 0$ to the final speed v and the right side from $\theta = \theta_0$ to $\theta = 0$:

$$\int_0^v v'dv'=\int_{\theta_0}^0 -gL\sin\theta' d\theta'$$

Evaluate the limits of integration to obtain:

$$\tfrac{1}{2}v^2 = gL(1-\cos\theta_0)$$

Note, from the figure, that $h = L(1 - \cos\theta_0)$. Substitute and solve for v:

$$v=\boxed{\sqrt{2gh}}$$

93 •••

Picture the Problem The potential energy of the climber is the sum of his gravitational potential energy and the potential energy stored in the spring-like bungee cord. Let θ be the angle which the position of the rock climber on the cliff face makes with a vertical axis and choose the zero of gravitational potential energy to be at the bottom of the cliff. We can use the definitions of U_g and U_{spring} to express the climber's total potential energy.

(*a*) Express the total potential energy of the climber:

$$U(s)=U_{\text{bungee cord}}+U_g$$

Substitute to obtain:

$$U(s)=\tfrac{1}{2}k(s-L)^2+Mgy$$
$$=\tfrac{1}{2}k(s-L)^2+MgH\cos\theta$$
$$=\tfrac{1}{2}k(s-L)^2+MgH\cos\left(\frac{s}{H}\right)$$

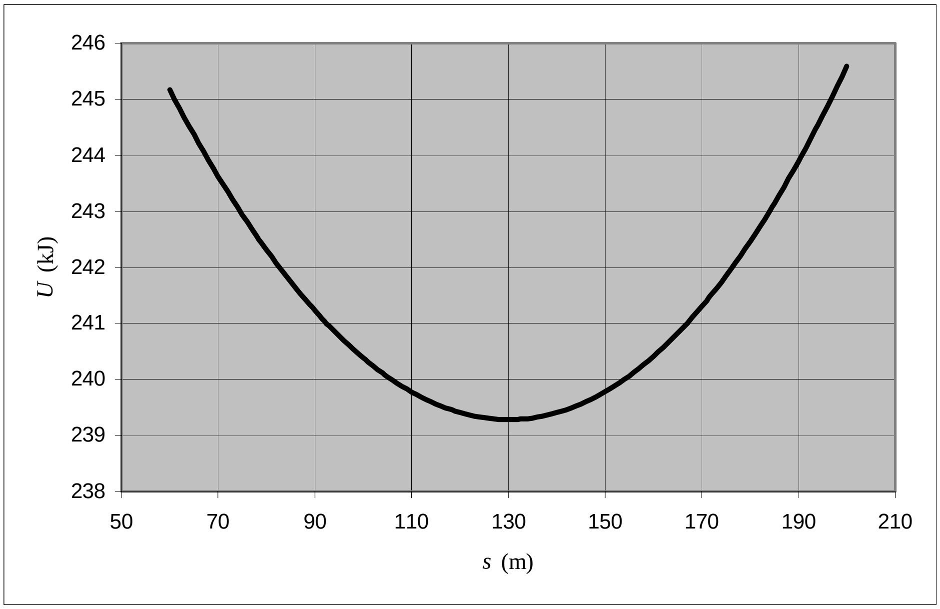
A spreadsheet solution is shown below. The constants used in the potential energy function and the formulas used to calculate the potential energy are as follows:

Cell	Content/Formula	Algebraic Form
B3	300	*H*
B4	5	*k*
B5	60	*L*

B6	85	M
B7	9.81	g
D11	60	s
D12	D11+1	$s + 1$
E11	0.5*$B$4*(D11–$B$5)^2 +$B$6*$B$7*$B$3*(cos(D11/$B$3))	$\frac{1}{2}k(s-L)^2 + MgH\cos\left(\frac{s}{H}\right)$
G11	E11–E61	$U(60\,\mathrm{m}) - U(110\,\mathrm{m})$

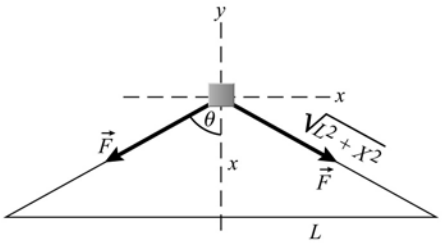
	A	B	C	D	E
1					
2					
3	H =	300	m		
4	k =	5	N/m		
5	L =	60	m		
6	m =	85	kg		
7	g =	9.81	m/s^2		
8					
9					
10				s	U(s)
11				60	2.45E+05
12				61	2.45E+05
13				62	2.45E+05
14				63	2.45E+05
15				64	2.45E+05
147				196	2.45E+05
148				197	2.45E+05
149				198	2.45E+05
150				199	2.45E+05
151				200	2.46E+05

The following graph was plotted using the data from columns D (s) and E ($U(s)$).



***94** •••

Picture the Problem The diagram to the right shows the forces each of the springs exerts on the block. The change in the potential energy stored in the springs is due to the elongation of both springs when the block is displaced a distance x from its equilibrium position and we can find ΔU using $\frac{1}{2}k(\Delta L)^2$. We can find the magnitude of the force pulling the block back toward its equilibrium position by finding the sum of the magnitudes of the y components of the forces exerted by the springs. In Part (d) we can use conservation of energy to find the speed of the block as it passes through its equilibrium position.



(a) Express the change in the potential energy stored in the springs when the block is displaced a distance x:

$$\Delta U = 2\left[\tfrac{1}{2}k(\Delta L)^2\right] = k(\Delta L)^2$$

where ΔL is the change in length of a spring.

Referring to the force diagram, express ΔL:

$$\Delta L = \sqrt{L^2 + x^2} - L$$

Substitute to obtain:

$$\Delta U = \boxed{k\left(\sqrt{L^2 + x^2} - L\right)^2}$$

(*b*) Sum the forces acting on the block to express $F_{\text{restoring}}$:

$$F_{\text{restoring}} = 2F\cos\theta = 2k\Delta L\cos\theta = 2k\Delta L\frac{x}{\sqrt{L^2+x^2}}$$

Substitute for ΔL to obtain:

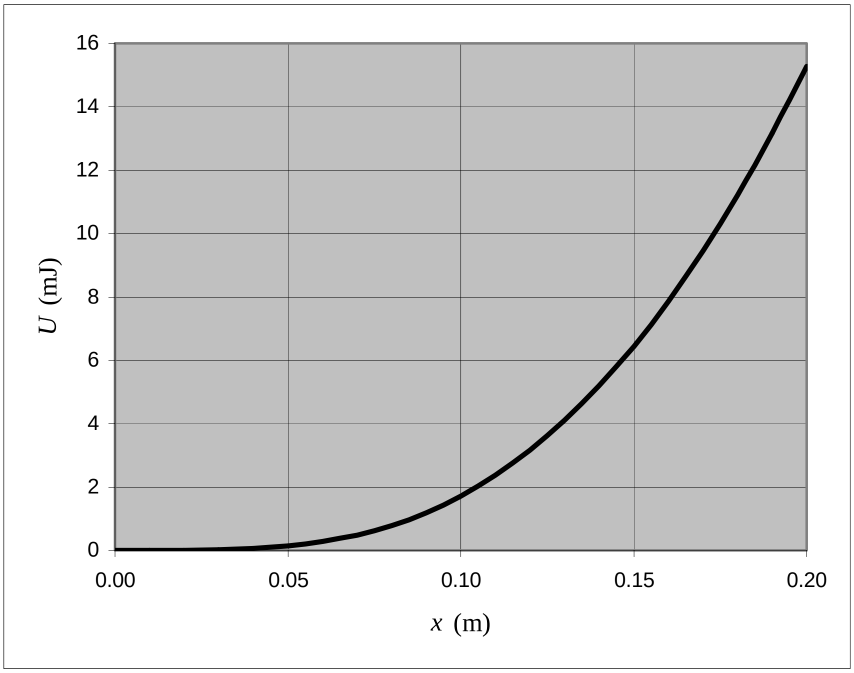
$$F_{\text{restoring}} = 2k\left(\sqrt{L^2+x^2}-L\right)\frac{x}{\sqrt{L^2+x^2}} = \boxed{2kx\left(1-\frac{L}{\sqrt{L^2+x^2}}\right)}$$

(*c*) A spreadsheet program to calculate *U*(*x*) is shown below. The constants used in the potential energy function and the formulas used to calculate the potential energy are as follows:

Cell	Content/Formula	Algebraic Form
B1	1	*L*
B2	1	*k*
B3	1	*M*
C8	C7+0.01	*x*
D7	$B$2*((C7^2+$B$1^2)^0.5–$B$1)^2	*U*(*x*)

	A	B	C	D
1	L =	0.1	m	
2	k =	1	N/m	
3	M =	1	kg	
4				
5				
6			*x*	*U*(*x*)
7			0	0
8			0.01	2.49E–07
9			0.02	3.92E–06
10			0.03	1.94E–05
11			0.04	5.93E–05
12			0.05	1.39E–04
23			0.16	7.86E–03
24			0.17	9.45E–03
25			0.18	1.12E–02
26			0.19	1.32E–02
27			0.20	1.53E–02

The following graph was plotted using the data from columns C (x) and D ($U(x)$).



(d) Use conservation of energy to relate the kinetic energy of the block as it passes through the equilibrium position to the change in its potential energy as it returns to its equilibrium position:

$$K_{\text{equilibrium}} = \Delta U$$

or

$$\tfrac{1}{2}Mv^2 = \Delta U$$

Solve for v to obtain:

$$v = \sqrt{\frac{2\Delta U}{M}} = \sqrt{\frac{2k\left(\sqrt{L^2+x^2}-L\right)^2}{M}} = \left(\sqrt{L^2+x^2}-L\right)\sqrt{\frac{2k}{M}}$$

Substitute numerical values and evaluate v:

$$v = \left(\sqrt{(0.1\,\text{m})^2+(0.1\,\text{m})^2}-0.1\,\text{m}\right)\sqrt{\frac{2(1\,\text{N/m})}{1\,\text{kg}}} = \boxed{5.86\,\text{cm/s}}$$