Chapter 11
Gravity

Conceptual Problems

***1** •

(*a*) False. Kepler's law of equal areas is a consequence of the fact that the gravitational force acts along the line joining two bodies but is independent of the manner in which the force varies with distance.

(*b*) True. The periods of the planets vary with the three-halves power of their distances from the sun. So the shorter the distance from the sun, the shorter the period of the planet's motion.

2 •

Determine the Concept We can apply Newton's 2nd law and the law of gravity to the satellite to obtain an expression for its speed as a function of the radius of its orbit.

Apply Newton's 2nd law to the satellite to obtain:

$$\sum F_{\text{radial}} = \frac{GMm}{r^2} = m\frac{v^2}{r}$$

where M is the mass of the object the satellite is orbiting and m is the mass of the satellite.

Solve for v to obtain:

$$v = \sqrt{\frac{GM}{r}}$$

Thus the speed of the satellite is independent of its mass and:

$$\boxed{(c)\text{ is correct.}}$$

3 ••

Picture the Problem The acceleration due to gravity varies inversely with the square of the distance from the center of the moon.

Express the dependence of the acceleration due to the gravity of the moon on the distance from its center:

$$a' \propto \frac{1}{r^2}$$

Express the dependence of the acceleration due to the gravity of the moon at its surface on its radius:

$$a \propto \frac{1}{R_{\text{M}}^2}$$

Divide the first of these expressions by the second to obtain:

$$\frac{a'}{a} = \frac{R_M^2}{r^2}$$

Solve for a':

$$a' = \frac{R_M^2}{r^2} a = \frac{R_M^2}{(4R_M)^2} a = \tfrac{1}{16} a$$

and $\boxed{(d)\text{ is correct.}}$

4 •

Determine the Concept Measurement of G is difficult because masses accessible in the laboratory are very small compared to the mass of the earth.

5 •

Determine the Concept The escape speed for a planet is given by $v_e = \sqrt{2Gm/R}$.

Between v_e depends on the square root of M, doubling M increases the escape speed by a factor of $\sqrt{2}$ and $\boxed{(a)\text{ is correct.}}$

6 ••

Determine the Concept We can take careful measurements of its position in order to determine whether its trajectory is an ellipse, a hyperbola, or a parabola. If the path is an ellipse, it will return; if its path is hyperbolic or parabolic, it will not return.

7 ••

Determine the Concept The gravitational field is proportional to the mass within the sphere of radius r *and* inversely proportional to the square of r, i.e., proportional to $r^3/r^2 = r$.

***8** •

Determine the Concept Let m represent the mass of Mercury, M_S the mass of the sun, v the orbital speed of Mercury, and R the mean orbital radius of Mercury. We can use Newton's 2nd law of motion to relate the gravitational force acting on the Mercury to its orbital speed.

Use Newton's 2nd law to relate the gravitational force acting on Mercury to its orbital speed:

$$F_{net} = \frac{GM_S m}{R^2} = m\frac{v^2}{R}$$

Simplify to obtain:

$$\tfrac{1}{2}mv^2 = \tfrac{1}{2}\frac{GM_S m}{R} = -\tfrac{1}{2}\left(-\frac{GM_S m}{R}\right) = -\tfrac{1}{2}U$$

or $\boxed{K = -\tfrac{1}{2}U}$

9 ••

Picture the Problem We can use the definition of the gravitational field to express the ratio of the student's weight at an elevation of two earth radii to her weight at the surface of the earth.

Express the weight of the student at the surface of the earth:

$$w = mg = \frac{GM_{\mathrm{E}}m}{R_{\mathrm{E}}^2}$$

Express the weight of the student at an elevation of two earth radii:

$$w' = mg' = \frac{GM_{\mathrm{E}}m}{(3R_{\mathrm{E}})^2}$$

Express the ratio of w' to w:

$$\frac{w'}{w} = \frac{\dfrac{GM_{\mathrm{E}}m}{(3R_{\mathrm{E}})^2}}{\dfrac{GM_{\mathrm{E}}m}{R_{\mathrm{E}}^2}} = \frac{1}{9} \text{ and } \boxed{(d)\text{ is correct.}}$$

10 ••

Determine the Concept One such machine would be a balance wheel with weights attached to the rim with half of them shielded using Cavourite. The weights on one side would be pulled down by the force of gravity, while the other side would not, leading to rotation, which can be converted into useful work, in violation of the law of the conservation of energy.

Estimation and Approximation

11 •

Picture the Problem To approximate the mass of the galaxy we'll assume the galactic center to be a point mass with the sun in orbit about it and apply Kepler's 3rd law.

Using Kepler's 3rd law, relate the period of the sun T to its mean distance r from the center of the galaxy:

$$T^2 = \frac{4\pi^2}{GM_{\mathrm{galaxy}}}r^3 = \frac{\dfrac{4\pi^2}{M_{\mathrm{s}}}}{G\dfrac{M_{\mathrm{galaxy}}}{M_{\mathrm{s}}}}r^3$$

Solve for $\dfrac{r^3}{T^2}$ to obtain:

$$\frac{r^3}{T^2} = \frac{G\dfrac{M_{\mathrm{galaxy}}}{M_{\mathrm{s}}}}{\dfrac{4\pi^2}{M_{\mathrm{s}}}} = \frac{\dfrac{M_{\mathrm{galaxy}}}{M_{\mathrm{s}}}}{\dfrac{4\pi^2}{GM_{\mathrm{s}}}}$$

If we measure distances in AU and times in years:

$$\frac{4\pi^2}{GM_s} = 1 \text{ and } \frac{r^3}{T^2} = \frac{M_{\text{galaxy}}}{M_s}$$

Substitute numerical values and evaluate M_{galaxy}/M_s:

$$\frac{M_{\text{galaxy}}}{M_s} = \frac{\left(3\times10^4\,\text{LY}\times\frac{6.3\times10^4\,\text{AU}}{\text{LY}}\right)^3}{\left(250\times10^6\,\text{y}\right)^2}$$
$$= 1.08\times10^{11}$$

or

$$M_{\text{galaxy}} = \boxed{1.08\times10^{11}M_s}$$

***12 •••**

Picture the Problem We can use Kepler's 3rd law to find the size of the semi-major axis of the planet's orbit and the conservation of momentum to find its mass.

(*a*) Using Kepler's 3rd law, relate the period of this planet T to the length r of its semi-major axis:

$$T^2 = \frac{4\pi^2}{GM_{\text{Iota Draconis}}}r^3 = \frac{\frac{4\pi^2}{M_s}}{G\frac{M_{\text{Iota Draconis}}}{M_s}}r^3 = \frac{\frac{4\pi^2}{GM_s}}{\frac{M_{\text{Iota Draconis}}}{M_s}}r^3$$

If we measure time in years, distances in AU, and masses in terms of the mass of the sun:

$$\frac{4\pi^2}{MG_s} = 1 \text{ and } T^2 = \frac{1}{\frac{M_{\text{Iota Draconis}}}{M_s}}r^3$$

Solve for r to obtain:

$$r = \sqrt[3]{\frac{M_{\text{Iota Draconis}}}{M_s}T^2}$$

Substitute numerical values and evaluate r:

$$r = \sqrt[3]{\left(\frac{1.05M_s}{M_s}\right)(1.5\,\text{y})^2} = \boxed{1.33\,\text{AU}}$$

(*b*) Apply conservation of momentum to the planet (mass m and speed v) and the star (mass $M_{\text{Iota Draconis}}$ and speed V) to obtain:

$$mv = M_{\text{Iota Draconis}}V$$

Solve for m to obtain:

$$m = M_{\text{Iota Draconis}} \frac{V}{v}$$

Use its definition to find the speed of the orbiting planet:

$$v = \frac{\Delta d}{\Delta t} = \frac{2\pi r}{T}$$

$$= \frac{2\pi\left(1.33\,\text{AU} \times \frac{1.5 \times 10^{11}\,\text{m}}{\text{AU}}\right)}{1.50\,\text{y} \times \frac{365.25\,\text{d}}{\text{y}} \times \frac{24\,\text{h}}{\text{d}} \times \frac{3600\,\text{s}}{\text{h}}}$$

$$= 2.65 \times 10^4\ \text{m/s}$$

Substitute numerical values and evaluate v:

$$m = M_{\text{Iota Draconis}}\left(\frac{296\,\text{m/s}}{2.65 \times 10^4\ \text{m/s}}\right)$$

$$= 0.0112 M_{\text{Iota Draconis}}$$

$$= 0.0112(1.05 M_{\text{sun}})$$

$$= 0.0112(1.05)(1.99 \times 10^{30}\ \text{kg})$$

$$= 2.34 \times 10^{28}\ \text{kg}$$

Express m in terms of the mass M_{J} of Jupiter:

$$\frac{m}{M_{\text{J}}} = \frac{2.34 \times 10^{28}\ \text{kg}}{1.90 \times 10^{27}\ \text{kg}} = 12.3$$

or

$$m = \boxed{12.3 M_{\text{J}}}$$

Remarks: A more sophisticated analysis, using the eccentricity of the orbit, leads to a lower bound of 8.7 Jovian masses. (Only a lower bound can be established, as the plane of the orbit is not known.)

13 •••

Picture the Problem We can apply Newton's law of gravity to estimate the maximum angular velocity which the sun can have if it is to stay together and use the definition of angular momentum to find the orbital angular momenta of Jupiter and Saturn. In part (c) we can relate the final angular velocity of the sun to its initial angular velocity, its moment of inertia, and the orbital angular momenta of Jupiter and Saturn.

(a) Gravity must supply the centripetal force which keeps an element of the sun's mass m rotating around it. Letting the radius of the sun be R, apply Newton's law of gravity to an element of mass m to obtain:

$$m\omega^2 R < \frac{GMm}{R^2}$$

or

$$\omega^2 R < \frac{GM}{R^2}$$

where we've used the inequality because we're estimating the *maximum* angular velocity which the sun can have if it is to stay together.

Solve for ω:

$$\omega < \sqrt{\frac{GM}{R^3}}$$

Substitute numerical values and evaluate ω:

$$\omega < \sqrt{\frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.99\times10^{30}\ \text{kg})}{(6.96\times10^8\ \text{m})^3}} = \boxed{6.28\times10^{-4}\ \text{rad/s}}$$

Calculate the period of this motion from its angular velocity:

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{6.28\times10^{-4}\ \text{rad/s}}$$
$$= 1.00\times10^4\ \text{s}\times\frac{1\ \text{h}}{3600\ \text{s}} = \boxed{2.78\ \text{h}}$$

(*b*) Express the orbital angular momenta of Jupiter and Saturn:

$$L_\text{J} = m_\text{J} r_\text{J} v_\text{J} \text{ and } L_\text{S} = m_\text{S} r_\text{S} v_\text{S}$$

Express the orbital speeds of Jupiter and Saturn in terms of their periods and distances from the sun:

$$v_\text{J} = \frac{2\pi\, r_\text{J}}{T_\text{J}} \text{ and } v_\text{S} = \frac{2\pi\, r_\text{S}}{T_\text{S}}$$

Substitute to obtain:

$$L_\text{J} = \frac{2\pi\, m_\text{J} r_\text{J}^2}{T_\text{J}} \text{ and } L_\text{S} = \frac{2\pi\, m_\text{S} r_\text{S}^2}{T_\text{S}}$$

Substitute numerical values and evaluate L_J and L_S:

$$L_\text{J} = \frac{2\pi(318M_\text{E})r_\text{J}^2}{T_\text{J}} = \frac{2\pi(318)(5.98\times10^{24}\ \text{kg})(778\times10^9\ \text{m})^2}{11.9\ \text{y}\times\frac{365.25\ \text{d}}{\text{y}}\times\frac{24\ \text{h}}{\text{d}}\times\frac{3600\ \text{s}}{\text{h}}}$$
$$= \boxed{1.93\times10^{43}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

and

$$L_\text{S} = \frac{2\pi(95.1M_\text{E})r_\text{S}^2}{T_\text{S}} = \frac{2\pi(95.1)(5.98\times10^{24}\ \text{kg})(1430\times10^9\ \text{m})^2}{29.5\ \text{y}\times\frac{365.25\ \text{d}}{\text{y}}\times\frac{24\ \text{h}}{\text{d}}\times\frac{3600\ \text{s}}{\text{h}}}$$
$$= \boxed{7.85\times10^{42}\ \text{kg}\cdot\text{m}^2/\text{s}}$$

Express the angular momentum of the sun as a fraction of the sum of the angular momenta of Jupiter and Saturn:

$$f = \frac{L_\text{sun}}{L_\text{J}+L_\text{S}}$$
$$= \frac{1.91\times10^{41}\ \text{kg}\cdot\text{m}^2/\text{s}}{(19.3+7.85)\times10^{42}\ \text{kg}\cdot\text{m}^2/\text{s}}$$
$$= \boxed{0.703\%}$$

(*c*) Relate the final angular momentum of the sun to its initial angular momentum and the angular momenta of Jupiter and Saturn:

$$L_f = L_i + L_J + L_S$$

or

$$I_{sun}\omega_f = I_{sun}\omega_i + L_J + L_S$$

Solve for ω_f to obtain:

$$\omega_f = \omega_i + \frac{L_J + L_S}{I_{sun}}$$

Substitute for ω_i and I_{sun}:

$$\omega_f = \frac{2\pi}{T_{sun}} + \frac{L_J + L_S}{0.059 M_{sun} R_{sun}^2}$$

Substitute numerical values and evaluate ω_f:

$$\omega_f = \frac{2\pi}{30\,\text{d} \times \frac{24\,\text{h}}{\text{d}} \times \frac{3600\,\text{s}}{\text{h}}} + \frac{(19.3 + 7.85) \times 10^{42}\ \text{kg} \cdot \text{m}^2/\text{s}}{0.059(1.99 \times 10^{30}\ \text{kg})(6.96 \times 10^8\ \text{m})^2}$$
$$= \boxed{4.80 \times 10^{-4}\ \text{rad/s}}$$

Note that this result is about 76% of the maximum possible rotation allowed by gravity that we calculated in part (*a*).

Kepler's Laws

14 •

Picture the Problem We can use the relationship between the semi-major axis and the distances of closest approach and greatest separation, together with Kepler's 3rd law, to find the greatest separation of Alex-Casey from the sun.

Letting *x* represent the greatest distance from the sun, express the relationship between *x*, the distance of closest approach, and its semi-major axis *R*:

$$R = \frac{x + 0.1\,\text{AU}}{2}$$

Solve for *x* to obtain:

$$x = 2R - 0.1\,\text{AU} \qquad (1)$$

Apply Kepler's 3rd law, with the period *T* measured in years and *R* in AU to obtain:

$$T^2 = R^3$$

Solve for *R*:

$$R = \sqrt[3]{T^2}$$

Substitute numerical values and evaluate *R*:

$$R = \sqrt[3]{(127.4\,\text{y})^2} = 25.3\,\text{AU}$$

Substitute in equation (1) and evaluate *R*:

$$x = 2(25.3\,\text{AU}) - 0.1\,\text{AU} = \boxed{50.5\,\text{AU}}$$

15 •

Picture the Problem We can use Kepler's 3rd law to relate the period of Uranus to its mean distance from the sun.

Using Kepler's 3rd law, relate the period of Uranus to its mean distance from the sun:

$$T^2 = Cr^3$$

where $C = \frac{4\pi^2}{GM_s} = 2.973\times 10^{-19}\ \text{s}^2/\text{m}^3$.

Solve for T:

$$T = \sqrt{Cr^3}$$

Substitute numerical values and evaluate T:

$$T = \sqrt{\left(2.973\times 10^{-19}\ \text{s}^2/\text{m}^3\right)\left(2.87\times 10^{12}\ \text{m}\right)^3}$$
$$= 2.651\times 10^{9}\ \text{s}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1\,\text{d}}{24\,\text{h}}\times\frac{1\,\text{y}}{365.25\,\text{d}} = \boxed{84.0\,\text{y}}$$

16 •

Picture the Problem We can use Kepler's 3rd law to relate the period of Hektor to its mean distance from the sun.

Using Kepler's 3rd law, relate the period of Hektor to its mean distance from the sun:

$$T^2 = Cr^3$$

where $C = \frac{4\pi^2}{GM_s} = 2.973\times 10^{-19}\ \text{s}^2/\text{m}^3$.

Solve for T:

$$T = \sqrt{Cr^3}$$

Substitute numerical values and evaluate T:

$$T = \sqrt{\left(2.973\times 10^{-19}\ \text{s}^2/\text{m}^3\right)\left(5.16\,\text{AU}\times\frac{1.50\times 10^{11}\ \text{m}}{\text{AU}}\right)^3}$$
$$= 3.713\times 10^{8}\ \text{s}\times\frac{1\,\text{h}}{3600\,\text{s}}\times\frac{1\,\text{d}}{24\,\text{h}}\times\frac{1\,\text{y}}{365.25\,\text{d}} = \boxed{11.8\,\text{y}}$$

17 ••

Picture the Problem Kepler's 3rd law relates the period of Icarus to the length of its semimajor axis. The aphelion distance r_a is related to the perihelion distance r_p and the semimajor axis by $r_a + r_p = 2a$.

(*a*) Using Kepler's 3rd law, relate the period of Icarus to the length of its semimajor axis:

$$T^2 = Ca^3$$

where $C = \dfrac{4\pi^2}{GM_s} = 2.973\times10^{-19}\ \text{s}^2/\text{m}^3$.

Solve for a:

$$a = \sqrt[3]{\frac{T^2}{C}}$$

Substitute numerical values and evaluate a:

$$a = \sqrt[3]{\frac{\left(1.1\,\text{y}\times\dfrac{365.25\,\text{d}}{\text{y}}\times\dfrac{24\,\text{h}}{\text{d}}\times\dfrac{3600\,\text{s}}{\text{h}}\right)^2}{2.973\times10^{-19}\ \text{s}^2/\text{m}^3}} = \boxed{1.59\times10^{11}\ \text{m}}$$

(*b*) Use the definition of the eccentricity of an ellipse to determine the perihelion distance of Icarus:

$$r_\text{p} = a(1-e) = \left(1.59\times10^{11}\ \text{m}\right)(1-0.83) = \boxed{2.71\times10^{10}\ \text{m}}$$

Express the relationship between r_p and r_a for an ellipse:

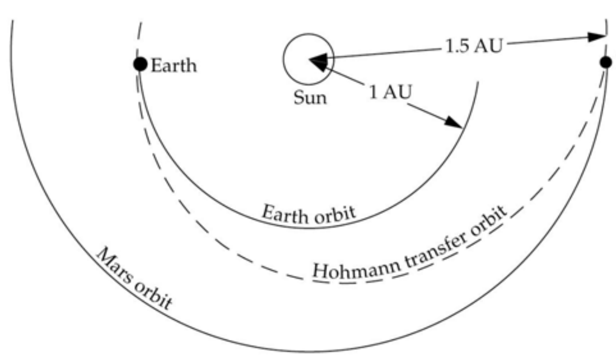
$$r_\text{a} + r_\text{p} = 2a$$

Solve for and evaluate r_a:

$$r_\text{a} = 2a - r_\text{p} = 2\left(1.59\times10^{11}\ \text{m}\right) - 2.71\times10^{10}\ \text{m} = \boxed{2.91\times10^{11}\ \text{m}}$$

18 ••

Picture the Problem The Hohmann transfer orbit is shown in the diagram. We can apply Kepler's 3rd law to relate the time-in-orbit to the period of the spacecraft in its Hohmann Earth-to-Mars orbit. The period of this orbit is, in turn, a function of its semi-major axis which we can find from the average of the lengths of the semi-major axes of the Earth and Mars orbits.



Using Kepler's 3rd law, relate the period T of the spacecraft to the semi-major axis of its orbit:

$$T^2 = R^3$$

Solve for T to obtain:

$$T = \sqrt{R^3}$$

Relate the transit time to the period of this orbit:

$$t_{\text{trip}} = \tfrac{1}{2}T = \tfrac{1}{2}\sqrt{R^3}$$

Express the semi-major axis of the Hohmann transfer orbit in terms of the mean sun-Mars and sun-Earth distances:

$$R = \frac{1.52\,\text{AU} + 1.00\,\text{AU}}{2} = 1.26\,\text{AU}$$

Substitute numerical values and evaluate t_{trip}:

$$t_{\text{trip}} = \tfrac{1}{2}\sqrt{(1.26\,\text{AU})^3}$$

$$= 0.707\,\text{y} \times \frac{365.24\,\text{d}}{1\,\text{y}} = \boxed{258\ \text{d}}$$

***19** ••

Picture the Problem We can use a property of lines tangent to a circle and radii drawn to the point of contact to show that $b = 90°$. Once we've established that b is a right angle we can use the definition of the sine function to relate the distance from the sun to Venus to the distance from the sun to the earth.

(a) The line from earth to Venus′ orbit is tangent to the orbit of Venus at the point of maximum extension. Venus will appear closer to the sun in earth's sky when it passes the line drawn from earth and tangent to its orbit. Hence:

$$b = \boxed{90°}$$

(b) Using trigonometry, relate the distance from the sun to Venus d_{SV} to the angle a:

$$\sin a = \frac{d_{\text{SV}}}{d_{\text{SE}}}$$

Solve for d_{SV}:

$$d_{\text{SV}} = d_{\text{SE}} \sin a$$

Substitute numerical values and evaluate d_{SV}:

$$d_{\text{SV}} = (1\,\text{AU})\sin 47° = \boxed{0.731\,\text{AU}}$$

Remarks: The correct distance from the sun to Venus is closer to 0.723 AU.

20 ••

Picture the Problem Because the gravitational force the Earth exerts on the moon is along the line joining their centers, the net torque acting on the moon is zero and its angular momentum is conserved in its orbit about the Earth. Because energy is also conserved, we can combine these two expressions to solve for either v_p or v_a initially and then substitute in the conservation of angular momentum equation to find the other.

Letting m be the mass of the moon, apply conservation of angular momentum to the moon at apogee and perigee to obtain:

$$mv_p r_p = mv_a r_a$$

or

$$v_p r_p = v_a r_a$$

Solve for v_a:

$$v_a = \frac{r_p}{r_a} v_p \qquad (1)$$

Apply conservation of energy to the moon-earth system to obtain:

$$\frac{1}{2}mv_p^2 - \frac{GMm}{r_p} = \frac{1}{2}mv_a^2 - \frac{GMm}{r_a}$$

or

$$\frac{1}{2}v_p^2 - \frac{GM}{r_p} = \frac{1}{2}v_a^2 - \frac{GM}{r_a}$$

Substitute for v_a to obtain:

$$\frac{1}{2}v_p^2 - \frac{GM}{r_p} = \frac{1}{2}\left(\frac{r_p}{r_a}v_p\right)^2 - \frac{GM}{r_a}$$

$$= \frac{1}{2}\left(\frac{r_p}{r_a}\right)^2 v_p^2 - \frac{GM}{r_a}$$

Solve for v_p to obtain:

$$v_p = \sqrt{\frac{2GM}{r_p}\left(\frac{1}{1 + r_p/r_a}\right)}$$

Substitute numerical values and evaluate v_p:

$$v_p = \sqrt{\frac{2\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)}{3.576\times10^{8}\ \text{m}}\left(\frac{1}{1+\dfrac{3.576\times10^{8}\ \text{m}}{4.064\times10^{8}\ \text{m}}}\right)} = \boxed{1.09\ \text{km/s}}$$

Substitute numerical values in equation (1) and evaluate v_a:

$$v_a = \frac{3.576\times10^{8}\ \text{m}}{4.064\times10^{8}\ \text{m}}\left(1.09\,\text{km/s}\right)$$

$$= \boxed{0.959\ \text{km/s}}$$

Newton's Law of Gravity

***21** ••

Picture the Problem We can use Kepler's 3rd law to find the mass of Jupiter in part (*a*). In part (*b*) we can express the centripetal accelerations of Europa and Callisto and compare their ratio to the square of the ratio of their distances from the center of Jupiter to show that the given data is consistent with an inverse square law for gravity.

(*a*) Assuming a circular orbit, apply Kepler's 3rd law to the motion of Europa to obtain:

$$T_E^2 = \frac{4\pi^2}{GM_J} R_E^3$$

Solve for the mass of Jupiter:

$$M_J = \frac{4\pi^2}{GT_E^2} R_E^3$$

Substitute numerical values and evaluate M_J:

$$M_J = \frac{4\pi^2}{\left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)} \times \frac{\left(6.71\times10^8\ \text{m}\right)^3}{\left(3.55\ \text{d}\times\frac{24\ \text{h}}{\text{d}}\times\frac{3600\ \text{s}}{\text{h}}\right)^2}$$

$$= \boxed{1.90\times10^{27}\ \text{kg}}$$, a result in excellent agreement with the accepted value of 1.902×10^{27} kg.

(*b*) Express the centripetal acceleration of both of the moons to obtain:

$$\frac{v^2}{R} = \frac{\left(\frac{2\pi R}{T}\right)^2}{R} = \frac{4\pi^2 R}{T^2}$$

where R and T are the radii and periods of their motion.

Using this result, express the centripetal accelerations of Europa and Callisto:

$$a_E = \frac{4\pi^2 R_E}{T_E^2} \text{ and } a_C = \frac{4\pi^2 R_C}{T_C^2}$$

Substitute numerical values and evaluate a_E:

$$a_E = \frac{4\pi^2\left(6.71\times10^8\ \text{m}\right)}{\left[(3.55\ \text{d})(24\ \text{h/d})(3600\ \text{s/h})\right]^2} = \boxed{0.282\ \text{m/s}^2}$$

Substitute numerical values and evaluate a_C:

$$a_C = \frac{4\pi^2\left(18.8\times10^8\ \text{m}\right)}{\left[(16.7\ \text{d})(24\ \text{h/d})(3600\ \text{s/h})\right]^2} = \boxed{0.0356\ \text{m/s}^2}$$

Evaluate the ratio of these accelerations:

$$\frac{a_E}{a_C} = \frac{0.282\ \text{m/s}^2}{0.0356\ \text{m/s}^2} = 7.91$$

Evaluate the square of the ratio of the distance of Callisto divided by the distance of Europa to obtain:

$$\left(\frac{R_C}{R_E}\right)^2 = \left(\frac{18.8\times10^8\ \text{m}}{6.71\times10^8\ \text{m}}\right)^2 = 7.85$$

$$\boxed{\begin{array}{l}\text{The close agreement (within 1\%) of our last two calculations strongly supports}\\ \text{the conclusion that the gravitational force varies inversely with the square of}\\ \text{the distance.}\end{array}}$$

***22 •**

Determine the Concept The weight of anything, including astronauts, is the reading of a scale from which the object is suspended or on which it rests. If the scale reads zero, then we say the object is ″weightless.″ The pull of the earth's gravity, on the other hand, depends on the local value of the acceleration of gravity and we can use Newton's law of gravity to find this acceleration at the elevation of the shuttle.

(*a*) Apply Newton's law of gravitation to an astronaut of mass *m* in a shuttle at a distance *h* above the surface of the earth:

$$mg_{\text{shuttle}} = \frac{GmM_E}{(h+R_E)^2}$$

Solve for g_{shuttle}:

$$g_{\text{shuttle}} = \frac{GM_E}{(h+R_E)^2}$$

Substitute numerical values and evaluate g_{shuttle}:

$$g_{\text{shuttle}} = \frac{(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}{(400\ \text{km}+6370\ \text{km})^2} = \boxed{8.71\ \text{m/s}^2}$$

$$(b)\ \boxed{\begin{array}{l}\text{Because they are in "free fall" everything on the shuttle is falling toward}\\ \text{the center of the earth with exactly the same acceleration, so the astronauts}\\ \text{will seem to be "weightless."}\end{array}}$$

23 •

Picture the Problem We can use Kepler's 3rd law to relate the periods of the moons of Saturn to their mean distances from its center.

(*a*) Using Kepler's 3rd law, relate the period of Mimas to its mean distance from the center of Saturn:

$$T_M^2 = \frac{4\pi^2}{GM_S}r_M^3$$

Solve for T_M:

$$T_M = \sqrt{\frac{4\pi^2}{GM_S}r_M^3}$$

(*b*) Using Kepler's 3rd law, relate the period of Titan to its mean distance from the center of Saturn:

$$T_T^2 = \frac{4\pi^2}{GM_S} r_T^3$$

Substitute numerical values and evaluate T_M:

$$T_M = \sqrt{\frac{4\pi^2 (1.86 \times 10^8\ \text{m})^3}{(5.69 \times 10^{26}\ \text{kg})(6.6726 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)}} = \boxed{8.18 \times 10^4\ \text{s}}$$

Solve for r_T:

$$r_T = \sqrt[3]{\frac{T_T^2 G M_S}{4\pi^2}}$$

Substitute numerical values and evaluate r_T:

$$r_T = \sqrt[3]{\frac{(1.38 \times 10^6\ \text{s})^2 (6.6726 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5.69 \times 10^{26}\ \text{kg})}{4\pi^2}} = \boxed{1.22 \times 10^9\ \text{m}}$$

24 •

Picture the Problem We can use Kepler's 3rd law to relate the period of the moon to the mass of the earth and the mean earth-moon distance.

(*a*) Using Kepler's 3rd law, relate the period of the moon to its mean orbital radius:

$$T_m^2 = \frac{4\pi^2}{GM_E} r_m^3$$

Solve for M_E:

$$M_E = \frac{4\pi^2}{GT_m^2} r_m^3$$

Substitute numerical values and evaluate M_E:

$$M_E = \frac{4\pi^2 (3.84 \times 10^8\ \text{m})^3}{(6.6726 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)\left(27.3\,\text{d} \times \frac{24\,\text{h}}{\text{d}} \times \frac{3600\ \text{s}}{\text{h}}\right)^2} = \boxed{6.02 \times 10^{24}\ \text{kg}}$$

Remarks: This analysis neglects the mass of the moon; consequently the mass calculated here is slightly too great.

25 •

Picture the Problem We can use Kepler's 3rd law to relate the period of the earth to the mass of the sun and the mean earth-sun distance.

(*a*) Using Kepler's 3rd law, relate the period of the earth to its mean orbital radius:

$$T_E^2 = \frac{4\pi^2}{GM_S} r_E^3$$

Solve for M_S:

$$M_S = \frac{4\pi^2}{GT_E^2} r_E^3$$

Substitute numerical values and evaluate M_S:

$$M_S = \frac{4\pi^2 \left(1.496\times10^{11}\,\text{m}\right)^3}{\left(6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1\,\text{y}\times\frac{365.25\,\text{d}}{\text{y}}\times\frac{24\,\text{h}}{\text{d}}\times\frac{3600\,\text{s}}{\text{h}}\right)^2}$$

$$= \boxed{1.99\times10^{30}\,\text{kg}}$$

***26 •**

Picture the Problem We can relate the acceleration of an object at any elevation to its acceleration at the surface of the earth through the law of gravity and Newton's 2nd law of motion.

Letting a represent the acceleration due to gravity at this altitude (R_E) and m the mass of the object, apply Newton's 2nd law and the law of gravity to obtain:

$$\sum F_{\text{radial}} = \frac{GmM_E}{(2R_E)^2} = ma$$

and

$$a = \frac{GM_E}{(2R_E)^2} \qquad (1)$$

Apply Newton's 2nd law to the same object when it is at the surface of the earth:

$$\sum F_{\text{radial}} = \frac{GmM_E}{R_E^2} = mg$$

and

$$g = \frac{GM_E}{R_E^2} \qquad (2)$$

Divide equation (1) by equation (2) and solve for a:

$$\frac{a}{g} = \frac{R_E^2}{4R_E^2}$$

and

$$a = \tfrac{1}{4}g = \tfrac{1}{4}\left(9.81\,\text{m/s}^2\right) = \boxed{2.45\,\text{m/s}^2}$$

27 •

Picture the Problem Your weight is the local gravitational force exerted on you. We can use the definition of density to relate the mass of the planet to the mass of earth and the law of gravity to relate your weight on the planet to your weight on earth.

Using the definition of density, relate the mass of the earth to its radius:

$$M_E = \rho V_E = \tfrac{4}{3}\rho\pi R_E^3$$

Relate the mass of the planet to its radius:

$$M_P = \rho V_P = \tfrac{4}{3}\rho\pi R_P^3$$
$$= \tfrac{4}{3}\rho\pi(10R_E)^3$$

Divide the second of these equations by the first to express M_P in terms of M_E:

$$\frac{M_P}{M_E} = \rho\frac{\tfrac{4}{3}\rho\pi(10R_E)^3}{\tfrac{4}{3}\rho\pi R_E^3}$$

and

$$M_P = 10^3 M_E$$

Letting w' represent your weight on the planet, use the law of gravity to relate w' to your weight on earth:

$$w' = \frac{GmM_P}{R_P^2} = \frac{Gm(10^3 M_E)}{(10R_E)^2}$$
$$= 10\frac{GmM_E}{R_E^2} = \boxed{10w}$$

where w is your weight on earth.

28 •

Picture the Problem We can relate the acceleration due to gravity of a test object at the surface of the new planet to the acceleration due to gravity at the surface of the earth through use of the law of gravity and Newton's 2nd law of motion.

Letting a represent the acceleration due to gravity at the surface of this new planet and m the mass of a test object, apply Newton's 2nd law and the law of gravity to obtain:

$$\sum F_{\text{radial}} = \frac{GmM_E}{(\tfrac{1}{2}R_E)^2} = ma$$

and

$$a = \frac{GM_E}{(\tfrac{1}{2}R_E)^2}$$

Simplify this expression to obtain:

$$a = 4\frac{GM_E}{R_E^2} = 4g = \boxed{39.2\ \text{m/s}^2}$$

29 •

Picture the Problem We can use conservation of angular momentum to relate the planet's speeds at aphelion and perihelion.

Using conservation of angular

$$L_a = L_p$$

momentum, relate the angular momenta of the planet at aphelion and perihelion:

or

$$mv_{\mathrm{p}}r_{\mathrm{p}} = mv_{\mathrm{a}}r_{\mathrm{a}}$$

Solve for the planet's speed at aphelion:

$$v_{\mathrm{a}} = \frac{v_{\mathrm{p}}r_{\mathrm{p}}}{r_{\mathrm{a}}}$$

Substitute numerical values and evaluate v_{a}:

$$v_{\mathrm{a}} = \frac{(5\times10^{4}\ \mathrm{m/s})(1.0\times10^{15}\ \mathrm{m})}{2.2\times10^{15}\ \mathrm{m}} = \boxed{2.27\times10^{4}\ \mathrm{m/s}}$$

30 •

Picture the Problem We can use Newton's law of gravity to express the gravitational force acting on an object at the surface of the neutron star in terms of the weight of the object. We can then simplify this expression be dividing out the mass of the object … leaving an expression for the acceleration due to gravity at the surface of the neutron star.

Apply Newton's law of gravity to an object of mass m at the surface of the neutron star to obtain:

$$\frac{GM_{\text{Neutron Star}}m}{R^{2}_{\text{Neutron Star}}} = mg$$

where g represents the acceleration due to gravity at the surface of the neutron star.

Solve for g and substitute for the mass of the neutron star:

$$g = \frac{GM_{\text{Neutron Star}}}{R^{2}_{\text{Neutron Star}}} = \frac{G(1.60M_{\text{sun}})}{R^{2}_{\text{Neutron Star}}}$$

Substitute numerical values and evaluate g:

$$g = \frac{1.60(6.673\times10^{-11}\ \mathrm{N\cdot m^{2}/kg^{2}})(1.99\times10^{30}\ \mathrm{kg})}{(10.5\ \mathrm{km})^{2}} = \boxed{1.93\times10^{12}\ \mathrm{m/s^{2}}}$$

***31** ••

Picture the Problem We can use conservation of angular momentum to relate the asteroid's aphelion and perihelion distances.

Using conservation of angular momentum, relate the angular momenta of the asteroid at aphelion and perihelion:

$$L_{\mathrm{a}} = L_{\mathrm{p}}$$

or

$$mv_{\mathrm{p}}r_{\mathrm{p}} = mv_{\mathrm{a}}r_{\mathrm{a}}$$

Solve for and evaluate the ratio of the asteroid's aphelion and perihelion distances:

$$\frac{r_{\mathrm{a}}}{r_{\mathrm{p}}} = \frac{v_{\mathrm{p}}}{v_{\mathrm{a}}} = \frac{20\ \mathrm{km/s}}{14\ \mathrm{km/s}} = \boxed{1.43}$$

32 ••

Picture the Problem We'll use the law of gravity to find the gravitational force acting on the satellite. The application of Newton's 2nd law will lead us to the speed of the satellite and its period can be found from its definition.

(*a*) Letting *m* represent the mass of the satellite and *h* its elevation, use the law of gravity to express the gravitational force acting on it:

$$F_g = \frac{GmM_E}{(R_E+h)^2} = \frac{mR_E^2 g}{(R_E+h)^2} = \frac{mg}{\left(1+\frac{h}{R_E}\right)^2}$$

Substitute numerical values and evaluate F_g:

$$F_g = \frac{mg}{\left(1+\frac{h}{R_E}\right)^2} = \frac{(300\,\text{kg})(9.81\,\text{N/kg})}{\left(1+\frac{5\times10^7\,\text{m}}{6.37\times10^6\,\text{m}}\right)^2} = \boxed{37.6\,\text{N}}$$

(*b*) Using Newton's 2nd law, relate the gravitational force acting on the satellite to its centripetal acceleration:

$$F_g = m\frac{v^2}{r}$$

Solve for *v*:

$$v = \sqrt{\frac{F_g r}{m}}$$

Substitute numerical values and evaluate *v*:

$$v = \sqrt{\frac{(37.6\,\text{N})(6.37\times10^6\,\text{m}+5\times10^7\,\text{m})}{300\,\text{kg}}} = \boxed{2.66\text{ km/s}}$$

(*c*) Express the period of the satellite:

$$T = \frac{2\pi r}{v}$$

Substitute numerical values and evaluate *T*:

$$T = \frac{2\pi(6.37\times10^6\,\text{m}+5\times10^7\,\text{m})}{2.66\times10^3\,\text{m/s}} = 1.33\times10^5\,\text{s}\times\frac{1\,\text{h}}{3600\,\text{s}} = \boxed{36.9\,\text{h}}$$

***33** ••

Picture the Problem We can determine the maximum range at which an object with a given mass can be detected by substituting the equation for the gravitational field in the expression for the resolution of the meter and solving for the distance. Differentiating $g(r)$ with respect to r, separating variables to obtain dg/g, and approximating Δr with dr will allow us to determine the vertical change in the position of the gravity meter in the earth's gravitational field is detectable.

(*a*) Express the gravitational field of the earth:

$$g_E = \frac{GM_E}{R_E^2}$$

Express the gravitational field due to the mass m (assumed to be a point mass) of your friend and relate it to the resolution of the meter:

$$g(r) = \frac{Gm}{r^2} = 10^{-11} g_E = 10^{-11} \frac{GM_E}{R_E^2}$$

Solve for r:

$$r = R_E \sqrt{\frac{10^{11} m}{M_E}}$$

Substitute numerical values and evaluate r:

$$r = (6.37 \times 10^6\ \text{m}) \sqrt{\frac{10^{11}(80\ \text{kg})}{5.98 \times 10^{24}\ \text{kg}}}$$

$$= \boxed{7.37\ \text{m}}$$

(*b*) Differentiate $g(r)$ and simplify to obtain:

$$\frac{dg}{dr} = \frac{-2Gm}{r^3} = -\frac{2}{r}\left(\frac{Gm}{r^2}\right) = -\frac{2}{r} g$$

Separate variables to obtain:

$$\frac{dg}{g} = -2\frac{dr}{r} = 10^{-11}$$

Approximating dr with Δr, evaluate Δr with $r = R_E$:

$$\Delta r = \left| -\tfrac{1}{2}(10^{-11})(6.37 \times 10^6\ \text{m}) \right|$$

$$= 3.19 \times 10^{-5}\ \text{m}$$

$$= \boxed{0.0319\ \text{mm}}$$

34 ••

Picture the Problem We can use the law of gravity and Newton's 2nd law to relate the force exerted on the planet by the star to its orbital speed and the definition of the period to relate it to the radius of the orbit.

Using the law of gravity and Newton's 2nd law, relate the force exerted on the planet by the star to its centripetal acceleration:

$$F_{\text{net}} = \frac{KMm}{r} = m\frac{v^2}{r}$$

Solve for v^2 to obtain:

$$v^2 = KM$$

Express the period of the planet:

$$T = \frac{2\pi r}{v} = \frac{2\pi r}{\sqrt{KM}} = \frac{2\pi}{\sqrt{KM}} r$$

or

$$\boxed{T \propto r}$$

***35 ••**

Picture the Problem We can use the definitions of the gravitational fields at the surfaces of the earth and the moon to express the accelerations due to gravity at these locations in terms of the average densities of the earth and the moon. Expressing the ratio of these accelerations will lead us to the ratio of the densities.

Express the acceleration due to gravity at the surface of the earth in terms of the earth's average density:

$$g_{\text{E}} = \frac{GM_{\text{E}}}{R_{\text{E}}^2} = \frac{G\rho_{\text{E}}V_{\text{E}}}{R_{\text{E}}^2} = \frac{G\rho_{\text{E}}\,\frac{4}{3}\pi R_{\text{E}}^3}{R_{\text{E}}^2} = \tfrac{4}{3}G\rho_{\text{E}}\pi R_{\text{E}}$$

Express the acceleration due to gravity at the surface of the moon in terms of the moon's average density:

$$g_{\text{M}} = \tfrac{4}{3}G\rho_{\text{M}}\pi R_{\text{M}}$$

Divide the second of these equations by the first to obtain:

$$\frac{g_{\text{M}}}{g_{\text{E}}} = \frac{\rho_{\text{M}} R_{\text{M}}}{\rho_{\text{E}} R_{\text{E}}}$$

Solve for $\frac{\rho_{\text{M}}}{\rho_{\text{E}}}$:

$$\frac{\rho_{\text{M}}}{\rho_{\text{E}}} = \frac{g_{\text{M}} R_{\text{E}}}{g_{\text{E}} R_{\text{M}}}$$

Substitute numerical values and evaluate $\frac{\rho_{\text{M}}}{\rho_{\text{E}}}$:

$$\frac{\rho_{\text{M}}}{\rho_{\text{E}}} = \frac{(1.62\,\text{m/s}^2)(6.37\times10^6\,\text{m})}{(9.81\,\text{m/s}^2)(1.738\times10^6\,\text{m})} = \boxed{0.605}$$

Measurement of G

36 •

Picture the Problem We can use the law of gravity to find the forces of attraction between the two masses and the definition of torque to determine the balancing torque required.

(*a*) Use the law of gravity to express the force of attraction between the two masses:

$$F = \frac{Gm_1m_2}{r^2}$$

Substitute numerical values and evaluate *F*:

$$F = \frac{\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(10\,\text{kg})(0.01\,\text{kg})}{(0.06\,\text{m})^2} = \boxed{1.85\times10^{-9}\ \text{N}}$$

(*b*) Use its definition to find the torque exerted by the suspension to balance these forces:

$$\tau = 2Fr = 2\left(1.85\times10^{-9}\ \text{N}\right)(0.1\,\text{m}) = \boxed{3.70\times10^{-10}\ \text{N}\cdot\text{m}}$$

Gravitational and Inertial Mass

37 •

Picture the Problem Newton's 2nd law of motion relates the masses and accelerations of these objects to their common accelerating force.

(*a*) Apply Newton's 2nd law to the standard object:

$$F = m_1a_1$$

Apply Newton's 2nd law to the object of unknown mass:

$$F = m_2a_2$$

Eliminate *F* between these two equations and solve for m_2:

$$m_2 = \frac{a_1}{a_2}m_1$$

Substitute numerical values and evaluate m_2:

$$m_2 = \frac{2.6587\ \text{m/s}^2}{1.1705\ \text{m/s}^2}(1\,\text{kg}) = \boxed{2.27\,\text{kg}}$$

(*b*) $\boxed{\text{It is the } \textit{inertial} \text{ mass of } m_2.}$

38 •

Picture the Problem Newton's 2nd law of motion relates the weights of these two objects to their masses and the acceleration due to gravity.

(*a*) Apply Newton's 2nd law to the standard object:

$$F_{\text{net}} = w_1 = m_1 g$$

Apply Newton's 2nd law to the object of unknown mass:

$$F_{\text{net}} = w_2 = m_2 g$$

Eliminate *g* between these two equations and solve for m_2:

$$m_2 = \frac{w_2}{w_1} m_1$$

Substitute numerical values and evaluate m_2:

$$m_2 = \frac{56.6\,\text{N}}{9.81\,\text{N}}(1\,\text{kg}) = \boxed{5.77\,\text{kg}}$$

(*b*) Since this result is determined by the effect on m_2 of the earth's gravitational field, it is the *gravitational* mass of m_2.

***39** •

Picture the Problem Noting that $g_1 \sim g_2 \sim g$, let the acceleration of gravity on the first object be g_1, and on the second be g_2. We can use a constant-acceleration equation to express the difference in the distances fallen by each object and then relate the average distance fallen by the two objects to obtain an expression from which we can approximate the distance they would have to fall before we might measure a difference in their fall distances greater than 1 mm.

Express the difference Δd in the distances fallen by the two objects in time *t*:

$$\Delta d = d_1 - d_2$$

Express the distances fallen by each of the objects in time *t*:

$$d_1 = \tfrac{1}{2} g_1 t^2$$

and

$$d_2 = \tfrac{1}{2} g_2 t^2$$

Substitute to obtain:

$$\Delta d = \tfrac{1}{2} g_1 t^2 - \tfrac{1}{2} g_2 t^2 = \tfrac{1}{2}(g_1 - g_2)\, t^2$$

Relate the average distance *d* fallen by the two objects to their time of fall:

$$d = \tfrac{1}{2} g t^2$$

or

$$t^2 = \frac{2d}{g}$$

Substitute to obtain:

$$\Delta d \approx \tfrac{1}{2}\Delta g \frac{2d}{g} = d\frac{\Delta g}{g}$$

Solve for d to obtain:

$$d = \Delta d \frac{g}{\Delta g}$$

Substitute numerical values and evaluate d:

$$d = \left(10^{-3}\text{ m}\right)\left(10^{12}\right) = \boxed{10^{9}\text{ m}}$$

Gravitational Potential Energy

40 •

Picture the Problem Choosing the zero of gravitational potential energy to be at infinite separation yields, as the potential energy of a two-body system in which the objects are separated by a distance r, $U(r) = -GMm/r$, where M and m are the masses of the two bodies. In order for an object to just escape a gravitational field from a particular location, it must have enough kinetic energy so that its total energy is zero.

(*a*) Letting $U(\infty) = 0$, express the gravitational potential energy of the earth-object system:

$$U(r) = -\frac{GM_{\text{E}}m}{r} \qquad (1)$$

Substitute for GM_{E} and simplify to obtain:

$$U(R_{\text{E}}) = -\frac{GM_{\text{E}}m}{R_{\text{E}}} = -\frac{gR_{\text{E}}^2 m}{R_{\text{E}}} = -mgR_{\text{E}}$$

Substitute numerical values and evaluate $U(R_{\text{E}})$:

$$U(R_{\text{E}}) = -(100\text{ kg})(9.81\text{ N/kg})\left(6.37\times10^{6}\text{ m}\right) = \boxed{-6.25\times10^{9}\text{ J}}$$

(*b*) Evaluate equation (1) with $r = 2R_{\text{E}}$:

$$U(2R_{\text{E}}) = -\frac{GM_{\text{E}}m}{2R_{\text{E}}} = -\frac{gR_{\text{E}}^2 m}{2R_{\text{E}}} = -\tfrac{1}{2}mgR_{\text{E}}$$

Substitute numerical values and evaluate $U(2R_{\text{E}})$:

$$U(2R_{\text{E}}) = -\tfrac{1}{2}(100\text{ kg})(9.81\text{ N/kg})\left(6.37\times10^{6}\text{ m}\right) = \boxed{-3.12\times10^{9}\text{ J}}$$

(*c*) Express the condition that an object must satisfy in order to escape from the earth's gravitational

$$K_{\text{esc}}(2R_{\text{E}}) + U(2R_{\text{E}}) = 0$$

or

$$\tfrac{1}{2}mv_{\text{esc}}^2 + U(2R_{\text{E}}) = 0$$

field from a height R_E above its surface:

Solve for v_{esc}:

$$v_{esc} = \sqrt{\frac{-2U(2R_E)}{m}}$$

Substitute numerical values and evaluate v_{esc}:

$$v_{esc} = \sqrt{\frac{-2(-3.12\times10^9\ \text{J})}{100\ \text{kg}}} = \boxed{7.90\ \text{km/s}}$$

41 •

Picture the Problem In order for an object to just escape a gravitational field from a particular location, an amount of work must be done on it that is equal to its potential energy in its initial position.

Express the work needed to remove the point mass from the surface of the sphere to a point a very large distance away:

$$W = \Delta U = U_f - U_i$$

or, because $U_f = 0$,

$$W = \Delta U = -U_i \qquad (1)$$

Express the initial potential energy of the system:

$$U_i = -\frac{GMm_0}{R}$$

Substitute in equation (1) to obtain:

$$W = \boxed{\frac{GMm_0}{R}}$$

42 •

Picture the Problem Let the zero of gravitational potential energy be at infinity and let m represent the mass of the spacecraft. We'll use conservation of energy to relate the initial kinetic and potential energies to the final potential energy of the earth-spacecraft system.

Use conservation of energy to relate the initial kinetic and potential energies of the system to its final energy when the spacecraft is one earth radius above the surface of the planet:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$-K(R_E) + U(2R_E) - U(R_E) = 0 \qquad (1)$$

Express the potential energy of the spacecraft-and-earth system when the spacecraft is at a distance r from the surface of the earth:

$$U(r) = -\frac{GM_{\mathrm{E}}m}{r}$$

Substitute in equation (1) to obtain:

$$-\tfrac{1}{2}mv^2 - \frac{GM_{\mathrm{E}}m}{2R_{\mathrm{E}}} + \frac{GM_{\mathrm{E}}m}{R_{\mathrm{E}}} = 0$$

Solve for v:

$$v = \sqrt{\frac{GM_{\mathrm{E}}}{R_{\mathrm{E}}}} = \sqrt{\frac{gR_{\mathrm{E}}^2}{R_{\mathrm{E}}}} = \sqrt{gR_{\mathrm{E}}}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{(9.81\,\mathrm{m/s^2})(6.37\times10^6\,\mathrm{m})} = \boxed{7.91\,\mathrm{km/s}}$$

***43 ••**

Picture the Problem Let the zero of gravitational potential energy be at infinity and let m represent the mass of the object. We'll use conservation of energy to relate the initial potential energy of the object-earth system to the final potential and kinetic energies.

Use conservation of energy to relate the initial potential energy of the system to its energy as the object is about to strike the earth:

$$K_{\mathrm{f}} - K_{\mathrm{i}} + U_{\mathrm{f}} - U_{\mathrm{i}} = 0$$

or, because $K_{\mathrm{i}} = 0$,

$$K(R_{\mathrm{E}}) + U(R_{\mathrm{E}}) - U(R_{\mathrm{E}} + h) = 0 \qquad (1)$$

where h is the initial height above the earth's surface.

Express the potential energy of the object-earth system when the object is at a distance r from the surface of the earth:

$$U(r) = -\frac{GM_{\mathrm{E}}m}{r}$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}mv^2 - \frac{GM_{\mathrm{E}}m}{R_{\mathrm{E}}} + \frac{GM_{\mathrm{E}}m}{R_{\mathrm{E}} + h} = 0$$

Solve for v:

$$v = \sqrt{2\left(\frac{GM_{\mathrm{E}}}{R_{\mathrm{E}}} - \frac{GM_{\mathrm{E}}}{R_{\mathrm{E}} + h}\right)} = \sqrt{2gR_{\mathrm{E}}\left(\frac{h}{R_{\mathrm{E}} + h}\right)}$$

Substitute numerical values and evaluate v:

$$v=\sqrt{\frac{2\left(9.81\,\text{m/s}^2\right)\left(6.37\times10^6\ \text{m}\right)\left(4\times10^6\ \text{m}\right)}{6.37\times10^6\ \text{m}+4\times10^6\ \text{m}}}=\boxed{6.94\,\text{km/s}}$$

44 ••

Picture the Problem Let the zero of gravitational potential energy be at infinity, m represent the mass of the object, and h the maximum height reached by the object. We'll use conservation of energy to relate the initial potential and kinetic energies of the object-earth system to the final potential energy.

Use conservation of energy to relate the initial potential energy of the system to its energy as the object is about to strike the earth:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$K(R_E)+U(R_E)-U(R_E+h)=0 \quad (1)$$

where h is the initial height above the earth's surface.

Express the potential energy of the object-earth system when the object is at a distance r from the surface of the earth:

$$U(r)=-\frac{GM_Em}{r}$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}mv^2-\frac{GM_Em}{R_E}+\frac{GM_Em}{R_E+h}=0$$

Solve for h:

$$h=\frac{R_E}{\dfrac{2gR_E}{v^2}-1}$$

Substitute numerical values and evaluate h:

$$h=\frac{6.37\times10^6\ \text{m}}{\dfrac{2\left(9.81\,\text{m/s}^2\right)\left(6.37\times10^6\ \text{m}\right)}{\left(4\times10^3\ \text{m}\right)^2}-1}=\boxed{935\,\text{km}}$$

45 ••

Picture the Problem When the point mass is inside the spherical shell, there is no mass between it and the center of the shell. On the other hand, when the point mass is outside the spherical shell we can use the law of gravity to express the force acting on it. In (*b*) we can derive $U(r)$ from $F(r)$.

(*a*) The force exerted by the shell on a point mass m_0 when m_0 is inside the shell is:

$$\vec{F}_{\text{inside}} = \boxed{0}$$

The force exerted by the shell on a point mass m_0 when m_0 is outside the shell is:

$$\vec{F}_{\text{outside}} = m_0\vec{g} = \boxed{-\frac{GMm_0}{r^2}\hat{r}}$$

where $\hat{r}$ is radially outward from the center of the spherical shell.

(*b*) Use its definition to express $U(r)$ for $r > R$:

$$U(r) = -\int_{\infty}^{r} F_r dr = GMm_0 \int_{\infty}^{r} r^{-2} dr$$

$$= \boxed{-\frac{GMm_0}{r}}$$

When $r = R$:

$$U(R) = \boxed{-\frac{GMm_0}{R}}$$

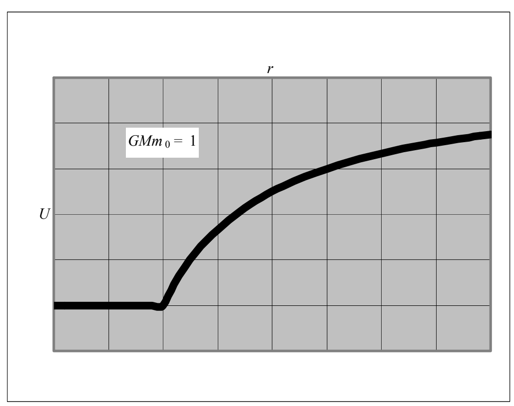
(*c*) For $r < R$, $F = 0$ and:

$$\frac{dU}{dr} = 0 \Rightarrow \boxed{U = \text{constant}}$$

(*d*) Because U is continuous, then for $r < R$:

$$U(r) = U(R) = \boxed{-\frac{GMm_0}{R}}$$

(*e*) A sketch of $U(r)$ with $GMm_0 = 1$ is shown to the right:



46 •

Picture the Problem The escape speed from a planet is related to its mass according to $v_e = \sqrt{2GM/R}$, where M and R represent the mass and radius of the planet, respectively.

Express the escape speed from Saturn:

$$v_{\text{e.S}} = \sqrt{\frac{2GM_{\text{S}}}{R_{\text{S}}}} \qquad (1)$$

Express the escape speed from Earth:

$$v_{\text{e.E}} = \sqrt{\frac{2GM_{\text{E}}}{R_{\text{E}}}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{v_{\text{e.S}}}{v_{\text{e.E}}} = \frac{\sqrt{\dfrac{2GM_{\text{S}}}{R_{\text{S}}}}}{\sqrt{\dfrac{2GM_{\text{E}}}{R_{\text{E}}}}} = \sqrt{\frac{R_{\text{E}}}{R_{\text{S}}} \cdot \frac{M_{\text{S}}}{M_{\text{E}}}}$$

Substitute numerical values and evaluate $\frac{v_{\text{e.S}}}{v_{\text{e.E}}}$:

$$\frac{v_{\text{e.S}}}{v_{\text{e.E}}} = \sqrt{\frac{1}{9.47} \times \frac{95.2}{1}} = 3.17$$

Solve for and evaluate $v_{\text{e,S}}$:

$$v_{\text{e.S}} = 3.17 v_{\text{e.E}} = 3.17(11.2\,\text{km/s}) = \boxed{35.5\,\text{km/s}}$$

47 •

Picture the Problem The escape speed from the moon or the earth is given by $v_{\text{e}} = \sqrt{2GM/R}$, where M and R represent the masses and radii of the moon or the earth.

Express the escape speed from the moon:

$$v_{\text{e.S}} = \sqrt{\frac{2GM_{\text{m}}}{R_{\text{m}}}} = \sqrt{2g_{\text{m}}R_{\text{m}}} \qquad (1)$$

Express the escape speed from earth:

$$v_{\text{e.E}} = \sqrt{\frac{2GM_{\text{E}}}{R_{\text{E}}}} = \sqrt{2g_{\text{E}}R_{\text{E}}} \qquad (2)$$

Divide equation (1) by equation (2) to obtain:

$$\frac{v_{\text{e.m}}}{v_{\text{e.E}}} = \frac{\sqrt{g_{\text{m}}R_{\text{m}}}}{\sqrt{g_{\text{E}}R_{\text{E}}}} = \sqrt{\frac{g_{\text{m}}R_{\text{m}}}{g_{\text{E}}R_{\text{E}}}}$$

Solve for $v_{\text{e,m}}$:

$$v_{\text{e.m}} = \sqrt{\frac{g_{\text{m}}R_{\text{m}}}{g_{\text{E}}R_{\text{E}}}}\, v_{\text{e.E}}$$

Substitute numerical values and evaluate $v_{\text{e,m}}$:

$$v_{\text{e.m}} = \sqrt{(0.166)(0.273)}(11.2\,\text{km/s}) = \boxed{2.38\,\text{km/s}}$$

***48 •**

Picture the Problem We'll consider a rocket of mass m which is initially on the surface of the earth (mass M and radius R) and compare the kinetic energy needed to get the rocket to its escape velocity with its kinetic energy in a low circular orbit around the earth. We can use conservation of energy to find the escape kinetic energy and Newton's law of gravity to derive an expression for the low earth-orbit kinetic energy.

Apply conservation of energy to relate the initial energy of the rocket to its escape kinetic energy:

$$K_f - K_i + U_f - U_i = 0$$

Letting the zero of gravitational potential energy be at infinity we have $U_f = K_f = 0$ and:

$$-K_i - U_i = 0$$

or

$$K_e = -U_i = \frac{GMm}{R}$$

Apply Newton's law of gravity to the rocket in orbit at the surface of the earth to obtain:

$$\frac{GMm}{R^2} = m\frac{v^2}{R}$$

Rewrite this equation to express the low-orbit kinetic energy E_o of the rocket:

$$K_o = \tfrac{1}{2}mv^2 = \frac{GMm}{2R}$$

Express the ratio of K_o to K_e:

$$\frac{K_o}{K_e} = \frac{\frac{GMm}{2R}}{\frac{GMm}{R}} = \frac{1}{2} \Rightarrow K_e = \boxed{2K_o}\text{, as}$$

asserted by Heinlein.

49 ••

Picture the Problem Let the zero of gravitational potential energy be at infinity, m represent the mass of the particle, and the subscript E refer to the earth. When the particle is very far from the earth, the gravitational potential energy of the earth-particle system will be zero. We'll use conservation of energy to relate the initial potential and kinetic energies of the particle-earth system to the final kinetic energy of the particle.

Use conservation of energy to relate the initial energy of the system to its energy when the particle is very far from the earth:

$$K_f - K_i + U_f - U_i = 0$$

or, because $U_f = 0$,

$$K(\infty) - K(R_E) - U(R_E) = 0 \qquad (1)$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}mv_\infty^2 - \tfrac{1}{2}m(2v_e)^2 + \frac{GM_E m}{R_E} = 0$$

or, because $GM_E = gR_E^2$,

$$\tfrac{1}{2}mv_\infty^2 - \tfrac{1}{2}mv^2 + mgR_E = 0$$

Solve for v_∞:

$$v_\infty = \sqrt{2\left(2v_e^2 - gR_E\right)}$$

Substitute numerical values and evaluate v_∞:

$$v_\infty = \sqrt{2\left[2\left(11.2\times10^3\ \text{m/s}\right)^2 - \left(9.81\,\text{m/s}^2\right)\left(6.37\times10^6\ \text{m}\right)\right]} = \boxed{19.4\,\text{km/s}}$$

50 ••

Picture the Problem Let the zero of gravitational potential energy be at infinity, m represent the mass of the particle, and the subscript E refer to the earth. When the particle is very far from the earth, the gravitational potential energy of the earth-particle system will be zero. We'll use conservation of energy to relate the initial potential and kinetic energies of the particle-earth system to the final kinetic energy of the particle.

Use conservation of energy to relate the initial energy of the system to its energy when the particle is very far away:

$$K_f - K_i + U_f - U_i = 0$$

or, because $U_f = 0$,

$$K(\infty) - K(R_E) - U(R_E) = 0 \qquad (1)$$

Substitute in equation (1) to obtain:

$$\tfrac{1}{2}mv_\infty^2 - \tfrac{1}{2}mv_i^2 + \frac{GM_E m}{R_E} = 0$$

or, because $GM_E = gR_E^2$,

$$\tfrac{1}{2}mv_\infty^2 - \tfrac{1}{2}mv_i^2 + mgR_E = 0$$

Solve for v_i:

$$v_i = \sqrt{v_\infty^2 + 2gR_E}$$

Substitute numerical values and evaluate v_i:

$$v_i = \sqrt{\left(11.2\times10^3\ \text{m/s}\right)^2 + 2\left(9.81\,\text{m/s}^2\right)\left(6.37\times10^6\ \text{m}\right)} = \boxed{15.8\,\text{km/s}}$$

51 ••

Picture the Problem We can use the definition of kinetic energy to find the energy necessary to launch a 1-kg object from the earth at escape speed.

(*a*) Using the definition of kinetic energy, find the energy required to launch a 1-kg object from the surface of the earth at escape speed:

$$K = \tfrac{1}{2}mv_e^2 = \tfrac{1}{2}(1\,\text{kg})\left(11.2\times10^3\ \text{m/s}\right)^2 = \boxed{62.7\,\text{MJ}}$$

(*b*) Using the conversion factor 1 kW·h = 3.6 MJ, convert 62.7 MJ to kW·h:

$$K = 62.7\,\text{MJ}\times\frac{1\,\text{kW}\cdot\text{h}}{3.6\,\text{MJ}} = \boxed{17.4\,\text{kW}\cdot\text{h}}$$

(*c*) Express the cost of this project in terms of the mass of the astronaut:

$$\text{Cost} = \text{rate}\times\frac{\text{required energy}}{\text{kg}}\times\text{mass}$$

Substitute numerical values and find the cost:

$$\text{Cost} = \frac{\$0.10}{\text{kW}\cdot\text{h}}\times\frac{17.4\,\text{kW}\cdot\text{h}}{\text{kg}}(80\,\text{kg}) = \boxed{\$139}$$

52 ••

Picture the Problem Let *m* represent the mass of the body that is projected vertically from the surface of the earth. We'll begin by using conservation of energy under the assumption that the gravitational field is constant to determine H'. We'll apply conservation of energy a second time, with the zero of gravitational potential energy at infinity, to express *H*. Finally, we'll solve these two equations simultaneously to express *H* in terms of H'.

Assuming the gravitational field to be constant and letting the zero of potential energy be at the surface of the earth, apply conservation of energy to relate the initial kinetic energy and the final potential energy of the object-earth system:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = U_i = 0$,

$$-K_i + U_f = 0$$

Substitute for K_i and U_f and solve for H':

$$-\tfrac{1}{2}mv^2 + mgH' = 0$$

and

$$H' = \frac{v^2}{2g} \qquad (1)$$

Letting the zero of gravitational potential energy be at infinity, use conservation of energy to relate the initial kinetic energy and the final potential energy of the object-earth system:

$$K_f - K_i + U_f - U_i = 0$$

or, because $K_f = 0$,

$$-K_i + U_f - U_i = 0$$

Substitute to obtain:

$$-\tfrac{1}{2}mv^2 - \frac{GMm}{R_E + H} + \frac{GMm}{R_E} = 0$$

or

$$-\tfrac{1}{2}v^2 - \frac{gR_E^2}{R_E + H} + \frac{gR_E^2}{R_E} = 0$$

Solve for v^2:

$$v^2 = 2gR_E^2\left(\frac{1}{R_E} - \frac{1}{R_E + H}\right) = 2gR_E\left(\frac{H}{R_E + H}\right)$$

Substitute in equation (1) to obtain:

$$H' = R_E\left(\frac{H}{R_E + H}\right)$$

Solve for H:

$$H = \boxed{\frac{H'R_E}{R_E - H'}}$$

Orbits

53 ••

Picture the Problem We can use its definition to express the period of the spacecraft's motion and apply Newton's 2nd law to the spacecraft to determine its orbital velocity. We can then use this orbital velocity to calculate the kinetic energy of the spacecraft. We can relate the spacecraft's angular momentum to its kinetic energy and moment of inertia.

(*a*) Express the period of the spacecraft's orbit about the earth:

$$T = \frac{2\pi R}{v} = \frac{2\pi(3R_E)}{v} = \frac{6\pi R_E}{v}$$

where v is the orbital speed of the spacecraft.

Use Newton's 2nd law to relate the gravitational force acting on the spacecraft to its orbital speed:

$$F_{\text{radial}} = \frac{GM_E m}{(3R_E)^2} = m\frac{v^2}{3R_E}$$

Solve for v to obtain:

$$v = \sqrt{\frac{gR_E}{3}}$$

Substitute for v in our expression for T to obtain:

$$T = 6\sqrt{3}\pi\sqrt{\frac{R_E}{g}}$$

Substitute numerical values and evaluate T:

$$T = 6\sqrt{3}\pi\sqrt{\frac{6.37\times10^{6}\text{ m}}{9.81\text{ m/s}^{2}}}$$
$$= 2.631\times10^{4}\text{ s}\times\frac{1\text{ h}}{3600\text{ s}} = \boxed{7.31\text{ h}}$$

(*b*) Using its definition, express the spacecraft's kinetic energy:

$$K = \tfrac{1}{2}mv^{2} = \tfrac{1}{2}m\left(\tfrac{1}{3}gR_{\text{E}}\right)$$

Substitute numerical values and evaluate K:

$$K = \tfrac{1}{6}(100\text{ kg})(9.81\text{ m/s}^{2})(6.37\times10^{6}\text{ m})$$
$$= \boxed{1.04\text{ GJ}}$$

(*c*) Express the kinetic energy of the spacecraft in terms of its angular momentum:

$$K = \frac{L^{2}}{2I}$$

Solve for L:

$$L = \sqrt{2IK}$$

Express the moment of inertia of the spacecraft with respect to an axis through the center of the earth:

$$I = m(3R_{\text{E}})^{2}$$
$$= 9mR_{\text{E}}^{2}$$

Substitute and solve for L:

$$L = \sqrt{18mR_{\text{E}}^{2}K} = 3R_{\text{E}}\sqrt{2mK}$$

Substitute numerical values and evaluate L:

$$L = 3(6.37\times10^{6}\text{ m})\sqrt{2(100\text{ kg})(1.04\times10^{9}\text{ J})} = \boxed{8.72\times10^{12}\text{ J}\cdot\text{s}}$$

***54** •

Picture the Problem Let the origin of our coordinate system be at the center of the earth and let the positive x direction be toward the moon. We can apply the definition of center of mass to find the center of mass of the earth-moon system and find the "orbital" speed of the earth using x_{cm} as the radius of its motion and the period of the moon as the period of this motion of the earth.

(*a*) Using its definition, express the x coordinate of the center of mass of the earth-moon system:

$$x_{\text{cm}} = \frac{M_{\text{E}}x_{\text{E}} + m_{\text{moon}}x_{\text{moon}}}{M_{\text{E}} + m_{\text{moon}}}$$

Substitute numerical values and evaluate x_{cm}:

$$x_{cm} = \frac{M_E(0) + (7.36\times10^{22}\ \text{kg})(3.82\times10^{8}\ \text{m})}{5.98\times10^{24}\ \text{kg} + 7.36\times10^{22}\ \text{kg}} = \boxed{4.64\times10^{6}\ \text{m}}$$

Note that, because the radius of the earth is 6.37×10^6 m, the center of mass is actually located about 1700 km *below* the surface of the earth.

(*b*) Express the "orbital" speed of the earth in terms of the radius of its circular orbit and its period of rotation:

$$v = \frac{2\pi x_{cm}}{T}$$

Substitute numerical values and evaluate v:

$$v = \frac{2\pi(4.64\times10^6\ \text{m})}{27.3\ \text{d}\times\frac{24\ \text{h}}{\text{d}}\times\frac{3600\ \text{s}}{\text{h}}} = \boxed{12.4\ \text{m/s}}$$

55 ••

Picture the Problem We can express the energy difference between these two orbits in terms of the total energy of a satellite at each elevation. The application of Newton's 2nd law to the force acting on a satellite will allow us to express the total energy of each satellite as function of its mass, the radius of the earth, and its orbital radius.

Express the energy difference:

$$\Delta E = E_{geo} - E_{1000} \qquad (1)$$

Express the total energy of an orbiting satellite:

$$E_{tot} = K + U = \tfrac{1}{2}mv^2 - \frac{GM_E m}{R} \qquad (2)$$

where R is the orbital radius.

Apply Newton's 2nd law to a satellite to relate the gravitational force to the orbital speed:

$$F_{radial} = \frac{GM_E m}{R^2} = m\frac{v^2}{R}$$

or

$$\frac{gR_E^2}{R^2} = \frac{v^2}{R}$$

Simplify and solve for v^2:

$$v^2 = \frac{gR_E^2}{R}$$

Substitute in equation (2) to obtain:

$$E_{tot} = \tfrac{1}{2}m\frac{gR_E^2}{R} - \frac{gR_E^2 m}{R} = -\frac{mgR_E^2}{2R}$$

Substitute in equation (1) and simplify to obtain:

$$\Delta E = -\frac{mgR_E^2}{2R_{geo}} + \frac{mgR_E^2}{2R_{1000}}$$
$$= \frac{mgR_E^2}{2}\left(\frac{1}{R_{1000}} - \frac{1}{R_{geo}}\right)$$

Substitute numerical values and evaluate ΔE:

$$\Delta E = \tfrac{1}{2}(500\,\text{kg})(9.81\,\text{N/kg})\left(6.37\times10^6\,\text{m}\right)^2\left(\frac{1}{7.37\times10^6\,\text{m}} - \frac{1}{4.22\times10^7\,\text{m}}\right) = \boxed{11.1\,\text{GJ}}$$

56 ••

Picture the Problem We can use Kepler's 3rd law to relate the periods of the moon and Earth, in their orbits about the earth and the sun, to their mean distances from the objects about which they are in orbit. We can solve these equations for the masses of the sun and the earth and then divide one by the other to establish a value for the ratio of the mass of the sun to the mass of the earth.

Using Kepler's 3rd law, relate the period of the moon to its mean distance from the earth:

$$T_m^2 = \frac{4\pi^2}{GM_E}r_m^3 \qquad (1)$$

where r_m is the distance between the centers of the earth and the moon.

Using Kepler's 3rd law, relate the period of the earth to its mean distance from the sun:

$$T_E^2 = \frac{4\pi^2}{GM_s}r_E^3 \qquad (2)$$

where r_E is the distance between the centers of the earth and the sun.

Solve equation (1) for M_E:

$$M_E = \frac{4\pi^2}{GT_m^2}r_m^3 \qquad (3)$$

Solve equation (2) for M_s:

$$M_s = \frac{4\pi^2}{GT_E^2}r_E^3 \qquad (4)$$

Divide equation (4) by equation (3) and simplify to obtain:

$$\frac{M_s}{M_E} = \left(\frac{r_E}{r_m}\right)^3\left(\frac{T_m}{T_E}\right)^2$$

Substitute numerical values and evaluate M_s/M_E:

$$\frac{M_s}{M_E} = \left(\frac{1.5\times10^{11}\,\text{m}}{3.82\times10^8\,\text{m}}\right)^3\left(\frac{27.3\,\text{d}}{365.24\,\text{d}}\right)^2$$
$$= \boxed{3.38\times10^5}$$

Express the difference between this value and the measured value of 3.33×10^5:

$$\%\,\text{diff} = \frac{3.38\times10^5 - 3.33\times10^5}{3.33\times10^5} = \boxed{1.50\,\%}$$

The Gravitational Field

57 •

Picture the Problem The gravitational field at any point is defined by $\vec{g} = \vec{F}/m$.

Using its definition, express the gravitational field at a point in space:

$$\vec{g} = \frac{\vec{F}}{m} = \frac{(12\,\text{N})\hat{i}}{3\,\text{kg}} = \boxed{(4\,\text{N/kg})\hat{i}}$$

***58** •

Picture the Problem The gravitational field at any point is defined by $\vec{g} = \vec{F}/m$.

Using its definition, express the gravitational field at a point in space:

$$\vec{g} = \frac{\vec{F}}{m}$$

Solve for $\vec{F}$ and substitute for m and $\vec{g}$ to obtain:

$$\vec{F} = m\vec{g} = (0.004\,\text{kg})(2.5\times10^{-6}\,\text{N/kg})\hat{j} = \boxed{(10^{-8}\,N)\hat{j}}$$

59 ••

Picture the Problem We can use the definition of the gravitational field due to a point mass to find the x and y components of the field at the origin and then add these components to find the resultant field. We can find the magnitude of the field from its components using the Pythagorean theorem.

(*a*) Express the gravitational field due to the point mass at $x = L$:

$$\vec{g}_x = \frac{Gm}{L^2}\hat{i}$$

Express the gravitational field due to the point mass at $y = L$:

$$\vec{g}_y = \frac{Gm}{L^2}\hat{j}$$

Add the two fields to obtain:

$$\vec{g} = \vec{g}_x + \vec{g}_y = \boxed{\frac{Gm}{L^2}\hat{i} + \frac{Gm}{L^2}\hat{j}}$$

(*b*) Find the magnitude of $\vec{g}$:

$$|\vec{g}| = \sqrt{g_x^2 + g_y^2} = \sqrt{\frac{Gm}{L^2} + \frac{Gm}{L^2}} = \boxed{\sqrt{2}\frac{Gm}{L^2}}$$

60 ••

Picture the Problem We can find the net force acting on *m* by superposition of the forces due to each of the objects arrayed on the circular arc. Once we have expressed the net force, we can find the gravitational field at the center of curvature from its definition.

(*a*) Express the net force acting on *m*:

$$\vec{F} = F_x\hat{i} + F_y\hat{j} \qquad (1)$$

Express F_x:

$$F_x = \frac{GMm}{R^2} - \frac{GMm}{R^2} + \frac{GMm}{R^2}\cos 45° - \frac{GMm}{R^2}\cos 45° = 0$$

Express F_y:

$$F_y = \frac{GMm}{R^2} + \frac{GMm}{R^2}\sin 45° + \frac{GMm}{R^2}\sin 45° = \frac{GMm}{R^2}(2\sin 45° + 1)$$

Substitute numerical values and evaluate F_y:

$$F_y = \frac{(6.673\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})}{(0.1\,\mathrm{m})^2} \times (3\,\mathrm{kg})(2\,\mathrm{kg})(2\sin 45° + 1) = 9.67\times10^{-8}\,\mathrm{N}$$

Substitute in equation (1) to obtain:

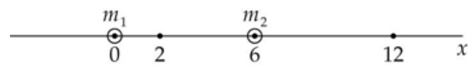
$$\vec{F} = \boxed{0\hat{i} + (9.67\times10^{-8}\,\mathrm{N})\hat{j}}$$

(*b*) Using its definition, express $\vec{g}$ at the center of curvature of the arc:

$$\vec{g} = \frac{\vec{F}}{m} = \frac{0\hat{i} + (9.67\times10^{-8}\,\mathrm{N})\hat{j}}{2\,\mathrm{kg}} = \boxed{(4.83\times10^{-8}\,\mathrm{N/kg})\hat{j}}$$

61 ••

Picture the Problem The configuration of point masses is shown to the right. The gravitational field at any point can be found by superimposing the fields due to each of the point masses.



(*a*) Express the gravitational field at $x = 2$ m as the sum of the fields due to the point masses m_1 and m_2:

$$\vec{g} = \vec{g}_1 + \vec{g}_2 \qquad (1)$$

Express $\vec{g}_1$ and $\vec{g}_2$:

$$\vec{g}_1 = -\frac{Gm_1}{x_1^2}\hat{i} \text{ and } \vec{g}_2 = \frac{Gm_2}{x_2^2}\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{g} = -\frac{Gm_1}{x_1^2}\hat{i} + \frac{Gm_2}{x_2^2}\hat{i} = -\frac{Gm_1}{x_1^2}\hat{i} + \frac{Gm_2}{(2x_1)^2}\hat{i} = -\frac{G}{x_1^2}\left(m_1 - \tfrac{1}{4}m_2\right)\hat{i}$$

Substitute numerical values and evaluate $\vec{g}$:

$$\vec{g} = -\frac{6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}{(2\ \text{m})^2}\times\left[2\ \text{kg} - \tfrac{1}{4}(4\ \text{kg})\right]\hat{i} = \boxed{\left(-1.67\times10^{-11}\ \text{N/kg}\right)\hat{i}}$$

(*b*) Express $\vec{g}_1$ and $\vec{g}_2$:

$$\vec{g}_1 = -\frac{Gm_1}{x_1^2}\hat{i} \text{ and } \vec{g}_2 = -\frac{Gm_2}{x_2^2}\hat{i}$$

Substitute in equation (1) to obtain:

$$\vec{g} = -\frac{Gm_1}{x_1^2}\hat{i} - \frac{Gm_2}{x_2^2}\hat{i} = -\frac{Gm_1}{(2x_2)^2}\hat{i} - \frac{Gm_2}{x_2^2}\hat{i} = -\frac{G}{x_2^2}\left(\tfrac{1}{4}m_1 + m_2\right)\hat{i}$$

Substitute numerical values and evaluate $\vec{g}$:

$$\vec{g} = -\frac{6.6726\times10^{-11}\ \mathrm{N\cdot m^2/kg^2}}{(6\ \mathrm{m})^2}\times\left[\tfrac{1}{4}(2\ \mathrm{kg})+4\ \mathrm{kg}\right]\hat{i} = \boxed{\left(-8.34\times10^{-12}\ \mathrm{N/kg}\right)\hat{i}}$$

(*c*) Express the condition that $\vec{g} = 0$:

$$\frac{Gm_1}{x^2}-\frac{Gm_2}{(6-x)^2}=0$$

or

$$\frac{2}{x^2}-\frac{4}{(6-x)^2}=0$$

Express this quadratic equation in standard form:

$x^2+12x-36=0$, where x is in meters.

Solve the equation to obtain:

$x = 2.48\ \mathrm{m}$ and $x=-14.5\ \mathrm{m}$

From the diagram it is clear that the physically meaningful root is the positive one at:

$$x=\boxed{2.48\ \mathrm{m}}$$

62 ••

Picture the Problem To show that the maximum value of $|g_x|$ for the field of Example 11-7 occurs at the points $x=\pm a/\sqrt{2}$, we can differentiate g_x with respect to x and set the derivative equal to zero.

From Example 11-7:

$$g_x = -\frac{2GMx}{\left(x^2+a^2\right)^{3/2}}$$

Differentiate g_x with respect to x and set the derivative equal to zero to find extreme values:

$$\frac{dg_x}{dx}=-2GM\left[\left(x^2+a^2\right)^{-3/2}-3x^2\left(x^2+a^2\right)^{-5/2}\right]=0 \text{ for extrema.}$$

Solve for x to obtain:

$$x=\boxed{\pm\frac{a}{\sqrt{2}}}$$

Remarks: To establish that this value for *x* corresponds to a relative maximum, we need to either evaluate the second derivative of g_x at $x = \pm a/\sqrt{2}$ or examine the graph of $|g_x|$ at $x = \pm a/\sqrt{2}$ for concavity downward.

63 ••

Picture the Problem We can find the mass of the rod by integrating *dm* over its length. The gravitational field at $x_0 > L$ can be found by integrating $d\vec{g}$ at x_0 over the length of the rod.

(*a*) Express the total mass of the stick:

$$M = \int_0^L \lambda\, dx = C\int_0^L x dx = \boxed{\tfrac{1}{2}CL^2}$$

(*b*) Express the gravitational field due to an element of the stick of mass *dm*:

$$d\vec{g} = -\frac{Gdm}{(x_0 - x)^2}\hat{i} = -\frac{G\lambda\, dx}{(x_0 - x)^2}\hat{i} = -\frac{GCxdx}{(x_0 - x)^2}\hat{i}$$

Integrate this expression over the length of the stick to obtain:

$$\vec{g} = -GC\int_0^L \frac{xdx}{(x_0 - x)^2}\hat{i} = \boxed{\frac{2GM}{L^2}\left[\ln\left(\frac{x_0}{x_0 - L}\right) - \left(\frac{L}{x_0 - L}\right)\right]\hat{i}}$$

64 •••

Picture the Problem Choose a mass element *dm* of the rod of thickness *dx* at a distance *x* from the origin. All such elements produce a gravitational field at a point *P* located a distance $x_0 > \frac{1}{2}L$ from the origin. We can calculate the total field by integrating the magnitude of the field produced by *dm* from $x = -L/2$ to $x = +L/2$.

(*a*) Express the gravitational field at *P* due to the element *dm*:

$$d\vec{g}_x = -\frac{Gdm}{r^2}\hat{i}$$

Relate *dm* to *dx*:

$$dm = \frac{M}{L}dx$$

Express the distance *r* between *dm* and point *P* in terms of *x* and x_0:

$$r = x_0 - x$$

Substitute these results to express $d\vec{g}_x$ in terms of *x* and x_0:

$$d\vec{g}_x = \boxed{\left\{-\frac{GM}{L(x_0 - x)^2}dx\right\}\hat{i}}$$

(*b*) Integrate to find the total field:

$$\vec{g}_x = -\frac{GM}{L}\int_{-L/2}^{L/2}\frac{dx}{(x_0 - x)^2}\hat{i}$$
$$= \left\{-\frac{GM}{L}\left[\frac{1}{x_0 - x}\right]_{-L/2}^{L/2}\right\}\hat{i}$$
$$= \boxed{-\frac{GM}{x_0^2 - \frac{1}{4}L^2}\hat{i}}$$

(*c*) Use the definition of $\vec{g}$ to express $\vec{F}$:

$$\vec{F} = m_0\vec{g} = \boxed{-\frac{GMm_0}{x_0^2 - \frac{1}{4}L^2}\hat{i}}$$

(*d*) Factor x_0^2 from the denominator of our expression for $\vec{g}_x$ to obtain:

$$\vec{g}_x = -\frac{GM}{x_0^2\left(1 - \frac{L^2}{4x_0^2}\right)}\hat{i}$$

For $x_0 >> L$ the second term in parentheses is very small and:

$$\vec{g}_x \approx \boxed{-\frac{GM}{x_0^2}\hat{i}}$$

which is the gravitational field of a point mass M located at the origin.

$\vec{g}$ due to Spherical Objects

65 •

Picture the Problem The gravitational field inside a spherical shell is zero and the field at the surface of and outside the shell is given by $g = GM/r^2$.

(*a*) Because 0.5 m < R:

$$g = \boxed{0}$$

(*b*) Because 1.9 m < R:

$$g = \boxed{0}$$

(*c*) Because 2.5 m > R:

$$g = \frac{GM}{r^2}$$
$$= \frac{\left(6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)(300\,\text{kg})}{(2.5\,\text{m})^2}$$
$$= \boxed{3.20\times10^{-9}\,\text{N/kg}}$$

66 •

Determine the Concept The gravitational attraction is zero. The gravitational field inside the 2 m shell due to that shell is zero; therefore, it exerts no force on the 1 m shell, and, by Newton's 3rd law, that shell exerts no force on the larger shell.

***67** •

Picture the Problem The gravitational field and acceleration of gravity at the surface of a sphere given by $g = GM/R^2$, where R is the radius of the sphere and M is its mass.

Express the acceleration of gravity on the surface of S_1:

$$g_1 = \frac{GM}{R^2}$$

Express the acceleration of gravity on the surface of S_2:

$$g_2 = \frac{GM}{R^2}$$

Divide the second of these equations by the first to obtain:

$$\frac{g_2}{g_1} = \frac{\frac{GM}{R^2}}{\frac{GM}{R^2}} = 1 \text{ or } \boxed{g_1 = g_2}$$

68 ••

Picture the Problem The gravitational field and acceleration of gravity at the surface of a sphere given by $g = GM/R^2$, where R is the radius of the sphere and M is its mass.

Express the acceleration of gravity on the surface of S_1:

$$g_1 = \frac{GM}{R_1^2}$$

Express the acceleration of gravity on the surface of S_2:

$$g_2 = \frac{GM}{R_2^2}$$

Divide the second of these equations by the first to obtain:

$$\frac{g_2}{g_1} = \frac{\frac{GM}{R_2^2}}{\frac{GM}{R_1^2}} = \frac{R_1^2}{R_2^2}$$

Solve for g_2:

$$g_2 = \boxed{\frac{R_1^2}{R_2^2} g_1}$$

Remarks: The accelerations depend only on the masses and radii because the points of interest are outside spherically symmetric distributions of mass.

69 ••

Picture the Problem The magnitude of the gravitational force is $F = mg$ where g inside a spherical shell is zero and outside is given by $g = GM/r^2$.

(*a*) At $r = 3a$, the masses of both spheres contribute to g:

$$F = mg = m\frac{G(M_1 + M_2)}{(3a)^2} = \boxed{\frac{Gm(M_1 + M_2)}{9a^2}}$$

(*b*) At $r = 1.9a$, g due to $M_2 = 0$:

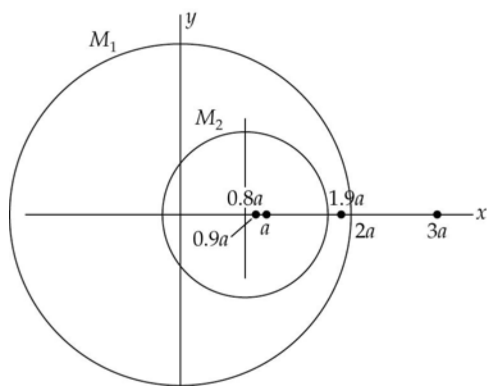
$$F = mg = m\frac{GM_1}{(1.9a)^2} = \boxed{\frac{GmM_1}{3.61a^2}}$$

(*c*) At $r = 0.9a$, $g = 0$:

$$F = \boxed{0}$$

70 ••

Picture the Problem The configuration is shown on the right. The centers of the spheres are indicated by the center-lines. The x coordinates of the mass m for parts (*a*), (*b*), and (*c*) are indicated along the x axis. The magnitude of the gravitational force is $F = mg$ where g inside a spherical shell is zero and outside is given by $g = \dfrac{GM}{r^2}$.



(*a*) Express the force acting on the object whose mass is m:

$$F = m(g_{1x} + g_{2x})$$

Find g_{1x} at $x = 3a$:

$$g_{1x} = \frac{GM_1}{(3a)^2} = \frac{GM_1}{9a^2}$$

Find g_{2x} at $x = 3a$:

$$g_{2x} = \frac{GM_2}{(3a - 0.8a)^2} = \frac{GM_2}{4.84a^2}$$

Substitute to obtain:

$$F = m\left(\frac{GM_1}{9a^2} + \frac{GM_2}{4.84a^2}\right) = \boxed{\frac{Gm}{a^2}\left(\frac{M_1}{9} + \frac{M_2}{4.84}\right)}$$

(*b*) Find g_{2x} at $x = 1.9a$:

$$g_{2x} = \frac{GM_2}{(1.9a - 0.8a)^2} = \frac{GM_2}{1.21a^2}$$

Find g_{1x} at $x = 1.9a$:

$$g_{1x} = 0$$

Substitute to obtain:

$$F = mg = \boxed{\frac{GmM_2}{1.21a^2}}$$

(*c*) At $x = 0.9a$, $g_{1x} = g_{2x} = 0$:

$$F = \boxed{0}$$

$\vec{g}$ Inside Solid Spheres

***71 ••**

Picture the Problem The "weight" as measured by a spring scale will be the normal force which the spring scale presses up against you. There are two forces acting on you as you stand at a distance r from the center of the planet: the normal force (F_N) and the force of gravity (mg). Because you are in equilibrium under the influence of these forces, your weight (the scale reading or normal force) will be equal to the gravitational force acting on you. We can use Newton's law of gravity to express this force.

(*a*) Express the force of gravity acting on you when you are a distance r from the center of the earth:

$$F_g = \frac{GM(r)m}{r^2} \qquad (1)$$

Using the definition of density, express the density of the earth between you and the center of the earth and the density of the earth as a whole:

$$\rho = \frac{M(r)}{V(r)} = \frac{M(r)}{\frac{4}{3}\pi r^3}$$

and

$$\rho = \frac{M_E}{V_E} = \frac{M_E}{\frac{4}{3}\pi R^3}$$

Because we're assuming the earth to of uniform-density and perfectly spherical:

$$\frac{M(r)}{\frac{4}{3}\pi r^3} = \frac{M_E}{\frac{4}{3}\pi R^3}$$

or

$$M(r) = M_E\left(\frac{r}{R}\right)^3$$

Substitute in equation (1) and simplify to obtain:

$$F_g = \frac{GM_E\left(\frac{r}{R}\right)^3 m}{r^2} = \frac{GM_E m}{R^2}\frac{r}{R}$$

Apply Newton's law of gravity to yourself at the surface of the earth to obtain:

$$mg = \frac{GM_E m}{R^2}$$

or

$$g = \frac{GM_E}{R^2}$$

where g is the magnitude of free-fall acceleration at the surface of the earth.

Substitute to obtain:

$$F_g = \boxed{\frac{mg}{R} r}$$

i.e., the force of gravity on you is proportional to your distance from the center of the earth.

(b) Apply Newton's 2nd law to your body to obtain:

$$F_N - mg\frac{r}{R} = -mr\omega^2$$

Solve for your "effective weight" (i.e., what a spring scale will measure) F_N:

$$F_N = \frac{mg}{R} r - mr\omega^2 = \boxed{\left(\frac{mg}{R} - m\omega^2\right) r}$$

Note that this equation tells us that your effective weight increases linearly with distance from the center of the earth. The second term can be interpreted as a "centrifugal force" pushing out, which increases the farther you get from the center of the earth.

(c) We can decide whether the change in mass with distance from the center of the earth or the rotational effect is more important by examining the ratio of the two terms in the expression for your effective weight:

$$\frac{\frac{mg}{R} r}{mr\omega^2} = \frac{\frac{g}{R}}{\omega^2} = \frac{g}{R\left(\frac{2\pi}{T}\right)^2} = \frac{gT^2}{4\pi^2 R}$$

Substitute numerical values and evaluate this ratio:

$$\frac{gT^2}{4\pi^2 R} = \frac{\left(9.81\,\text{m/s}^2\right)\left(24\,\text{h} \times \frac{3600\,\text{s}}{\text{h}}\right)^2}{4\pi^2 (6370\,\text{km})} = 291$$

$$\boxed{\text{The change in the mass between you and the center of the earth as you move away from the center is 291 times more important than the rotational effect.}}$$

72 ••

Picture the Problem We can find the loss in weight at this depth by taking the difference between the weight of the student at the surface of the earth and her weight at a depth d = 15 km. To find the gravitational field at depth d, we'll use its definition and the mass of the earth that is between the bottom of the shaft and the center of the earth. We'll assume (incorrectly) that the density of the earth is constant.

Express the loss in weight:

$$\Delta w = w(R_E) - w(R) \qquad (1)$$

Express the mass M inside $R = R_E - d$:

$$M = \rho V = \tfrac{4}{3}\rho\pi(R_E - d)^3$$

Express the mass of the earth:

$$M_E = \rho V_E = \tfrac{4}{3}\rho\pi R_E^3$$

Divide the first of these equations by the second to obtain:

$$\frac{M}{M_E} = \frac{\tfrac{4}{3}\rho\pi(R_E - d)^3}{\tfrac{4}{3}\rho\pi R_E^3} = \frac{(R_E - d)^3}{R_E^3}$$

Solve for M:

$$M = M_E\frac{(R_E - d)^3}{R_E^3}$$

Express the gravitational field at $R = R_E - d$:

$$g = \frac{GM}{R^2} = \frac{GM_E(R_E - d)^3}{(R_E - d)^2 R_E^2} \qquad (2)$$

Express the gravitational field at $R = R_E$:

$$g_E = \frac{GM_E}{R_E^2} \qquad (3)$$

Divide equation (2) by equation (3) to obtain:

$$\frac{g}{g_E} = \frac{\dfrac{GM_E(R_E - d)^3}{(R_E - d)^2 R_E^2}}{\dfrac{GM_E}{R_E^2}} = \frac{R_E - d}{R_E}$$

Solve for g:

$$g = \frac{R_E - d}{R_E}g_E$$

Express the weight of the student at $R = R_E - d$:

$$w(R) = mg(R) = \frac{R_E - d}{R_E}mg_E$$

$$= \left(1 - \frac{d}{\mathrm{R_E}}\right)mg_E$$

Substitute in equation (1) to obtain:

$$\Delta w = mg_E - \left(1 - \frac{d}{R_E}\right) mg_E = \frac{mg_E d}{R_E}$$

Substitute numerical values and evaluate Δw:

$$\Delta w = \frac{(800\,\text{N})(15\,\text{km})}{6370\,\text{km}} = \boxed{1.88\,\text{N}}$$

73 ••

Picture the Problem We can use the hint to find the gravitational field along the x axis.

Using the hint, express $g(x)$:

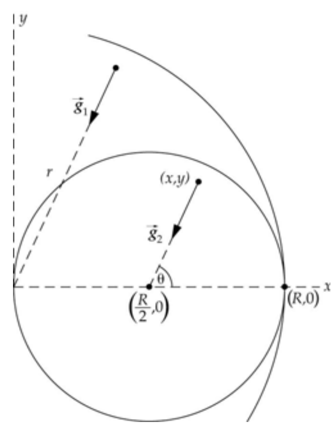
$$g(x) = g_{\text{solid sphere}} + g_{\text{hollow sphere}}$$

Substitute for $g_{\text{solid sphere}}$ and $g_{\text{hollow sphere}}$ and simplify to obtain:

$$\begin{aligned} g(x) &= \frac{GM_{\text{solid sphere}}}{x^2} + \frac{GM_{\text{hollow sphere}}}{\left(x - \frac{1}{2}R\right)^2} \\ &= \frac{G\rho_0\left(\frac{4}{3}\pi R^3\right)}{x^2} + \frac{G\rho_0\left[-\frac{4}{3}\pi\left(\frac{1}{2}R\right)^3\right]}{\left(x - \frac{1}{2}R\right)^2} \\ &= \boxed{G\left(\frac{4\pi\rho_0 R^3}{3}\right)\left[\frac{1}{x^2} - \frac{1}{8\left(x - \frac{1}{2}R\right)^2}\right]} \end{aligned}$$

74 •••

Picture the Problem The diagram shows the portion of the solid sphere in which the hollow sphere is embedded. $\vec{g}_1$ is the field due to the solid sphere of radius R and density ρ_0 and $\vec{g}_2$ is the field due to the sphere of radius ½R and negative density ρ_0 centered at ½R. We can find the resultant field by adding the x and y components of $\vec{g}_1$ and $\vec{g}_2$.



Use its definition to express $|\vec{g}_1|$:

$$|\vec{g}_1| = \frac{GM}{r^2} = \frac{G\rho_0 V}{r^2} = \frac{4\pi\rho_0 r^3 G}{3r^2} = \frac{4\pi\rho_0 r G}{3}$$

Find the x and y components of $\vec{g}_1$:

$$g_{1x} = -g_1\cos\theta = -g_1\left(\frac{x}{r}\right) = -\frac{4\pi\rho_0 Gx}{3}$$

and

$$g_{1y} = -g_1 \sin\theta = -g_1\left(\frac{y}{r}\right) = -\frac{4\pi\rho_0 Gy}{3}$$

where the negative signs indicate that the field points inward.

Use its definition to express $|\vec{g}_2|$:

$$|\vec{g}_2| = \frac{GM_2}{r^2} = \frac{G\rho_0 V_2}{r_2^2} = \frac{4\pi\rho_0 r_2^3 G}{3r_2^2}$$
$$= \frac{4\pi\rho_0 r_2 G}{3}$$

where $r_2 = \sqrt{\left(x - \frac{1}{2}R\right)^2 + y^2}$

Express the x and y components of $\vec{g}_2$:

$$g_{2x} = g_2\left(\frac{x - \frac{1}{2}R}{r_2}\right) = \frac{4\pi\rho_0 G\left(x - \frac{1}{2}R\right)}{3}$$

$$g_{2y} = g_2\left(\frac{y}{r_2}\right) = \frac{4\pi\rho_0 Gy}{3}$$

Add the x components to obtain the x component of the resultant field:

$$g_x = g_{1x} + g_{2x}$$
$$= -\frac{4\pi\rho_0 Gx}{3} + \frac{4\pi\rho_0 G\left(x - \frac{1}{2}R\right)}{3}$$
$$= -\frac{2\pi\rho_0 GR}{3}$$

where the negative sign indicates that the field points inward.

Add the y components to obtain the y component of the resultant field:

$$g_y = g_{1y} + g_{2y}$$
$$= -\frac{4\pi\rho_0 Gy}{3} + \frac{4\pi\rho_0 Gy}{3} = 0$$

Express $\vec{g}$ in vector form and evaluate $|\vec{g}|$:

$$\vec{g} = g_x\hat{i} + G_y\hat{j} = \boxed{\left(-\frac{2\pi\rho_0 GR}{3}\right)\hat{i}}$$

and

$$|\vec{g}| = \boxed{\frac{2\pi\rho_0 GR}{3}}$$

75 •••

Picture the Problem The gravitational field will exert an inward radial force on the objects in the tunnel. We can relate this force to the angular velocity of the planet by using Newton's 2nd law of motion.

Letting r be the distance from the objects to the center of the planet, use Newton's 2nd law to relate the gravitational force acting on the objects to their angular velocity:

$$F_{\text{net}} = F_{\text{g}} = mr\omega^2$$
or
$$mg = mr\omega^2$$

Solve for ω to obtain:

$$\omega = \sqrt{\frac{g}{r}} \qquad (1)$$

Use its definition to express g:

$$g = \frac{GM}{r^2} = \frac{G\rho_0 V}{r^2} = \frac{4\pi\rho_0 r^3 G}{3r^2} = \frac{4\pi\rho_0 rG}{3}$$

Substitute in equation (1) and simplify:

$$\omega = \sqrt{\frac{\frac{4\pi\rho_0 rG}{3}}{r}} = \boxed{\sqrt{\frac{4\pi\rho_0 G}{3}}}$$

76 •••

Picture the Problem Because we're given the mass of the sphere, we can find C by expressing the mass of the sphere in terms of C. We can use its definition to find the gravitational field of the sphere both inside and outside its surface.

(a) Express the mass of a differential element of the sphere:

$$dm = \rho\, dV = \rho\left(4\pi r^2 dr\right)$$

Integrate to express the mass of the sphere in terms of C:

$$M = 4\pi C\int_0^{5\,\text{m}} r dr = \left(50\,\text{m}^2\right)\pi C$$

Solve for C:

$$C = \frac{M}{\left(50\,\text{m}^2\right)\pi}$$

Substitute numerical values and evaluate C:

$$C = \frac{1011\,\text{kg}}{\left(50\,\text{m}^2\right)\pi} = \boxed{6.436\,\text{kg/m}^2}$$

(b) Use its definition to express the gravitational field of the sphere at a distance from its center greater than its radius:

$$g = \frac{GM}{r^2}$$

(1) For $r > 5$ m:

$$g = \frac{\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(1011\,\text{kg})}{r^2} = \boxed{\frac{6.75\times10^{-8}\ \text{N/kg}}{r^2}}$$

Use its definition to express the gravitational field of the sphere at a distance from its center less than its radius:

$$g = G\frac{\int_0^r 4\pi r^2\rho\,dr}{r^2} = G\frac{\int_0^r 4\pi r^2\frac{C}{r}dr}{r^2} = G\frac{4\pi C\int_0^r r\,dr}{r^2} = 2\pi GC$$

(2) For $r < 5$ m:

$$g = 2\pi\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\times\left(6.436\,\text{kg/m}^2\right) = \boxed{2.70\times10^{-9}\ \text{N/kg}}$$

Remarks: Note that *g* is continuous at *r* = 5 m.

***77 •••**

Picture the Problem We can use conservation of energy to relate the work done by the gravitational field to the speed of the small object as it strikes the bottom of the hole. Because we're given the mass of the sphere, we can find C by expressing the mass of the sphere in terms of C. We can then use its definition to find the gravitational field of the sphere inside its surface. The work done by the field equals the negative of the change in the potential energy of the system as the small object falls in the hole.

Use conservation of energy to relate the work done by the gravitational field to the speed of the small object as it strikes the bottom of the hole:

$$K_\text{f} - K_\text{i} + \Delta U = 0$$

or, because $K_\text{i} = 0$ and $W = -\Delta U$,

$$W = \tfrac{1}{2}mv^2$$

where v is the speed with which the object strikes the bottom of the hole and W is the work done by the gravitational field.

Solve for v:

$$v = \sqrt{\frac{2W}{m}} \qquad (1)$$

Express the mass of a differential element of the sphere:

$$dm = \rho\,dV = \rho\left(4\pi r^2 dr\right)$$

Integrate to express the mass of the sphere in terms of C:

$$M = 4\pi C\int_0^{5\,\mathrm{m}} r dr = \left(50\,\mathrm{m}^2\right)\pi C$$

Solve for and evaluate C:

$$C = \frac{M}{\left(50\,\mathrm{m}^2\right)\pi} = \frac{1011\,\mathrm{kg}}{\left(50\,\mathrm{m}^2\right)\pi} = 6.436\,\mathrm{kg/m^2}$$

Use its definition to express the gravitational field of the sphere at a distance from its center less than its radius:

$$g = G\frac{\int_0^r 4\pi r^2 \rho\, dr}{r^2} = G\frac{\int_0^r 4\pi r^2 \frac{C}{r} dr}{r^2} = G\frac{4\pi C\int_0^r r\, dr}{r^2} = 2\pi GC$$

Express the work done on the small object by the gravitational force acting on it:

$$W = -\int_{5\,\mathrm{m}}^{3\,\mathrm{m}} mg dr = \left(2\,\mathrm{m}\right)mg$$

Substitute in equation (1) and simplify to obtain:

$$v = \sqrt{\frac{2\left(2\,\mathrm{m}\right)m\left(2\pi GC\right)}{m}} = \sqrt{\left(8\,\mathrm{m}\right)\pi GC}$$

Substitute numerical values and evaluate v:

$$v = \sqrt{\left(8\,\mathrm{m}\right)\pi\left(6.6726\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(6.436\,\mathrm{kg/m^2}\right)} = \boxed{0.104\,\mathrm{mm/s}}$$

78 •••

Picture the Problem The spherical deposit of heavy metals will increase the gravitational field at the surface of the earth. We can express this increase in terms of the difference in densities of the deposit and the earth and then form the quotient $\Delta g/g$.

Express Δg due to the spherical deposit:

$$\Delta g = \frac{G\Delta M}{r^2} \qquad (1)$$

Express the mass of the spherical deposit:

$$M = \Delta\rho V = \Delta\rho\left(\tfrac{4}{3}\pi R^3\right) = \tfrac{4}{3}\pi\,\Delta\rho\, R^3$$

Substitute in equation (1):

$$\Delta g = \frac{\tfrac{4}{3}G\pi\,\Delta\rho\, R^3}{r^2}$$

Express $\Delta g/g$:

$$\frac{\Delta g}{g}=\frac{\frac{\frac{4}{3}G\pi\,\Delta\rho\,R^3}{r^2}}{g}=\frac{\frac{4}{3}G\pi\,\Delta\rho\,R^3}{gr^2}$$

Substitute numerical values and evaluate $\Delta g/g$:

$$\frac{\Delta g}{g}=\frac{\frac{4}{3}\pi\left(6.6726\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}\right)\left(5000\,\mathrm{kg/m^3}\right)\left(1000\,\mathrm{m}\right)^3}{\left(9.81\,\mathrm{N/kg}\right)\left(2000\,\mathrm{m}\right)^2}=\boxed{3.56\times10^{-5}}$$

***79 •••**

Picture the Problem The force of attraction of the small sphere of mass m to the lead sphere is the sum of the forces due to the solid sphere ($\vec{F}_{\mathrm{S}}$) and the cavities ($\vec{F}_{\mathrm{C}}$) of negative mass.

(a) Express the force of attraction:

$$\vec{F}=\vec{F}_{\mathrm{S}}+\vec{F}_{\mathrm{C}}\qquad(1)$$

Use the law of gravity to express the force due to the solid sphere:

$$\vec{F}_{\mathrm{S}}=-\frac{GMm}{d^2}\hat{i}$$

Express the magnitude of the force acting on the small sphere due to one cavity:

$$F_{\mathrm{C}}=\frac{GM'm}{d^2+\left(\frac{R}{2}\right)^2}$$

where M' is the negative mass of a cavity.

Relate the negative mass of a cavity to the mass of the sphere before hollowing:

$$M'=-\rho V=-\rho\left[\frac{4}{3}\pi\left(\frac{R}{2}\right)^3\right]$$
$$=-\frac{1}{8}\left(\frac{4}{3}\pi\rho R^3\right)=-\frac{1}{8}M$$

Letting θ be the angle between the x axis and the line joining the center of the small sphere to the center of either cavity, use the law of gravity to express the force due to the two cavities:

$$\vec{F}_{\mathrm{C}}=2\frac{GMm}{8\left(d^2+\frac{R^2}{4}\right)}\cos\theta\,\hat{i}$$

because, by symmetry, the y components add to zero.

Express $\cos\theta$:

$$\cos\theta=\frac{d}{\sqrt{d^2+\frac{R^2}{4}}}$$

Substitute to obtain:

$$\vec{F}_C = \frac{GMm}{4\left(d^2+\frac{R^2}{4}\right)}\frac{d}{\sqrt{d^2+\frac{R^2}{4}}}\hat{i} = \frac{GMmd}{4\left(d^2+\frac{R^2}{4}\right)^{3/2}}\hat{i}$$

Substitute in equation (1) and simplify:

$$\vec{F} = -\frac{GMm}{d^2}\hat{i} + \frac{GMmd}{4\left(d^2+\frac{R^2}{4}\right)^{3/2}}\hat{i} = \boxed{-\frac{GMm}{d^2}\left[1-\frac{\frac{d^3}{4}}{\left\{d^2+\frac{R^2}{4}\right\}^{3/2}}\right]\hat{i}}$$

(*b*) Evaluate $\vec{F}$ at $d = R$:

$$\vec{F}(R) = -\frac{GMm}{R^2}\left[1-\frac{\frac{R^3}{4}}{\left\{R^2+\frac{R^2}{4}\right\}^{3/2}}\right]\hat{i} = \boxed{-0.821\frac{GMm}{R^2}\hat{i}}$$

80 ••

Picture the Problem Let R be the size of the cluster, and N the total number of stars in it. We can apply Newton's law of gravity and the 2nd law of motion to relate the net force (which depends on the number of stars $N(r)$ in a sphere whose radius is equal to the distance between the star of interest and the center of the cluster) acting on a star at a distance r from the center of the cluster to its speed. We can use the definition of density, in conjunction with the assumption of uniform distribution of the starts within the cluster, to find $N(r)$ and, ultimately, express the orbital speed v of a star in terms of the total mass of the cluster.

Using Newton's law of gravity and 2nd law, express the force acting on a star at a distance r from the center of the cluster:

$$F(r) = \frac{GN(r)M^2}{r^2} = M\frac{v^2}{r}$$

where $N(r)$ is the number of stars within a distance r of the center of the cluster and M is the mass of an individual star.

Using the uniform distribution assumption and the definition of density, relate the number of stars $N(r)$ within a distance r of the center of the cluster to the total number N of stars in the cluster:

$$\rho = \frac{N(r)M}{\frac{4}{3}\pi r^3} = \frac{NM}{\frac{4}{3}\pi R^3}$$

or

$$N(r) = N\frac{r^3}{R^3}$$

Substitute to obtain:

$$\frac{GNM^2}{r^2}\frac{r^3}{R^3} = M\frac{v^2}{r}$$

or

$$GNM\frac{r^2}{R^3} = v^2$$

Solve for v to obtain:

$$v = r\sqrt{\frac{GNM}{R^3}} \Rightarrow v \propto r$$

$\boxed{\text{i.e., the mean velocity } v \text{ of a star in a circular orbit around the center of the cluster increases linearly with distance } r \text{ from the center.}}$

General Problems

***81 •**

Picture the Problem We can use Kepler's 3rd law to relate Pluto's period to its mean distance from the sun.

Using Kepler's 3rd law, relate the period of Pluto to its mean distance from the sun:

$$T^2 = Cr^3$$

where $C = \dfrac{4\pi^2}{GM_s} = 2.973\times10^{-19}\ \text{s}^2/\text{m}^3$.

Solve for T:

$$T = \sqrt{Cr^3}$$

Substitute numerical values and evaluate T:

$$T = \sqrt{\left(2.973\times10^{-19}\ \text{s}^2/\text{m}^3\right)\left(39.5\ \text{AU}\times\frac{1.50\times10^{11}\ \text{m}}{\text{AU}}\right)^3}$$

$$= 7.864\times10^9\ \text{s}\times\frac{1\ \text{h}}{3600\ \text{s}}\times\frac{1\ \text{d}}{24\ \text{h}}\times\frac{1\ \text{y}}{365.25\ \text{d}}$$

$$= \boxed{249\ \text{y}}$$

82 •

Picture the Problem Consider an object of mass m at the surface of the earth. We can relate the weight of this object to the gravitational field of the earth and to the mass of the earth.

Using Newton's 2nd law, relate the weight of an object at the surface of the earth to the gravitational force acting on it:

$$w = mg = \frac{GM_{\mathrm{E}}m}{R_{\mathrm{E}}^2}$$

Solve for M_{E}:

$$M_{\mathrm{E}} = \frac{gR_{\mathrm{E}}^2}{\mathrm{G}}$$

Substitute numerical values and evaluate M_{E}:

$$M_{\mathrm{E}} = \frac{(9.81\,\mathrm{N/kg})(6.37\times10^{6}\,\mathrm{m})^2}{6.6726\times10^{-11}\,\mathrm{N\cdot m^2/kg^2}} = \boxed{5.97\times10^{24}\,\mathrm{kg}}$$

83 ••

Picture the Problem The work you must do against gravity to move the particle from a distance r_1 to r_2 is the negative of the change in the particle's gravitational potential energy.

(a) Relate the work you must do to the change in the gravitational potential energy of the earth-particle system:

$$W = -\Delta U = -\int_{r_1}^{r_2} F_{\mathrm{g}}\,dr = GM_{\mathrm{E}}m\int_{r_1}^{r_2}\frac{dr}{r^2}$$
$$= -GM_{\mathrm{E}}m\left(\frac{1}{r_2}-\frac{1}{r_1}\right)$$
$$= \boxed{GM_{\mathrm{E}}m\left(\frac{1}{r_1}-\frac{1}{r_2}\right)}$$

(b) Substitute gR_{E}^2 for GM_{E}, R_{E} for r_1, and $R_{\mathrm{E}} + h$ for r_2 to obtain:

$$W = \boxed{mgR_{\mathrm{E}}^2\left(\frac{1}{R_{\mathrm{E}}}-\frac{1}{R_{\mathrm{E}}+h}\right)} \qquad (1)$$

(*c*) Rewrite equation (1) with a common denominator and simplify to obtain:

$$W = mgR_E^2\left(\frac{R_E + h - R_E}{R_E(R_E + h)}\right)$$

$$= mgh\left(\frac{R_E}{R_E + h}\right) = mgh\left(\frac{1}{1+\dfrac{h}{R_E}}\right)$$

$$\approx \boxed{mgh}$$

when $h << R_E$.

84 ••

Picture the Problem The gravitational field outside a uniform sphere is given by $g = -GM/r^2$ and the field inside the sphere by $g = -(GM/R^3)r$.

(*a*) Express *g* outside the sphere:

$$g = -\frac{GM}{r^2}$$

Find the mass of the sphere:

$$M = \rho V = \rho\left(\tfrac{4}{3}\pi R^3\right)$$

Substitute and simplify to obtain:

$$g = -\frac{G\rho\left(\frac{4}{3}\pi R^3\right)}{r^2} = -\tfrac{4}{3}\frac{G\rho R^3}{r^2}$$

Substitute numerical values and evaluate *g*:

$$g = -\tfrac{4}{3}\frac{\left(6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2000\,\text{kg/m}^3\right)\left(100\,\text{m}\right)^3}{r^2} = \boxed{-\frac{0.559\,\text{N}\cdot\text{m}^2/\text{kg}}{r^2}}$$

(*b*) Express the gravitational field inside the uniform sphere:

$$g = -\frac{GM}{R^3}r = -\frac{G\rho\left(\frac{4}{3}\pi R^3\right)}{R^3}r$$

$$= -\tfrac{4}{3}\pi\rho Gr$$

Substitute numerical values and evaluate *g*:

$$g = -\tfrac{4}{3}\pi\left(2000\,\text{kg/m}^3\right)\left(6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)r = \boxed{-\left(5.59\times10^{-7}\,\text{N/kg}\cdot\text{m}\right)r}$$

85 ••

Picture the Problem We can use Kepler's 3rd law to relate the period of the satellite to its mean distance from the center of Jupiter.

Use Kepler's 3rd law to relate the period of the satellite to its mean distance from the center of Jupiter:

$$T^2 = \frac{4\pi^2}{GM_J}(R_J + h)^3$$

Solve for h:

$$h = \sqrt[3]{\frac{T^2 GM_J}{4\pi^2}} - R_J \qquad (1)$$

Express the mass of Jupiter in terms of the mass of the earth:

$$M_J = 320 M_E$$

Express the volume of Jupiter in terms of the mass of the earth:

$$V_J = 1320 V_E$$

Express the volumes of Jupiter and Earth in terms of their radii and solve for R_J:

$$R_J = \sqrt[3]{1320} R_E$$

Substitute in equation (1) to obtain:

$$h = \sqrt[3]{\frac{T^2 G\{320 M_E\}}{4\pi^2}} - \sqrt[3]{1320} R_E$$

Express the period of the satellite in seconds:

$$\begin{aligned} T &= 9\,\text{h} + 50\,\text{min} \\ &= 9\,\text{h} \times \frac{3600\,\text{s}}{\text{h}} + 50\,\text{min} \times \frac{60\,\text{s}}{\text{min}} \\ &= 3.54 \times 10^4\,\text{s} \end{aligned}$$

Substitute numerical values and evaluate h:

$$\begin{aligned} h &= \sqrt[3]{\frac{(3.54\times10^4\,\text{s})^2 (6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2)\{320(5.98\times10^{24}\,\text{kg})\}}{4\pi^2}} \\ &\quad - \sqrt[3]{1320}(6.37\times10^6\,\text{m}) \\ &= \boxed{8.96\times10^7\,\text{m}} \end{aligned}$$

86 ••

Picture the Problem Let *m* represent the mass of the spacecraft. From Kepler's 3rd law we know that its period will be a minimum when it is in orbit just above the surface of the moon. We'll use Newton's 2nd law to relate the angular velocity of the spacecraft to the gravitational force acting on it.

Relate the period of the spacecraft to its angular velocity:

$$T = \frac{2\pi}{\omega} \qquad (1)$$

Using Newton's 2nd law of motion, relate the gravitational force acting on the spacecraft when it is in orbit at the surface of the moon to the angular velocity of the spacecraft:

$$\sum F_{\text{radial}} = \frac{GM_{\text{M}}m}{R_{\text{M}}^2} = mR_{\text{M}}\omega^2$$

Solve for ω:

$$\omega = \sqrt{\frac{GM_{\text{M}}}{R_{\text{M}}^3}} = \sqrt{\frac{G\left(\frac{4}{3}\pi\rho R_{\text{M}}^3\right)}{R_{\text{M}}^3}} = \sqrt{\tfrac{4}{3}G\pi\rho}$$

Substitute in equation (1) and simplify to obtain:

$$T_{\text{min}} = \frac{2\pi}{\sqrt{\frac{4}{3}G\pi\rho}} = \sqrt{\frac{3\pi}{\rho G}}$$

Substitute numerical values and evaluate T_{min}:

$$T_{\text{min}} = \sqrt{\frac{3\pi}{\left(6.6726\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(3340\ \text{kg/m}^3\right)}} = 6503\ \text{s} = \boxed{1\ \text{h}\ 48\ \text{min}}$$

87 ••

Picture the Problem We can use conservation of energy to establish a relationship between the height *h* to which the projectile will rise and its initial speed. The application of Newton's 2nd law will relate the orbital speed, which is equal to the initial speed of the projectile, to the mass and radius of the moon.

Use conservation of energy to relate the initial energies of the projectile to its final energy:

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

or, because $K_{\text{f}} = 0$,

$$-\tfrac{1}{2}mv^2 - \frac{GM_{\text{M}}m}{R_{\text{M}} + h} + \frac{GM_{\text{M}}m}{R_{\text{M}}} = 0$$

Solve for h:

$$h = R\left(\frac{1}{1-\frac{v^2 R_{\text{M}}}{2GM_{\text{M}}}} - 1\right) \qquad (1)$$

Use Newton's 2nd law to relate velocity of the satellite to the gravitational force acting on it:

$$\sum F_{\text{radial}} = \frac{GM_{\text{M}}m}{R_{\text{M}}^2} = m\frac{v^2}{R_{\text{M}}}$$

Solve for v^2:

$$v^2 = \frac{GM_{\text{M}}}{R_{\text{M}}}$$

Substitute for v^2 in equation (1) and simplify to obtain:

$$h = R\left(\frac{1}{1-\frac{1}{2}} - 1\right) = R = \boxed{1.70\,\text{Mm}}$$

***88** ••

Picture the Problem If we assume the astronauts experience a constant acceleration in the barrel of the cannon, we can use a constant-acceleration equation to relate their exit speed (the escape speed from the earth) to the acceleration they would need to undergo in order to reach that speed. We can use conservation of energy to express their escape speed in terms of the mass and radius of the earth and then substitute in the constant-acceleration equation to find their acceleration. To find the balance point between the earth and the moon we can equate the gravitational forces exerted by the earth and the moon at that point.

(*a*) Assuming constant acceleration down the cannon barrel, relate the ship's speed as it exits the barrel to the length of the barrel and the acceleration required to get the ship to escape speed:

$$v_{\text{e}}^2 = 2a\Delta\ell$$

where ℓ is the length of the cannon.

Solve for the acceleration:

$$a = \frac{v_{\text{e}}^2}{2\Delta\ell} \qquad (1)$$

Use conservation of energy to relate the initial energy of astronaut's ship to its energy when it has escaped the earth's gravitational field:

$$\Delta K + \Delta U = 0$$

or

$$K_{\text{f}} - K_{\text{i}} + U_{\text{f}} - U_{\text{i}} = 0$$

When the ship has escaped the earth's gravitational field:

$$K_{\text{f}} = U_{\text{f}} = 0$$

and

$$-K_{\text{i}} - U_{\text{i}} = 0$$

or

$$-\tfrac{1}{2}mv_e^2 - \left(-\frac{GM_E m}{R}\right) = 0$$

where m is the mass of the spaceship.

Solve for v_e^2 to obtain:

$$v_e^2 = \frac{2GM_E}{R}$$

Substitute in equation (1) to obtain:

$$a = \frac{GM_E}{\Delta\ell R}$$

Substitute numerical values and evaluate a:

$$a = \left(6.673\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right) \times \frac{\left(5.98\times10^{24}\ \text{kg}\right)}{(274\ \text{m})(6370\ \text{km})} = 2.29\times10^5\ \text{m/s}^2 \approx 23{,}300g$$

$\boxed{\text{Survival is extremely unlikely!}}$

(*b*) Let the distance from the center of the earth to the center of the moon be R, and the distance from the center of the spaceship to the earth be x. If M is the mass of the earth and m the mass of the moon, the forces will balance out when:

$$\frac{GM}{x^2} = \frac{Gm}{(R-x)^2}$$

or

$$\frac{x}{\sqrt{M}} = \frac{R-x}{\sqrt{m}}$$

where we've ignored the negative solution, as it doesn't indicate a point between the two bodies.

Solve for x to obtain:

$$x = \frac{R}{1+\sqrt{\dfrac{m}{M}}}$$

Substitute numerical values and evaluate x:

$$x = \frac{3.84\times10^8\ \text{m}}{1+\sqrt{\dfrac{7.36\times10^{22}\ \text{kg}}{5.98\times10^{24}\ \text{kg}}}} = \boxed{3.46\times10^8\ \text{m}}$$

(*c*) $\boxed{\text{No it is not. During the entire trip, the astronauts would be in free-fall, and so would not seem to weigh anything.}}$

89 ••

Picture the Problem Let the origin of our coordinate system be at the center of mass of the binary star system and let the distances of the stars from their center of mass be r_1 and r_2. The period of rotation is related to the angular velocity of the star system and we can use Newton's 2nd law of motion to relate this velocity to the separation of the stars.

Relate the square of the period of the motion of the stars to their angular velocity:

$$T^2 = \frac{4\pi^2}{\omega^2} \quad (1)$$

Using Newton's 2nd law of motion, relate the gravitational force acting on the star whose mass is m_2 to the angular velocity of the system:

$$\sum F_{\text{radial}} = \frac{Gm_1m_2}{(r_1+r_2)^2} = m_2r_2\omega^2$$

Solve for ω^2:

$$\omega^2 = \frac{Gm_1}{r_2(r_1+r_2)^2} \quad (2)$$

From the definition of the center of mass we have:

$$m_1r_1 = m_2r_2 \quad (3)$$

where $r = r_1 + r_2 \quad (4)$

Eliminate r_1 from equations (3) and (4) and solve for r_2:

$$r_2 = \frac{rm_1}{m_1+m_2}$$

Eliminate r_2 from equations (3) and (4) and solve for r_1:

$$r_1 = \frac{rm_2}{m_1+m_2}$$

Substitute for r_1 and r_2 in equation (2) to obtain:

$$\omega^2 = \frac{G(m_1+m_2)}{r^3}$$

Finally, substitute in equation (1) and simplify:

$$T^2 = \frac{4\pi^2}{\frac{G(m_1+m_2)}{r^3}} = \boxed{\frac{4\pi^2r^3}{G(m_1+m_2)}}$$

90 ••

Picture the Problem Because the two-particle system has zero initial energy and zero initial linear momentum; we can use energy and momentum conservation to obtain simultaneous equations in the variables r, v_1 and v_2. We'll assume that initial separation distance of the particles and their final separation r *is large compared to the size of the particles* so that we can treat them as though they are point particles.

Use conservation of energy to relate the speeds of the particles when their separation distance is r:

$$E_i = E_f$$

or

$$0 = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 - \frac{Gm_1m_2}{r} \quad (1)$$

Use conservation of linear momentum to obtain a second relationship between the speeds of the particles and their masses:

$$p_i = p_f$$

or

$$0 = m_1v_1 + m_2v_2 \quad (2)$$

Solve equation (2) for v_1 and substitute in equation (1) to obtain:

$$v_2^2\left(m_2 + \frac{m_2^2}{m_1}\right) = \frac{2Gm_1m_2}{r} \quad (3)$$

Solve equation (3) for v_2:

$$v_2 = \boxed{\sqrt{\frac{2Gm_1^2}{r(m_1+m_2)}}}$$

Solve equation (2) for v_1 and substitute for v_2 to obtain:

$$v_1 = \boxed{\sqrt{\frac{2Gm_2^2}{r(m_1+m_2)}}}$$

***91** ••

Picture the Problem We can find the orbital speeds of the planets from their distance from the center of mass of the system and the period of their motion. Application of Kepler's 3rd law will allow us to express the period of their motion T in terms of the effective mass of the system … which we can find from its definition.

Express the orbital speeds of the planets in terms of their period T:

$$v = \frac{2\pi R}{T}$$

where R is the distance to the center of mass of the four-planet system.

Apply Kepler's 3rd law to express the period of the planets:

$$T = \sqrt{\frac{4\pi^2}{GM_{eff}}R^3}$$

where M_{eff} is the effective mass of the four planets.

Substitute to obtain:

$$v = \frac{2\pi R}{\sqrt{\frac{4\pi^2}{GM_{\text{eff}}}R^3}} = \sqrt{\frac{GM_{\text{eff}}}{R}}$$

The distance of each planet from the effective mass is:

$$R = \frac{a}{\sqrt{2}}$$

Find M_{eff} from its definition:

$$\frac{1}{M_{\text{eff}}} = \frac{1}{M} + \frac{1}{M} + \frac{1}{M} + \frac{1}{M}$$

and

$$M_{\text{eff}} = \tfrac{1}{4}M$$

Substitute for R and M_{eff} to obtain:

$$v = \boxed{\sqrt{\frac{\sqrt{2}GM}{4a}}}$$

92 ••

Picture the Problem Let r represent the separation of the particle from the center of the earth and assume a uniform density for the earth. The work required to lift the particle from the center of the earth to its surface is the integral of the gravitational force function. This function can be found from the law of gravity and by relating the mass of the earth between the particle and the center of the earth to the earth's mass. We can use the work-kinetic energy theorem to find the speed with which the particle, when released from the surface of the earth, will strike the center of the earth. Finally, the energy required for the particle to escape the earth from the center of the earth is the sum of the energy required to get it to the surface of the earth and the kinetic energy it must have to escape from the surface of the earth.

(*a*) Express the work required to lift the particle from the center of the earth to the earth's surface:

$$W = \int_0^{R_E} F\,dr \qquad (1)$$

where F is the gravitational force acting on the particle.

Using the law of gravity, express the force acting on the particle as a function of its distance from the center of the earth:

$$F = \frac{GmM}{r^2} \qquad (2)$$

where M is the mass of a sphere whose radius is r.

Express the ratio of M to M_E:

$$\frac{M}{M_E} = \frac{\rho\left(\frac{4}{3}\pi r^3\right)}{\rho\left(\frac{4}{3}\pi R_E^3\right)} \Rightarrow M = M_E\frac{r^3}{R_E^3}$$

Substitute for M in equation (2) to obtain:

$$F = \frac{GmM_E}{R_E^3}r = \frac{mgR_E^2}{R_E^3}r = \frac{mg}{R_E}r$$

Substitute for F in equation (1) and evaluate the integral:

$$W = \frac{mg}{R_E}\int_0^{R_E} r\,dr = \boxed{\frac{gmR_E}{2}}$$

(*b*) Use the work-kinetic energy theorem to relate the kinetic energy of the particle as it reaches the center of the earth to the work done on it in moving it to the surface of the earth:

$$W = \Delta K = \tfrac{1}{2}mv^2$$

Substitute for W and solve for v:

$$v = \boxed{\sqrt{gR_E}}$$

(*c*) Express the total energy required for the particle to escape when projected from the center of the earth:

$$E_{esc} = W + \tfrac{1}{2}mv_e^2 = \tfrac{1}{2}mv_{esc}^2$$

where v_e is the escape speed from the surface of the earth.

Substitute for W and solve for v_{esc}:

$$v_{esc} = \sqrt{3gR_E}$$

Substitute numerical values and evaluate v_{esc}:

$$v_{esc} = \sqrt{3(9.81\,\text{N/kg})(6.37\times10^6\,\text{m})} = \boxed{13.7\,\text{km/s}}$$

93 ••

Picture the Problem We need to find the gravitational field in three regions: $r < R_1$, $R_1 < r < R_2$, and $r > R_2$.

For $r < R_1$:

$$g = \boxed{0}$$

For $r > R_2$, $g(r)$ is the field of a mass M centered at the origin:

$$g(r) = \boxed{\frac{GM}{r^2}}$$

For $R_1 < r < R_2$, $g(r)$ is determined by the mass within the shell of radius r:

$$g(r) = \frac{Gm}{r^2} \quad (1)$$

where $m = \tfrac{4}{3}\pi\rho\left(r^3 - R_1^3\right) \quad (2)$

Express the density of the spherical shell:

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi\left(R_2^3 - R_1^3\right)}$$

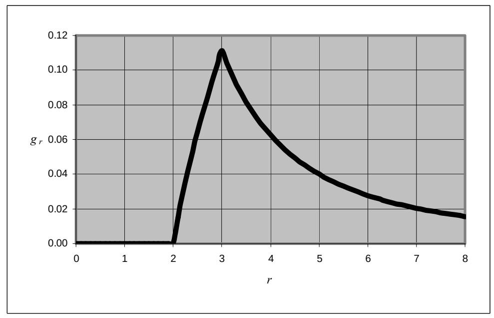
Substitute for ρ in equation (2) and simplify to obtain:

$$m = \frac{M\left(r^3 - R_1^3\right)}{R_2^3 - R_1^3}$$

Substitute for m in equation (1) to obtain:

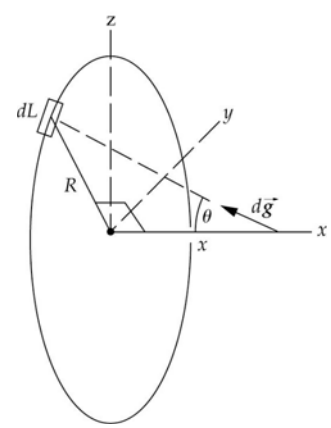
$$g(r) = \boxed{\frac{GM\left(r^3 - R_1^3\right)}{r^2\left(R_2^3 - R_1^3\right)}}$$

A graph of g_r with $R_1 = 2$, $R_2 = 3$, and $GM = 1$ is shown to the right.



94 ••

Picture the Problem A ring of radius R is shown to the right. Choose a coordinate system in which the origin is at the center of the ring and x axis is as shown. An element of length dL and mass dm is responsible for the field dg at a distance x from the center of the ring. We can express the x component of dg and then integrate over the circumference of the ring to find the total field as a function of x.



(a) Express the differential gravitational field at a distance x from the center of the ring in terms of the mass of elemental length dL:

$$dg = \frac{Gdm}{R^2 + x^2}$$

Relate the mass of the element to its length:

$$dm = \lambda\, dL$$

where λ is the linear density of the ring.

Substitute to obtain:

$$dg = \frac{G\lambda\, dL}{R^2 + x^2}$$

By symmetry, the y and z components of g vanish. Express the x component of dg:

$$dg_x = dg\cos\theta = \frac{G\lambda\,dL}{R^2+x^2}\cos\theta$$

Referring to the figure, express $\cos\theta$:

$$\cos\theta = \frac{x}{\sqrt{R^2+x^2}}$$

Substitute to obtain:

$$dg_x = \frac{G\lambda\,dL}{R^2+x^2}\times\frac{x}{\sqrt{R^2+x^2}} = \frac{G\lambda\,x dL}{\left(R^2+x^2\right)^{3/2}}$$

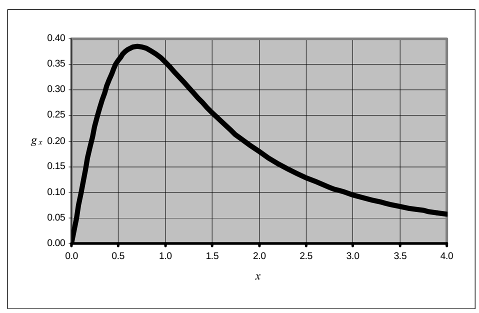
Because $\lambda = \dfrac{M}{2\pi R}$:

$$dg_x = \frac{GM\,xdL}{2\pi R\left(R^2+x^2\right)^{3/2}}$$

Integrate to find $g(x)$:

$$g(x) = \frac{GM\,x}{2\pi R\left(R^2+x^2\right)^{3/2}}\int_0^{2\pi R} dL = \boxed{\frac{GM}{\left(R^2+x^2\right)^{3/2}}\,x}$$

A plot of g_x is shown to the right. The curve is normalized for $R = 1$ and $GM = 1$.



(*b*) Differentiate $g(x)$ with respect to x and set the derivative equal to zero to identify extreme values:

$$\frac{dg}{dx} = GM\left[\frac{\left(x^2+R^2\right)^{3/2} - x\left(\frac{3}{2}\right)\left(x^2+R^2\right)^{1/2}(2x)}{\left(R^2+x^2\right)^3}\right] = 0 \text{ for extrema}$$

Simplify to obtain:

$$\left(x^2+R^2\right)^{3/2} - 3x^2\left(x^2+R^2\right)^{1/2} = 0$$

Solve for x to obtain:

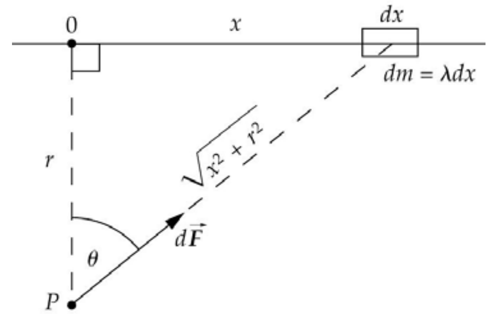
$$x = \boxed{\pm\frac{R}{\sqrt{2}}}$$

Because the curve is concave downward,

we can conclude that this result corresponds to a maximum. Note that this result agrees with our graphical maximum.

95 •••

Picture the Problem The diagram shows a segment of the wire of length *dx* and mass $dm = \lambda dx$ at a distance *x* from the origin of our coordinate system. We can find the magnitude of the gravitational field at a distance *r* from the wire from the resultant gravitational force acting on a particle of mass m' located at point *P* and then integrating over the length of the wire.



Express the gravitational force acting on a particle of mass m' at a distance *r* from the wire due to the segment of the wire of length *dx*:

$$dF = m'dg$$

or

$$dg = \frac{dF}{m'}$$

Using Newton's law of gravity, express *dF*:

$$dF = \frac{Gm'\lambda dx}{R^2}$$

or, because $R^2 = x^2 + r^2$,

$$dF = \frac{Gm'\lambda dx}{x^2 + r^2}$$

Substitute and simplify to express the gravitational field due to the segment of the wire of length *dx*:

$$dg = \frac{G\lambda dx}{x^2 + r^2}$$

By symmetry, the segment on the opposite side of the origin at the same distance from the origin will cancel out all but the radial component of the field, so the gravitational field will be given by:

$$dg = \frac{G\lambda dx}{x^2 + r^2}\cos\theta = \frac{G\lambda dx}{x^2 + r^2}\frac{r}{\sqrt{x^2 + r^2}} = \frac{G\lambda r}{\left(x^2 + r^2\right)^{3/2}}dx$$

Integrate *dg* from $x' = -\infty$ to $x' = +\infty$ to obtain:

$$g = \int_{-\infty}^{\infty}\frac{G\lambda r}{\left(x'^2 + r^2\right)^{3/2}}dx' = 2G\lambda\int_{0}^{\infty}\frac{r}{\left(x'^2 + r^2\right)^{3/2}}dx' = \frac{2G\lambda}{r}\left[\frac{x}{\sqrt{x'^2 + r^2}}\right]_0^{\infty} = \boxed{\frac{2G\lambda}{r}}$$

96 •••

Picture the Problem We can use the relationship between the angular velocity of an orbiting object and its tangential velocity to express the speeds v_{in} and v_{out} of the innermost and outermost portions of the ring. In part (*b*) we can use Newton's law of gravity, in conjunction with the 2nd law of motion, to relate the tangential speed of a chunk of the ring to the gravitational force acting on it. As in part (*a*), once we know v_{in} and v_{out}, we can express the difference between them to obtain the desired results.

(*a*) Express the speed of a point in the ring at a distance R' from the center of the planet under the assumption that the ring is solid and rotates with an angular velocity ω:

$$v(R') = \omega R$$

Express the speeds v_{in} and v_{out} of the innermost and outermost portions of the ring:

$$v_{in} = \left(R - \tfrac{1}{2}r\right)\omega$$
and
$$v_{out} = \left(R + \tfrac{1}{2}r\right)\omega$$

Express the difference between v_{out} and v_{in}:

$$v_{out} - v_{in} = \left(R + \tfrac{1}{2}r\right)\omega - \left(R - \tfrac{1}{2}r\right)\omega$$
$$= \omega r = \frac{v}{R}r = \boxed{v\frac{r}{R}}$$

(*b*) Assume that a chunk of the ring is moving in a circular orbit around the center of the planet under the force of gravity. Then, we can find its velocity by equating the force of gravity to the centripetal force needed to keep it in orbit:

$$\frac{GMm}{R'^2} = \frac{mv^2}{R'}$$
or
$$v = \sqrt{\frac{GM}{R'}}$$
where M is the mass of the planet and R' the distance from the center.

Substitute for R' to express v_{out}:

$$v_{out} = \sqrt{\frac{GM}{R + \frac{1}{2}r}} = \sqrt{\frac{GM}{R\left(1 + \frac{1}{2}\frac{r}{R}\right)}}$$
$$= \sqrt{\frac{GM}{R}}\left(1 + \frac{1}{2}\frac{r}{R}\right)^{-1/2}$$

Expand binomially to obtain:

$$v_{out} = \sqrt{\frac{GM}{R}}\left(1 - \frac{1}{2}\frac{1}{2}\frac{r}{R} + \text{higher order terms}\right)$$
$$\approx \sqrt{\frac{GM}{R}}\left(1 - \frac{1}{4}\frac{r}{R}\right)$$

Proceed similarly to obtain, for v_{in}:

$$v_{in} \approx \sqrt{\frac{GM}{R}}\left(1 + \frac{1}{4}\frac{r}{R}\right)$$

Express the difference between v_{out} and v_{in}:

$$v_{out} - v_{in} \approx \sqrt{\frac{GM}{R}}\left(1-\frac{1}{4}\frac{r}{R}\right)-\sqrt{\frac{GM}{R}}\left(1+\frac{1}{4}\frac{r}{R}\right)=\sqrt{\frac{GM}{R}}\left(-\frac{1}{2}\frac{r}{R}\right)$$

$$\text{and, because } v=\sqrt{\frac{GM}{R}},\ v_{out}-v_{in}\approx \boxed{-\frac{1}{2}\frac{r}{R}v}$$

97 •••

Picture the Problem Let $U = 0$ at $x = \infty$. The potential energy of an element of the stick dm and the point mass m_0 is given by the definition of gravitational potential energy: $dU = -Gm_0dm/r$ where r is the separation of dm and m_0.

(*a*) Express the potential energy of the masses m_0 and dm:

$$dU = -\frac{Gm_0dm}{x_0 - x}$$

The mass dm is proportional to the size of the element dx:

$$dm = \lambda\, dx$$

where $\lambda = \dfrac{M}{L}$.

Substitute these results to express dU in terms of x:

$$dU = -\frac{Gm_0\lambda\, dx}{x_0 - x} = \boxed{-\frac{GMm_0\, dx}{L(x_0 - x)}}$$

(*b*) Integrate to find the total potential energy for the system:

$$U = -\frac{GMm_0}{L}\int_{-L/2}^{L/2}\frac{dx}{x_0 - x} = \frac{GMm_0}{L}\left[\ln\left(x_0 - \frac{L}{2}\right) - \ln\left(x_0 + \frac{L}{2}\right)\right]$$

$$= \boxed{-\frac{GMm_0}{L}\ln\left(\frac{x_0 + L/2}{x_0 - L/2}\right)}$$

(*c*) Because x_0 is a general point along the x axis:

$$F(x_0) = -\frac{dU}{dx_0} = \frac{Gmm_0}{L}\left[\frac{1}{x_0 + \frac{L}{2}} - \frac{1}{x_0 - \frac{L}{2}}\right]$$

Simplify this expression to obtain:

$$F(x_0) = \boxed{-\frac{Gmm_0}{x^2 - L^2/4}}$$

in agreement with the result of Example 11-8.

***98 •••**

Picture the Problem Choose a mass element *dm* of the rod of thickness *dx* at a distance *x* from the origin. All such elements of the rod experience a gravitational force *dF* due to presence of the sphere centered at the origin. We can find the total gravitational force of attraction experienced by the rod by integrating *dF* from $x = a$ to $x = a + L$.

Express the gravitational force *dF* acting on the element of the rod of mass *dm*:

$$dF = \frac{GMdm}{x^2}$$

Express *dm* in terms of the mass *m* and length *L* of the rod:

$$dm = \frac{m}{L}dx$$

Substitute to obtain:

$$dF = \frac{GMm}{L}\frac{dx}{x^2}$$

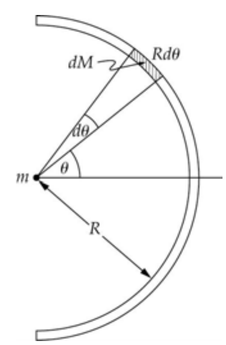
Integrate *dF* from $x = a$ to $x = a + L$ to find the total gravitational force acting on the rod:

$$F = \frac{GMm}{L}\int_a^{a+L} x^{-2}dx = -\frac{GMm}{L}\left[\frac{1}{x}\right]_a^{a+L}$$

$$= \boxed{\frac{GMm}{a(a+L)}}$$

99 •••

Picture the Problem The semicircular rod is shown in the figure. We'll use an element of length $Rd\theta = \frac{L}{\pi}d\theta$ whose mass *dM* is $\frac{M}{\pi}d\theta$. By symmetry, $F_y = 0$. We'll first find dF_x and then integrate over θ from $-\pi/2$ to $\pi/2$.



Express dF_x:

$$dF_x = \frac{GmdM}{R^2}$$

$$= \frac{GMm}{\pi\left(\frac{L}{\pi}\right)^2}d\theta\cos\theta$$

Integrate dF_x over θ from $-\pi/2$ to $\pi/2$:

$$F_x = \frac{\pi\, GMm}{L^2}\int_{-\pi/2}^{\pi/2}\cos\theta\, d\theta = \frac{2\pi\, GMm}{L^2}$$

Substitute numerical values and evaluate F_x:

$$F_x = \frac{2\pi\left(6.6726\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)(20\,\text{kg})(0.1\,\text{kg})}{(5\,\text{m})^2} = \boxed{33.5\,\text{pN}}$$

***100** •••

Picture the Problem We can begin by expressing the forces exerted by the sun and the moon on a body of water of mass *m* and taking the ratio of these forces. In (*b*) we'll simply follow the given directions and in (*c*) we can approximate differential quantities with finite quantities to establish the given ratio.

(*a*) Express the force exerted by the sun on a body of water of mass *m*:

$$F_\text{S} = \frac{GM_\text{S}m}{r_\text{S}^2}$$

Express the force exerted by the moon on a body of water of mass *m*:

$$F_\text{m} = \frac{GM_\text{m}m}{r_\text{m}^2}$$

Divide the first of these equations by the second and simplify to obtain:

$$\frac{F_\text{S}}{F_\text{m}} = \boxed{\frac{M_\text{S}r_\text{m}^2}{M_\text{m}r_\text{S}^2}}$$

Substitute numerical values and evaluate this ratio:

$$\frac{F_\text{S}}{F_\text{m}} = \frac{\left(1.99\times10^{30}\,\text{kg}\right)\left(3.84\times10^{8}\,\text{m}\right)^2}{\left(7.36\times10^{22}\,\text{kg}\right)\left(1.50\times10^{11}\,\text{m}\right)^2} = \boxed{177}$$

(*b*) Find $\dfrac{dF}{dr}$:

$$\frac{dF}{dr} = -\frac{2Gm_1m_2}{r^3} = -2\frac{F}{r}$$

Solve for the ratio $\dfrac{dF}{F}$:

$$\frac{dF}{F} = \boxed{-2\frac{dr}{r}}$$

(*c*) Express the change in force ΔF for a small change in distance Δr:

$$\Delta F = -2\frac{F}{r}\Delta r$$

Express ΔF_S:

$$\Delta F_S = -2\frac{\frac{GmM_S}{r_S^2}}{r_S}\Delta r_S = -2\frac{GmM_S}{r_S^3}\Delta r_S$$

Express ΔF_m:

$$\Delta F_m = -2\frac{GmM_m}{r_m^3}\Delta r_m$$

Divide the first of these equations by the second and simplify:

$$\frac{\Delta F_S}{\Delta F_m} = \frac{\frac{M_S}{r_S^3}\Delta r_S}{\frac{M_m}{r_m^3}\Delta r_m} = \frac{M_S r_m^3}{M_m r_S^3}\frac{\Delta r_S}{\Delta r_m} = \boxed{\frac{M_S r_m^3}{M_m r_S^3}}$$

because $\frac{\Delta r_S}{\Delta r_m} = 1$.

Substitute numerical values and evaluate this ratio:

$$\frac{\Delta F_S}{\Delta F_m} = \frac{(1.99\times10^{30}\text{ kg})(3.84\times10^{8}\text{ m})^3}{(7.36\times10^{22}\text{ kg})(1.50\times10^{11}\text{ m})^3} = \boxed{0.454}$$

101 ••

Picture the Problem Let M_{NS} be the mass of the Neutron Star and m the mass of each robot. We can use Newton's law of gravity to express the difference in the tidal-like forces acting on the coupled robots. Expanding the expression for the force on the robot further from the Neutron Star binomially will lead us to an expression for the distance at which the breaking tension in the connecting cord will be exceeded.

(*a*) $\boxed{\text{The gravitational force is greater on the lower robot, so if it were not for the cable its acceleration would be greater than that of the upper robot, and they would separate. In opposing this separation the cable is stressed.}}$

(*b*) Letting the separation of the two robots be Δr, and the distance from the center of the star to the lower robot be r, use Newton's law of gravity to express the difference in the forces acting on the robots:

$$F_{\text{tide}} = \frac{GM_{\text{NS}}m}{r^2} - \frac{GM_{\text{NS}}m}{(r+\Delta r)^2}$$

$$= GM_{\text{NS}}m\left[\frac{1}{r^2} - \frac{1}{r^2\left(1+\frac{\Delta r}{r}\right)^2}\right]$$

$$= \frac{GM_{\text{NS}}m}{r^2}\left[1-\left(1+\frac{\Delta r}{r}\right)^{-2}\right]$$

Expand the expression in the square brackets binomially to obtain:

$$1-\left(1+\frac{\Delta r}{r}\right)^{-2} \approx 1-\left(1-2\frac{\Delta r}{r}\right)$$

$$= 2\frac{\Delta r}{r}$$

Substitute to obtain:

$$F_{\text{tide}} \approx \frac{2GM_{\text{NS}}m}{r^3}\Delta r$$

Letting F_{B} be the breaking tension of the cord, substitute for F_{tide} and solve for the value of r corresponding to the breaking strain being exceeded:

$$r = \sqrt[3]{\frac{2GM_{\text{NS}}m}{F_B}\Delta r}$$

Substitute numerical values and evaluate r:

$$r = \sqrt[3]{\frac{2\left(6.673\times10^{-11}\,\text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\,\text{kg}\right)(1\,\text{kg})}{25\,\text{kN}}(1\,\text{m})} = \boxed{220\ \text{km}}$$