Chapter 9
Rotation

Conceptual Problems

***1 •**

Determine the Concept Because r is greater for the point on the rim, it moves the greater distance. Both turn through the same angle. Because r is greater for the point on the rim, it has the greater speed. Both have the same angular velocity. Both have zero tangential acceleration. Both have zero angular acceleration. Because r is greater for the point on the rim, it has the greater centripetal acceleration.

2 •

(*a*) False. Angular velocity has the dimensions $\left[\frac{1}{T}\right]$ whereas linear velocity has dimensions $\left[\frac{L}{T}\right]$.

(*b*) True. The angular velocity of all points on the wheel is $d\theta/dt$.

(*c*) True. The angular acceleration of all points on the wheel is $d\omega/dt$.

3 ••

Picture the Problem The constant-acceleration equation that relates the given variables is $\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$. We can set up a proportion to determine the number of revolutions required to double ω and then subtract to find the number of additional revolutions to accelerate the disk to an angular speed of 2ω.

Using a constant-acceleration equation, relate the initial and final angular velocities to the angular acceleration:

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

or, because $\omega_0^2 = 0$,

$$\omega^2 = 2\alpha\Delta\theta$$

Let $\Delta\theta_{10}$ represent the number of revolutions required to reach an angular velocity ω:

$$\omega^2 = 2\alpha\Delta\theta_{10} \qquad (1)$$

Let $\Delta\theta_{2\omega}$ represent the number of revolutions required to reach an angular velocity ω:

$$(2\omega)^2 = 2\alpha\Delta\theta_{2\omega} \qquad (2)$$

Divide equation (2) by equation (1) and solve for $\Delta\theta_{2\omega}$:

$$\Delta\theta_{2\omega} = \frac{(2\omega)^2}{\omega^2}\Delta\theta_{10} = 4\Delta\theta_{10}$$

The number of *additional* revolutions is: $4\Delta\theta_{10} - \Delta\theta_{10} = 3\Delta\theta_{10} = 3(10\,\text{rev}) = 30\,\text{rev}$

and $\boxed{(c)\text{ is correct.}}$

***4 •**

Determine the Concept Torque has the dimension $\left[\frac{ML^2}{T^2}\right]$.

(*a*) Impulse has the dimension $\left[\frac{ML}{T}\right]$.

(*b*) Energy has the dimension $\left[\frac{ML^2}{T^2}\right]$. $\boxed{(b)\text{ is correct.}}$

(*c*) Momentum has the dimension $\left[\frac{ML}{T}\right]$.

5 •

Determine the Concept The moment of inertia of an object is the product of a constant that is characteristic of the object's distribution of matter, the mass of the object, and the square of the distance from the object's center of mass to the axis about which the object is rotating. Because both (*b*) and (*c*) are correct $\boxed{(d)\text{ is correct.}}$

***6 •**

Determine the Concept Yes. A net torque is required to *change* the rotational state of an object. In the absence of a net torque an object continues in whatever state of rotational motion it was at the instant the net torque became zero.

7 •

Determine the Concept No. A net torque is required to *change* the rotational state of an object. A net torque may decrease the angular speed of an object. All we can say for sure is that a net torque will *change* the angular speed of an object.

8 •

(*a*) False. The net torque acting on an object determines the angular acceleration of the object. At any given instant, the angular velocity may have any value including zero.

(*b*) True. The moment of inertia of a body is *always* dependent on one's choice of an axis of rotation.

(*c*) False. The moment of inertia of an object is the product of a constant that is characteristic of the object's distribution of matter, the mass of the object, and the square of the distance from the object's center of mass to the axis about which the object is rotating.

9 •

Determine the Concept The angular acceleration of a rotating object is proportional to the *net* torque acting on it. The net torque is the product of the tangential force and its lever arm.

Express the angular acceleration of the disk as a function of the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{Fd}{I} = \frac{F}{I}d$$

i.e., $\alpha \propto d$

Because $\alpha \propto d$, doubling d will double the angular acceleration.

$\boxed{(b)\text{ is correct.}}$

***10** •

Determine the Concept From the parallel-axis theorem we know that $I = I_{\text{cm}} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, M is the mass of the object, and h is the distance between the parallel axes. Therefore, I is always greater than I_{cm} by Mh^2. $\boxed{(d)\text{ is correct.}}$

11 •

Determine the Concept The power delivered by the constant torque is the product of the torque and the angular velocity of the merry-go-round. Because the constant torque causes the merry-go-round to accelerate, neither the power input nor the angular velocity of the merry-go-round is constant. $\boxed{(b)\text{ is correct.}}$

12 •

Determine the Concept Let's make the simplifying assumption that the object and the surface do not deform when they come into contact, i.e., we'll assume that the system is rigid. A force does no work if and only if it is perpendicular to the velocity of an object, and exerts no torque on an extended object if and only if it's directed toward the center of the object. Because neither of these conditions is satisfied, the statement is *false*.

13 •

Determine the Concept For a given applied force, this increases the torque about the hinges of the door, which increases the door's angular acceleration, leading to the door being opened more quickly. It is clear that putting the knob far from the hinges means that the door can be opened with less effort (force). However, it also means that the hand on the knob must move through the greatest distance to open the door, so it may not be the quickest way to open the door. Also, if the knob were at the center of the door, you would have to walk around the door after opening it, assuming the door is opening toward you.

***14 •**

Determine the Concept If the wheel is rolling without slipping, a point at the top of the wheel moves with a speed twice that of the center of mass of the wheel, but the bottom of the wheel is momentarily at rest. $\boxed{(c) \text{ is correct.}}$

15 ••

Picture the Problem The kinetic energies of both objects is the sum of their translational and rotational kinetic energies. Their speed dependence will differ due to the differences in their moments of inertia. We can express the total kinetic of both objects and equate them to decide which of their translational speeds is greater.

Express the kinetic energy of the cylinder:

$$K_{\text{cyl}} = \tfrac{1}{2} I_{\text{cyl}} \omega_{\text{cyl}}^2 + \tfrac{1}{2} m v_{\text{cyl}}^2 = \tfrac{1}{2}\left(\tfrac{1}{2} m r^2\right)\frac{v_{\text{cyl}}^2}{r^2} + \tfrac{1}{2} m v_{\text{cyl}}^2 = \tfrac{3}{4} m v_{\text{cyl}}^2$$

Express the kinetic energy of the sphere:

$$K_{\text{sph}} = \tfrac{1}{2} I_{\text{sph}} \omega_{\text{sphl}}^2 + \tfrac{1}{2} m v_{\text{sph}}^2 = \tfrac{1}{2}\left(\tfrac{2}{5} m r^2\right)\frac{v_{\text{sph}}^2}{r^2} + \tfrac{1}{2} m v_{\text{sph}}^2 = \tfrac{7}{10} m v_{\text{sph}}^2$$

Equate the kinetic energies and simplify to obtain:

$$v_{\text{cyl}} = \sqrt{\tfrac{14}{15}}\, v_{\text{sph}} < v_{\text{sph}}$$

and $\boxed{(b) \text{ is correct.}}$

***16 •**

Determine the Concept You could spin the pipes about their center. The one which is easier to spin has its mass concentrated closer to the center of mass and, hence, has a smaller moment of inertia.

17 ••

Picture the Problem Because the coin and the ring begin from the same elevation, they will have the same kinetic energy at the bottom of the incline. The kinetic energies of both objects is the sum of their translational and rotational kinetic energies. Their speed dependence will differ due to the differences in their moments of inertia. We can express the total kinetic of both objects and equate them to their common potential energy loss to decide which of their translational speeds is greater at the bottom of the incline.

Express the kinetic energy of the coin at the bottom of the incline:

$$K_{\text{coin}} = \tfrac{1}{2}I_{\text{cyl}}\omega_{\text{coin}}^2 + \tfrac{1}{2}m_{\text{coin}}v_{\text{coin}}^2 = \tfrac{1}{2}\left(\tfrac{1}{2}m_{\text{coin}}r^2\right)\frac{v_{\text{coin}}^2}{r^2} + \tfrac{1}{2}m_{\text{coin}}v_{\text{coin}}^2 = \tfrac{3}{4}m_{\text{coin}}v_{\text{coin}}^2$$

Express the kinetic energy of the ring at the bottom of the incline:

$$K_{\text{ring}} = \tfrac{1}{2}I_{\text{ring}}\omega_{\text{ring}}^2 + \tfrac{1}{2}m_{\text{ring}}v_{\text{ring}}^2 = \tfrac{1}{2}\left(m_{\text{ring}}r^2\right)\frac{v_{\text{ring}}^2}{r^2} + \tfrac{1}{2}m_{\text{ring}}v_{\text{ring}}^2 = m_{\text{ring}}v_{\text{ring}}^2$$

Equate the kinetic of the coin to its change in potential energy as it rolled down the incline and solve for v_{coin}:

$$\tfrac{3}{4}m_{\text{coin}}v_{\text{coin}}^2 = m_{\text{coin}}gh$$

and

$$v_{\text{coin}}^2 = \tfrac{4}{3}gh$$

Equate the kinetic of the ring to its change in potential energy as it rolled down the incline and solve for v_{ring}:

$$m_{\text{ring}}v_{\text{ring}}^2 = m_{\text{ring}}gh$$

and

$$v_{\text{ring}}^2 = gh$$

Therefore, $v_{\text{coin}} > v_{\text{ring}}$ and (b) is correct.

18 ••

Picture the Problem We can use the definitions of the translational and rotational kinetic energies of the hoop and the moment of inertia of a hoop (ring) to express and compare the kinetic energies.

Express the translational kinetic energy of the hoop:

$$K_{\text{trans}} = \tfrac{1}{2}mv^2$$

Express the rotational kinetic energy of the hoop:

$$K_{\text{rot}} = \tfrac{1}{2}I_{\text{hoop}}\omega^2 = \tfrac{1}{2}\left(mr^2\right)\frac{v^2}{r^2} = \tfrac{1}{2}mv^2$$

Therefore, the translational and rotational kinetic energies are the same and (c) is correct.

19 ••

Picture the Problem We can use the definitions of the translational and rotational kinetic energies of the disk and the moment of inertia of a disk (cylinder) to express and compare the kinetic energies.

Express the translational kinetic energy of the disk:

$$K_{\text{trans}} = \tfrac{1}{2}mv^2$$

Express the rotational kinetic energy of the disk:

$$K_{\text{rot}} = \tfrac{1}{2}I_{\text{hoop}}\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v^2}{r^2} = \tfrac{1}{4}mv^2$$

Therefore, the translational kinetic energy is greater and $\boxed{(a)\text{ is correct.}}$

20 ••

Picture the Problem Let us assume that $f \neq 0$ and acts along the direction of motion. Now consider the acceleration of the center of mass and the angular acceleration about the point of contact with the plane. Because $F_{\text{net}} \neq 0$, $a_{\text{cm}} \neq 0$. However, $\tau = 0$ because $\ell = 0$, so $\alpha = 0$. But $\alpha = 0$ is not consistent with $a_{\text{cm}} \neq 0$. Consequently, $f = 0$.

21 •

Determine the Concept True. If the sphere is slipping, then there is kinetic friction which dissipates the mechanical energy of the sphere.

22 •

Determine the Concept Because the ball is struck high enough to have topspin, the frictional force is forward; reducing ω until the nonslip condition is satisfied.

$\boxed{(a)\text{ is correct.}}$

Estimation and Approximation

23 ••

Picture the Problem Assume the wheels are hoops, i.e., neglect the mass of the spokes, and express the total kinetic energy of the bicycle and rider. Let M represent the mass of the rider, m the mass of the bicycle, m_{w} the mass of each bicycle wheel, and r the radius of the wheels.

Express the ratio of the kinetic energy associated with the rotation of the wheels to that associated with the total kinetic energy of the bicycle and rider:

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{K_{\text{rot}}}{K_{\text{trans}} + K_{\text{rot}}} \qquad (1)$$

Express the translational kinetic energy of the bicycle and rider:

$$K_{\text{trans}} = K_{\text{bicycle}} + K_{\text{rider}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}Mv^2$$

Express the rotational kinetic energy of the bicycle wheels:

$$K_{\text{rot}} = 2K_{\text{rot, 1 wheel}} = 2\left(\tfrac{1}{2}I_{\text{w}}\omega^2\right) = \left(m_{\text{w}}r^2\right)\frac{v^2}{r^2} = m_{\text{w}}v^2$$

Substitute in equation (1) to obtain:

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{m_{\text{w}}v^2}{\tfrac{1}{2}mv^2 + \tfrac{1}{2}Mv^2 + m_{\text{w}}v^2} = \frac{m_{\text{w}}}{\tfrac{1}{2}m + \tfrac{1}{2}M + m_{\text{w}}} = \frac{2}{2 + \dfrac{m+M}{m_{\text{w}}}}$$

Substitute numerical values and evaluate $K_{\text{rot}}/K_{\text{tot}}$:

$$\frac{K_{\text{rot}}}{K_{\text{tot}}} = \frac{2}{2 + \dfrac{14\,\text{kg} + 38\,\text{kg}}{3\,\text{kg}}} = \boxed{10.3\,\%}$$

24 ••

Picture the Problem We can apply the definition of angular velocity to find the angular orientation of the slice of toast when it has fallen a distance of 0.5 m from the edge of the table. We can then interpret the orientation of the toast to decide whether it lands jelly-side up or down.

Relate the angular orientation θ of the toast to its initial angular orientation, its angular velocity ω, and time of fall Δt:

$$\theta = \theta_0 + \omega\Delta t \qquad (1)$$

Use the equation given in the problem statement to find the angular velocity corresponding to this length of toast:

$$\omega = 0.956\sqrt{\frac{9.81\,\text{m/s}^2}{0.1\,\text{m}}} = 9.47\,\text{rad/s}$$

Using a constant-acceleration equation, relate the distance the toast falls Δy to its time of fall Δt:

$$\Delta y = v_{0y}\Delta t + \tfrac{1}{2}a_y(\Delta t)^2$$

or, because $v_{0y} = 0$ and $a_y = g$,

$$\Delta y = \tfrac{1}{2}g(\Delta t)^2$$

Solve for Δt:

$$\Delta t = \sqrt{\frac{2\Delta y}{g}}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \sqrt{\frac{2(0.5\,\text{m})}{9.81\,\text{m/s}^2}} = 0.319\,\text{s}$$

$\frac{(v_f')^2}{2gL} + \cos\theta_0$ Substitute in equation (1) to find θ:

$$\theta = \frac{\pi}{6} + (9.47\,\text{rad/s})(0.319\,\text{s})$$
$$= 3.54\,\text{rad} \times \frac{180°}{\pi\,\text{rad}} = 203°$$

The orientation of the slice of toast will therefore be at an angle of 203° with respect to the ground, i.e. with the jelly-side down.

***25 ••**

Picture the Problem Assume that the mass of an average adult male is about 80 kg, and that we can model his body when he is standing straight up with his arms at his sides as a cylinder. From experience in men's clothing stores, a man's average waist circumference seems to be about 34 inches, and the average chest circumference about 42 inches. We'll also assume that about 20% of the body's mass is in the two arms, and each has a length L = 1 m, so that each arm has a mass of about m = 8 kg.

Letting I_{out} represent his moment of inertia with his arms straight out and I_{in} his moment of inertia with his arms at his side, the ratio of these two moments of inertia is:

$$\frac{I_{out}}{I_{in}} = \frac{I_{body} + I_{arms}}{I_{in}} \qquad (1)$$

Express the moment of inertia of the ″man as a cylinder″:

$$I_{in} = \tfrac{1}{2}MR^2$$

Express the moment of inertia of his arms:

$$I_{arms} = 2\left(\tfrac{1}{3}\right)mL^2$$

Express the moment of inertia of his body-less-arms:

$$I_{body} = \tfrac{1}{2}(M - m)R^2$$

Substitute in equation (1) to obtain:

$$\frac{I_{out}}{I_{in}} = \frac{\tfrac{1}{2}(M - m)R^2 + 2\left(\tfrac{1}{3}\right)mL^2}{\tfrac{1}{2}MR^2}$$

Assume the circumference of the cylinder to be the average of the average waist circumference and the average chest circumference:

$$c_{av} = \frac{34\,\text{in} + 42\,\text{in}}{2} = 38\,\text{in}$$

Find the radius of a circle whose circumference is 38 in:

$$R = \frac{c_{av}}{2\pi} = \frac{38\,\text{in} \times \frac{2.54\,\text{cm}}{\text{in}} \times \frac{1\,\text{m}}{100\,\text{cm}}}{2\pi}$$
$$= 0.154\,\text{m}$$

Substitute numerical values and evaluate I_{out}/I_{in}:

$$\frac{I_{\text{out}}}{I_{\text{in}}} = \frac{\frac{1}{2}(80\,\text{kg} - 16\,\text{kg})(0.154\,\text{m})^2 + \frac{2}{3}(8\,\text{kg})(1\,\text{m})^2}{\frac{1}{2}(80\,\text{kg})(0.154\,\text{m})^2} = \boxed{6.42}$$

Angular Velocity and Angular Acceleration

26 •

Picture the Problem The tangential and angular velocities of a particle moving in a circle are directly proportional. The number of revolutions made by the particle in a given time interval is proportional to both the time interval and its angular speed.

(*a*) Relate the angular velocity of the particle to its speed along the circumference of the circle:

$$v = r\omega$$

Solve for and evaluate ω:

$$\omega = \frac{v}{r} = \frac{25\,\text{m/s}}{90\,\text{m}} = \boxed{0.278\,\text{rad/s}}$$

(*b*) Using a constant-acceleration equation, relate the number of revolutions made by the particle in a given time interval to its angular velocity:

$$\Delta\theta = \omega\Delta t = \left(0.278\frac{\text{rad}}{\text{s}}\right)(30\,\text{s})\left(\frac{1\,\text{rev}}{2\pi\,\text{rad}}\right)$$

$$= \boxed{1.33\,\text{rev}}$$

27 •

Picture the Problem Because the angular acceleration is constant; we can find the various physical quantities called for in this problem by using constant-acceleration equations.

(*a*) Using a constant-acceleration equation, relate the angular velocity of the wheel to its angular acceleration and the time it has been accelerating:

$$\omega = \omega_0 + \alpha\Delta t$$

or, when $\omega_0 = 0$,

$$\omega = \alpha\Delta t$$

Evaluate ω when $\Delta t = 6$ s:

$$\omega = \left(2.6\,\text{rad/s}^2\right)(6\,\text{s}) = \boxed{15.6\,\text{rad/s}}$$

(*b*) Using another constant-acceleration equation, relate the angular displacement to the wheel's angular acceleration and the time it

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, when $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2$$

has been accelerating:

Evaluate $\Delta\theta$ when $\Delta t = 6$ s:

$$\Delta\theta(6\,\text{s}) = \tfrac{1}{2}\left(2.6\,\text{rad/s}^2\right)(6\,\text{s})^2 = \boxed{46.8\,\text{rad}}$$

(*c*) Convert $\Delta\theta(6\,\text{s})$ from rad to revolutions:

$$\Delta\theta(6\,\text{s}) = 46.8\,\text{rad} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} = \boxed{7.45\,\text{rev}}$$

(*d*) Relate the angular velocity of the particle to its tangential speed and evaluate the latter when $\Delta t = 6$ s:

$$v = r\omega = (0.3\,\text{m})(15.6\,\text{rad/s}) = \boxed{4.68\,\text{m/s}}$$

Relate the resultant acceleration of the point to its tangential and centripetal accelerations when $\Delta t = 6$ s:

$$a = \sqrt{a_\text{t}^2 + a_\text{c}^2} = \sqrt{(r\alpha)^2 + \left(r\omega^2\right)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Substitute numerical values and evaluate *a*:

$$a = (0.3\,\text{m})\sqrt{\left(2.6\,\text{rad/s}^2\right)^2 + (15.6\,\text{rad/s})^4} = \boxed{73.0\,\text{m/s}^2}$$

***28** •

Picture the Problem Because we're assuming constant angular acceleration; we can find the various physical quantities called for in this problem by using constant-acceleration equations.

(*a*) Using its definition, express the angular acceleration of the turntable:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{0 - 33\tfrac{1}{3}\dfrac{\text{rev}}{\text{min}} \times \dfrac{2\pi\,\text{rad}}{\text{rev}} \times \dfrac{1\,\text{min}}{60\,\text{s}}}{26\,\text{s}} = \boxed{0.134\,\text{rad/s}^2}$$

(*b*) Because the angular acceleration is constant, the average angular velocity is the average of its initial and final values:

$$\omega_{av} = \frac{\omega_0 + \omega}{2} = \frac{33\tfrac{1}{3}\dfrac{\text{rev}}{\text{min}} \times \dfrac{2\pi\,\text{rad}}{\text{rev}} \times \dfrac{1\,\text{min}}{60\,\text{s}}}{2} = \boxed{1.75\,\text{rad/s}}$$

(*c*) Using the definition of ω_{av}, find the number or revolutions the turntable makes before stopping:

$$\Delta\theta = \omega_{av}\Delta t = (1.75\,\text{rad/s})(26\,\text{s}) = 45.5\,\text{rad} \times \frac{1\,\text{rev}}{2\pi\,\text{rad}} = \boxed{7.24\,\text{rev}}$$

29 •

Picture the Problem Because the angular acceleration of the disk is constant, we can use a constant-acceleration equation to relate its angular velocity to its acceleration and the time it has been accelerating. We can find the tangential and centripetal accelerations from their relationships to the angular velocity and angular acceleration of the disk.

(*a*) Using a constant-acceleration equation, relate the angular velocity of the disk to its angular acceleration and time during which it has been accelerating:

$$\omega = \omega_0 + \alpha\,\Delta t$$

or, because $\omega_0 = 0$,

$$\omega = \alpha\,\Delta t$$

Evaluate ω when $t = 5$ s:

$$\omega(5\,\text{s}) = (8\,\text{rad/s}^2)(5\,\text{s}) = \boxed{40.0\,\text{rad/s}}$$

(*b*) Express a_t in terms of α:

$$a_t = r\alpha$$

Evaluate a_t when $t = 5$ s:

$$a_t(5\,\text{s}) = (0.12\,\text{m})(8\,\text{rad/s}^2) = \boxed{0.960\,\text{m/s}^2}$$

Express a_c in terms of ω:

$$a_c = r\omega^2$$

Evaluate a_c when $t = 5$ s:

$$a_c(5\,\text{s}) = (0.12\,\text{m})(40.0\,\text{rad/s})^2 = \boxed{192\,\text{m/s}^2}$$

30 •

Picture the Problem We can find the angular velocity of the Ferris wheel from its definition and the linear speed and centripetal acceleration of the passenger from the relationships between those quantities and the angular velocity of the Ferris wheel.

(*a*) Find ω from its definition:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{27\,\text{s}} = \boxed{0.233\,\text{rad/s}}$$

(*b*) Find the linear speed of the passenger from his/her angular speed:

$$v = r\omega = (12\,\text{m})(0.233\,\text{rad/s}) = \boxed{2.79\,\text{m/s}}$$

Find the passenger's centripetal acceleration from his/her angular velocity:

$$a_c = r\omega^2 = (12\,\text{m})(0.233\,\text{rad/s})^2 = \boxed{0.651\,\text{m/s}^2}$$

31 •

Picture the Problem Because the angular acceleration of the wheels is constant, we can use constant-acceleration equations in rotational form to find their angular acceleration and their angular velocity at any given time.

(*a*) Using a constant-acceleration equation, relate the angular displacement of the wheel to its angular acceleration and the time it has been accelerating:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2$$

Solve for α:

$$\alpha = \frac{2\Delta\theta}{(\Delta t)^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(3\,\text{rev})\left(\frac{2\pi\,\text{rad}}{\text{rev}}\right)}{(8\,\text{s})^2} = \boxed{0.589\,\text{rad/s}^2}$$

(*b*) Using a constant-acceleration equation, relate the angular velocity of the wheel to its angular acceleration and the time it has been accelerating:

$$\omega = \omega_0 + \alpha\Delta t$$

or, when $\omega_0 = 0$,

$$\omega = \alpha\Delta t$$

Evaluate ω when $\Delta t = 8$ s:

$$\omega(8\,\text{s}) = (0.589\,\text{rad/s}^2)(8\,\text{s}) = \boxed{4.71\,\text{rad/s}}$$

32 •

Picture the Problem The earth rotates through 2π radians every 24 hours.

Find ω using its definition:

$$\omega \equiv \frac{\Delta\theta}{\Delta t} = \frac{2\pi\ \text{rad}}{24\ \text{h} \times \frac{3600\ \text{s}}{\text{h}}} = \boxed{7.27 \times 10^{-5}\ \text{rad/s}}$$

33 •

Picture the Problem When the angular acceleration of a wheel is constant, its average angular velocity is the average of its initial and final angular velocities. We can combine this relationship with the always applicable definition of angular velocity to find the initial angular velocity of the wheel.

Express the average angular velocity of the wheel in terms of its initial and final angular speeds:

$$\omega_{\text{av}} = \frac{\omega_0 + \omega}{2}$$

or, because $\omega = 0$,

$$\omega_{\text{av}} = \tfrac{1}{2}\omega_0$$

Express the definition of the average angular velocity of the wheel:

$$\omega_{\text{av}} \equiv \frac{\Delta\theta}{\Delta t}$$

Equate these two expressions and solve for ω_0:

$$\omega_0 = \frac{2\Delta\theta}{\Delta t} = \frac{2(5\ \text{rad})}{2.8\ \text{s}} = 3.57\ \text{s and}$$

$$\boxed{(d)\ \text{is correct.}}$$

34 •

Picture the Problem The tangential and angular accelerations of the wheel are directly proportional to each other with the radius of the wheel as the proportionality constant. Provided there is no slippage, the acceleration of a point on the rim of the wheel is the same as the acceleration of the bicycle. We can use its defining equation to determine the acceleration of the bicycle.

Relate the tangential acceleration of a point on the wheel (equal to the acceleration of the bicycle) to the wheel's angular acceleration and solve for its angular acceleration:

$$a = a_{\text{t}} = r\alpha$$

and

$$\alpha = \frac{a}{r}$$

Use its definition to express the acceleration of the wheel:

$$a = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{\Delta t}$$

or, because $v_0 = 0$,

$$a = \frac{v}{\Delta t}$$

Substitute in the expression for α to obtain:

$$\alpha = \frac{v}{r\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{\left(24\,\frac{\text{km}}{\text{h}}\right)\left(\frac{1\,\text{h}}{3600\,\text{s}}\right)\left(\frac{1000\,\text{m}}{\text{km}}\right)}{(0.6\,\text{m})(14.0\,\text{s})} = \boxed{0.794\,\text{rad/s}^2}$$

***35 ••**

Picture the Problem The two tapes will have the same tangential and angular velocities when the two reels are the same size, i.e., have the same area. We can calculate the tangential speed of the tape from its length and running time and relate the angular velocity to the constant tangential speed and the radius of the reels when they are turning with the same angular velocity.

Relate the angular velocity of the tape to its tangential speed:

$$\omega = \frac{v}{r} \quad (1)$$

Letting R_f represent the outer radius of the reel when the reels have the same area, express the condition that they have the same speed:

$$\pi R_f^2 - \pi r^2 = \tfrac{1}{2}\left(\pi R^2 - \pi r^2\right)$$

Solve for R_f:

$$R_f = \sqrt{\frac{R^2 + r^2}{2}}$$

Substitute numerical values and evaluate R_f:

$$R_f = \sqrt{\frac{(45\,\text{mm})^2 + (12\,\text{mm})^2}{2}} = 32.9\,\text{mm}$$

Find the tangential speed of the tape from its length and running time:

$$v = \frac{L}{\Delta t} = \frac{246\,\text{m} \times \frac{100\,\text{cm}}{\text{m}}}{2\,\text{h} \times \frac{3600\,\text{s}}{\text{h}}} = 3.42\,\text{cm/s}$$

Substitute in equation (1) and evaluate ω:

$$\omega = \frac{v}{R_f} = \frac{3.42\,\text{cm/s}}{32.9\,\text{mm}\times\dfrac{1\,\text{cm}}{10\,\text{mm}}} = \boxed{1.04\,\text{rad/s}}$$

Convert 1.04 rad/s to rev/min:

$$1.04\,\text{rad/s} = 1.04\,\frac{\text{rad}}{\text{s}}\times\frac{1\,\text{rev}}{2\pi\,\text{rad}}\times\frac{60\,\text{s}}{\text{min}} = \boxed{9.93\,\text{rev/min}}$$

Torque, Moment of Inertia, and Newton's Second Law for Rotation

36 •

Picture the Problem The force that the woman exerts through her axe, because it does not act at the axis of rotation, produces a net torque that changes (decreases) the angular velocity of the grindstone.

(*a*) From the definition of angular acceleration we have:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{\omega - \omega_0}{\Delta t}$$

or, because $\omega = 0$,

$$\alpha = \frac{-\omega_0}{\Delta t}$$

Substitute numerical values and evaluate α:

$$\alpha = -\frac{730\,\dfrac{\text{rev}}{\text{min}}\times\dfrac{2\pi\,\text{rad}}{\text{rev}}\times\dfrac{1\,\text{min}}{60\,\text{s}}}{9\,\text{s}} = \boxed{-8.49\,\text{rad/s}^2}$$

where the minus sign means that the grindstone is slowing down.

(*b*) Use Newton's 2nd law in rotational form to relate the angular acceleration of the grindstone to the net torque slowing it:

$$\tau_{\text{net}} = I\alpha$$

Express the moment of inertia of disk with respect to its axis of rotation:

$$I = \tfrac{1}{2}MR^2$$

Substitute to obtain:

$$\tau_{\text{net}} = \tfrac{1}{2} MR\alpha$$

Substitute numerical values and evaluate τ_{net}:

$$\tau_{\text{net}} = \tfrac{1}{2}(1.7\,\text{kg})(0.08\,\text{m})^2(8.49\,\text{rad/s}^2) = \boxed{0.0462\,\text{N}\cdot\text{m}}$$

***37** •

Picture the Problem We can find the torque exerted by the 17-N force from the definition of torque. The angular acceleration resulting from this torque is related to the torque through Newton's 2nd law in rotational form. Once we know the angular acceleration, we can find the angular velocity of the cylinder as a function of time.

(*a*) Calculate the torque from its definition:

$$\tau = F\ell = (17\,\text{N})(0.11\,\text{m}) = \boxed{1.87\,\text{N}\cdot\text{m}}$$

(*b*) Use Newton's 2nd law in rotational form to relate the acceleration resulting from this torque to the torque:

$$\alpha = \frac{\tau}{I}$$

Express the moment of inertia of the cylinder with respect to its axis of rotation:

$$I = \tfrac{1}{2} MR^2$$

Substitute to obtain:

$$\alpha = \frac{2\tau}{MR^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(1.87\,\text{N}\cdot\text{m})}{(2.5\,\text{kg})(0.11\,\text{m})^2} = \boxed{124\,\text{rad/s}^2}$$

(*c*) Using a constant-acceleration equation, express the angular velocity of the cylinder as a function of time:

$$\omega = \omega_0 + \alpha t$$

or, because $\omega_0 = 0$,

$$\omega = \alpha t$$

Evaluate ω (5 s):

$$\omega(5\,\text{s}) = (124\,\text{rad/s}^2)(5\,\text{s}) = \boxed{620\,\text{rad/s}}$$

38 ••

Picture the Problem We can find the angular acceleration of the wheel from its definition and the moment of inertia of the wheel from Newton's 2nd law.

(*a*) Express the moment of inertia of the wheel in terms of the angular acceleration produced by the applied torque:

$$I = \frac{\tau}{\alpha}$$

Find the angular acceleration of the wheel:

$$\alpha = \frac{\Delta\omega}{\Delta t} = \frac{600\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}}{20\,\text{s}}$$

$$= 3.14\,\text{rad/s}^2$$

Substitute and evaluate *I*:

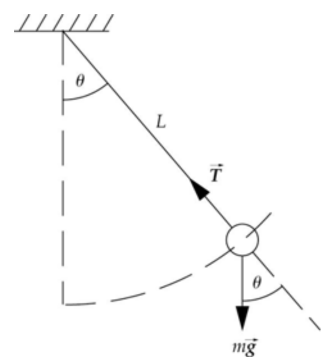
$$I = \frac{50\,\text{N}\cdot\text{m}}{3.14\,\text{rad/s}^2} = \boxed{15.9\,\text{kg}\cdot\text{m}^2}$$

(*b*) Because the wheel takes 120 s to slow to a stop (it took 20 s to acquire an angular velocity of 600 rev/min) and its angular acceleration is directly proportional to the accelerating torque:

$$\tau_{\text{fr}} = \tfrac{1}{6}\tau = \tfrac{1}{6}(50\,\text{N}\cdot\text{m}) = \boxed{8.33\,\text{N}\cdot\text{m}}$$

39 ••

Picture the Problem The pendulum and the forces acting on it are shown in the free-body diagram. Note that the tension in the string is radial, and so exerts no tangential force on the ball. We can use Newton's 2nd law in both translational and rotational form to find the tangential component of the acceleration of the bob.



(*a*) Referring to the FBD, express the component of $m\vec{g}$ that is tangent to the circular path of the bob:

$$F_t = mg\sin\theta$$

Use Newton's 2nd law to express the tangential acceleration of the bob:

$$a_t = \frac{F_t}{m} = \boxed{g\sin\theta}$$

(*b*) Noting that, because the line-of-action of the tension passes through the pendulum's pivot point, its lever arm is zero and the net torque is due

$$\sum\tau_{\text{pivot point}} = \boxed{mgL\sin\theta}$$

to the weight of the bob, sum the torques about the pivot point to obtain:

(*c*) Use Newton's 2nd law in rotational form to relate the angular acceleration of the pendulum to the net torque acting on it:

$$\tau_{\text{net}} = mgL\sin\theta = I\alpha$$

Solve for α to obtain:

$$\alpha = \frac{mgL\sin\theta}{I}$$

Express the moment of inertia of the bob with respect to the pivot point:

$$I = mL^2$$

Substitute to obtain:

$$\alpha = \frac{mgL\sin\theta}{mL^2} = \frac{g\sin\theta}{L}$$

Relate α to a_t:

$$a_t = r\alpha = L\left(\frac{g\sin\theta}{L}\right) = \boxed{g\sin\theta}$$

***40 •••**

Picture the Problem We can express the velocity of the center of mass of the rod in terms of its distance from the pivot point and the angular velocity of the rod. We can find the angular velocity of the rod by using Newton's 2nd law to find its angular acceleration and then a constant-acceleration equation that relates ω to α. We'll use the impulse-momentum relationship to derive the expression for the force delivered to the rod by the pivot. Finally, the location of the *center of percussion* of the rod will be verified by setting the force exerted by the pivot to zero.

(*a*) Relate the velocity of the center of mass to its distance from the pivot point:

$$v_{\text{cm}} = \frac{L}{2}\omega \qquad (1)$$

Express the torque due to F_0:

$$\tau = F_0 x = I_{\text{pivot}}\alpha$$

Solve for α:

$$\alpha = \frac{F_0 x}{I_{\text{pivot}}}$$

Express the moment of inertia of the rod with respect to an axis through

$$I_{\text{pivot}} = \tfrac{1}{3}ML^2$$

its pivot point:

Substitute to obtain:

$$\alpha = \frac{3F_0 x}{ML^2}$$

Express the angular velocity of the rod in terms of its angular acceleration:

$$\omega = \alpha\,\Delta t = \frac{3F_0 x\Delta t}{ML^2}$$

Substitute in equation (1) to obtain:

$$v_{\rm cm} = \boxed{\frac{3F_0 x\Delta t}{2ML}}$$

(*b*) Let I_P be the impulse exerted by the pivot on the rod. Then the total impulse (equal to the change in momentum of the rod) exerted on the rod is:

$$I_{\rm P} + F_0\Delta t = Mv_{\rm cm}$$

and

$$I_{\rm P} = Mv_{\rm cm} - F_0\Delta t$$

Substitute our result from (a) to obtain:

$$I_{\rm P} = \frac{3F_0 x\Delta t}{2L} - F_0\Delta t = F_0\Delta t\left(\frac{3x}{2L} - 1\right)$$

Because $I_{\rm P} = F_{\rm P}\Delta t$:

$$F_{\rm P} = \boxed{F_0\left(\frac{3x}{2L} - 1\right)}$$

In order for F_P to be zero:

$$\frac{3x}{2L} - 1 = 0 \Rightarrow x = \boxed{\frac{2L}{3}}$$

41 •••

Picture the Problem We'll first express the torque exerted by the force of friction on the elemental disk and then integrate this expression to find the torque on the entire disk. We'll use Newton's 2nd law to relate this torque to the angular acceleration of the disk and then to the stopping time for the disk.

(*a*) Express the torque exerted on the elemental disk in terms of the friction force and the distance to the elemental disk:

$$d\tau_{\rm f} = rdf_{\rm k} \qquad (1)$$

Using the definition of the coefficient of friction, relate the

$$df_{\rm k} = \mu_{\rm k} g dm \qquad (2)$$

force of friction to μ_k and the weight of the circular element:

Letting σ represent the mass per unit area of the disk, express the mass of the circular element:

$$dm = 2\pi r\sigma\, dr \qquad (3)$$

Substitute equations (2) and (3) in (1) to obtain:

$$d\tau_f = 2\pi\mu_k\sigma g r^2 dr \qquad (4)$$

Because $\sigma = \dfrac{M}{\pi R^2}$:

$$d\tau_f = \boxed{\frac{2\mu_k M g}{R^2} r^2 dr}$$

(*b*) Integrate $d\tau_f$ to obtain the total torque on the elemental disk:

$$\tau_f = \frac{2\mu_k M g}{R^2}\int_0^R r^2 dr = \boxed{\tfrac{2}{3} MR\mu_k g}$$

(*c*) Relate the disk's stopping time to its angular velocity and acceleration:

$$\Delta t = \frac{\omega}{\alpha}$$

Using Newton's 2nd law, express α in terms of the net torque acting on the disk:

$$\alpha = \frac{\tau_f}{I}$$

The moment of inertia of the disk, with respect to its axis of rotation, is:

$$I = \tfrac{1}{2} MR^2$$

Substitute and simplify to obtain:

$$\Delta t = \boxed{\frac{3R\omega}{4\mu_k g}}$$

Calculating the Moment of Inertia

42 •

Picture the Problem One can find the formula for the moment of inertia of a thin spherical shell in Table 9-1.

The moment of inertia of a thin spherical shell about its diameter is:

$$I = \tfrac{2}{3} MR^2$$

Substitute numerical values and evaluate I:

$$I = \tfrac{2}{3}(0.057\,\text{kg})(0.035\,\text{m})^2 = \boxed{4.66\times10^{-5}\,\text{kg}\cdot\text{m}^2}$$

***43 •**

Picture the Problem The moment of inertia of a system of particles with respect to a given axis is the sum of the products of the mass of each particle and the square of its distance from the given axis.

Use the definition of the moment of inertia of a system of particles to obtain:

$$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate I:

$$I = (3\,\text{kg})(2\,\text{m})^2 + (4\,\text{kg})\left(2\sqrt{2}\,\text{m}\right)^2 + (4\,\text{kg})(0)^2 + (3\,\text{kg})(2\,\text{m})^2 = \boxed{56.0\,\text{kg}\cdot\text{m}^2}$$

44 •

Picture the Problem Note, from symmetry considerations, that the center of mass of the system is at the intersection of the diagonals connecting the four masses. Thus the distance of each particle from the axis through the center of mass is $\sqrt{2}$ m. According to the parallel-axis theorem, $I = I_{\text{cm}} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, M is the mass of the object, and h is the distance between the parallel axes.

Express the parallel axis theorem:

$$I = I_{\text{cm}} + Mh^2$$

Solve for I_{cm} and substitute from Problem 44:

$$I_{\text{cm}} = I - Mh^2 = 56.0\,\text{kg}\cdot\text{m}^2 - (14\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 = \boxed{28.0\,\text{kg}\cdot\text{m}^2}$$

Use the definition of the moment of inertia of a system of particles to express I_{cm}:

$$I_{\text{cm}} = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate I_{cm}:

$$I_{\text{cm}} = (3\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 + (4\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 + (4\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 + (3\,\text{kg})\left(\sqrt{2}\,\text{m}\right)^2 = \boxed{28.0\,\text{kg}\cdot\text{m}^2}$$

45 •

Picture the Problem The moment of inertia of a system of particles with respect to a given axis is the sum of the products of the mass of each particle and the square of its distance from the given axis.

(*a*) Apply the definition of the moment of inertia of a system of particles to express I_x:

$$I_x = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate I_x:

$$I_x = (3\,\text{kg})(2\,\text{m})^2 + (4\,\text{kg})(2\,\text{m})^2 + (4\,\text{kg})(0) + (3\,\text{kg})(0) = \boxed{28.0\,\text{kg}\cdot\text{m}^2}$$

(*b*) Apply the definition of the moment of inertia of a system of particles to express I_y:

$$I_y = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate I_y:

$$I_y = (3\,\text{kg})(0) + (4\,\text{kg})(2\,\text{m})^2 + (4\,\text{kg})(0) + (3\,\text{kg})(2\,\text{m})^2 = \boxed{28.0\,\text{kg}\cdot\text{m}^2}$$

Remarks: We could also use a symmetry argument to conclude that $I_y = I_x$.

46 •

Picture the Problem According to the parallel-axis theorem, $I = I_{\text{cm}} + Mh^2$, where I_{cm} is the moment of inertia of the object with respect to an axis through its center of mass, M is the mass of the object, and h is the distance between the parallel axes.

Use Table 9-1 to find the moment of inertia of a sphere with respect to an axis through its center of mass:

$$I_{\text{cm}} = \tfrac{2}{5}MR^2$$

Express the parallel axis theorem:

$$I = I_{\text{cm}} + Mh^2$$

Substitute for I_{cm} and simplify to obtain:

$$I = \tfrac{2}{5}MR^2 + MR^2 = \boxed{\tfrac{7}{5}MR^2}$$

47 ••

Picture the Problem The moment of inertia of the wagon wheel is the sum of the moments of inertia of the rim and the six spokes.

Express the moment of inertia of the wagon wheel as the sum of the moments of inertia of the rim and the spokes:

$$I_{\text{wheel}} = I_{\text{rim}} + I_{\text{spokes}}$$

Using Table 9-1, find formulas for the moments of inertia of the rim and spokes:

$$I_{\text{rim}} = M_{\text{rim}} R^2$$

and

$$I_{\text{spoke}} = \tfrac{1}{3} M_{\text{spoke}} L^2$$

Substitute to obtain:

$$I_{\text{wheel}} = M_{\text{rim}} R^2 + 6\left(\tfrac{1}{3} M_{\text{spoke}} L^2\right) = M_{\text{rim}} R^2 + 2M_{\text{spoke}} L^2$$

Substitute numerical values and evaluate I_{wheel}:

$$I_{\text{wheel}} = (8\,\text{kg})(0.5\,\text{m})^2 + 2(1.2\,\text{kg})(0.5\,\text{m})^2 = \boxed{2.60\,\text{kg}\cdot\text{m}^2}$$

***48 ••**

Picture the Problem The moment of inertia of a system of particles depends on the axis with respect to which it is calculated. Once this choice is made, the moment of inertia is the sum of the products of the mass of each particle and the square of its distance from the chosen axis.

(*a*) Apply the definition of the moment of inertia of a system of particles:

$$I = \sum_{\text{i}} m_{\text{i}} r_{\text{i}}^2 = \boxed{m_1 x^2 + m_2 (L - x)^2}$$

(*b*) Set the derivative of I with respect to x equal to zero in order to identify values for x that correspond to either maxima or minima:

$$\frac{dI}{dx} = 2m_1 x + 2m_2 (L - x)(-1) = 2(m_1 x + m_2 x - m_2 L) = 0 \text{ for extrema}$$

If $\dfrac{dI}{dx} = 0$, then:

$$m_1 x + m_2 x - m_2 L = 0$$

Solve for x:

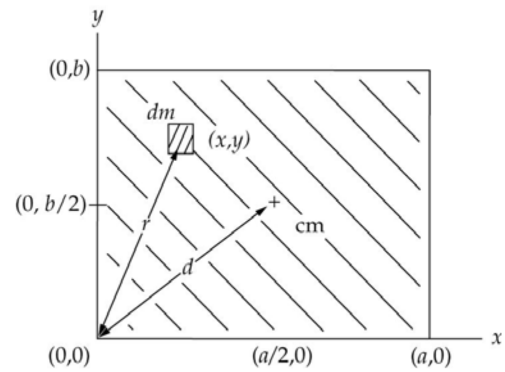
$$x = \frac{m_2 L}{m_1 + m_2}$$

Convince yourself that you've found a minimum by showing that $\frac{d^2 I}{dx^2}$ is positive at this point.

$x = \frac{m_2 L}{m_1 + m_2}$ is, by definition, the distance of the center of mass from m.

49 ••

Picture the Problem Let σ be the mass per unit area of the uniform rectangular plate. Then the elemental unit has mass $dm = \sigma\, dxdy$. Let the corner of the plate through which the axis runs be the origin. The distance of the element whose mass is dm from the corner r is related to the coordinates of dm through the Pythagorean relationship $r^2 = x^2 + y^2$.



(*a*) Express the moment of inertia of the element whose mass is dm with respect to an axis perpendicular to it and passing through one of the corners of the uniform rectangular plate:

$$dI = \sigma\left(x^2 + y^2\right)dxdy$$

Integrate this expression to find I:

$$I = \sigma\int_0^a\int_0^b \left(x^2 + y^2\right)dxdy$$

$$= \tfrac{1}{3}\sigma\left(a^3 b + ab^3\right) = \boxed{\tfrac{1}{3}m\left(a^2 + b^3\right)}$$

(*b*) Letting d represent the distance from the origin to the center of mass of the plate, use the parallel axis theorem to relate the moment of inertia found in (*a*) to the moment of inertia with respect to an axis through the center of mass:

$$I = I_{\text{cm}} + md^2$$

or

$$I_{\text{cm}} = I - md^2 = \tfrac{1}{3}m\left(a^2 + b^2\right) - md^2$$

Using the Pythagorean theorem, relate the distance d to the center of

$$d^2 = \left(\tfrac{1}{2}a\right)^2 + \left(\tfrac{1}{2}b\right)^2 = \tfrac{1}{4}\left(a^2 + b^2\right)$$

mass to the lengths of the sides of the plate:

Substitute for d^2 in the expression for I_{cm} and simplify to obtain:

$$I_{cm} = \tfrac{1}{3}m\left(a^2+b^2\right)-\tfrac{1}{4}m\left(a^2+b^2\right)^2$$
$$= \boxed{\tfrac{1}{12}m\left(a^2+b^2\right)}$$

***50** ••

Picture the Problem Corey will use the point-particle relationship $I_{app} = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2$ for his calculation whereas Tracey's calculation will take into account not only the rod but also the fact that the spheres are not point particles.

(*a*) Using the point-mass approximation and the definition of the moment of inertia of a system of particles, express I_{app}:

$$I_{app} = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2$$

Substitute numerical values and evaluate I_{app}:

$$I_{app} = (0.5\,\text{kg})(0.2\,\text{m})^2 + (0.5\,\text{kg})(0.2\,\text{m})^2$$
$$= \boxed{0.0400\,\text{kg}\cdot\text{m}^2}$$

Express the moment of inertia of the two spheres and connecting rod system:

$$I = I_{spheres} + I_{rod}$$

Use Table 9-1 to find the moments of inertia of a sphere (with respect to its center of mass) and a rod (with respect to an axis through its center of mass):

$$I_{sphere} = \tfrac{2}{5}M_{sphere}R^2$$

and

$$I_{rod} = \tfrac{1}{12}M_{rod}L^2$$

Because the spheres are not on the axis of rotation, use the parallel axis theorem to express their moment of inertia with respect to the axis of rotation:

$$I_{sphere} = \tfrac{2}{5}M_{sphere}R^2 + M_{sphere}h^2$$

where h is the distance from the center of mass of a sphere to the axis of rotation.

Substitute to obtain:

$$I = 2\left\{\tfrac{2}{5}M_{sphere}R^2 + M_{sphere}h^2\right\} + \tfrac{1}{12}M_{rod}L^2$$

Substitute numerical values and evaluate I:

$$I = 2\left\{\tfrac{2}{5}(0.5\,\text{kg})(0.05\,\text{m})^2 + (0.5\,\text{kg})(0.2\,\text{m})^2\right\} + \tfrac{1}{12}(0.06\,\text{kg})(0.3\,\text{m})^2$$
$$= \boxed{0.0415\,\text{kg}\cdot\text{m}^2}$$

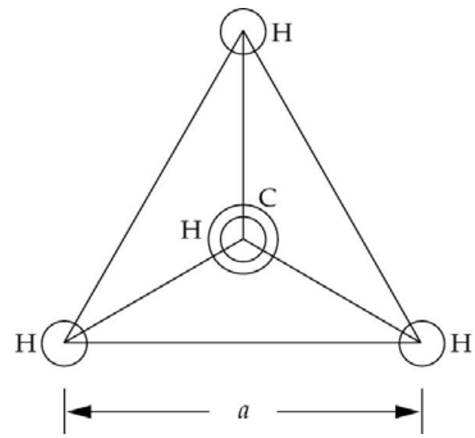
Compare I and I_{app} by taking their ratio:

$$\frac{I_{\text{app}}}{I} = \frac{0.0400\,\text{kg}\cdot\text{m}^2}{0.0415\,\text{kg}\cdot\text{m}^2} = \boxed{0.964}$$

(*b*) The rotational inertia would increase because I_{cm} of a hollow sphere is greater than I_{cm} of a solid sphere.

51 ••

Picture the Problem The axis of rotation passes through the center of the base of the tetrahedron. The carbon atom and the hydrogen atom at the apex of the tetrahedron do not contribute to I because the distance of their nuclei from the axis of rotation is zero. From the geometry, the distance of the three H nuclei from the rotation axis is $a/\sqrt{3}$, where a is the length of a side of the tetrahedron.



Apply the definition of the moment of inertia for a system of particles to obtain:

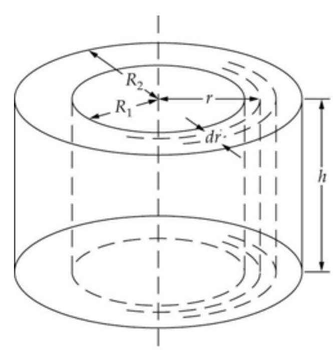
$$I = \sum_i m_i r_i^2 = m_H r_1^2 + m_H r_2^2 + m_H r_3^2$$
$$= 3m_H\left(\frac{a}{\sqrt{3}}\right)^2 = m_H a^2$$

Substitute numerical values and evaluate I:

$$I = (1.67\times10^{-27}\,\text{kg})(0.18\times10^{-9}\,\text{m})^2$$
$$= \boxed{5.41\times10^{-47}\,\text{kg}\cdot\text{m}^2}$$

52 ••

Picture the Problem Let the mass of the element of volume dV be $dm = \rho dV = 2\pi\rho hrdr$ where h is the height of the cylinder. We'll begin by expressing the moment of inertia dI for the element of volume and then integrating it between R_1 and R_2.



Express the moment of inertia of the element of mass dm:

$$dI = r^2 dm = 2\pi\rho h r^3 dr$$

Integrate dI from R_1 to R_2 to obtain:

$$I = 2\pi\rho h \int_{R_1}^{R_2} r^3 dr = \tfrac{1}{2}\pi\rho h\left(R_2^4 - R_1^4\right)$$
$$= \tfrac{1}{2}\pi\rho h\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right)$$

The mass of the hollow cylinder is $m = \pi\rho h\left(R_2^2 - R_1^2\right)$, so:

$$\rho = \frac{m}{\pi h\left(R_2^2 - R_1^2\right)}$$

Substitute for ρ and simplify to obtain:

$$I = \tfrac{1}{2}\pi\left(\frac{m}{\pi h\left(R_2^2 - R_1^2\right)}\right)h\left(R_2^2 - R_1^2\right)\left(R_2^2 + R_1^2\right) = \boxed{\tfrac{1}{2}m\left(R_2^2 + R_1^2\right)}$$

53 •••

Picture the Problem We can derive the given expression for the moment of inertia of a spherical shell by following the procedure outlined in the problem statement.

Find the moment of inertia of a sphere, with respect to an axis through a diameter, in Table 9-1:

$$I = \tfrac{2}{5}mR^2$$

Express the mass of the sphere as a function of its density and radius:

$$m = \tfrac{4}{3}\pi\rho R^3$$

Substitute to obtain:

$$I = \tfrac{8}{15}\pi\rho R^5$$

Express the differential of this expression:

$$dI = \tfrac{8}{3}\pi\rho R^4 dR \qquad (1)$$

Express the increase in mass dm as the radius of the sphere increases by dR:

$$dm = 4\pi\rho R^2 dR \qquad (2)$$

Eliminate dR between equations (1) and (2) to obtain:

$$dI = \tfrac{2}{3}R^2 dm$$

Therefore, the moment of inertia of the spherical shell of mass m is $\tfrac{2}{3}mR^2$.

***54 •••**

Picture the Problem We can find C in terms of M and R by integrating a spherical shell of mass dm with the given density function to find the mass of the earth as a function of M and then solving for C. In part (*b*), we'll start with the moment of inertia of the same spherical shell, substitute the earth's density function, and integrate from 0 to R.

(*a*) Express the mass of the earth using the given density function:

$$M = \int dm = \int_0^R 4\pi\,\rho\,r^2 dr$$

$$= 4\pi\,C\int_0^R 1.22r^2 dr - \frac{4\pi\,C}{R}\int_0^R r^3 dr$$

$$= \frac{4\pi}{3}1.22CR^3 - \pi\,CR^3$$

Solve for C as a function of M and R to obtain:

$$C = \boxed{0.508\frac{M}{R^3}}$$

(*b*) From Problem 9-40 we have:

$$dI = \tfrac{8}{3}\pi\,\rho\,r^4 dr$$

Integrate to obtain:

$$I = \tfrac{8}{3}\pi\int_0^R \rho\,r^4 dr$$

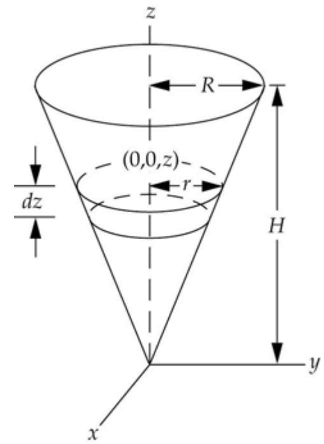
$$= \frac{8\pi(0.508)M}{3R^3}\left[\int_0^R 1.22r^4 dr - \frac{1}{R}\int_0^R r^5 dr\right]$$

$$= \frac{4.26M}{R^3}\left[\frac{1.22}{5}R^5 - \frac{1}{6}R^5\right]$$

$$= \boxed{0.329MR^2}$$

55 •••

Picture the Problem Let the origin be at the apex of the cone, with the *z* axis along the cone's symmetry axis. Then the radius of the elemental ring, at a distance *z* from the apex, can be obtained from the proportion $\frac{r}{z}=\frac{R}{H}$. The mass *dm* of the elemental disk is $\rho dV = \rho\pi r^2 dz$. We'll integrate $r^2 dm$ to find the moment of inertia of the disk in terms of *R* and *H* and then integrate *dm* to obtain a second equation in *R* and *H* that we can use to eliminate *H* in our expression for *I*.



Express the moment of inertia of the cone in terms of the moment of inertia of the elemental disk:

$$I=\tfrac{1}{2}\int r^2 dm$$
$$=\tfrac{1}{2}\int_0^H \frac{R^2}{H^2}z^2\left(\rho\pi\frac{R^2}{H^2}z^2\right)dz$$
$$=\frac{\pi\rho R^4}{2H^4}\int_0^H z^4 dz=\frac{\pi\rho R^4 H}{10}$$

Express the total mass of the cone in terms of the mass of the elemental disk:

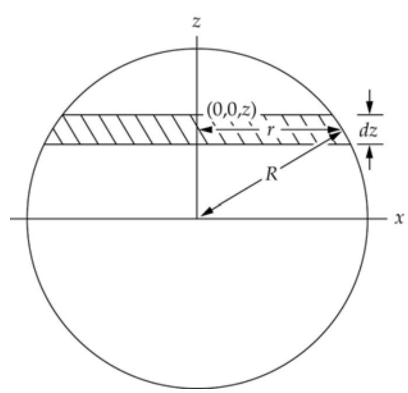
$$M=\pi\rho\int_0^H r^2 dz=\pi\rho\int_0^H \frac{R^2}{H^2}z^2 dz$$
$$=\tfrac{1}{3}\pi\rho R^2 H$$

Divide *I* by *M*, simplify, and solve for *I* to obtain:

$$I=\boxed{\tfrac{3}{10}MR^2}$$

56 •••

Picture the Problem Let the axis of rotation be the *x* axis. The radius *r* of the elemental area is $\sqrt{R^2-z^2}$ and its mass, *dm*, is $\sigma\, dA = 2\sigma\sqrt{R^2-z^2}\,dz$. We'll integrate $z^2\, dm$ to determine *I* in terms of σ and then divide this result by *M* in order to eliminate σ and express *I* in terms of *M* and *R*.



Express the moment of inertia about the x axis:

$$I = \int z^2 dm = \int z^2 \sigma\, dA$$
$$= \int_{-R}^{R} z^2 \left(2\sigma\sqrt{R^2 - z^2}\, dz\right)$$
$$= \tfrac{1}{4}\sigma\pi R^4$$

The mass of the thin uniform disk is:

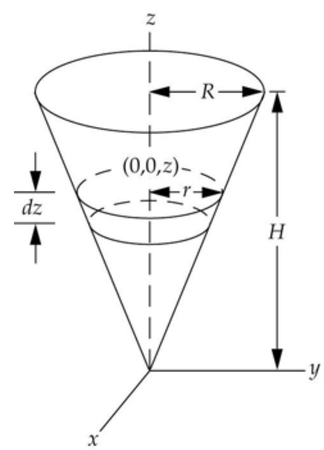
$$M = \sigma\pi R^2$$

Divide I by M, simplify, and solve for I to obtain:

$I = \boxed{\tfrac{1}{4}MR^2}$, a result in agreement with the expression given in Table 9-1 for a cylinder of length $L = 0$.

57 •••

Picture the Problem Let the origin be at the apex of the cone, with the z axis along the cone's symmetry axis, and the axis of rotation be the x rotation. Then the radius of the elemental disk, at a distance z from the apex, can be obtained from the proportion $\frac{r}{z} = \frac{R}{H}$. The mass dm of the elemental disk is $\rho dV = \rho\pi r^2 dz$. Each elemental disk rotates about an axis that is parallel to its diameter but removed from it by a distance z. We can use the result from Problem 9-57 for the moment of inertia of the elemental disk with respect to a diameter and then use the parallel axis theorem to express the moment of inertia of the cone with respect to the x axis.



Using the parallel axis theorem, express the moment of inertia of the elemental disk with respect to the x axis:

$$dI_x = dI_{\text{disk}} + dm\, z^2 \qquad (1)$$

where

$$dm = \rho\, dV = \rho\pi\, r^2 dz$$

In Problem 9-57 it was established that the moment of inertia of a thin uniform disk of mass M and radius R rotating about a diameter is $\tfrac{1}{4}MR^2$. Express this result in

$$dI_{\text{disk}} = \tfrac{1}{4}\left(\rho\pi\, r^2 dz\right) r^2$$
$$= \tfrac{1}{4}\rho\pi\left(\frac{R^2}{H^2}z^2\right)^2 dz$$

terms of our elemental disk:

Substitute in equation (1) to obtain:

$$dI_x = \pi\rho\left[\frac{1}{4}\left(\frac{R^2}{H^2}z^2\right)^2\right]dz + \left(\pi\rho\left(\frac{R}{H}z\right)^2 dz\right)z^2$$

Integrate from 0 to H to obtain:

$$I_x = \pi\rho\int_0^H\left[\frac{1}{4}\left(\frac{R^2}{H^2}z^2\right)^2 + \frac{R^2}{H^2}z^4\right]dz = \pi\rho\left(\frac{R^4H}{20} + \frac{R^2H^3}{5}\right)$$

Express the total mass of the cone in terms of the mass of the elemental disk:

$$M = \pi\rho\int_0^H r^2dz = \pi\rho\int_0^H \frac{R^2}{H^2}z^2dz = \tfrac{1}{3}\pi\rho R^2H$$

Divide I_x by M, simplify, and solve for I_x to obtain:

$$I_x = \boxed{3M\left(\frac{H^2}{5} + \frac{R^2}{20}\right)}$$

Remarks: Because both *H* and *R* appear in the numerator, the larger the cones are, the greater their moment of inertia and the greater the energy consumption required to set them into motion.

Rotational Kinetic Energy

58 •

Picture the Problem The kinetic energy of this rotating system of particles can be calculated either by finding the tangential velocities of the particles and using these values to find the kinetic energy or by finding the moment of inertia of the system and using the expression for the rotational kinetic energy of a system.

(*a*) Use the relationship between v and ω to find the speed of each particle:

$$v_3 = r_3\omega = (0.2\,\text{m})(2\,\text{rad/s}) = 0.4\,\text{m/s}$$

and

$$v_1 = r_1\omega = (0.4\,\text{m})(2\,\text{rad/s}) = 0.8\,\text{m/s}$$

Find the kinetic energy of the system:

$$K = 2K_3 + 2K_1 = m_3 v_3^2 + m_1 v_1^2$$
$$= (3\,\text{kg})(0.4\,\text{m/s})^2 + (1\,\text{kg})(0.8\,\text{m/s})^2$$
$$= \boxed{1.12\,\text{J}}$$

(*b*) Use the definition of the moment of inertia of a system of particles to obtain:

$$I = \sum_i m_i r_i^2$$
$$= m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + m_4 r_4^2$$

Substitute numerical values and evaluate *I*:

$$I = (1\,\text{kg})(0.4\,\text{m})^2 + (3\,\text{kg})(0.2\,\text{m})^2 + (1\,\text{kg})(0.4\,\text{m})^2 + (3\,\text{kg})(0.2\,\text{m})^2$$
$$= 0.560\,\text{kg}\cdot\text{m}^2$$

Calculate the kinetic energy of the system of particles:

$$K = \tfrac{1}{2} I\omega^2 = \tfrac{1}{2}\left(0.560\,\text{kg}\cdot\text{m}^2\right)(2\,\text{rad/s})^2$$
$$= \boxed{1.12\,\text{J}}$$

***59 •**

Picture the Problem We can find the kinetic energy of this rotating ball from its angular speed and its moment of inertia. We can use the same relationship to find the new angular speed of the ball when it is supplied with additional energy.

(*a*) Express the kinetic energy of the ball:

$$K = \tfrac{1}{2} I\omega^2$$

Express the moment of inertia of ball with respect to its diameter:

$$I = \tfrac{2}{5} MR^2$$

Substitute for *I*:

$$K = \tfrac{1}{5} MR^2\omega^2$$

Substitute numerical values and evaluate *K*:

$$K = \tfrac{1}{5}(1.4\,\text{kg})(0.075\,\text{m})^2 \times \left(70\,\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}\right)^2$$
$$= \boxed{84.6\,\text{mJ}}$$

(*b*) Express the new kinetic energy with $K' = 2.0846$ J:

$$K' = \tfrac{1}{2} I\omega'^2$$

Express the ratio of *K* to *K'*:

$$\frac{K'}{K} = \frac{\tfrac{1}{2} I\omega'^2}{\tfrac{1}{2} I\omega'^2} = \left(\frac{\omega'}{\omega}\right)^2$$

Solve for ω':

$$\omega' = \omega\sqrt{\frac{K'}{K}}$$

Substitute numerical values and evaluate ω':

$$\omega' = (70\,\text{rev/min})\sqrt{\frac{2.0846\,\text{J}}{0.0846\,\text{J}}} = \boxed{347\,\text{rev/min}}$$

60 •

Picture the Problem The power delivered by an engine is the product of the torque it develops and the angular speed at which it delivers the torque.

Express the power delivered by the engine as a function of the torque it develops and the angular speed at which it delivers this torque:

$$P = \tau\omega$$

Substitute numerical values and evaluate P:

$$P = (400\,\text{N}\cdot\text{m})\left(3700\frac{\text{rev}}{\text{min}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\times\frac{1\,\text{min}}{60\,\text{s}}\right) = \boxed{155\,\text{kW}}$$

61 ••

Picture the Problem Let r_1 and r_2 be the distances of m_1 and m_2 from the center of mass. We can use the definition of rotational kinetic energy and the definition of the center of mass of the two point masses to show that $K_1/K_2 = m_2/m_1$.

Use the definition of rotational kinetic energy to express the ratio of the rotational kinetic energies:

$$\frac{K_1}{K_2} = \frac{\frac{1}{2}I\omega_1^2}{\frac{1}{2}I\omega_2^2} = \frac{m_1r_1^2\omega^2}{m_2r_2^2\omega^2} = \frac{m_1r_1^2}{m_2r_2^2}$$

Use the definition of the center of mass to relate m_1, m_2, r_1, and r_2:

$$r_1m_1 = r_2m_2$$

Solve for $\frac{r_1}{r_2}$, substitute and simplify to obtain:

$$\frac{K_1}{K_2} = \frac{m_1}{m_2}\left(\frac{m_2}{m_1}\right)^2 = \boxed{\frac{m_2}{m_1}}$$

62 ••

Picture the Problem The earth's rotational kinetic energy is given by $K_{\text{rot}} = \frac{1}{2}I\omega^2$ where I is its moment of inertia with respect to its axis of rotation. The center of mass of the earth-sun system is so close to the center of the sun and the earth-sun distance so large that we can use the earth-sun distance as the separation of their centers of mass and assume each to be point mass.

Express the rotational kinetic energy of the earth:

$$K_{rot} = \tfrac{1}{2}I\omega^2 \qquad (1)$$

Find the angular speed of the earth's rotation using the definition of ω:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{24\,\text{h}\times\frac{3600\,\text{s}}{\text{h}}} = 7.27\times10^{-5}\ \text{rad/s}$$

From Table 9-1, for the moment of inertia of a homogeneous sphere, we find:

$$I = \tfrac{2}{5}MR^2 = \tfrac{2}{5}\left(6.0\times10^{24}\ \text{kg}\right)\left(6.4\times10^{6}\ \text{m}\right)^2 = 9.83\times10^{37}\ \text{kg}\cdot\text{m}^2$$

Substitute numerical values in equation (1) to obtain:

$$K_{rot} = \tfrac{1}{2}\left(9.83\times10^{37}\ \text{kg}\cdot\text{m}^2\right)\times\left(7.27\times10^{-5}\ \text{rad/s}\right)^2 = \boxed{2.60\times10^{29}\ \text{J}}$$

Express the earth's orbital kinetic energy:

$$K_{orb} = \tfrac{1}{2}I\omega_{orb}^2 \qquad (2)$$

Find the angular speed of the center of mass of the earth-sun system:

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\,\text{rad}}{365.25\,\text{days}\times24\frac{\text{h}}{\text{day}}\times\frac{3600\,\text{s}}{\text{h}}} = 1.99\times10^{-7}\ \text{rad/s}$$

Express and evaluate the orbital moment of inertia of the earth:

$$I = M_E R_{orb}^2 = \left(6.0\times10^{24}\ \text{kg}\right)\left(1.50\times10^{11}\ \text{m}\right)^2 = 1.35\times10^{47}\ \text{kg}\cdot\text{m}^2$$

Substitute in equation (2) to obtain:

$$K_{orb} = \tfrac{1}{2}\left(1.35\times10^{47}\ \text{kg}\cdot\text{m}^2\right)\times\left(1.99\times10^{-7}\ \text{rad/s}\right)^2 = 2.67\times10^{33}\ \text{J}$$

Evaluate the ratio $\frac{K_{\text{orb}}}{K_{\text{rot}}}$:

$$\frac{K_{\text{orb}}}{K_{\text{rot}}} = \frac{2.67\times10^{33}\ \text{J}}{2.60\times10^{29}\ \text{J}} \approx \boxed{10^4}$$

***63** ••

Picture the Problem Because the load is not being accelerated, the tension in the cable equals the weight of the load. The role of the massless pulley is to change the direction the force (tension) in the cable acts.

(*a*) Because the block is lifted at constant speed:

$$T = mg = (2000\,\text{kg})(9.81\,\text{m/s}^2) = \boxed{19.6\,\text{kN}}$$

(*b*) Apply the definition of torque at the winch drum:

$$\tau = Tr = (19.6\,\text{kN})(0.30\,\text{m}) = \boxed{5.89\,\text{kN}\cdot\text{m}}$$

(*c*) Relate the angular speed of the winch drum to the rate at which the load is being lifted (the tangential speed of the cable on the drum):

$$\omega = \frac{v}{r} = \frac{0.08\,\text{m/s}}{0.30\,\text{m}} = \boxed{0.267\,\text{rad/s}}$$

(*d*) Express the power developed by the motor in terms of the tension in the cable and the speed with which the load is being lifted:

$$P = Tv = (19.6\,\text{kN})(0.08\,\text{m/s}) = \boxed{1.57\,\text{kW}}$$

64 ••

Picture the Problem Let the zero of gravitational potential energy be at the lowest point of the small particle. We can use conservation of energy to find the angular velocity of the disk when the particle is at its lowest point and Newton's 2nd law to find the force the disk will have to exert on the particle to keep it from falling off.

(*a*) Use conservation of energy to relate the initial potential energy of the system to its rotational kinetic energy when the small particle is at its lowest point:

$$\Delta K + \Delta U = 0$$

or, because $U_f = K_i = 0$,

$$\tfrac{1}{2}\left(I_{\text{disk}} + I_{\text{particle}}\right)\omega_f^2 - mg\Delta h = 0$$

Solve for ω_f:

$$\omega_f = \sqrt{\frac{2mg\Delta h}{I_{\text{disk}} + I_{\text{particle}}}}$$

Substitute for I_{disk}, $I_{particle}$, and Δh and simplify to obtain:

$$\omega_f = \sqrt{\frac{2mg(2R)}{\frac{1}{2}MR^2 + mR^2}} = \boxed{\sqrt{\frac{8mg}{R(2m+M)}}}$$

(*b*) The mass is in uniform circular motion at the bottom of the disk, so the sum of the force *F* exerted by the disk and the gravitational force must be the centripetal force:

$$F - mg = mR\omega_f^2$$

Solve for *F* and simplify to obtain:

$$F = mg + mR\omega_f^2$$
$$= mg + mR\left(\frac{8mg}{R(2mM)}\right)$$
$$= \boxed{mg\left(1 + \frac{8m}{2m+M}\right)}$$

65 ••

Picture the Problem Let the zero of gravitational potential energy be at the center of mass of the ring when it is directly below the point of support. We'll use conservation of energy to relate the maximum angular velocity and the initial angular velocity required for a complete revolution to the changes in the potential energy of the ring.

(*a*) Use conservation of energy to relate the initial potential energy of the ring to its rotational kinetic energy when its center of mass is directly below the point of support:

$$\Delta K + \Delta U = 0$$

or, because $U_f = K_i = 0$,

$$\tfrac{1}{2}I_P\omega_{max}^2 - mg\Delta h = 0 \qquad (1)$$

Use the parallel axis theorem and Table 9-1 to express the moment of inertia of the ring with respect to its pivot point *P*:

$$I_P = I_{cm} + mR^2$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}\left(mR^2 + mR^2\right)\omega_{max}^2 - mgR = 0$$

Solve for ω_{max}:

$$\omega_{max} = \sqrt{\frac{g}{R}}$$

Substitute numerical values and evaluate ω_{max}:

$$\omega_{max} = \sqrt{\frac{9.81\,\text{m/s}^2}{0.75\,\text{m}}} = \boxed{3.62\,\text{rad/s}}$$

(*b*) Use conservation of energy to relate the final potential energy of the ring to its initial rotational kinetic energy:

$$\Delta K + \Delta U = 0$$

or, because $U_i = K_f = 0$,

$$-\tfrac{1}{2} I_P \omega_i^2 + mg\Delta h = 0$$

Noting that the center of mass must rise a distance R if the ring is to make a complete revolution, substitute for I_P and Δh to obtain:

$$-\tfrac{1}{2}\left(mR^2 + mR^2\right)\omega_i^2 + mgR = 0$$

Solve for ω_i:

$$\omega_i = \sqrt{\frac{g}{R}}$$

Substitute numerical values and evaluate ω_i:

$$\omega_i = \sqrt{\frac{9.81\,\text{m/s}^2}{0.75\,\text{m}}} = \boxed{3.62\,\text{rad/s}}$$

66 ••

Picture the Problem We can find the energy that must be stored in the flywheel and relate this energy to the radius of the wheel and use the definition of rotational kinetic energy to find the wheel's radius.

Relate the kinetic energy of the flywheel to the energy it must deliver:

$$K_{\text{rot}} = \tfrac{1}{2} I_{\text{cyl}}\omega^2 = (2\,\text{MJ/km})(300\,\text{km}) = 600\,\text{MJ}$$

Express the moment of inertia of the flywheel:

$$I_{\text{cyl}} = \tfrac{1}{2} MR^2$$

Substitute for I_{cyl} and solve for ω:

$$R = \frac{2}{\omega}\sqrt{\frac{K_{\text{rot}}}{M}}$$

Substitute numerical values and evaluate R:

$$R = \frac{2}{400\,\frac{\text{rev}}{\text{s}} \times \frac{2\pi\,\text{rad}}{\text{rev}}}\sqrt{\frac{600\,\text{MJ} \times \frac{10^6\,\text{J}}{\text{MJ}}}{100\,\text{kg}}} = \boxed{1.95\,\text{m}}$$

67 ••

Picture the Problem We'll solve this problem for the general case of a ladder of length L, mass M, and person of mass m. Let the zero of gravitational potential energy be at floor level and include you, the ladder, and the earth in the system. We'll use conservation of energy to relate your impact speed falling freely to your impact speed riding the ladder to the ground.

Use conservation of energy to relate the speed with which a person will strike the ground to the fall distance L:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2} m v_f^2 - mgL = 0$$

Solve for v_f^2:

$$v_f^2 = 2gL$$

Letting ω_r represent the angular velocity of the ladder+person system as it strikes the ground, use conservation of energy to relate the initial and final momenta of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}\left(I_{\text{person}} + I_{\text{ladder}}\right)\omega_r^2 - \left(mgL + Mg\frac{L}{2}\right) = 0$$

Substitute for the moments of inertia to obtain:

$$\tfrac{1}{2}\left(m + \frac{1}{3}M\right)L^2\omega_f^2 - \left(mgL + Mg\frac{L}{2}\right) = 0$$

Substitute v_r for $L\omega_f$ and solve for v_r^2:

$$v_r^2 = \frac{2gL\left(m + \dfrac{M}{2}\right)}{m + \dfrac{M}{3}}$$

Express the ratio $\dfrac{v_r^2}{v_f^2}$:

$$\frac{v_r^2}{v_f^2} = \frac{m + \dfrac{M}{2}}{m + \dfrac{M}{3}}$$

Unless M, the mass of the ladder, is zero, $v_r > v_f$. It is better to let go and fall to the ground.

Pulleys, Yo-Yos, and Hanging Things

***68 ••**

Picture the Problem We'll solve this problem for the general case in which the mass of the block on the ledge is M, the mass of the hanging block is m, and the mass of the pulley is M_p, and R is the radius of the pulley. Let the zero of gravitational potential energy be 2.5 m below the initial position of the 2-kg block and R represent the radius of the pulley. Let the system include both blocks, the shelf and pulley, and the earth. The initial potential energy of the 2-kg block will be transformed into the translational kinetic energy of both blocks plus rotational kinetic energy of the pulley.

(*a*) Use energy conservation to relate the speed of the 2 kg block when it has fallen a distance Δh to its initial potential energy and the kinetic energy of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}I_{\text{pulley}}\omega^2 - mgh = 0$$

Substitute for I_{pulley} and ω to obtain:

$$\tfrac{1}{2}(m+M)v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{v^2}{R^2} - mgh = 0$$

Solve for v:

$$v = \sqrt{\frac{2mgh}{M + m + \frac{1}{2}M_p}}$$

Substitute numerical values and evaluate v:

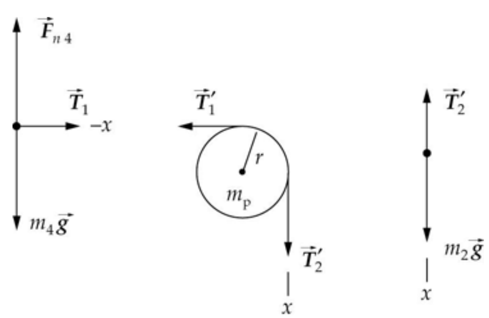
$$v = \sqrt{\frac{2(2\,\text{kg})(9.81\,\text{m/s}^2)(2.5\,\text{m})}{4\text{kg} + 2\,\text{kg} + \frac{1}{2}(0.6\,\text{kg})}} = \boxed{3.95\,\text{m/s}}$$

(*b*) Find the angular velocity of the pulley from its tangential speed:

$$\omega = \frac{v}{R} = \frac{3.95\,\text{m/s}}{0.08\,\text{m}} = \boxed{49.3\,\text{rad/s}}$$

69 ••

Picture the Problem The diagrams show the forces acting on each of the masses and the pulley. We can apply Newton's 2nd law to the two blocks and the pulley to obtain three equations in the unknowns T_1, T_2, and a.



Apply Newton's 2nd law to the two blocks and the pulley:

$$\sum F_x = T_1 = m_4 a\,, \qquad (1)$$

$$\sum \tau_p = (T_2 - T_1)r = I_p\alpha\,, \qquad (2)$$

and

$$\sum F_x = m_2 g - T_2 = m_2 a \qquad (3)$$

Eliminate α in equation (2) to obtain:

$$T_2 - T_1 = \tfrac{1}{2}M_p a \qquad (4)$$

Eliminate T_1 and T_2 between equations (1), (3) and (4) and solve for a:

$$a = \frac{m_2 g}{m_2 + m_4 + \frac{1}{2}M_p}$$

Substitute numerical values and evaluate a:

$$a = \frac{(2\,\text{kg})(9.81\,\text{m/s}^2)}{2\,\text{kg} + 4\,\text{kg} + \frac{1}{2}(0.6\,\text{kg})} = \boxed{3.11\,\text{m/s}^2}$$

Using equation (1), evaluate T_1:

$$T_1 = (4\,\text{kg})(3.11\,\text{m/s}^2) = \boxed{12.5\,\text{N}}$$

Solve equation (3) for T_2:

$$T_2 = m_2(g - a)$$

Substitute numerical values and evaluate T_2:

$$T_2 = (2\,\text{kg})(9.81\,\text{m/s}^2 - 3.11\,\text{m/s}^2) = \boxed{13.4\,\text{N}}$$

70 ••

Picture the Problem We'll solve this problem for the general case in which the mass of the block on the ledge is M, the mass of the hanging block is m, the mass of the pulley is M_p, and R is the radius of the pulley. Let the zero of gravitational potential energy be 2.5 m below the initial position of the 2-kg block. The initial potential energy of the 2-kg block will be transformed into the translational kinetic energy of both blocks plus rotational kinetic energy of the pulley plus work done against friction.

(*a*) Use energy conservation to relate the speed of the 2 kg block when it has fallen a distance Δh to its initial potential energy, the kinetic energy of the system and the work done against friction:

$$\Delta K + \Delta U + W_f = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}(m + M)v^2 + \tfrac{1}{2}I_{\text{pulley}}\omega^2 - mgh + \mu_k Mgh = 0$$

Substitute for I_{pulley} and ω to obtain:

$$\tfrac{1}{2}(m + M)v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}M_p\right)\frac{v^2}{R^2} - mgh + \mu_k Mgh = 0$$

Solve for v:

$$v=\sqrt{\frac{2gh\left(m-\mu_{\mathrm{k}}M\right)}{M+m+\frac{1}{2}M_p}}$$

Substitute numerical values and evaluate v:

$$v=\sqrt{\frac{2\left(9.81\,\mathrm{m/s^2}\right)\left(2.5\,\mathrm{m}\right)\left[2\,\mathrm{kg}-\left(0.25\right)\left(4\,\mathrm{kg}\right)\right]}{4\mathrm{kg}+2\,\mathrm{kg}+\frac{1}{2}\left(0.6\,\mathrm{kg}\right)}}=\boxed{2.79\,\mathrm{m/s}}$$

(*b*) Find the angular velocity of the pulley from its tangential speed:

$$\omega=\frac{v}{R}=\frac{2.79\,\mathrm{m/s}}{0.08\,\mathrm{m}}=\boxed{34.9\,\mathrm{rad/s}}$$

71 ••

Picture the Problem Let the zero of gravitational potential energy be at the water's surface and let the system include the winch, the car, and the earth. We'll apply energy conservation to relate the car's speed as it hits the water to its initial potential energy. Note that some of the car's initial potential energy will be transformed into rotational kinetic energy of the winch and pulley.

Use energy conservation to relate the car's speed as it hits the water to its initial potential energy:

$$\Delta K+\Delta U=0$$

or, because $K_{\mathrm{i}}=U_{\mathrm{f}}=0$,

$$\tfrac{1}{2}mv^2+\tfrac{1}{2}I_{\mathrm{w}}\omega_{\mathrm{w}}^2+\tfrac{1}{2}I_{\mathrm{p}}\omega_{\mathrm{p}}^2-mg\Delta h=0$$

Express ω_{w} and ω_{p} in terms of the speed v of the rope, which is the same throughout the system:

$$\omega_{\mathrm{w}}=\frac{v^2}{r_{\mathrm{w}}^2}\text{ and }\omega_{\mathrm{p}}=\frac{v^2}{r_{\mathrm{p}}^2}$$

Substitute to obtain:

$$\tfrac{1}{2}mv^2+\tfrac{1}{2}I_{\mathrm{w}}\frac{v^2}{r_{\mathrm{w}}^2}+\tfrac{1}{2}I_{\mathrm{p}}\frac{v^2}{r_{\mathrm{p}}^2}-mg\Delta h=0$$

Solve for v:

$$v=\sqrt{\frac{2mg\Delta h}{m+\frac{I_{\mathrm{w}}}{r_{\mathrm{w}}^2}+\frac{I_{\mathrm{p}}}{r_{\mathrm{p}}^2}}}$$

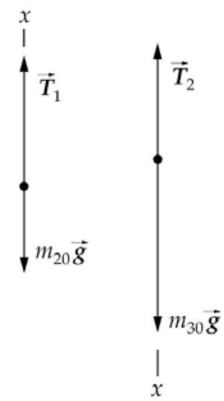
Substitute numerical values and evaluate v:

$$v=\sqrt{\frac{2\left(1200\,\mathrm{kg}\right)\left(9.81\,\mathrm{m/s^2}\right)\left(5\,\mathrm{m}\right)}{1200\,\mathrm{kg}+\frac{320\,\mathrm{kg\cdot m^2}}{\left(0.8\,\mathrm{m}\right)^2}+\frac{4\,\mathrm{kg\cdot m^2}}{\left(0.3\,\mathrm{m}\right)^2}}}$$

$$=\boxed{8.21\,\mathrm{m/s}}$$

***72 ••**

Picture the Problem Let the system include the blocks, the pulley and the earth. Choose the zero of gravitational potential energy to be at the ledge and apply energy conservation to relate the impact speed of the 30-kg block to the initial potential energy of the system. We can use a constant-acceleration equations and Newton's 2nd law to find the tensions in the strings and the descent time.



(*a*) Use conservation of energy to relate the impact speed of the 30-kg block to the initial potential energy of the system:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$\tfrac{1}{2}m_{30}v^2 + \tfrac{1}{2}m_{20}v^2 + \tfrac{1}{2}I_p\omega_p^2 + m_{20}g\Delta h - m_{30}g\Delta h = 0$$

Substitute for ω_p and I_p to obtain:

$$\tfrac{1}{2}m_{30}v^2 + \tfrac{1}{2}m_{20}v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}M_p r^2\right)\left(\frac{v^2}{r^2}\right) + m_{20}g\Delta h - m_{30}g\Delta h = 0$$

Solve for v:

$$v = \sqrt{\frac{2g\Delta h(m_{30} - m_{20})}{m_{20} + m_{30} + \frac{1}{2}M_p}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\frac{2(9.81\,\text{m/s}^2)(2\,\text{m})(30\,\text{kg} - 20\,\text{kg})}{20\,\text{kg} + 30\,\text{kg} + \frac{1}{2}(5\,\text{kg})}} = \boxed{2.73\,\text{m/s}}$$

(*b*) Find the angular speed at impact from the tangential speed at impact and the radius of the pulley:

$$\omega = \frac{v}{r} = \frac{2.73\,\text{m/s}}{0.1\,\text{m}} = \boxed{27.3\,\text{rad/s}}$$

(*c*) Apply Newton's 2nd law to the blocks:

$$\sum F_x = T_1 - m_{20}g = m_{20}a \qquad (1)$$

$$\sum F_x = m_{30}g - T_2 = m_{30}a \qquad (2)$$

Using a constant-acceleration equation, relate the speed at impact to the fall distance and the

$$v^2 = v_0^2 + 2a\Delta h$$

or, because $v_0 = 0$,

acceleration and solve for and evaluate a:

$$a=\frac{v^2}{2\Delta h}=\frac{(2.73\,\text{m/s})^2}{2(2\,\text{m})}=1.87\,\text{m/s}^2$$

Substitute in equation (1) to find T_1:

$$T_1=m_{20}(g+a)$$
$$=(20\,\text{kg})(9.81\,\text{m/s}^2+1.87\,\text{m/s}^2)$$
$$=\boxed{234\,\text{N}}$$

Substitute in equation (2) to find T_2:

$$T_2=m_{30}(g-a)$$
$$=(30\,\text{kg})(9.81\,\text{m/s}^2-1.87\,\text{m/s}^2)$$
$$=\boxed{238\,\text{N}}$$

(*d*) Noting that the initial speed of the 30-kg block is zero, express the time-of-fall in terms of the fall distance and the block's average speed:

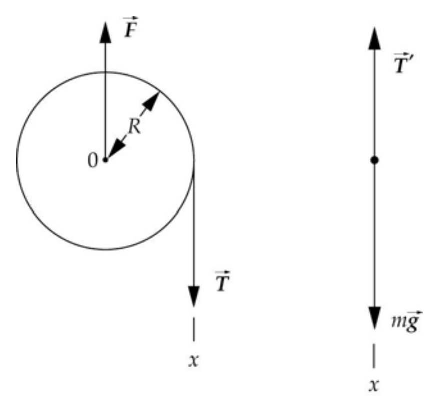
$$\Delta t=\frac{\Delta h}{v_{\text{av}}}=\frac{\Delta h}{\frac{1}{2}v}=\frac{2\Delta h}{v}$$

Substitute numerical values and evaluate Δt:

$$\Delta t=\frac{2(2\,\text{m})}{2.73\,\text{m/s}}=\boxed{1.47\,\text{s}}$$

73 ••

Picture the Problem The force diagram shows the forces acting on the sphere and the hanging object. The tension in the string is responsible for the angular acceleration of the sphere and the difference between the weight of the object and the tension is the net force acting on the hanging object. We can use Newton's 2nd law to obtain two equations in a and T that we can solve simultaneously.



(*a*)Apply Newton's 2nd law to the sphere and the hanging object:

$$\sum\tau_0=TR=I_{\text{sphere}}\alpha \qquad (1)$$

and

$$\sum F_x=mg-T=ma \qquad (2)$$

Substitute for I_{sphere} and α in equation (1) to obtain:

$$TR=\left(\tfrac{2}{5}MR^2\right)\frac{a}{R} \qquad (3)$$

Eliminate T between equations (2) and (3) and solve for a to obtain:

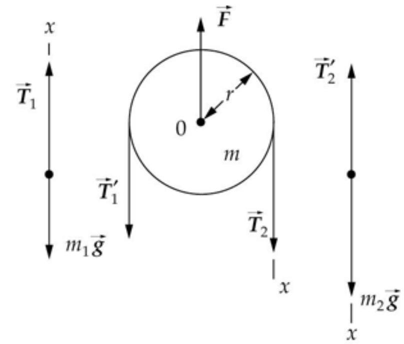
$$a = \boxed{\frac{g}{1+\frac{2M}{5m}}}$$

(b) Substitute for a in equation (2) and solve for T to obtain:

$$T = \boxed{\frac{2mMg}{5m+2M}}$$

74 ••

Picture the Problem The diagram shows the forces acting on both objects and the pulley. By applying Newton's 2nd law of motion, we can obtain a system of three equations in the unknowns T_1, T_2, and a that we can solve simultaneously.



(a) Apply Newton's 2nd law to the pulley and the two objects:

$$\sum F_x = T_1 - m_1 g = m_1 a, \qquad (1)$$

$$\sum \tau_0 = (T_2 - T_1)r = I_0\alpha, \qquad (2)$$

and

$$\sum F_x = m_2 g - T_2 = m_2 a \qquad (3)$$

Substitute for $I_0 = I_{\text{pulley}}$ and α in equation (2) to obtain:

$$(T_2 - T_1)r = \left(\tfrac{1}{2}mr^2\right)\frac{a}{r} \qquad (4)$$

Eliminate T_1 and T_2 between equations (1), (3) and (4) and solve for a to obtain:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + \frac{1}{2}m}$$

Substitute numerical values and evaluate a:

$$a = \frac{(510\,\text{g} - 500\,\text{g})(981\,\text{cm/s}^2)}{500\,\text{g} + 510\,\text{g} + \frac{1}{2}(50\,\text{g})} = \boxed{9.478\,\text{cm/s}^2}$$

(b) Substitute for a in equation (1) and solve for T_1 to obtain:

$$T_1 = m_1(g+a) = (0.500\,\text{kg})\left(9.81\,\text{m/s}^2 + 0.09478\,\text{m/s}^2\right) = \boxed{4.9524\,\text{N}}$$

Substitute for a in equation (3) and solve for T_2 to obtain:

$$T_2 = m_2(g-a) = (0.510\,\text{kg})(9.81\,\text{m/s}^2 - 0.09478\,\text{m/s}^2) = \boxed{4.9548\,\text{N}}$$

Find ΔT:

$$\Delta T = T_2 - T_1 = 4.9548\,\text{N} - 4.9524\,\text{N} = \boxed{0.0024\,\text{N}}$$

(c) If we ignore the mass of the pulley, our acceleration equation is:

$$a = \frac{(m_2 - m_1)g}{m_1 + m_2}$$

Substitute numerical values and evaluate a:

$$a = \frac{(510\,\text{g} - 500\,\text{g})(981\,\text{cm/s}^2)}{500\,\text{g} + 510\,\text{g}} = \boxed{9.713\,\text{cm/s}^2}$$

Substitute for a in equation (1) and solve for T_1 to obtain:

$$T_1 = m_1(g+a)$$

Substitute numerical values and evaluate T_1:

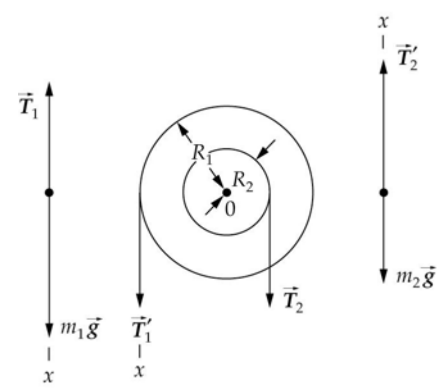
$$T_1 = (0.500\,\text{kg})(9.81\,\text{m/s}^2 + 0.09713\,\text{m/s}^2) = \boxed{4.9536\,\text{N}}$$

From equation (4), if $m = 0$:

$$\boxed{T_1 = T_2}$$

***75 ••**

Picture the Problem The diagram shows the forces acting on both objects and the pulley. By applying Newton's 2nd law of motion, we can obtain a system of three equations in the unknowns T_1, T_2, and α that we can solve simultaneously.



(a) Express the condition that the system does not accelerate:

$$\tau_{\text{net}} = m_1 g R_1 - m_2 g R_2 = 0$$

Solve for m_2:

$$m_2 = m_1 \frac{R_1}{R_2}$$

Substitute numerical values and evaluate m_2:

$$m_2 = (24\,\text{kg})\frac{1.2\,\text{m}}{0.4\,\text{m}} = \boxed{72.0\,\text{kg}}$$

(*b*) Apply Newton's 2nd law to the objects and the pulley:

$$\sum F_x = m_1 g - T_1 = m_1 a\,, \qquad (1)$$

$$\sum \tau_0 = T_1 R_1 - T_2 R_2 = I_0 \alpha\,, \qquad (2)$$

and

$$\sum F_x = T_2 - m_2 g = m_2 a \qquad (3)$$

Eliminate a in favor of α in equations (1) and (3) and solve for T_1 and T_2:

$$T_1 = m_1 (g - R_1 \alpha) \qquad (4)$$

and

$$T_2 = m_2 (g + R_2 \alpha) \qquad (5)$$

Substitute for T_1 and T_2 in equation (2) and solve for α to obtain:

$$\alpha = \frac{(m_1 R_1 - m_2 R_2) g}{m_1 R_1^2 + m_2 R_2^2 + I_0}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{[(36\,\text{kg})(1.2\,\text{m}) - (72\,\text{kg})(0.4\,\text{m})](9.81\,\text{m/s}^2)}{(36\,\text{kg})(1.2\,\text{m})^2 + (72\,\text{kg})(0.4\,\text{m})^2 + 40\,\text{kg}\cdot\text{m}^2} = \boxed{1.37\,\text{rad/s}^2}$$

Substitute in equation (4) to find T_1:

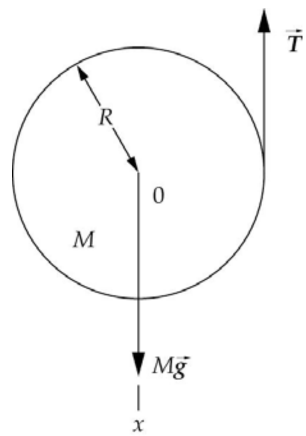
$$T_1 = (36\,\text{kg})\left[9.81\,\text{m/s}^2 - (1.2\,\text{m})(1.37\,\text{rad/s}^2)\right] = \boxed{294\,\text{N}}$$

Substitute in equation (5) to find T_2:

$$T_2 = (72\,\text{kg})\left[9.81\,\text{m/s}^2 + (0.4\,\text{m})(1.37\,\text{rad/s}^2)\right] = \boxed{746\,\text{N}}$$

76 ••

Picture the Problem Choose the coordinate system shown in the diagram. By applying Newton's 2nd law of motion, we can obtain a system of two equations in the unknowns T and a. In (b) we can use the torque equation from (a) and our value for T to find α. In (c) we use the condition that the acceleration of a point on the rim of the cylinder is the same as the acceleration of the hand, together with the angular acceleration of the cylinder, to find the acceleration of the hand.



(a) Apply Newton's 2nd law to the cylinder about an axis through its center of mass:

$$\sum \tau_0 = TR = I_0 \frac{a}{R} \qquad (1)$$

and

$$\sum F_x = Mg - T = 0 \qquad (2)$$

Solve for T to obtain:

$$T = \boxed{Mg}$$

(b) Rewrite equation (1) in terms of α:

$$TR = I_0 \alpha$$

Solve for α:

$$\alpha = \frac{TR}{I_0}$$

Substitute for T and I_0 to obtain:

$$\alpha = \frac{MgR}{\frac{1}{2}MR^2} = \boxed{\frac{2g}{R}}$$

(c) Relate the acceleration a of the hand to the angular acceleration of the cylinder:

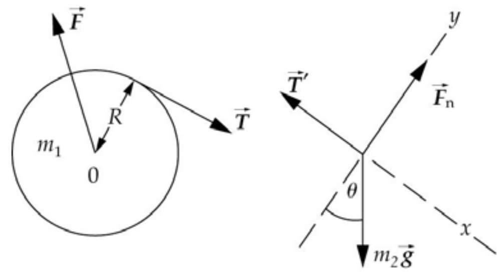
$$a = R\alpha$$

Substitute for α to obtain:

$$a = R\left(\frac{2g}{R}\right) = \boxed{2g}$$

77 ••

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the incline. By applying Newton's 2nd law to the cylinder and the block we can obtain simultaneous equations in a, T, and α from which we can express a and T. By applying the conservation of energy, we can derive an expression for the speed of the block when it reaches the bottom of the incline.



(*a*) Apply Newton's 2nd law to the cylinder and the block:

$$\sum \tau_0 = TR = I_0\alpha \qquad (1)$$

and

$$\sum F_x = m_2 g \sin\theta - T = m_2 a \qquad (2)$$

Substitute for α in equation (1), solve for T, and substitute in equation (2) and solve for a to obtain:

$$a = \boxed{\frac{g\sin\theta}{1+\dfrac{m_1}{2m_2}}}$$

(*b*) Substitute for a in equation (2) and solve for T:

$$T = \boxed{\frac{\frac{1}{2}m_1 g\sin\theta}{1+\dfrac{m_1}{2m_2}}}$$

(*c*) Noting that the block is released from rest, express the total energy of the system when the block is at height h:

$$E = U + K = \boxed{m_2 gh}$$

(*d*) Use the fact that this system is conservative to express the total energy at the bottom of the incline:

$$E_{\text{bottom}} = \boxed{m_2 gh}$$

(*e*) Express the total energy of the system when the block is at the bottom of the incline in terms of its kinetic energies:

$$E_{\text{bottom}} = K_{\text{tran}} + K_{\text{rot}} = \tfrac{1}{2}m_2 v^2 + \tfrac{1}{2}I_0\omega^2$$

Substitute for ω and I_0 to obtain:

$$\tfrac{1}{2}m_2v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m_1r^2\right)\frac{v^2}{r^2} = m_2gh$$

Solve for v to obtain:

$$\boxed{v = \sqrt{\frac{2gh}{1+\dfrac{m_1}{2m_2}}}}$$

(*f*) For $\theta = 0$:

$$\boxed{a = T = 0}$$

For $\theta = 90°$:

$$a = \boxed{\frac{g}{1+\dfrac{m_1}{2m_2}}},$$

$$T = \frac{\tfrac{1}{2}m_1g}{1+\dfrac{m_1}{2m_2}} = \boxed{\tfrac{1}{2}m_1a},$$

and

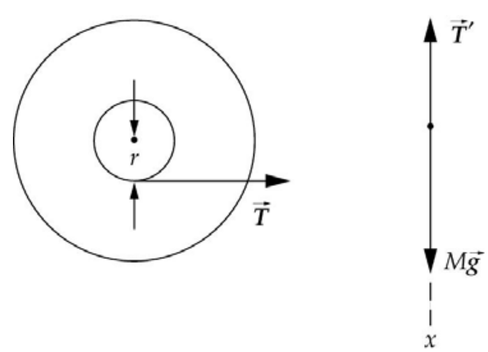
$$\boxed{v = \sqrt{\frac{2gh}{1+\dfrac{m_1}{2m_2}}}}$$

For $m_1 = 0$:

$$a = \boxed{g\sin\theta},\ T = \boxed{0},\ \text{and}$$
$$v = \boxed{\sqrt{2gh}}$$

***78 ••**

Picture the Problem Let r be the radius of the concentric drum (10 cm) and let I_0 be the moment of inertia of the drum plus platform. We can use Newton's 2nd law in both translational and rotational forms to express I_0 in terms of a and a constant-acceleration equation to express a and then find I_0. We can use the same equation to find the total moment of inertia when the object is placed on the platform and then subtract to find its moment of inertia.



(*a*) Apply Newton's 2nd law to the platform and the weight:

$$\sum \tau_0 = Tr = I_0\alpha \qquad (1)$$
$$\sum F_x = Mg - T = Ma \qquad (2)$$

Substitute a/r for α in equation (1) and solve for T:

$$T = \frac{I_0}{r^2}a$$

Substitute for T in equation (2) and solve for a to obtain:

$$I_0 = \frac{Mr^2(g-a)}{a} \qquad (3)$$

Using a constant-acceleration equation, relate the distance of fall to the acceleration of the weight and the time of fall and solve for the acceleration:

$$\Delta x = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$ and $\Delta x = D$,

$$a = \frac{2D}{(\Delta t)^2}$$

Substitute for a in equation (3) to obtain:

$$I_0 = Mr^2\left(\frac{g}{a}-1\right) = Mr^2\left(\frac{g(\Delta t)^2}{2D}-1\right)$$

Substitute numerical values and evaluate I_0:

$$I_0 = (2.5\,\text{kg})(0.1\,\text{m})^2 \times \left[\frac{(9.81\,\text{m/s}^2)(4.2\,\text{s})^2}{2(1.8\,\text{m})}-1\right] = \boxed{1.177\,\text{kg}\cdot\text{m}^2}$$

(*b*) Relate the moments of inertia of the platform, drum, shaft, and pulley (I_0) to the moment of inertia of the object and the total moment of inertia:

$$I_{\text{tot}} = I_0 + I = Mr^2\left(\frac{g}{a}-1\right) = Mr^2\left(\frac{g(\Delta t)^2}{2D}-1\right)$$

Substitute numerical values and evaluate I_{tot}:

$$I_{\text{tot}} = (2.5\,\text{kg})(0.1\,\text{m})^2 \times \left[\frac{(9.81\,\text{m/s}^2)(6.8\,\text{s})^2}{2(1.8\,\text{m})}-1\right] = \boxed{3.125\,\text{kg}\cdot\text{m}^2}$$

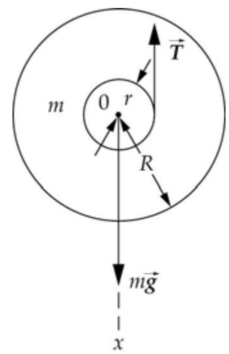
Solve for and evaluate I:

$$I = I_{\text{tot}} - I_0 = 3.125\,\text{kg}\cdot\text{m}^2 - 1.177\,\text{kg}\cdot\text{m}^2 = \boxed{1.948\,\text{kg}\cdot\text{m}^2}$$

Objects Rolling Without Slipping

***79 ••**

Picture the Problem The forces acting on the yo-yo are shown in the figure. We can use a constant-acceleration equation to relate the velocity of descent at the end of the fall to the yo-yo's acceleration and Newton's 2nd law in both translational and rotational form to find the yo-yo's acceleration.



Using a constant-acceleration equation, relate the yo-yo's final speed to its acceleration and fall distance:

$$v^2 = v_0^2 + 2a\Delta h$$

or, because $v_0 = 0$,

$$v = \sqrt{2a\Delta h} \qquad (1)$$

Use Newton's 2nd law to relate the forces that act on the yo-yo to its acceleration:

$$\sum F_x = mg - T = ma \qquad (2)$$

and

$$\sum \tau_0 = Tr = I_0\alpha \qquad (3)$$

Use $a = r\alpha$ to eliminate α in equation (3)

$$Tr = I_0\frac{a}{r} \qquad (4)$$

Eliminate T between equations (2) and (4) to obtain:

$$mg - \frac{I_0}{r^2}a = ma \qquad (5)$$

Substitute $\frac{1}{2}mR^2$ for I_0 in equation (5):

$$mg - \frac{\frac{1}{2}mR^2}{r^2}a = ma$$

Solve for a:

$$a = \frac{g}{1 + \frac{R^2}{2r^2}}$$

Substitute numerical values and evaluate a:

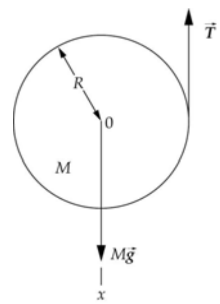
$$a = \frac{9.81\,\text{m/s}^2}{1 + \frac{(1.5\,\text{m})^2}{2(0.1\,\text{m})^2}} = 0.0864\,\text{m/s}^2$$

Substitute in equation (1) and evaluate v:

$$v = \sqrt{2(0.0864\,\text{m/s}^2)(57\,\text{m})} = \boxed{3.14\,\text{m/s}}$$

80 ••

Picture the Problem The diagram shows the forces acting on the cylinder. By applying Newton's 2nd law of motion, we can obtain a system of two equations in the unknowns T, a, and α that we can solve simultaneously.



(*a*) Apply Newton's 2nd law to the cylinder:

$$\sum \tau_0 = TR = I_0\alpha \qquad (1)$$

and

$$\sum F_x = Mg - T = Ma \qquad (2)$$

Substitute for α and I_0 in equation (1) to obtain:

$$TR = \left(\tfrac{1}{2}MR^2\right)\left(\frac{a}{R}\right)$$

Solve for T:

$$T = \tfrac{1}{2}Ma \qquad (3)$$

Substitute for T in equation (2) and solve for a to obtain:

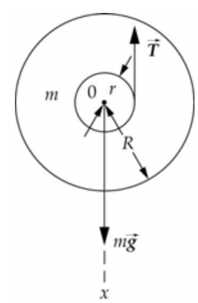
$$a = \boxed{\tfrac{2}{3}g}$$

(*b*) Substitute for a in equation (3) to obtain:

$$T = \tfrac{1}{2}M\left(\tfrac{2}{3}g\right) = \boxed{\tfrac{1}{3}Mg}$$

81 ••

Picture the Problem The forces acting on the yo-yo are shown in the figure. Apply Newton's 2nd law in both translational and rotational form to obtain simultaneous equations in T, a, and α from which we can eliminate α and solve for T and a.



Apply Newton's 2nd law to the yo-yo:

$$\sum F_x = mg - T = ma \qquad (1)$$

and

$$\sum \tau_0 = Tr = I_0\alpha \qquad (2)$$

Use $a = r\alpha$ to eliminate α in equation (2)

$$Tr = I_0\frac{a}{r} \qquad (3)$$

Eliminate T between equations (1) and (3) to obtain:

$$mg - \frac{I_0}{r^2}a = ma \quad (4)$$

Substitute $\frac{1}{2}mR^2$ for I_0 in equation (4):

$$mg - \frac{\frac{1}{2}mR^2}{r^2}a = ma$$

Solve for a:

$$a = \frac{g}{1 + \frac{R^2}{2r^2}}$$

Substitute numerical values and evaluate a:

$$a = \frac{9.81\,\text{m/s}^2}{1 + \frac{(0.1\,\text{m})^2}{2(0.01\,\text{m})^2}} = \boxed{0.192\,\text{m/s}^2}$$

Use equation (1) to solve for and evaluate T:

$$T = m(g - a)$$
$$= (0.1\,\text{kg})(9.81\,\text{m/s}^2 - 0.192\,\text{m/s}^2)$$
$$= \boxed{0.962\,\text{N}}$$

***82 •**

Picture the Problem We can determine the kinetic energy of the cylinder that is due to its rotation about its center of mass by examining the ratio K_{rot}/K .

Express the rotational kinetic energy of the homogeneous solid cylinder:

$$K_{\text{rot}} = \tfrac{1}{2}I_{\text{cyl}}\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v^2}{r^2} = \tfrac{1}{4}mv^2$$

Express the total kinetic energy of the homogeneous solid cylinder:

$$K = K_{\text{rot}} + K_{\text{trans}} = \tfrac{1}{4}mv^2 + \tfrac{1}{2}mv^2 = \tfrac{3}{4}mv^2$$

Express the ratio $\frac{K_{\text{rot}}}{K}$:

$$\frac{K_{\text{rot}}}{K} = \frac{\frac{1}{4}mv^2}{\frac{3}{4}mv^2} = \tfrac{1}{3} \text{ and } \boxed{(b)\text{ is correct.}}$$

83 •

Picture the Problem Any work done on the cylinder by a net force will change its kinetic energy. Therefore, the work needed to give the cylinder this motion is equal to its kinetic energy.

Express the relationship between the work needed to stop the cylinder and its kinetic energy:

$$|W| = |\Delta K| = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

Because the cylinder is rolling without slipping, its translational and angular speeds are related according to:

$$v = r\omega$$

Substitute for I (see Table 9-1) and ω and simplify to obtain:

$$|W| = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\frac{v^2}{r^2} = \tfrac{3}{4}mv^2$$

Substitute for m and v to obtain:

$$|W| = \tfrac{3}{4}(60\,\text{kg})(5\,\text{m/s})^2 = \boxed{1.13\,\text{kJ}}$$

84 •

Picture the Problem The total kinetic energy of any object that is rolling without slipping is given by $K = K_{\text{trans}} + K_{\text{rot}}$. We can find the percentages associated with each motion by expressing the moment of inertia of the objects as kmr^2 and deriving a general expression for the ratios of rotational kinetic energy to total kinetic energy and translational kinetic energy to total kinetic energy and substituting the appropriate values of k.

Express the total kinetic energy associated with a rotating and translating object:

$$K = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(kmr^2\right)\frac{v^2}{r^2} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}kmv^2 = \tfrac{1}{2}mv^2(1+k)$$

Express the ratio $\frac{K_{\text{rot}}}{K}$:

$$\frac{K_{\text{rot}}}{K} = \frac{\tfrac{1}{2}kmv^2}{\tfrac{1}{2}mv^2(1+k)} = \frac{k}{1+k} = \frac{1}{1+\frac{1}{k}}$$

Express the ratio $\frac{K_{\text{trans}}}{K}$:

$$\frac{K_{\text{trans}}}{K} = \frac{\tfrac{1}{2}mv^2}{\tfrac{1}{2}mv^2(1+k)} = \frac{1}{1+k}$$

(*a*) Substitute $k = 2/5$ for a uniform sphere to obtain:

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1+\frac{1}{0.4}} = 0.286 = \boxed{28.6\%}$$

and

$$\frac{K_{\text{trans}}}{K} = \frac{1}{1+0.4} = 0.714 = \boxed{71.4\%}$$

(*b*) Substitute $k = 1/2$ for a uniform cylinder to obtain:

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1+\frac{1}{0.5}} = \boxed{33.3\%}$$

and

$$\frac{K_{\text{trans}}}{K} = \frac{1}{1+0.5} = \boxed{66.7\%}$$

(*c*) Substitute $k = 1$ for a hoop to obtain:

$$\frac{K_{\text{rot}}}{K} = \frac{1}{1+\frac{1}{1}} = \boxed{50.0\%}$$

and

$$\frac{K_{\text{trans}}}{K} = \frac{1}{1+1} = \boxed{50.0\%}$$

85 •

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the incline. As the hoop rolls up the incline its translational and rotational kinetic energies are transformed into gravitational potential energy. We can use energy conservation to relate the distance the hoop rolls up the incline to its total kinetic energy at the bottom of the incline.

Using energy conservation, relate the distance the hoop will roll up the incline to its kinetic energy at the bottom of the incline:

$$\Delta K + \Delta U = 0$$

or, because $K_{\text{f}} = U_{\text{i}} = 0$,

$$-K_{\text{i}} + U_{\text{f}} = 0 \qquad (1)$$

Express K_{i} as the sum of the translational and rotational kinetic energies of the hoop:

$$K_{\text{i}} = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2$$

When a rolling object moves with speed v, its outer surface turns with a speed v also. Hence $\omega = v/r$. Substitute for I and ω to obtain:

$$K_{\text{i}} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(mr^2\right)\frac{v^2}{r^2} = mv^2$$

Letting Δh be the change in elevation of the hoop as it rolls up the incline and ΔL the distance it rolls along the incline, express U_{f}:

$$U_{\text{f}} = mg\Delta h = mg\Delta L\sin\theta$$

Substitute in equation (1) to obtain:

$$-mv^2 + mg\Delta L\sin\theta = 0$$

Solve for ΔL:

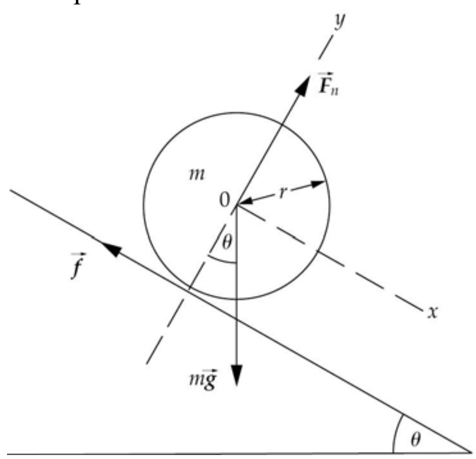
$$\Delta L = \frac{v^2}{g\sin\theta}$$

Substitute numerical values and evaluate ΔL:

$$\Delta L = \frac{(15\,\text{m/s})^2}{(9.81\,\text{m/s}^2)\sin 30°} = \boxed{45.9\,\text{m}}$$

***86 ••**

Picture the Problem From Newton's 2nd law, the acceleration of the center of mass equals the net force divided by the mass. The forces acting on the sphere are its weight $m\vec{g}$ downward, the normal force $\vec{F}_n$ that balances the normal component of the weight, and the force of friction $\vec{f}$ acting up the incline. As the sphere accelerates down the incline, the angular velocity of rotation must increase to maintain the nonslip condition. We can apply Newton's 2nd law for rotation about a horizontal axis through the center of mass of the sphere to find α, which is related to the acceleration by the nonslip condition. The only torque about the center of mass is due to $\vec{f}$ because both $m\vec{g}$ and $\vec{F}_n$ act through the center of mass. Choose the positive direction to be down the incline.



Apply $\sum \vec{F} = m\vec{a}$ to the sphere:

$$mg\sin\theta - f = ma_{cm} \qquad (1)$$

Apply $\sum \tau = I_{cm}\alpha$ to the sphere:

$$fr = I_{cm}\alpha$$

Use the nonslip condition to eliminate α and solve for f:

$$fr = I_{cm}\frac{a_{cm}}{r}$$

and

$$f = \frac{I_{cm}}{r^2}a_{cm}$$

Substitute this result for f in equation (1) to obtain:

$$mg\sin\theta - \frac{I_{cm}}{r^2}a_{cm} = ma_{cm}$$

From Table 9-1 we have, for a solid sphere:

$$I_{cm} = \tfrac{2}{5}mr^2$$

Substitute in equation (1) and simplify to obtain:

$$mg\sin\theta - \tfrac{2}{5}a_{cm} = ma_{cm}$$

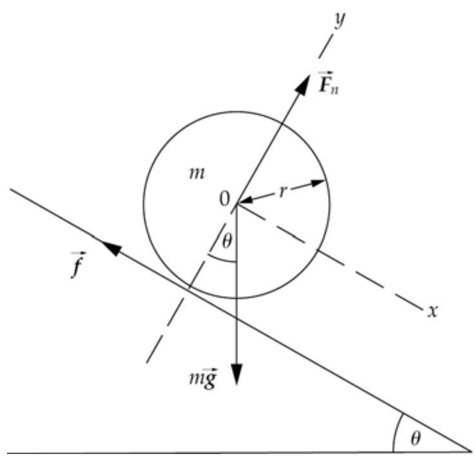
Solve for and evaluate θ:

$$\theta = \sin^{-1}\left(\frac{7a_{cm}}{5g}\right)$$

$$= \sin^{-1}\left[\frac{7(0.2g)}{5g}\right] = \boxed{16.3°}$$

87 ••

Picture the Problem From Newton's 2nd law, the acceleration of the center of mass equals the net force divided by the mass. The forces acting on the thin spherical shell are its weight $m\vec{g}$ downward, the normal force $\vec{F}_n$ that balances the normal component of the weight, and the force of friction $\vec{f}$ acting up the incline. As the spherical shell accelerates down the incline, the angular velocity of rotation must increase to maintain the nonslip condition. We can apply Newton's 2nd law for rotation about a horizontal axis through the center of mass of the sphere to find α, which is related to the acceleration by the nonslip condition. The only torque about the center of mass is due to $\vec{f}$ because both $m\vec{g}$ and $\vec{F}_n$ act through the center of mass. Choose the positive direction to be down the incline.



Apply $\sum\vec{F} = m\vec{a}$ to the thin spherical shell:

$$mg\sin\theta - f = ma_{cm} \qquad (1)$$

Apply $\sum\tau = I_{cm}\alpha$ to the thin spherical shell:

$$fr = I_{cm}\alpha$$

Use the nonslip condition to eliminate α and solve for f:

$$fr = I_{cm}\frac{a_{cm}}{r} \text{ and } f = \frac{I_{cm}}{r^2}a_{cm}$$

Substitute this result for f in equation (1) to obtain:

$$mg\sin\theta - \frac{I_{cm}}{r^2}a_{cm} = ma_{cm}$$

From Table 9-1 we have, for a thin

$$I_{cm} = \tfrac{2}{3}mr^2$$

spherical shell:

Substitute in equation (1) and simplify to obtain:

$$mg\sin\theta - \tfrac{2}{3}a_{cm} = ma_{cm}$$

Solve for and evaluate θ:

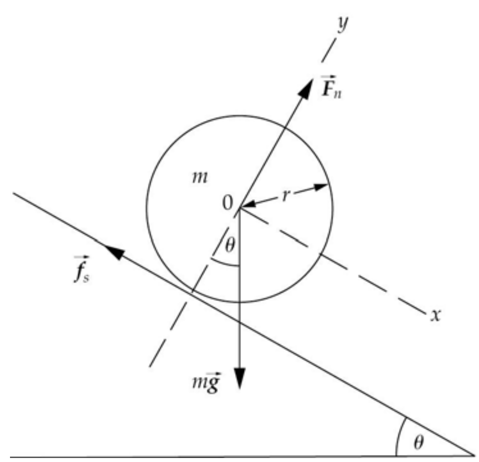
$$\theta = \sin^{-1}\frac{5a_{cm}}{3g}$$

$$= \sin^{-1}\frac{5(0.2g)}{3g} = \boxed{19.5°}$$

Remarks: This larger angle makes sense, as the moment of inertia for a given mass is larger for a hollow sphere than for a solid one.

88 ••

Picture the Problem The three forces acting on the basketball are the weight of the ball, the normal force, and the force of friction. Because the weight can be assumed to be acting at the center of mass, and the normal force acts through the center of mass, the only force which exerts a torque about the center of mass is the frictional force. We can use Newton's 2nd law to find a system of simultaneous equations that we can solve for the quantities called for in the problem statement.



(*a*) Apply Newton's 2nd law in both translational and rotational form to the ball:

$$\sum F_x = mg\sin\theta - f_s = ma, \qquad (1)$$

$$\sum F_y = F_n - mg\cos\theta = 0 \qquad (2)$$

and

$$\sum \tau_0 = f_s r = I_0\alpha \qquad (3)$$

Because the basketball is rolling without slipping we know that:

$$\alpha = \frac{a}{r}$$

Substitute in equation (3) to obtain:

$$f_s r = I_0\frac{a}{r} \qquad (4)$$

From Table 9-1 we have:

$$I_0 = \tfrac{2}{3}mr^2$$

Substitute for I_0 and α in equation (4) and solve for f_s:

$$f_s r = \left(\tfrac{2}{3}mr^2\right)\frac{a}{r} \Rightarrow f_s = \tfrac{2}{3}ma \qquad (5)$$

Substitute for f_s in equation (1) and solve for a:

$$a = \boxed{\tfrac{3}{5} g \sin\theta}$$

(*b*) Find f_s using equation (5):

$$f_s = \tfrac{2}{3} m\left(\tfrac{3}{5} g \sin\theta\right) = \boxed{\tfrac{2}{5} mg \sin\theta}$$

(*c*) Solve equation (2) for F_n:

$$F_n = mg\cos\theta$$

Use the definition of $f_{s,max}$ to obtain:

$$f_{s,max} = \mu_s F_n = \mu_s mg\cos\theta_{max}$$

Use the result of part (*b*) to obtain:

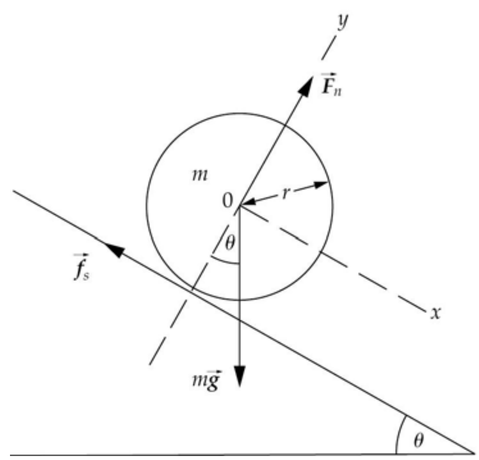
$$\tfrac{2}{5} mg \sin\theta_{max} = \mu_s mg\cos\theta_{max}$$

Solve for θ_{max}:

$$\theta_{max} = \boxed{\tan^{-1}\left(\tfrac{5}{2}\mu_s\right)}$$

89 ••

Picture the Problem The three forces acting on the cylinder are the weight of the cylinder, the normal force, and the force of friction. Because the weight can be assumed to be acting at the center of mass, and the normal force acts through the center of mass, the only force which exerts a torque about the center of mass is the frictional force. We can use Newton's 2nd law to find a system of simultaneous equations that we can solve for the quantities called for in the problem statement.



(*a*) Apply Newton's 2nd law in both translational and rotational form to the cylinder:

$$\sum F_x = mg\sin\theta - f_s = ma, \quad (1)$$

$$\sum F_y = F_n - mg\cos\theta = 0 \quad (2)$$

and

$$\sum \tau_0 = f_s r = I_0 \alpha \quad (3)$$

Because the cylinder is rolling without slipping we know that:

$$\alpha = \frac{a}{r}$$

Substitute in equation (3) to obtain:

$$f_s r = I_0 \frac{a}{r} \quad (4)$$

From Table 9-1 we have:

$$I_0 = \tfrac{1}{2} mr^2$$

Substitute for I_0 and α in equation (4) and solve for f_s:

$$f_s r = \left(\tfrac{1}{2}mr^2\right)\frac{a}{r} \Rightarrow f_s = \tfrac{1}{2}ma \qquad (5)$$

Substitute for f_s in equation (1) and solve for a:

$$a = \boxed{\tfrac{2}{3}g\sin\theta}$$

(*b*) Find f_s using equation (5):

$$f_s = \tfrac{1}{2}m\left(\tfrac{2}{3}g\sin\theta\right) = \boxed{\tfrac{1}{3}mg\sin\theta}$$

(*c*) Solve equation (2) for F_n:

$$F_n = mg\cos\theta$$

Use the definition of $f_{s,max}$ to obtain:

$$f_{s,max} = \mu_s F_n = \mu_s mg\cos\theta_{max}$$

Use the result of part (*b*) to obtain:

$$\tfrac{1}{3}mg\sin\theta_{max} = \mu_s mg\cos\theta_{max}$$

Solve for θ_{max}:

$$\theta_{max} = \boxed{\tan^{-1}(3\mu_s)}$$

***90 ••**

Picture the Problem Let the zero of gravitational potential energy be at the elevation where the spheres leave the ramp. The distances the spheres will travel are directly proportional to their speeds when they leave the ramp.

Express the ratio of the distances traveled by the two spheres in terms of their speeds when they leave the ramp:

$$\frac{L'}{L} = \frac{v'\Delta t}{v\Delta t} = \frac{v'}{v} \qquad (1)$$

Use conservation of mechanical energy to find the speed of the spheres when they leave the ramp:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0 \qquad (2)$$

Express K_f for the spheres:

$$\begin{aligned} K_f &= K_{trans} + K_{rot} \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{cm}\omega^2 \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(kmR^2\right)\frac{v^2}{R^2} \\ &= \tfrac{1}{2}mv^2 + \tfrac{1}{2}kmv^2 \\ &= (1+k)\tfrac{1}{2}mv^2 \end{aligned}$$

where k is 2/3 for the spherical shell and 2/5 for the uniform sphere.

Substitute in equation (2) to obtain:

$$(1+k)\tfrac{1}{2}mv^2 = mgH$$

Solve for v:

$$v = \sqrt{\frac{2gH}{1+k}}$$

Substitute in equation (1) to obtain:

$$\frac{L'}{L} = \sqrt{\frac{1+k}{1+k'}} = \sqrt{\frac{1+\frac{2}{3}}{1+\frac{2}{5}}} = 1.09$$

or

$$L' = \boxed{1.09L}$$

91 ••

Picture the Problem Let the subscripts u and h refer to the uniform and thin-walled spheres, respectively. Because the cylinders climb to the same height, their kinetic energies at the bottom of the incline must be equal.

Express the total kinetic energy of the thin-walled cylinder at the bottom of the inclined plane:

$$K_h = K_{trans} + K_{rot} = \tfrac{1}{2}m_h v^2 + \tfrac{1}{2}I_h\omega^2$$
$$= \tfrac{1}{2}m_h v^2 + \tfrac{1}{2}\left(m_h r^2\right)\frac{v^2}{r^2} = m_h v^2$$

Express the total kinetic energy of the solid cylinder at the bottom of the inclined plane:

$$K_u = K_{trans} + K_{rot} = \tfrac{1}{2}m_u v'^2 + \tfrac{1}{2}I_u\omega^2$$
$$= \tfrac{1}{2}m_u v'^2 + \tfrac{1}{2}\left(\tfrac{1}{2}m_u r^2\right)\frac{v'^2}{r^2} = \tfrac{3}{4}m_u v'^2$$

Because the cylinders climb to the same height:

$$\tfrac{3}{4}m_u v'^2 = m_u gh$$

and

$$m_h v^2 = m_h gh$$

Divide the first of these equations by the second:

$$\frac{\tfrac{3}{4}m_u v'^2}{m_h v^2} = \frac{m_u gh}{m_h gh}$$

Simplify to obtain:

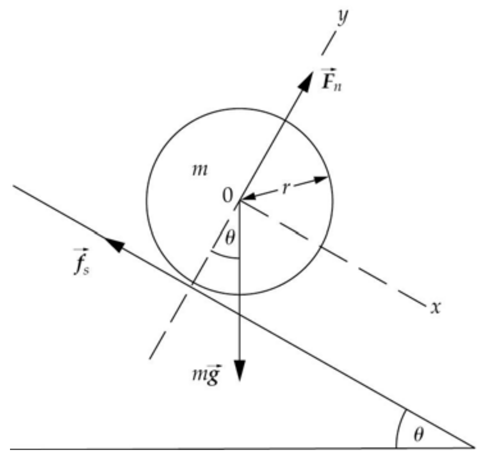
$$\frac{3v'^2}{4v^2} = 1$$

Solve for v':

$$v' = \boxed{\sqrt{\frac{4}{3}}v}$$

92 ••

Picture the Problem Let the subscripts s and c refer to the solid sphere and thin-walled cylinder, respectively. Because the cylinder and sphere descend from the same height, their kinetic energies at the bottom of the incline must be equal. The force diagram shows the forces acting on the solid sphere. We'll use Newton's 2nd law to relate the accelerations to the angle of the incline and use a constant acceleration to relate the accelerations to the distances traveled down the incline.



Apply Newton's 2nd law to the sphere:

$$\sum F_x = mg\sin\theta - f_s = ma_s, \quad (1)$$

$$\sum F_y = F_n - mg\cos\theta = 0, \quad (2)$$

and

$$\sum \tau_0 = f_s r = I_0\alpha \quad (3)$$

Substitute for I_0 and α in equation (3) and solve for f_s:

$$f_s r = \left(\tfrac{2}{5}mr^2\right)\frac{a}{r} \Rightarrow f_s = \tfrac{2}{5}ma_s$$

Substitute for f_s in equation (1) and solve for a:

$$a_s = \tfrac{5}{7}g\sin\theta$$

Proceed as above for the thin-walled cylinder to obtain:

$$a_c = \tfrac{1}{2}g\sin\theta$$

Using a constant-acceleration equation, relate the distance traveled down the incline to its acceleration and the elapsed time:

$$\Delta s = v_0\Delta t + \tfrac{1}{2}a(\Delta t)^2$$

or, because $v_0 = 0$,

$$\Delta s = \tfrac{1}{2}a(\Delta t)^2 \quad (4)$$

Because Δs is the same for both objects:

$$a_s t_s^2 = a_c t_c^2$$

where

$$t_c^2 = (t_s + 2.4)^2 = t_s^2 + 4.8t_s + 5.76$$

provided t_c and t_s are in seconds.

Substitute for a_s and a_c to obtain the quadratic equation:

$$t_s^2 + 4.8t_s + 5.76 = \tfrac{10}{7}t_s^2$$

Solve for the positive root to obtain: $t_s = 12.3\,\text{s}$

Substitute in equation (4), simplify, and solve for θ:

$$\theta = \sin^{-1}\left[\frac{14\Delta s}{5gt_s^2}\right]$$

Substitute numerical values and evaluate θ:

$$\theta = \sin^{-1}\left[\frac{14(3\,\text{m})}{5(9.81\,\text{m/s}^2)(12.3\,\text{s})^2}\right] = \boxed{0.324°}$$

93 •••

Picture the Problem The kinetic energy of the wheel is the sum of its translational and rotational kinetic energies. Because the wheel is a composite object, we can model its moment of inertia by treating the rim as a cylindrical shell and the spokes as rods.

Express the kinetic energy of the wheel:

$$K = K_{\text{trans}} + K_{\text{rot}} = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}I_{\text{cm}}\omega^2 = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}I_{\text{cm}}\frac{v^2}{R^2}$$

where $M_{\text{tot}} = M_{\text{rim}} + 4M_{\text{spoke}}$

Express the moment of inertia of the wheel:

$$I_{\text{cm}} = I_{\text{rim}} + I_{\text{spokes}} = M_{\text{rim}}R^2 + 4\left(\tfrac{1}{3}M_{\text{spoke}}R^2\right) = \left(M_{\text{rim}} + \tfrac{4}{3}M_{\text{spoke}}\right)R^2$$

Substitute for I_{cm} in the equation for K:

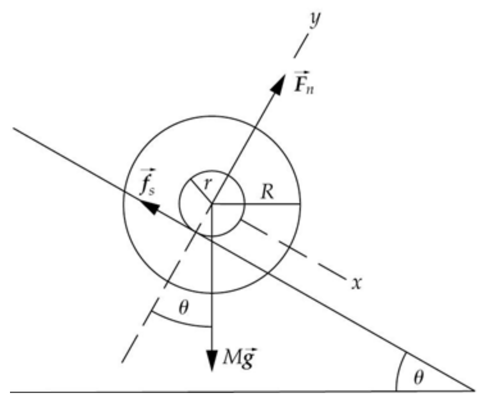
$$K = \tfrac{1}{2}M_{\text{tot}}v^2 + \tfrac{1}{2}\left[\left(M_{\text{rim}} + \tfrac{4}{3}M_{\text{spoke}}\right)R^2\right]\frac{v^2}{R^2} = \left[\tfrac{1}{2}\left(M_{\text{tot}} + M_{\text{rim}}\right) + \tfrac{2}{3}M_{\text{spoke}}\right]v^2$$

Substitute numerical values and evaluate K:

$$K = \left[\tfrac{1}{2}(7.8\,\text{kg} + 3\,\text{kg}) + \tfrac{2}{3}(1.2\,\text{kg})\right](6\,\text{m/s})^2 = \boxed{223\,\text{J}}$$

94 •••

Picture the Problem Let M represent the combined mass of the two disks and their connecting rod and I their moment of inertia. The object's initial potential energy is transformed into translational and rotational kinetic energy as it rolls down the incline. The force diagram shows the forces acting on this composite object as it rolls down the incline. Application of Newton's 2nd law will allow us to derive an expression for the acceleration of the object.



(*a*) Apply Newton's 2nd law to the disks and rod:

$$\sum F_x = Mg\sin\theta - f_s = Ma, \quad (1)$$

$$\sum F_y = F_n - Mg\cos\theta = 0, \quad (2)$$

and

$$\sum \tau_0 = f_s r = I\alpha \quad (3)$$

Eliminate f_s and α between equations (1) and (3) and solve for a to obtain:

$$a = \frac{Mg\sin\theta}{M + \frac{I}{r^2}} \quad (4)$$

Express the moment of inertia of the two disks plus connecting rod:

$$\begin{aligned} I &= 2I_{disk} + I_{rod} \\ &= 2\left(\tfrac{1}{2}m_{disk}R^2\right) + \tfrac{1}{2}m_{rod}r^2 \\ &= m_{disk}R^2 + \tfrac{1}{2}m_{rod}r^2 \end{aligned}$$

Substitute numerical values and evaluate I:

$$\begin{aligned} I &= (20\,\text{kg})(0.3\,\text{m})^2 + \tfrac{1}{2}(1\,\text{kg})(0.02\,\text{m})^2 \\ &= 1.80\,\text{kg}\cdot\text{m}^2 \end{aligned}$$

Substitute in equation (4) and evaluate a:

$$a = \frac{(41\,\text{kg})(9.81\,\text{m/s}^2)\sin 30°}{41\,\text{kg} + \frac{1.80\,\text{kg}\cdot\text{m}^2}{(0.02\,\text{m})^2}} = \boxed{0.0443\,\text{m/s}^2}$$

(*b*) Find α from a:

$$\alpha = \frac{a}{r} = \frac{0.0443\,\text{m/s}^2}{0.02\,\text{m}} = \boxed{2.21\,\text{rad/s}^2}$$

(*c*) Express the kinetic energy of translation of the disks-plus-rod when it has rolled a distance Δs down the incline:

$$K_{\text{trans}} = \tfrac{1}{2}Mv^2$$

Using a constant-acceleration equation, relate the speed of the disks-plus-rod to their acceleration and the distance moved:

$$v^2 = v_0^2 + 2a\Delta s$$

or, because $v_0 = 0$,

$$v^2 = 2a\Delta s$$

Substitute to obtain:

$$K_{\text{trans}} = Ma\Delta s = (41\,\text{kg})(0.0443\,\text{m/s}^2)(2\,\text{m}) = \boxed{3.63\,\text{J}}$$

(*d*) Express the rotational kinetic energy of the disks after rolling 2 m in terms of their initial potential energy and their translational kinetic energy:

$$K_{\text{rot}} = U_{\text{i}} - K_{\text{trans}} = Mgh - K_{\text{trans}}$$

Substitute numerical values and evaluate K_{rot}:

$$K_{\text{rot}} = (41\,\text{kg})(9.81\,\text{m/s}^2)(2\,\text{m})\sin 30° - 3.63\,\text{J} = \boxed{399\,\text{J}}$$

95 •••

Picture the Problem We can express the coordinates of point *P* as the sum of the coordinates of the center of the wheel and the coordinates, relative to the center of the wheel, of the tip of the vector $\vec{r}_0$. Differentiation of these expressions with respect to time will give us the *x* and *y* components of the velocity of point *P*.

(*a*) Express the coordinates of point *P* relative to the center of the wheel:

$$x = r_0 \cos\theta$$

and

$$y = r_0 \sin\theta$$

Because the coordinates of the center of the circle are *X* and *R*:

$$(x_P, y_P) = \boxed{(X + r_0\cos\theta, R + r_0\sin\theta)}$$

(b) Differentiate x_P to obtain:

$$v_{Px} = \frac{d}{dt}\left(X + r_0 \cos\theta\right) = \frac{dX}{dt} - r_0 \sin\theta \cdot \frac{d\theta}{dt}$$

Note that $\frac{dX}{dt} = V$ and $\frac{d\theta}{dt} = -\omega = -\frac{V}{R}$ so:

$$v_{Px} = \boxed{V + \frac{r_0 V}{R}\sin\theta}$$

Differentiate y_P to obtain:

$$v_{Py} = \frac{d}{dt}\left(R + r_0 \sin\theta\right) = r_0 \cos\theta \cdot \frac{d\theta}{dt}$$

Because $\frac{d\theta}{dt} = -\omega = -\frac{V}{R}$:

$$v_{Py} = \boxed{-\frac{r_0 V}{R}\cos\theta}$$

(c) Calculate $\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{r}}$:

$$\begin{aligned}\vec{\boldsymbol{v}} \cdot \vec{\boldsymbol{r}} &= v_{Px} r_x + v_{Py} r_y \\ &= \left(V + \frac{r_0 V}{R}\sin\theta\right)\left(r_0 \cos\theta\right) \\ &\quad - \left(\frac{r_0 V}{R}\cos\theta\right)\left(R + r_0 \sin\theta\right) \\ &= \boxed{0}\end{aligned}$$

(d) Express v in terms of its components:

$$\begin{aligned}v &= \sqrt{v_x^2 + v_y^2} \\ &= \sqrt{\left(V + \frac{r_0 V}{R}\sin\theta\right)^2 + \left(-\frac{r_0 V}{R}\cos\theta\right)^2} \\ &= V\sqrt{1 + 2\frac{r_0}{R}\sin\theta + \frac{r_0^2}{R^2}}\end{aligned}$$

Express r in terms of its components:

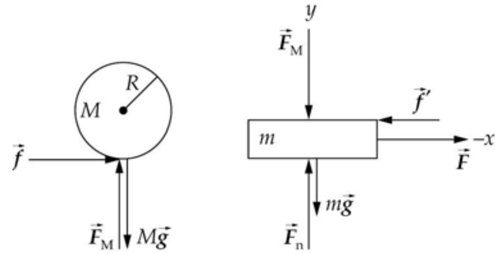
$$\begin{aligned}r &= \sqrt{r_x^2 + r_y^2} \\ &= \sqrt{\left(r_0 \cos\theta\right)^2 + \left(R + r_0 \sin\theta\right)^2} \\ &= R\sqrt{1 + 2\frac{r_0}{R}\sin\theta + \frac{r_0^2}{R^2}}\end{aligned}$$

Divide v by r to obtain:

$$\omega = \frac{v}{r} = \boxed{\frac{V}{R}}$$

***96** •••

Picture the Problem Let the letter B identify the block and the letter C the cylinder. We can find the accelerations of the block and cylinder by applying Newton's 2nd law and solving the resulting equations simultaneously.



Apply $\sum F_x = ma_x$ to the block: $F - f' = ma_B$ (1)

Apply $\sum F_x = ma_x$ to the cylinder: $f = Ma_C$, (2)

Apply $\sum \tau_{CM} = I_{CM}\alpha$ to the cylinder: $fR = I_{CM}\alpha$ (3)

Substitute for I_{CM} in equation (3) and solve for $f = f'$ to obtain: $f = \frac{1}{2}MR\alpha$ (4)

Relate the acceleration of the block to the acceleration of the cylinder:

$a_C = a_B + a_{CB}$

or, because $a_{CB} = -R\alpha$ is the acceleration of the cylinder relative to the block,

$a_C = a_B - R\alpha$

and

$R\alpha = a_B - a_C$ (5)

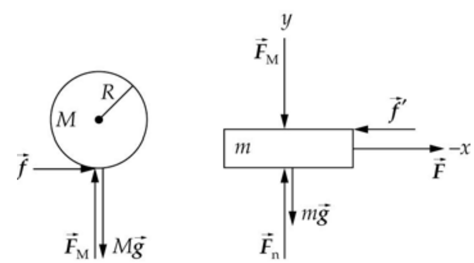
Equate equations (2) and (4) and substitute from (5) to obtain: $a_B = 3a_C$

Substitute equation (4) in equation (1) and substitute for a_C to obtain: $F - \frac{1}{3}Ma_B = ma_B$

Solve for a_B: $a_B = \boxed{\dfrac{3F}{M + 3m}}$

97 •••

Picture the Problem Let the letter B identify the block and the letter C the cylinder. In this problem, as in Problem 97, we can find the accelerations of the block and cylinder by applying Newton's 2nd law and solving the resulting equations simultaneously.



Apply $\sum F_x = ma_x$ to the block: $F - f = ma_\text{B}$ (1)

Apply $\sum F_x = ma_x$ to the cylinder: $f = Ma_\text{C}$, (2)

Apply $\sum \tau_\text{CM} = I_\text{CM}\alpha$ to the cylinder: $fR = I_\text{CM}\alpha$ (3)

Substitute for I_CM in equation (3) and solve for f: $f = \frac{1}{2}MR\alpha$ (4)

Relate the acceleration of the block to the acceleration of the cylinder:

$a_\text{C} = a_\text{B} + a_\text{CB}$

or, because $a_\text{CB} = -R\alpha$,

$a_\text{C} = a_\text{B} - R\alpha$

and

$R\alpha = a_\text{B} - a_\text{C}$ (5)

(*a*) Solve for α and substitute for a_B to obtain:

$$\alpha = \frac{a_\text{B} - a_\text{C}}{R} = \frac{3a_\text{C} - a_\text{C}}{R} = \frac{2a_\text{C}}{R} = \boxed{\frac{2F}{R(M + 3m)}}$$

From the force diagram it is evident that the torque and, therefore, α is in the counterclockwise direction.

(*b*) Equate equations (2) and (4) and substitute (5) to obtain: $a_\text{B} = 3a_\text{C}$

From equations (1) and (4) we obtain: $F - \frac{1}{3}Ma_\text{B} = ma_\text{B}$

Solve for a_B:

$$a_\text{B} = \frac{3F}{M + 3m}$$

Substitute to obtain the linear acceleration of the cylinder relative to the table:

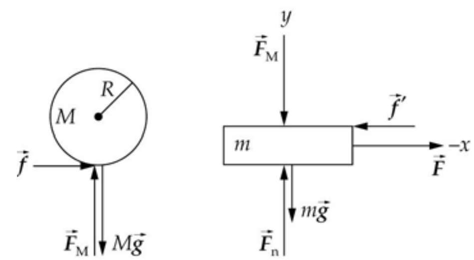
$$a_\text{C} = \frac{1}{3}a_B = \boxed{\frac{F}{M + 3m}}$$

(*c*) Express the acceleration of the cylinder relative to the block:

$$a_{CB} = a_C - a_B = a_C - 3a_C = -2a_C$$
$$= \boxed{-\frac{2F}{M+3m}}$$

98 •••

Picture the Problem Let the system include the earth, the cylinder, and the block. Then $\vec{F}$ is an external force that changes the energy of the system by doing work on it. We can find the kinetic energy of the block from its speed when it has traveled a distance *d*. We can find the kinetic energy of the cylinder from the sum of its translational and rotational kinetic energies. In part (c) we can add the kinetic energies of the block and the cylinder to show that their sum is the work done by $\vec{F}$ in displacing the system a distance *d*.



(*a*) Express the kinetic energy of the block:

$$K_B = W_{\text{on block}} = \tfrac{1}{2}mv_B^2$$

Using a constant-acceleration equation, relate the velocity of the block to its acceleration and the distance traveled:

$$v_B^2 = v_0^2 + 2a_B d$$

or, because the block starts from rest,

$$v_B^2 = 2a_B d$$

Substitute to obtain:

$$K_B = \tfrac{1}{2}m(2a_B d) = ma_B d \qquad (1)$$

Apply $\sum F_x = ma_x$ to the block:

$$F - f = ma_B \qquad (2)$$

Apply $\sum F_x = ma_x$ to the cylinder:

$$f = Ma_C\,, \qquad (3)$$

Apply $\sum \tau_{CM} = I_{CM}\alpha$ to the cylinder:

$$fR = I_{CM}\alpha \qquad (4)$$

Substitute for I_{CM} in equation (4) and solve for *f*:

$$f = \tfrac{1}{2}MR\alpha \qquad (5)$$

Relate the acceleration of the block to the acceleration of the cylinder:

$$a_C = a_B + a_{CB}$$

or, because $a_{CB} = -R\alpha$,

$$a_C = a_B - R\alpha$$
and
$$R\alpha = a_B - a_C \qquad (6)$$

Equate equations (3) and (5) and substitute in (6) to obtain:

$$a_B = 3a_C$$

Substitute equation (5) in equation (2) and use $a_B = 3a_C$ to obtain:

$$F - Ma_C = ma_B$$
or
$$F - \tfrac{1}{3}Ma_B = ma_B$$

Solve for a_B:

$$a_B = \frac{F}{m + \frac{1}{3}M}$$

Substitute in equation (1) to obtain:

$$K_B = \boxed{\frac{mFd}{m + \frac{1}{3}M}}$$

(*b*) Express the total kinetic energy of the cylinder:

$$K_{cyl} = K_{trans} + K_{rot} = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}I_{CM}\omega^2 = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}I_{CM}\frac{v_{CB}^2}{R^2} \qquad (7)$$

where $v_{CB} = v_C - v_B$.

In part (*a*) it was established that:

$$a_B = 3a_C$$

Integrate both sides of the equation with respect to time to obtain:

$$v_B = 3v_C + \text{constant}$$

where the constant of integration is determined by the initial conditions that $v_C = 0$ when $v_B = 0$.

Substitute the initial conditions to obtain:

$$\text{constant} = 0$$
and
$$v_B = 3v_C$$

Substitute in our expression for v_{CB} to obtain:

$$v_{CB} = v_C - v_B = v_C - 3v_C = -2v_C$$

Substitute for I_{CM} and v_{CB} in equation (7) to obtain:

$$K_{cyl} = \tfrac{1}{2}Mv_C^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{(-2v_C)^2}{R^2} = \tfrac{3}{2}Mv_C^2 \qquad (8)$$

Because $v_C = \frac{1}{3} v_B$: $v_C^2 = \frac{1}{9} v_B^2$

It part (*a*) it was established that: $v_B^2 = 2a_B d$

and

$$a_B = \frac{F}{m + \frac{1}{3}M}$$

Substitute to obtain:

$$v_C^2 = \frac{1}{9}(2a_B d) = \frac{2}{9}\left(\frac{F}{m + \frac{1}{3}M}\right)d = \frac{2Fd}{9\left(m + \frac{1}{3}M\right)}$$

Substitute in equation (8) to obtain:

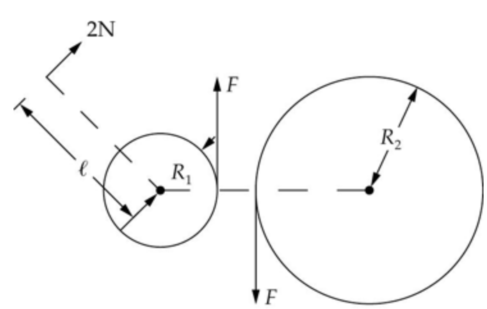
$$K_{cyl} = \frac{3}{2}M\left(\frac{2Fd}{9\left(m + \frac{1}{3}M\right)}\right) = \boxed{\frac{MFd}{3\left(m + \frac{1}{3}M\right)}}$$

(*c*) Express the total kinetic energy of the system and simplify to obtain:

$$K_{tot} = K_B + K_{cyl} = \frac{mFd}{m + \frac{1}{3}M} + \frac{MFd}{3\left(m + \frac{1}{3}M\right)} = \frac{(3m + M)}{3\left(m + \frac{1}{3}M\right)}Fd = \boxed{Fd}$$

99 ••

Picture the Problem The forces responsible for the rotation of the gears are shown in the diagram to the right. The forces acting through the centers of mass of the two gears have been omitted because they produce no torque. We can apply Newton's 2nd law in rotational form to obtain the equations of motion of the gears and the not slipping condition to relate their angular accelerations.



(*a*) Apply $\sum \tau = I\alpha$ to the gears to obtain their equations of motion:

$$2\ \text{N}\cdot\text{m} - FR_1 = I_1\alpha_1 \qquad (1)$$

and

$$FR_2 = I_2\alpha_2 \qquad (2)$$

where F is the force keeping the gears from slipping with respect to each other.

Because the gears do not slip $R_1\alpha_1 = R_2\alpha_2$

relative to each other, the tangential accelerations of the points where they are in contact must be the same:

or

$$\alpha_2 = \frac{R_1}{R_2}\alpha_1 = \tfrac{1}{2}\alpha_1 \qquad (3)$$

Divide equation (1) by R_1 to obtain:

$$\frac{2\ \text{N}\cdot\text{m}}{R_1} - F = \frac{I_1}{R_1}\alpha_1$$

Divide equation (2) by R_2 to obtain:

$$F = \frac{I_2}{R_2}\alpha_2$$

Add these equations to obtain:

$$\frac{2\ \text{N}\cdot\text{m}}{R_1} = \frac{I_1}{R_1}\alpha_1 + \frac{I_2}{R_2}\alpha_2$$

Use equation (3) to eliminate α_2:

$$\frac{2\ \text{N}\cdot\text{m}}{R_1} = \frac{I_1}{R_1}\alpha_1 + \frac{I_2}{2R_2}\alpha_1$$

Solve for α_1 to obtain:

$$\alpha_1 = \frac{2\ \text{N}\cdot\text{m}}{I_1 + \frac{R_1}{R_2}I_2}$$

Substitute numerical values and evaluate α_1:

$$\alpha_1 = \frac{2\ \text{N}\cdot\text{m}}{1\,\text{kg}\cdot\text{m}^2 + \frac{0.5\,\text{m}}{1\,\text{m}}\left(16\,\text{kg}\cdot\text{m}^2\right)} = \boxed{0.222\ \text{rad/s}^2}$$

Use equation (3) to evaluate α_2:

$$\alpha_2 = \tfrac{1}{2}\left(0.222\ \text{rad/s}^2\right) = \boxed{0.111\ \text{rad/s}^2}$$

(*b*) To counterbalance the 2-N·m torque, a counter torque of 2 N·m must be applied to the first gear. Use equation (2) with $\alpha_1 = 0$ to find F:

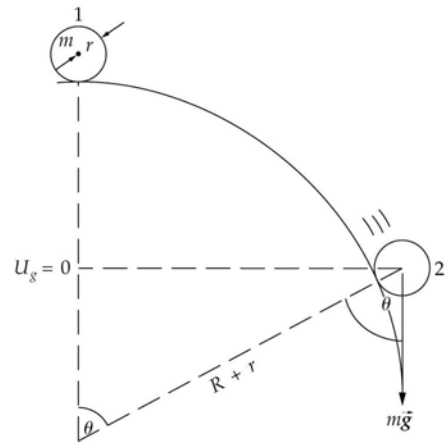
$$2\ \text{N}\cdot\text{m} - FR_1 = 0$$

and

$$F = \frac{2\ \text{N}\cdot\text{m}}{R_1} = \frac{2\ \text{N}\cdot\text{m}}{0.5\,\text{m}} = \boxed{4.00\ \text{N}}$$

***100** ••

Picture the Problem Let r be the radius of the marble, m its mass, R the radius of the large sphere, and v the speed of the marble when it breaks contact with the sphere. The numeral 1 denotes the initial configuration of the sphere-marble system and the numeral 2 is configuration as the marble separates from the sphere. We can use conservation of energy to relate the initial potential energy of the marble to the sum of its translational and rotational kinetic energies as it leaves the sphere. Our choice of the zero of potential energy is shown on the diagram.



(*a*) Apply conservation of energy:

$$\Delta U + \Delta K = 0$$

or

$$U_2 - U_1 + K_2 - K_1 = 0$$

Because $U_2 = K_1 = 0$:

$$-mg[R + r - (R + r)\cos\theta] + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = 0$$

or

$$-mg[(R + r)(1 - \cos\theta)] + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = 0$$

Use the rolling-without-slipping condition to eliminate ω:

$$-mg[(R + r)(1 - \cos\theta)] + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\frac{v^2}{r^2} = 0$$

From Table 9-1 we have:

$$I = \tfrac{2}{5}mr^2$$

Substitute to obtain:

$$-mg[(R + r)(1 - \cos\theta)] + \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mr^2\right)\frac{v^2}{r^2} = 0$$

or

$$-mg[(R + r)(1 - \cos\theta)] + \tfrac{1}{2}mv^2 + \tfrac{1}{5}mv^2 = 0$$

Solve for v^2 to obtain:

$$v^2 = \frac{10}{7}g(R + r)(1 - \cos\theta)$$

Apply $\sum F_r = ma_r$ to the marble as it separates from the sphere:

$$mg\cos\theta = m\frac{v^2}{R + r}$$

or

$$\cos\theta = \frac{v^2}{g(R+r)}$$

Substitute for v^2:

$$\cos\theta = \frac{1}{g(R+r)}\left[\frac{10}{7}g(R+r)(1-\cos\theta)\right] = \left[\frac{10}{7}(1-\cos\theta)\right]$$

Solve for and evaluate θ:

$$\theta = \cos^{-1}\left(\frac{10}{17}\right) = \boxed{54.0°}$$

(*b*) The force of friction is always less than μ_s multiplied by the normal force on the marble. However, the normal force decreases to 0 at the point where the ball leaves the sphere, meaning that the force of friction must be less than the force needed to keep the ball rolling without slipping before it leaves the sphere.

Rolling With Slipping

101 •

Picture the Problem Part (*a*) of this problem is identical to Example 9-16. In part (*b*) we can use the definitions of translational and rotational kinetic energy to find the ratio of the final and initial kinetic energies.

(*a*) From Example 9-16:

$$s_1 = \boxed{\frac{12}{49}\frac{v_0^2}{\mu_k g}},$$

$$t_1 = \boxed{\frac{2}{7}\frac{v_0}{\mu_k g}}, \text{ and}$$

$$v_1 = \tfrac{5}{2}\mu_k g t_1 = \boxed{\frac{5}{7}v_0}$$

(*b*) When the ball rolls without slipping, $v_1 = r\omega$. Express the final kinetic energy of the ball:

$$K_f = K_{trans} + K_{rot}$$
$$= \tfrac{1}{2}Mv_1^2 + \tfrac{1}{2}I\omega^2$$
$$= \tfrac{1}{2}Mv_1^2 + \tfrac{1}{2}\left(\tfrac{2}{5}Mr^2\right)\frac{v_1^2}{r^2}$$
$$= \tfrac{7}{10}Mv_1^2 = \tfrac{5}{14}Mv_0^2$$

Express the ratio of the final and initial kinetic energies:

$$\frac{K_f}{K_i} = \frac{\frac{5}{14}Mv_0^2}{\frac{1}{2}Mv_0^2} = \boxed{\frac{5}{7}}$$

(*c*) Substitute in the expressions in (*a*) to obtain:

$$s_1 = \frac{12}{49}\frac{(8\,\text{m/s})^2}{(0.06)(9.81\,\text{m/s}^2)} = \boxed{26.6\,\text{m}}$$

$$t_1 = \frac{2}{7}\frac{8\,\text{m/s}}{(0.06)(9.81\,\text{m/s}^2)} = \boxed{3.88\,\text{s}}$$

$$v_1 = \frac{5}{7}(8\,\text{m/s}) = \boxed{5.71\,\text{m/s}}$$

***102 ••**

Picture the Problem The cue stick's blow delivers a rotational impulse as well as a translational impulse to the cue ball. The rotational impulse changes the angular momentum of the ball and the translational impulse changes its linear momentum.

Express the rotational impulse P_{rot} as the product of the average torque and the time during which the rotational impulse acts:

$$P_{rot} = \tau_{av}\Delta t$$

Express the average torque it produces about an axis through the center of the ball:

$$\tau_{av} = P_0(h-r)\sin\theta = P_0(h-r)$$

where θ (= 90°) is the angle between F and the lever arm $h - r$.

Substitute in the expression for P_{rot} to obtain:

$$P_{rot} = P_0(h-r)\Delta t = (P_0\Delta t)(h-r)$$
$$= P_{trams}(h-r) = \Delta L = I\omega_0$$

The translational impulse is also given by:

$$P_{trans} = P_0\Delta t = \Delta p = mv_0$$

Substitute to obtain:

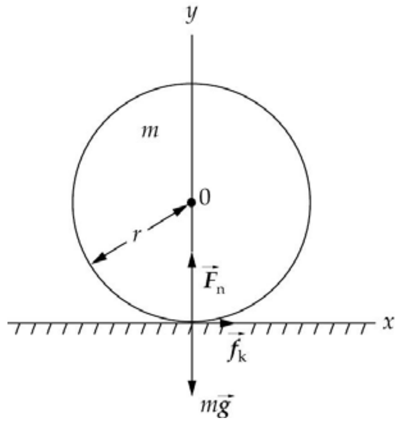
$$mv_0(h-r) = \tfrac{2}{5}mr^2\omega_0$$

Solve for ω_0:

$$\omega_0 = \boxed{\frac{5v_0(h-r)}{2r^2}}$$

103 ••

Picture the Problem The angular velocity of the rotating sphere will decrease until the condition for rolling without slipping is satisfied and then it will begin to roll. The force diagram shows the forces acting on the sphere. We can apply Newton's 2nd law to the sphere and use the condition for rolling without slipping to find the speed of the center of mass when the sphere begins to roll without slipping.



Relate the velocity of the sphere when it begins to roll to its acceleration and the elapsed time:

$$v = a\Delta t \quad (1)$$

Apply Newton's 2nd law to the sphere:

$$\sum F_x = f_k = ma\ , \quad (2)$$

$$\sum F_y = F_n - mg = 0\,, \quad (3)$$

and

$$\sum \tau_0 = f_k r = I_0\alpha \quad (4)$$

Using the definition of f_k and F_n from equation (3), substitute in equation (2) and solve for a:

$$a = \mu_k g$$

Substitute in equation (1) to obtain:

$$v = a\Delta t = \mu_k g\Delta t \quad (5)$$

Solve for α in equation (4):

$$\alpha = \frac{f_k r}{I_0} = \frac{mar}{\frac{2}{5}mr^2} = \frac{5}{2}\frac{\mu_k g}{r}$$

Express the angular speed of the sphere when it has been moving for a time Δt:

$$\omega = \omega_0 - \alpha\,\Delta t = \omega_0 - \frac{5\mu_k g}{2r}\Delta t \quad (6)$$

Express the condition that the sphere rolls without slipping:

$$v = r\omega$$

Substitute from equations (5) and (6) and solve for the elapsed time until the sphere begins to roll:

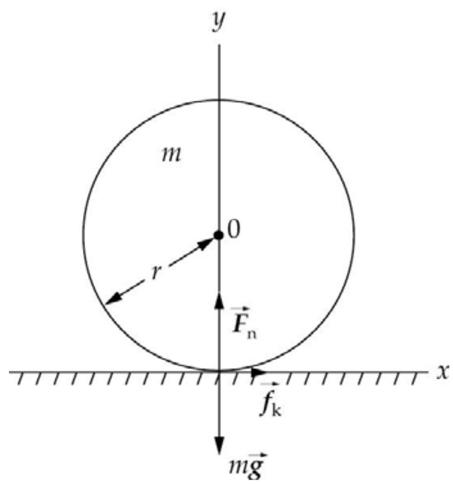
$$\Delta t = \frac{2}{7}\frac{r\omega_0}{\mu_k g}$$

Use equation (4) to find v when the sphere begins to roll:

$$v = \mu_k g\Delta t = \frac{2}{7}\frac{r\omega_0\mu_k g}{\mu_k g} = \boxed{\frac{2r\omega_0}{7}}$$

104 ••

Picture the Problem The sharp force delivers a rotational impulse as well as a translational impulse to the ball. The rotational impulse changes the angular momentum of the ball and the translational impulse changes its linear momentum. In parts (*c*) and (*d*) we can apply Newton's 2nd law to the ball to obtain equations describing both the translational and rotational motion of the ball. We can then solve these equations to find the constant accelerations that allow us to apply constant-acceleration equations to find the velocity of the ball when it begins to roll and its sliding time.



(*a*) Relate the translational impulse delivered to the ball to its change in its momentum:

$$P_{\text{trans}} = F_{\text{av}}\Delta t = \Delta p = mv_0$$

Solve for v_0:

$$v_0 = \frac{F_{\text{av}}\Delta t}{m}$$

Substitute numerical values and evaluate v_0:

$$v_0 = \frac{(20\,\text{kN})(2\times10^{-4}\,\text{s})}{0.02\,\text{kg}} = \boxed{200\,\text{m/s}}$$

(*b*) Express the rotational impulse P_{rot} as the product of the average torque and the time during which the rotational impulse acts:

$$P_{\text{rot}} = \tau_{\text{av}}\Delta t$$

Letting h be the height at which the impulsive force is delivered, express the average torque it produces about an axis through the center of the ball:

$$\tau_{\text{av}} = F\ell\sin\theta$$

where θ is the angle between F and the lever arm ℓ.

Substitute $h - r$ for ℓ and 90° for θ

$$\tau_{\text{av}} = F(h - r)$$

to obtain:

Substitute in the expression for P_{rot} to obtain:

$$P_{\text{rot}} = F(h-r)\Delta t$$

Because $P_{\text{trans}} = F\Delta t$:

$$P_{\text{rot}} = P_{\text{trans}}(h-r) = \Delta L = I\omega_0$$
$$= \tfrac{2}{5}mr^2\omega_0$$

Express the translational impulse delivered to the cue ball:

$$P_{\text{trans}} = P_0\Delta t = \Delta p = mv_0$$

Substitute for P_{trans} to obtain:

$$\tfrac{2}{5}mr^2\omega_0 = mv_0$$

Solve for ω_0:

$$\omega_0 = \frac{5v_0(h-r)}{2r^2}$$

Substitute numerical values and evaluate ω_0:

$$\omega_0 = \frac{5(200\,\text{m/s})(0.09\,\text{m} - 0.05\,\text{m})}{2(.05\,\text{m})^2}$$
$$= \boxed{8000\,\text{rad/s}}$$

(*c*) and (*d*) Relate the velocity of the ball when it begins to roll to its acceleration and the elapsed time:

$$v = a\Delta t \qquad (1)$$

Apply Newton's 2nd law to the ball:

$$\sum F_x = f_{\text{k}} = ma\,, \qquad (2)$$
$$\sum F_y = F_{\text{n}} - mg = 0\,, \qquad (3)$$

and

$$\sum \tau_0 = f_{\text{k}}r = I_0\alpha \qquad (4)$$

Using the definition of f_{k} and F_{n} from equation (3), substitute in equation (2) and solve for a:

$$a = \mu_{\text{k}}g$$

Substitute in equation (1) to obtain:

$$v = a\Delta t = \mu_{\text{k}}g\Delta t \qquad (5)$$

Solve for α in equation (4):

$$\alpha = \frac{f_{\text{k}}r}{I_0} = \frac{mar}{\tfrac{2}{5}mr^2} = \frac{5}{2}\frac{\mu_{\text{k}}g}{r}$$

Express the angular speed of the ball when it has been moving for a time Δt:

$$\omega = \omega_0 - \alpha\,\Delta t = \omega_0 - \frac{5\mu_k g}{2r}\Delta t \quad (6)$$

Express the speed of the ball when it has been moving for a time Δt:

$$v = v_0 + \mu_k g\Delta t \quad (7)$$

Express the condition that the ball rolls without slipping:

$$v = r\omega$$

Substitute from equations (6) and (7) and solve for the elapsed time until the ball begins to roll:

$$\Delta t = \frac{2}{7}\frac{r\omega_0 - v_0}{\mu_k g}$$

Substitute numerical values and evaluate Δt:

$$\Delta t = \frac{2}{7}\left[\frac{(0.05\,\text{m})(8000\,\text{rad/s}) - 200\,\text{m/s}}{(0.5)(9.81\,\text{m/s}^2)}\right] = \boxed{11.6\,\text{s}}$$

Use equation (4) to express v when the ball begins to roll:

$$v = v_0 + \mu_k g\Delta t$$

Substitute numerical values and evaluate v:

$$v = 200\,\text{m/s} + (0.5)(9.81\,\text{m/s}^2)(11.6\,\text{s}) = \boxed{257\,\text{m/s}}$$

105 ••

Picture the Problem Because the impulse is applied through the center of mass, $\omega_0 = 0$. We can use the results of Example 9-16 to find the rolling time without slipping, the distance traveled to rolling without slipping, and the velocity of the ball once it begins to roll without slipping.

(*a*) From Example 9-16 we have:

$$t_1 = \frac{2}{7}\frac{v_0}{\mu_k g}$$

Substitute numerical values and evaluate t_1:

$$t_1 = \frac{2}{7}\frac{4\,\text{m/s}}{(0.6)(9.81\,\text{m/s}^2)} = \boxed{0.194\,\text{s}}$$

(*b*) From Example 9-16 we have:

$$s_1 = \frac{12}{49}\frac{v_0^2}{\mu_k g}$$

Substitute numerical values and evaluate s_1:

$$s_1 = \frac{12}{49}\frac{(4\,\text{m/s})^2}{(0.6)(9.81\,\text{m/s}^2)} = \boxed{0.666\,\text{m}}$$

(*c*) From Example 9-16 we have:

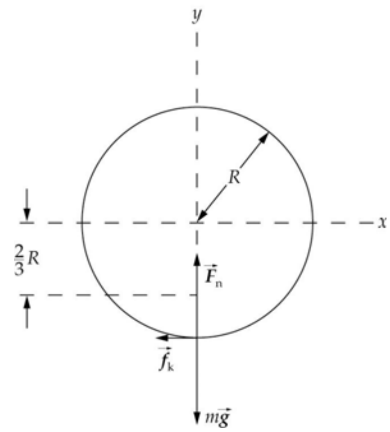
$$v_1 = \frac{5}{7}v_0$$

Substitute numerical values and evaluate v_1:

$$v_1 = \frac{5}{7}(4\,\text{m/s}) = \boxed{2.86\,\text{m/s}}$$

106 ••

Picture the Problem Because the impulsive force is applied below the center line, the spin is backward, i.e., the ball will slow down. We'll use the impulse-momentum theorem and Newton's 2nd law to find the linear and rotational velocities and accelerations of the ball and constant-acceleration equations to relate these quantities to each other and to the elapsed time to rolling without slipping.



(*a*) Express the rotational impulse delivered to the ball:

$$P_{\text{rot}} = mv_0 r = mv_0\frac{2R}{3} = I_{\text{cm}}\omega_0 = \left(\tfrac{2}{5}mR^2\right)\omega_0$$

Solve for ω_0:

$$\omega_0 = \boxed{\frac{5}{3}\frac{v_0}{R}}$$

(*b*) Apply Newton's 2nd law to the ball to obtain:

$$\sum \tau_0 = f_k R = I_{\text{cm}}\alpha\,, \qquad (1)$$

$$\sum F_y = F_n - mg = 0\,, \qquad (2)$$

and

$$\sum F_x = -f_k = ma \qquad (3)$$

Using the definition of f_k and F_n from equation (2), solve for α:

$$\alpha = \frac{\mu_k mgR}{I_{\text{cm}}} = \frac{\mu_k mgR}{\frac{2}{5}mR^2} = \frac{5\mu_k g}{2R}$$

Using a constant-acceleration equation, relate the angular speed of the ball to its acceleration:

$$\omega = \omega_0 + \alpha\Delta t = \omega_0 + \frac{5\mu_k g}{2R}\Delta t$$

Using the definition of f_k and F_n from equation (2), solve equation (3) for a:

$$a = -\mu_k g$$

Using a constant-acceleration equation, relate the speed of the ball to its acceleration:

$$v = v_0 + a\Delta t = v_0 - \mu_k g\Delta t \qquad (4)$$

Impose the condition for rolling without slipping to obtain:

$$R\left(\omega_0 + \frac{5\mu_k g}{2R}\Delta t\right) = v_0 - \mu_k g\Delta t$$

Solve for Δt:

$$\Delta t = \frac{16}{21}\frac{v_0}{\mu_k g}$$

Substitute in equation (4) to obtain:

$$v = v_0 - \mu_k g\left(\frac{16}{21}\frac{v_0}{\mu_k}g\right) = \frac{5}{21}v_0$$

$$= \boxed{0.238v_0}$$

(*c*) Express the initial kinetic energy of the ball:

$$K_i = K_{trans} + K_{rot} = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2$$

$$= \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\left(\frac{5v_0}{3R}\right)^2 = \frac{19}{18}mv_0^2$$

$$= \boxed{1.056mv_0^2}$$

(*d*) Express the work done by friction in terms of the initial and final kinetic energies of the ball:

$$W_{fr} = K_i - K_f$$

Express the final kinetic energy of the ball:

$$K_f = \tfrac{1}{2}mv^2 + \tfrac{1}{2}I_{cm}\omega^2$$

$$= \tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{2}{5}mR^2\right)\frac{v^2}{R^2} = \tfrac{7}{10}mv^2$$

$$= \tfrac{7}{10}m(0.238v_0)^2 = 0.0397mv_0^2$$

Substitute to find W_{fr}:

$$W_{fr} = 1.056mv_0^2 - 0.0397mv_0^2$$

$$= \boxed{1.016mv_0^2}$$

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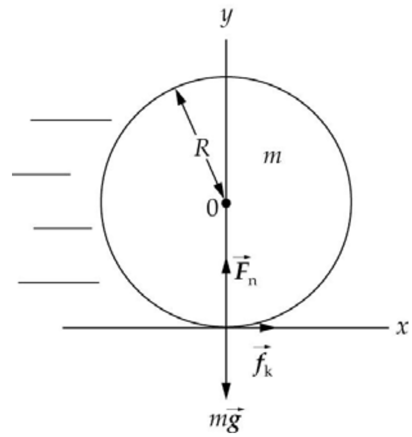
Picture the Problem The figure shows the forces acting on the bowling during the sliding phase of its motion. Because the ball has a forward spin, the friction force is in the direction of motion and will cause the ball's translational speed to increase. We'll apply Newton's 2nd law to find the linear and rotational velocities and accelerations of the ball and constant-acceleration equations to relate these quantities to each other and to the elapsed time to rolling without slipping.



(*a*) and (*b*) Relate the velocity of the ball when it begins to roll to its acceleration and the elapsed time:

$$v = v_0 + a\Delta t \quad (1)$$

Apply Newton's 2nd law to the ball:

$$\sum F_x = f_k = ma\,, \quad (2)$$

$$\sum F_y = F_n - mg = 0\,, \quad (3)$$

and

$$\sum \tau_0 = f_k R = I_0 \alpha \quad (4)$$

Using the definition of f_k and F_n from equation (3), substitute in equation (2) and solve for a:

$$a = \mu_k g$$

Substitute in equation (1) to obtain:

$$v = v_0 + a\Delta t = v_0 + \mu_k g \Delta t \quad (5)$$

Solve for α in equation (4):

$$\alpha = \frac{f_k R}{I_0} = \frac{maR}{\frac{2}{5}mR^2} = \frac{5}{2}\frac{\mu_k g}{R}$$

Relate the angular speed of the ball to its acceleration:

$$\omega = \omega_0 - \frac{5}{2}\frac{\mu_k g}{R}\Delta t$$

Apply the condition for rolling without slipping:

$$v = R\omega = R\left(\omega_0 - \frac{5}{2}\frac{\mu_k g}{R}\Delta t\right)$$

$$= R\left(\frac{3v_0}{R} - \frac{5}{2}\frac{\mu_k g}{R}\Delta t\right)$$

$$\therefore v = 3v_0 - \frac{5}{2}\mu_k g\Delta t \qquad (6)$$

Equate equations (5) and (6) and solve Δt:

$$\Delta t = \boxed{\frac{4}{7}\frac{v_0}{\mu_k g}}$$

Substitute for Δt in equation (6) to obtain:

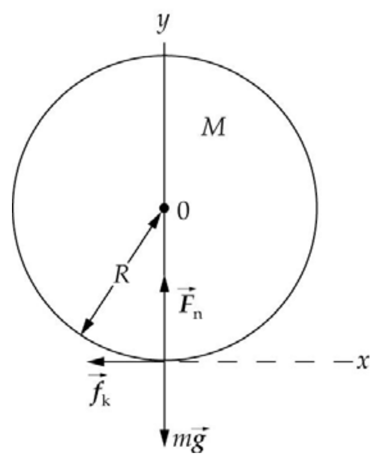
$$v = \frac{11}{7}v_0 = \boxed{1.57v_0}$$

(c) Relate Δx to the average speed of the ball and the time it moves before beginning to roll without slipping:

$$\Delta x = v_{av}\Delta t = \tfrac{1}{2}(v_0 + v)\Delta t$$
$$= \tfrac{1}{2}\left(v_0 + \frac{11}{7}v_0\right)\left(\frac{4v_0}{7\mu_k g}\right)$$
$$= \frac{36}{49}\frac{v_0^2}{\mu_k g} = \boxed{0.735\frac{v_0^2}{\mu_k g}}$$

***108 ••**

Picture the Problem The figure shows the forces acting on the cylinder during the sliding phase of its motion. The friction force will cause the cylinder's translational speed to decrease and eventually satisfy the condition for rolling without slipping. We'll use Newton's 2nd law to find the linear and rotational velocities and accelerations of the ball and constant-acceleration equations to relate these quantities to each other and to the distance traveled and the elapsed time until the satisfaction of the condition for rolling without slipping.



(a) Apply Newton's 2nd law to the cylinder:

$$\sum F_x = -f_k = Ma\,, \qquad (1)$$
$$\sum F_y = F_n - Mg = 0\,, \qquad (2)$$

and

$$\sum \tau_0 = f_k R = I_0\alpha \qquad (3)$$

Use $f_k = \mu_k F_n$ to eliminate F_n between equations (1) and (2) and solve for a:

$$a = -\mu_k g$$

Using a constant-acceleration equation, relate the speed of the cylinder to its acceleration and the elapsed time:

$$v = v_0 + a\Delta t = v_0 - \mu_k g\Delta t$$

Similarly, eliminate f_k between equations (2) and (3) and solve for α:

$$\alpha = \frac{2\mu_k g}{R}$$

Using a constant-acceleration equation, relate the angular speed of the cylinder to its acceleration and the elapsed time:

$$\omega = \omega_0 + \alpha\Delta t = \frac{2\mu_k g}{R}\Delta t$$

Apply the condition for rolling without slipping:

$$v = v_0 - \mu_k g\Delta t = R\omega = R\left(\frac{2\mu_k g}{R}\Delta t\right)$$
$$= 2\mu_k g\Delta t$$

Solve for Δt:

$$\Delta t = \frac{v_0}{3\mu_k g}$$

Substitute for Δt in the expression for v:

$$v = v_0 - \mu_k g\frac{v_0}{3\mu_k g} = \boxed{\frac{2}{3}v_0}$$

(*b*) Relate the distance the cylinder travels to its average speed and the elapsed time:

$$\Delta x = v_{av}\Delta t = \tfrac{1}{2}\left(v_0 + \tfrac{2}{3}v_0\right)\left(\frac{v_0}{3\mu_k g}\right)$$
$$= \boxed{\frac{5}{18}\frac{v_0^2}{\mu_k g}}$$

(*c*) Express the ratio of the energy dissipated in friction to the cylinder's initial mechanical energy:

$$\frac{W_{fr}}{K_i} = \frac{K_i - K_f}{K_i}$$

Express the kinetic energy of the cylinder as it begins to roll without slipping:

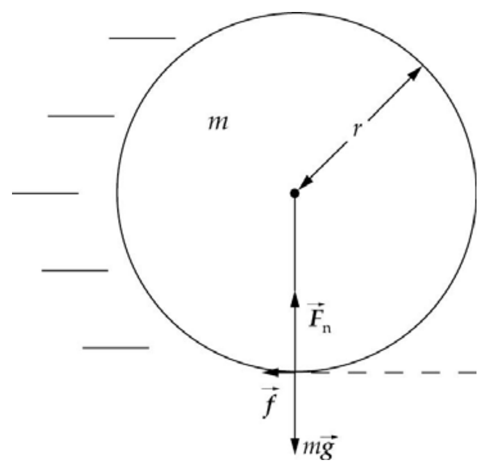
$$K_f = \tfrac{1}{2}Mv^2 + \tfrac{1}{2}I_{cm}\omega^2$$
$$= \tfrac{1}{2}Mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{v^2}{R^2}$$
$$= \frac{3}{4}Mv^2 = \frac{3}{4}M\left(\frac{2}{3}v_0\right)^2 = \frac{1}{3}Mv_0^2$$

Substitute for K_i and K_f and simplify to obtain:

$$\frac{W_{fr}}{K_i} = \frac{\frac{1}{2}Mv_0^2 - \frac{1}{3}Mv_0^2}{\frac{1}{2}Mv_0^2} = \boxed{\frac{1}{3}}$$

109 ••

Picture the Problem The forces acting on the ball as it slides across the floor are its weight $m\vec{g}$, the normal force $\vec{F}_n$ exerted by the floor, and the friction force $\vec{f}$. Because the weight and normal force act through the center of mass of the ball and are equal in magnitude, the friction force is the net (decelerating) force. We can apply Newton's 2nd law in both translational and rotational form to obtain a set of equations that we can solve for the acceleration of the ball. Once we have determined the ball's acceleration, we can use constant-acceleration equations to obtain its velocity when it begins to roll without slipping.



(*a*) Apply $\sum \vec{F} = m\vec{a}$ to the ball:

$$\sum F_x = -f = ma \quad (1)$$

and

$$\sum F_y = F_n - mg = 0 \quad (2)$$

From the definition of the coefficient of kinetic friction we have:

$$f = \mu_k F_n \quad (3)$$

Solve equation (2) for F_n:

$$F_n = mg$$

Substitute in equation (3) to obtain:

$$f = \mu_k mg$$

Substitute in equation (1) to obtain:

$$-\mu_k mg = ma$$

or

$$a = -\mu_k g$$

Apply $\sum \tau = I\alpha$ to the ball:

$$fr = I\alpha$$

Solve for α to obtain:

$$\alpha = \frac{fr}{I} = \frac{\mu_k mgr}{I}$$

Assuming that the coefficient of kinetic friction is constant*, we can use constant-acceleration equations to describe how long it will take the ball to begin

$$v_f - v = a\Delta t = -\mu_k g\Delta t \quad (4)$$

and

$$\omega_f = \frac{\mu_k gmr}{I}\Delta t \quad (5)$$

rolling without slipping:

Once rolling without slipping has been established, we also have:

$$\omega_f = \frac{v_f}{r} \quad (6)$$

Equate equations (5) and (6):

$$\frac{v_f}{r} = \frac{\mu_k gmr}{I}\Delta t$$

Solve for Δt:

$$\Delta t = \frac{v_f I}{\mu_k gmr^2}$$

Substitute in equation (4) to obtain:

$$v_f - v = -\mu_k g\left(\frac{v_f I}{\mu_k gmr^2}\right)$$

$$= -\frac{I}{mr^2}v_f$$

Solve for v_f:

$$\boxed{v_f = \frac{1}{1+\frac{I}{mr^2}}v}$$

(*b*) Express the total kinetic energy of the ball:

$$K = \frac{1}{2}mv_f^2 + \frac{1}{2}I\omega_f^2$$

Because the ball is now rolling without slipping, $v = r\omega_f$ and:

$$K = \frac{1}{2}m\left(\frac{1}{1+I/mr^2}\right)^2 v^2 + \frac{1}{2}I\left(\frac{1}{1+I/mr^2}\right)^2\frac{v^2}{r^2} = \frac{1}{2}mv^2\left((1+I/mr^2)\left(\frac{1}{1+I/mr^2}\right)^2\right)$$

$$= \boxed{\frac{1}{2}mv^2\left(\frac{1}{1+I/mr^2}\right)}$$

*** Remarks: This assumption is not necessary. One can use the impulse-momentum theorem and the related theorem for torque and change in angular momentum to prove that the result holds for an *arbitrary* frictional force acting on the ball, so long as the ball moves along a straight line and the force is directed opposite to the direction of motion of the ball.**

General Problems

***110** •

Picture the Problem The angular velocity of an object is the ratio of the number of revolutions it makes in a given period of time to the elapsed time.

The moon's angular velocity is:

$$\omega = \frac{1\,\text{rev}}{27.3\,\text{days}} = \frac{1\,\text{rev}}{27.3\,\text{days}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{day}}{24\,\text{h}} \times \frac{1\,\text{h}}{3600\,\text{s}} = \boxed{2.66 \times 10^{-6}\,\text{rad/s}}$$

111 •

Picture the Problem The moment of inertia of the hoop, about an axis perpendicular to the plane of the hoop and through its edge, is related to its moment of inertia with respect to an axis through its center of mass by the parallel axis theorem.

Apply the parallel axis theorem:

$$I = I_{\text{cm}} + Mh^2 = MR^2 + MR^2 = \boxed{2mR^2}$$

112 ••

Picture the Problem The force you exert on the rope results in a net torque that accelerates the merry-go-round. The moment of inertia of the merry-go-round, its angular acceleration, and the torque you apply are related through Newton's 2nd law.

(*a*) Using a constant-acceleration equation, relate the angular displacement of the merry-go-round to its angular acceleration and acceleration time:

$$\Delta\theta = \omega_0 \Delta t + \tfrac{1}{2}\alpha(\Delta t)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha(\Delta t)^2$$

Solve for and evaluate α:

$$\alpha = \frac{2\Delta\theta}{(\Delta t)^2} = \frac{2(2\pi\,\text{rad})}{(12\,\text{s})^2} = \boxed{0.0873\,\text{rad/s}^2}$$

(*b*) Use the definition of torque to obtain:

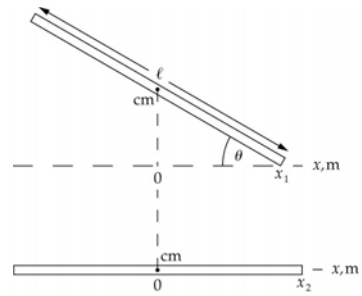
$$\tau = Fr = (260\,\text{N})(2.2\,\text{m}) = \boxed{572\,\text{N}\cdot\text{m}}$$

(*c*) Use Newton's 2nd law to find the moment of inertia of the merry-go-round:

$$I = \frac{\tau_{\text{net}}}{\alpha} = \frac{572\,\text{N}\cdot\text{m}}{0.0873\,\text{rad/s}^2} = \boxed{6.55 \times 10^3\,\text{kg}\cdot\text{m}^2}$$

113 •

Picture the Problem Because there are no horizontal forces acting on the stick, the center of mass of the stick will not move in the horizontal direction. Choose a coordinate system in which the origin is at the horizontal position of the center of mass. The diagram shows the stick in its initial raised position and when it has fallen to the ice.



Express the displacement of the right end of the stick Δx as the difference between the position coordinates x_2 and x_2:

$$\Delta x = x_2 - x_1$$

Using trigonometry, find the initial coordinate of the right end of the stick:

$$x_1 = \ell\cos\theta = (1\,\text{m})\cos 30° = 0.866\,\text{m}$$

Because the center of mass has not moved horizontally:

$$x_2 = \ell = 1\,\text{m}$$

Substitute to find the displacement of the right end of the stick:

$$\Delta x = 1\,\text{m} - 0.866\,\text{m} = \boxed{0.134\,\text{m}}$$

114 ••

Picture the Problem The force applied to the string results in a torque about the center of mass of the disk that accelerates it. We can relate these quantities to the moment of inertia of the disk through Newton's 2nd law and then use constant-acceleration equations to find the disk's angular velocity the angle through which it has rotated in a given period of time. The disk's rotational kinetic energy can be found from its definition.

(*a*) Use the definition of torque to obtain:

$$\tau \equiv FR = (20\,\text{N})(0.12\,\text{m}) = \boxed{2.40\,\text{N}\cdot\text{m}}$$

(*b*) Use Newton's 2nd law to express the angular acceleration of the disk in terms of the net torque acting on it and its moment of inertia:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{\tau_{\text{net}}}{\frac{1}{2}MR^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{2(2.40\,\text{N}\cdot\text{m})}{(5\,\text{kg})(0.12\,\text{m})^2} = \boxed{66.7\,\text{rad/s}^2}$$

(*c*) Using a constant-acceleration equation, relate the angular velocity of the disk to its angular

$$\omega = \omega_0 + \alpha\Delta t$$

or, because $\omega_0 = 0$,

$$\omega = \alpha\Delta t$$

acceleration and the elapsed time:

Substitute numerical values and evaluate ω:

$$\omega = \left(66.7\,\text{rad/s}^2\right)\left(5\,\text{s}\right) = \boxed{333\,\text{rad/s}}$$

(d) Use the definition of rotational kinetic energy to obtain:

$$K_{\text{rot}} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2$$

Substitute numerical values and evaluate K_{rot}:

$$K_{\text{rot}} = \tfrac{1}{4}\left(5\,\text{kg}\right)\left(0.12\,\text{m}\right)^2\left(333\,\text{rad/s}\right)^2 = \boxed{2.00\,\text{kJ}}$$

(e) Using a constant-acceleration equation, relate the angle through which the disk turns to its angular acceleration and the elapsed time:

$$\Delta\theta = \omega_0\Delta t + \tfrac{1}{2}\alpha\left(\Delta t\right)^2$$

or, because $\omega_0 = 0$,

$$\Delta\theta = \tfrac{1}{2}\alpha\left(\Delta t\right)^2$$

Substitute numerical values and evaluate $\Delta\theta$:

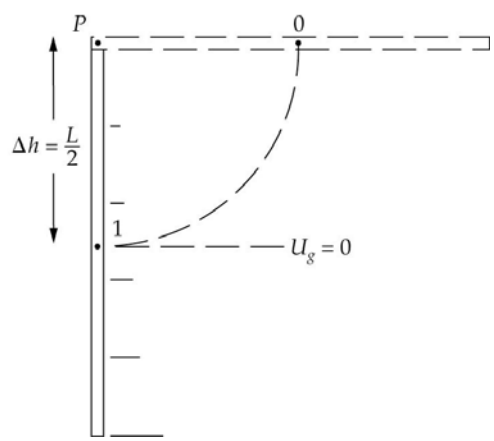
$$\Delta\theta = \tfrac{1}{2}\left(66.7\,\text{rad/s}^2\right)\left(5\,\text{s}\right)^2 = \boxed{834\,\text{rad}}$$

(f) Express K in terms of τ and θ:

$$K = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\frac{\tau}{\alpha}\right)\left(\alpha\Delta t\right)^2 = \tfrac{1}{2}\alpha\tau\left(\Delta t\right)^2 = \boxed{\tau\,\Delta\theta}$$

115 ••

Picture the Problem The diagram shows the rod in its initial horizontal position and then, later, as it swings through its vertical position. The center of mass is denoted by the numerals 0 and 1. Let the length of the rod be represented by L and its mass by m. We can use Newton's 2nd law in rotational form to find, first, the angular acceleration of the rod and then, from α, the acceleration of any point on the rod. We can use conservation of energy to find the angular velocity of the center of mass of the rod when it is vertical and then use this value to find its linear velocity.



(a) Relate the acceleration of the center of the rod to the angular

$$a = \ell\alpha = \frac{L}{2}\alpha$$

acceleration of the rod:

Use Newton's 2nd law to relate the torque about the suspension point of the rod (exerted by the weight of the rod) to the rod's angular acceleration:

$$\alpha = \frac{\tau}{I} = \frac{Mg\frac{L}{2}}{\frac{1}{3}ML^2} = \frac{3g}{2L}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{3(9.81\,\text{m/s}^2)}{2(0.8\,\text{m})} = 18.4\,\text{rad/s}^2$$

Substitute numerical values and evaluate a:

$$a = \tfrac{1}{2}(0.8\,\text{m})(18.4\,\text{rad/s}^2) = \boxed{7.36\,\text{m/s}^2}$$

(*b*) Relate the acceleration of the end of the rod to α:

$$a_{\text{end}} = L\alpha = (0.8\,\text{m})(18.4\,\text{rad/s}^2) = \boxed{14.7\,\text{m/s}^2}$$

(*c*) Relate the linear velocity of the center of mass of the rod to its angular velocity as it passes through the vertical:

$$v = \omega\Delta h = \tfrac{1}{2}\omega L$$

Use conservation of energy to relate the changes in the kinetic and potential energies of the rod as it swings from its initial horizontal orientation through its vertical orientation:

$$\Delta K + \Delta U = K_1 - K_0 + U_1 - U_0 = 0$$

or, because $K_0 = U_1 = 0$,

$$K_1 - U_0 = 0$$

Substitute to obtain:

$$\tfrac{1}{2} I_P\omega^2 = mg\Delta h$$

Substitute for Δh and solve for ω:

$$\omega = \sqrt{\frac{3g}{L}}$$

Substitute to obtain:

$$v = \tfrac{1}{2}L\sqrt{\frac{3g}{L}} = \tfrac{1}{2}\sqrt{3gL}$$

Substitute numerical values and evaluate v:

$$v = \tfrac{1}{2}\sqrt{3(9.81\,\text{m/s}^2)(0.8\,\text{m})} = \boxed{2.43\,\text{m/s}}$$

116 ••

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the track. The initial potential energy of the marble is transformed into translational and rotational kinetic energy as it rolls down the track to its lowest point and then, because the portion of the track to the right is frictionless, into translational kinetic energy and, eventually, into gravitational potential energy.

Using conservation of energy, relate h_2 to the kinetic energy of the marble at the bottom of the track:

$$\Delta K + \Delta U = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f to obtain:

$$-\tfrac{1}{2}Mv^2 - Mgh_2 = 0$$

Solve for h_2:

$$h_2 = \frac{v^2}{2g} \qquad (1)$$

Using conservation of energy, relate h_1 to the kinetic energy of the marble at the bottom of the track:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_f - U_i = 0$$

Substitute for K_f and U_i to obtain:

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 - Mgh_1 = 0$$

Substitute for I and solve for v^2 to obtain:

$$v^2 = \tfrac{10}{7}gh_1$$

Substitute in equation (1) to obtain:

$$h_2 = \frac{\tfrac{10}{7}gh_1}{2g} = \boxed{\tfrac{5}{7}h_1}$$

***117** ••

Picture the Problem To stop the wheel, the tangential force will have to do an amount of work equal to the initial rotational kinetic energy of the wheel. We can find the stopping torque and the force from the average power delivered by the force during the slowing of the wheel. The number of revolutions made by the wheel as it stops can be found from a constant-acceleration equation.

(*a*) Relate the work that must be done to stop the wheel to its kinetic energy:

$$W = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{1}{2}mr^2\right)\omega^2 = \tfrac{1}{4}mr^2\omega^2$$

Substitute numerical values and evaluate W:

$$W = \tfrac{1}{4}(120\,\text{kg})(1.4\,\text{m})^2 \times \left[1100\frac{\text{rev}}{\text{min}} \times \frac{2\pi\,\text{rad}}{\text{rev}} \times \frac{1\,\text{min}}{60\,\text{s}}\right]^2 = \boxed{780\,\text{kJ}}$$

(*b*) Express the stopping torque is terms of the average power required:

$$P_{\text{av}} = \tau\omega_{\text{av}}$$

Solve for τ:

$$\tau = \frac{P_{\text{av}}}{\omega_{\text{av}}}$$

Substitute numerical values and evaluate τ:

$$\tau = \frac{\dfrac{780\,\text{kJ}}{(2.5\,\text{min})(60\,\text{s/min})}}{\dfrac{(1100\,\text{rev/min})(2\pi\,\text{rad/rev})(1\,\text{min}/60\,\text{s})}{2}} = \boxed{90.3\,\text{N}\cdot\text{m}}$$

Relate the stopping torque to the magnitude of the required force and solve for F:

$$F = \frac{\tau}{R} = \frac{90.3\,\text{N}\cdot\text{m}}{0.6\,\text{m}} = \boxed{151\text{ N}}$$

(*c*) Using a constant-acceleration equation, relate the angular displacement of the wheel to its average angular velocity and the stopping time:

$$\Delta\theta = \omega_{\text{av}}\Delta t$$

Substitute numerical values and evaluate $\Delta\theta$:

$$\Delta\theta = \left(\frac{1100\,\text{rev/min}}{2}\right)(2.5\,\text{min}) = \boxed{1380\,\text{rev}}$$

118 ••

Picture the Problem The work done by the four children on the merry-go-round will change its kinetic energy. We can use the work-energy theorem to relate the work done by the children to the distance they ran and Newton's 2nd law to find the angular acceleration of the merry-go-round.

(*a*) Use the work-kinetic energy theorem to relate the work done by the children to the kinetic energy of the merry-go-round:

$$W_{\text{net force}} = \Delta K$$
$$= K_{\text{f}}$$

or

$$4F\Delta s = \tfrac{1}{2}I\omega^2$$

Substitute for I and solve for Δs to obtain:

$$\Delta s = \frac{I\omega^2}{8F} = \frac{\tfrac{1}{2}mr^2\omega^2}{8F} = \frac{mr^2\omega^2}{16F}$$

Substitute numerical values and evaluate Δs:

$$\Delta s = \frac{(240\,\text{kg})(2\,\text{m})^2\left[\frac{1\,\text{rev}}{2.8\,\text{s}}\times\frac{2\pi\,\text{rad}}{\text{rev}}\right]^2}{16(26\,\text{N})}$$
$$= \boxed{11.6\,\text{m}}$$

(*b*) Apply Newton's 2nd law to express the angular acceleration of the merry-go-round:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{4Fr}{\tfrac{1}{2}mr^2} = \frac{8F}{mr}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{8(26\,\text{N})}{(240\,\text{kg})(2\,\text{m})} = \boxed{0.433\,\text{rad/s}^2}$$

(*c*) Use the definition of work to relate the force exerted by each child to the distance over which that force is exerted:

$$W = F\Delta s = (26\,\text{N})(11.6\,\text{m}) = \boxed{302\,\text{J}}$$

(*d*) Relate the kinetic energy of the merry-go-round to the work that was done on it:

$$W_{\text{net force}} = \Delta K = K_{\text{f}} - 0 = 4F\Delta s$$

Substitute numerical values and evaluate $W_{\text{net force}}$:

$$W_{\text{net force}} = 4(26\,\text{N})(11.6\,\text{m}) = \boxed{1.21\,\text{kJ}}$$

119 ••

Picture the Problem Because the center of mass of the hoop is at its center, we can use Newton's second law to relate the acceleration of the hoop to the net force acting on it. The distance moved by the center of the hoop can be determined using a constant-acceleration equation, as can the angular velocity of the hoop.

(*a*) Using a constant-acceleration equation, relate the distance the

$$\Delta s = \tfrac{1}{2}a_{\text{cm}}(\Delta t)^2$$

center of the travels in 3 s to the acceleration of its center of mass:

Relate the acceleration of the center of mass of the hoop to the net force acting on it:

$$a_{cm} = \frac{F_{net}}{m}$$

Substitute to obtain:

$$\Delta s = \frac{F(\Delta t)^2}{2m}$$

Substitute numerical values and evaluate Δs:

$$\Delta s = \frac{(5\,\text{N})(3\,\text{s})^2}{2(1.5\,\text{kg})} = \boxed{15.0\,\text{m}}$$

(*b*) Relate the angular velocity of the hoop to its angular acceleration and the elapsed time:

$$\omega = \alpha\,\Delta t$$

Use Newton's 2nd law to relate the angular acceleration of the hoop to the net torque acting on it:

$$\alpha = \frac{\tau_{net}}{I} = \frac{FR}{mR^2} = \frac{F}{mR}$$

Substitute to obtain:

$$\omega = \frac{F\Delta t}{mR}$$

Substitute numerical values and evaluate ω:

$$\omega = \frac{(5\,\text{N})(3\,\text{s})}{(1.5\,\text{kg})(0.65\,\text{m})} = \boxed{15.4\,\text{rad/s}}$$

120 ••

Picture the Problem Let R represent the radius of the grinding wheel, M its mass, r the radius of the handle, and m the mass of the load attached to the handle. In the absence of information to the contrary, we'll treat the 25-kg load as though it were concentrated at a point. Let the zero of gravitational potential energy be where the 25-kg load is at its lowest point. We'll apply Newton's 2nd law and the conservation of mechanical energy to determine the initial angular acceleration and the maximum angular velocity of the wheel.

(*a*) Use Newton's 2nd law to relate the acceleration of the wheel to the net torque acting on it:

$$\alpha = \frac{\tau_{net}}{I} = \frac{mgr}{\frac{1}{2}MR^2 + mr^2}$$

Substitute numerical values and evaluate α:

$$\alpha = \frac{(25\,\text{kg})(9.81\,\text{m/s}^2)(0.65\,\text{m})}{\frac{1}{2}(60\,\text{kg})(0.45\,\text{m})^2 + (25\,\text{kg})(0.65\,\text{m})^2} = \boxed{9.58\,\text{rad/s}^2}$$

(*b*) Use the conservation of mechanical energy to relate the initial potential energy of the load to its kinetic energy and the rotational kinetic energy of the wheel when the load is directly below the center of mass of the wheel:

$$\Delta K + \Delta U = 0$$

or, because $K_\text{i} = U_\text{f} = 0$,

$$K_\text{f,trans} + K_\text{f,rot} - U_\text{i} = 0\,.$$

Substitute and solve for ω:

$$\tfrac{1}{2}mv^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\omega^2 - mgr = 0\,,$$

$$\tfrac{1}{2}mr^2\omega^2 + \tfrac{1}{4}MR^2\omega^2 - mgr = 0\,,$$

and

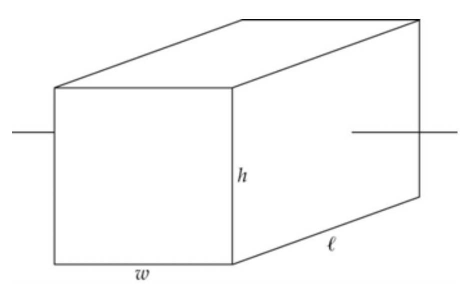
$$\omega = \sqrt{\frac{4mgr}{2mr^2 + MR^2}}$$

Substitute numerical values and evaluate ω:

$$\omega = \sqrt{\frac{4(25\,\text{kg})(9.81\,\text{m/s}^2)(0.65\,\text{m})}{2(25\,\text{kg})(0.65\,\text{m})^2 + (60\,\text{kg})(0.45\,\text{m})^2}} = \boxed{4.38\,\text{rad/s}}$$

***121** ••

Picture the Problem Let the smaller block have the dimensions shown in the diagram. Then the length, height, and width of the larger block are $S\ell$, Sh, and Sw, respectively. Let the numeral 1 denote the smaller block and the numeral 2 the larger block and express the ratios of the surface areas, masses, and moments of inertia of the two blocks.



(*a*) Express the ratio of the surface areas of the two blocks:

$$\frac{A_2}{A_1} = \frac{2(Sw)(S\ell) + 2(S\ell)(Sh) + 2(Sw)(Sh)}{2w\ell + 2\ell h + 2wh} = \frac{S^2(2w\ell + 2\ell h + 2wh)}{2w\ell + 2\ell h + 2wh} = \boxed{S^2}$$

(*b*) Express the ratio of the masses of the two blocks:

$$\frac{M_2}{M_1}=\frac{\rho V_2}{\rho V_1}=\frac{V_2}{V_1}=\frac{(Sw)(S\ell)(Sh)}{w\ell h}$$
$$=\frac{S^3(w\ell h)}{w\ell h}=\boxed{S^3}$$

(*c*) Express the ratio of the moments of inertia, about the axis shown in the diagram, of the two blocks:

$$\frac{I_2}{I_1}=\frac{\frac{1}{12}M_2\left[(S\ell)^2+(Sh)^2\right]}{\frac{1}{12}M_1\left[\ell^2+h^2\right]}$$
$$=\frac{M_2}{M_1}\frac{S^2\left[\ell^2+h^2\right]}{\left[\ell^2+h^2\right]}=\left(\frac{M_2}{M_1}\right)\left(S^2\right)$$

In part (*b*) we showed that:

$$\frac{M_2}{M_1}=S^3$$

Substitute to obtain:

$$\frac{I_2}{I_1}=\left(S^3\right)\left(S^2\right)=\boxed{S^5}$$

122 ••

Picture the Problem We can derive the perpendicular-axis theorem for planar objects by following the step-by-step procedure outlined in the problem.

(*a*) and (*b*)

$$I_z=\int r^2dm=\int\left(x^2+y^2\right)dm$$
$$=\int x^2dm+\int y^2dm$$
$$=\boxed{I_x+I_y}$$

(*c*) Let the *z* axis be the axis of rotation of the disk. By symmetry:

$$I_x=I_y$$

Express I_z in terms of I_x:

$$I_z=2I_x$$

Letting *M* represent the mass of the disk, solve for I_x:

$$I_x=\tfrac{1}{2}I_z=\tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)=\boxed{\tfrac{1}{4}MR^2}$$

123 ••

Picture the Problem Let the zero of gravitational potential energy be at the center of the disk when it is directly below the pivot. The initial gravitational potential energy of the disk is transformed into rotational kinetic energy when its center of mass is directly below the pivot. We can use Newton's 2nd law to relate the force exerted by the pivot to the weight of the disk and the centripetal force acting on it at its lowest point.

(*a*) Use the conservation of mechanical energy to relate the initial potential energy of the disk to its kinetic energy when its center of mass is directly below the pivot:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_{f,rot} - U_i = 0$$

Substitute for $K_{f,rot}$ and U_i:

$$\tfrac{1}{2} I\omega^2 - Mgr = 0 \qquad (1)$$

Use the parallel-axis theorem to relate the moment of inertia of the disk about the pivot to its moment of inertia with respect to an axis through its center of mass:

$$I = I_{cm} + Mh^2$$

or

$$I = \tfrac{1}{2}Mr^2 + Mr^2 = \tfrac{3}{2}Mr^2$$

Solve equation (1) for ω and substitute for I to obtain:

$$\omega = \boxed{\sqrt{\frac{4g}{3r}}}$$

(*b*) Letting F represent the force exerted by the pivot, use Newton's 2nd law to express the net force acting on the swinging disk as it passes through its lowest point:

$$F_{net} = F - Mg = Mr\omega^2$$

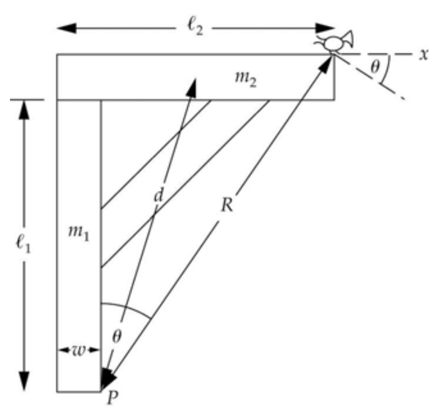
Solve for F and simplify to obtain:

$$F = Mg + Mr\omega^2 = Mg + Mr\frac{4g}{3r}$$

$$= Mg + \tfrac{4}{3}Mg = \boxed{\tfrac{7}{3}Mg}$$

124 ••

Picture the Problem The diagram shows a vertical cross-piece. Because we'll need to take moments about the point of rotation (point *P*), we'll need to use the parallel-axis theorem to find the moments of inertia of the two parts of this composite structure. Let the numeral 1 denote the vertical member and the numeral 2 the horizontal member. We can apply Newton's 2nd law in rotational form to the structure to express its angular acceleration in terms of the net torque causing it to fall and its moment of inertia with respect to point *P*.



(*a*) Taking clockwise rotation to be positive (this is the direction the structure is going to rotate), apply $\sum \tau = I_P \alpha$:

$$m_2 g\left(\frac{\ell_2}{2}\right) - m_1 g\left(\frac{w}{2}\right) = I_P \alpha$$

Solve for α to obtain:

$$\alpha = \frac{m_2 g \ell_2 - m_1 g w}{2 I_P}$$

or

$$\alpha = \frac{g(m_2 \ell_2 - m_1 w)}{2(I_{1P} + I_{2P})} \qquad (1)$$

Convert $\ell_1, \ell_2,$ and w to SI units:

$$\ell_1 = 12\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 3.66\,\text{m},$$

$$\ell_2 = 6\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 1.83\,\text{m}, \text{ and}$$

$$w = 2\,\text{ft} \times \frac{1\,\text{m}}{3.281\,\text{ft}} = 0.610\,\text{m}$$

Using Table 9-1 and the parallel-axis theorem, express the moment of inertia of the vertical member about an axis through point *P*:

$$I_{1P} = \tfrac{1}{3} m_1 \ell_1^2 + m_1\left(\frac{w}{2}\right)^2 = m_1\left(\tfrac{1}{3}\ell_1^2 + \tfrac{1}{4}w^2\right)$$

Substitute numerical values and evaluate I_{1P}:

$$I_{1P} = (350\,\text{kg})\left[\tfrac{1}{3}(3.66\,\text{m})^2 + \tfrac{1}{4}(0.610\,\text{m})^2\right] = 1.60 \times 10^3\,\text{kg}\cdot\text{m}^2$$

Using the parallel-axis theorem, express the moment of inertia of the horizontal member about an axis through point *P*:

$$I_{2P} = I_{2,\text{cm}} + m_2 d^2 \qquad (2)$$

where

$$d^2 = \left(\ell_1 + \tfrac{1}{2}w\right)^2 + \left(\tfrac{1}{2}\ell_2 - w\right)^2$$

Solve for *d*:

$$d = \sqrt{\left(\ell_1 + \tfrac{1}{2}w\right)^2 + \left(\tfrac{1}{2}\ell_2 - w\right)^2}$$

Substitute numerical values and evaluate *d*:

$$d = \sqrt{\left[3.66\,\text{m} + \tfrac{1}{2}(0.610\,\text{m})\right]^2 + \left[\tfrac{1}{2}(1.83\,\text{m}) - 0.610\,\text{m}\right]^2} = 3.86\,\text{m}$$

From Table 9-1 we have:

$$I_{2,\text{cm}} = \tfrac{1}{12} m_2 \ell_2^2$$

Substitute in equation (2) to obtain:

$$I_{2P} = \tfrac{1}{12} m_2 \ell_2^2 + m_2 d^2 = m_2\left(\tfrac{1}{12}\ell_2^2 + d^2\right)$$

Evaluate I_{2P}:

$$I_{2P} = (175\,\text{kg})\left[\tfrac{1}{12}(1.83\,\text{m})^2 + (3.86\,\text{m})^2\right] = 2.66\times10^3\ \text{kg}\cdot\text{m}^2$$

Substitute in equation (1) and evaluate α:

$$\alpha = \frac{(9.81\,\text{m/s}^2)[(175\,\text{kg})(1.83\,\text{m}) - (350\,\text{kg})(0.61\,\text{m})]}{2(1.60+2.66)\times10^3\ \text{kg}\cdot\text{m}^2} = \boxed{0.123\,\text{rad/s}^2}$$

(*b*) Express the magnitude of the acceleration of the sparrow:

$$a = \alpha R$$

where R is the distance of the sparrow from the point of rotation and

$$R^2 = (\ell_1 + w)^2 + (\ell_2 - w)^2$$

Solve for R:

$$R = \sqrt{(\ell_1 + w)^2 + (\ell_2 - w)^2}$$

Substitute numerical values and evaluate R:

$$R = \sqrt{(3.66\,\text{m} + 0.610\,\text{m})^2 + (1.83\,\text{m} - 0.610\,\text{m})^2} = 4.44\,\text{m}$$

Substitute numerical values and evaluate a:

$$a = (0.123\,\text{rad/s}^2)(4.44\,\text{m}) = \boxed{0.546\,\text{m/s}^2}$$

(*c*) Refer to the diagram to express a_x in terms of a:

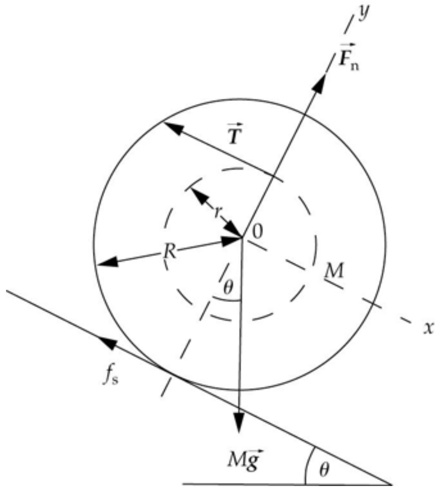
$$a_x = a\cos\theta = a\frac{\ell_1 + w}{R}$$

Substitute numerical values and evaluate a_x:

$$a_x = (0.546\,\text{m/s}^2)\frac{3.66\,\text{m} + 0.61\,\text{m}}{4.44\,\text{m}} = \boxed{0.525\,\text{m/s}^2}$$

125 ••

Picture the Problem Let the zero of gravitational potential energy be at the bottom of the incline. The initial potential energy of the spool is transformed into rotational and translational kinetic energy when the spool reaches the bottom of the incline. We can apply the conservation of mechanical energy to find an expression for its speed at that location. The force diagram shows the forces acting on the spool when there is enough friction to keep it from slipping. We'll use Newton's 2nd law in both translational and rotational form to derive an expression for the static friction force.



(*a*) In the absence of friction, the forces acting on the spool will be its weight, the normal force exerted by the incline, and the tension in the string. A component of its weight will cause the spool to accelerate down the incline and the tension in the string will exert a torque that will cause counterclockwise rotation of the spool.

The spool will move down the plane at constant acceleration, spinning in a counterclockwise direction as string unwinds.

Use the conservation of mechanical energy to relate the speed of the center of mass of the spool at the bottom of the slope to its initial potential energy:

$$\Delta K + \Delta U = 0$$

or, because $K_i = U_f = 0$,

$$K_{f,trans} + K_{f,rot} - U_i = 0.$$

Substitute for $K_{f,trans}$, $K_{f,rot}$ and U_i:

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\omega^2 - MgD\sin\theta = 0 \quad (1)$$

Substitute for ω and solve for v to obtain:

$$\tfrac{1}{2}Mv^2 + \tfrac{1}{2}I\frac{v^2}{r^2} - MgD\sin\theta = 0$$

and

$$v = \sqrt{\frac{2MgD\sin\theta}{M + \frac{I}{r^2}}}$$

(*b*) Apply Newton's 2nd law to the spool:

$$\sum F_x = Mg\sin\theta - T - f_s = 0$$

$$\sum \tau_0 = Tr - f_s R = 0$$

Eliminate T between these equations to obtain:

$$f_s = \boxed{\frac{Mg\sin\theta}{1+\frac{R}{r}}}\text{, up the incline.}$$

126 ••

Picture the Problem While the angular acceleration of the rod is the same at each point along its length, the linear acceleration and, hence, the force exerted on each coin by the rod, varies along its length. We can relate this force the linear acceleration of the rod through Newton's 2nd law and the angular acceleration of the rod.

Letting x be the distance from the pivot, use Newton's 2nd law to express the force F acting on a coin:

$$F_{\text{net}} = mg - F(x) = ma(x)$$

or

$$F(x) = m(g - a(x)) \qquad (1)$$

Use Newton's 2nd law to relate the angular acceleration of the system to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{Mg\frac{L}{2}}{\frac{1}{3}ML^2} = \frac{3g}{2L}$$

Relate $a(x)$ and α:

$$a(x) = x\alpha = x\frac{3g}{2(1.5\,\text{m})} = gx$$

Substitute in equation (1) to obtain:

$$F(x) = m(g - gx) = mg(1 - x)$$

Evaluate $F(0.25\text{ m})$:

$$F(0.25\,\text{m}) = mg(1 - 0.25\,\text{m}) = \boxed{0.75mg}$$

Evaluate $F(0.5\text{ m})$:

$$F(0.5\,\text{m}) = mg(1 - 0.5\,\text{m}) = \boxed{0.5mg}$$

Evaluate $F(0.75\text{ m})$:

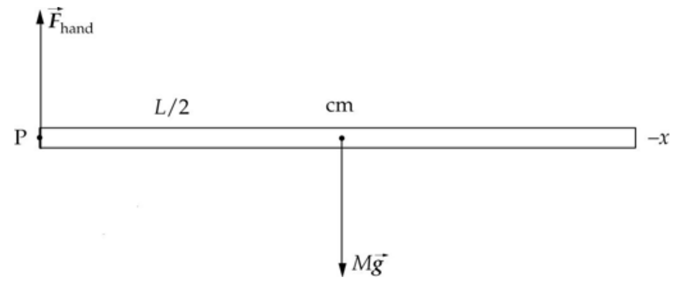
$$F(0.75\,\text{m}) = mg(1 - 0.75\,\text{m}) = \boxed{0.25mg}$$

Evaluate $F(1\text{ m})$:

$$F(1\,\text{m}) = F(1.25\,\text{m}) = F(1.5\,\text{m}) = \boxed{0}$$

***127** ••

Picture the Problem The diagram shows the force the hand supporting the meterstick exerts at the pivot point and the force the earth exerts on the meterstick acting at the center of mass. We can relate the angular acceleration to the acceleration of the end of the meterstick using $a = L\alpha$ and use Newton's 2nd law in rotational form to relate α to the moment of inertia of the meterstick.



(*a*) Relate the acceleration of the far end of the meterstick to the angular acceleration of the meterstick:

$$a = L\alpha \quad (1)$$

Apply $\sum \tau_P = I_P\alpha$ to the meterstick:

$$Mg\left(\frac{L}{2}\right) = I_P\alpha$$

Solve for α:

$$\alpha = \frac{MgL}{2I_P}$$

From Table 9-1, for a rod pivoted at one end, we have:

$$I_P = \frac{1}{3}ML^2$$

Substitute to obtain:

$$\alpha = \frac{3MgL}{2ML^2} = \frac{3g}{2L}$$

Substitute in equation (1) to obtain:

$$a = \frac{3g}{2}$$

Substitute numerical values and evaluate *a*:

$$a = \frac{3(9.81\,\text{m/s}^2)}{2} = \boxed{14.7\,\text{m/s}^2}$$

(*b*) Express the acceleration of a point on the meterstick a distance *x* from the pivot point:

$$a = \alpha x = \frac{3g}{2L}x$$

Express the condition that the meterstick leaves the penny behind:

$$a > g$$

Substitute to obtain:

$$\frac{3g}{2L}x > g$$

Solve for and evaluate *x*:

$$x > \frac{2L}{3} = \frac{2(1\,\text{m})}{3} = \boxed{66.7\,\text{cm}}$$

128 ••

Picture the Problem Let *m* represent the 0.2-kg mass, *M* the 0.8-kg mass of the cylinder, *L* the 1.8-m length, and $x + \Delta x$ the distance from the center of the objects whose mass is *m*. We can use Newton's 2nd law to relate the radial forces on the masses to the spring's stiffness constant and use the work-energy theorem to find the work done as the system accelerates to its final angular speed.

(*a*) Express the net inward force acting on each of the 0.2-kg masses:

$$\sum F_{\text{radial}} = k\Delta x = m(x+\Delta x)\omega^2$$

Solve for *k*:

$$k = \frac{m(x+\Delta x)\omega^2}{\Delta x}$$

Substitute numerical values and evaluate *k*:

$$k = \frac{(0.2\,\text{kg})(0.8\,\text{m})(24\,\text{rad/s})^2}{0.4\,\text{m}} = \boxed{230\,\text{N/m}}$$

(*b*) Using the work-energy theorem, relate the work done to the change in energy of the system:

$$W = K_{\text{rot}} + \Delta U_{\text{spring}} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}k(\Delta x)^2 \qquad (1)$$

Express *I* as the sum of the moments of inertia of the cylinder and the masses:

$$I = I_M + I_{2m} = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2I_m$$

From Table 9-1 we have, for a solid cylinder about a diameter through its center:

$$I = \tfrac{1}{4}mr^2 + \tfrac{1}{12}mL^2$$

where *L* is the length of the cylinder.

For a disk (thin cylinder), *L* is small and:

$$I = \tfrac{1}{4}mr^2$$

Apply the parallel-axis theorem to obtain:

$$I_m = \tfrac{1}{4}mr^2 + mx^2$$

Substitute to obtain:

$$I = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2\left(\tfrac{1}{4}mr^2 + mx^2\right) = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2m\left(\tfrac{1}{4}r^2 + x^2\right)$$

Substitute numerical values and evaluate *I*:

$$I = \tfrac{1}{2}(0.8\,\text{kg})(0.2\,\text{m})^2 + \tfrac{1}{12}(0.8\,\text{kg})(1.8\,\text{m})^2 + 2(0.2\,\text{kg})\left[\tfrac{1}{4}(0.2\,\text{m})^2 + (0.8\,\text{m})^2\right]$$
$$= 0.492\,\text{N}\cdot\text{m}^2$$

Substitute in equation (1) to obtain:

$$W = \tfrac{1}{2}(0.492\,\text{N}\cdot\text{m}^2)(24\,\text{rad/s})^2 + \tfrac{1}{2}(230\,\text{N/m})(0.4\,\text{m})^2 = \boxed{160\,\text{J}}$$

129 ••

Picture the Problem Let m represent the 0.2-kg mass, M the 0.8-kg mass of the cylinder, L the 1.8-m length, and $x + \Delta x$ the distance from the center of the objects whose mass is m. We can use Newton's 2nd law to relate the radial forces on the masses to the spring's stiffness constant and use the work-energy theorem to find the work done as the system accelerates to its final angular speed.

Using the work-energy theorem, relate the work done to the change in energy of the system:

$$W = K_{\text{rot}} + \Delta U_{\text{spring}} = \tfrac{1}{2}I\omega^2 + \tfrac{1}{2}k(\Delta x)^2 \qquad (1)$$

Express I as the sum of the moments of inertia of the cylinder and the masses:

$$I = I_M + I_{2m} = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2I_m$$

From Table 9-1 we have, for a solid cylinder about a diameter through its center:

$$I = \tfrac{1}{4}mr^2 + \tfrac{1}{12}mL^2$$

where L is the length of the cylinder.

For a disk (thin cylinder), L is small and:

$$I = \tfrac{1}{4}mr^2$$

Apply the parallel-axis theorem to obtain:

$$I_m = \tfrac{1}{4}mr^2 + mx^2$$

Substitute to obtain:

$$I = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2\left(\tfrac{1}{4}mr^2 + mx^2\right) = \tfrac{1}{2}Mr^2 + \tfrac{1}{12}ML^2 + 2m\left(\tfrac{1}{4}r^2 + x^2\right)$$

Substitute numerical values and evaluate I:

$$I = \tfrac{1}{2}(0.8\,\text{kg})(0.2\,\text{m})^2 + \tfrac{1}{12}(0.8\,\text{kg})(1.8\,\text{m})^2 + 2(0.2\,\text{kg})\left[\tfrac{1}{4}(0.2\,\text{m})^2 + (0.8\,\text{m})^2\right]$$
$$= 0.492\,\text{N}\cdot\text{m}^2$$

Express the net inward force acting on each of the 0.2-kg masses:

$$\sum F_{\text{radial}} = k\Delta x = m(x+\Delta x)\omega^2$$

Solve for ω:

$$\omega = \sqrt{\frac{k\Delta x}{m(x+\Delta x)}}$$

Substitute numerical values and evaluate ω:

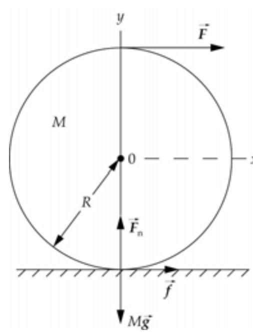
$$\omega = \sqrt{\frac{(60\,\text{N/m})(0.4\,\text{m})}{(0.2\,\text{kg})(0.8\,\text{m})}} = 12.2\,\text{rad/s}$$

Substitute numerical values in equation (1) to obtain:

$$W = \tfrac{1}{2}\left(0.492\,\text{N}\cdot\text{m}^2\right)(12.2\,\text{rad/s})^2 + \tfrac{1}{2}(60\,\text{N/m})(0.4\,\text{m})^2 = \boxed{41.4\,\text{J}}$$

130 ••

Picture the Problem The force diagram shows the forces acting on the cylinder. Because F causes the cylinder to rotate clockwise, f, which opposes this motion, is to the right. We can use Newton's 2nd law in both translational and rotational forms to relate the linear and angular accelerations to the forces acting on the cylinder.



(*a*) Use Newton's 2nd law to relate the angular acceleration of the center of mass of the cylinder to F:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{FR}{\tfrac{1}{2}MR^2} = \frac{2F}{MR}$$

Use Newton's 2nd law to relate the acceleration of the center of mass of the cylinder to F:

$$a_{\text{cm}} = \frac{F_{\text{net}}}{M} = \frac{F}{M}$$

Express the rolling-without-slipping condition to the accelerations:

$$\alpha' = \frac{a_{\text{cm}}}{R} = \frac{F}{MR} = \boxed{2\alpha}$$

(*b*) Take the point of contact with the floor as the "pivot" point, express the net torque about that point, and solve for α:

$$\tau_{\text{net}} = 2FR = I\alpha$$

and

$$\alpha = \frac{2FR}{I}$$

Express the moment of inertia of the cylinder with respect to the pivot point:

$$I = \tfrac{1}{2}MR^2 + MR^2 = \tfrac{3}{2}MR^2$$

Substitute to obtain:

$$\alpha = \frac{2FR}{\tfrac{3}{2}MR^2} = \frac{4F}{3MR}$$

Express the linear acceleration of the cylinder:

$$a_{\text{cm}} = R\alpha = \boxed{\frac{4F}{3M}}$$

Apply Newton's 2nd law to the forces acting on the cylinder:

$$\sum F_x = F + f = Ma_{\text{cm}}$$

Solve for *f*:

$$f = Ma_{\text{cm}} - F = \frac{4F}{3} - F$$

$$= \boxed{\tfrac{1}{3}F \text{ in the positive } x \text{ direction.}}$$

131 ••

Picture the Problem As the load falls, mechanical energy is conserved. As in Example 9-7, choose the initial potential energy to be zero. Apply conservation of mechanical energy to obtain an expression for the speed of the bucket as a function of its position and use the given expression for *t* to determine the time required for the bucket to travel a distance *y*.

Apply conservation of mechanical energy:

$$U_{\text{f}} + K_{\text{f}} = U_{\text{i}} + K_{\text{i}} = 0 + 0 = 0 \quad (1)$$

Express the total potential energy when the bucket has fallen a distance *y*:

$$U_{\text{f}} = U_{\text{bf}} + U_{\text{cf}} + U_{\text{wf}}$$

$$= -mgy - m_{\text{c}}'g\left(\frac{y}{2}\right)$$

where m_{c}' is the mass of the hanging part of the cable.

Assume the cable is uniform and express m_{c}' in terms of m_{c}, y, and L:

$$\frac{m_{\text{c}}'}{y} = \frac{m_{\text{c}}}{L} \text{ or } m_{\text{c}}' = \frac{m_{\text{c}}}{L}y$$

Substitute to obtain:

$$U_{\text{f}} = -mgy - \frac{m_{\text{c}}gy^2}{2L}$$

Noting that bucket, cable, and rim of the winch have the same speed v, express the total kinetic energy when the bucket is falling with speed v:

$$K_f = K_{bf} + K_{cf} + K_{wf} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{2}I\omega_f^2 = \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{2}\left(\tfrac{1}{2}MR^2\right)\frac{v^2}{R^2} = \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{4}Mv^2$$

Substitute in equation (1) to obtain:

$$-mgy - \frac{m_c g y^2}{2L} + \tfrac{1}{2}mv^2 + \tfrac{1}{2}m_c v^2 + \tfrac{1}{4}Mv^2 = 0$$

Solve for v:

$$v = \sqrt{\frac{4mgy + \dfrac{2m_c g y^2}{L}}{M + 2m + 2m_c}}$$

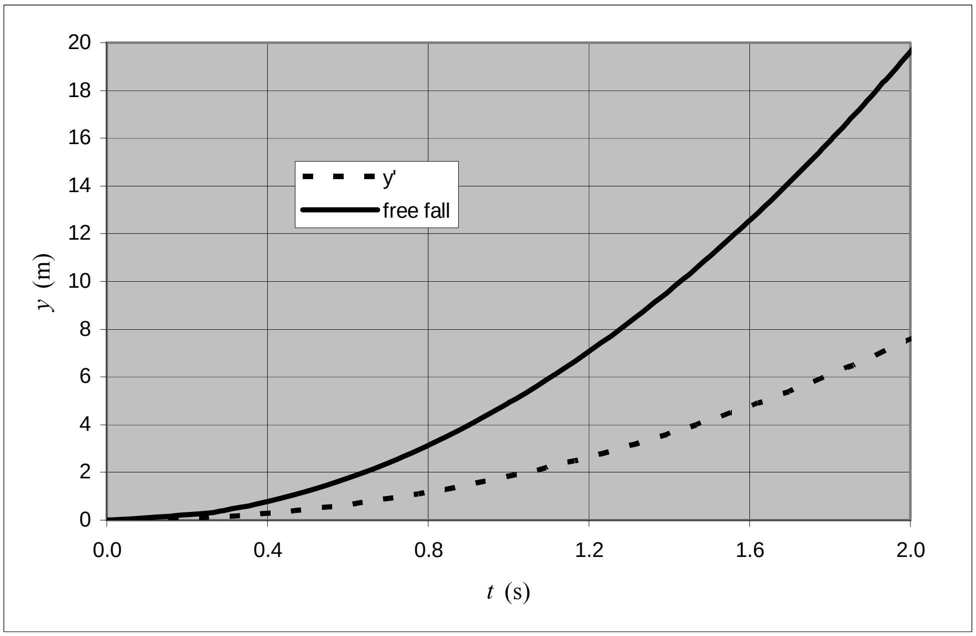
A spreadsheet solution is shown below. The formulas used to calculate the quantities in the columns are as follows:

Cell	Formula/Content	Algebraic Form
D9	0	y_0
D10	D9+$B$8	$y + \Delta y$
E9	0	v_0
E10	((4*$B$3*$B$7*D10+2*$B$7*D10^2/(2*$B$5))/ ($B$1+2*$B$3+2*$B$4))^0.5	$\sqrt{\frac{4mgy + \frac{2m_c g y^2}{L}}{M + 2m + 2m_c}}$
F10	F9+$B$8/((E10+E9)/2)	$t_{n-1} + \left(\frac{v_{n-1} + v_n}{2}\right)\Delta y$
J9	0.5*$B$7*H9^2	$\tfrac{1}{2}gt^2$

	A	B	C	D	E	F	G	H	I	J
1	M=	10	kg							
2	R=	0.5	m							
3	m=	5	kg							
4	mc=	3.5	kg							
5	L=	10	m							
6										
7	g=	9.81	m/s^2							
8	dy=	0.1	m	y	v(y)	t(y)		t(y)	y	1/2gt^2
9				0.0	0.00	0.00		0.00	0.0	0.00
10				0.1	0.85	0.23		0.23	0.1	0.27
11				0.2	1.21	0.33		0.33	0.2	0.54
12				0.3	1.48	0.41		0.41	0.3	0.81
13				0.4	1.71	0.47		0.47	0.4	1.08
15				0.5	1.91	0.52		0.52	0.5	1.35

105				9.6	9.03	2.24		2.24	9.6	24.61
106				9.7	9.08	2.25		2.25	9.7	24.85
107				9.8	9.13	2.26		2.26	9.8	25.09
108				9.9	9.19	2.27		2.27	9.9	25.34
109				10.0	9.24	2.28		2.28	10.0	25.58

The solid line on the graph shown below shows the position y of the bucket when it is in free fall and the dashed line shows y under the conditions modeled in this problem.



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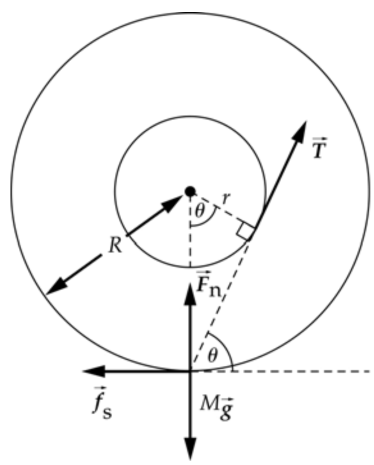
Picture the Problem The pictorial representation shows the forces acting on the cylinder when it is stationary. First, we note that if the tension is small, then there can be no slipping, and the system must roll. Now consider the point of contact of the cylinder with the surface as the "pivot" point. If τ about that point is zero, the system will not roll. This will occur if the line of action of the tension passes through the pivot point.



From the diagram we see that:

$$\theta = \boxed{\cos^{-1}\left(\frac{r}{R}\right)}$$

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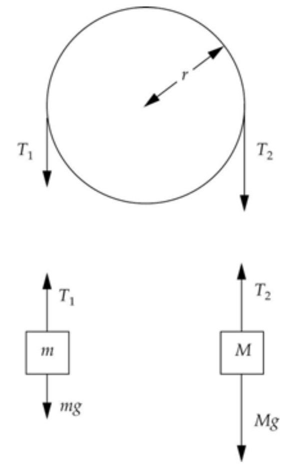
Picture the Problem Free-body diagrams for the pulley and the two blocks are shown to the right. Choose a coordinate system in which the direction of motion of the block whose mass is M (downward) is the positive y direction. We can use the given relationship $T'_{max} = Te^{\mu_s \Delta\theta}$ to relate the tensions in the rope on either side of the pulley and apply Newton's 2nd law in both rotational form (to the pulley) and translational form (to the blocks) to obtain a system of equations that we can solve simultaneously for a, T_1, T_2, and M.



(a) Use $T'_{max} = Te^{\mu_s \Delta\theta}$ to evaluate the maximum tension required to prevent the rope from slipping on the pulley:

$$T'_{max} = (10\,\text{N})e^{(0.3)\pi} = \boxed{25.7\,\text{N}}$$

(c) Given that the angle of wrap is π radians, express T_2 in terms of T_1:

$$T_2 = T_1 e^{0.3\pi} = 2.57T_1 \quad (1)$$

Because the rope doesn't slip, we can relate the angular acceleration, α, of the pulley to the acceleration, a, of the hanging masses by:

$$\alpha = \frac{a}{r}$$

Apply $\sum F_y = ma_y$ to the two blocks to obtain:

$$T_1 - mg = ma \quad (2)$$

and

$$Mg - T_2 = Ma \quad (3)$$

Apply $\sum \tau = I\alpha$ to the pulley to obtain:

$$(T_2 - T_1)r = I\frac{a}{r} \quad (4)$$

Substitute for T_2 from equation (1) in equation (4) to obtain:

$$(2.57T_1 - T_1)r = I\frac{a}{r}$$

Solve for T_1 and substitute numerical values to obtain:

$$T_1 = \frac{I}{1.57r^2}a = \frac{0.35\,\text{kg}\cdot\text{m}^2}{1.57(0.15\,\text{m})^2}a \quad (5)$$
$$= (9.91\,\text{kg})a$$

Substitute in equation (2) to obtain:

$$(9.91\,\text{kg})a - mg = ma$$

Solve for and evaluate a:

$$a = \frac{mg}{9.91\,\text{kg} - m} = \frac{g}{\dfrac{9.91\,\text{kg}}{m} - 1}$$

$$= \frac{9.81\,\text{m/s}^2}{\dfrac{9.91\,\text{kg}}{1\,\text{kg}} - 1} = \boxed{1.10\,\text{m/s}^2}$$

(b) Solve equation (3) for M:

$$M = \frac{T_2}{g - a}$$

Substitute in equation (5) to find T_1:

$$T_1 = (9.91\,\text{kg})(1.10\,\text{m/s}^2) = 10.9\,\text{N}$$

Substitute in equation (1) to find T_2:

$$T_2 = (2.57)(10.9\,\text{N}) = 28.0\,\text{N}$$

Evaluate M:

$$M = \frac{28.0\,\text{N}}{9.81\,\text{m/s}^2 - 1.10\,\text{m/s}^2} = \boxed{3.21\,\text{kg}}$$

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Picture the Problem When the tension is horizontal, the cylinder will roll forward and the friction force will be in the direction of $\vec{T}$. We can use Newton's 2nd law to obtain equations that we can solve simultaneously for a and f.

(a) Apply Newton's 2nd law to the cylinder:

$$\sum F_x = T + f = ma \qquad (1)$$

and

$$\sum \tau = Tr - fR = I\alpha \qquad (2)$$

Substitute for I and α in equation (2) to obtain:

$$Tr - fR = \tfrac{1}{2} mR^2 \frac{a}{R} = \tfrac{1}{2} mRa \qquad (3)$$

Solve equation (3) for f:

$$f = \frac{Tr}{R} - \tfrac{1}{2} ma \qquad (4)$$

Substitute equation (4) in equation (1) and solve for a:

$$a = \frac{2T}{3m}\left(1 + \frac{r}{R}\right) \qquad (5)$$

Substitute equation (5) in equation (4) to obtain:

$$f = \boxed{\frac{T}{3}\left(\frac{2r}{R} - 1\right)}$$

(b) Equation (4) gives the acceleration of the center of mass:

$$a = \boxed{\frac{2T}{3m}\left(1 + \frac{r}{R}\right)}$$

(*c*) Express the condition that $a > \dfrac{T}{m}$:

$$\frac{2T}{3m}\left(1+\frac{r}{R}\right) > \frac{T}{m} \Rightarrow \frac{2}{3}\left(1+\frac{r}{R}\right) > 1$$

or

$$r > \boxed{\tfrac{1}{2}R}$$

(*d*) If $r > \frac{1}{2}R$:

$$\boxed{f > 0\text{, i.e., in the direction of } \vec{T}.}$$

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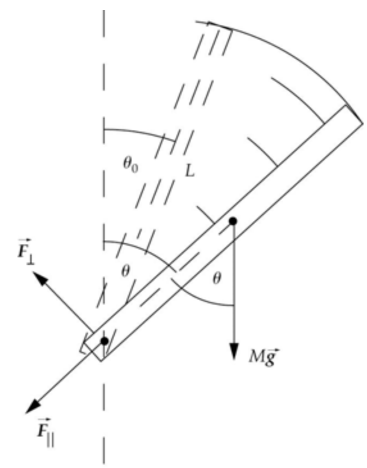
Picture the Problem The system is shown in the drawing in two positions, with angles θ_0 and θ with the vertical. The drawing also shows all the forces that act on the stick. These forces result in a rotation of the stick—and its center of mass—about the pivot, and a tangential acceleration of the center of mass. We'll apply the conservation of mechanical energy and Newton's 2nd law to relate the radial and tangential forces acting on the stick.



Use the conservation of mechanical energy to relate the kinetic energy of the stick when it makes an angle θ with the vertical and its initial potential energy:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$-\tfrac{1}{2}I\omega^2 + Mg\frac{L}{2}\cos\theta - Mg\frac{L}{2}\cos\theta_0 = 0$$

Substitute for I and solve for ω^2:

$$\omega^2 = \frac{3g}{L}\left(\cos\theta - \cos\theta_0\right)$$

Express the centripetal force acting on the center of mass:

$$F_c = M\frac{L}{2}\omega^2 = M\frac{L}{2}\frac{3g}{L}\left(\cos\theta - \cos\theta_0\right) = \frac{3Mg}{2}\left(\cos\theta - \cos\theta_0\right)$$

Express the radial component of $M\vec{g}$:

$$(Mg)_{\text{radial}} = Mg\cos\theta$$

Express the total radial force at the hinge:

$$F_{||} = F_c + (Mg)_{\text{radial}}$$

$$= \frac{3Mg}{2}(\cos\theta - \cos\theta_0) + Mg\cos\theta$$

$$= \boxed{\tfrac{1}{2}Mg(5\cos\theta - 3\cos\theta_0)}$$

Relate the tangential acceleration of the center of mass to its angular acceleration:

$$a_\perp = \tfrac{1}{2}L\alpha$$

Use Newton's 2nd law to relate the angular acceleration of the stick to the net torque acting on it:

$$\alpha = \frac{\tau_{\text{net}}}{I} = \frac{Mg\frac{L}{2}\sin\theta}{\frac{1}{3}ML^2} = \frac{3g\sin\theta}{2L}$$

Express $a_\perp$ in terms of α:

$$a_\perp = \tfrac{1}{2}L\alpha = \tfrac{3}{4}g\sin\theta = g\sin\theta + F_\perp/M$$

Solve for $F_\perp$ to obtain:

$F_\perp = \boxed{-\tfrac{1}{4}Mg\sin\theta}$ where the minus sign indicates that the force is directed oppositely to the tangential component of $M\vec{g}$.