Chapter 8
Systems of Particles and Conservation of Momentum

Conceptual Problems

1 •

Determine the Concept A doughnut. The definition of the center of mass of an object does not require that there be any matter at its location. Any hollow sphere (such as a basketball) or an empty container with any geometry are additional examples of three-dimensional objects that have no mass at their center of mass.

***2 •**

Determine the Concept The center of mass is midway between the two balls and is in free-fall along with them (all forces can be thought to be concentrated at the center of mass.) The center of mass will initially rise, then fall.

Because the initial velocity of the center of mass is half of the initial velocity of the ball thrown upwards, the mass thrown upwards will rise for twice the time that the center of mass rises. Also, the center of mass will rise until the velocities of the two balls are equal but opposite. $\boxed{(b)\text{ is correct.}}$

3 •

Determine the Concept The acceleration of the center of mass of a system of particles is described by $\vec{F}_{\text{net,ext}} = \sum_{\text{i}} \vec{F}_{\text{i,ext}} = M\vec{a}_{\text{cm}}$, where M is the total mass of the system.

Express the acceleration of the center of mass of the two pucks:

$$a_{\text{cm}} = \frac{F_{\text{net,ext}}}{M} = \frac{F_1}{m_1 + m_2}$$

and $\boxed{(b)\text{ is correct.}}$

4 •

Determine the Concept The acceleration of the center of mass of a system of particles is described by $\vec{F}_{\text{net,ext}} = \sum_{\text{i}} \vec{F}_{\text{i,ext}} = M\vec{a}_{\text{cm}}$, where M is the total mass of the system.

Express the acceleration of the center of mass of the two pucks:

$$a_{\text{cm}} = \frac{F_{\text{net,ext}}}{M} = \frac{F_1}{m_1 + m_2}$$

because the spring force is an internal force.

$\boxed{(b)\text{ is correct.}}$

***5** •

Determine the Concept No. Consider a 1-kg block with a speed of 1 m/s and a 2- kg block with a speed of 0.707 m/s. The blocks have equal kinetic energies but momenta of magnitude 1 kg·m /s and 1.414 kg·m/s, respectively.

6 •

(*a*) True. The momentum of an object is the product of its mass and velocity. Therefore, if we are considering just the magnitudes of the momenta, the momentum of a heavy object is greater than that of a light object moving at the same speed.

(*b*) True. Consider the collision of two objects of equal mass traveling in opposite directions with the same speed. Assume that they collide inelastically. The mechanical energy of the system is not conserved (it is transformed into other forms of energy), but the momentum of the system is the same after the collision as before the collision, i.e., zero. Therefore, for any inelastic collision, the momentum of a system may be conserved even when mechanical energy is not.

(*c*) True. This is a restatement of the expression for the total momentum of a system of particles.

7 •

Determine the Concept To the extent that the system in which the rifle is being fired is an isolated system, i.e., the net external force is zero, momentum is conserved during its firing.

Apply conservation of momentum to the firing of the rifle:

$$\vec{p}_{\text{rifle}} + \vec{p}_{\text{bullet}} = 0$$

or

$$\boxed{\vec{p}_{\text{rifle}} = -\vec{p}_{\text{bullet}}}$$

***8** •

Determine the Concept When she jumps from a boat to a dock, she must, in order for momentum to be conserved, give the boat a recoil momentum, i.e., her forward momentum must be the same as the boat's backward momentum. The energy she imparts to the boat is $E_{\text{boat}} = p_{\text{boat}}^2 / 2m_{\text{boat}}$.

When she jumps from one dock to another, the mass of the dock plus the earth is so large that the energy she imparts to them is essentially zero.

***9** ••

Determine the Concept Conservation of momentum requires only that the net external force acting on the system be zero. It does not require the presence of a medium such as air.

10 •

Determine the Concept The kinetic energy of the sliding ball is $\frac{1}{2}mv_{\text{cm}}^2$. The kinetic energy of the rolling ball is $\frac{1}{2}mv_{\text{cm}}^2 + K_{\text{rel}}$, where its kinetic energy relative to its center of mass is K_{rel}. Because the bowling balls are identical and have the same velocity, the rolling ball has more energy.

11 •

Determine the Concept Think of someone pushing a box across a floor. Her push on the box is equal but opposite to the push of the box on her, but the action and reaction forces act on *different objects*. You can only add forces when they act on the same object.

12 •

Determine the Concept It's not possible for both to remain at rest after the collision, as that wouldn't satisfy the requirement that momentum is conserved. It is possible for one to remain at rest: This is what happens for a one-dimensional collision of two identical particles colliding elastically.

13 •

Determine the Concept It violates the conservation of momentum! To move forward requires pushing something backwards, which Superman doesn't appear to be doing when flying around. In a similar manner, if Superman picks up a train and throws it at Lex Luthor, he (Superman) ought to be tossed backwards at a pretty high speed to satisfy the conservation of momentum.

***14** ••

Determine the Concept There is only one force which can cause the car to move forward–the friction of the road! The car's engine causes the tires to rotate, but if the road were frictionless (as is closely approximated by icy conditions) the wheels would simply spin without the car moving anywhere. Because of friction, the car's tire pushes backwards against the road–from Newton's third law, the frictional force acting on the tire must then push it forward. This may seem odd, as we tend to think of friction as being a retarding force only, but true.

15 ••

Determine the Concept The friction of the tire against the road causes the car to slow down. This is rather subtle, as the tire is in contact with the ground without slipping at all times, and so as you push on the brakes harder, the force of static friction of the road against the tires must increase. Also, of course, the brakes heat up, and not the tires.

16 •

Determine the Concept Because $\Delta p = F\Delta t$ is constant, a safety net reduces the force acting on the performer by increasing the time Δt during which the slowing force acts.

17 •

Determine the Concept Assume that the ball travels at 80 mi/h $\approx$ 36 m/s. The ball stops in a distance of about 1 cm. So the distance traveled is about 2 cm at an average speed of about 18 m/s. The collision time is $\dfrac{0.02\,\text{m}}{18\,\text{m/s}} \approx 1\,\text{ms}$.

18 •

Determine the Concept The average force on the glass is less when falling on a carpet because Δt is longer.

19 •

(*a*) False. In a perfectly inelastic collision, the colliding bodies stick together but may or may not continue moving, depending on the momentum each brings to the collision.

(*b*) True. In a head-on elastic collision both kinetic energy and momentum are conserved and the relative speeds of approach and recession are equal.

(*c*) True. This is the definition of an elastic collision.

***20** ••

Determine the Concept All the initial kinetic energy of the isolated system is lost in a perfectly inelastic collision in which the velocity of the center of mass is zero.

21 ••

Determine the Concept We can find the loss of kinetic energy in these two collisions by finding the initial and final kinetic energies. We'll use conservation of momentum to find the final velocities of the two masses in each perfectly elastic collision.

(*a*) Letting *V* represent the velocity of the masses after their perfectly inelastic collision, use conservation of momentum to determine *V*:

$$p_{\text{before}} = p_{\text{after}}$$

or

$$mv - mv = 2mV \Rightarrow V = 0$$

Express the loss of kinetic energy for the case in which the two objects have oppositely directed velocities of magnitude *v*/2:

$$\Delta K = K_\text{f} - K_\text{i} = 0 - 2\left(\tfrac{1}{2}m\left(\frac{v}{2}\right)^2\right)$$

$$= -\frac{mv^2}{4}$$

Letting *V* represent the velocity of the masses after their perfectly inelastic collision, use conservation of momentum to determine *V*:

$$p_{\text{before}} = p_{\text{after}}$$

or

$$mv = 2mV \Rightarrow V = \tfrac{1}{2}v$$

Express the loss of kinetic energy for the case in which the one object is initially at rest and the other has an initial velocity *v*:

$$\Delta K = K_\text{f} - K_\text{i} = \tfrac{1}{2}(2m)\left(\frac{v}{2}\right)^2 - \tfrac{1}{2}mv^2 = -\frac{mv^2}{4}$$

$$\boxed{\text{The loss of kinetic energy is the same in both cases.}}$$

(*b*) Express the percentage loss for the case in which the two objects have oppositely directed velocities of magnitude *v*/2:

$$\frac{\Delta K}{K_\text{before}} = \frac{\frac{1}{4}mv^2}{\frac{1}{4}mv^2} = 100\%$$

Express the percentage loss for the case in which the one object is initially at rest and the other has an initial velocity *v*:

$$\frac{\Delta K}{K_\text{before}} = \frac{\frac{1}{4}mv^2}{\frac{1}{2}mv^2} = 50\%$$

$$\boxed{\begin{array}{l}\text{The percentage loss is greatest for}\\ \text{the case in which the two objects}\\ \text{have oppositely directed velocities}\\ \text{of magnitude } v/2.\end{array}}$$

***22 ••**

Determine the Concept A will travel farther. Both peas are acted on by the same force, but pea A is acted on by that force for a longer time. By the impulse-momentum theorem, its momentum (and, hence, speed) will be higher than pea B's speed on leaving the shooter.

23 ••

Determine the Concept Refer to the particles as particle 1 and particle 2. Let the direction particle 1 is moving before the collision be the positive *x* direction. We'll use both conservation of momentum and conservation of mechanical energy to obtain an expression for the velocity of particle 2 after the collision. Finally, we'll examine the ratio of the final kinetic energy of particle 2 to that of particle 1 to determine the condition under which there is maximum energy transfer from particle 1 to particle 2.

Use conservation of momentum to obtain one relation for the final velocities:

$$m_1 v_{1,\text{i}} = m_1 v_{1,\text{f}} + m_2 v_{2,\text{f}} \qquad (1)$$

Use conservation of mechanical energy to set the velocity of

$$v_{2,\text{f}} - v_{1,\text{f}} = -\left(v_{2,\text{i}} - v_{1,\text{i}}\right) = v_{1,\text{i}} \qquad (2)$$

recession equal to the negative of the velocity of approach:

To eliminate $v_{1,f}$, solve equation (2) for $v_{1,f}$, and substitute the result in equation (1):

$$v_{1,f} = v_{2,f} + v_{1,i}$$
$$m_1 v_{1,i} = m_1(v_{2,f} - v_{1,i}) + m_2 v_{2,f}$$

Solve for $v_{2,f}$:

$$v_{2,f} = \frac{2m_1}{m_1 + m_2} v_{1,i}$$

Express the ratio R of $K_{2,f}$ to $K_{1,i}$ in terms of m_1 and m_2:

$$R = \frac{K_{2,f}}{K_{1,i}} = \frac{\frac{1}{2} m_2 \left(\frac{2m_1}{m_1 + m_2}\right)^2 v_{1,i}^2}{\frac{1}{2} m_1 v_{1,i}^2}$$
$$= \frac{m_2}{m_1} \frac{4m_1^2}{(m_1 + m_2)^2}$$

Differentiate this ratio with respect to m_2, set the derivative equal to zero, and obtain the quadratic equation:

$$-\frac{m_2^2}{m_1^2} + 1 = 0$$

Solve this equation for m_2 to determine its value for maximum energy transfer:

$$m_2 = m_1$$

$\therefore$ (b) is correct because all of 1's kinetic energy is transferred to 2 when $m_2 = m_1$.

24 •

Determine the Concept In the center-of-mass reference frame the two objects approach with equal but opposite momenta and remain at rest after the collision.

25 •

Determine the Concept The water is changing direction when it rounds the corner in the nozzle. Therefore, the nozzle must exert a force on the stream of water to change its direction, and, from Newton's 3rd law, the water exerts an equal but opposite force on the nozzle.

26 •

Determine the Concept The collision usually takes place in such a short period of time that the impulse delivered by gravity or friction is negligible.

27 •

Determine the Concept No. $\vec{F}_{\text{ext,net}} = d\vec{p}/dt$ defines the relationship between the net force acting on a system and the rate at which its momentum changes. The net external force acting on the pendulum bob is the sum of the force of gravity and the tension in the string and these forces do not add to zero.

***28** ••

Determine the Concept We can apply conservation of momentum and Newton's laws of motion to the analysis of these questions.

(*a*) Yes, the car should slow down. An easy way of seeing this is to imagine a "packet" of grain being dumped into the car all at once: This is a completely inelastic collision, with the packet having an initial horizontal velocity of 0. After the collision, it is moving with the same horizontal velocity that the car does, so the car must slow down.

(*b*) When the packet of grain lands in the car, it initially has a horizontal velocity of 0, so it must be accelerated to come to the same speed as the car of the train. Therefore, the train must exert a force on it to accelerate it. By Newton's 3$^{\text{rd}}$ law, the grain exerts an equal but opposite force on the car, slowing it down. In general, this is a frictional force which causes the grain to come to the same speed as the car.

(*c*) No it doesn't speed up. Imagine a packet of grain being "dumped" out of the railroad car. This can be treated as a collision, too. It has the same horizontal speed as the railroad car when it leaks out, so the train car doesn't have to speed up or slow down to conserve momentum.

***29** ••

Determine the Concept Think of the stream of air molecules hitting the sail. Imagine that they bounce off the sail elastically–their net change in momentum is then roughly twice the change in momentum that they experienced going through the fan. Another way of looking at it: Initially, the air is at rest, but after passing through the fan and bouncing off the sail, it is moving backward–therefore, the boat must exert a net force on the air pushing it backward, and there must be a force on the boat pushing it forward.

Estimation and Approximation

30 ••

Picture the Problem We can estimate the time of collision from the average speed of the car and the distance traveled by the center of the car during the collision. We'll assume a car length of 6 m. We can calculate the average force exerted by the wall on the car from the car's change in momentum and it's stopping time.

(*a*) Relate the stopping time to the assumption that the center of the car travels halfway to the wall with constant deceleration:

$$\Delta t = \frac{d_{\text{stopping}}}{v_{\text{av}}} = \frac{\frac{1}{2}\left(\frac{1}{2}L_{\text{car}}\right)}{v_{\text{av}}} = \frac{\frac{1}{4}L_{\text{car}}}{v_{\text{av}}}$$

Express and evaluate v_{av}:

$$v_{av} = \frac{v_i + v_f}{2} = \frac{0 + 90\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1000\,\text{m}}{\text{km}}}{2} = 12.5\,\text{m/s}$$

Substitute for v_{av} and evaluate Δt:

$$\Delta t = \frac{\frac{1}{4}(6\,\text{m})}{12.5\,\text{m/s}} = \boxed{0.120\,\text{s}}$$

(*b*) Relate the average force exerted by the wall on the car to the car's change in momentum:

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{(2000\,\text{kg})\left(90\,\frac{\text{km}}{\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} \times \frac{1000\,\text{m}}{\text{km}}\right)}{0.120\,\text{s}} = \boxed{417\,\text{kN}}$$

31 ••

Picture the Problem Let the direction the railcar is moving be the positive *x* direction and the system include the earth, the pumpers, and the railcar. We'll also denote the railcar with the letter c and the pumpers with the letter p. We'll use conservation of momentum to relate the center of mass frame velocities of the car and the pumpers and then transform to the earth frame of reference to find the time of fall of the car.

(*a*) Relate the time of fall of the railcar to the distance it falls and its velocity as it leaves the bank:

$$\Delta t = \frac{\Delta y}{v_c}$$

Use conservation of momentum to find the speed of the car relative to the velocity of its center of mass:

$$\vec{p}_i = \vec{p}_f$$

or

$$m_c u_c + m_p u_p = 0$$

Relate u_c to u_p and solve for u_c:

$$u_c - u_p = 4\,\text{m/s}$$

$$\therefore u_p = u_c - 4\,\text{m/s}$$

Substitute for u_p to obtain:

$$m_c u_c + m_p (u_c - 4\,\text{m/s}) = 0$$

Solve for and evaluate u_c:

$$u_c = \frac{4\,\text{m/s}}{1+\dfrac{m_c}{m_p}} = \frac{4\,\text{m/s}}{1+\dfrac{350\,\text{kg}}{4(75\,\text{kg})}} = 1.85\,\text{m/s}$$

Relate the speed of the car to its speed relative to the center of mass of the system:

$$v_c = u_c + v_{cm} = 1.85\,\frac{\text{m}}{\text{s}} + 32\,\frac{\text{km}}{\text{h}}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1000\,\text{m}}{\text{km}} = 10.74\,\text{m/s}$$

Substitute and evaluate Δt:

$$\Delta t = \frac{25\,\text{m}}{10.74\,\text{m/s}} = \boxed{2.33\,\text{s}}$$

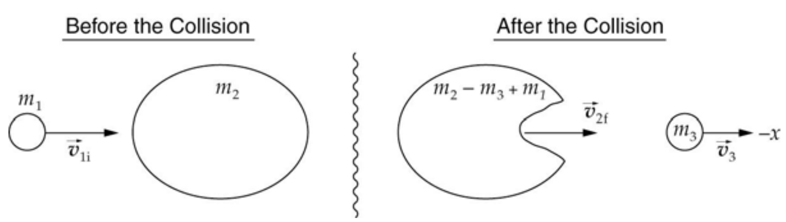
(*b*) Find the speed with which the pumpers hit the ground:

$$v_p = v_c - u_p = 10.74\,\text{m/s} - 4\,\text{m/s} = \boxed{6.74\,\text{m/s}}$$

Hitting the ground at this speed, they may be injured.

***32**

Picture the Problem The diagram depicts the bullet just before its collision with the melon and the motion of the melon-and-bullet-less-jet and the jet just after the collision. We'll assume that the bullet stays in the watermelon after the collision and use conservation of momentum to relate the mass of the bullet and its initial velocity to the momenta of the melon jet and the melon less the plug after the collision.



Apply conservation of momentum to the collision to obtain:

$$m_1v_{1i} = (m_2 - m_3 + m_1)v_{2f} + \sqrt{2m_3K_3}$$

Solve for v_{2f}:

$$v_{2f} = \frac{m_1v_{1i} - \sqrt{2m_3K_3}}{m_2 - m_3 + m_1}$$

Express the kinetic energy of the jet of melon in terms of the initial kinetic energy of the bullet:

$$K_3 = \tfrac{1}{10}K_1 = \tfrac{1}{10}\left(\tfrac{1}{2}m_1v_{1i}^2\right) = \tfrac{1}{20}m_1v_{1i}^2$$

Substitute and simplify to obtain:

$$v_{2f} = \frac{m_1 v_{1i} - \sqrt{2m_3\left(\frac{1}{20} m_1 v_{1i}^2\right)}}{m_2 - m_3 + m_1} = \frac{v_{1i}\left(m_1 - \sqrt{0.1 m_1 m_3}\right)}{m_2 - m_3 + m_1}$$

Substitute numerical values and evaluate v_{2f}:

$$v_{2f} = \left(1800\,\frac{\text{ft}}{\text{s}} \times \frac{1\,\text{m}}{3.281\text{ ft}}\right)\frac{\left(0.0104\,\text{kg} - \sqrt{0.1(0.0104\,\text{kg})(0.14\,\text{kg})}\right)}{2.50\,\text{kg} - 0.14\,\text{kg} + 0.0104\,\text{kg}} = -0.386\,\text{m/s}$$

$$= \boxed{-1.27\,\text{ft/s}}$$

Note that this result is in reasonably good agreement with experimental results.

Finding the Center of Mass

33 •

Picture the Problem We can use its definition to find the center of mass of this system.

Apply its definition to find x_{cm}:

$$x_{cm} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3} = \frac{(2\,\text{kg})(0) + (2\,\text{kg})(0.2\,\text{m}) + (2\,\text{kg})(0.5\,\text{m})}{2\,\text{kg} + 2\,\text{kg} + 2\,\text{kg}} = 0.233\,\text{m}$$

Because the point masses all lie along the x axis:

$y_{cm} = 0$ and the center of mass of this system of particles is at $\boxed{(0.233\,\text{m}, 0)}$.

***34** •

Picture the Problem Let the left end of the handle be the origin of our coordinate system. We can disassemble the club-ax, find the center of mass of each piece, and then use these coordinates and the masses of the handle and stone to find the center of mass of the club-ax.

Express the center of mass of the handle plus stone system:

$$x_{cm} = \frac{m_{stick} x_{cm,stick} + m_{stone} x_{cm,stone}}{m_{stick} + m_{stone}}$$

Assume that the stone is drilled and the stick passes through it. Use symmetry considerations to locate the center of mass of the stick:

$$x_{cm,stick} = 45.0\,\text{cm}$$

Use symmetry considerations to locate the center of mass of the stone:

$$x_{\text{cm,stone}} = 89.0\,\text{cm}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{\text{cm}} = \frac{(2.5\,\text{kg})(45\,\text{cm}) + (8\,\text{kg})(89\,\text{cm})}{2.5\,\text{kg} + 8\,\text{kg}} = \boxed{78.5\text{cm}}$$

35 •

Picture the Problem We can treat each of balls as though they are point objects and apply the definition of the center of mass to find (x_{cm}, y_{cm}).

Use the definition of x_{cm}:

$$x_{\text{cm}} = \frac{m_A x_A + m_B x_B + m_C x_C}{m_A + m_B + m_C} = \frac{(3\,\text{kg})(2\,\text{m}) + (1\,\text{kg})(1\,\text{m}) + (1\,\text{kg})(3\,\text{m})}{3\,\text{kg} + 1\,\text{kg} + 1\,\text{kg}} = 2.00\,\text{m}$$

Use the definition of y_{cm}:

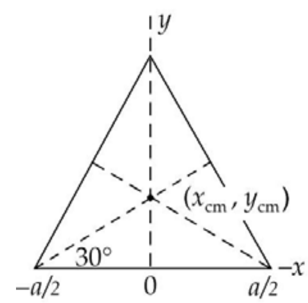
$$y_{\text{cm}} = \frac{m_A y_A + m_B y_B + m_C y_C}{m_A + m_B + m_C} = \frac{(3\,\text{kg})(2\,\text{m}) + (1\,\text{kg})(1\,\text{m}) + (1\,\text{kg})(0)}{3\,\text{kg} + 1\,\text{kg} + 1\,\text{kg}} = 1.40\,\text{m}$$

The center of mass of this system of particles is at:

$$\boxed{(2.00\,\text{m}, 1.40\,\text{m})}$$

36 •

Picture the Problem The figure shows an equilateral triangle with its y-axis vertex above the x axis. The bisectors of the vertex angles are also shown. We can find x coordinate of the center-of-mass by inspection and the y coordinate using trigonometry.



From symmetry considerations:

$$x_{\text{cm}} = 0$$

Express the trigonometric relationship between $a/2$, 30°, and y_{cm}:

$$\tan 30° = \frac{y_{cm}}{a/2}$$

Solve for y_{cm}:

$$y_{cm} = \tfrac{1}{2} a \tan 30° = 0.289a$$

The center of mass of an equilateral triangle oriented as shown above is at $\boxed{(0, 0.289a)}$.

***37 ••**

Picture the Problem Let the subscript 1 refer to the 3-m by 3-m sheet of plywood before the 2-m by 1-m piece has been cut from it. Let the subscript 2 refer to 2-m by 1-m piece that has been removed and let σ be the area density of the sheet. We can find the center-of-mass of these two regions; treating the missing region as though it had negative mass, and then finding the center-of-mass of the U-shaped region by applying its definition.

Express the coordinates of the center of mass of the sheet of plywood:

$$x_{cm} = \frac{m_1 x_{cm,1} - m_2 x_{cm,2}}{m_1 - m_2}$$

$$y_{cm} = \frac{m_1 y_{cm,1} - m_2 y_{cm,2}}{m_1 - m_2}$$

Use symmetry to find $x_{cm,1}$, $y_{cm,1}$, $x_{cm,2}$, and $y_{cm,2}$:

$$x_{cm,1} = 1.5\,\text{m},\; y_{cm,1} = 1.5\,\text{m}$$

and

$$x_{cm,2} = 1.5\,\text{m},\, y_{cm,2} = 2.0\,\text{m}$$

Determine m_1 and m_2:

$$m_1 = \sigma A_1 = 9\sigma\,\text{kg}$$

and

$$m_2 = \sigma A_2 = 2\sigma\, kg$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{(9\sigma\,\text{kg})(1.5\,\text{m}) - (2\sigma\,\text{kg})(1.5\,\text{kg})}{9\sigma\,\text{kg} - 2\sigma\,\text{kg}} = 1.50\,\text{m}$$

Substitute numerical values and evaluate y_{cm}:

$$y_{cm} = \frac{(9\sigma\,\text{kg})(1.5\,\text{m}) - (2\sigma\,\text{kg})(2\,\text{m})}{9\sigma\,\text{kg} - 2\sigma\,\text{kg}} = 1.36\,\text{m}$$

The center of mass of the U-shaped sheet of plywood is at $\boxed{(1.50\,\text{m}, 1.36\,\text{m})}$.

38 ••

Picture the Problem We can use its definition to find the center of mass of the can plus water. By setting the derivative of this function equal to zero, we can find the value of x that corresponds to the minimum height of the center of mass of the water as it drains out and then use this extreme value to express the minimum height of the center of mass.

(a) Using its definition, express the location of the center of mass of the can + water:

$$x_{cm}=\frac{M\left(\frac{H}{2}\right)+m\left(\frac{x}{2}\right)}{M+m}$$

Let the cross-sectional area of the cup be A and use the definition of density to relate the mass m of water remaining in the can at any given time to its depth x:

$$\rho=\frac{M}{AH}=\frac{m}{Ax}$$

Solve for m to obtain:

$$m=\frac{x}{H}M$$

Substitute to obtain:

$$x_{cm}=\frac{M\left(\frac{H}{2}\right)+\left(\frac{x}{H}M\right)\left(\frac{x}{2}\right)}{M+\frac{x}{H}M}=\boxed{\frac{H}{2}\left(\frac{1+\left(\frac{x}{H}\right)^2}{1+\frac{x}{H}}\right)}$$

(b) Differentiate x_{cm} with respect to x and set the derivative equal to zero for extrema:

$$\frac{dx_{cm}}{dx}=\frac{H}{2}\frac{d}{dx}\left(\frac{1+\left(\frac{x}{H}\right)^2}{1+\frac{x}{H}}\right)=\frac{H}{2}\left\{\frac{\left(1+\frac{x}{H}\right)\frac{d}{dx}\left[1+\left(\frac{x}{H}\right)^2\right]}{\left(1+\frac{x}{H}\right)^2}-\frac{\left[1+\left(\frac{x}{H}\right)^2\right]\frac{d}{dx}\left(1+\frac{x}{H}\right)}{\left(1+\frac{x}{H}\right)^2}\right\}$$

$$=\frac{H}{2}\left\{\frac{\left(1+\frac{x}{H}\right)2\left(\frac{x}{H}\right)\left(\frac{1}{H}\right)}{\left(1+\frac{x}{H}\right)^2}-\frac{\left[1+\left(\frac{x}{H}\right)^2\right]\left(\frac{1}{H}\right)}{\left(1+\frac{x}{H}\right)^2}\right\}=0$$

Simplify this expression to obtain:

$$\left(\frac{x}{H}\right)^2+2\left(\frac{x}{H}\right)-1=0$$

Solve for x/H to obtain:

$$x = H\left(\sqrt{2}-1\right) \approx 0.414H$$

where we've kept the positive solution because a negative value for x/H would make no sense.

Use your graphing calculator to convince yourself that the graph of x_{cm} as a function of x is concave upward at $x \approx 0.414H$ and that, therefore, the minimum value of x_{cm} occurs at $x \approx 0.414H$.

Evaluate x_{cm} at $x = H\left(\sqrt{2}-1\right)$ to obtain:

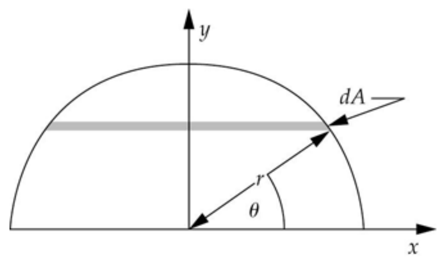
$$x_{cm}\Big|_{x=H\left(\sqrt{2}-1\right)} = \frac{H}{2}\left(\frac{1+\left(\frac{H\left(\sqrt{2}-1\right)}{H}\right)^2}{1+\frac{H\left(\sqrt{2}-1\right)}{H}}\right)$$

$$= \boxed{H\left(\sqrt{2}-1\right)}$$

Finding the Center of Mass by Integration

***39** ••

Picture the Problem A semicircular disk and a surface element of area dA is shown in the diagram. Because the disk is a continuous object, we'll use $M\vec{r}_{cm} = \int \vec{r}\,dm$ and symmetry to find its center of mass.



Express the coordinates of the center of mass of the semicircular disk:

$$x_{cm} = 0 \text{ by symmetry.}$$

$$y_{cm} = \frac{\int y\sigma\, dA}{M}$$

Express y as a function of r and θ:

$$y = r\sin\theta$$

Express dA in terms of r and θ:

$$dA = r\,d\theta\,dr$$

Express M as a function of r and θ:

$$M = \sigma A_{\text{half disk}} = \tfrac{1}{2}\sigma\pi R^2$$

Substitute and evaluate y_{cm}:

$$y_{\text{cm}} = \frac{\sigma \int_0^R \int_0^\pi r^2 \sin\theta \, d\theta \, dr}{M} = \frac{2\sigma}{M}\int_0^R r^2 dr = \frac{2\sigma}{3M}R^3 = \boxed{\frac{4}{3\pi}R}$$

40 •••

Picture the Problem Because a solid hemisphere is a continuous object, we'll use $M\vec{r}_{\text{cm}} = \int \vec{r}\,dm$ to find its center of mass. The volume element for a sphere is $dV = r^2 \sin\theta \, d\theta \, d\phi \, dr$, where θ is the polar angle and ϕ the azimuthal angle.

Let the base of the hemisphere be the *xy* plane and ρ be the mass density. Then:

$$z = r\cos\theta$$

Express the *z* coordinate of the center of mass:

$$z_{\text{cm}} = \frac{\int r\rho dV}{\int \rho dV}$$

Evaluate $M = \int \rho dV$:

$$M = \int \rho dV = \tfrac{1}{2}\rho V_{\text{sphere}} = \tfrac{1}{2}\rho\left(\tfrac{4}{3}\pi R^3\right) = \tfrac{2}{3}\pi\rho R^3$$

Evaluate $\int r\rho dV$:

$$\int r\rho dV = \int_0^R \int_0^{\pi/2} \int_0^{2\pi} r^3 \sin\theta\cos\theta d\theta d\phi dr = \frac{\pi\rho R^4}{2}\left[\tfrac{1}{2}\sin^2\theta\right]_0^{\pi/2} = \frac{\pi\rho R^4}{4}$$

Substitute and simplify to find z_{cm}:

$$z_{\text{cm}} = \frac{\frac{1}{4}\pi\rho R^4}{\frac{2}{3}\pi\rho R^3} = \boxed{\tfrac{3}{8}R}$$

41 •••

Picture the Problem Because a thin hemisphere shell is a continuous object, we'll use $M\vec{r}_{\text{cm}} = \int \vec{r}\,dm$ to find its center of mass. The element of area on the shell is $dA = 2\pi R^2 \sin\theta \, d\theta$, where R is the radius of the hemisphere.

Let σ be the surface mass density and express the *z* coordinate of the center of mass:

$$z_{\text{cm}} = \frac{\int z\sigma \, dA}{\int \sigma \, dA}$$

Evaluate $M = \int \sigma\, dA$:

$$M = \int \sigma\, dA = \tfrac{1}{2}\sigma A_{\text{spherical shell}} = \tfrac{1}{2}\sigma\left(4\pi R^2\right) = 2\pi\sigma R^2$$

Evaluate $\int z\sigma\, dA$:

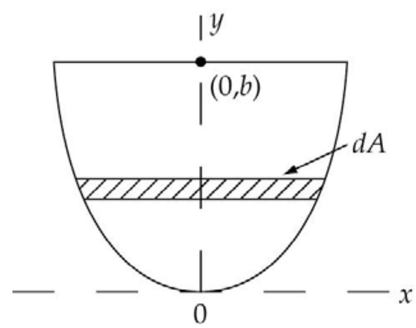
$$\int z\sigma\, dA = 2\pi R^3\sigma \int_0^{\pi/2} \sin\theta\cos\theta\, d\theta = \pi R^3\sigma \int_0^{\pi/2} \sin 2\theta\, d\theta = \pi R^3\sigma$$

Substitute and simplify to find z_{cm}:

$$z_{cm} = \frac{\pi R^3\sigma}{2\pi\sigma R^2} = \boxed{\tfrac{1}{2}R}$$

42 •••

Picture the Problem The parabolic sheet is shown to the right. Because the area of the sheet is distributed symmetrically with respect to the y axis, $x_{cm} = 0$. We'll integrate the element of area dA (= xdy) to obtain the total area of the sheet and $yxdy$ to obtain the numerator of the definition of the center of mass.



Express y_{cm}:

$$y_{cm} = \frac{\int_0^b xydy}{\int_0^b xdy}$$

Evaluate $\int_0^b xydy$:

$$\int_0^b xydy = \int_0^b \frac{y^{1/2}}{\sqrt{a}}ydy = \frac{1}{\sqrt{a}}\int_0^b y^{3/2}dy = \frac{2}{5\sqrt{a}}b^{5/2}$$

Evaluate $\int_0^b xdy$:

$$\int_0^b xdy = \int_0^b \frac{y^{1/2}}{\sqrt{a}}dy = \frac{1}{\sqrt{a}}\int_0^b y^{1/2}dy = \frac{2}{3\sqrt{a}}b^{3/2}$$

Substitute and simplify to determine y_{cm}:

$$y_{cm} = \frac{\frac{2}{5\sqrt{a}}b^{5/2}}{\frac{2}{3\sqrt{a}}b^{3/2}} = \tfrac{3}{5}b$$

Note that, by symmetry: $x_{cm} = 0$

The center of mass of the parabolic sheet is at:

$$\boxed{\left(0, \tfrac{3}{5}b\right)}$$

Motion of the Center of Mass

43 •

Picture the Problem The velocity of the center of mass of a system of particles is related to the total momentum of the system through $\vec{\boldsymbol{P}} = \sum_i m_i\vec{\boldsymbol{v}}_i = M\vec{\boldsymbol{v}}_{cm}$.

Use the expression for the total momentum of a system to relate the velocity of the center of mass of the two-particle system to the momenta of the individual particles:

$$\vec{\boldsymbol{v}}_{cm} = \frac{\sum_i m_i\vec{\boldsymbol{v}}_i}{M} = \frac{m_1\vec{\boldsymbol{v}}_1 + m_2\vec{\boldsymbol{v}}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{\boldsymbol{v}}_{cm}$:

$$\vec{\boldsymbol{v}}_{cm} = \frac{(3\,\text{kg})(\vec{\boldsymbol{v}}_1 + \vec{\boldsymbol{v}}_2)}{6\,\text{kg}} = \tfrac{1}{2}(\vec{\boldsymbol{v}}_1 + \vec{\boldsymbol{v}}_2)$$
$$= \tfrac{1}{2}\left[(6\,\text{m/s})\hat{\boldsymbol{i}} - (3\,\text{m/s})\hat{\boldsymbol{j}}\right]$$
$$= \boxed{(3\,\text{m/s})\hat{\boldsymbol{i}} - (1.5\,\text{m/s})\hat{\boldsymbol{j}}}$$

***44** •

Picture the Problem Choose a coordinate system in which east is the positive *x* direction and use the relationship $\vec{\boldsymbol{P}} = \sum_i m_i\vec{\boldsymbol{v}}_i = M\vec{\boldsymbol{v}}_{cm}$ to determine the velocity of the center of mass of the system.

Use the expression for the total momentum of a system to relate the velocity of the center of mass of the two-vehicle system to the momenta of the individual vehicles:

$$\vec{\boldsymbol{v}}_{cm} = \frac{\sum_i m_i\vec{\boldsymbol{v}}_i}{M} = \frac{m_t\vec{\boldsymbol{v}}_t + m_c\vec{\boldsymbol{v}}_c}{m_t + m_c}$$

Express the velocity of the truck:

$$\vec{\boldsymbol{v}}_t = (16\,\text{m/s})\hat{\boldsymbol{i}}$$

Express the velocity of the car: $\vec{v}_c = (-20\,\text{m/s})\hat{i}$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3000\,\text{kg})(16\,\text{m/s})\,\hat{i} + (1500\,\text{kg})(-20\,\text{m/s})\,\hat{i}}{3000\,\text{kg} + 1500\,\text{kg}} = \boxed{(4.00\,\text{m/s})\,\hat{i}}$$

45 •

Picture the Problem The acceleration of the center of mass of the ball is related to the net external force through Newton's 2nd law: $\vec{F}_{\text{net,ext}} = M\vec{a}_{cm}$.

Use Newton's 2nd law to express the acceleration of the ball:

$$\vec{a}_{cm} = \frac{\vec{F}_{\text{net,ext}}}{M}$$

Substitute numerical values and evaluate $\vec{a}_{cm}$:

$$\vec{a}_{cm} = \frac{(12\,\text{N})\hat{i}}{3\,\text{kg} + 1\,\text{kg} + 1\,\text{kg}} = \boxed{(2.4\,\text{m/s}^2)\hat{i}}$$

46 ••

Picture the Problem Choose a coordinate system in which upward is the positive y direction. We can use Newton's 2nd law $\vec{F}_{\text{net,ext}} = M\vec{a}_{cm}$ to find the acceleration of the center of mass of this two-body system.

(*a*) $\boxed{\text{Yes; initially the scale reads } (M+m)g\text{; while } m \text{ is in free fall, the reading is } Mg.}$

(*b*) Using Newton's 2nd law, express the acceleration of the center of mass of the system:

$$\vec{a}_{cm} = \frac{\vec{F}_{\text{net,ext}}}{m_{\text{tot}}}$$

Substitute to obtain:

$$\vec{a}_{cm} = \boxed{-\frac{mg}{M+m}\,\hat{j}}$$

(*c*) Use Newton's 2nd law to express the net force acting on the scale while the object of mass m is falling:

$$F_{\text{net,ext}} = (M+m)g - (M+m)a_{cm}$$

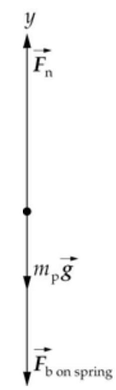
Substitute and simplify to obtain:

$$F_{\text{net,ext}} = (M+m)g - (M+m)\left(\frac{mg}{M+m}\right) = \boxed{Mg}$$

as expected, given our answer to part (*a*).

***47 ••**

Picture the Problem The free-body diagram shows the forces acting on the platform when the spring is partially compressed. The scale reading is the force the scale exerts on the platform and is represented on the FBD by F_n. We can use Newton's 2nd law to determine the scale reading in part (*a*) and the work-energy theorem in conjunction with Newton's 2nd law in parts (*b*) and (*c*).



(*a*) Apply $\sum F_y = ma_y$ to the spring when it is compressed a distance *d*:

$$\sum F_y = F_n - m_p g - F_{\text{ball on spring}} = 0$$

Solve for F_n:

$$F_n = m_p g + F_{\text{ball on spring}} = m_p g + kd = m_p g + k\left(\frac{m_b g}{k}\right) = \boxed{m_p g + m_b g = (m_p + m_b)g}$$

(*b*) Use conservation of mechanical energy, with $U_g = 0$ at the position at which the spring is fully compressed, to relate the gravitational potential energy of the system to the energy stored in the fully compressed spring:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Because $\Delta K = U_{g,f} = U_{s,i} = 0$,

$$U_{g,i} - U_{s,f} = 0$$

or

$$m_b g d - \tfrac{1}{2}kd^2 = 0$$

Solve for *d*:

$$d = \frac{2m_b g}{k}$$

Evaluate our force equation in (*a*) with $d = \frac{2m_b g}{k}$:

$$F_n = m_p g + F_{\text{ball on spring}}$$
$$= m_p g + kd = m_p g + k\left(\frac{2m_b g}{k}\right)$$
$$= \boxed{m_p g + 2m_b g = (m_p + 2m_b)g}$$

(*c*) When the ball is in its original position, the spring is relaxed and exerts no force on the ball. Therefore:

$$F_n = \text{scale reading}$$
$$= \boxed{m_p g}$$

***48 ••**

Picture the Problem Assume that the object whose mass is m_1 is moving downward and take that direction to be the positive direction. We'll use Newton's 2nd law for a system of particles to relate the acceleration of the center of mass to the acceleration of the individual particles.

(*a*) Relate the acceleration of the center of mass to m_1, m_2, m_c and their accelerations:

$$M\vec{a}_{cm} = m_1\vec{a}_1 + m_2\vec{a}_2 + m_c\vec{a}_c$$

Because m_1 and m_2 have a common acceleration a and $a_c = 0$:

$$a_{cm} = a\frac{m_1 - m_2}{m_1 + m_2 + m_c}$$

From Problem 4-81 we have:

$$a = g\frac{m_1 - m_2}{m_1 + m_2}$$

Substitute to obtain:

$$a_{cm} = \left(\frac{m_1 - m_2}{m_1 + m_2}g\right)\left(\frac{m_1 - m_2}{m_1 + m_2 + m_c}\right)$$
$$= \boxed{\frac{(m_1 - m_2)^2}{(m_1 + m_2)(m_1 + m_2 + m_c)}g}$$

(*b*) Use Newton's 2nd law for a system of particles to obtain:

$$F - Mg = -Ma_{cm}$$

where $M = m_1 + m_2 + m_c$ and F is positive upwards.

Solve for F and substitute for a_{cm} from part (a):

$$F = Mg - Ma_{cm} = Mg - \frac{(m_1 - m_2)^2}{m_1 + m_2} g = \boxed{\left[\frac{4m_1 m_2}{m_1 + m_2} + m_c\right] g}$$

(c) From Problem 4-81:

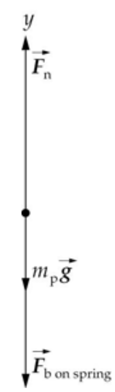
$$T = \frac{2m_1 m_2}{m_1 + m_2} g$$

Substitute in our result from part (b) to obtain:

$$F = \left[2\frac{2m_1 m_2}{m_1 + m_2} + m_c\right] g = \left[2\frac{T}{g} + m_c\right] g = \boxed{2T + m_c g}$$

49 ••

Picture the Problem The free-body diagram shows the forces acting on the platform when the spring is partially compressed. The scale reading is the force the scale exerts on the platform and is represented on the FBD by F_n. We can use Newton's 2nd law to determine the scale reading in part (a) and the result of Problem 7-96 part (b) to obtain the scale reading when the ball is dropped from a height h above the cup.



(a) Apply $\sum F_y = ma_y$ to the spring when it is compressed a distance d:

$$\sum F_y = F_n - m_p g - F_{\text{ball on spring}} = 0$$

Solve for F_n:

$$F_n = m_p g + F_{\text{ball on spring}} = m_p g + kd = m_p g + k\left(\frac{m_b g}{k}\right) = \boxed{m_p g + m_b g = (m_p + m_b) g}$$

(b) From Problem 7-96, part (b):

$$x_{max} = \frac{m_b g}{k}\left(1 + \sqrt{1 + \frac{2kh}{m_b g}}\right)$$

From part (*a*):

$$F_n = m_p g + F_{\text{ball on spring}} = m_p g + kx_{max}$$
$$= \boxed{m_p g + m_b g\left(1+\sqrt{1+\frac{2kh}{m_b g}}\right)}$$

The Conservation of Momentum

50 •

Picture the Problem Let the system include the woman, the canoe, and the earth. Then the *net* external force is zero and linear momentum is conserved as she jumps off the canoe. Let the direction she jumps be the positive *x* direction.

Apply conservation of momentum to the system:

$$\sum m_i \vec{v}_i = m_{girl}\vec{v}_{girl} + m_{canoe}\vec{v}_{canoe} = 0$$

Substitute to obtain:

$$(55\,\text{kg})(2.5\,\text{m/s})\hat{i} + (75\,\text{kg})\vec{v}_{canoe} = 0$$

Solve for $\vec{v}_{canoe}$:

$$\vec{v}_{canoe} = \boxed{(-1.83\,\text{m/s})\,\hat{i}}$$

51 •

Picture the Problem If we include the earth in our system, then the net external force is zero and linear momentum is conserved as the spring delivers its energy to the two objects.

Apply conservation of momentum to the system:

$$\sum m_i \vec{v}_i = m_5\vec{v}_5 + m_{10}\vec{v}_{10} = 0$$

Substitute numerical values to obtain:

$$(5\,\text{kg})(-8\,\text{m/s})\hat{i} + (10\,\text{kg})\vec{v}_{10} = 0$$

Solve for $\vec{v}_{10}$:

$$\vec{v}_{10} = \boxed{(4\,\text{m/s})\hat{i}}$$

***52** •

Picture the Problem This is an explosion-like event in which linear momentum is conserved. Thus we can equate the initial and final momenta in the *x* direction and the initial and final momenta in the *y* direction. Choose a coordinate system in the positive *x* direction is to the right and the positive *y* direction is upward.

Equate the momenta in the *y* direction before and after the explosion:

$$\sum p_{y,i} = \sum p_{y,f} = mv_2 - 2mv_1$$
$$= m(2v_1) - 2mv_1 = 0$$

We can conclude that the momentum was entirely in the x direction before the particle exploded.

Equate the momenta in the x direction before and after the explosion:

$$\sum p_{x,i} = \sum p_{x,f}$$
$$\therefore 4mv_i = mv_3$$

Solve for v_3:

$$v_i = \tfrac{1}{4}v_3 \text{ and } \boxed{(c)\text{ is correct.}}$$

53 •

Picture the Problem Choose the direction the shell is moving just before the explosion to be the positive x direction and apply conservation of momentum.

Use conservation of momentum to relate the masses of the fragments to their velocities:

$$\vec{\boldsymbol{p}}_i = \vec{\boldsymbol{p}}_f$$
or
$$mv\hat{\boldsymbol{i}} = \tfrac{1}{2}mv\hat{\boldsymbol{j}} + \tfrac{1}{2}m\vec{\boldsymbol{v}}'$$

Solve for $\vec{\boldsymbol{v}}'$:

$$\vec{\boldsymbol{v}}' = \boxed{2v\hat{\boldsymbol{i}} - v\hat{\boldsymbol{j}}}$$

***54** ••

Picture the Problem Let the system include the earth and the platform, gun and block. Then $\vec{\boldsymbol{F}}_{\text{net,ext}} = 0$ and momentum is conserved within the system.

(a) Apply conservation of momentum to the system just before and just after the bullet leaves the gun:

$$\vec{\boldsymbol{p}}_{\text{before}} = \vec{\boldsymbol{p}}_{\text{after}}$$
or
$$0 = \vec{\boldsymbol{p}}_{\text{bullet}} + \vec{\boldsymbol{p}}_{\text{platform}}$$

Substitute for $\vec{\boldsymbol{p}}_{\text{bullet}}$ and $\vec{\boldsymbol{p}}_{\text{platform}}$ and solve for $\vec{\boldsymbol{v}}_{\text{platform}}$:

$$0 = m_b v_b\hat{\boldsymbol{i}} + m_p\vec{\boldsymbol{v}}_{\text{platform}}$$
and
$$\vec{\boldsymbol{v}}_{\text{platform}} = \boxed{-\frac{m_b}{m_p}v_b\hat{\boldsymbol{i}}}$$

(b) Apply conservation of momentum to the system just before the bullet leaves the gun and just after it comes to rest in the block:

$$\vec{\boldsymbol{p}}_{\text{before}} = \vec{\boldsymbol{p}}_{\text{after}}$$
or
$$0 = \vec{\boldsymbol{p}}_{\text{platform}} \Rightarrow \vec{\boldsymbol{v}}_{\text{platform}} = 0$$

(c) Express the distance Δs traveled by the platform:

$$\Delta s = v_{\text{platform}}\Delta t$$

Express the velocity of the bullet relative to the platform:

$$v_{\text{rel}} = v_{\text{b}} - v_{\text{platform}} = v_{\text{b}} + \frac{m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}$$

$$= \left(1 + \frac{m_{\text{b}}}{m_{\text{p}}}\right) v_{\text{b}} = \frac{m_{\text{p}} + m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}$$

Relate the time of flight Δt to L and v_{rel}:

$$\Delta t = \frac{L}{v_{\text{rel}}}$$

Substitute to find the distance Δs moved by the platform in time Δt:

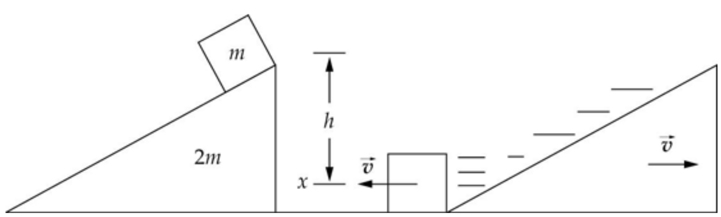
$$\Delta s = v_{\text{platform}} \Delta t = \left(\frac{m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}\right)\left(\frac{L}{v_{\text{rel}}}\right)$$

$$= \left(\frac{m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}\right)\left(\frac{L}{\frac{m_{\text{p}} + m_{\text{b}}}{m_{\text{p}}} v_{\text{b}}}\right)$$

$$= \boxed{\frac{m_{\text{b}}}{m_{\text{p}} + m_{\text{b}}} L}$$

55 ••

Picture the Problem The pictorial representation shows the wedge and small object, initially at rest, to the left, and, to the right, both in motion as the small object leaves the wedge. Choose the direction the small object is moving when it leaves the wedge be the positive *x* direction and the zero of potential energy to be at the surface of the table. Let the speed of the small object be *v* and that of the wedge *V*. We can use conservation of momentum to express *v* in terms of *V* and conservation of energy to express *v* in terms of *h*.



Apply conservation of momentum to the small object and the wedge:

$$\vec{p}_{\text{i},x} = \vec{p}_{\text{f},x}$$

or

$$0 = mv\hat{i} + 2m\vec{V}$$

Solve for $\vec{V}$:

$$\vec{V} = -\tfrac{1}{2} v\hat{i} \qquad (1)$$

and

$$V = \tfrac{1}{2}v$$

Use conservation of energy to determine the speed of the small object when it exits the wedge:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $U_f = K_i = 0$:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}(2m)V^2 - mgh = 0$$

Substitute for V to obtain:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}(2m)\left(\tfrac{1}{2}v\right)^2 - mgh = 0$$

Solve for v to obtain:

$$v = 2\sqrt{\frac{gh}{3}}$$

Substitute in equation (1) to determine $\vec{\boldsymbol{V}}$:

$$\vec{\boldsymbol{V}} = -\tfrac{1}{2}\left(2\sqrt{\frac{gh}{3}}\right)\hat{\boldsymbol{i}} = \boxed{-\sqrt{\frac{gh}{3}}\,\hat{\boldsymbol{i}}}$$

i.e., the wedge moves in the direction opposite to that of the small object with a speed of $\sqrt{\frac{gh}{3}}$.

***56** ••

Picture the Problem Because no external forces act on either cart, the center of mass of the two-cart system can't move. We can use the data concerning the masses and separation of the gliders initially to calculate its location and then apply the definition of the center of mass a second time to relate the positions X_1 and X_2 of the centers of the carts when they first touch. We can also use the separation of the centers of the gliders when they touch to obtain a second equation in X_1 and X_2 that we can solve simultaneously with the equation obtained from the location of the center of mass.

(*a*) Apply its definition to find the center of mass of the 2-glider system:

$$x_{cm} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{(0.1\,\text{kg})(0.1\,\text{m}) + (0.2\,\text{kg})(1.6\,\text{m})}{0.1\,\text{kg} + 0.2\,\text{kg}} = 1.10\,\text{m}$$

from the left end of the air track.

Use the definition of the center of mass to relate the coordinates of the centers of the two gliders when they first touch to the location of the center of mass:

$$1.10\,\text{m} = \frac{m_1X_1 + m_2X_2}{m_1 + m_2} = \frac{(0.1\,\text{kg})X_1 + (0.2\,\text{kg})X_2}{0.1\,\text{kg} + 0.2\,\text{kg}} = \tfrac{1}{3}X_1 + \tfrac{2}{3}X_2$$

Also, when they first touch, their centers are separated by half their combined lengths:

$$X_2 - X_1 = \tfrac{1}{2}(10\,\text{cm} + 20\,\text{cm}) = 0.15\,\text{m}$$

Thus we have:

$$0.333X_1 + 0.667X_2 = 1.10\,\text{m}$$
and
$$X_2 - X_1 = 0.15\,\text{m}$$

Solve these equations simultaneously to obtain:

$$X_1 = \boxed{1.00\,\text{m}} \text{ and } X_2 = \boxed{1.15\,\text{m}}$$

(*b*)

No. The initial momentum of the system is zero, so it must be zero after the collision.

Kinetic Energy of a System of Particles

***57 •**

Picture the Problem Choose a coordinate system in which the positive *x* direction is to the right. Use the expression for the total momentum of a system to find the velocity of the center of mass and the definition of relative velocity to express the sum of the kinetic energies relative to the center of mass.

(*a*) Find the sum of the kinetic energies:

$$K = K_1 + K_2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 = \tfrac{1}{2}(3\,\text{kg})(5\,\text{m/s})^2 + \tfrac{1}{2}(3\,\text{kg})(2\,\text{m/s})^2 = \boxed{43.5\,\text{J}}$$

(*b*) Relate the velocity of the center of mass of the system to its total momentum:

$$M\vec{v}_{\text{cm}} = m_1\vec{v}_1 + m_2\vec{v}_2$$

Solve for $\vec{v}_{\text{cm}}$:

$$\vec{v}_{\text{cm}} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{v}_{\text{cm}}$:

$$\vec{v}_{\text{cm}} = \frac{(3\,\text{kg})(5\,\text{m/s})\hat{i} - (3\,\text{kg})(2\,\text{m/s})\hat{i}}{3\,\text{kg} + 3\,\text{kg}} = \boxed{(1.50\,\text{m/s})\hat{i}}$$

(*c*) The velocity of an object relative to the center of mass is given by:

$$\vec{v}_{\text{rel}} = \vec{v} - \vec{v}_{\text{cm}}$$

$$\vec{v}_{1,\text{rel}} = (5\,\text{m/s})\hat{i} - (1.5\,\text{m/s})\hat{i} = \boxed{(3.50\,\text{m/s})\hat{i}}$$

$$\vec{v}_{2,\text{rel}} = (-2\,\text{m/s})\hat{i} - (1.5\,\text{m/s})\hat{i} = \boxed{(-3.50\,\text{m/s})\hat{i}}$$

(*d*) Express the sum of the kinetic energies relative to the center of mass:

$$K_{\text{rel}} = K_{1,\text{rel}} + K_{2,\text{rel}} = \tfrac{1}{2}m_1 v_{1,\text{rel}}^2 + \tfrac{1}{2}m_2 v_{2,\text{rel}}^2$$

Substitute numerical values and evaluate K_{rel}:

$$K_{\text{rel}} = \tfrac{1}{2}(3\,\text{kg})(3.5\,\text{m/s})^2 + \tfrac{1}{2}(3\,\text{kg})(-3.5\,\text{m/s})^2 = \boxed{36.75\,\text{J}}$$

(*e*) Find K_{cm}:

$$K_{\text{cm}} = \tfrac{1}{2}m_{\text{tot}}v_{\text{cm}}^2 = \tfrac{1}{2}(6\,\text{kg})(1.5\,\text{m/s})^2 = 6.75\,\text{J} = 43.5\,\text{J} - 36.75\,\text{J} = \boxed{K - K_{\text{rel}}}$$

58 •

Picture the Problem Choose a coordinate system in which the positive *x* direction is to the right. Use the expression for the total momentum of a system to find the velocity of the center of mass and the definition of relative velocity to express the sum of the kinetic energies relative to the center of mass.

(*a*) Express the sum of the kinetic energies:

$$K = K_1 + K_2 = \tfrac{1}{2}m_1 v_1^2 + \tfrac{1}{2}m_2 v_2^2$$

Substitute numerical values and evaluate *K*:

$$K = \tfrac{1}{2}(3\,\text{kg})(5\,\text{m/s})^2 + \tfrac{1}{2}(5\,\text{kg})(3\,\text{m/s})^2 = \boxed{60.0\,\text{J}}$$

(*b*) Relate the velocity of the center of mass of the system to its total momentum:

$$M\vec{v}_{cm} = m_1\vec{v}_1 + m_2\vec{v}_2$$

Solve for $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3\,\text{kg})(5\,\text{m/s})\hat{i} + (5\,\text{kg})(3\,\text{m/s})\hat{i}}{3\,\text{kg} + 5\,\text{kg}} = \boxed{(3.75\,\text{m/s})\hat{i}}$$

(*c*) The velocity of an object relative to the center of mass is given by:

$$\vec{v}_{rel} = \vec{v} - \vec{v}_{cm}$$

Substitute numerical values and evaluate the relative velocities:

$$\vec{v}_{1,rel} = (5\,\text{m/s})\hat{i} - (3.75\,\text{m/s})\hat{i} = \boxed{(1.25\,\text{m/s})\hat{i}}$$

and

$$\vec{v}_{2,rel} = (3\,\text{m/s})\hat{i} - (3.75\,\text{m/s})\hat{i} = \boxed{(-0.750\,\text{m/s})\hat{i}}$$

(*d*) Express the sum of the kinetic energies relative to the center of mass:

$$K_{rel} = K_{1,rel} + K_{2,rel} = \tfrac{1}{2}m_1 v_{1,rel}^2 + \tfrac{1}{2}m_2 v_{2,rel}^2$$

Substitute numerical values and evaluate K_{rel}:

$$K_{rel} = \tfrac{1}{2}(3\,\text{kg})(1.25\,\text{m/s})^2 + \tfrac{1}{2}(5\,\text{kg})(-0.75\,\text{m/s})^2 = \boxed{3.75\,\text{J}}$$

(*e*) Find K_{cm}:

$$K_{cm} = \tfrac{1}{2}m_{tot}v_{cm}^2 = \tfrac{1}{2}(8\,\text{kg})(3.75\,\text{m/s})^2 = 56.3\,\text{J} = \boxed{K - K_{rel}}$$

Impulse and Average Force

59 •

Picture the Problem The impulse imparted to the ball by the kicker equals the *change* in the ball’s momentum. The impulse is also the product of the average force exerted on the ball by the kicker and the time during which the average force acts.

(*a*) Relate the impulse delivered to the ball to its change in momentum:

$$I = \Delta p = p_f - p_i$$
$$= mv_f \text{ since } v_i = 0$$

Substitute numerical values and evaluate *I*:

$$I = (0.43\,\text{kg})(25\,\text{m/s}) = \boxed{10.8\,\text{N}\cdot\text{s}}$$

(*b*) Express the impulse delivered to the ball as a function of the average force acting on it and solve for and evaluate F_{av}:

$$I = F_{av}\Delta t$$
and
$$F_{av} = \frac{I}{\Delta t} = \frac{10.8\,\text{N}\cdot\text{s}}{0.008\,\text{s}} = \boxed{1.34\,\text{kN}}$$

60 •

Picture the Problem The impulse exerted by the ground on the brick equals the *change* in momentum of the brick and is also the product of the average force exerted by the ground on the brick and the time during which the average force acts.

(*a*) Express the impulse exerted by the ground on the brick:

$$I = |\Delta p_{brick}| = |p_{f,brick} - p_{i,brick}|$$

Because $p_{f,brick} = 0$:

$$I = p_{i,brick} = m_{brick}v \qquad (1)$$

Use conservation of energy to determine the speed of the brick at impact:

$$\Delta K + \Delta U = 0$$
or
$$K_f - K_i + U_f - U_i = 0$$

Because $U_f = K_i = 0$:

$$K_f - U_i = 0$$
or
$$\tfrac{1}{2}m_{brick}v^2 - m_{brick}gh = 0$$

Solve for *v*:

$$v = \sqrt{2gh}$$

Substitute in equation (1) to obtain:

$$I = m_{brick}\sqrt{2gh}$$

Substitute numerical values and evaluate *I*:

$$I = (0.3\,\text{kg})\sqrt{2(9.81\,\text{m/s}^2)(8\,\text{m})}$$
$$= \boxed{3.76\,\text{N}\cdot\text{s}}$$

(*c*) Express the impulse delivered to the brick as a function of the

$$I = F_{av}\Delta t$$
and

average force acting on it and solve for and evaluate F_{av}:

$$F_{av} = \frac{I}{\Delta t} = \frac{3.76\,\text{N}\cdot\text{s}}{0.0013\,\text{s}} = \boxed{2.89\,\text{kN}}$$

***61** •

Picture the Problem The impulse exerted by the ground on the meteorite equals the *change* in momentum of the meteorite and is also the product of the average force exerted by the ground on the meteorite and the time during which the average force acts.

Express the impulse exerted by the ground on the meteorite:

$$I = \Delta p_{\text{meteorite}} = p_f - p_i$$

Relate the kinetic energy of the meteorite to its initial momentum and solve for its initial momentum:

$$K_i = \frac{p_i^2}{2m} \Rightarrow p_i = \sqrt{2mK_i}$$

Express the ratio of the initial and final kinetic energies of the meteorite:

$$\frac{K_i}{K_f} = \frac{\frac{p_i^2}{2m}}{\frac{p_f^2}{2\text{m}}} = \frac{p_i^2}{p_f^2} = 2$$

Solve for p_f:

$$p_f = \frac{p_i}{\sqrt{2}}$$

Substitute in our expression for I and simplify:

$$I = \frac{p_i}{\sqrt{2}} - p_i = p_i\left(\frac{1}{\sqrt{2}} - 1\right)$$
$$= \sqrt{2mK_i}\left(\frac{1}{\sqrt{2}} - 1\right)$$

Because our interest is in its magnitude, evaluate $|I|$:

$$|I| = \left|\sqrt{2(30.8\times10^3\,\text{kg})(617\times10^6\,\text{J})}\left(\frac{1}{\sqrt{2}} - 1\right)\right| = \boxed{1.81\,\text{MN}\cdot\text{s}}$$

Express the impulse delivered to the meteorite as a function of the average force acting on it and solve for and evaluate F_{av}:

$$I = F_{av}\Delta t$$

and

$$F_{av} = \frac{I}{\Delta t} = \frac{1.81\,\text{MN}\cdot\text{s}}{3\,\text{s}} = \boxed{0.602\,\text{MN}}$$

62 ••

Picture the Problem The impulse exerted by the bat on the ball equals the *change* in momentum of the ball and is also the product of the average force exerted by the bat on the ball and the time during which the bat and ball were in contact.

(*a*) Express the impulse exerted by the bat on the ball in terms of the change in momentum of the ball:

$$\vec{I} = \Delta\vec{p}_{ball} = \vec{p}_f - \vec{p}_i = mv_f\hat{i} - (-mv_i\hat{i}) = 2mv\hat{i}$$

where $v = v_f = v_i$

Substitute for m and v and evaluate I:

$$I = 2(0.15\,\text{kg})(20\,\text{m/s}) = \boxed{6.00\,\text{N}\cdot\text{s}}$$

(*b*) Express the impulse delivered to the ball as a function of the average force acting on it and solve for and evaluate F_{av}:

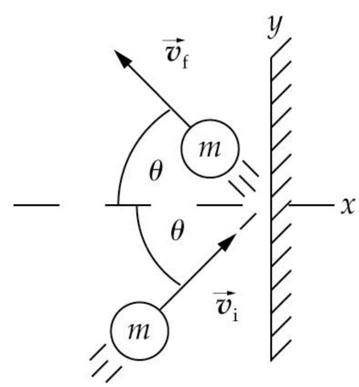
$$I = F_{av}\Delta t$$

and

$$F_{av} = \frac{I}{\Delta t} = \frac{6.00\,\text{N}\cdot\text{s}}{1.3\,\text{ms}} = \boxed{4.62\,\text{kN}}$$

***63** ••

Picture the Problem The figure shows the handball just before and immediately after its collision with the wall. Choose a coordinate system in which the positive x direction is to the right. The wall changes the momentum of the ball by exerting a force on it during the ball's collision with it. The reaction to this force is the force the ball exerts on the wall. Because these action and reaction forces are equal in magnitude, we can find the average force exerted on the ball by finding the change in momentum of the ball.



Using Newton's 3rd law, relate the average force exerted by the ball on the wall to the average force exerted by the wall on the ball:

$$\vec{F}_{av\,on\,wall} = -\vec{F}_{av\,on\,ball}$$

and

$$F_{av\,on\,wall} = F_{av\,on\,ball} \qquad (1)$$

Relate the average force exerted by the wall on the ball to its change in momentum:

$$\vec{F}_{av\,on\,ball} = \frac{\Delta\vec{p}}{\Delta t} = \frac{m\Delta\vec{v}}{\Delta t}$$

Express $\Delta\vec{v}_x$ for the ball:

$$\Delta\vec{v}_x = v_{f,x}\hat{i} - v_{i,x}\hat{i}$$

or, because $v_{i,x} = v\cos\theta$ and $v_{f,x} = -v\cos\theta$,

$$\Delta\vec{v}_x = -v\cos\theta\,\hat{i} - v\cos\theta\,\hat{i} = -2v\cos\theta\,\hat{i}$$

Substitute in our expression for $\vec{F}_{\text{av on ball}}$:

$$\vec{F}_{\text{av on ball}} = \frac{m\Delta\vec{v}}{\Delta t} = -\frac{2mv\cos\theta}{\Delta t}\hat{i}$$

Evaluate the magnitude of $\vec{F}_{\text{av on ball}}$:

$$F_{\text{av on ball}} = \frac{2mv\cos\theta}{\Delta t} = \frac{2(0.06\,\text{kg})(5\,\text{m/s})\cos 40°}{2\,\text{ms}} = 230\,\text{N}$$

Substitute in equation (1) to obtain:

$$F_{\text{av on wall}} = \boxed{230\,\text{N}}$$

64 ••

Picture the Problem The pictorial representation shows the ball during the interval of time you are exerting a force on it to accelerate it upward. The average force you exert can be determined from the change in momentum of the ball. The change in the velocity of the ball can be found by applying conservation of mechanical energy to its rise in the air once it has left your hand.

(*a*) Relate the average force exerted by your hand on the ball to the change in momentum of the ball:

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{p_2 - p_1}{\Delta t} = \frac{mv_2}{\Delta t}$$

because v_1 and, hence, $p_1 = 0$.

Letting $U_g = 0$ at the initial elevation of your hand, use conservation of mechanical energy to relate the initial kinetic energy of the ball to its potential energy when it is at its highest point:

$$\Delta K + \Delta U = 0$$

or

$$-K_i + U_f = 0$$

since $K_f = U_i = 0$

Substitute for K_f *and* U_i and solve for v_2:

$$-\tfrac{1}{2}mv_2^2 + mgh = 0$$

and

$$v_2 = \sqrt{2gh}$$

Relate Δt to the average speed of the ball while you are throwing it upward:

$$\Delta t = \frac{d}{v_{av}} = \frac{d}{\frac{v_2}{2}} = \frac{2d}{v_2}$$

Substitute for Δt and v_2 in the expression for F_{av} to obtain:

$$F_{av} = \frac{mgh}{d}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = \frac{(0.15\,\text{kg})(9.81\,\text{m/s}^2)(40\,\text{m})}{0.7\,\text{m}} = \boxed{84.1\,\text{N}}$$

(*b*) Express the ratio of the weight of the ball to the average force acting on it:

$$\frac{w}{F_{av}} = \frac{mg}{F_{av}} = \frac{(0.15\,\text{kg})(9.81\,\text{m/s}^2)}{84.1\,\text{N}} < 2\%$$

Because the weight of the ball is less than 2% of the average force exerted on the ball, it is reasonable to have neglected its weight.

65 ••

Picture the Problem Choose a coordinate system in which the direction the ball is moving *after* its collision with the wall is the positive *x* direction. The impulse delivered to the wall or received by the player equals the change in the momentum of the ball. We can find the average forces from the rate of change in the momentum of the ball.

(*a*) Relate the impulse delivered to the wall to the change in momentum of the handball:

$$\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i = (0.06\,\text{kg})(8\,\text{m/s})\hat{i} - \left[-(0.06\,\text{kg})(10\,\text{m/s})\hat{i}\right] = \boxed{(1.08\,\text{N}\cdot\text{s})\hat{i}\text{ directed into wall.}}$$

(*b*) Find F_{av} from the change in the ball's momentum:

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{1.08\,\text{N}\cdot\text{s}}{0.003\,\text{s}} = \boxed{360\,\text{N, into wall.}}$$

(*c*) Find the impulse received by the player from the change in momentum of the ball:

$$I = \Delta p_{ball} = m\Delta v$$
$$= (0.06\,\text{kg})(8\,\text{m/s})$$
$$= \boxed{0.480\,\text{N}\cdot\text{s, away from wall.}}$$

(*d*) Relate F_{av} to the change in the ball's momentum:

$$F_{av} = \frac{\Delta p_{ball}}{\Delta t}$$

Express the stopping time in terms of the average speed v_{av} of the ball and its stopping distance d:

$$\Delta t = \frac{d}{v_{av}}$$

Substitute to obtain:

$$F_{av} = \frac{v_{av}\Delta p_{ball}}{d}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = \frac{(4\,\text{m/s})(0.480\,\text{N}\cdot\text{s})}{0.5\,\text{m}}$$
$$= \boxed{3.84\,\text{N, away from wall.}}$$

66 •••

Picture the Problem The average force exerted on the limestone by the droplets of water equals the rate at which momentum is being delivered to the floor. We're given the number of droplets that arrive per minute and can use conservation of mechanical energy to determine their velocity as they reach the floor.

(*a*) Letting *N* represent the rate at which droplets fall, relate F_{av} to the change in the droplet's momentum:

$$F_{av} = \frac{\Delta p_{droplets}}{\Delta t} = N\frac{m\Delta v}{\Delta t}$$

Find the mass of the droplets:

$$m = \rho V = (1\,\text{kg/L})(0.03\,\text{mL})$$
$$= 3\times10^{-5}\,\text{kg}$$

Letting $U_g = 0$ at the point of impact of the droplets, use conservation of mechanical energy to relate their speed at impact to their fall distance:

$$\Delta K + \Delta U = 0$$

or

$$K_f - K_i + U_f - U_i = 0$$

Because $K_i = U_f = 0$:

$$\tfrac{1}{2}mv_f^2 - mgh = 0$$

Solve for and evaluate $v = v_f$:

$$v = \sqrt{2gh} = \sqrt{2\left(9.81\,\text{m/s}^2\right)(5\,\text{m})} = 9.90\,\text{m/s}$$

Substitute numerical values and evaluate F_{av}:

$$F_{av} = \left(\frac{N}{\Delta t}\right)m\Delta v = \left(10\,\frac{\text{droplets}}{\text{min}} \times \frac{1\,\text{min}}{60\,\text{s}}\right) \times \left(3\times10^{-5}\,\text{kg}\right)(9.90\,\text{m/s}) = \boxed{4.95\times10^{-5}\,\text{N}}$$

(*b*) Calculate the ratio of the weight of a droplet to F_{av}:

$$\frac{w}{F_{av}} = \frac{mg}{F_{av}} = \frac{\left(3\times10^{-5}\,\text{kg}\right)\left(9.81\,\text{m/s}^2\right)}{4.95\times10^{-5}\,\text{N}} \approx \boxed{6}$$

Collisions in One Dimension

***67 •**

Picture the Problem We can apply conservation of momentum to this perfectly inelastic collision to find the after-collision speed of the two cars. The ratio of the transformed kinetic energy to kinetic energy before the collision is the fraction of kinetic energy lost in the collision.

(*a*) Letting *V* be the velocity of the two cars after their collision, apply conservation of momentum to their perfectly inelastic collision:

$$p_{\text{initial}} = p_{\text{final}}$$

or

$$mv_1 + mv_2 = (m+m)V$$

Solve for and evaluate *V*:

$$V = \frac{v_1 + v_2}{2} = \frac{30\,\text{m/s} + 10\,\text{m/s}}{2} = \boxed{20.0\,\text{m/s}}$$

(*b*) Express the ratio of the kinetic energy that is lost to the kinetic energy of the two cars before the collision and simplify:

$$\frac{\Delta K}{K_{\text{initial}}}=\frac{K_{\text{final}}-K_{\text{initial}}}{K_{\text{initial}}}=\frac{K_{\text{final}}}{K_{\text{initial}}}-1=\frac{\frac{1}{2}(2m)V^2}{\frac{1}{2}mv_1^2+\frac{1}{2}mv_2^2}-1=\frac{2V^2}{v_1^2+v_2^2}-1$$

Substitute numerical values to obtain:

$$\frac{\Delta K}{K_{\text{initial}}}=\frac{2(20\,\text{m/s})^2}{(30\,\text{m/s})^2+(10\,\text{m/s})^2}-1=-0.200$$

$$\boxed{\text{20\% of the initial kinetic energy is transformed into heat, sound, and the deformation of metal.}}$$

68 •

Picture the Problem We can apply conservation of momentum to this perfectly inelastic collision to find the after-collision speed of the two players.

Letting the subscript 1 refer to the running back and the subscript 2 refer to the linebacker, apply conservation of momentum to their perfectly inelastic collision:

$$p_{\text{i}}=p_{\text{f}}$$

or

$$m_1v_1=(m_1+m_2)V$$

Solve for V:

$$V=\frac{m_1}{m_1+m_2}v_1$$

Substitute numerical values and evaluate V:

$$V=\frac{85\,\text{kg}}{85\,\text{kg}+105\,\text{kg}}(7\,\text{m/s})=\boxed{3.13\,\text{m/s}}$$

69 •

Picture the Problem We can apply conservation of momentum to this collision to find the after-collision speed of the 5-kg object. Let the direction the 5-kg object is moving before the collision be the positive direction. We can decide whether the collision was elastic by examining the initial and final kinetic energies of the system.

(*a*) Letting the subscript 5 refer to the 5-kg object and the subscript 2 refer to the 10-kg object, apply conservation of momentum to obtain:

$$p_i = p_f$$

or

$$m_5 v_{i,5} - m_{10} v_{i,10} = m_5 v_{f,5}$$

Solve for $v_{f,5}$:

$$v_{f,5} = \frac{m_5 v_{i,5} - m_{10} v_{i,10}}{m_5}$$

Substitute numerical values and evaluate $v_{f,5}$:

$$v_{f,5} = \frac{(5\,\text{kg})(4\,\text{m/s}) - (10\,\text{kg})(3\,\text{m/s})}{5\,\text{kg}}$$

$$= \boxed{-2.00\,\text{m/s}}$$

where the minus sign means that the 5-kg object is moving to the left after the collision.

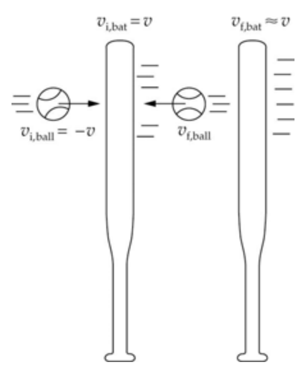
(*b*) Evaluate ΔK for the collision:

$$\Delta K = K_f - K_i = \tfrac{1}{2}(5\,\text{kg})(2\,\text{m/s})^2 - \left[\tfrac{1}{2}(5\,\text{kg})(4\,\text{m/s})^2 + \tfrac{1}{2}(10\,\text{kg})(3\,\text{m/s})^2\right] = -75.0\,\text{J}$$

$$\boxed{\text{Because } \Delta K \neq 0, \text{ the collision was inelastic.}}$$

70 •

Picture the Problem The pictorial representation shows the ball and bat just before and just after their collision. Take the direction the bat is moving to be the positive direction. Because the collision is elastic, we can equate the speeds of recession and approach, with the approximation that $v_{i,bat} \approx v_{f,bat}$ to find $v_{f,ball}$.



Express the speed of approach of the bat and ball:

$$v_{f,bat} - v_{f,ball} = -(v_{i,bat} - v_{i,ball})$$

Because the mass of the bat is much greater than that of the ball:

$$v_{i,bat} \approx v_{f,bat}$$

Substitute to obtain:

$$v_{f,bat} - v_{f,ball} = -(v_{f,bat} - v_{i,ball})$$

Solve for and evaluate $v_{\text{f,ball}}$:

$$v_{\text{f,ball}} = v_{\text{f,bat}} + \left(v_{\text{f,bat}} - v_{\text{i,ball}}\right) = -v_{\text{i,ball}} + 2v_{\text{f,bat}} = v + 2v = \boxed{3v}$$

***71 ••**

Picture the Problem Let the direction the proton is moving before the collision be the positive x direction. We can use both conservation of momentum and conservation of mechanical energy to obtain an expression for velocity of the proton after the collision.

(a) Use the expression for the total momentum of a system to find v_{cm}:

$$\vec{P} = \sum_i m_i \vec{v}_i = M\vec{v}_{\text{cm}}$$

and

$$\vec{v}_{\text{cm}} = \frac{m\vec{v}_{\text{p,i}}}{m + 12m} = \tfrac{1}{13}(300\,\text{m/s})\hat{i} = \boxed{(23.1\,\text{m/s})\hat{i}}$$

(b) Use conservation of momentum to obtain one relation for the final velocities:

$$m_{\text{p}} v_{\text{p,i}} = m_{\text{p}} v_{\text{p,f}} + m_{\text{nuc}} v_{\text{nuc,f}} \qquad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{\text{nuc,f}} - v_{\text{p,f}} = -\left(v_{\text{nuc,i}} - v_{\text{p,i}}\right) = v_{\text{p,i}} \qquad (2)$$

To eliminate $v_{\text{nuc,f}}$, solve equation (2) for $v_{\text{nuc,f}}$, and substitute the result in equation (1):

$$v_{\text{nuc,f}} = v_{\text{p,i}} + v_{\text{p,f}}$$
$$m_{\text{p}} v_{\text{p,i}} = m_{\text{p}} v_{\text{p,f}} + m_{\text{nuc}}\left(v_{\text{p,i}} + v_{\text{p,f}}\right)$$

Solve for and evaluate $v_{\text{p,f}}$:

$$v_{\text{p,f}} = \frac{m_{\text{p}} - m_{\text{nuc}}}{m_{\text{p}} + m_{\text{nuc}}} v_{\text{p,i}} = \frac{m - 12m}{13m}(300\,\text{m/s}) = \boxed{-254\,\text{m/s}}$$

72 ••

Picture the Problem We can use conservation of momentum and the definition of an elastic collision to obtain two equations in $v_{2\text{f}}$ and $v_{3\text{f}}$ that we can solve simultaneously.

Use conservation of momentum to obtain one relation for the final

$$m_3 v_{3\text{i}} = m_3 v_{3\text{f}} + m_2 v_{2\text{f}} \qquad (1)$$

velocities:

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{2f} - v_{3f} = -(v_{2i} - v_{3i}) = v_{3i} \qquad (2)$$

Solve equation (2) for v_{3f} , substitute in equation (1) to eliminate v_{3f}, and solve for and evaluate v_{2f}:

$$v_{2f} = \frac{2m_3v_{3i}}{m_2 + m_3} = \frac{2(3\,\text{kg})(4\,\text{m/s})}{2\,\text{kg} + 3\,\text{kg}}$$
$$= \boxed{4.80\,\text{m/s}}$$

Use equation (2) to find v_{3f}:

$$v_{3f} = v_{2f} - v_{3i} = 4.80\,\text{m/s} - 4.00\,\text{m/s}$$
$$= \boxed{0.800\,\text{m/s}}$$

Evaluate K_i and K_f:

$$K_i = K_{3i} = \tfrac{1}{2}m_3v_{3i}^2 = \tfrac{1}{2}(3\,\text{kg})(4\,\text{m/s})^2$$
$$= 24.0\,\text{J}$$

and

$$K_f = K_{3f} + K_{2f} = \tfrac{1}{2}m_3v_{3f}^2 + \tfrac{1}{2}m_2v_{2f}^2$$
$$= \tfrac{1}{2}(3\,\text{kg})(0.8\,\text{m/s})^2$$
$$+ \tfrac{1}{2}(2\,\text{kg})(4.8\,\text{m/s})^2$$
$$= 24.0\,\text{J}$$

Because $K_i = K_f$, we can conclude that the values obtained for v_{2f} and v_{3f} are consistent with the collision having been elastic.

73 ••

Picture the Problem We can find the velocity of the center of mass from the definition of the total momentum of the system. We'll use conservation of energy to find the maximum compression of the spring and express the initial (i.e., before collision) and final (i.e., at separation) velocities. Finally, we'll transform the velocities from the center of mass frame of reference to the table frame of reference.

(*a*) Use the definition of the total momentum of a system to relate the initial momenta to the velocity of the center of mass:

$$\vec{P} = \sum_i m_i\vec{v}_i = M\vec{v}_{cm}$$

or

$$m_1v_{1i} = (m_1 + m_2)v_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(2\,\text{kg})(10\,\text{m/s}) + (5\,\text{kg})(3\,\text{m/s})}{2\,\text{kg} + 5\,\text{kg}} = \boxed{5.00\,\text{m/s}}$$

(*b*) Find the kinetic energy of the system at maximum compression ($u_1 = u_2 = 0$):

$$K = K_{cm} = \tfrac{1}{2} M v_{cm}^2 = \tfrac{1}{2}(7\,\text{kg})(5\,\text{m/s})^2 = 87.5\,\text{J}$$

Use conservation of energy to relate the kinetic energy of the system to the potential energy stored in the spring at maximum compression:

$$\Delta K + \Delta U_s = 0$$

or

$$K_f - K_i + U_{sf} - U_{si} = 0$$

Because $K_f = K_{cm}$ and $U_{si} = 0$:

$$K_{cm} - K_i + \tfrac{1}{2} k (\Delta x)^2 = 0$$

Solve for Δx:

$$\Delta x = \sqrt{\frac{2(K_i - K_{cm})}{k}} = \sqrt{\frac{2\left[\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 - K_{cm}\right]}{k}} = \sqrt{\frac{m_1 v_{1i}^2 + m_2 v_{2i}^2 - 2K_{cm}}{k}}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \sqrt{\frac{(2\,\text{kg})(10\,\text{m/s})^2 + (5\,\text{kg})(3\,\text{m/s})^2}{1120\,\text{N/m}} - \frac{2(87.5\,\text{J})}{1120\,\text{N/m}}} = \boxed{0.250\,\text{m}}$$

(*c*) Find u_{1i}, u_{2i}, and u_{1f} for this elastic collision:

$$u_{1i} = v_{1i} - v_{cm} = 10\,\text{m/s} - 5\,\text{m/s} = 5\,\text{m/s},$$

$$u_{2i} = v_{2i} - v_{cm} = 3\,\text{m/s} - 5\,\text{m/s} = -2\,\text{m/s},$$

and

$$u_{1f} = v_{1f} - v_{cm} = 0 - 5\,\text{m/s} = -5\,\text{m/s}$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach and solve

$$u_{2f} - u_{1f} = -(u_{2i} - u_{1i})$$

and

for u_{2f}:

$$u_{2f} = -u_{2i} + u_{1i} + u_{1f} = -(-2\,\text{m/s}) + 5\,\text{m/s} - 5\,\text{m/s} = 2\,\text{m/s}$$

Transform u_{1f} and u_{2f} to the table frame of reference:

$$v_{1f} = u_{1f} + v_{cm} = -5\,\text{m/s} + 5\,\text{m/s} = \boxed{0}$$

and

$$v_{2f} = u_{2f} + v_{cm} = 2\,\text{m/s} + 5\,\text{m/s} = \boxed{7.00\,\text{m/s}}$$

***74 ••**

Picture the Problem Let the system include the earth, the bullet, and the sheet of plywood. Then $W_{ext} = 0$. Choose the zero of gravitational potential energy to be where the bullet enters the plywood. We can apply both conservation of energy and conservation of momentum to obtain the various physical quantities called for in this problem.

(*a*) Use conservation of mechanical energy after the bullet exits the sheet of plywood to relate its exit speed to the height to which it rises:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-\tfrac{1}{2}mv_m^2 + mgh = 0$$

Solve for v_m:

$$v_m = \boxed{\sqrt{2gh}}$$

Proceed similarly to relate the initial velocity of the plywood to the height to which it rises:

$$v_M = \boxed{\sqrt{2gH}}$$

(*b*) Apply conservation of momentum to the collision of the bullet and the sheet of plywood:

$$\vec{p}_i = \vec{p}_f$$

or

$$mv_{mi} = mv_m + Mv_M$$

Substitute for v_m and v_M and solve for v_{mi}:

$$v_{mi} = \boxed{\sqrt{2gh} + \frac{M}{m}\sqrt{2gH}}$$

(*c*) Express the initial mechanical energy of the system (i.e., just before the collision):

$$E_i = \tfrac{1}{2}mv_{mi}^2 = \boxed{mg\left[h + \frac{2M}{m}\sqrt{hH} + \left(\frac{M}{m}\right)^2 H\right]}$$

Express the final mechanical energy of the system (i.e., when the bullet and block have reached their maximum heights):

$$E_f = mgh + MgH = \boxed{g(mh + MH)}$$

(*d*) Use the work-energy theorem with $W_{ext} = 0$ to find the energy dissipated by friction in the inelastic collision:

$$E_f - E_i + W_{friction} = 0$$

and

$$W_{friction} = E_i - E_f = \boxed{gMH\left[2\sqrt{\frac{h}{H}} + \frac{M}{m} - 1\right]}$$

75 ••

Picture the Problem We can find the velocity of the center of mass from the definition of the total momentum of the system. We'll use conservation of energy to find the speeds of the particles when their separation is least and when they are far apart.

(*a*) Noting that when the distance between the two particles is least, both move at the same speed, namely v_{cm}, use the definition of the total momentum of a system to relate the initial momenta to the velocity of the center of mass:

$$\vec{\boldsymbol{P}} = \sum_i m_i \vec{\boldsymbol{v}}_i = M\vec{\boldsymbol{v}}_{cm}$$

or

$$m_p v_{pi} = (m_p + m_\alpha) v_{cm}.$$

Solve for and evaluate v_{cm}:

$$v_{cm} = v' = \frac{m_p v_{pi} + m_\alpha v_{\alpha i}}{m_1 + m_2} = \frac{mv_0 + 0}{m + 4m} = \boxed{0.200\, v_0}$$

(*b*) Use conservation of momentum to obtain one relation for the final velocities:

$$m_p v_0 = m_p v_{pf} + m_\alpha v_{\alpha f} \qquad (1)$$

Use conservation of mechanical energy to set the velocity of recession equal to the negative of the velocity of approach:

$$v_{pf} - v_{\alpha f} = -(v_{pi} - v_{\alpha i}) = -v_{pi} \qquad (2)$$

Solve equation (2) for v_{pf}, substitute in equation (1) to eliminate v_{pf}, and solve for $v_{\alpha f}$:

$$v_{\alpha f} = \frac{2m_p v_0}{m_p + m_\alpha} = \frac{2mv_0}{m + 4m} = \boxed{0.400\, v_0}$$

76 •

Picture the Problem Let the numeral 1 denote the electron and the numeral 2 the hydrogen atom. We can find the final velocity of the electron and, hence, the fraction of its initial kinetic energy that is transferred to the atom, by transforming to the center-of-mass reference frame, calculating the post-collision velocity of the electron, and then transforming back to the laboratory frame of reference.

Express *f*, the fraction of the electron's initial kinetic energy that is transferred to the atom:

$$f = \frac{K_i - K_f}{K_i} = 1 - \frac{K_f}{K_i} = 1 - \frac{\frac{1}{2}m_1 v_{1f}^2}{\frac{1}{2}m_1 v_{1i}^2} = 1 - \left(\frac{v_{1f}}{v_{1i}}\right)^2 \quad (1)$$

Find the velocity of the center of mass:

$$v_{cm} = \frac{m_1 v_{1i}}{m_1 + m_2}$$

or, because $m_2 = 1840m_1$,

$$v_{cm} = \frac{m_1 v_{1i}}{m_1 + 1840m_1} = \frac{1}{1841}v_{1i}$$

Find the initial velocity of the electron in the center-of-mass reference frame:

$$u_{1i} = v_{1i} - v_{cm} = v_{1i} - \frac{1}{1841}v_{1i} = \left(1 - \frac{1}{1841}\right)v_{1i}$$

Find the post-collision velocity of the electron in the center-of-mass reference frame by reversing its velocity:

$$u_{1f} = -u_{1i} = \left(\frac{1}{1841} - 1\right)v_{1i}$$

To find the final velocity of the electron in the original frame, add v_{cm} to its final velocity in the center-of-mass reference frame:

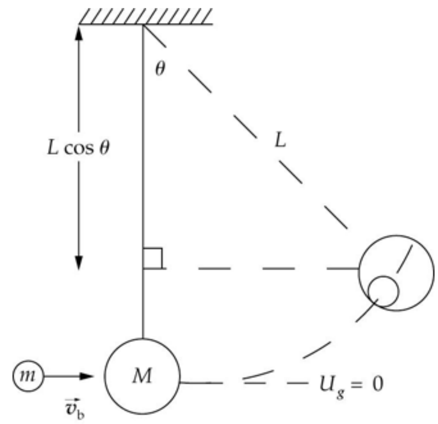
$$v_{1f} = u_{1f} + v_{cm} = \left(\frac{2}{1841} - 1\right)v_{1i}$$

Substitute in equation (1) to obtain:

$$f = 1 - \left(\frac{\left(\frac{2}{1841} - 1\right)v_{1i}}{v_{1i}}\right)^2 = 1 - \left(\frac{2}{1841} - 1\right)^2 = 2.17\times10^{-3} = \boxed{0.217\%}$$

77 ••

Picture the Problem The pictorial representation shows the bullet about to imbed itself in the bob of the ballistic pendulum and then, later, when the bob plus bullet have risen to their maximum height. We can use conservation of momentum during the collision to relate the speed of the bullet to the initial speed of the bob plus bullet (V). The initial kinetic energy of the bob plus bullet is transformed into gravitational potential energy when they reach their maximum height. Hence we apply conservation of mechanical energy to relate V to the angle through which the bullet plus bob swings and then solve the momentum and energy equations simultaneously for the speed of the bullet.



Use conservation of momentum to relate the speed of the bullet just before impact to the initial speed of the bob plus bullet:

$$mv_b = (m+M)V$$

Solve for the speed of the bullet:

$$v_b = \left(1+\frac{M}{m}\right)V \qquad (1)$$

Use conservation of energy to relate the initial kinetic energy of the bullet to the final potential energy of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f and solve for V:

$$-\tfrac{1}{2}(m+M)V^2 + (m+M)gL(1-\cos\theta) = 0$$

and

$$V = \sqrt{2gL(1-\cos\theta)}$$

Substitute for V in equation (1) to obtain:

$$v_b = \left(1+\frac{M}{m}\right)\sqrt{2gL(1-\cos\theta)}$$

Substitute numerical values and evaluate v_b:

$$v_b = \left(1 + \frac{1.5\,\text{kg}}{0.016\,\text{kg}}\right)\sqrt{2(9.81\,\text{m/s}^2)(2.3\,\text{m})(1-\cos 60°)} = \boxed{450\,\text{m/s}}$$

***78** ••

Picture the Problem We can apply conservation of momentum and the definition of an elastic collision to obtain equations relating the initial and final velocities of the colliding objects that we can solve for v_{1f} and v_{2f}.

Apply conservation of momentum to the elastic collision of the particles to obtain:

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i} \quad (1)$$

Relate the initial and final kinetic energies of the particles in an elastic collision:

$$\tfrac{1}{2} m_1 v_{1f}^2 + \tfrac{1}{2} m_2 v_{2f}^2 = \tfrac{1}{2} m_1 v_{1i}^2 + \tfrac{1}{2} m_2 v_{2i}^2$$

Rearrange this equation and factor to obtain:

$$m_2\left(v_{2f}^2 - v_{2i}^2\right) = m_1\left(v_{1i}^2 - v_{1f}^2\right)$$

or

$$m_2(v_{2f} - v_{2i})(v_{2f} + v_{2i}) = m_1(v_{1i} - v_{1f})(v_{1i} + v_{1f}) \quad (2)$$

Rearrange equation (1) to obtain:

$$m_2(v_{2f} - v_{2i}) = m_1(v_{1i} - v_{1f}) \quad (3)$$

Divide equation (2) by equation (3) to obtain:

$$v_{2f} + v_{2i} = v_{1i} + v_{1f}$$

Rearrange this equation to obtain equation (4):

$$v_{1f} - v_{2f} = v_{2i} - v_{1i} \quad (4)$$

Multiply equation (4) by m_2 and add it to equation (1) to obtain:

$$(m_1 + m_2)v_{1f} = (m_1 - m_2)v_{1i} + 2m_2 v_{2i}$$

Solve for v_{1f} to obtain:

$$v_{1f} = \boxed{\frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}}$$

Multiply equation (4) by m_1 and subtract it from equation (1) to obtain:

$$(m_1 + m_2)v_{2f} = (m_2 - m_1)v_{2i} + 2m_1 v_{1i}$$

Solve for v_{2f} to obtain:

$$v_{2f} = \boxed{\frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}}$$

Remarks: Note that the velocities satisfy the condition that $v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$. This verifies that the speed of recession equals the speed of approach.

79 ••

Picture the Problem As in this problem, Problem 78 involves an elastic, one-dimensional collision between two objects. Both solutions involve using the conservation of momentum equation $m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i}$ and the elastic collision equation $v_{1f} - v_{2f} = v_{2i} - v_{1i}$. In part (*a*) we can simply set the masses equal to each other and substitute in the equations in Problem 78 to show that the particles "swap" velocities. In part (*b*) we can divide the numerator and denominator of the equations in Problem 78 by m_2 and use the condition that $m_2 >> m_1$ to show that $v_{1f} \approx -v_{1i}+2v_{2i}$ and $v_{2f} \approx v_{2i}$.

(*a*) From Problem 78 we have:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}v_{1i} + \frac{2m_2}{m_1 + m_2}v_{2i} \qquad (1)$$

and

$$v_{2f} = \frac{2m_1}{m_1 + m_2}v_{1i} + \frac{m_2 - m_1}{m_1 + m_2}v_{2i} \qquad (2)$$

Set $m_1 = m_2 = m$ to obtain:

$$v_{1f} = \frac{2m}{m+m}v_{2i} = \boxed{v_{2i}}$$

and

$$v_{2f} = \frac{2m}{m+m}v_{1i} = \boxed{v_{1i}}$$

(*b*) Divide the numerator and denominator of both terms in equation (1) by m_2 to obtain:

$$v_{1f} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}v_{1i} + \frac{2}{\frac{m_1}{m_2} + 1}v_{2i}$$

If $m_2 >> m_1$:

$$v_{1f} \approx \boxed{-v_{1i} + 2v_{2i}}$$

Divide the numerator and denominator of both terms in equation (2) by m_2 to obtain:

$$v_{2f} = \frac{2\frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}v_{1i} + \frac{1 - \frac{m_1}{m_2}}{\frac{m_1}{m_2} + 1}v_{2i}$$

If $m_2 >> m_1$:

$$v_{2f} \approx \boxed{v_{2i}}$$

Remarks: Note that, in both parts of this problem, the velocities satisfy the condition that $v_{2f} - v_{1f} = -(v_{2i} - v_{1i})$. This verifies that the speed of recession equals the speed of approach.

Perfectly Inelastic Collisions and the Ballistic Pendulum

80 ••

Picture the Problem Choose $U_g = 0$ at the bob's equilibrium position. Momentum is conserved in the collision of the bullet with bob and the initial kinetic energy of the bob plus bullet is transformed into gravitational potential energy as it swings up to the top of the circle. If the bullet plus bob just makes it to the top of the circle with zero speed, it will swing through a complete circle.

Use conservation of momentum to relate the speed of the bullet just before impact to the initial speed of the bob plus bullet:

$$m_1 v = (m_1 + m_2)V$$

Solve for the speed of the bullet:

$$v = \left(1 + \frac{m_2}{m_1}\right)V \qquad (1)$$

Use conservation of energy to relate the initial kinetic energy of the bob plus bullet to their potential energy at the top of the circle:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f:

$$-\tfrac{1}{2}(m_1 + m_2)V^2 + (m_1 + m_2)g(2L) = 0$$

Solve for V:

$$V = \sqrt{gL}$$

Substitute for V in equation (1) and simplify to obtain:

$$v = \boxed{\left(1 + \frac{m_2}{m_1}\right)\sqrt{gL}}$$

***81 ••**

Picture the Problem Choose $U_g = 0$ at the equilibrium position of the ballistic pendulum. Momentum is conserved in the collision of the bullet with the bob and kinetic energy is transformed into gravitational potential energy as the bob swings up to its maximum height.

Letting V represent the initial speed of the bob as it begins its upward swing, use conservation of momentum to relate this speed to the speeds of the bullet just before and

$$m_1 v = m_1\left(\tfrac{1}{2}v\right) + m_2 V$$

after its collision with the bob:

Solve for the speed of the bob:

$$V = \frac{m_1}{2m_2}v \qquad (1)$$

Use conservation of energy to relate the initial kinetic energy of the bob to its potential energy at its maximum height:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f:

$$-\tfrac{1}{2}m_2V^2 + m_2gh = 0$$

Solve for h:

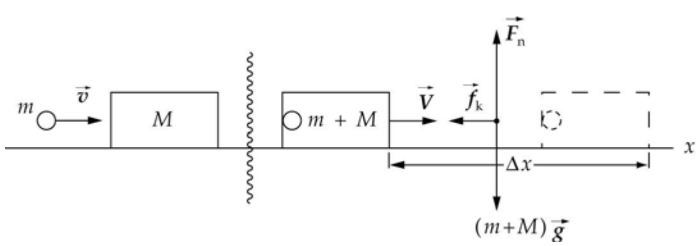
$$h = \frac{V^2}{2g} \qquad (2)$$

Substitute V from equation (1) in equation (2) and simplify to obtain:

$$h = \frac{\left(\frac{m_1}{2m_2}v\right)^2}{2g} = \boxed{\frac{v^2}{8g}\left(\frac{m_1}{m_2}\right)^2}$$

82 •

Picture the Problem Let the mass of the bullet be m, that of the wooden block M, the pre-collision velocity of the bullet v, and the post-collision velocity of the block+bullet be V. We can use conservation of momentum to find the velocity of the block with the bullet imbedded in it just after their perfectly inelastic collision. We can use Newton's 2nd law to find the acceleration of the sliding block and a constant-acceleration equation to find the distance the block slides.



Using a constant-acceleration equation, relate the velocity of the block+bullet just after their collision to their acceleration and displacement before stopping:

$$0 = V^2 + 2a\Delta x$$

because the final velocity of the block+bullet is zero.

Solve for the distance the block slides before coming to rest:

$$\Delta x = -\frac{V^2}{2a} \qquad (1)$$

Use conservation of momentum to relate the pre-collision velocity of the bullet to the post-collision velocity of the block+bullet:

$$mv = (m+M)V$$

Solve for V:

$$V = \frac{m}{m+M}v$$

Substitute in equation (1) to obtain:

$$\Delta x = -\frac{1}{2a}\left(\frac{m}{m+M}v\right)^2 \quad (2)$$

Apply $\sum \vec{F} = m\vec{a}$ to the block+bullet (see the FBD in the diagram):

$$\sum F_x = -f_\text{k} = (m+M)a \quad (3)$$

and

$$\sum F_y = F_\text{n} - (m+M)g = 0 \quad (4)$$

Use the definition of the coefficient of kinetic friction and equation (4) to obtain:

$$f_\text{k} = \mu_\text{k} F_\text{n} = \mu_\text{k}(m+M)g$$

Substitute in equation (3):

$$-\mu_\text{k}(m+M)g = (m+M)a$$

Solve for a to obtain:

$$a = -\mu_\text{k} g$$

Substitute in equation (2) to obtain:

$$\Delta x = \frac{1}{2\mu_\text{k} g}\left(\frac{m}{m+M}v\right)^2$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{1}{2(0.22)(9.81\,\text{m/s}^2)}\left(\frac{0.0105\,\text{kg}}{0.0105\,\text{kg} + 10.5\,\text{kg}}(750\,\text{m/s})\right)^2 = \boxed{0.130\,\text{m}}$$

83 ••

Picture the Problem The collision of the ball with the box is perfectly inelastic and we can find the speed of the box-and-ball immediately after their collision by applying conservation of momentum. If we assume that the kinetic friction force is constant, we can use a constant-acceleration equation to find the acceleration of the box and ball combination and the definition of μ_k to find its value.

Using its definition, express the coefficient of kinetic friction of the table:

$$\mu_\text{k} = \frac{f_\text{k}}{F_\text{n}} = \frac{(M+m)|a|}{(M+m)g} = \frac{|a|}{g} \quad (1)$$

Use conservation of momentum to relate the speed of the ball just before the collision to the speed of the ball+box immediately after the

$$MV = (m+M)v$$

collision:

Solve for v:

$$v = \frac{MV}{m+M} \qquad (2)$$

Use a constant-acceleration equation to relate the sliding distance of the ball+box to its initial and final velocities and its acceleration:

$$v_f^2 = v_i^2 + 2a\Delta x$$

or, because $v_f = 0$ and $v_i = v$,

$$0 = v^2 + 2a\Delta x$$

Solve for a:

$$a = -\frac{v^2}{2\Delta x}$$

Substitute in equation (1) to obtain:

$$\mu_k = \frac{v^2}{2g\Delta x}$$

Use equation (2) to eliminate v:

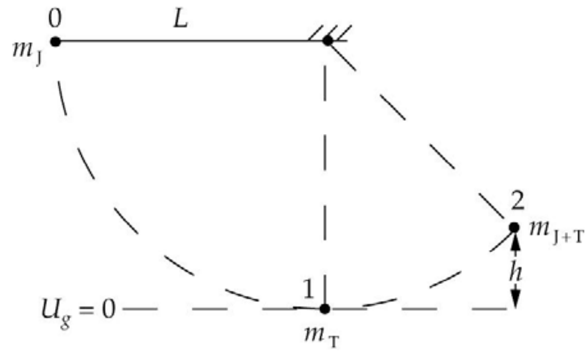
$$\mu_k = \frac{1}{2g\Delta x}\left(\frac{MV}{m+M}\right)^2 = \frac{1}{2g\Delta x}\left(\frac{V}{\frac{m}{M}+1}\right)^2$$

Substitute numerical values and evaluate μ_k:

$$\mu_k = \frac{1}{2(9.81\,\text{m/s}^2)(0.52\,\text{m})}\left(\frac{1.3\,\text{m/s}}{\frac{0.327\,\text{kg}}{0.425\,\text{kg}}+1}\right)^2 = \boxed{0.0529}$$

***84** ••

Picture the Problem Jane's collision with Tarzan is a perfectly inelastic collision. We can find her speed v_1 just before she grabs Tarzan from conservation of energy and their speed V just after she grabs him from conservation of momentum. Their kinetic energy just after their collision will be transformed into gravitational potential energy when they have reached their greatest height h.



Use conservation of energy to relate the potential energy of Jane and Tarzan at their highest point (2) to their kinetic energy immediately after Jane grabbed Tarzan:

$$U_2 = K_1$$
or
$$m_{J+T}gh = \tfrac{1}{2}m_{J+T}V^2$$

Solve for h to obtain:

$$h = \frac{V^2}{2g} \qquad (1)$$

Use conservation of momentum to relate Jane's velocity just before she collides with Tarzan to their velocity just after their perfectly inelastic collision:

$$m_J v_1 = m_{J+T}V$$

Solve for V:

$$V = \frac{m_J}{m_{J+T}}v_1 \qquad (2)$$

Apply conservation of energy to relate Jane's kinetic energy at 1 to her potential energy at 0:

$$K_1 = U_0$$
or
$$\tfrac{1}{2}m_J v_1^2 = m_J gL$$

Solve for v_1:

$$v_1 = \sqrt{2gL}$$

Substitute in equation (2) to obtain:

$$V = \frac{m_J}{m_{J+T}}\sqrt{2gL}$$

Substitute in equation (1) and simplify:

$$h = \frac{1}{2g}\left(\frac{m_J}{m_{J+T}}\right)^2 2gL = \left(\frac{m_J}{m_{J+T}}\right)^2 L$$

Substitute numerical values and evaluate h:

$$h = \left(\frac{54\,\text{kg}}{54\,\text{kg} + 82\,\text{kg}}\right)^2 (25\,\text{m}) = \boxed{3.94\,\text{m}}$$

Exploding Objects and Radioactive Decay

85 ••

Picture the Problem This nuclear reaction is $^4\text{Be} \rightarrow 2\alpha + 1.5\times10^{-14}$ J. In order to conserve momentum, the alpha particles will have move in opposite directions with the same velocities. We'll use conservation of energy to find their speeds.

Letting E represent the energy released in the reaction, express conservation of energy for this process:

$$2K_\alpha = 2\left(\tfrac{1}{2}m_\alpha v_\alpha^2\right) = E$$

Solve for v_α:

$$v_\alpha = \sqrt{\frac{E}{m_\alpha}}$$

Substitute numerical values and evaluate v_α:

$$v_\alpha = \sqrt{\frac{1.5\times10^{-14}\text{ J}}{6.68\times10^{-27}\text{ kg}}} = \boxed{1.50\times10^{6}\text{ m/s}}$$

86 ••

Picture the Problem This nuclear reaction is $^5\text{Li} \rightarrow \alpha + \text{p} + 3.15\times10^{-13}$ J. To conserve momentum, the alpha particle and proton must move in opposite directions. We'll apply both conservation of energy and conservation of momentum to find the speeds of the proton and alpha particle.

Use conservation of momentum in this process to express the alpha particle's velocity in terms of the proton's:

$$p_\text{i} = p_\text{f} = 0$$

and

$$0 = m_\text{p}v_\text{p} - m_\alpha v_\alpha$$

Solve for v_α and substitute for m_α to obtain:

$$v_\alpha = \frac{m_\text{p}}{m_\alpha}v_\text{p} = \frac{m_\text{p}}{4m_\text{p}}v_\text{p} = \tfrac{1}{4}v_\text{p}$$

Letting E represent the energy released in the reaction, apply conservation of energy to the process:

$$K_\text{p} + K_\alpha = E$$

or

$$\tfrac{1}{2}m_\text{p}v_\text{p}^2 + \tfrac{1}{2}m_\alpha v_\alpha^2 = E$$

Substitute for v_α:

$$\tfrac{1}{2}m_\text{p}v_\text{p}^2 + \tfrac{1}{2}m_\alpha\left(\tfrac{1}{4}v_\text{p}\right)^2 = E$$

Solve for v_p and substitute for m_α to obtain:

$$v_p = \sqrt{\frac{32E}{16m_p + m_\alpha}} = \sqrt{\frac{32E}{16m_p + 4m_p}}$$

Substitute numerical values and evaluate v_p:

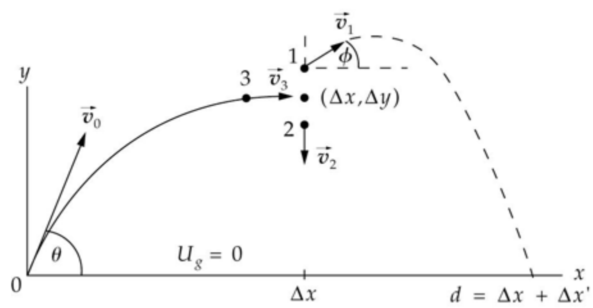
$$v_p = \sqrt{\frac{32(3.15\times10^{-13}\text{ J})}{20(1.67\times10^{-27}\text{ kg})}} = \boxed{1.74\times10^{7}\text{ m/s}}$$

Use the relationship between v_p and v_α to obtain v_α:

$$v_\alpha = \tfrac{1}{4}v_p = \tfrac{1}{4}(1.74\times10^{7}\text{ m/s}) = \boxed{4.34\times10^{6}\text{ m/s}}$$

87 •••

Picture the Problem The pictorial representation shows the projectile at its maximum elevation and is moving horizontally. It also shows the two fragments resulting from the explosion. We chose the system to include the projectile and the earth so that no external forces act to change the momentum of the system during the explosion. With this choice of system we can also use conservation of energy to determine the elevation of the projectile when it explodes. We'll also find it useful to use constant-acceleration equations in our description of the motion of the projectile and its fragments.



(*a*) Use conservation of momentum to relate the velocity of the projectile before its explosion to the velocities of its two parts after the explosion:

$$\vec{p}_i = \vec{p}_f$$
$$m_3\vec{v}_3 = m_1\vec{v}_1 + m_2\vec{v}_2$$
$$m_3v_3\hat{i} = m_1v_{x1}\hat{i} + m_1v_{y1}\hat{j} - m_2v_{y2}\hat{j}$$

The only way this equality can hold is if:

$$m_3v_3 = m_1v_{x1}$$
and
$$m_1v_{y1} = m_2v_{y2}$$

Express v_3 in terms of v_0 and substitute for the masses to obtain:

$$v_{x1} = 3v_3 = 3v_0\cos\theta = 3(120\,\text{m/s})\cos30° = 312\,\text{m/s}$$

and

$$v_{y1} = 2v_{y2} \quad (1)$$

Using a constant-acceleration equation with the downward direction positive, relate v_{y2} to the time it takes the 2-kg fragment to hit the ground:

$$\Delta y = v_{y2}\Delta t + \tfrac{1}{2} g(\Delta t)^2$$

$$v_{y2} = \frac{\Delta y - \frac{1}{2} g(\Delta t)^2}{\Delta t} \quad (2)$$

With $U_g = 0$ at the launch site, apply conservation of energy to the climb of the projectile to its maximum elevation:

$$\Delta K + \Delta U = 0$$

Because $K_f = U_i = 0$, $-K_i + U_f = 0$

or

$$-\tfrac{1}{2} m_3 v_{y0}^2 + m_3 g\Delta y = 0$$

Solve for Δy:

$$\Delta y = \frac{v_{y0}^2}{2g} = \frac{(v_0 \sin 30°)^2}{2g}$$

Substitute numerical values and evaluate Δy:

$$\Delta y = \frac{[(120\,\text{m/s})\sin 30°]^2}{2(9.81\,\text{m/s}^2)} = 183.5\,\text{m}$$

Substitute in equation (2) and evaluate v_{y2}:

$$v_{y2} = \frac{183.5\,\text{m} - \frac{1}{2}(9.81\,\text{m/s}^2)(3.6\,\text{s})^2}{3.6\,\text{s}} = 33.3\,\text{m/s}$$

Substitute in equation (1) and evaluate v_{y1}:

$$v_{y1} = 2(33.3\,\text{m/s}) = 66.6\,\text{m/s}$$

Express $\vec{v}_1$ in vector form:

$$\vec{v}_1 = v_{x1}\hat{i} + v_{y1}\hat{j} = \boxed{(312\,\text{m/s})\hat{i} + (66.6\,\text{m/s})\hat{j}}$$

(*b*) Express the total distance d traveled by the 1-kg fragment:

$$d = \Delta x + \Delta x' \quad (3)$$

Relate Δx to v_0 and the time-to-explosion:

$$\Delta x = (v_0 \cos\theta)(\Delta t_{\text{exp}}) \quad (4)$$

Using a constant-acceleration equation, express Δt_{exp}:

$$\Delta t_{\text{exp}} = \frac{v_{y0}}{g} = \frac{v_0 \sin\theta}{g}$$

Substitute numerical values and evaluate Δt_{exp}:

$$\Delta t_{exp} = \frac{(120\,\text{m/s})\sin 30°}{9.81\,\text{m/s}^2} = 6.12\,\text{s}$$

Substitute in equation (4) and evaluate Δx:

$$\Delta x = (120\,\text{m/s})(\cos 30°)(6.12\,\text{s}) = 636.5\,\text{m}$$

Relate the distance traveled by the 1-kg fragment after the explosion to the time it takes it to reach the ground:

$$\Delta x' = v_{x1}\Delta t'$$

Using a constant-acceleration equation, relate the time $\Delta t'$ for the 1-kg fragment to reach the ground to its initial speed in the y direction and the distance to the ground:

$$\Delta y = v_{y1}\Delta t' - \tfrac{1}{2}g(\Delta t')^2$$

Substitute to obtain the quadratic equation:

$$(\Delta t')^2 - (13.6\,\text{s})\Delta t' - 37.4\,\text{s}^2 = 0$$

Solve the quadratic equation to find $\Delta t'$:

$$\Delta t' = 15.9\text{ s}$$

Substitute in equation (3) and evaluate d:

$$\begin{aligned} d &= \Delta x + \Delta x' = \Delta x + v_{x1}\Delta t' \\ &= 636.5\,\text{m} + (312\,\text{m/s})(15.9\,\text{s}) \\ &= \boxed{5.61\,\text{km}} \end{aligned}$$

(*c*) Express the energy released in the explosion:

$$E_{exp} = \Delta K = K_f - K_i \qquad (5)$$

Find the kinetic energy of the fragments after the explosion:

$$\begin{aligned} K_f &= K_1 + K_2 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 \\ &= \tfrac{1}{2}(1\,\text{kg})\left[(312\,\text{m/s})^2 + (66.6\,\text{m/s})^2\right] \\ &\quad + \tfrac{1}{2}(2\,\text{kg})(33.3\,\text{m/s})^2 \\ &= 52.0\,\text{kJ} \end{aligned}$$

Find the kinetic energy of the projectile before the explosion:

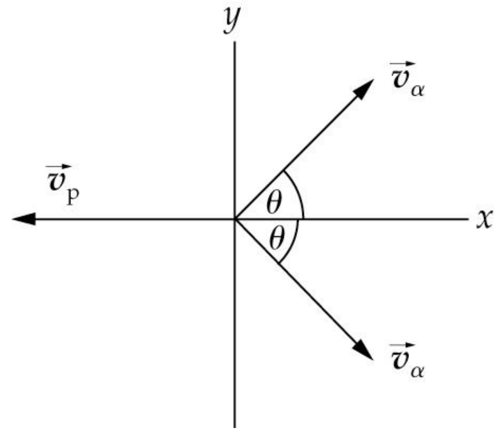
$$\begin{aligned} K_i &= \tfrac{1}{2}m_3v_3^2 = \tfrac{1}{2}m_3(v_0\cos\theta)^2 \\ &= \tfrac{1}{2}(3\,\text{kg})\left[(120\,\text{m/s})\cos 30°\right]^2 \\ &= 16.2\,\text{kJ} \end{aligned}$$

Substitute in equation (5) to determine the energy released in the explosion:

$$E_{exp} = K_f - K_i = 52.0\,\text{kJ} - 16.2\,\text{kJ} = \boxed{35.8\,\text{kJ}}$$

***88 •••**

Picture the Problem This nuclear reaction is $^{9}\text{B} \rightarrow 2\alpha + \text{p} + 4.4\times10^{-14}$ J. Assume that the proton moves in the –*x* direction as shown in the figure. The sum of the kinetic energies of the decay products equals the energy released in the decay. We'll use conservation of momentum to find the angle between the velocities of the proton and the alpha particles. Note that $v_\alpha = v_\alpha'$.



Express the energy released to the kinetic energies of the decay products:

$$K_\text{p} + 2K_\alpha = E_\text{rel}$$

or

$$\tfrac{1}{2}m_\text{p}v_\text{p}^2 + 2\left(\tfrac{1}{2}m_\alpha v_\alpha^2\right) = E_\text{rel}$$

Solve for v_α:

$$v_\alpha = \sqrt{\frac{E_\text{rel} - \frac{1}{2}m_\text{p}v_\text{p}^2}{m_\alpha}}$$

Substitute numerical values and evaluate v_α:

$$v_\alpha = \sqrt{\frac{4.4\times10^{-14}\,\text{J}}{6.68\times10^{-27}\,\text{kg}} - \frac{\frac{1}{2}\left(1.67\times10^{-27}\,\text{kg}\right)\left(6\times10^{6}\,\text{m/s}\right)^2}{6.68\times10^{-27}\,\text{kg}}} = \boxed{1.44\times10^{6}\,\text{m/s}}$$

Given that the boron isotope was at rest prior to the decay, use conservation of momentum to relate the momenta of the decay products:

$$\vec{p}_\text{f} = \vec{p}_\text{i} = 0 \Rightarrow p_{x\text{f}} = 0$$

$$\therefore 2\left(m_\alpha v_\alpha \cos\theta\right) - m_\text{p}v_\text{p} = 0$$

or

$$2\left(4m_\text{p}v_\alpha \cos\theta\right) - m_\text{p}v_\text{p} = 0$$

Solve for θ:

$$\theta = \cos^{-1}\left[\frac{v_\text{p}}{8v_\alpha}\right] = \cos^{-1}\left[\frac{6\times10^{6}\,\text{m/s}}{8\left(1.44\times10^{6}\,\text{m/s}\right)}\right] = \pm58.7°$$

Let θ' equal the angle the velocities of the alpha particles make with that of the proton:

$$\theta' = \pm(180° - 58.7°) = \boxed{\pm 121°}$$

Coefficient of Restitution

89 •

Picture the Problem The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which the ball was dropped and the height to which it rebounded by using conservation of mechanical energy.

Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_{rec}}{v_{app}}$$

Letting $U_g = 0$ at the surface of the steel plate, apply conservation of energy to express the velocity of approach:

$$\Delta K + \Delta U = 0$$

Because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

or

$$\tfrac{1}{2} m v_{app}^2 - mgh_{app} = 0$$

Solve for v_{app}:

$$v_{app} = \sqrt{2gh_{app}}$$

In like manner, show that:

$$v_{rec} = \sqrt{2gh_{rec}}$$

Substitute in the equation for e to obtain:

$$e = \frac{\sqrt{2gh_{rec}}}{\sqrt{2gh_{app}}} = \sqrt{\frac{h_{rec}}{h_{app}}}$$

Substitute numerical values and evaluate e:

$$e = \sqrt{\frac{2.5\,\text{m}}{3\,\text{m}}} = \boxed{0.913}$$

***90** •

Picture the Problem The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which an object was dropped and the height to which it rebounded by using conservation of mechanical energy.

Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_{\text{rec}}}{v_{\text{app}}}$$

Letting $U_g = 0$ at the surface of the steel plate, apply conservation of energy to express the velocity of approach:

$$\Delta K + \Delta U = 0$$

Because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

or

$$\tfrac{1}{2}mv_{\text{app}}^2 - mgh_{\text{app}} = 0$$

Solve for v_{app}:

$$v_{\text{app}} = \sqrt{2gh_{\text{app}}}$$

In like manner, show that:

$$v_{\text{rec}} = \sqrt{2gh_{\text{rec}}}$$

Substitute in the equation for e to obtain:

$$e = \frac{\sqrt{2gh_{\text{rec}}}}{\sqrt{2gh_{\text{app}}}} = \sqrt{\frac{h_{\text{rec}}}{h_{\text{app}}}}$$

Find $e_{\min}$:

$$e_{\min} = \sqrt{\frac{173\,\text{cm}}{254\,\text{cm}}} = 0.825$$

Find $e_{\max}$:

$$e_{\max} = \sqrt{\frac{183\,\text{cm}}{254\,\text{cm}}} = 0.849$$

and $\boxed{0.825 \le e \le 0.849}$

91 •

Picture the Problem Because the rebound kinetic energy is proportional to the rebound height, the percentage of mechanical energy lost in one bounce can be inferred from knowledge of the rebound height. The coefficient of restitution is defined as the ratio of the velocity of recession to the velocity of approach. These velocities can be determined from the heights from which an object was dropped and the height to which it rebounded by using conservation of mechanical energy.

(a) We know, from conservation of energy, that the kinetic energy of an object dropped from a given height h is proportional to h:

$$K \;\alpha\; h.$$

If, for each bounce of the ball, $h_{rec} = 0.8h_{app}$:

$$\boxed{20\%\text{ of its mechanical energy is lost.}}$$

(*b*) Use its definition to relate the coefficient of restitution to the velocities of approach and recession:

$$e = \frac{v_{rec}}{v_{app}}$$

Letting $U_g = 0$ at the surface from which the ball is rebounding, apply conservation of energy to express the velocity of approach:

$$\Delta K + \Delta U = 0$$

Because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

or

$$\tfrac{1}{2}mv_{app}^2 - mgh_{app} = 0$$

Solve for v_{app}:

$$v_{app} = \sqrt{2gh_{app}}$$

In like manner, show that:

$$v_{rec} = \sqrt{2gh_{rec}}$$

Substitute in the equation for e to obtain:

$$e = \frac{\sqrt{2gh_{rec}}}{\sqrt{2gh_{app}}} = \sqrt{\frac{h_{rec}}{h_{app}}}$$

Substitute for $\frac{h_{rec}}{h_{app}}$ to obtain:

$$e = \sqrt{0.8} = \boxed{0.894}$$

92 ••

Picture the Problem Let the numeral 2 refer to the 2-kg object and the numeral 4 to the 4-kg object. Choose a coordinate system in which the direction the 2-kg object is moving before the collision is the positive *x* direction and let the system consist of the earth, the surface on which the objects slide, and the objects. Then we can use conservation of momentum to find the velocity of the recoiling 4-kg object. We can find the energy transformed in the collision by calculating the difference between the kinetic energies before and after the collision and the coefficient of restitution from its definition.

(*a*) Use conservation of momentum in one dimension to relate the initial and final momenta of the participants in the collision:

$$\vec{p}_i = \vec{p}_f$$

or

$$m_2 v_{2i} = m_4 v_{4f} - m_2 v_{2f}$$

Solve for and evaluate the final velocity of the 4-kg object:

$$v_{4f} = \frac{m_2 v_{2i} + m_2 v_{2f}}{m_4} = \frac{(2\,\text{kg})(6\,\text{m/s} + 1\,\text{m/s})}{4\,\text{kg}} = \boxed{3.50\,\text{m/s}}$$

(*b*) Express the energy lost in terms of the kinetic energies before and after the collision:

$$E_{\text{lost}} = K_i - K_f = \tfrac{1}{2} m_2 v_{2i}^2 - \left(\tfrac{1}{2} m_2 v_{2f}^2 + \tfrac{1}{2} m_4 v_{4f}^2\right) = \tfrac{1}{2}\left[m_2\left(v_{2i}^2 - v_{2f}^2\right) - m_4 v_{4f}^2\right]$$

Substitute numerical values and evaluate E_{lost}:

$$E_{\text{lost}} = \tfrac{1}{2}\left[\left((2\,\text{kg})\left\{(6\,\text{m/s})^2 - (1\,\text{m/s})^2\right\}\right) - (4\,\text{kg})(3.5\,\text{m/s})^2\right] = \boxed{10.5\,\text{J}}$$

(*c*) Use the definition of the coefficient of restitution:

$$e = \frac{v_{\text{rec}}}{v_{\text{app}}} = \frac{v_{4f} - v_{2f}}{v_{2i}} = \frac{3.5\,\text{m/s} - (-1\,\text{m/s})}{6\,\text{m/s}} = \boxed{0.750}$$

93 ••

Picture the Problem Let the numeral 2 refer to the 2-kg block and the numeral 3 to the 3-kg block. Choose a coordinate system in which the direction the blocks are moving before the collision is the positive *x* direction and let the system consist of the earth, the surface on which the blocks move, and the blocks. Then we can use conservation of momentum find the velocity of the 2-kg block after the collision. We can find the coefficient of restitution from its definition.

(*a*) Use conservation of momentum in one dimension to relate the initial and final momenta of the participants in the collision:

$$\vec{p}_i = \vec{p}_f$$

or

$$m_2 v_{2i} + m_3 v_{3i} = m_2 v_{2f} + m_3 v_{3f}$$

Solve for the final velocity of the 2-kg object:

$$v_{2f} = \frac{m_2 v_{2i} + m_3 v_{3i} - m_3 v_{3f}}{m_2}$$

Substitute numerical values and evaluate v_{2f}:

$$v_{2f} = \frac{(2\,\text{kg})(5\,\text{m/s}) + (3\,\text{kg})(2\,\text{m/s} - 4.2\,\text{m/s})}{2\,\text{kg}} = \boxed{1.70\,\text{m/s}}$$

(*b*) Use the definition of the coefficient of restitution:

$$e = \frac{v_{\text{rec}}}{v_{\text{app}}} = \frac{v_{3\text{f}} - v_{2\text{f}}}{v_{2\text{i}} - v_{3\text{i}}} = \frac{4.2\,\text{m/s} - 1.7\,\text{m/s}}{5\,\text{m/s} - 2\,\text{m/s}}$$

$$= \boxed{0.833}$$

Collisions in Three Dimensions

***94 ••**

Picture the Problem We can use the definition of the magnitude of a vector and the definition of the dot product to establish the result called for in (*a*). In part (*b*) we can use the result of part (*a*), the conservation of momentum, and the definition of an elastic collision (kinetic energy is conserved) to show that the particles separate at right angles.

(*a*) Find the dot product of $\vec{B} + \vec{C}$ with itself:

$$\left(\vec{B} + \vec{C}\right) \cdot \left(\vec{B} + \vec{C}\right) = B^2 + C^2 + 2\vec{B} \cdot \vec{C}$$

Because $\vec{A} = \vec{B} + \vec{C}$:

$$A^2 = \left|\vec{B} + \vec{C}\right|^2 = \left(\vec{B} + \vec{C}\right) \cdot \left(\vec{B} + \vec{C}\right)$$

Substitute to obtain:

$$\boxed{A^2 = B^2 + C^2 + 2\vec{B} \cdot \vec{C}}$$

(*b*) Apply conservation of momentum to the collision of the particles:

$$\vec{p}_1 + \vec{p}_2 = \vec{P}$$

Form the dot product of each side of this equation with itself to obtain:

$$\left(\vec{p}_1 + \vec{p}_2\right) \cdot \left(\vec{p}_1 + \vec{p}_2\right) = \vec{P} \cdot \vec{P}$$

or

$$p_1^2 + p_2^2 + 2\vec{p}_1 \cdot \vec{p}_2 = P^2 \qquad (1)$$

Apply the definition of an elastic collision to obtain:

$$\frac{p_1^2}{2m} + \frac{p_2^2}{2m} = \frac{P^2}{2m}$$

or

$$p_1^2 + p_2^2 = P^2 \qquad (2)$$

Subtract equation (1) from equation (2) to obtain:

$$2\vec{p}_1 \cdot \vec{p}_2 = 0 \text{ or } \boxed{\vec{p}_1 \cdot \vec{p}_2 = 0}$$

i.e., the particles move apart along paths that are at right angles to each other.

95 •

Picture the Problem Let the initial direction of motion of the cue ball be the positive *x* direction. We can apply conservation of energy to determine the angle the cue ball makes with the positive *x* direction and the conservation of momentum to find the final velocities of the cue ball and the eight ball.

(*a*) Use conservation of energy to relate the velocities of the collision participants before and after the collision:

$$\frac{1}{2}mv_{ci}^2 = \frac{1}{2}mv_{cf}^2 + \frac{1}{2}mv_8^2$$

or

$$v_{ci}^2 = v_{cf}^2 + v_8^2$$

This Pythagorean relationship tells us that $\vec{v}_{ci}$, $\vec{v}_{cf}$, and $\vec{v}_8$ form a right triangle. Hence:

$$\theta_{cf} + \theta_8 = 90°$$

and

$$\theta_{cf} = \boxed{60°}$$

(*b*) Use conservation of momentum in the *x* direction to relate the velocities of the collision participants before and after the collision:

$$\vec{p}_{xi} = \vec{p}_{xf}$$

or

$$mv_{ci} = mv_{cf}\cos\theta_{cf} + mv_8\cos\theta_8$$

Use conservation of momentum in the *y* direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$\vec{p}_{yi} = \vec{p}_{yf}$$

or

$$0 = mv_{cf}\sin\theta_{cf} + mv_8\sin\theta_8$$

Solve these equations simultaneously to obtain:

$$v_{cf} = \boxed{2.50\text{ m/s}}$$

and

$$v_8 = \boxed{4.33\text{ m/s}}$$

96 ••

Picture the Problem We can find the final velocity of the object whose mass is M_1 by using the conservation of momentum. Whether the collision was elastic can be decided by examining the difference between the initial and final kinetic energy of the interacting objects.

(*a*) Use conservation of momentum to relate the initial and final velocities of the two objects:

$$\vec{p}_i = \vec{p}_f$$

or

$$mv_0\hat{\boldsymbol{i}} + 2m\left(\tfrac{1}{2}v_0\hat{\boldsymbol{j}}\right) = 2m\left(\tfrac{1}{4}v_0\hat{\boldsymbol{i}}\right) + m\vec{v}_{1f}$$

Simplify to obtain:

$$v_0\hat{\boldsymbol{i}} + v_0\hat{\boldsymbol{j}} = \tfrac{1}{2}v_0\hat{\boldsymbol{i}} + \vec{v}_{1f}$$

Solve for $\vec{v}_{1f}$:

$$\vec{v}_{1f} = \boxed{\tfrac{1}{2}v_0\hat{\boldsymbol{i}} + v_0\hat{\boldsymbol{j}}}$$

(*b*) Express the difference between the kinetic energy of the system before the collision and its kinetic energy after the collision:

$$\Delta E = K_\text{i} - K_\text{f} = K_\text{1i} + K_\text{2i} - (K_\text{1f} + K_\text{2f}) = \tfrac{1}{2}\left[M_1 v_\text{1i}^2 + M_2 v_\text{2i}^2 - M_1 v_\text{1f}^2 - M_2 v_\text{2f}^2\right]$$
$$= \tfrac{1}{2}\left[mv_\text{1i}^2 + 2mv_\text{2i}^2 - mv_\text{1f}^2 - 2mv_\text{2f}^2\right] = \tfrac{1}{2}m\left[v_\text{1i}^2 + 2v_\text{2i}^2 - v_\text{1f}^2 - 2v_\text{2f}^2\right]$$
$$= \tfrac{1}{2}m\left[v_0^2 + 2\left(\tfrac{1}{4}v_0^2\right) - \tfrac{5}{4}v_0^2 - 2\left(\tfrac{1}{16}v_0^2\right)\right] = \boxed{\tfrac{1}{16}mv_0^2}$$

Because $\Delta E \neq 0$, the collision is *inelastic*.

***97 ••**

Picture the Problem Let the direction of motion of the puck that is moving before the collision be the positive *x* direction. Applying conservation of momentum to the collision in both the *x* and *y* directions will lead us to two equations in the unknowns v_1 and v_2 that we can solve simultaneously. We can decide whether the collision was elastic by either calculating the system's kinetic energy before and after the collision or by determining whether the angle between the final velocities is 90°.

(*a*) Use conservation of momentum in the *x* direction to relate the velocities of the collision participants before and after the collision:

$$p_\text{xi} = p_\text{xf}$$
or
$$mv = mv_1\cos 30° + mv_2\cos 60°$$
or
$$v = v_1\cos 30° + v_2\cos 60°$$

Use conservation of momentum in the *y* direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$p_\text{yi} = p_\text{yf}$$
or
$$0 = mv_1\sin 30° - mv_2\sin 60°$$
or
$$0 = v_1\sin 30° - v_2\sin 60°$$

Solve these equations simultaneously to obtain:

$$v_1 = \boxed{1.73\,\text{m/s}} \text{ and } v_2 = \boxed{1.00\,\text{m/s}}$$

(*b*) Because the angle between $\vec{v}_1$ and $\vec{v}_2$ is 90°, the collision was *elastic*.

98 ••

Picture the Problem Let the direction of motion of the object that is moving before the collision be the positive *x* direction. Applying conservation of momentum to the motion in both the *x* and *y* directions will lead us to two equations in the unknowns v_2 and θ_2 that we can solve simultaneously. We can show that the collision was elastic by showing that the system's kinetic energy before and after the collision is the same.

(*a*) Use conservation of momentum in the *x* direction to relate the velocities of the collision participants before and after the collision:

$$p_{\mathrm{xi}} = p_{\mathrm{xf}}$$

or

$$3mv_0 = \sqrt{5}mv_0 \cos\theta_1 + 2mv_2 \cos\theta_2$$

or

$$3v_0 = \sqrt{5}v_0 \cos\theta_1 + 2v_2 \cos\theta_2$$

Use conservation of momentum in the *y* direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$p_{\mathrm{yi}} = p_{\mathrm{yf}}$$

or

$$0 = \sqrt{5}mv_0 \sin\theta_1 - 2mv_2 \sin\theta_2$$

or

$$0 = \sqrt{5}v_0 \sin\theta_1 - 2v_2 \sin\theta_2$$

Note that if $\tan\theta_1 = 2$, then:

$$\cos\theta_1 = \frac{1}{\sqrt{5}} \text{ and } \sin\theta_1 = \frac{2}{\sqrt{5}}$$

Substitute in the momentum equations to obtain:

$$3v_0 = \sqrt{5}v_0 \frac{1}{\sqrt{5}} + 2v_2 \cos\theta_2$$

or

$$v_0 = v_2 \cos\theta_2$$

and

$$0 = \sqrt{5}v_0 \frac{2}{\sqrt{5}} - 2v_2 \sin\theta_2$$

or

$$0 = v_0 - v_2 \sin\theta_2$$

Solve these equations simultaneously for θ_2 :

$$\theta_2 = \tan^{-1} 1 = \boxed{45.0°}$$

Substitute to find v_2:

$$v_2 = \frac{v_0}{\cos\theta_2} = \frac{v_0}{\cos 45°} = \boxed{\sqrt{2}v_0}$$

(*b*) To show that the collision was elastic, find the before-collision and after-collision kinetic energies:

$$K_{\mathrm{i}} = \tfrac{1}{2}m(3v_0)^2 = 4.5mv_0^2$$

and

$$K_{\mathrm{f}} = \tfrac{1}{2}m\left(\sqrt{5}v_0\right)^2 + \tfrac{1}{2}(2m)\left(\sqrt{2}v_0\right)^2$$
$$= 4.5mv_0^2$$

Because $K_i = K_f$, the collision is elastic.

***99** ••

Picture the Problem Let the direction of motion of the ball that is moving before the collision be the positive x direction. Let v represent the velocity of the ball that is moving before the collision, v_1 its velocity after the collision and v_2 the velocity of the initially-at-rest ball after the collision. We know that because the collision is elastic and the balls have the same mass, v_1 and v_2 are 90° apart. Applying conservation of momentum to the collision in both the x and y directions will lead us to two equations in the unknowns v_1 and v_2 that we can solve simultaneously.

Noting that the angle of deflection for the recoiling ball is 60°, use conservation of momentum in the x direction to relate the velocities of the collision participants before and after the collision:

$$p_{xi} = p_{xf}$$

or

$$mv = mv_1 \cos 30° + mv_2 \cos 60°$$

or

$$v = v_1 \cos 30° + v_2 \cos 60°$$

Use conservation of momentum in the y direction to obtain a second equation relating the velocities of the collision participants before and after the collision:

$$p_{yi} = p_{yf}$$

or

$$0 = mv_1 \sin 30° - mv_2 \sin 60°$$

or

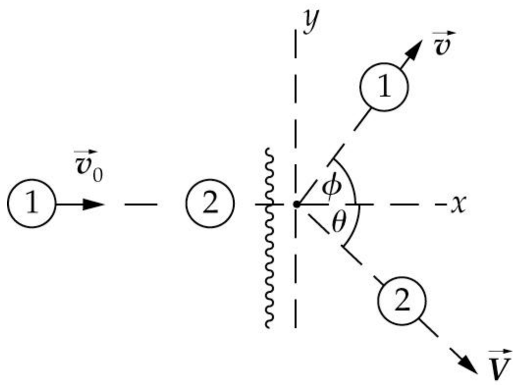
$$0 = v_1 \sin 30° - v_2 \sin 60°$$

Solve these equations simultaneously to obtain:

$$v_1 = \boxed{8.66\,\text{m/s}} \text{ and } v_2 = \boxed{5.00\,\text{m/s}}$$

100 ••

Picture the Problem Choose the coordinate system shown in the diagram below with the x-axis the axis of initial approach of the first particle. Call V the speed of the target particle after the collision. In part (a) we can apply conservation of momentum in the x and y directions to obtain two equations that we can solve simultaneously for $\tan\theta$. In part (b) we can use conservation of momentum in vector form and the elastic-collision equation to show that $v = v_0\cos\phi$.



(*a*) Apply conservation of momentum in the *x* direction to obtain:

$$v_0 = v\cos\phi + V\cos\theta \qquad (1)$$

Apply conservation of momentum in the *y* direction to obtain:

$$v\sin\phi = V\sin\theta \qquad (2)$$

Solve equation (1) for $V\cos\theta$:

$$V\cos\theta = v_0 - v\cos\phi \qquad (3)$$

Divide equation (2) by equation (3) to obtain:

$$\frac{V\sin\theta}{V\cos\theta} = \frac{v\sin\phi}{v_0 - v\cos\phi}$$

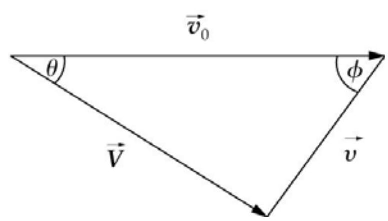
or

$$\tan\theta = \boxed{\frac{v\sin\phi}{v_0 - v\cos\phi}}$$

(*b*) Apply conservation of momentum to obtain:

$$\vec{v}_0 = \vec{v} + \vec{V}$$

Draw the vector diagram representing this equation:



Use the definition of an elastic collision to obtain:

$$v_0^2 = v^2 + V^2$$

If this Pythagorean condition is to hold, the third angle of the triangle must be a right angle and, using the definition of the cosine function:

$$v = \boxed{v_0\cos\phi}$$

Center-of-Mass Frame

101 ••

Picture the Problem The total kinetic energy of a system of particles is the sum of the kinetic energy of the center of mass and the kinetic energy relative to the center of mass. The kinetic energy of a particle of mass *m* is related to momentum according to $K = p^2/2m$.

Express the total kinetic energy of the system:

$$K = K_{\text{rel}} + K_{\text{cm}} \qquad (1)$$

Relate the kinetic energy relative to the center of mass to the momenta of the two particles:

$$K_{\text{rel}} = \frac{p_1^2}{2m_1} + \frac{p_1^2}{2m_2} = \frac{p_1^2(m_1 + m_2)}{2m_1 m_2}$$

Express the kinetic energy of the center of mass of the two particles:

$$K_{\text{cm}} = \frac{(2p_1)^2}{2(m_1 + m_2)} = \frac{2p_1^2}{m_1 + m_2}$$

Substitute in equation (1) and simplify to obtain:

$$K = \frac{p_1^2(m_1 + m_2)}{2m_1 m_2} + \frac{2p_1^2}{m_1 + m_2} = \frac{p_1^2}{2}\left[\frac{m_1^2 + 6m_1 m_2 + m_2^2}{m_1^2 m_2 + m_1 m_2^2}\right]$$

In an elastic collision:

$$K_{\text{i}} = K_{\text{f}}$$

$$= \boxed{\frac{p_1^2}{2}\left[\frac{m_1^2 + 6m_1 m_2 + m_2^2}{m_1^2 m_2 + m_1 m_2^2}\right]}$$

$$= \boxed{\frac{p_1'^2}{2}\left[\frac{m_1^2 + 6m_1 m_2 + m_2^2}{m_1^2 m_2 + m_1 m_2^2}\right]}$$

Simplify to obtain:

$$(p_1')^2 = (p_1)^2 \Rightarrow \boxed{p_1' = \pm p_1}$$

and

$$\boxed{\text{If } p_1' = +p_1, \text{the particles do not collide.}}$$

***102 ••**

Picture the Problem Let the numerals 3 and 1 denote the blocks whose masses are 3 kg and 1 kg respectively. We can use $\sum_i m_i \vec{v}_i = M\vec{v}_{\text{cm}}$ to find the velocity of the center-of-mass of the system and simply follow the directions in the problem step by step.

(*a*) Express the total momentum of this two-particle system in terms of the velocity of its center of mass:

$$\vec{P} = \sum_i m_i \vec{v}_i = m_1\vec{v}_1 + m_3\vec{v}_3 = M\vec{v}_{cm} = (m_1 + m_3)\vec{v}_{cm}$$

Solve for $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{m_3\vec{v}_3 + m_1\vec{v}_1}{m_3 + m_1}$$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3\,\text{kg})(-5\,\text{m/s})\hat{i} + (1\,\text{kg})(3\,\text{m/s})\hat{i}}{3\,\text{kg} + 1\,\text{kg}} = \boxed{(-3.00\,\text{m/s})\hat{i}}$$

(*b*) Find the velocity of the 3-kg block in the center of mass reference frame:

$$\vec{u}_3 = \vec{v}_3 - \vec{v}_{cm} = (-5\,\text{m/s})\hat{i} - (-3\,\text{m/s})\hat{i} = \boxed{(-2.00\,\text{m/s})\hat{i}}$$

Find the velocity of the 1-kg block in the center of mass reference frame:

$$\vec{u}_1 = \vec{v}_1 - \vec{v}_{cm} = (3\,\text{m/s})\hat{i} - (-3\,\text{m/s})\hat{i} = \boxed{(6.00\,\text{m/s})\hat{i}}$$

(*c*) Express the after-collision velocities of both blocks in the center of mass reference frame:

$$\vec{u}_3' = \boxed{(2.00\,\text{m/s})\hat{i}}$$

and

$$\vec{u}_1' = \boxed{(-6.00\,\text{m/s})\hat{i}}$$

(*d*) Transform the after-collision velocity of the 3-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_3' = \vec{u}_3' + \vec{v}_{cm} = (2\,\text{m/s})\hat{i} + (-3\,\text{m/s})\hat{i} = \boxed{(-1.00\,\text{m/s})\hat{i}}$$

Transform the after-collision velocity of the 1-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_1' = \vec{u}_1' + \vec{v}_{cm} = (-6\,\text{m/s})\hat{i} + (-3\,\text{m/s})\hat{i} = \boxed{(-9.00\,\text{m/s})\hat{i}}$$

(*e*) Express K_i in the original frame of reference:

$$K_i = \tfrac{1}{2}m_3v_3^2 + \tfrac{1}{2}m_1v_1^2$$

Substitute numerical values and evaluate K_i:

$$K_i = \tfrac{1}{2}\left[(3\,\text{kg})(5\,\text{m/s})^2 + (1\,\text{kg})(3\,\text{m/s})^2\right] = \boxed{42.0\,\text{J}}$$

Express K_f in the original frame of reference:

$$K_f = \tfrac{1}{2}m_3 v'^2_3 + \tfrac{1}{2}m_1 v'^2_1$$

Substitute numerical values and evaluate K_f:

$$K_f = \tfrac{1}{2}\left[(3\,\text{kg})(1\,\text{m/s})^2 + (1\,\text{kg})(9\,\text{m/s})^2\right] = \boxed{42.0\,\text{J}}$$

103 ••

Picture the Problem Let the numerals 3 and 1 denote the blocks whose masses are 3 kg and 1 kg respectively. We can use $\sum_i m_i\vec{v}_i = M\vec{v}_{cm}$ to find the velocity of the center-of-mass of the system and simply follow the directions in the problem step by step.

(*a*) Express the total momentum of this two-particle system in terms of the velocity of its center of mass:

$$\vec{P} = \sum_i m_i\vec{v}_i = m_3\vec{v}_3 + m_5\vec{v}_5 = M\vec{v}_{cm} = (m_3 + m_5)\vec{v}_{cm}$$

Solve for $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{m_3\vec{v}_3 + m_5\vec{v}_5}{m_3 + m_5}$$

Substitute numerical values and evaluate $\vec{v}_{cm}$:

$$\vec{v}_{cm} = \frac{(3\,\text{kg})(-5\,\text{m/s})\hat{i} + (5\,\text{kg})(3\,\text{m/s})\hat{i}}{3\,\text{kg} + 5\,\text{kg}} = \boxed{0}$$

(*b*) Find the velocity of the 3-kg block in the center of mass reference frame:

$$\vec{u}_3 = \vec{v}_3 - \vec{v}_{cm} = (-5\,\text{m/s})\,\hat{i} - 0 = \boxed{(-5\,\text{m/s})\,\hat{i}}$$

Find the velocity of the 5-kg block in the center of mass reference frame:

$$\vec{u}_5 = \vec{v}_5 - \vec{v}_{cm} = (3\,\text{m/s})\hat{i} - 0 = \boxed{(3\,\text{m/s})\hat{i}}$$

(*c*) Express the after-collision velocities of both blocks in the center of mass reference frame:

$$\vec{u}_3' = \boxed{(5\,\text{m/s})\hat{i}}$$

and

$$u_5' = \boxed{0.75\,\text{m/s}}$$

(*d*) Transform the after-collision velocity of the 3-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_3' = \vec{u}_3' + \vec{v}_{cm} = (5\,\text{m/s})\hat{i} + 0 = \boxed{(5\,\text{m/s})\hat{i}}$$

Transform the after-collision velocity of the 5-kg block from the center of mass reference frame to the original reference frame:

$$\vec{v}_5' = \vec{u}_5' + \vec{v}_{cm} = (-3\,\text{m/s})\hat{i} + 0$$
$$= \boxed{(-3\,\text{m/s})\hat{i}}$$

(*e*) Express K_i in the original frame of reference:

$$K_i = \tfrac{1}{2}m_3v_3^2 + \tfrac{1}{2}m_5v_5^2$$

Substitute numerical values and evaluate K_i:

$$K_i = \tfrac{1}{2}\left[(3\,\text{kg})(5\,\text{m/s})^2 + (5\,\text{kg})(3\,\text{m/s})^2\right]$$
$$= \boxed{60.0\,\text{J}}$$

Express K_f in the original frame of reference:

$$K_f = \tfrac{1}{2}m_3v_3'^2 + \tfrac{1}{2}m_5v_5'^2$$

Substitute numerical values and evaluate K_f:

$$K_f = \tfrac{1}{2}\left[(3\,\text{kg})(5\,\text{m/s})^2 \quad + (5\,\text{kg})(3\,\text{m/s})^2\right] = \boxed{60.0\,\text{J}}$$

Systems With Continuously Varying Mass: Rocket Propulsion

104 ••

Picture the Problem The thrust of a rocket F_{th} depends on the burn rate of its fuel *dm/dt* and the relative speed of its exhaust gases u_{ex} according to $F_{th} = |dm/dt|u_{ex}$.

Using its definition, relate the rocket's thrust to the relative speed of its exhaust gases:

$$F_{th} = \left|\frac{dm}{dt}\right|u_{ex}$$

Substitute numerical values and evaluate F_{th}:

$$F_{th} = (200\,\text{kg/s})(6\,\text{km/s}) = \boxed{1.20\,\text{MN}}$$

105 ••

Picture the Problem The thrust of a rocket F_{th} depends on the burn rate of its fuel *dm/dt* and the relative speed of its exhaust gases u_{ex} according to $F_{th} = |dm/dt|u_{ex}$. The final velocity v_f of a rocket depends on the relative speed of its exhaust gases u_{ex}, its payload to initial mass ratio m_f/m_0 and its burn time according to $v_f = -u_{ex}\ln(m_f/m_0) - gt_b$.

(*a*) Using its definition, relate the rocket's thrust to the relative speed

$$F_{th} = \left|\frac{dm}{dt}\right|u_{ex}$$

of its exhaust gases:

Substitute numerical values and evaluate F_{th}:

$$F_{th} = (200\,\text{kg/s})(1.8\,\text{km/s}) = \boxed{360\,\text{kN}}$$

(*b*) Relate the time to burnout to the mass of the fuel and its burn rate:

$$t_b = \frac{m_{fuel}}{dm/dt} = \frac{0.8m_0}{dm/dt}$$

Substitute numerical values and evaluate t_b:

$$t_b = \frac{0.8(30{,}000\,\text{kg})}{200\,\text{kg/s}} = \boxed{120\,\text{s}}$$

(*c*) Relate the final velocity of a rocket to its initial mass, exhaust velocity, and burn time:

$$v_f = -u_{ex}\ln\left(\frac{m_f}{m_0}\right) - gt_b$$

Substitute numerical values and evaluate v_f:

$$v_f = -(1.8\,\text{km/s})\ln\left(\frac{1}{5}\right) - (9.81\,\text{m/s}^2)(120\,\text{s}) = \boxed{1.72\,\text{km/s}}$$

***106 ••**

Picture the Problem We can use the dimensions of thrust, burn rate, and acceleration to show that the dimension of specific impulse is time. Combining the definitions of rocket thrust and specific impulse will lead us to $u_{ex} = gI_{sp}$.

(*a*) Express the dimension of specific impulse in terms of the dimensions of F_{th}, R, and g:

$$[I_{sp}] = \frac{[F_{th}]}{[R][g]} = \frac{\dfrac{\text{M}\cdot\text{L}}{\text{T}^2}}{\dfrac{\text{M}}{\text{T}}\cdot\dfrac{\text{L}}{\text{T}^2}} = \boxed{\text{T}}$$

(*b*) From the definition of rocket thrust we have:

$$F_{th} = Ru_{ex}$$

Solve for u_{ex}:

$$u_{ex} = \frac{F_{th}}{R}$$

Substitute for F_{th} to obtain:

$$u_{ex} = \frac{RgI_{sp}}{R} = \boxed{gI_{sp}} \qquad (1)$$

(*c*) Solve equation (1) for I_{sp} and substitute for u_{ex} to obtain:

$$I_{sp} = \frac{F_{th}}{Rg}$$

From Example 8-21 we have:

$R = 1.384\times10^4$ kg/s and $F_{th} = 3.4\times10^6$ N

Substitute numerical values and evaluate I_{sp}:

$$I_{sp} = \frac{3.4\times10^6\ \text{N}}{(1.384\times10^4\ \text{kg/s})(9.81\text{m/s}^2)} = \boxed{25.0\text{s}}$$

***107** •••

Picture the Problem We can use the rocket equation and the definition of rocket thrust to show that $\tau_0 = 1 + a_0/g$. In part (*b*) we can express the burn time t_b in terms of the initial and final masses of the rocket and the rate at which the fuel burns, and then use this equation to express the rocket's final velocity in terms of I_{sp}, τ_0, and the mass ratio m_0/m_f. In part (*d*) we'll need to use trial-and-error methods or a graphing calculator to solve the transcendental equation giving v_f as a function of m_0/m_f.

(*a*) Express the rocket equation:

$$-mg + Ru_{ex} = ma$$

From the definition of rocket thrust we have:

$$F_{th} = Ru_{ex}$$

Substitute to obtain:

$$-mg + F_{th} = ma$$

Solve for F_{th} at takeoff:

$$F_{th} = m_0 g + m_0 a_0$$

Divide both sides of this equation by $m_0 g$ to obtain:

$$\frac{F_{th}}{m_0 g} = 1 + \frac{a_0}{g}$$

Because $\tau_0 = F_{th}/(m_0 g)$:

$$\tau_0 = \boxed{1 + \frac{a_0}{g}}$$

(*b*) Use equation 8-42 to express the final speed of a rocket that starts from rest with mass m_0:

$$v_f = u_{ex} \ln\frac{m_0}{m_f} - gt_b, \qquad (1)$$

where t_b is the burn time.

Express the burn time in terms of the burn rate R (assumed constant):

$$t_b = \frac{m_0 - m_f}{R} = \frac{m_0}{R}\left(1 - \frac{m_f}{m_0}\right)$$

Multiply t_b by one in the form gT/gT and simplify to obtain:

$$t_b = \frac{gF_{th}}{gF_{th}}\frac{m_0}{R}\left(1 - \frac{m_f}{m_0}\right) = \frac{gm_0}{F_{th}}\frac{F_{th}}{gR}\left(1 - \frac{m_f}{m_0}\right) = \frac{I_{sp}}{\tau_0}\left(1 - \frac{m_f}{m_0}\right)$$

Substitute in equation (1):

$$v_f = u_{ex} \ln\frac{m_0}{m_f} - \frac{gI_{sp}}{\tau_0}\left(1 - \frac{m_f}{m_0}\right)$$

From Problem 32 we have:

$$u_{ex} = gI_{sp},$$

where u_{ex} is the exhaust velocity of the propellant.

Substitute and factor to obtain:

$$v_f = gI_{sp} \ln\frac{m_0}{m_f} - \frac{gI_{sp}}{\tau_0}\left(1 - \frac{m_f}{m_0}\right)$$

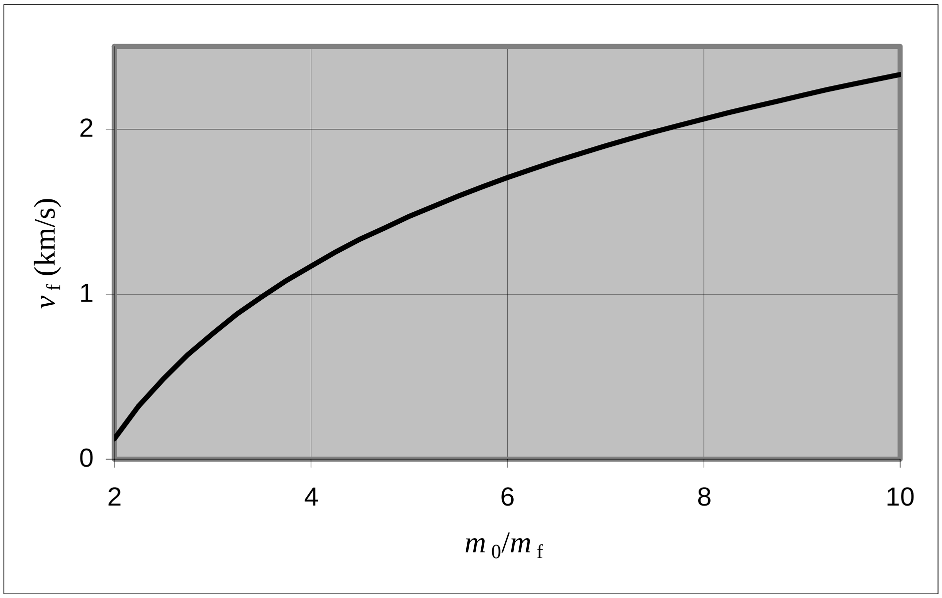
$$= \boxed{gI_{sp}\left[\ln\left(\frac{m_0}{m_f}\right) - \frac{1}{\tau_0}\left(1 - \frac{m_f}{m_0}\right)\right]}$$

(*c*) A spreadsheet program to calculate the final velocity of the rocket as a function of the mass ratio m_0/m_f is shown below. The constants used in the velocity function and the formulas used to calculate the final velocity are as follows:

Cell	Content/Formula	Algebraic Form
B1	250	I_{sp}
B2	9.81	g
B3	2	τ
D9	D8 + 0.25	m_0/m_f
E8	B2*B1*(LOG(D8) – (1/B3)*(1/D8))	$gI_{sp}\left[\ln\left(\frac{m_0}{m_f}\right) - \frac{1}{\tau_0}\left(1 - \frac{m_f}{m_0}\right)\right]$

	A	B	C	D	E
1	Isp =	250	s		
2	g =	9.81	m/s^2		
3	tau =	2			
4					
5					
6					
7				mass ratio	vf
8				2.00	1.252E+02
9				2.25	3.187E+02
10				2.50	4.854E+02
11				2.75	6.316E+02
12				3.00	7.614E+02
36				9.00	2.204E+03
37				9.25	2.237E+03
38				9.50	2.269E+03
39				9.75	2.300E+03
40				10.00	2.330E+03
41				725.00	7.013E+03

A graph of final velocity as a function of mass ratio is shown below.



(*d*) Substitute the data given in part (*c*) in the equation derived in part (*b*) to obtain:

$$7\,\text{km/s} = \left(9.81\,\text{m/s}^2\right)\left(250\,\text{s}\right)\left(\ln\frac{m_0}{m_f} - \frac{1}{2}\left(1 - \frac{m_f}{m_0}\right)\right)$$

or

$$2.854 = \ln x - 0.5 + \frac{0.5}{x} \text{ where } x = m_0/m_f.$$

Use trial-and-error methods or a graphing calculator to solve this transcendental equation for the root greater than 1:

$$x = \boxed{28.1},$$

a value considerably larger than the practical limit of 10 for single-stage rockets.

108 ••

Picture the Problem We can use the velocity-at-burnout equation from Problem 106 to find v_f and constant-acceleration equations to approximate the maximum height the rocket will reach and its total flight time.

(*a*) Assuming constant acceleration, relate the maximum height reached by the model rocket to its time-to-top-of-trajectory:

$$h = \tfrac{1}{2} g t_{top}^2 \qquad (1)$$

From Problem 106 we have:

$$v_f = gI_{sp}\left(\ln\left(\frac{m_0}{m_f}\right) - \frac{1}{\tau}\left(1 - \frac{m_f}{m_0}\right)\right)$$

Evaluate the velocity at burnout v_f for I_{sp} = 100 s, m_0/m_f = 1.2, and τ = 5:

$$v_f = (9.81\,\text{m/s}^2)(100\,\text{s}) \times \left[\ln(1.2) - \frac{1}{5}\left(1 - \frac{1}{1.2}\right)\right] = 146\,\text{m/s}$$

Assuming that the time for the fuel to burn up is short compared to the total flight time, find the time to the top of the trajectory:

$$t_{top} = \frac{v_f}{g} = \frac{146\,\text{m/s}}{9.81\,\text{m/s}^2} = 14.9\,\text{s}$$

Substitute in equation (1) and evaluate h:

$$h = \tfrac{1}{2}(9.81\,\text{m/s}^2)(14.9\,\text{s})^2 = \boxed{1.09\,\text{km}}$$

(*b*) Find the total flight time from the time it took the rocket to reach its maximum height:

$$t_{flight} = 2t_{top} = 2(14.9\,\text{s}) = \boxed{29.8\,\text{s}}$$

(*c*) Express and evaluate the fuel burn time t_b:

$$t_b = \frac{I_{sp}}{\tau}\left(1 - \frac{m_f}{m_0}\right) = \frac{100\,\text{s}}{5}\left(1 - \frac{1}{1.2}\right) = 3.33\,\text{s}$$

Because this burn time is approximately 1/5 of the total flight time, we can't expect the answer we obtained in Part (*b*) to be very accurate. It should, however, be good to about 30% accuracy, as the maximum distance the model rocket could possibly move in this time is $\frac{1}{2}vt_b = 243\,\text{m}$, assuming constant acceleration until burnout.

General Problems

109 •

Picture the Problem Let the direction of motion of the 250-g car before the collision be the positive *x* direction. Let the numeral 1 refer to the 250-kg car, the numeral 2 refer to the 400-kg car, and *V* represent the velocity of the linked cars. Let the system include the earth and the cars. We can use conservation of momentum to find their speed after they have linked together and the definition of kinetic energy to find their initial and final kinetic energies.

Use conservation of momentum to relate the speeds of the cars immediately before and immediately after their collision:

$$p_{ix} = p_{fx}$$

or

$$m_1 v_1 = (m_1 + m_2)V$$

Solve for V:

$$V = \frac{m_1 v_1}{m_1 + m_2}$$

Substitute numerical values and evaluate V:

$$V = \frac{(0.250\,\text{kg})(0.50\,\text{m/s})}{0.250\,\text{kg} + 0.400\,\text{kg}} = \boxed{0.192\,\text{m/s}}$$

Find the initial kinetic energy of the cars:

$$K_\text{i} = \tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2}(0.250\,\text{kg})(0.50\,\text{m/s})^2 = \boxed{31.3\,\text{mJ}}$$

Find the final kinetic energy of the coupled cars:

$$K_\text{f} = \tfrac{1}{2}(m_1 + m_2)V^2 = \tfrac{1}{2}(0.250\,\text{kg} + 0.400\,\text{kg})(0.192\,\text{m/s})^2 = \boxed{12.0\,\text{mJ}}$$

110 •

Picture the Problem Let the direction of motion of the 250-g car before the collision be the positive x direction. Let the numeral 1 refer to the 250-kg car and the numeral 2 refer to the 400-g car and the system include the earth and the cars. We can use conservation of momentum to find their speed after they have linked together and the definition of kinetic energy to find their initial and final kinetic energies.

(*a*) Express and evaluate the initial kinetic energy of the cars:

$$K_\text{i} = \tfrac{1}{2} m_1 v_1^2 = \tfrac{1}{2}(0.250\,\text{kg})(0.50\,\text{m/s})^2 = \boxed{31.3\,\text{mJ}}$$

(*b*) Relate the velocity of the center of mass to the total momentum of the system:

$$\vec{\boldsymbol{P}} = \sum_\text{i} m_\text{i} \vec{\boldsymbol{v}}_\text{i} = m\vec{\boldsymbol{v}}_\text{cm}$$

Solve for v_cm:

$$v_\text{cm} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2}$$

Substitute numerical values and evaluate v_cm:

$$v_\text{cm} = \frac{(0.250\,\text{kg})(0.50\,\text{m/s})}{0.250\,\text{kg} + 0.400\,\text{kg}} = 0.192\,\text{m/s}$$

Find the initial velocity of the 250-g car relative to the velocity of the center of mass:

$$u_1 = v_1 - v_\text{cm} = 0.50\,\text{m/s} - 0.192\,\text{m/s} = \boxed{0.308\,\text{m/s}}$$

Find the initial velocity of the 400-g car relative to the velocity of the center of mass:

$$u_2 = v_2 - v_{cm} = 0\,\text{m/s} - 0.192\,\text{m/s} = \boxed{-0.192\,\text{m/s}}$$

Express the initial kinetic energy of the system relative to the center of mass:

$$K_{i,rel} = \tfrac{1}{2}m_1u_1^2 + \tfrac{1}{2}m_2u_2^2$$

Substitute numerical values and evaluate $K_{i,rel}$:

$$K_{i,rel} = \tfrac{1}{2}(0.250\,\text{kg})(0.308\,\text{m/s})^2 + \tfrac{1}{2}(0.400\,\text{kg})(-0.192\,\text{m/s})^2 = \boxed{19.2\,\text{mJ}}$$

(*c*) Express the kinetic energy of the center of mass:

$$K_{cm} = \tfrac{1}{2}Mv_{cm}^2$$

Substitute numerical values and evaluate K_{cm}:

$$K_{cm} = \tfrac{1}{2}(0.650\,\text{kg})(0.192\,\text{m/s})^2 = \boxed{12.0\,\text{mJ}}$$

(*d*) Relate the initial kinetic energy of the system to its initial kinetic energy relative to the center of mass and the kinetic energy of the center of mass:

$$K_i = K_{i,rel} + K_{cm} = 19.2\,\text{mJ} + 12.0\,\text{mJ} = 31.2\,\text{mJ}$$

$$\therefore \boxed{K_i = K_{i,rel} + K_{cm}}$$

***111** •

Picture the Problem Let the direction the 4-kg fish is swimming be the positive *x* direction and the system include the fish, the water, and the earth. The velocity of the larger fish immediately after its lunch is the velocity of the center of mass in this perfectly inelastic collision.

Relate the velocity of the center of mass to the total momentum of the system:

$$\vec{\boldsymbol{P}} = \sum_i m_i\vec{\boldsymbol{v}}_i = m\vec{\boldsymbol{v}}_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_4v_4 - m_{1.2}v_{1.2}}{m_4 + m_{1.2}}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(4\,\text{kg})(1.5\,\text{m/s}) - (1.2\,\text{kg})(3\,\text{m/s})}{4\,\text{kg} + 1.2\,\text{kg}} = \boxed{0.462\,\text{m/s}}$$

112 •

Picture the Problem Let the direction the 3-kg block is moving be the positive *x* direction and include both blocks and the earth in the system. The total kinetic energy of the two-block system is the sum of the kinetic energies of the blocks. We can relate the momentum of the system to the velocity of its center of mass and use this relationship to find v_{cm}. Finally, we can use the definition of kinetic energy to find the kinetic energy relative to the center of mass.

(*a*) Express the total kinetic energy of the system in terms of the kinetic energy of the blocks:

$$K_{tot} = \tfrac{1}{2} m_3 v_3^2 + \tfrac{1}{2} m_6 v_6^2$$

Substitute numerical values and evaluate K_{tot}:

$$K_{tot} = \tfrac{1}{2}(3\,\text{kg})(6\,\text{m/s})^2 + \tfrac{1}{2}(6\,\text{kg})(3\,\text{m/s})^2 = \boxed{81.0\,\text{J}}$$

(*b*) Relate the velocity of the center of mass to the total momentum of the system:

$$\vec{\boldsymbol{P}} = \sum_i m_i \vec{\boldsymbol{v}}_i = m\vec{\boldsymbol{v}}_{cm}$$

Solve for v_{cm}:

$$v_{cm} = \frac{m_3 v_3 + m_6 v_6}{m_1 + m_2}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(3\,\text{kg})(6\,\text{m/s}) + (6\,\text{kg})(3\,\text{m/s})}{3\,\text{kg} + 6\,\text{kg}} = \boxed{4.00\,\text{m/s}}$$

(*c*) Find the center of mass kinetic energy from the velocity of the center of mass:

$$K_{cm} = \tfrac{1}{2} M v_{cm}^2 = \tfrac{1}{2}(9\,\text{kg})(4\,\text{m/s})^2 = \boxed{72.0\,\text{J}}$$

(*d*) Relate the initial kinetic energy of the system to its initial kinetic energy relative to the center of mass and the kinetic energy of the center of mass:

$$K_{rel} = K_{tot} - K_{cm} = 81.0\,\text{J} - 72.0\,\text{J} = \boxed{9.00\,\text{J}}$$

113 •

Picture the Problem Let east be the positive *x* direction and north the positive *y* direction. Include both cars and the earth in the system and let the numeral 1 denote the 1500-kg car and the numeral 2 the 2000-kg car. Because the net external force acting on the system is zero, momentum is conserved in this perfectly inelastic collision.

(*a*) Express the total momentum of the system:

$$\vec{p} = \vec{p}_1 + \vec{p}_2 = m_1\vec{v}_1 + m_2\vec{v}_2$$
$$= m_1 v_1 \hat{j} - m_2 v_2 \hat{i}$$

Substitute numerical values and evaluate $\vec{p}$:

$$\vec{p} = (1500\,\text{kg})(70\,\text{km/h})\hat{j} - (2000\,\text{kg})(55\,\text{km/h})\hat{i}$$
$$= \boxed{-(1.10\times10^5\,\text{kg}\cdot\text{km/h})\,\hat{i} + (1.05\times10^5\,\text{kg}\cdot\text{km/h})\hat{j}}$$

(*b*) Express the velocity of the wreckage in terms of the total momentum of the system:

$$\vec{v}_\text{f} = \vec{v}_\text{cm} = \frac{\vec{p}}{M}$$

Substitute numerical values and evaluate $\vec{v}_\text{f}$:

$$\vec{v}_\text{f} = \frac{-(1.10\times10^5\,\text{kg}\cdot\text{km/h})\,\hat{i}}{1500\,\text{kg} + 2000\,\text{kg}} + \frac{(1.05\times10^5\,\text{kg}\cdot\text{km/h})\hat{j}}{1500\,\text{kg} + 2000\,\text{kg}}$$
$$= -(31.4\,\text{km/h})\,\hat{i} + (30.0\,\text{km/h})\hat{j}$$

Find the magnitude of the velocity of the wreckage:

$$v_\text{f} = \sqrt{(31.4\,\text{km/h})^2 + (30.0\,\text{km/h})^2}$$
$$= \boxed{43.4\,\text{km/h}}$$

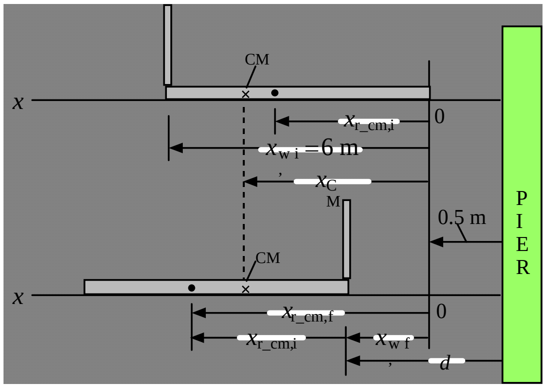
Find the direction of the velocity of the wreckage:

$$\theta = \tan^{-1}\left[\frac{30.0\,\text{km/h}}{-31.4\,\text{km/h}}\right] = -43.7°$$

$$\boxed{\begin{array}{l}\text{The direction of the wreckage is}\\ 46.3°\text{ west of north.}\end{array}}$$

***114** ••

Picture the Problem Take the origin to be at the initial position of the right-hand end of raft and let the positive *x* direction be to the left. Let ″*w*″ denote the woman and ″*r*″ the raft, *d* be the distance of the end of the raft from the pier after the woman has walked to its front. The raft moves to the left as the woman moves to the right; with the center of mass of the woman-raft system remaining fixed (because $F_\text{ext,net} = 0$). The diagram shows the initial ($x_{w,i}$) and final ($x_{w,f}$) positions of the woman as well as the initial ($x_{r\_cm,i}$) and final ($x_{r\_cm,f}$) positions of the center of mass of the raft both before and after the woman has walked to the front of the raft.



(*a*) Express the distance of the raft from the pier after the woman has walked to the front of the raft:

$$d = 0.5\,\text{m} + x_{f,w} \qquad (1)$$

Express x_{cm} before the woman has walked to the front of the raft:

$$x_{cm} = \frac{m_w x_{w,i} + m_r x_{r\_cm,\,i}}{m_w + m_r}$$

Express x_{cm} after the woman has walked to the front of the raft:

$$x_{cm} = \frac{m_w x_{w,f} + m_r x_{r\_cm,f}}{m_w + m_r}$$

Because $F_{ext,net} = 0$, the center of mass remains fixed and we can equate these two expressions for x_{cm} to obtain:

$$m_w x_{w,i} + m_r x_{r\_cm,i} = m_w x_{w,f} + m_r x_{r\_cm,f}$$

Solve for $x_{w,f}$:

$$x_{w,f} = x_{w,i} - \frac{m_r}{m_w}\left(x_{r\_cm,f} - x_{r\_cm,i}\right)$$

From the figure it can be seen that $x_{r\_cm,f} - x_{r\_cm,i} = x_{w,f}$. Substitute $x_{w,f}$ for $x_{r\_cm,f} - x_{r\_cm,i}$ and to obtain:

$$x_{w,f} = \frac{m_w x_{w,i}}{m_w + m_r}$$

Substitute numerical values and evaluate $x_{w,f}$:

$$x_{w,f} = \frac{(60\,\text{kg})(6\,\text{m})}{60\,\text{kg} + 120\,\text{kg}} = 2.00\,\text{m}$$

Substitute in equation (1) to obtain:

$$d = 2.00\,\text{m} + 0.5\,\text{m} = \boxed{2.50\,\text{m}}$$

(*b*) Express the total kinetic energy of the system:

$$K_{\text{tot}} = \tfrac{1}{2}m_{\text{w}}v_{\text{w}}^2 + \tfrac{1}{2}m_{\text{r}}v_{\text{r}}^2$$

Noting that the elapsed time is 2 s, find v_{w} and v_{r}:

$$v_{\text{w}} = \frac{x_{\text{w,f}} - x_{\text{w,i}}}{\Delta t} = \frac{2\,\text{m} - 6\,\text{m}}{2\,\text{s}} = -2\,\text{m/s}$$

relative to the dock, and

$$v_{\text{r}} = \frac{x_{\text{r,f}} - x_{\text{r,i}}}{\Delta t} = \frac{2.50\,\text{m} - 0.5\,\text{m}}{2\,\text{s}} = 1\,\text{m/s},$$

also relative to the dock.

Substitute numerical values and evaluate K_{tot}:

$$K_{\text{tot}} = \tfrac{1}{2}(60\,\text{kg})(-2\,\text{m/s})^2 + \tfrac{1}{2}(120\,\text{kg})(1\,\text{m/s})^2 = \boxed{180\,\text{J}}$$

Evaluate K with the raft tied to the pier:

$$K_{\text{tot}} = \tfrac{1}{2}m_{\text{w}}v_{\text{w}}^2 = \tfrac{1}{2}(60\,\text{kg})(3\,\text{m/s})^2 = \boxed{270\,\text{J}}$$

(*c*) All the kinetic energy derives from the chemical energy of the woman and, assuming she stops via static friction, the kinetic energy is transformed into her internal energy.

(*d*) After the shot leaves the woman's hand, the raft - woman system constitutes an inertial reference frame. In that frame the shot has the same initial velocity as did the shot that had a range of 6 m in the reference frame of the land. Thus, in the raft - woman frame, the shot also has a range of 6 m and lands at the front of the raft.

115 ••

Picture the Problem Let the zero of gravitational potential energy be at the elevation of the 1-kg block. We can use conservation of energy to find the speed of the bob just before its perfectly elastic collision with the block and conservation of momentum to find the speed of the block immediately after the collision. We'll apply Newton's 2nd law to find the acceleration of the sliding block and use a constant-acceleration equation to find how far it slides before coming to rest.

(*a*) Use conservation of energy to find the speed of the bob just before its collision with the block:

$$\Delta K + \Delta U = 0$$
or
$$K_f - K_i + U_f - U_i = 0$$

Because $K_i = U_f = 0$:

$$\tfrac{1}{2} m_{ball} v_{ball}^2 + m_{ball} g \Delta h = 0$$
and
$$v_{ball} = \sqrt{2g\Delta h}$$

Substitute numerical values and evaluate v_{ball}:

$$v_{ball} = \sqrt{2(9.81\,\text{m/s}^2)(2\,\text{m})} = 6.26\ \text{m/s}$$

Because the collision is perfectly elastic and the ball and block have the same mass:

$$v_{block} = v_{ball} = \boxed{6.26\,\text{m/s}}$$

(*b*) Using a constant-acceleration equation, relate the displacement of the block to its acceleration and initial speed and solve for its displacement:

$$v_f^2 = v_i^2 + 2a_{block}\Delta x$$
Since $v_f = 0$,
$$\Delta x = \frac{-v_i^2}{2a_{block}} = \frac{-v_{block}^2}{2a_{block}}$$

Apply $\sum \vec{F} = m\vec{a}$ to the sliding block:

$$\sum F_x = -f_k = ma_{block}$$
and
$$\sum F_y = F_n - m_{block} g = 0$$

Using the definition of f_k ($\mu_k F_n$) eliminate f_k and F_n between the two equations and solve for a_{block}:

$$a_{block} = -\mu_k g$$

Substitute for a_{block} to obtain:

$$\Delta x = \frac{-v_{block}^2}{-2\mu_k g} = \frac{v_{block}^2}{2\mu_k g}$$

Substitute numerical values and evaluate Δx:

$$\Delta x = \frac{(6.26\,\text{m/s})^2}{2(0.1)(9.81\,\text{m/s}^2)} = \boxed{20.0\,\text{m}}$$

***116** ••

Picture the Problem We can use conservation of momentum in the horizontal direction to find the recoil velocity of the car along the track after the firing. Because the shell will neither rise as high nor be moving as fast at the top of its trajectory as it would be in the absence of air friction, we can apply the work-energy theorem to find the amount of thermal energy produced by the air friction.

(*a*) **No. The vertical reaction force of the rails is an external force and so the momentum of the system will not be conserved.**

(*b*) Use conservation of momentum in the horizontal (x) direction to obtain:

$$\Delta p_x = 0$$

or

$$mv\cos 30° - Mv_{recoil} = 0$$

Solve for and evaluate v_{recoil}:

$$v_{recoil} = \frac{mv\cos 30°}{M}$$

Substitute numerical values and evaluate v_{recoil}:

$$v_{recoil} = \frac{(200\,\text{kg})(125\,\text{m/s})\cos 30°}{5000\,\text{kg}} = \boxed{4.33\,\text{m/s}}$$

(*c*) Using the work-energy theorem, relate the thermal energy produced by air friction to the change in the energy of the system:

$$W_{ext} = W_f = \Delta E_{sys} = \Delta U + \Delta K$$

Substitute for ΔU and ΔK to obtain:

$$W_{ext} = mgy_f - mgy_i + \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_i^2$$
$$= mg(y_f - y_i) + \tfrac{1}{2}m\left(v_f^2 - v_i^2\right)$$

Substitute numerical values and evaluate W_{ext}:

$$W_{ext} = (200\,\text{kg})(9.81\,\text{m/s}^2)(180\,\text{m}) + \tfrac{1}{2}(200\,\text{kg})\left[(80\,\text{m/s})^2 - (125\,\text{m/s})^2\right] = \boxed{-569\,\text{kJ}}$$

117 ••

Picture the Problem Because this is a perfectly inelastic collision, the velocity of the block after the collision is the same as the velocity of the center of mass before the collision. The distance the block travels before hitting the floor is the product of its velocity and the time required to fall 0.8 m; which we can find using a constant-acceleration equation.

Relate the distance D to the velocity of the center of mass and the time for the block to fall to the floor:

$$D = v_{cm}\Delta t$$

Relate the velocity of the center of mass to the total momentum of the system and solve for v_{cm}:

$$\vec{P} = \sum_i m_i \vec{v}_i = M\vec{v}_{cm}$$

and

$$v_{cm} = \frac{m_{bullet}v_{bullet} + m_{block}v_{block}}{m_{bullet} + m_{block}}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \frac{(0.015\,kg)(500\,m/s)}{0.015\,kg + 0.8\,kg} = 9.20\,m/s$$

Using a constant-acceleration equation, find the time for the block to fall to the floor:

$$\Delta y = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

Because $v_0 = 0$, $\Delta t = \sqrt{\dfrac{2\Delta y}{g}}$

Substitute to obtain:

$$D = v_{cm}\sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate D:

$$D = (9.20\,m/s)\sqrt{\frac{2(0.8\,m)}{9.81\,m/s^2}} = \boxed{3.72\,m}$$

118 ••

Picture the Problem Let the direction the particle whose mass is *m* is moving initially be the positive *x* direction and the direction the particle whose mass is 4*m* is moving initially be the negative *y* direction. We can determine the impulse delivered by $\vec{F}$ and, hence, the change in the momentum of the system from the change in the momentum of the particle whose mass is *m*. Knowing $\Delta\vec{p}$, we can express the final momentum of the particle whose mass is 4*m* and solve for its final velocity.

Express the impulse delivered by the force $\vec{F}$:

$$\vec{I} = \vec{F}T = \Delta\vec{p} = \vec{p}_f - \vec{p}_i = m(4v)\hat{i} - mv\,\hat{i} = 3mv\,\hat{i}$$

Express $\vec{p}\,'_{4m}$:

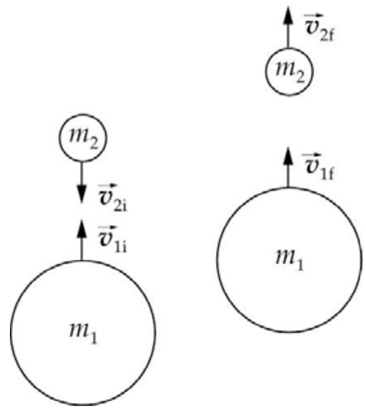
$$\vec{p}\,'_{4m} = 4m\vec{v}\,' = \vec{p}_{4m}(0) + \Delta\vec{p} = -4mv\,\hat{j} + 3mv\,\hat{i}$$

Solve for $\vec{v}\,'$:

$$\vec{v}\,' = \boxed{\tfrac{3}{4}v\,\hat{i} - v\,\hat{j}}$$

119 ••

Picture the Problem Let the numeral 1 refer to the basketball and the numeral 2 to the baseball. The left-hand side of the diagram shows the balls after the basketball's elastic collision with the floor and just before they collide. The right-hand side of the diagram shows the balls just after their collision. We can apply conservation of momentum and the definition of an elastic collision to obtain equations relating the initial and final velocities of the masses of the colliding objects that we can solve for v_{1f} and v_{2f}.



(*a*) Because both balls are in free-fall, and both are in the air for the same amount of time, they have the same velocity just before the basketball rebounds. After the basketball rebounds elastically, its velocity will have the same magnitude, but the opposite direction than just before it hit the ground.

The velocity of the basketball will be equal in magnitude but opposite in direction to the velocity of the baseball.

(*b*) Apply conservation of momentum to the collision of the balls to obtain:

$$m_1v_{1f} + m_2v_{2f} = m_1v_{1i} + m_2v_{2i} \quad (1)$$

Relate the initial and final kinetic energies of the balls in their elastic collision:

$$\tfrac{1}{2}m_1v_{1f}^2 + \tfrac{1}{2}m_2v_{2f}^2 = \tfrac{1}{2}m_1v_{1i}^2 + \tfrac{1}{2}m_2v_{2i}^2$$

Rearrange this equation and factor to obtain:

$$m_2\left(v_{2f}^2 - v_{2i}^2\right) = m_1\left(v_{1i}^2 - v_{1f}^2\right)$$

or

$$m_2\left(v_{2f} - v_{2i}\right)\left(v_{2f} + v_{2i}\right) = m_1\left(v_{1i} - v_{1f}\right)\left(v_{1i} + v_{1f}\right) \quad (2)$$

Rearrange equation (1) to obtain:

$$m_2\left(v_{2f} - v_{2i}\right) = m_1\left(v_{1i} - v_{1f}\right) \quad (3)$$

Divide equation (2) by equation (3) to obtain:

$$v_{2f} + v_{2i} = v_{1i} + v_{1f}$$

Rearrange this equation to obtain equation (4):

$$v_{1f} - v_{2f} = v_{2i} - v_{1i} \quad (4)$$

Multiply equation (4) by m_2 and add it to equation (1) to obtain:

$$\left(m_1 + m_2\right)v_{1f} = \left(m_1 - m_2\right)v_{1i} + 2m_2v_{2i}$$

Solve for v_{1f} to obtain:

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_{1i}+\frac{2m_2}{m_1+m_2}v_{2i}$$

or, because $v_{2i}=-v_{1i}$,

$$v_{1f}=\frac{m_1-m_2}{m_1+m_2}v_{1i}-\frac{2m_2}{m_1+m_2}v_{1i}=\frac{m_1-3m_2}{m_1+m_2}v_{1i}$$

For $m_1 = 3m_2$ and $v_{1i} = v$:

$$v_{1f}=\frac{3m_2-3m_2}{3m_2+m_2}v=\boxed{0}$$

(*c*) Multiply equation (4) by m_1 and subtract it from equation (1) to obtain:

$$(m_1+m_2)v_{2f}=(m_2-m_1)v_{2i}+2m_1v_{1i}$$

Solve for v_{2f} to obtain:

$$v_{2f}=\frac{2m_1}{m_1+m_2}v_{1i}+\frac{m_2-m_1}{m_1+m_2}v_{2i}$$

or, because $v_{2i}=-v_{1i}$,

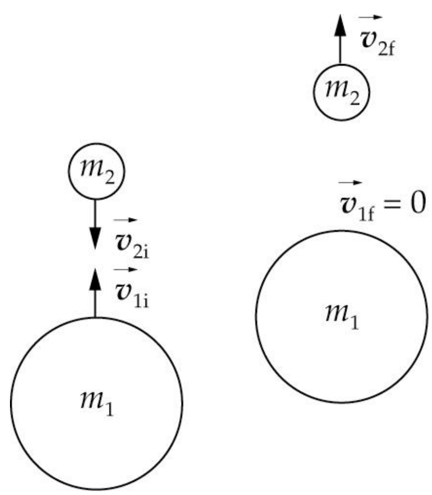
$$v_{2f}=\frac{2m_1}{m_1+m_2}v_{1i}-\frac{m_2-m_1}{m_1+m_2}v_{1i}=\frac{3m_1-m_2}{m_1+m_2}v_{1i}$$

For $m_1 = 3m_2$ and $v_{1i} = v$:

$$v_{2f}=\frac{3(3m_2)-m_2}{3m_2+m_2}v=\boxed{2v}$$

120 •••

Picture the Problem In Problem 119 only two balls are dropped. They collide head on, each moving at speed v, and the collision is elastic. In this problem, as it did in Problem 119, the solution involves using the conservation of momentum equation $m_1v_{1f}+m_2v_{2f}=m_1v_{1i}+m_2v_{2i}$ and the elastic collision equation $v_{1f}-v_{2f}=v_{2i}-v_{1i}$, where the numeral 1 refers to the baseball, and the numeral 2 to the top ball. The diagram shows the balls just before and just after their collision. From Problem 119 we know that that $v_{1i} = 2v$ and $v_{2i} = -v$.



(a) Express the final speed v_{1f} of the baseball as a function of its initial speed v_{1i} and the initial speed of the top ball v_{2i} (see Problem 78):

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2} v_{1i} + \frac{2m_2}{m_1 + m_2} v_{2i}$$

Substitute for v_{1i} and , v_{2i} to obtain:

$$v_{1f} = \frac{m_1 - m_2}{m_1 + m_2}(2v) + \frac{2m_2}{m_1 + m_2}(-v)$$

Divide the numerator and denominator of each term by m_2 to introduce the mass ratio of the upper ball to the lower ball:

$$v_{1f} = \frac{\frac{m_1}{m_2} - 1}{\frac{m_1}{m_2} + 1}(2v) + \frac{2}{\frac{m_1}{m_2} + 1}(-v)$$

Set the final speed of the baseball v_{1f} equal to zero, let x represent the mass ratio m_1/m_2, and solve for x:

$$0 = \frac{x-1}{x+1}(2v) + \frac{2}{x+1}(-v)$$

and

$$x = \frac{m_1}{m_2} = \boxed{\frac{1}{2}}$$

(b) Apply the second of the two equations in Problem 78 to the collision between the top ball and the baseball:

$$v_{2f} = \frac{2m_1}{m_1 + m_2} v_{1i} + \frac{m_2 - m_1}{m_1 + m_2} v_{2i}$$

Substitute $v_{1i} = 2v$ and are given that $v_{2i} = -v$ to obtain:

$$v_{2f} = \frac{2m_1}{m_1 + m_2}(2v) + \frac{m_2 - m_1}{m_1 + m_2}(-v)$$

In part (a) we showed that $m_2 = 2m_1$. Substitute and simplify:

$$v_{3f} = \frac{2(2m_1)}{m_1 + 2m_1}(2v) - \frac{2m_1 - m_1}{m_1 + 2m_1} v$$

$$= \frac{4m_1}{3m_1}(2v) - \frac{m_1}{3m_1} v = \tfrac{8}{3} v - \tfrac{1}{3} v$$

$$= \boxed{\tfrac{7}{3} v}$$

***121** ••

Picture the Problem Let the direction the probe is moving after its elastic collision with Saturn be the positive direction. The probe gains kinetic energy at the expense of the kinetic energy of Saturn. We'll relate the velocity of approach relative to the center of mass to u_{rec} and then to v.

(a) Relate the velocity of recession to the velocity of recession relative to the center of mass:

$$v = u_{rec} + v_{cm}$$

Find the velocity of approach:

$$u_{app} = -9.6\,\text{km/s} - 10.4\,\text{km/s} = -20.0\,\text{km/s}$$

Relate the relative velocity of approach to the relative velocity of recession for an elastic collision:

$$u_{rec} = -u_{app} = 20.0\,\text{km/s}$$

Because Saturn is so much more massive than the space probe:

$$v_{cm} = v_{Saturn} = 9.6\,\text{km/s}$$

Substitute and evaluate v:

$$v = u_{rec} + v_{cm} = 20\,\text{km/s} + 9.6\,\text{km/s} = \boxed{29.6\,\text{km/s}}$$

(*b*) Express the ratio of the final kinetic energy to the initial kinetic energy:

$$\frac{K_f}{K_i} = \frac{\frac{1}{2}Mv_{rec}^2}{\frac{1}{2}Mv_i^2} = \left(\frac{v_{rec}}{v_i}\right)^2 = \left(\frac{29.6\,\text{km/s}}{10.4\,\text{km/s}}\right)^2 = \boxed{8.10}$$

$$\boxed{\text{The energy comes from an immeasurably small slowing of Saturn.}}$$

***122** ••

Picture the Problem We can use the relationships $P = c\Delta m$ and $\Delta E = \Delta mc^2$ to show that $P = \Delta E/c$. We can then equate this expression with the change in momentum of the flashlight to find the latter's final velocity.

(*a*) Express the momentum of the mass lost (i.e., carried away by the light) by the flashlight:

$$P = c\Delta m$$

Relate the energy carried away by the light to the mass lost by the flashlight:

$$\Delta m = \frac{\Delta E}{c^2}$$

Substitute to obtain:

$$P = c\frac{\Delta E}{c^2} = \boxed{\frac{\Delta E}{c}}$$

(*b*) Relate the final momentum of the flashlight to ΔE:

$$\frac{\Delta E}{c} = \Delta p = mv$$

because the flashlight is initially at rest.

Solve for v:

$$v = \frac{\Delta E}{mc}$$

Substitute numerical values and evaluate v:

$$v = \frac{1.5 \times 10^3\ \text{J}}{(1.5\,\text{kg})(2.998 \times 10^8\ \text{m/s})} = 3.33 \times 10^{-6}\ \text{m/s} = \boxed{3.33\,\mu\text{m/s}}$$

123 •

Picture the Problem We can equate the change in momentum of the block to the momentum of the beam of light and relate the momentum of the beam of light to the mass converted to produce the beam. Combining these expressions will allow us to find the speed attained by the block.

Relate the change in momentum of the block to the momentum of the beam:

$$(M - m)v = P_{\text{beam}}$$

because the block is initially at rest.

Express the momentum of the mass converted into a well-collimated beam of light:

$$P_{\text{beam}} = mc$$

Substitute to obtain:

$$(M - m)v = mc$$

Solve for v:

$$v = \frac{mc}{M - m}$$

Substitute numerical values and evaluate v:

$$v = \frac{(0.001\text{kg})(2.998 \times 10^8\ \text{m/s})}{1\text{kg} - 0.001\text{kg}} = \boxed{3.00 \times 10^5\ \text{m/s}}$$

124 ••

Picture the Problem Let the origin of the coordinate system be at the end of the boat at which your friend is sitting prior to changing places. If we let the system include you and your friend, the boat, the water and the earth, then $F_{\text{ext,net}} = 0$ and the center of mass is at the same location after you change places as it was before you shifted.

Express the center of mass of the system prior to changing places:

$$x_{\text{cm}} = \frac{m_{\text{boat}} x_{\text{boat}} + m_{\text{you}} x_{\text{you}} + m x_{\text{friend}}}{m_{\text{boat}} + m_{\text{you}} + m_{\text{friend}}} = \frac{x_{\text{you}}(m_{\text{boat}} + m_{\text{you}}) + m x_{\text{friend}}}{m_{\text{boat}} + m_{\text{you}} + m}$$

Substitute numerical values and simplify to obtain an expression for x_{cm} in terms of m:

$$x_{cm} = \frac{(2\,\text{m})(60\,\text{kg} + 80\,\text{kg}) + (0)m}{60\,\text{kg} + 80\,\text{kg} + m} = \frac{280\,\text{kg}\cdot\text{m}}{140\,\text{kg} + m}$$

Find the center of mass of the system after changing places:

$$x'_{cm} = \frac{m_{boat}x_{boat} + m_{you}x_{you} + mx_{friend}}{m_{boat} + m_{you} + m_{friend}} = \frac{(m_{boat} + m)(2\,\text{m} \pm 0.2\,\text{m})}{m_{boat} + m_{you} + m} + \frac{m_{you}(\pm 0.2\,\text{m})}{m_{boat} + m_{you} + m}$$

Substitute numerical values and simplify to obtain:

$$x'_{cm} = \frac{(60\,\text{kg} + m)(2\,\text{m} \pm 0.2\,\text{m})}{60\,\text{kg} + 80\,\text{kg} + m} + \frac{(80\,\text{kg})(\pm 0.2\,\text{m})}{60\,\text{kg} + 80\,\text{kg} + m} = \frac{120\,\text{kg}\cdot\text{m} \pm 12\,\text{kg}\cdot\text{m}}{140\,\text{kg} + m} + \frac{(2\,\text{m})m \pm 0.2m\,\text{m} \pm 16\,\text{kg}\cdot\text{m}}{140\,\text{kg} + m}$$

Because $F_{ext,net} = 0$, $x'_{cm} = x_{cm}$. Equate the two expressions and solve for m to obtain:

$$m = \frac{(160 \pm 28)}{(2 \pm 0.2)}\,\text{kg}$$

Calculate the largest possible mass for your friend:

$$m = \frac{(160 + 28)}{(2 - 0.2)}\,\text{kg} = \boxed{104\,\text{kg}}$$

Calculate the smallest possible mass for your friend:

$$m = \frac{(160 - 28)}{(2 + 0.2)}\,\text{kg} = \boxed{60.0\,\text{kg}}$$

125 ••

Picture the Problem Let the system include the woman, both vehicles, and the earth. Then $F_{ext,net} = 0$ and $a_{cm} = 0$. Include the mass of the man in the mass of the truck. We can use Newton's 2nd and 3rd laws to find the acceleration of the truck and net force acting on both the car and the truck.

(*a*) Relate the action and reaction forces acting on the car and truck:

$$F_{car} = F_{truck}$$

or

$$m_{car}a_{car} = m_{truck+woman}a_{truck}$$

Solve for the acceleration of the truck:

$$a_{truck} = \frac{m_{car}a_{car}}{m_{truck+woman}}$$

Substitute numerical values and evaluate a_{truck}:

$$a_{truck} = \frac{(800\,kg)(1.2\,m/s^2)}{1600\,kg} = \boxed{0.600\,m/s^2}$$

(*b*) Apply Newton's 2nd law to either vehicle to obtain:

$$F_{net} = m_{car}a_{car}$$

Substitute numerical values and evaluate F_{net}:

$$F_{net} = (800\,kg)(1.2\,m/s^2) = \boxed{960\,N}$$

126 ••

Picture the Problem Let the system include the block, the putty, and the earth. Then $F_{ext,net} = 0$ and momentum is conserved in this perfectly inelastic collision. We'll use conservation of momentum to relate the after-collision velocity of the block plus blob and conservation of energy to find their after-collision velocity.

Noting that, because this is a perfectly elastic collision, the final velocity of the block plus blob is the velocity of the center of mass, use conservation of momentum to relate the velocity of the center of mass to the velocity of the glob before the collision:

$$p_i = p_f$$

or

$$m_{gl}v_{gl} = Mv_{cm}$$

where $M = m_{gl} + m_{bl}$.

Solve for v_{gl} to obtain:

$$v_{gl} = \frac{M}{m_{gl}}v_{cm} \qquad (1)$$

Use conservation of energy to find the initial energy of the block plus glob:

$$\Delta K + \Delta U + W_f = 0$$

Because $\Delta U = K_f = 0$,

$$-\tfrac{1}{2}Mv_{cm}^2 + f_k\Delta x = 0$$

Use $f_k = \mu_k Mg$ to eliminate f_k and solve for v_{cm}:

$$v_{cm} = \sqrt{2\mu_k g\Delta x}$$

Substitute numerical values and evaluate v_{cm}:

$$v_{cm} = \sqrt{2(0.4)(9.81\,m/s^2)(0.15\,m)}$$

$$= 1.08\,m/s$$

Substitute numerical values in equation (1) and evaluate v_{gl}:

$$v_{gl} = \frac{13\,\text{kg} + 0.4\,\text{kg}}{0.4\,\text{kg}}(1.08\,\text{m/s}) = \boxed{36.2\,\text{m/s}}$$

***127 ••**

Picture the Problem Let the direction the moving car was traveling before the collision be the positive *x* direction. Let the numeral 1 denote this car and the numeral 2 the car that is stopped at the stop sign and the system include both cars and the earth. We can use conservation of momentum to relate the speed of the initially-moving car to the speed of the meshed cars immediately after their perfectly inelastic collision and conservation of energy to find the initial speed of the meshed cars.

Using conservation of momentum, relate the before-collision velocity to the after-collision velocity of the meshed cars:

$$p_i = p_f$$

or

$$m_1 v_1 = (m_1 + m_2)V$$

Solve for v_1:

$$v_1 = \frac{m_1 + m_2}{m_1}V = \left(1 + \frac{m_2}{m_1}\right)V$$

Using conservation of energy, relate the initial kinetic energy of the meshed cars to the work done by friction in bringing them to a stop:

$$\Delta K + \Delta E_{\text{thermal}} = 0$$

or, because $K_f = 0$ and $\Delta E_{\text{thermal}} = f\Delta s$,

$$-K_i + f_k \Delta s = 0$$

Substitute for *Ki* and, using $f_k = \mu_k F_n = \mu_k Mg$, eliminate f_k to obtain:

$$-\tfrac{1}{2}MV^2 + \mu_k Mg\Delta x = 0$$

Solve for *V*:

$$V = \sqrt{2\mu_k g\Delta x}$$

Substitute to obtain:

$$v_1 = \left(1 + \frac{m_2}{m_1}\right)\sqrt{2\mu_k g\Delta x}$$

Substitute numerical values and evaluate v_1:

$$v_1 = \left(1 + \frac{900\,\text{kg}}{1200\,\text{kg}}\right)\sqrt{2(0.92)(9.81\,\text{m/s}^2)(0.76\,\text{m})} = 6.48\,\text{m/s} = 23.3\,\text{km/h}$$

The driver was not telling the truth. He was traveling at 23.3 km/h.

128 ••

Picture the Problem Let the zero of gravitational potential energy be at the lowest point of the bob's swing and note that the bob can swing either forward or backward after the collision. We'll use both conservation of momentum and conservation of energy to relate the velocities of the bob and the block before and after their collision.

Express the kinetic energy of the block in terms of its after-collision momentum:

$$K_m = \frac{p_m^2}{2m}$$

Solve for m to obtain:

$$m = \frac{p_m^2}{2K_m} \qquad (1)$$

Use conservation of energy to relate K_m to the change in the potential energy of the bob:

$$\Delta K + \Delta U = 0$$

or, because $K_i = 0$,

$$K_m + U_f - U_i = 0$$

Solve for K_m:

$$\begin{aligned} K_m &= -U_f + U_i \\ &= m_{bob} g\left[L(1-\cos\theta_i) - L(1-\cos\theta_f)\right] \\ &= m_{bob} gL\left[\cos\theta_f - \cos\theta_i\right] \end{aligned}$$

Substitute numerical values and evaluate K_m:

$$K_m = (0.4\,\text{kg})(9.81\,\text{m/s}^2)(1.6\,\text{m})[\cos 5.73° - \cos 53°] = 2.47\,\text{J}$$

Use conservation of energy to find the velocity of the bob just before its collision with the block:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

$$\therefore \tfrac{1}{2} m_{bob} v^2 - m_{bob} gL(1-\cos\theta_i) = 0$$

or

$$v = \sqrt{2gL(1-\cos\theta_i)}$$

Substitute numerical values and evaluate v:

$$\begin{aligned} v &= \sqrt{2(9.81\,\text{m/s}^2)(1.6\,\text{m})(1-\cos 53°)} \\ &= 3.544\,\text{m/s} \end{aligned}$$

Use conservation of energy to find

$$\Delta K + \Delta U = 0$$

the velocity of the bob just after its collision with the block:

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f to obtain:

$$-\tfrac{1}{2} m_{bob} v'^2 + m_{bob} gL(1-\cos\theta_f) = 0$$

Solve for v':

$$v' = \sqrt{2gL(1-\cos\theta_f)}$$

Substitute numerical values and evaluate v':

$$v' = \sqrt{2(9.81\,\text{m/s}^2)(1.6\,\text{m})(1-\cos 5.73°)} = 0.396\,\text{m/s}$$

Use conservation of momentum to relate p_m after the collision to the momentum of the bob just before and just after the collision:

$$p_i = p_f$$

or

$$m_{bob} v = m_{bob} v' \pm p_m$$

Solve for and evaluate p_m:

$$p_m = m_{bob} v \pm m_{bob} v' = (0.4\,\text{kg})(3.544\,\text{m/s} \pm 0.396\,\text{m/s}) = 1.418\,\text{kg}\cdot\text{m/s} \pm 0.158\,\text{kg}\cdot\text{m/s}$$

Find the larger value for p_m:

$$p_m = 1.418\,\text{kg}\cdot\text{m/s} + 0.158\,\text{kg}\cdot\text{m/s} = 1.576\,\text{kg}\cdot\text{m/s}$$

Find the smaller value for p_m:

$$p_m = 1.418\,\text{kg}\cdot\text{m/s} - 0.158\,\text{kg}\cdot\text{m/s} = 1.260\,\text{kg}\cdot\text{m/s}$$

Substitute in equation (1) to determine the two values for m:

$$m = \frac{(1.576\,\text{kg}\cdot\text{m/s})^2}{2(2.47\,\text{J})} = \boxed{0.503\,\text{kg}}$$

or

$$m = \frac{(1.260\,\text{kg}\cdot\text{m/s})^2}{2(2.47\,\text{J})} = \boxed{0.321\,\text{kg}}$$

129 ••

Picture the Problem Choose the zero of gravitational potential energy at the location of the spring's maximum compression. Let the system include the spring, the blocks, and the earth. Then the net external force is zero as is work done against friction. We can use conservation of energy to relate the energy transformations taking place during the evolution of this system.

Apply conservation of energy:

$$\Delta K + \Delta U_g + \Delta U_s = 0$$

Because $\Delta K = 0$:

$$\Delta U_g + \Delta U_s = 0$$

Express the change in the gravitational potential energy:

$$\Delta U_g = -mg\Delta h - Mgx\sin\theta$$

Express the change in the potential energy of the spring:

$$\Delta U_s = \tfrac{1}{2}kx^2$$

Substitute to obtain:

$$-mg\Delta h - Mgx\sin\theta + \tfrac{1}{2}kx^2 = 0$$

Solve for M:

$$M = \frac{\frac{1}{2}kx^2 - mg\Delta h}{gx\sin 30°} = \frac{kx}{g} - \frac{2m\Delta h}{x}$$

Relate Δh to the initial and rebound positions of the block whose mass is m:

$$\Delta h = (4\,\text{m} - 2.56\,\text{m})\sin 30° = 0.720\,\text{m}$$

Substitute numerical values and evaluate M:

$$M = \frac{(11\times10^3\,\text{N/m})(0.04\,\text{m})}{9.81\,\text{m/s}^2} - \frac{2(1\,\text{kg})(0.72\,\text{m})}{0.04\,\text{m}} = \boxed{8.85\,\text{kg}}$$

***130 ••**

Picture the Problem By symmetry, $x_{cm} = 0$. Let σ be the mass per unit area of the disk. The mass of the modified disk is the difference between the mass of the whole disk and the mass that has been removed.

Start with the definition of y_{cm}:

$$y_{cm} = \frac{\sum_i m_i y_i}{M - m_{hole}} = \frac{m_{disk}y_{disk} - m_{hole}y_{hole}}{M - m_{hole}}$$

Express the mass of the complete disk:

$$M = \sigma A = \sigma\pi r^2$$

Express the mass of the material removed:

$$m_{hole} = \sigma\pi\left(\frac{r}{2}\right)^2 = \tfrac{1}{4}\sigma\pi r^2 = \tfrac{1}{4}M$$

Substitute and simplify to obtain:

$$y_{cm} = \frac{M(0) - \left(\frac{1}{4}M\right)\left(-\frac{1}{2}r\right)}{M - \frac{1}{4}M} = \boxed{\tfrac{1}{6}r}$$

131 ••

Picture the Problem Let the horizontal axis by the *y* axis and the vertical axis the *z* axis. By symmetry, $x_{cm} = y_{cm} = 0$. Let ρ be the mass per unit volume of the sphere. The mass of the modified sphere is the difference between the mass of the whole sphere and the mass that has been removed.

Start with the definition of y_{cm}:

$$z_{cm} = \frac{\sum_i m_i y_i}{M - m_{hole}} = \frac{m_{sphere} y_{sphere} - m_{hole} y_{hole}}{M - m_{hole}}$$

Express the mass of the complete sphere:

$$M = \rho V = \tfrac{4}{3}\rho\pi r^3$$

Express the mass of the material removed:

$$m_{hole} = \tfrac{4}{3}\rho\pi\left(\frac{r}{2}\right)^3 = \tfrac{1}{8}\left(\tfrac{4}{3}\rho\pi r^3\right) = \tfrac{1}{8}M$$

Substitute and simplify to obtain:

$$z_{cm} = \frac{M(0) - \left(\frac{1}{8}M\right)\left(-\frac{1}{2}r\right)}{M - \frac{1}{8}M} = \boxed{\tfrac{1}{14}r}$$

***132** ••

Picture the Problem In this elastic head-on collision, the kinetic energy of recoiling nucleus is the difference between the initial and final kinetic energies of the neutron. We can derive the indicated results by using both conservation of energy and conservation of momentum and writing the kinetic energies in terms of the momenta of the particles before and after the collision.

(*a*) Use conservation of energy to relate the kinetic energies of the particles before and after the collision:

$$\frac{p_{ni}^2}{2m} = \frac{p_{nf}^2}{2m} + \frac{p_{nucleus}^2}{2M} \qquad (1)$$

Apply conservation of momentum to obtain a second relationship between the initial and final momenta:

$$p_{ni} = p_{nf} + p_{nucleus} \qquad (2)$$

Eliminate p_{nf} in equation (1) using equation (2):

$$\frac{p_{nucleus}}{2M} + \frac{p_{nucleus}}{2m} - \frac{p_{ni}}{m} = 0 \qquad (3)$$

Use equation (3) to write $p_{ni}^2/2m$ in terms of $p_{nucleus}$:

$$\frac{p_{ni}^2}{2m} = K_n = \frac{p_{nucleus}^2 (M+m)^2}{8M^2 m} \qquad (4)$$

Use equation (4) to express $K_{\text{nucleus}} = p_{\text{nucleus}}^2 / 2M$ in terms of K_{n}:

$$K_{\text{nucleus}} = \boxed{K_{\text{n}}\left[\frac{4Mm}{(M+m)^2}\right]} \qquad (5)$$

(*b*) Relate the *change* in the kinetic energy of the neutron to the after-collision kinetic energy of the nucleus:

$$\Delta K_{\text{n}} = -K_{\text{nucleus}}$$

Using equation (5), express the fraction of the energy lost in the collision:

$$\frac{-\Delta K_{\text{n}}}{K_{\text{n}}} = \boxed{\frac{4Mm}{(M+m)^2} = \frac{4\dfrac{m}{M}}{\left(1+\dfrac{m}{M}\right)^2}}$$

133 ••

Picture the Problem Problem 132 (*b*) provides an expression for the fractional loss of energy per collision.

(*a*) Using the result of Problem 132 (*b*), express the fractional loss of energy per collision:

$$\frac{K_{\text{nf}}}{K_{\text{ni}}} = \frac{K_{\text{ni}} - \Delta K_{\text{n}}}{E_0} = \frac{(M-m)^2}{(M+m)^2}$$

Evaluate this fraction to obtain:

$$\frac{K_{\text{nf}}}{E_0} = \frac{(12m-m)^2}{(12m+m)^2} = 0.716$$

Express the kinetic energy of one neutron after N collisions:

$$K_{\text{nf}} = \boxed{0.716^N E_0}$$

(*b*) Substitute for K_{nf} and E_0 to obtain:

$$0.716^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for N:

$$\text{N} = \frac{-8}{\log 0.716} \approx \boxed{55}$$

134 ••

Picture the Problem We can relate the number of collisions needed to reduce the energy of a neutron from 2 MeV to 0.02 eV to the fractional energy loss per collision and solve the resulting exponential equation for *N*.

(*a*) Using the result of Problem 132 (*b*), express the fractional loss of energy per collision:

$$\frac{K_{nf}}{K_{ni}} = \frac{K_{ni} - \Delta K_n}{E_0} = \frac{K_{ni} - 0.63K_{ni}}{K_{ni}} = 0.37$$

Express the kinetic energy of one neutron after *N* collisions:

$$K_{nf} = \boxed{0.37^N E_0}$$

Substitute for K_{nf} and E_0 to obtain:

$$0.37^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for *N*:

$$N = \frac{-8}{\log 0.37} \approx \boxed{19}$$

(*b*) Proceed as in (*a*) to obtain:

$$\frac{K_{nf}}{K_{ni}} = \frac{K_{ni} - \Delta K_n}{E_0} = \frac{K_{ni} - 0.11K_{ni}}{K_{ni}} = 0.89$$

Express the kinetic energy of one neutron after *N* collisions:

$$K_{nf} = \boxed{0.89^N E_0}$$

Substitute for K_{nf} and E_0 to obtain:

$$0.89^N = 10^{-8}$$

Take the logarithm of both sides of the equation and solve for *N*:

$$N = \frac{-8}{\log 0.89} \approx \boxed{158}$$

135 ••

Picture the Problem Let $\lambda = M/L$ be the mass per unit length of the rope and *y* the length of rope supported by *F* at any instant and use the definition of the center of mass.

(*a*) Letting *m* represent the mass of the rope that is being supported by the force at any given time and y' its center of mass, express y_{cm}:

$$y_{cm} = \frac{my'}{\lambda L} = \frac{\lambda y\left(\frac{1}{2}y\right)}{\lambda L} = \frac{y^2}{2L}$$

Relate *y* to *v*:

$$y = vt$$

Substitute to obtain:

$$y_{cm} = \frac{y^2}{2L} = \frac{(vt)^2}{2L} = \boxed{\frac{v^2}{2L}t^2}$$

(*b*) Differentiate y_{cm} twice to a_{cm}:

$$\frac{dy_{cm}}{dt} = 2\frac{v^2}{2L}t = \frac{v^2}{L}t$$

and

$$\frac{d^2 y_{cm}}{dt^2} = a_{cm} = \boxed{\frac{v^2}{L}}$$

(*c*) Apply $\sum F_y = ma_{cm}$ to the rope to obtain:

$$F - mg = ma_{cm}$$

Solve for *F*, substitute and simplify to obtain:

$$F = ma_{cm} + mg = \lambda vt\left(\frac{v^2}{L}\right) + \lambda vtg$$

$$= \frac{Mv^3}{L^2}t - \frac{Mvg}{L}t$$

$$= \boxed{\left(\frac{Mv^3}{L^2} + \frac{Mvg}{L}\right)t}$$

136 ••

Picture the Problem The free-body diagram shows the forces acting on the platform when the spring is partially compressed. The scale reading is the force the scale exerts on the platform and is represented on the FBD by F_n.
We can use Newton's 2nd law to determine the scale reading in part (*a*). We'll use both conservation of energy and momentum to obtain the scale reading when the ball collides inelastically with the cup.

(*a*) Apply $\sum F_y = ma_y$ to the spring when it is compressed a distance *d*:

$$F_n - m_p g - F_{ball\ on\ spring} = 0$$

Solve for F_n:

$$F_n = m_p g + F_{ball\ on\ spring}$$

$$= m_p g + kd$$

$$= m_p g + k\left(\frac{m_b g}{k}\right)$$

$$= \boxed{m_p g + m_b g = (m_p + m_b)g}$$

(*b*) Letting the zero of gravitational energy be at the initial elevation of the cup and v_{bi} represent the velocity of the ball just before it hits the cup, use conservation of energy to find this velocity:

$$\Delta K + \Delta U_g = 0 \text{ where } K_i = U_{gf} = 0$$

$$\therefore \tfrac{1}{2} m_b v_{bi}^2 - mgh = 0$$

and

$$v_{bi} = \sqrt{2gh}$$

Use conservation of momentum to find the velocity of the center of mass:

$$\vec{p}_i = \vec{p}_f$$

$$\therefore v_{cm} = \frac{m_b v_{bi}}{m_b + m_c} = \sqrt{2gh}\left[\frac{m_b}{m_b + m_c}\right]$$

Apply conservation of energy to the collision to obtain:

$$\Delta K_{cm} + \Delta U_s = 0$$

or, with $K_f = U_{si} = 0$,

$$-\tfrac{1}{2}(m_b + m_c)v_{cm}^2 + \tfrac{1}{2}kx^2 = 0$$

Substitute for v_{cm} and solve for kx^2:

$$kx^2 = (m_b + m_c)v_{cm}^2 = 2gh(m_b + m_c)\left[\frac{m_b}{m_b + m_c}\right]^2 = \frac{2ghm_b^2}{m_b + m_c}$$

Solve for x:

$$x = m_b\sqrt{\frac{2gh}{k(m_b + m_c)}}$$

From part (*a*):

$$F_n = m_p g + kx = m_p g + km_b\sqrt{\frac{2gh}{k(m_b + m_c)}} = \boxed{g\left(m_p + m_b\sqrt{\frac{2kh}{g(m_b + m_c)}}\right)}$$

(*c*) Because the collision is inelastic, the ball never returns to its original height.

137 ••

Picture the Problem Let the direction that astronaut 1 first throws the ball be the positive direction and let v_b be the initial speed of the ball in the laboratory frame. Note that each collision is perfectly inelastic. We can apply conservation of momentum and the definition of the speed of the ball relative to the thrower to each of the perfectly inelastic collisions to express the final speeds of each astronaut after one throw and one catch.

Use conservation of momentum to relate the speeds of astronaut 1 and the ball after the first throw:

$$m_1 v_1 + m_b v_b = 0 \quad (1)$$

Relate the speed of the ball in the laboratory frame to its speed relative to astronaut 1:

$$v = v_b - v_1 \quad (2)$$

Eliminate v_b between equations (1) and (2) and solve for v_1:

$$v_1 = -\frac{m_b}{m_1 + m_b} v \quad (3)$$

Substitute equation (3) in equation (2) and solve for v_b:

$$v_b = \frac{m_1}{m_1 + m_b} v \quad (4)$$

Apply conservation of momentum to express the speed of astronaut 2 and the ball after the first catch:

$$0 = m_b v_b = (m_2 + m_b) v_2 \quad (5)$$

Solve for v_2:

$$v_2 = \frac{m_b}{m_2 + m_b} v_b \quad (6)$$

Express v_2 in terms of v by substituting equation (4) in equation (6):

$$v_2 = \frac{m_b}{m_2 + m_b} \frac{m_1}{m_1 + m_b} v = \left[\frac{m_b m_1}{(m_2 + m_b)(m_1 + m_b)}\right] v \quad (7)$$

Use conservation of momentum to express the speed of astronaut 2 and the ball after she throws the ball:

$$(m_2 + m_b) v_2 = m_b v_{bf} + m_2 v_{2f} \quad (8)$$

Relate the speed of the ball in the laboratory frame to its speed relative to astronaut 2:

$$v = v_{2f} - v_{bf} \quad (9)$$

Eliminate v_{bf} between equations (8) and (9) and solve for v_{2f}:

$$v_{2f} = \boxed{\left(\frac{m_b}{m_2 + m_b}\right)\left[1 + \frac{m_1}{m_1 + m_b}\right] v} \quad (10)$$

Substitute equation (10) in equation (9) and solve for v_{bf}:

$$v_{\text{bf}} = -\left[1 - \frac{m_{\text{b}}}{m_2 + m_{\text{b}}}\right] \times \left[1 + \frac{m_1}{m_1 + m_{\text{b}}}\right] v \qquad (11)$$

Apply conservation of momentum to express the speed of astronaut 1 and the ball after she catches the ball:

$$(m_1 + m_{\text{b}})v_{1\text{f}} = m_{\text{b}} v_{\text{bf}} + m_1 v_1 \qquad (12)$$

Using equations (3) and (11), eliminate v_{bf} and v_1 in equation (12) and solve for $v_{1\text{f}}$:

$$v_{1\text{f}} = \boxed{-\frac{m_2 m_{\text{b}}(2m_1 + m_{\text{b}})}{(m_1 + m_{\text{b}})^2 (m_2 + m_{\text{b}})} v}$$

***138 ••**

Picture the Problem We can use the definition of the center of mass of a system containing multiple objects to locate the center of mass of the earth–moon system. Any object external to the system will exert accelerating forces on the system.

(*a*) Express the center of mass of the earth–moon system relative to the center of the earth:

$$M\vec{r}_{\text{cm}} = \sum_{\text{i}} m_{\text{i}} \vec{r}_{\text{i}}$$

or

$$r_{\text{cm}} = \frac{M_{\text{e}}(0) + m_{\text{m}} r_{\text{em}}}{M_{\text{e}} + m_{\text{m}}} = \frac{m_{\text{m}} r_{\text{em}}}{M_{\text{e}} + m_{\text{m}}} = \frac{r_{\text{em}}}{\frac{M_{\text{e}}}{m_{\text{m}}} + 1}$$

Substitute numerical values and evaluate r_{cm}:

$$r_{\text{cm}} = \frac{3.84 \times 10^5\ \text{km}}{81.3 + 1} = \boxed{4670\ \text{km}}$$

Because this distance is less than the radius of the earth, the position of the center of mass of the earth – moon system is below the surface of the earth.

(*b*) Any object not in the earth – moon system exerts forces on the system, e.g., the sun and other planets.

(*c*) Because the sun exerts the dominant external force on the earth – moon system, the acceleration of the system is toward the sun.

(*d*) Because the center of mass is at a fixed distance from the sun, the distance *d* moved by the earth in this time interval is:

$$d = 2r_{em} = 2(4670\,\text{km}) = \boxed{9340\,\text{km}}$$

139 ••

Picture the Problem Let the numeral 2 refer to you and the numeral 1 to the water leaving the hose. Apply conservation of momentum to the system consisting of yourself, the water, and the earth and then differentiate this expression to relate your recoil acceleration to your mass, the speed of the water, and the rate at which the water is leaving the hose.

Use conservation of momentum to relate your recoil velocity to the velocity of the water leaving the hose:

$$\vec{p}_1 + \vec{p}_2 = 0$$

or

$$m_1 v_1 + m_2 v_2 = 0$$

Differentiate this expression with respect to t:

$$m_1 \frac{dv_1}{dt} + v_1 \frac{dm_1}{dt} + m_2 \frac{dv_2}{dt} + v_2 \frac{dm_2}{dt} = 0$$

or

$$m_1 a_1 + v_1 \frac{dm_1}{dt} + m a_2 + v_2 \frac{dm_2}{dt} = 0$$

Because the acceleration of the water leaving the hose, a_1, is zero … as is $\frac{dm_2}{dt}$, the rate at which you are losing mass:

$$v_1 \frac{dm_1}{dt} + m_2 a_2 = 0$$

and

$$a_2 = -\frac{v_1}{m_2}\frac{dm_1}{dt}$$

Substitute numerical values and evaluate a_2:

$$a_2 = -\frac{30\,\text{m/s}}{75\,\text{kg}}(2.4\,\text{kg/s}) = \boxed{-0.960\,\text{m/s}^2}$$

***140 •••**

Picture the Problem Take the zero of gravitational potential energy to be at the elevation of the pan and let the system include the balance, the beads, and the earth. We can use conservation of energy to find the vertical component of the velocity of the beads as they hit the pan and then calculate the net downward force on the pan from Newton's 2nd law.

Use conservation of energy to relate the y component of the bead's velocity as it hits the pan to its height of fall:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}mv_y^2 - mgh = 0$$

Solve for v_y:

$$v_y = \sqrt{2gh}$$

Substitute numerical values and evaluate v_y:

$$v_y = \sqrt{2(9.81\,\text{m/s}^2)(0.5\,\text{m})} = 3.13\,\text{m/s}$$

Express the change in momentum in the y direction per bead:

$$\Delta p_y = p_{yf} - p_{yi} = mv_y - (-mv_y) = 2mv_y$$

Use Newton's 2nd law to express the net force in the y direction exerted on the pan by the beads:

$$F_{\text{net},y} = N\frac{\Delta p_y}{\Delta t}$$

Letting M represent the mass to be placed on the other pan, equate its weight to the net force exerted by the beads, substitute for Δp_y, and solve for M:

$$Mg = N\frac{\Delta p_y}{\Delta t}$$

and

$$M = \frac{N}{\Delta t}\left(\frac{2mv_y}{g}\right)$$

Substitute numerical values and evaluate M:

$$M = (100/\text{s})\frac{[2(0.0005\,\text{kg})(3.13\,\text{m/s})]}{9.81\,\text{m/s}^2}$$

$$= \boxed{31.9\,\text{g}}$$

141 •••

Picture the Problem Assume that the connecting rod goes halfway through both balls, i.e., the centers of mass of the balls are separated by L. Let the system include the dumbbell, the wall and floor, and the earth. Let the zero of gravitational potential be at the center of mass of the lower ball and use conservation of energy to relate the speeds of the balls to the potential energy of the system. By symmetry, the speeds will be equal when the angle with the vertical is 45°.

Use conservation of energy to express the relationship between the initial and final energies of the system:

$$E_i = E_f$$

Express the initial energy of the system:

$$E_i = mgL$$

Express the energy of the system when the angle with the vertical is 45°:

$$E_f = mgL\sin 45° + \tfrac{1}{2}(2m)v^2$$

Substitute to obtain:

$$gL = gL\left(\frac{1}{\sqrt{2}}\right) + v^2$$

Solve for *v*:

$$v = \sqrt{gL\left(1 - \frac{1}{\sqrt{2}}\right)}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{(9.81\,\text{m/s}^2)L\left(1 - \frac{1}{\sqrt{2}}\right)}$$

$$= \boxed{(1.70\,\text{m}^{1/2}/\text{s})\sqrt{L}}$$